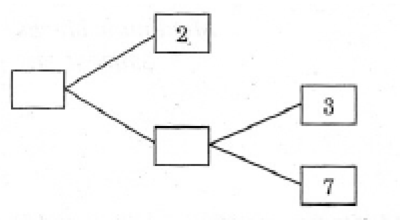
Previous Year Question Paper 2008

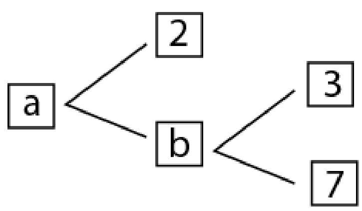
***General Instuctions** :*
(i) All questions are compulsory.
(ii) The question paper consists of 30 questions divided into four sections — A, B, C and D. Section A comprises of ten questions of .1 mark each, Section B comprises of live questions of 2 marks each, Section C comprises of ten questions of 3 marks each and Section D comprises of five questions of 6 marks each
(iii) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
(iv) There is no overall choice. However, an internal choice has been provided in one question of 2 marks each, three questions of 3 marks each and two questions of 6 marks each You have to attempt only one of the alternatives in all such questions.
(v) In question on construction, the drawings should be neat and exactly as per the given measurement.
(vi) Use of calculators is not permitted.

SECTION – A

1. Complete the missing entries in the following factor tree :



Solution:



Lets assume the missing entries be a, b
$b = 3 \times 7 = 21$
$a = 2 \times b = 2 \times 21 = 42$

2. If (x + a) is a factor of $2x^2 + 2ax + 5x + 10$, find a.
Solution:
(x + a) is factor of the polynomial $p(x) = 2x^2 + 2ax + 5x + 10$
$\therefore p(-a) = 0$ {By factor theorem}

$$2(-a)^2 + 2a(-a) + 5(-a) + 10 = 0$$
$$2a^2 - 2a^2 - 5a + 10 = 0$$
$$a = 2$$

3. Show that x = - 3 is a solution of $x^2 + 6x + 9 = 0$.

Solution:

$x^2 + 6x + 9 = 0$

$$x^2 + 2.3x + (3)^2 = 0$$
$$(x+3)^2 = 0$$

$\Rightarrow x = -3$ is the solution of $x^2 + 6x + 9 = 0$

4. The first term of an A.P. is p and its common difference is q. Find its 10^{th} term.

Solution:

First term of an A.P. = p

Common difference = q

$$T_{10} = P + (10-1)q$$
$$T_{10} = P + 9q$$

5. If tan A = $\frac{5}{12}$, find the value of (sin A + cos A) sec A.

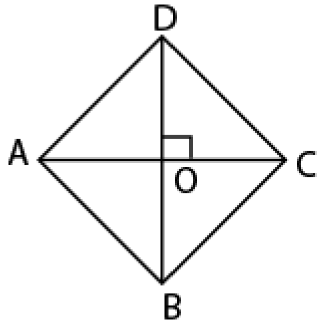
Solution:

$$\tan A = \frac{5}{12}$$
$$\begin{aligned}(\sin A + \cos A)\sec A &= \frac{\sin A}{\cos A} + \frac{\cos A}{\cos A} \\ &= \tan A + 1 \\ &= \frac{5}{12} + 1 \\ &= \frac{17}{12}\end{aligned}$$

6. The lengths of the diagonals of a rhombus are 30 cm and 40 cm. Find the side of the rhombus.

Solution:

Length of the diagonals of a rhombus are 30cm and 40cm



i.e., BD = 30 cm

AC = 40 cm

$\therefore OD = OB = 15cm$

$OA = OC = 20cm$

ΔAOD

$OA^2 + OD^2 = AD^2$

$(20)^2 + (15)^2 = AD^2$

$AD = 25cm$

Side of rhombus = 25 cm

7. In Figure 1, PQ II BC and AP : PB = 1 : 2. Find $\frac{ar(\Delta APQ)}{ar(\Delta ABC)}$.

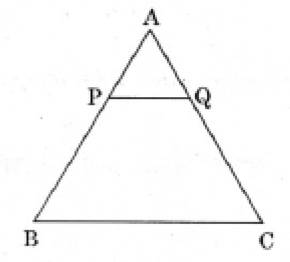
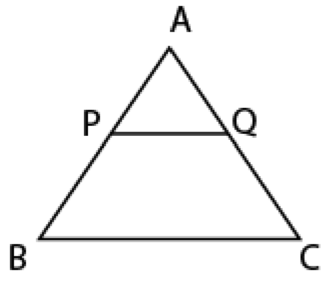


Figure 1

Solution:



PQ || BC

$$\frac{AP}{PB}=\frac{1}{2}$$
$$\frac{PB}{AP}=\frac{2}{1}$$
$$\frac{PB}{AP}+1=\frac{2}{1}+1$$
$$\frac{PB+AP}{AP}=\frac{3}{1}$$
$$\boxed{\frac{AP}{AB}=\frac{1}{3}}$$
$$\therefore \frac{ar(\Delta APQ)}{ar(\Delta ABC)}=\left(\frac{AP}{AB}\right)^2=\frac{1}{9}$$

8. The surface area of a sphere is 616 cm^2. Find its radius.

Solution:

Surface area of sphere = 616 cm^2

$$4\pi r^2=616$$
$$4\times\frac{22}{7}\times r^2=616$$
$$\boxed{r=7\,cm}$$

9. A die is thrown once. Find the probability of getting a number less than 3.

Solution:

Total possible outcomes = 6

Outcomes which are less than 3 = 1, 2

Probability $\frac{2}{6}$

= $\frac{1}{3}$

10. Find the class marks of classes 10 —25 and 35 —55.

Solution:

Class	Class marks
10-25	$\frac{10+25}{2}=17.5$
35-55	$\frac{35+55}{2}=45$

SECTION – B

Questions number 11 to 15 carry 2 marks each.

11. Find all the zeros of the polynomial $x^4 + x^3 - 34x^2 - 4x + 120$, if two of its zeros are 2 and -2.

Solution:

$p(x) = x^4 + x^3 - 34x^2 - 4x + 120$

Lets assume other two zeroes are α, β

Sum of all zeroes $= \alpha + \beta + 2 - 2$

$= \alpha + \beta$

$\alpha + \beta = -1$

$\Rightarrow \boxed{\alpha = -1 - \beta}$ ....(1)

Product of zeroes = 120

$\alpha.\beta.2.(-2) = 120$

$\boxed{\alpha\beta = -30}$ ....(2)

Substituting (1) in (2)

$\beta(-1 - \beta) = -30$

$\beta + \beta^2 = 30$

$\beta^2 + \beta - 30 = 0$

$\therefore \beta = -6, 5$

$\alpha = 5, -6$

Zeroes of the polynomial are $-6, -2, 2, 5$

12. A pair of dice is thrown once. Find the probability of getting the same number on each dice.

Solution:

Total outcomes $= 6 \times 6 = 36$

Outcomes with same number on each = {(1, 1)(2, 2)(3, 3)(4, 4)(5, 5)(6, 6)}

No. of favorable outcomes = 6

Probability $= \frac{6}{36} = \frac{1}{6}$

13. If sec 4A = cosec (A - 20°), where 4A is an acute angle, find the value of A.

OR

In a ΔABC, right-angled at C, if tan A = $\frac{1}{\sqrt{3}}$, find the value of sin A cos B + cos A sin B.

Solution:

$\sec 4A = \text{cosec}(A - 20)$

$\sec 4A = \sec(90 - (A - 20))\,[\sec(90 - x) = \text{cosec}\, x]$

$\sec A = \sec(110 - A)$

$4A = 110 - A$

$5A = 110°$

$A = 22°$

OR

In $\Delta ABC, \angle c = 90^\circ \tan$

$$\tan A = \frac{1}{\sqrt{3}}$$

$$\begin{aligned}\sin A \cos B + \cos A \sin B &= \sin(A+B)\\ &= \sin(180 - c)\\ &= \sin c\\ &= \sin 90^\circ\\ &= 1\end{aligned}$$

14. Find the value of k if the points (k, 3), (6, -2) and (-3, 4) are collinear.

Solution:

Given points (K, 3), (6, -2), (-3, 4) are collinear

$\therefore$ Area of the triangle formed by these points = 0

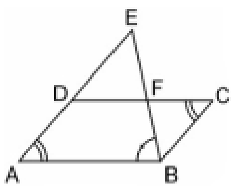
$$\frac{1}{2}\left|k(-2-4)+6(4-3)-3(3+2)\right| = 0$$

$$-6k + 6 - 15 = 0$$

$$k = \frac{-3}{2}$$

15. E is a point on the side AD produced of a ||^{gm} ABCD and BE intersects CD at F. Show that Δ ABE ~ Δ CFB.

Solution:



In ΔABE and ΔCFB,

$\angle A = \angle C$ (Opposite angles of a parallelogram)

$\angle AEB = \angle CBF$ (Alternate interior angles as AE||BC)

$\therefore \Delta ABE \sim \Delta CFB$ (By AA similarly criterion)

SECTION C

Questions number 16 to 25 carry 3 marks each.

16. Use Euclid's Division Lemma to show that the square of any positive integer is either of the form 3m or (3m + 1) for some integer m.

Solution:

Let’s ‘a’ be any positive integer and b = 3.

We know a = bq + r, 0 $\leq$ r <b.

Now, $a = 3q + r, 0 \leq r < 3$.

The possible of remainder = 0, 1 or 2

Case I – a =3q

$a^2 = 9q^2$

$= 3 \times (3q^2)$

$= 3m$ (where $m = 3q^2$)

Case II – a = 3q+1

$a^2 = (3q+1)^2$

$= 9q^2 + 6q + 1$

$= 3(3q^2 + 2q) + 1$

= 3m + 1 (where m = $3q^2 + 2q$)

Case III – a = 3q + 2

$a^2 = (3q+2)^2$

$= 9q^2 + 12q + 4$

$= 3(3q^2 + 4q + 1) + 1$

= 3m + 1 (where $m = 3q^2 + 4q + 1$)

From all the above cases it is clear that square of any positive integer (as in this a^2) is either of the form 3m or 3m + 1.

17. Represent the following pair of equations graphically and write the coordinates of points where the lines intersect y-axis :

x + 3y = 6

2x - 3y = 12

Solution:

$x + 3y = 6$

$2x - 3y = 12$

Graph of x + 3y = 6:

When x= 0, we have y= 2 and when y = 0, we have x= 6.

Therefore, two points on the line are (0, 2) and (6, 0).

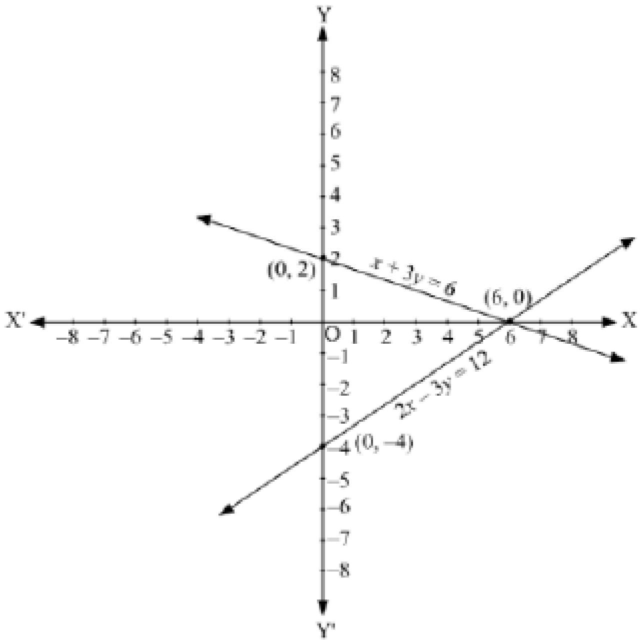
The line x + 3y= 6 is represented in the given graph.

Graph of 2x - 3y = 12:

When x= 0, we have y = - 4 and when y = 0, we have x = 6.

Hence, the two points on the line are (0, - 4) and (6, 0).

The line 2x - 3y =12 is shown in the graph.



The line x + 3y = 6 intersects y-axis at (0,2) and the line 2x - 3y = 12 intersects y-axis at (0, - 4).

18. For what value of n are the n^{th} terms of two A.P.'s 63, 65, 67, ... and 3, 10, 17, ... equal ?

OR

If m times the m^{th} term of an A.P. is equal to n times its n^{th} term, find the $(m + n)^{th}$ term of the AP.

Solution:

n^{th} term of 63, 65, 67, …..

$=63+(n-1)(2)$

$=63+2n-2$

$=61+2n$ ....(1)

n^{th} term of 3, 10, 17, ……….

$=3+(n-1)7$

$=3+7n-7$

$=7n-4$ ....(2)

Given that n^{th} terms of two AP's are equal.

(1) = (2)

$61+2n=7n-4$

$65=5n$

$\boxed{n=13}$

OR

Lets assume first term = a

Common difference = d

$T_m=a+(m-1)d$

$T_n=a+(n-1)d$

Given $m.T_m=n.T_n$

$m(a+(m-1)d)=n(a+(n-1)d)$
$ma+m(m-1)d=na+n(n-1)d$
$(m-n)a+d(m^2-m-n^2+n)=0$
$a(m-n)+d(m-n)(m+n-1)=0$
$(m-n)[a+(m+n-1)d]=0$
$m\neq n \quad \therefore a+(m+n-1)d=0$
$\boxed{T_{m+n}=0}$

19. In an A.P., the first term is 8, n[th] term is 33 and sum to first n terms is 123. Find n and d, the common difference.

Solution:

. First term (a) = 5
$T_n=33$
Sum of first n terms = 123
$\therefore \frac{n}{2}\left[a+T_n\right]=123$
$\frac{n}{2}\left[8+33\right]=123$
$\boxed{n=6}$
$T_n=a+(n-1)d$
$33=8+(5)d$
$\boxed{d=5}$

20. Prove that : (1 + cot A + tan A) (sin A— cos A) = sin A tan A— cot A cos A.

OR

Without using trigonometric tables, evaluate the following :

$$2\left(\frac{\cos 58°}{\sin 32°}\right)-\sqrt{3}\left(\frac{\cos 38° \operatorname{cosec} 52°}{\tan 15° \tan 60° \tan 75°}\right)$$

Solution:

$(1+\cos A+\tan A)(\sin A-\cos A)$
$=\left(1+\frac{1}{\tan A}+\tan A\right)\left(\frac{\sin A}{\cos A}-1\right)\cos A$
$=\frac{\left(1+\tan^2 A+\tan A\right)\left(\tan A-1\right)\cos A}{\tan A}$
$=\frac{\left(\tan^3 A-1\right)\cos A}{\tan A}$
$=\tan^2 A\cos A-\cot A\cos A$
$=\tan A.\frac{\sin A}{\cos A}.\cos A-\cot A\cos A$
$=\sin A\tan A-\cot A\cos A$

OR

$$2\left(\frac{\cos 58^\circ}{\sin 32^\circ}\right)-\sqrt{3}\left(\frac{\cos 38^\circ \operatorname{cosec} 72^\circ}{\tan 15^\circ \tan 60^\circ \tan 75^\circ}\right)$$

$\tan 75^\circ = \cot(90^\circ - 15^\circ) = \cot 15^\circ$

$\tan 15^\circ \tan 75^\circ = 1,\ \tan 60^\circ = \sqrt{3}$

$\sin 32^\circ = \cos 58^\circ,\ \cos 38^\circ = \sin 72^\circ$

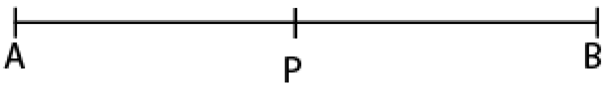
$$\Rightarrow 2\left(\frac{\sin 32^\circ}{\sin 32^\circ}\right)-\sqrt{3}\left(\frac{\cos 38^\circ \sec 38^\circ}{\sqrt{3}}\right)$$

$\Rightarrow 2-1$

$\Rightarrow 1$

21. If P divides the join of A(-2, -2) and B(2, -4) such that $\frac{AP}{AB}=\frac{3}{7}$, find the coordinates of P.

Solution:



$\frac{AP}{AB}=\frac{3}{7}$

As, AB = 7, AP = 3

$\Rightarrow AB = AP + PB$

$\Rightarrow 7 = 3 + PB$

$\Rightarrow PB = 7 - 3 = 4$

Let the point P(x, y) divide the line segment joining the points $A(-2,-2)$ and $B(2,-4)$ in the ratio AP : PB = 3:4

$$\Rightarrow x = \frac{2(3)+(-2)(4)}{3+4} \text{ and } y = \frac{(-4)(3)+(4)(-2)}{3+4}$$

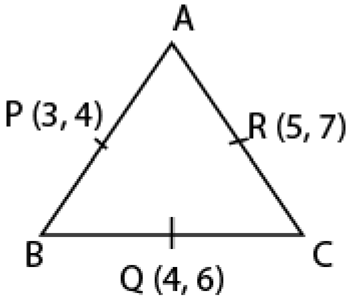
$$\Rightarrow x = \frac{6-8}{7} \text{ and } y = \frac{-12-8}{7}$$

$$\Rightarrow x = \frac{-2}{7} \text{ and } y = \frac{-20}{7}$$

$\Rightarrow$ The coordinate of $P(x,y)=\left(\frac{-2}{7},\frac{-20}{7}\right)$

22. The mid-points of the sides of a triangle are (3, 4), (4, 6) and (5, 7). Find the coordinates of the vertices of the triangle.

Solution:



Consider a ΔABC with A(x_1, y_1),B(x_2, y_2) and C(x_3, y_3). If P(3, 4), Q(4,6) and R(5,7) are the midpoints of AB, BC and CA. Then,

$3=\frac{x_1+x_2}{2}\Rightarrow x_1+x_2=6$ ...(i)

$4=\frac{y_1+y_2}{2}\Rightarrow y_1+y_2=8$ ...(ii)

$4=\frac{x_2+x_3}{2}\Rightarrow x_2+x_3=8$ ...(iii)

$5=\frac{y_2+y_3}{2}\Rightarrow y_2+y_3=12$ ...(iv)

$6=\frac{x_3+x_1}{2}\Rightarrow x_3+x_1=10$ ...(v)

$7=\frac{y_3+y_1}{2}\Rightarrow y_2+y_1=14$ ...(vi)

Adding (i), (ii) and (v) we get

$2(x_1+x_2+x_3)=6+8+10=24$

$\Rightarrow x_1+x_2+x_3=12$ ...(iv)

From (i) and (vii), we get x_3 - 12 - 6 = 6

From (iii) and (vii) we get v_1, = 12 - 8 = 4

From (v) and (vii), we get x_2 = 12 - 10 = 2

Now, adding (ii), (iv) and (vi), we get

20(y_1 + y_2 + y_3)= 8 + 12 + 14 = 34

$\Rightarrow y_1+y_2+y_3=17$ (viii)

From (ii) and (viii), we get y_3 = 17 - 8 = 9

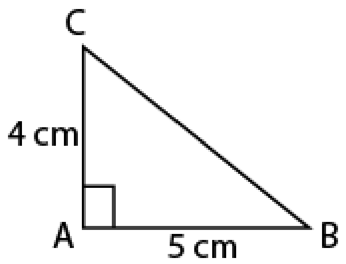
From (iv) and (viii), we get y_1 = 17 - 12 = 5

From (vi) and (viii), we get y_2 = 17 – 14 = 3

Hence, the vertices of ΔABC are A (4, 5), B(2, 3), C(6, 9)

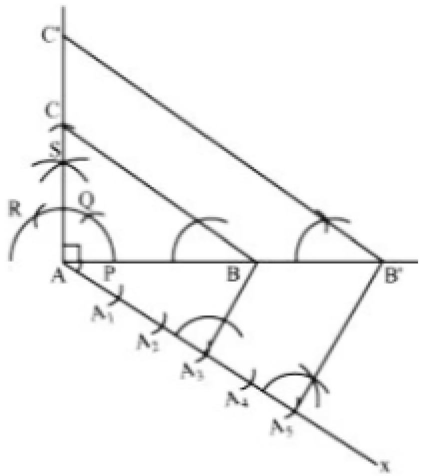
23. Draw a right triangle in which the sides containing the right angle are 5 cm and 4 cm. Construct a similar triangle whose sides are $\frac{5}{3}$ times the sides of the above triangle.

Solution:



Steps:

1) Draw a line segment AB = 5 cm, Draw a ray SA making 90 with it.
2) Draw an arc with radius 4 cm to cut ray SA at C. Join BC. ΔABC is required Δ
3) Draw a ray AX making an acute angle with AB, opposite to vertex C.
4) Locate 5 points (as 5 is greater in 5 and 3), A_1, A_2, A_3, A_4, A_5, on line segment AX such that AA = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5
5) Join A_3B. Draw a line through A_5 parallel to A_3B intersecting line segment AB at B'.
6) Through B', draw a line parallel to BC intersecting extended line segment AC at C'. $\Delta AB'C'$ is the required triangle.



Justification:

The construction can be justified by provided that

$AB' = \frac{5}{3}AB, B'C' = \frac{5}{3}BC, AC' = \frac{5}{3}AC$

In ΔABC and $\Delta AB'C'$,

$\angle ABC = \angle AB'C'$ (Corresponding angles)

$\angle BAC = \angle B'AC'$ (Common)

$\therefore \Delta ABC \sim \Delta AB'C'$ (AA similarly criterion)

$\Rightarrow \frac{AB}{AB'} = \frac{BC}{B'C'} = \frac{AC}{AC'}$ ....(1)

In ΔAA_3B and $\Delta AA_5B'$,

$\angle A_3AB = \angle A_5AB'$ (Common)

$\angle AA_3B = \angle AA_5B'$ (Corresponding angles)

$\therefore \Delta AA_3B \sim \Delta AA_5B'$ (AA similarly criterion)

$\Rightarrow \frac{AB}{AB'} = \frac{AA_3}{AA_5}$

$\Rightarrow \frac{AB}{AB'} = \frac{3}{5}$ ....(2)

On comparing equations (1) and (2), we obtain

$\frac{AB}{AB'} = \frac{BC}{B'C'} = \frac{AC}{AC'} = \frac{3}{5}$

$\Rightarrow AB' = \frac{5}{3}AB, B'C' = \frac{5}{3}BC, AC' = \frac{5}{3}AC$

This justifies the construction.

24. Prove that a parallelogram circumscribing a circle is a rhombus.

OR

In Figure 2, AD $\perp$ BC. Prove that $AB^2 + CD^2 = BD^2 + AC^2$.

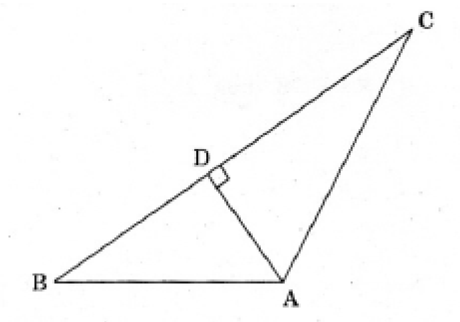
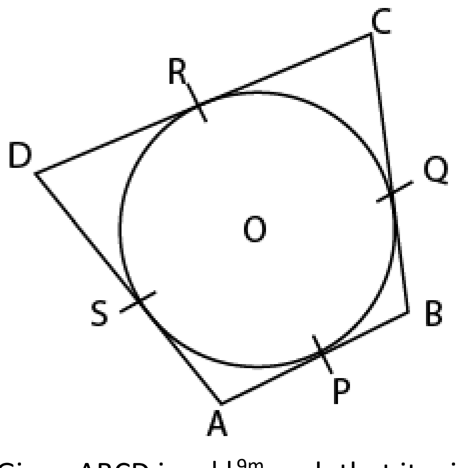


Figure 2

Solution:



Given ABCD is a $||^{9m}$ such that its sides touch a circle with centre O.

$\therefore$ AB = CD and AB || CD,

AD = BC and AD II BC

Now, P Q, R and S are the touching point of both the circle and the $||9^{th}$

We know that, tangents to a circle from an exterior point are equal in length.

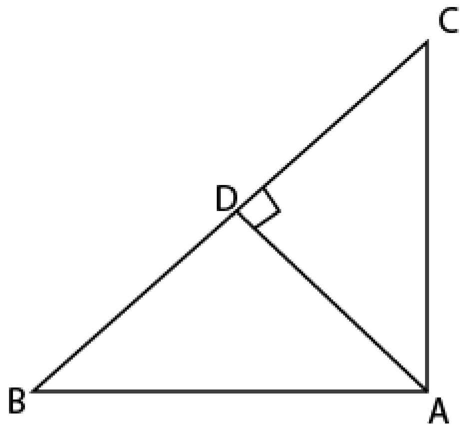
$\therefore$ AP = AS [Tangents from point A] ... (1)

BP = BQ [Tangents from point B] ...(2)

CR = CQ [Tangents from point C] .....(3)

DR = DS [Tangents from point D] …. (4)
On adding (1), (2), (3) and (4), we get
AP+BP+CR+DR=AS+BQ+CQ+DS
$\Rightarrow$ (AP + BP) + (CR +DR) = (AS + DS) + (BQ + CQ)
$\Rightarrow$ AB+CD = AD + BC
$\Rightarrow$ AB + AB = BC + BC [$\because$ ABCD is a $||^{9m}$ $\therefore$ AB = CD and AD= BC]
$\Rightarrow$ 2AB = 2BC
$\Rightarrow$ AB = BC
Therefore, AB = BC implies
AB = BC = CD = AD
Hence, ABCD is a rhombus.

OR



In ΔABD.
by Pythagoras theorem,
$AB^2 = BD^2 + AD^2$ ....(i)
and in ΔADC,
by Pythagoras theorem,
$AC^2 = CD^2 + AD^2$
$CD^2 = AC^2 - AD^2$ ....(ii)
On adding (i) &(ii), we get,
$\Rightarrow AB^2 + CD^2 = BD^2 + AD^2 + AC^2 - AD^2$
$\Rightarrow AB^2 + CD^2 = BD^2 + AC^2$
Hence, proved.

25. In Figure 3, ABC is a quadrant of a circle of radius 14 cm and a semi-circle is drawn with BC as diameter. Find the area of the shaded region.

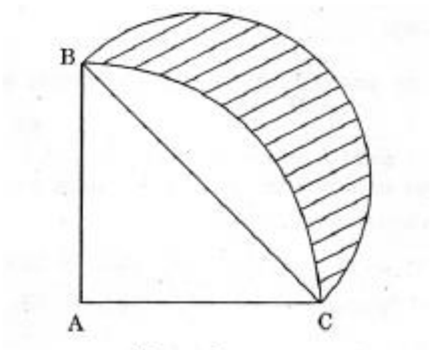
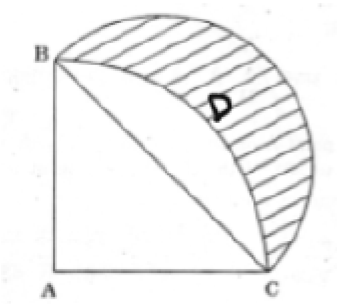


Figure 3

Solution:



Given AC = AB = 14 cm

$\therefore BC = \sqrt{14^2 + 14^2} = 14\sqrt{2}$ cm

Area of shaded region = Area of semi-circle – (Area of sector ABDC – Area of ΔABC)

$\therefore$ Area of $\Delta ABC = \frac{1}{2} \times 14 \times 14 = 98\ \text{cm}^2$

Area of Quadrant ABC = $\frac{1}{4} . \frac{22}{7} . (14)^2 = 154\ cm^2$

Area of segment BC = ar (Quadrant ABC) – ar(ΔABC)

$= 154 - 98$

$= 56\ cm^2$

Area of semicircle BC = $\frac{1}{2} \pi \left(\frac{BC}{1} \right)^2$

$= \frac{1}{2} \times \frac{22}{7} \times \frac{1}{4} \times 14\sqrt{2} \times 14\sqrt{2}$

$= 154\ cm^2$

Area of shaded region = Area of semicircle BC – Area of segment BC

$= 154 - 56$

$= 98\ cm^2$

SECTION D

Questions number 26 to 30 carry 6 marks each.

26. A peacock is sitting on the top of a pillar, which is 9 m high. From a point 27 in away from the bottom of the pillar, a snake is coming to its hole at the base of the pillar. Seeing the snake the peacock pounces on it. If their speeds are equal, at what distance from the hole is the snake caught ?

OR

The difference of two numbers is 4. If the difference of their reciprocals is $\frac{4}{21}$, find the two numbers.

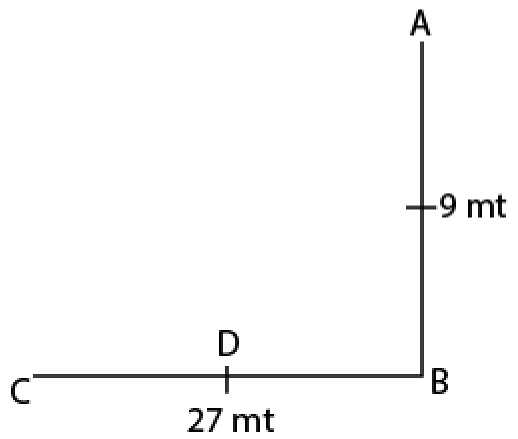
Solution:

Let AB be the pillar of height 9 meter. The peacock is sitting at point A on the pillar and B is the foot of the pillar. (AB = 9)

Let C be the position of the snake which is at 27 meters from B. (BC = 27 and ABC = 90)

As the speed of the snake and of the peacock is same they will travel the same distance in the same time

Now take a point D on BC that is equidistant from A and C (Please note that snake is moving towards the pillar)



Hence by condition 1 AD = DC = y (say)

Take BD = x

Now consider triangle ABD which is a right angled triangle

Using Pythagorus theorem (AB + BD = AD)

9 + x = y

9 = y - x = (y-x)(y + x)

81/(y + x) = (y-x)

y + x = BC = 27

Hence 81/27 = (y - x) = 3

y - x = 3

y + x = 27

Adding gives 2y = 30 or y = 15

x = 27 · y = 12

Thus the snake is caught at a distance of x meters or 12 meters from the hole.

OR

Lets assume the two numbers to be x, y (y > x)

Given that $y-x=4y=4+x$ ...(1)

$\frac{1}{x} - \frac{1}{y} = \frac{4}{21}$

$\Rightarrow \frac{y-x}{xy} = \frac{4}{21}$

$\Rightarrow \frac{+4}{xy} = \frac{4}{21}$

$xy = +21$

$x(4+x) = +21$

$x^2 + 4x - 21 = 0$

$(x+7)(x-3) = 0$

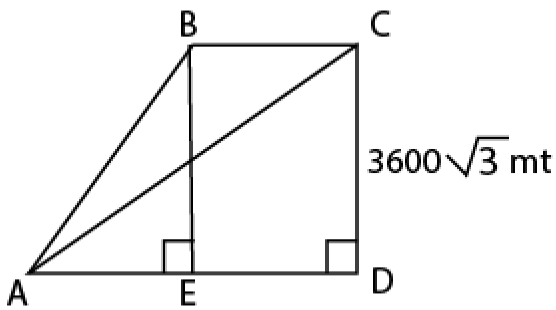
$x = -7, 3$

$y = -3, 7$

$\therefore$ Numbers are $-7, -3$ or $3, 7$

27. The angle of elevation of an aeroplane from a point A on the ground is 60°. After a flight of 30 seconds, the angle of elevation changes to 30°. If the plane is flying at a constant height of $3600\sqrt{3}$ m find the speed, in km/hour, of the plane.

Solution:



Height of aeroplane (CD) = 3600 $\sqrt{3}$ mt = BE

$\angle BAD = 60^\circ$ and $\angle CAD = 30^\circ$

In ΔABE

$\tan 60^\circ = \frac{BE}{AE}$

$AE = \frac{BE}{\tan 60^\circ}$

$AE = 3600\, mt$

In ΔACD

$$\tan 30^\circ = \frac{CD}{AD}$$

$$AD = \frac{3600\sqrt{3}}{\frac{1}{\sqrt{3}}}$$

$$AD = 10800\, mt$$

$$\therefore BC = AD - AE = 10800 - 3600$$

$$BC = 7200\, mt$$

Speed of aeroplane $= \dfrac{\text{distance}}{\text{time}}$

$$= \frac{7200}{30} = 240\, m/s$$

Speed (in km/hr) = 864 km/s

28. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, prove that the other two sides are divided in the same ratio.

Using the above, prove the following

In Figure 4, AB II DE and BC || EF. Prove that AC || DF.

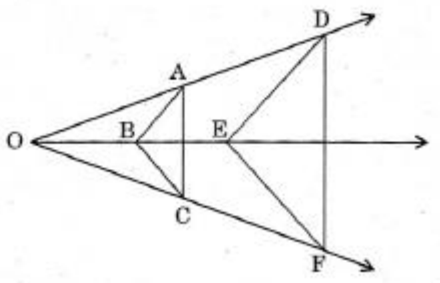


Figure 4

OR

Prove that the lengths of tangents drawn from an external point to a circle are equal. Using the above, prove the following :

ABC is an isosceles triangle in which AB = AC, circumscribed about a circle, as shown in Figure 5. Prove that the base is bisected by the point of contact.

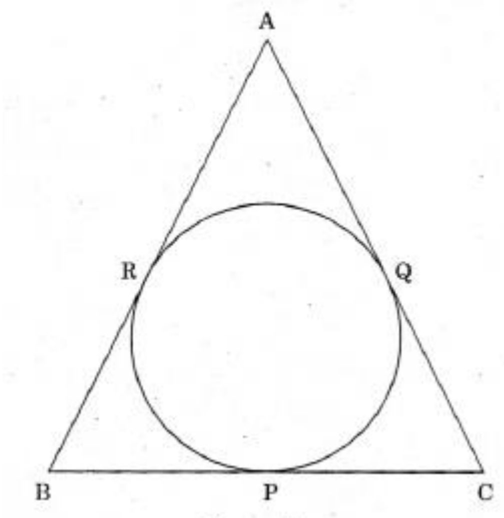
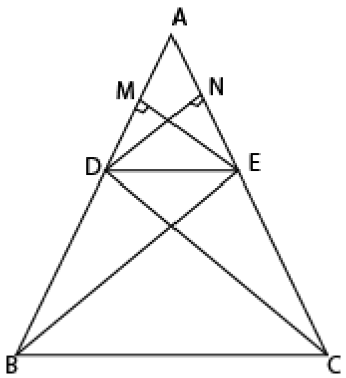


Figure 5

Solution:



Construction: Join BE and CD and draw perpendicular DN & EM to AC and AD respectively.

$$\frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{\frac{1}{2}\times EM\times AD}{\frac{1}{2}\times BD\times EM}$$

$$\Rightarrow \frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{AD}{BD} \qquad \text{........(1)}$$

Similarly,

$$\frac{ar(\Delta ADE)}{ar(\Delta CDE)} = \frac{\frac{1}{2}\times AB\times DN}{\frac{1}{2}\times EC\times DN}$$

$$\Rightarrow \frac{ar(\Delta ADE)}{ar(\Delta CDE)} = \frac{AE}{EC} \qquad \text{........(2)}$$

But $ar(\Delta BDE) = ar(\Delta CDE)$ (triangle on same base DE and between the same parallels DE and BC)

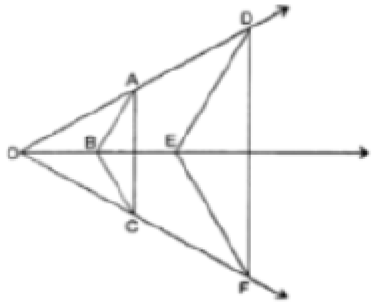
Thus, equation (2) becomes,

$$\frac{ar(\Delta ADE)}{ar(\Delta EDE)} = \frac{AE}{EC} \qquad \text{............(3)}$$

From equations (2) and (3), we have,

$\frac{AD}{BD} = \frac{AE}{EC}$

In the given AB||DE and BC||EF. Prove that AC||DF.



In $\Delta OPE, AB||DE$ (Given)

$\therefore$ By basic proportionality Theorem,

$\frac{OA}{AD} = \frac{OB}{BE}$ ....(1)

Similarly, in $\Delta OEF, BC||EF$ (Given)

$\therefore \quad \frac{OB}{BE} = \frac{OC}{CF}$ .....(2)

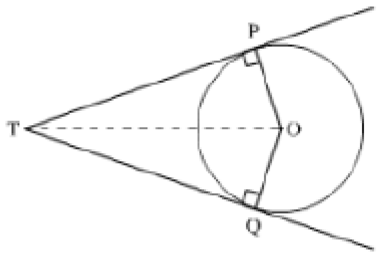
Comparing (1) and (2), we get

$\frac{OA}{AD} = \frac{OC}{CF}$

Hence, $AC||DF$

[By the converse of BPT]

OR



Given: PT and TQ we two tangent drawn from an external point T to the circle C (0, r).

To prove: 1. PT = TQ

2. $\angle$ OTP = $\angle$ OTQ

Construction: Join OT.

Proof: We know that a tangent to circle is perpendicular to the radius through the point of contact.

$\therefore \angle$ OPT = $\angle$ OQT = 90

In Δ OPT and Δ OQT,

OT = OT (Common)

OP = OQ (Radius of the circle)

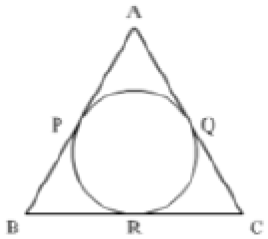
$\angle$ OPT = $\angle$ OQT (90)

$\therefore \Delta$ OPT

$\cong$

Δ OQT (RHS congruence criterion)

$\Rightarrow$ PT = TQ and $\angle$OTP = ΔOTQ (CPCT)
PT = TQ,
$\therefore$ The lengths of the tangents drawn from an external point to a circle are equal.



We know that the tangents drawn from an exterior point to a circle are equal in length.
$\therefore$ AP = AQ (Tangents from A) (1)
BP = RR (Tangents from B) (2)
CQ - CR (Tangents from C) (3)
Now, the given triangle is isosceles. so given AB = AC
Subtract AP from both sides, we get
AB- AP = AC-AP
= AB – AP = AC - AQ (Using (1))
BP = CQ
$\Rightarrow$ BR = CQ (Using (2))
$\Rightarrow$ BR = CR (Using (3))
So BR = CR, shows that BC is bisected at the point of contact.

29. If the radii of the circular ends of a conical bucket, which is 16 cm high, are 20 cm and 8 cm. find the capacity and total surface area of the bucket. (Use $\pi = \frac{22}{7}$]

Solution:
Radius of the bigger end of the frustum (bucket) of cone = R = 20 cm
Radius of the smaller end of the frustum (bucket) of the cone = r = 8 cm
Height = 16 cm
Volume = $1/3\pi h[R^2 + r^2 + R*r]$
= $1/3*22/7*16[20^2 + 8^2 + 20*8]$
= 352/21[400 + 64 + 160]
=(352*624)/21
= 219648/21
= 10459.43 cu cm
Now,
Slant height of the cone = l = $\sqrt{(R-r)^2 + h^2}$

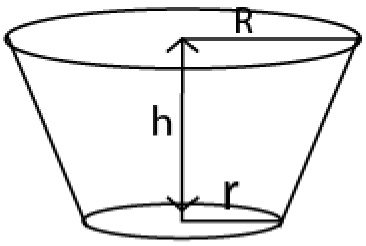
$l=\sqrt{(20-8)^2+16^2}$

$l=\sqrt{12^2+16^2}$

$l=\sqrt{144+256}$

$l=\sqrt{400}$

$l=20\ cm$



Slant height is 20 cm

Now,

Surface area = $=\pi[R^2+r^2+(R+r)*]$

= 22/7[20^2 + 8^2 + (20 + 8)*16]

= 22/7[400 + 64 + 448]

= 22/7*912

= 20064/7

= 2866.29 sq cm

30. Find mean, median and mode of the following data :

Classes	Frequency
0-20	6
20 -40	8
40 – 60	10
60 -80	12
80 – 100	6
100 – 120	5
120 -140	3

Solution:

Class	Frequency	class mark (x_i)	$x_i f_i$
0-20	6	10	60
20-40	8	30	240
40-60	10	50	500
60-80	12	70	840
80-100	6	90	540
100-120	5	110	550
120-140	3	130	390
	$\Sigma f_i=50$	$\Sigma f_i x_i=3120$	

Mean $= \frac{\Sigma x_i f_i}{\Sigma f_i}$

$= \frac{3120}{50}$

$= 62.4$

Class	f	(less than) cumulative frequency
0-20	6	6
20-40	8	14
40-60	10	24
60-80	12	36
80-100	6	42
100-120	5	47
120-140	3	50

$n = \Sigma f_i = 50$

$\frac{n}{2} = 25$

$\therefore$ Median class = 60 – 80

$$M = l + \left(\frac{\frac{n}{2} - c.f}{f} \right) \times h$$

$$M = 60 + \left(\frac{25 - 24}{12} \right) \times 20$$

$M = 61.66$

Mode:

Maximum class frequency = 12

$\therefore$ Model class = 60 – 80

Mode = $l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$

$= 60 + \left(\frac{12 - 10}{2 \times 12 - 10 - 6} \right) \times 20$

$= 65$