PREVIOUS YEAR QUESTION PAPER
2015

General Instructions:

*(i) **All** questions are compulsory.*
*(ii) Please check that this Question Paper contains **26** Questions.*
(iii) Marks for each question are indicated against it.
*(iv) Questions **1** to **6** in Section-**A** are Very Short Answer Type Questions carrying **one** mark each.*
*(v) Questions **7** to **19** in Section-**B** are Long Answer **I** Type Questions carrying **4** marks each.*
*(vi) Questions **20** to **26** in Section-**C** are Long Answer **II** Type Questions carrying **6** marks each*
(vii) Please write down the serial number of the Question before attempting it.

SECTION – A

Question numbers **1** to **6** carry **1** mark each.

1. If $\vec{a} = 7\hat{i} + \hat{j} - 4k$ and $\vec{b} = 2\hat{i} + 6\hat{j} + 3k$, then find the projection of $\vec{a}$ on $\vec{b}$.

Solution:

$$p = \frac{\vec{a}.\vec{b}}{|\vec{b}|} = \frac{8}{7}$$

2. Find λ, if the vectors $\vec{a} = \hat{i} + 3\hat{j} + \hat{k}, \vec{b} = 2\hat{i} - \hat{j} - \hat{k}$ and $\vec{c} = \lambda\hat{j} + 3\hat{k}$ are coplanar.

Solution:

$$\begin{vmatrix} 1 & 3 & 1 \\ 2 & -1 & -1 \\ 0 & \lambda & 3 \end{vmatrix} = 0 \Rightarrow \lambda = 7$$

3. If a line makes angles $90^0, 60^0$ and θ with x, y and z- axis respectively, where θ is Acute, then find θ.

Solution:

$$\cos^2 \pi/2 + \cos^2 \pi/3 + \cos^2 \theta = 1 \Rightarrow \theta = \frac{\pi}{6}$$

4. Write the element a_{23} of a 3 × 3 matrix A = (a_{ij}) whose elements a_{ij} are given $a_{ij} = \frac{|i-j|}{2}$.

Solution:

$$a_{23} = \frac{|2-3|}{2} = \frac{1}{2}$$

5. Find the differential equation representing the family of curves v = $\frac{A}{r}$ + B, where A and B are arbitrary constants.

Solution:

$$\frac{dv}{dr} = -\frac{A}{r^2}, \Rightarrow r^2 \frac{d^2v}{dr^2} + 2r\frac{dv}{dr} = 0$$

6. Find the integrating factor of the differential equation

$$\left(\frac{e^{-2}\sqrt{x}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dx}{dy} = 1.$$

Solution:

$$I.F = \int_e \frac{1}{\sqrt{x}}dx = e^{2\sqrt{x}}$$

SECTION – B

Question numbers **7** to **19** carry **4** marks each.

7. If $A = \begin{pmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{pmatrix}$ find $A^2 - 5A + 4I$ and hence find a matrix X such that $A^2 - 5A + 4I + X = O$

OR

If $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix}$, find $(A')^{-1}$.

Solution:

Getting $A^2 = \begin{pmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{pmatrix}$

$$A^2 - 5A + 4I = \begin{pmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{pmatrix} + \begin{pmatrix} -10 & 0 & -5 \\ -10 & -5 & -15 \\ -5 & 5 & 0 \end{pmatrix} + \begin{pmatrix} 4 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & -1 & -3 \\ -1 & -3 & -10 \\ -5 & 4 & 2 \end{pmatrix}$$

$$\therefore X = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & 10 \\ 5 & -4 & -2 \end{pmatrix}$$

OR

$$A' = \begin{pmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{pmatrix}$$

$$|A'| = 1(-9) - 2(-5) = -9 + 10 = 1 \neq 0$$

$$\text{Adj } A' = \begin{pmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{pmatrix}$$

$$\therefore (A')^{-1} = \begin{pmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{pmatrix}$$

8. if f(x) $= \begin{bmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{bmatrix}$, using properties of determinants find the value of F(2x) - f(x).

Solution:

$$f(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$$

$R_2 \to R_2 - xR_1 \; and \; R_3 \to R_3 - x^2R^1$

$$f(x) = \begin{vmatrix} a & -1 & 0 \\ 0 & a+x & -1 \\ 0 & ax+x^2 & a \end{vmatrix}$$ (For bringing 2 zeroes in any row/column

$$\therefore f(x) = a(a^2 + 2ax + x^2) = a(x+a)^2$$

$$\therefore f(2x) - f(x) = a[2x+a]^2 - a(x+a)^2$$

$$= a\,x\,(3x + 2a)$$

9. Find : $\int \frac{dx}{\sin x + \sin 2x}$

OR

Integrate the following w. r. t. x

$$\frac{x^2 - 3x + 1}{\sqrt{1-x^2}}$$

Solution:

$$\int \frac{dx}{\sin x + \sin 2x} = \int \frac{dx}{\sin x\,(1 + 2\cos x)} = \int \frac{\sin x \cdot dx}{(1-\cos x)(1+\cos x)(1+2\cos x)}$$

$$= -\int \frac{dt}{(1-t)(1+t)(1+2t)}$$ where cos x = t

$$= \int \left(\frac{-1/6}{1-t} + \frac{1/2}{1+t} - \frac{4/3}{1+2t} \right) dt$$

$$= +\frac{1}{6}\log|1-t| + \frac{1}{2}\log|1+t| - \frac{2}{3}\log|1+2\cos x| + c$$

OR

$$\int \frac{x^2 - 3x + 1}{\sqrt{1-x^2}}\,dx = \int \frac{2 - 3x - (1-x^2)}{\sqrt{1-x^2}}\,dx$$

$$= 2\int \frac{1}{\sqrt{1-x^2}}\,dx - 3\int \frac{x}{\sqrt{1-x^2}}\,dx - \int \sqrt{1-x^2}\,dx$$

$$= 2\sin^{-1}x + 3\sqrt{1-x^2} - \frac{x}{2}\sqrt{1-x^2} - \frac{1}{2}\sin^{-1}x + c$$

or $= \frac{3}{2}\sin^{-1}x + \frac{1}{2}(6-x)\sqrt{1-x^2} + c$

10. Evaluate : $\int_{-\pi}^{\pi} (\cos ax - \sin bx)^2\,dx$

Solution:

$$I = \int_{-\pi}^{\pi} \left(\cos ax - \sin bx\right)^2\,dx = \int_{-\pi}^{\pi} \left(\cos^2 ax + \sin^2 bx\right)dx - \int_{-\pi}^{\pi} 2\cos ax \sin bx\,dx$$

$$= I_1 - I_2$$

$I_1 = 2\int_0^{\pi} \left(\cos^2 ax + \sin^2 bx\right)dx$ (being an even fun.)

$I_2 = 0$ (being an odd fun.)

$$\therefore I = I_1 = \int_0^{\pi} \left(1 + \cos 2ax + 1 - \cos 2bx\right)dx$$

$$= \left[2x + \frac{\sin 2ax}{2a} - \frac{\sin 2bx}{2b}\right]_0^{\pi}$$

$$= \left[2\pi + \frac{1}{2a}\cdot \sin 2a\pi - \frac{\sin 2b\pi}{2b}\right] \text{ or } 2\pi$$

11. A bag A contains 4 black and 6 red balls and bag B contains 7 black and 3 red balls. A die is thrown. If 1 or 2 appears on it, then bag A is chosen, otherwise bag b. If two balls are drawn at random (without replacement) from the select bag, find the probability of one of them being red and another black.

OR

An unbiased coin is tossed 4 times. Find the mean and variance of the number of heads obtained.

Solution:

. Let E_1 : selecting bag A, and E_2 : selecting bag B.

$$\therefore P(E_1) = 1/3,\ P(E_2) = 2/3$$

Let A : Getting one Red and one balck ball

$$\therefore P\left(A/E_1\right) = \frac{{}^4C_1 \cdot {}^6C_1}{{}^{10}C_2} = \frac{8}{15},\ P\left(A/E_2\right) = \frac{{}^7C_1 \cdot {}^3C_1}{{}^{10}C_2} = \frac{7}{15}$$

$$P(A) = P(E_1)\cdot P(A/E_1) + P(E_2)\cdot P\left(A/E_2\right)$$

$$= \frac{1}{3}\cdot\frac{8}{15} + \frac{2}{3}\cdot\frac{7}{15} = \frac{22}{45}$$

OR

x	:	0	1	2	3	4
P (x)	:	${}^4C_0\left(\frac{1}{2}\right)^4$	${}^4C_1\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)$	${}^4C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2$	${}^4C_3\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^3$	${}^4C_4\left(\frac{1}{2}\right)^4$
	:	$=\frac{1}{16}$	$=\frac{4}{16}$	$=\frac{6}{16}$	$=\frac{4}{16}$	$=\frac{1}{16}$
x P (x)	:	0	$\frac{4}{16}$	$\frac{12}{16}$	$\frac{12}{16}$	$\frac{4}{16}$
x^2P (x)	:	0	$\frac{4}{16}$	$\frac{24}{16}$	$\frac{36}{16}$	$\frac{16}{16}$

$$Mean = \sum x\ P(x) = \frac{32}{16} = 2$$

Variance $= \sum x^2\ P(x) - \left(\sum xP(x)\right)^2 = \frac{80}{16} - (2)^2 = 1$

12. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, find $(\vec{r}\times\hat{i})\ .\ (\vec{r}\times j) + xy$

Solution:

$$\vec{r}\times\vec{i} = \left(x\hat{i} + y\hat{j} + z\hat{k}\right)x\hat{i} = -y\hat{k} + z\hat{j}$$

$$\vec{r}\times\vec{j} = \left(x\hat{i} + y\hat{j} + z\hat{k}\right)\hat{j} = x\hat{k} - z\hat{i}$$

$$\left(\vec{r}\times\hat{i}\right), \left(\vec{r}\times\vec{j}\right) = \left(o\hat{i} + z\hat{j} - y\hat{k}\right)\cdot\left(-z\hat{i} + o\hat{j} + x\hat{k}\right) = -xy$$

$$\left(\vec{r}\times\hat{i}\right)\cdot\left(\vec{r}\times\vec{j}\right) + xy = -xy + xy + 0$$

13. Find the distance between the point (-1 , -5, -10) and the point of intersection of the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ and the plane x – y + z = 5.

Solution:

. Any point on the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ $is\ (3\lambda+2, 4\lambda-1, 12\lambda+2)$

If this is the point of intersection with plane x – y + z = 5

Then $3\lambda + 2 - 4\lambda + 1 + 12\lambda + 2 - 5 = 0 \Rightarrow \lambda = 0$

$\therefore$ Point of intersection is (2, –1, 2)

Required distance $= \sqrt{(2+1)^2 + (-1+5)^2 + (2+10)}^2 = 13$

14. If sin $\left[\cot^{-1}(x+1)\right]$ = cos $(\tan^{-1}x)$, then find x.

OR

If $(\tan^{-1}x)^2 + (\cot^{-1}x)^2 = \frac{5\pi^2}{8}$, then find x.

Solution:

Writing $\cot^{-1}(x+1)=\sin^{-1}\frac{1}{\sqrt{1+(x+1)^2}}$

and $\tan^{-1}x=\cos^{-1}\frac{1}{\sqrt{1+x^2}}$

$$\therefore \sin\left(\sin^{-1}\frac{1}{\sqrt{1+(x+1)^2}}\right)=\cos\left(\cos^{-1}\frac{1}{\sqrt{1+x^2}}\right)$$

$$1+x^2+2x+1=1+x^2 \Rightarrow x=-\frac{1}{2}$$

OR

$$\left(\tan^{-1}x\right)^2+\left(\cot^{-1}x\right)^2=\frac{5\pi^2}{8}\Rightarrow\left(\tan^{-1}x\right)^2+\left(\frac{\pi}{2}-\tan^{-1}x\right)^2=\frac{5\pi^2}{8}$$

$$\therefore 2\left(\tan^{-1}x\right)^2-\pi\tan^{-1}x-\frac{3\pi^2}{8}=0$$

$$\tan^{-1}x=\frac{\pi\pm\sqrt{\pi^2+3\pi^2}}{4}=3\pi/4,\ -\pi/4$$

$$\Rightarrow x=-1$$

15. If y = $\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right), x^2\le 1,$ then find $\frac{dy}{dx}$.

Solution:

Putting $x^2=\cos\theta$, *we get*

$$y=\tan^{-1}\left(\frac{\sqrt{1+\cos\theta}+\sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta}-\sqrt{1-\cos\theta}}\right)$$

$$=\tan^{-1}\left(\frac{\cos\theta/2+\sin\theta/2}{\cos\theta/2-\sin\theta/2}\right)=\tan^{-1}\left(\frac{1+\tan\theta/2}{1-\tan\theta/2}\right)$$

$$y=\frac{\pi}{4}+\theta/2=\frac{\pi}{4}+\frac{1}{2}\cos^{-1}x^2$$

$$\frac{dy}{dx}=-\frac{1}{2}\frac{1}{\sqrt{1-x^4}}\cdot 2x=-\frac{x}{\sqrt{1-x^4}}$$

16. If x = a cosθ + b sinθ, y = a sinθ - b cosθ, show that $y^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+y=0.$

Solution:

$$\frac{dx}{d\theta}=-a\sin\theta+b\cos\theta$$

$$\frac{dy}{d\theta}=a\cos\theta+b\sin\theta$$

$$\therefore \frac{dy}{dx} = \frac{a\cos\theta + b\sin\theta}{a\sin\theta + b\cos\theta} = -\frac{x}{y}$$

Or $y\frac{dy}{dx} + x = 0$

$$\therefore y\frac{d^2y}{dx^2} + \frac{dy}{dx}\cdot\frac{dy}{dx} + 1 = 0$$

17. The side of an equilateral triangle is increasing at the rate of 2 cm/s. At what rate is its area increasing when the side of the triangle is cm ?

Solution:

Let x be the side of an equilateral triangle

$$\therefore \frac{dx}{dt} = 2cm/s.$$

Area (A) $= \frac{\sqrt{3x^2}}{4}$

$$\Rightarrow \frac{dA}{dt} = \frac{\sqrt{3}}{2}x\frac{dx}{dt}$$

$$\Rightarrow \frac{dA}{dt} = \frac{\sqrt{3}}{2}\cdot(20)\cdot(2) = 20\sqrt{3}\,cm^2/_s$$

18. Find : $\int (x+3)\sqrt{3-4x-x^2}\,dx.$

Solution:

Writing $x+3 = -\frac{1}{2}(-4-2x)+1$

$$\therefore \int (x+3)\sqrt{3-4x-x^2}\ dx = -\frac{1}{2}\int(-4-2x)\sqrt{3-4x-x^2}\ dx + \int\sqrt{7-(x+2)^2}\ dx$$

$$= -\frac{1}{3}(3-4x-x^2)^{3/2} + \frac{x+2}{2}\sqrt{3-4x-x^2} + \frac{7}{2}\sin^{-1}\left(\frac{x+2}{\sqrt{7}}\right) + c$$

19. Three schools A, B and C organized a mela for collecting found for helping the rehabilitation of flood victims. They sold hand made fans, mats and plates from recycled material at a cost of Rs. 25 , Rs. 100, Rs. 50, each. The number of articles sold are given below :

Article \ School	A	B	C
Hand-fans	40	25	35
Mats	50	40	50
Plates	20	30	40

Find the found collected by each school separately by selling the above articles, Also Find the total founds collected for the purpose. Write one value generated by the above situation.

Solution:

$$\begin{matrix} & HF. & M & P \end{matrix}$$

$$\begin{matrix} A \\ B \\ C \end{matrix}\begin{pmatrix} 40 & 50 & 20 \\ 25 & 40 & 30 \\ 35 & 50 & 40 \end{pmatrix}\begin{pmatrix} 25 \\ 100 \\ 50 \end{pmatrix} = \begin{pmatrix} 7000 \\ 6125 \\ 7875 \end{pmatrix}$$

Funds collected by school A : Rs. 7000,
School B : Rs. 6125, School C : Rs. 7875
Total collected : Rs. 21000
For writing one value

SECTION – C

Question numbers **20** to **26** carry **6** marks each.

20. Let N denote the set of all natural numbers and R be the relation on N × N defined by (a, b) R (c, d) if ad (b + c) = bc (a + d). show that R is an equivalence relation.

Solution:

$\forall\, a, b \in N, (a,b) R (a,b)\ as\ ab\,(b+a) = ba(a+b)$

$\therefore$ R is reflexive .... (i)

Let (a, b) R (c, d) for (a, b), (c, d) $\in N \times N$

$\therefore$ ad (b + c) = bc (a + d) .... (ii)

Also (c, d) R (a, b) $\because$ cb (d + a) = da (c + b) (using ii)

$\therefore$ R is symmetric .... (iii)

Let (a, b) R (c, d) and (c, d) R (e, f), for a, b, c, d, e, f $\in N$

$\therefore$ ad (b + c) = bc (a + d) and cf (d + e) = de (c + f)

$\therefore \frac{b+c}{bc} = \frac{a+d}{ad}\ and\ \frac{d+e}{de} = \frac{c+f}{cf}$

i.e $\frac{1}{c} + \frac{1}{b} = \frac{1}{d} + \frac{1}{a}\ and\ \frac{1}{e} + \frac{1}{d} = \frac{1}{f} + \frac{1}{c}$

adding we get $\frac{1}{c} + \frac{1}{b} + \frac{1}{e} + \frac{1}{d} = \frac{1}{d} + \frac{1}{a} + \frac{1}{f} + \frac{1}{c}$

$\Rightarrow$ af (b + e) = be (a + f)

Hence (a, b) R (e, f) $\therefore$ R is transitive .... (iv)

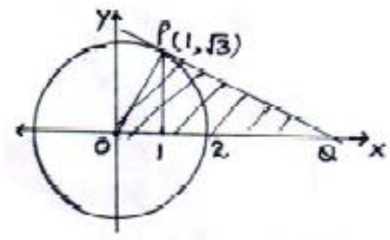
Form (i), (iii) and (iv) R is an equivalence relation

21. Using integration find the area of the triangle formed by positive x- zxis and tangent and normal to the circle $x^2 + y^2 = 4$ at $(1, \sqrt{3})$.

OR

Evaluate $\int_1^3 (e^{2-3x} + x^2 + 1)$ as a limit of a sum.

Solution:



Eqn. of normal (OP) : y = $\sqrt{3}x$

Eqn. of tangent (PQ) is

$$y-\sqrt{3}=-\frac{1}{\sqrt{3}}(x-1)\ i.e\ y=\frac{1}{\sqrt{3}}(4-x)$$

Coordinates of Q (4, 0)

$$\therefore \text{ Req. area } =\int_0^1 \sqrt{3}x\,dx+\int_1^4 \frac{1}{\sqrt{3}}(4-x)\,dx$$

$$=\sqrt{3}\frac{x^2}{2}\Big]_0^1+\frac{1}{\sqrt{3}}\left[4x-\frac{x^2}{2}\right]_1^4$$

$$=\frac{\sqrt{3}}{2}+\frac{1}{\sqrt{3}}\left[16-8-4+\frac{1}{2}\right]=2\sqrt{3}\ \text{sq. units}$$

OR

$$\int_1^3\left(e^{2-3x}+x^2+1\right)dx\ here\ h=\frac{2}{n}$$

$$=\lim_{h\to 0}h\left[f(1)+f(1+h)+f(1+2h)+.......+f(1+(n-1)h)\right]$$

$$=\lim_{h\to 0}h\Big[\left(e^{-1}+2\right)+\left(e^{-1-3h}+2+2h+h^2\right)+\left(e^{-1-6h}+2+4h+4h^2\right)+.......$$

$$+\left(e^{-1-3(n-1)h}+2+2(n-1)h+(n-1)^2h^2\right)\Big]$$

$$=\lim_{h\to 0}h\left[e^{-1}\left(1+e^{-3h}+e^{-6h}+.....+e^{-3(n-1)h}\right)+2n+2h\left(1+2+....+(n-1)^2\right)\right]$$

$$=\lim_{h\to 0}h\left(e^{-1}\cdot\frac{e^{-3nh}-1}{e^{-3n}-1}\cdot h+2nh+2\frac{nh(nh-h)}{2}+\frac{nh(nh-h)(2nh-h)}{6}\right)$$

$$=e^{-1}\frac{\left(e^{-6}-1\right)}{-3}+4+4+\frac{8}{3}=-e^{-1}\frac{e^{-6}-1}{3}+\frac{32}{3}$$

22. Solve the differential equation : $(\tan^{-1}y-x)dy=(1+y^2)dx.$

OR

Find the particular solution of the differential equation $\frac{dy}{dx}=\frac{xy}{x^2+y^2}$ given that y = 1, When x = 0 .

Solution:

Given differential equation can be written as

$$\frac{dx}{dy}+\frac{1}{1+y^2}\cdot x=\frac{\tan^{-1}y}{1+y^2}$$

$\therefore$ Integrating factor is $e^{\tan^{-1}}y$

$\therefore$ Solution is : $x \cdot e^{\tan^{-1}}y = \int \frac{\tan^{-1} y \cdot e^{\tan^{-1}y}}{1+y^2} dy$

$\Rightarrow x \cdot e^{\tan^{-1}}y = \int t\, e^t dt \ where \tan^{-1} y = t$

$= t\, e^t - e^t + c = e^{\tan^{-1}y}\left(\tan^{-1} y - 1\right) + c$

$or\ x = \tan^{-1} y - 1 + c\, e^{-\tan^{-1} y}$

OR

Given differential equation is $\frac{dy}{dx} = \frac{y/x}{1+\left(y/x\right)^2}$

Putting $\frac{y}{x} = v$ to get $v + x\frac{dv}{dx} = \frac{v}{1+v^2}$

$\therefore x\frac{dv}{dx} = \frac{v}{1+v^2} - v = \frac{|-v^3}{1+v^2}$

$\Rightarrow \int \frac{v^2+1}{v^3} dv = -\int \frac{dx}{x}$

$\Rightarrow \log|v| - \frac{1}{2v^2} = -\log|x| + c$

$\therefore \log y - \frac{x^2}{2y^2} = c$

$x = 0,\ y = 1 \Rightarrow c = 0 \therefore \log y - \frac{x^2}{2y^2} = 0$

23. If lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y+k}{2} = \frac{z}{1}$ intersect , then find the value of K and hence find the equation of the plane containing these lines.

Solution:

. Any point on line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ $is\left(2\lambda+1, 3\lambda-1, 4\lambda+1\right)$

$\therefore \frac{2\lambda+1-3}{1} = \frac{3\lambda-1-k}{2} = \frac{4\lambda+1}{1} \Rightarrow \lambda = -\frac{3}{2}$, hence k $= \frac{9}{2}$

Eqn. of plane containing three lines is

$$\begin{vmatrix} x-1 & y+1 & z-1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

$\Rightarrow -5(x-1) + 2(y+1) + 1(z-1) = 0$

i.e. $5x - 2y - z - 6 = 0$

24. If A and B are two independent events such that $P(\bar{A}\cap B)=\dfrac{2}{15}$ and $P(A\cap\bar{B})=\dfrac{1}{6}$, then find P (A) and P (B).

Solution:

$P\left(\bar{A}\cap B\right)=\dfrac{2}{15}\Rightarrow P\left(\bar{A}\right)\cdot P\left(B\right)=\dfrac{2}{15}$

$P\left(A\cap\bar{B}\right)=\dfrac{1}{6}\Rightarrow P\left(\bar{A}\right)\cdot P\left(\bar{B}\right)=\dfrac{1}{6}$

$\therefore\left(1-P\left(A\right)\right)P\left(B\right)=\dfrac{2}{15}\ or\ P\left(B\right)-P\left(A\right)\cdot P\left(B\right)=\dfrac{2}{15}$ …………………(i)

$P\left(A\right)\left(1-P\left(B\right)\right)=\dfrac{1}{6}\ or\ P\left(A\right)-P\left(A\right)\cdot P\left(B\right)=\dfrac{1}{6}$ ………………………..(ii)

From (i) and (ii) $P\left(A\right)-P\left(B\right)=\dfrac{1}{6}-\dfrac{2}{15}=\dfrac{1}{30}$

Let P (A) = x, P (B) = y $\therefore x=\left(\dfrac{1}{30}+y\right)$

(i)$\Rightarrow y|-\left(\dfrac{1}{30}+y\right)y=\dfrac{2}{15}\ \therefore\ 30y^2-29y+4=0$

Solving to get $y={}^{1}\!/_{6}\ or\ y={}^{4}\!/_{5}$

$\therefore x={}^{1}\!/_{5}\ or\ x={}^{5}\!/_{6}$

Hence $P\left(A\right)={}^{1}\!/_{5},\ \ P\left(B={}^{1}\!/_{6}\right)$ OR $P\left(A\right)={}^{5}\!/_{6},\ \ P\left(B={}^{4}\!/_{5}\right)$

25. Find the local maxima and local minima, of the function $f(x)=\sin x,\ 0<x<2\pi.$ Also find the local maximum and local minimum values.

Solution:

$f\left(x\right)=\sin x-\cos x,\ 0<x<2\pi$

$f'\left(x\right)=0\ \Rightarrow\cos x+\sin x=0\ or\ \tan x=-1,$

$\therefore x={}^{3\pi}\!/_{4},\dfrac{7\pi}{4}$

$f''\left({}^{3\pi}\!/_{4}\right)=-\dfrac{1}{\sqrt{2}}-\dfrac{1}{\sqrt{2}}$ i.e ve so, $x={}^{3\pi}\!/_{4}$ is LocalMaxima

and $f''\left({}^{7\pi}\!/_{4}\right)=-\dfrac{1}{\sqrt{2}}+\dfrac{1}{2}$ i.e ve so, $x={}^{7\pi}\!/_{4}$ is LocalMinima

Local Maximum value $=\dfrac{1}{\sqrt{2}}+\dfrac{1}{\sqrt{2}}=\sqrt{2}$

Local Minimum value $=-\dfrac{1}{\sqrt{2}}-\dfrac{1}{\sqrt{2}}=\sqrt{2}$

26. Find graphically, the maximum value of z = 2x + 5y, subject to constraints given below :

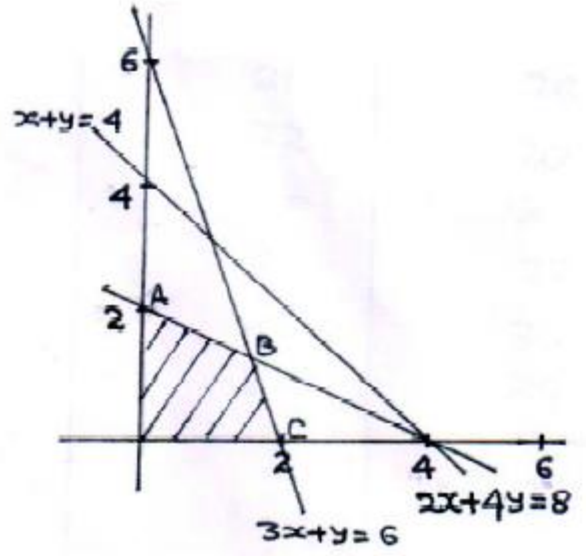
$2x + 4y \le 8.$

$3x + y \le 6$

$x + y \le 4$

$x \ge 0,\ y \ge 0$

Solution:



Correct graphs of three lines
Correctly shading
feasible region
Vertices are
A (0, 2), B (1.6, 1.2), C (2, .0)
Z = 2x + 5y is maximum
at A (0, 2) and maximum value = 10