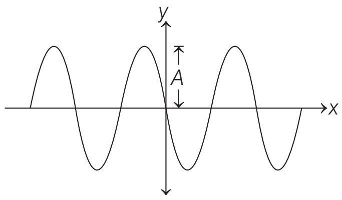
10

Wave Motion

Topic 1 Wave Pulse and Travelling Wave

Objective Questions I (Only one correct option)

1. A progressive wave travelling along the positive x-direction is represented by $y(x, t) = A \sin(kx - \omega t + \phi)$. Its snapshot at $t = 0$ is given in the figure. **(2019 Main, 12 April I)**



For this wave, the phase ϕ is

(a) $-\frac{\pi}{2}$ (b) π (c) 0 (d) $\frac{\pi}{2}$

2. A submarine A travelling at 18 km/h is being chased along the line of its velocity by another submarine B travelling at 27 km/h. B sends a sonar signal of 500 Hz to detect A and receives a reflected sound of frequency ν. The value of ν is close to (Speed of sound in water $= 1500\,\text{ms}^{-1}$) **(2019 Main, 12 April I)**

(a) 504 Hz (b) 507 Hz (c) 499 Hz (d) 502 Hz

3. A string of length 1 m and mass 5 g is fixed at both ends. The tension in the string is 8.0 N. The string is set into vibration using an external vibrator of frequency 100 Hz. The separation between successive nodes on the string is close to **(2019 Main, 10 Jan I)**

(a) 16.6 cm (b) 33.3 cm (c) 10.0 cm (d) 20.0 cm

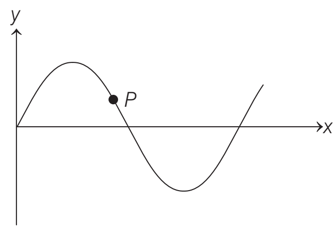
4. A heavy ball of mass M is suspended from the ceiling of a car by a light string of mass $m(m << M)$. When the car is at rest, the speed of transverse waves in the string is $60\,\text{ms}^{-1}$. When the car has acceleration a, the wave speed increases to $60.5\,\text{ms}^{-1}$. The value of a, in terms of gravitational acceleration g is closest to **(2019 Main, 9 Jan I)**

(a) $\frac{g}{20}$ (b) $\frac{g}{5}$ (c) $\frac{g}{30}$ (d) $\frac{g}{10}$

5. A uniform string of length 20 m is suspended from a rigid support. A short wave pulse is introduced at its lowest end. It starts moving up the string. The time taken to reach the support is (Take, $g = 10\,\text{ms}^{-2}$) **(2016 Main)**

(a) $2\pi\sqrt{2}$ s (b) 2 s
(c) $2\sqrt{2}$ s (d) $\sqrt{2}$ s

6. A transverse sinusoidal wave moves along a string in the positive x-direction at a speed of 10 cm/s. The wavelength of the wave is 0.5 m and its amplitude is 10 cm. At a particular time t, the snap-shot of the wave is shown in figure. The velocity of point P when its displacement is 5 cm is **(2008)**



(a) $\frac{\sqrt{3}\pi}{50}\hat{\mathbf{j}}$ m/s (b) $-\frac{\sqrt{3}\pi}{50}\hat{\mathbf{j}}$ m/s (c) $\frac{\sqrt{3}\pi}{50}\hat{\mathbf{i}}$ m/s (d) $-\frac{\sqrt{3}\pi}{50}\hat{\mathbf{i}}$ m/s

7. A travelling wave in a stretched string is described by the equation ; $y = A \sin(kx - \omega t)$

The maximum particle velocity is **(1997, 1M)**

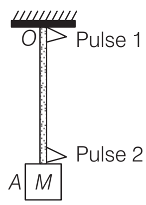
(a) $A\omega$ (b) ω/k (c) $d\omega/dk$ (d) x/ω

8. A transverse wave is described by the equation $y = y_0 \sin 2\pi\left(ft - \frac{x}{\lambda}\right)$. The maximum particle velocity is equal to four times the wave velocity if **(1984, 2M)**

(a) $\lambda = \pi y_0/4$ (b) $\lambda = \pi y_0/2$
(c) $\lambda = \pi y_0$ (d) $\lambda = 2\pi y_0$Objective

Questions II (One or more correct option)

9. A block M hangs vertically at the bottom end of a uniform rope of constant mass per unit length. The top end of the rope is attached to a fixed rigid support at O. A transverse wave pulse (Pulse 1) of wavelength λ_0 is produced at point O on the rope. The pulse takes time T_{OA} to reach point A. If the wave pulse of wavelength λ_0 is produced at point A (Pulse 2) without disturbing the position of M it takes time T_{AO} to reach point O. Which of the following options is/are correct? **(2017 Adv.)**



(a) The time $T_{AO} = T_{OA}$
(b) The wavelength of Pulse 1 becomes longer when it reaches point A
(c) The velocity of any pulse along the rope is independent of its frequency and wavelength
(d) The velocities of the two pulses (Pulse 1 and Pulse 2) are the same at the mid-point of rope

10. $Y(x,t) = \dfrac{0.8}{[(4x+5t)^2+5]}$ represents a moving pulse where x and y are in metre and t is in second. Then, **(1999, 3M)**

(a) pulse is moving in positive x-direction
(b) in 2 s it will travel a distance of 2.5 m
(c) its maximum displacement is 0.16 m
(d) it is a symmetric pulse

11. A transverse sinusoidal wave of amplitude a, wavelength λ and frequency f is travelling on a stretched string. The maximum speed of any point on the string is $v/10$, where v is the speed of propagation of the wave. If $a = 10^{-3}$ m and $v = 10\,\text{m/s}$, then λ and f are given by **(1998, 2M)**

(a) $\lambda = 2\pi \times 10^{-2}$ m
(b) $\lambda = 10^{-3}$ m
(c) $f = \dfrac{10^3}{2\pi}$ Hz
(d) $f = 10^4$ Hz

12. A wave is represented by the equation ;

$$y = A \sin(10\pi x + 15\pi t + \pi/3)$$

where, x is in metre and t is in second. The expression represents **(1990, 2M)**

(a) a wave travelling in the positive x-direction with a velocity 1.5 m/s
(b) a wave travelling in the negative x-direction with a velocity 1.5 m/s
(c) a wave travelling in the negative x-direction with a wavelength 0.2 m
(d) a wave travelling in the positive x-direction with a wavelength 0.2 m

13. A wave equation which gives the displacement along the y-direction is given by : $y = 10^{-4}\sin(60t + 2x)$ where, x and y are in metre and t is time in second. This represents a wave **(1981, 3M)**

(a) travelling with a velocity of 30 m/s in the negative x-direction
(b) of wavelength (π) m
(c) of frequency $\left(\dfrac{30}{\pi}\right)$ Hz
(d) of amplitude 10^{-4} m

Fill in the Blanks

14. The amplitude of a wave disturbance travelling in the positive x-direction is given by $y = \dfrac{1}{(1+x)^2}$ at time $t = 0$ and by $y = \dfrac{1}{[1+(x-1)^2]}$ at $t = 2$ s, where x and y are in metre. The shape of the wave disturbance does not change during the propagation. The velocity of the wave is …… m/s. **(1990, 2M)**

15. A travelling wave has the frequency ν and the particle displacement amplitude A. For the wave the particle velocity amplitude is …… and the particle acceleration amplitude is …… . **(1983, 2M)**

Analytical & Descriptive Question

16. A harmonically moving transverse wave on a string has a maximum particle velocity and acceleration of 3m/s and 90 m/s^2 respectively. Velocity of the wave is 20 m/s. Find the waveform. **(2005, 2M)**

Topic 2 Standing Waves, Stretched Wire and Organ Pipes

Objective Questions I (Only one correct option)

1. A tuning fork of frequency 480 Hz is used in an experiment for measuring speed of sound (v) in air by resonance tube method. Resonance is observed to occur at two successive lengths of the air column $l_1 = 30$ cm and $l_2 = 70$ cm.

Then, v is equal to **(Main 2019, 12 April II)**

(a) 332 ms^{-1} (b) 384 ms^{-1} (c) 338 ms^{-1} (d) 379 ms^{-1}

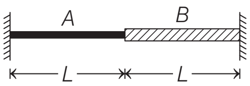
2. A string 2.0 m long and fixed at its ends is driven by a 240 Hz vibrator. The string vibrates in its third harmonic mode. The speed of the wave and its fundamental frequency is **(Main 2019, 9 April II)**

(a) 180 m/s, 80 Hz (b) 320 m/s, 80 Hz
(c) 320 m/s, 120 Hz (d) 180 m/s, 120 Hz

3. A string is clamped at both the ends and it is vibrating in its 4th harmonic. The equation of the stationary wave is $Y = 0.3\sin(0.157x)\cos(200\pi t)$. The length of the string is (All quantities are in SI units) **(Main 2019, 9 April I)**

(a) 60 m (b) 40 m (c) 80 m (d) 20 m

4. A wire of length $2L$, is made by joining two wires A and B of same length but different radii r and $2r$ and made of the same material. It is vibrating at a frequency such that the joint of the two wires forms a node. If the number of antinodes in wire A is p and that in B is q, then the ratio $p:q$ is **(2019 Main, 8 April I)**



(a) 3 : 5 (b) 4 : 9 (c) 1 : 2 (d) 1 : 4

5. A resonance tube is old and has jagged end. It is still used in the laboratory to determine velocity of sound in air. A tuning fork of frequency 512 Hz produces first resonance when the tube is filled with water to a mark 11 cm below a reference mark. near the open end of the tube. The experiment is repeated with another fork of frequency 256 Hz which produces first resonance when water reaches a mark 27 cm below the reference mark. The velocity of sound in air, obtained in the experiment is close to **(Main 2019, 12 Jan II)**

(a) $328\ ms^{-1}$ (b) $341\ ms^{-1}$ (c) $322\ ms^{-1}$ (d) $335\ ms^{-1}$

6. Equation of travelling wave on a stretched string of linear density 5 g/m is $y = 0.03\sin(450t - 9x)$, where distance and time are measured in SI units. The tension in the string is **(Main 2019, 11 Jan I)**

(a) 5 N (b) 12.5 N (c) 7.5 N (d) 10 N

7. A closed organ pipe has a fundamental frequency of 1.5 kHz. The number of overtones that can be distinctly heard by a person with this organ pipe will be (Assume that the highest frequency a person can hear is 20,000 Hz) **(Main 2019, 10 Jan II)**

(a) 7 (b) 4 (c) 5 (d) 6

8. A string of length 1 m and mass 5 g is fixed at both ends. The tension in the string is 8.0 N. The string is set into vibration using an external vibrator of frequency 100 Hz. The separation between successive nodes on the string is close to **(2019 Main, 10 Jan I)**

(a) 16.6 cm (b) 33.3 cm (c) 10.0 cm (d) 20.0 cm

9. A granite rod of 60 cm length is clamped at its middle point and is set into longitudinal vibrations. The density of granite is $2.7 \times 10^3\ kg/m^3$ and its Young's modulus is 9.27×10^{10} Pa. What will be the fundamental frequency of the longitudinal vibrations? **(2018 Main)**

(a) 7.5 kHz (b) 5 kHz (c) 2.5 kHz (d) 10 kHz

10. A pipe open at both ends has a fundamental frequency f in air. The pipe is dipped vertically in water, so that half of it is in water. The fundamental frequency of the air column is now **(2016 Main)**

(a) $\frac{f}{2}$ (b) $\frac{3f}{4}$ (c) $2f$ (d) f

11. A pipe of length 85 cm is closed from one end. Find the number of possible natural oscillations of air column in the pipe whose frequencies lie below 1250 Hz. The velocity of sound in air is 340 m/s. **(2014 Main)**

(a) 12 (b) 8 (c) 6 (d) 4

12. A sonometer wire of length 1.5 m is made of steel. The tension in it produces an elastic strain of 1%. What is the fundamental frequency of steel if density and elasticity of steel are $7.7 \times 10^3\ kg/m^3$ and $2.2 \times 10^{11}\ N/m^2$ respectively?

(a) 188.5 Hz (b) 178.2 Hz **(2013 Main)**
(c) 200.5 Hz (d) 770 Hz

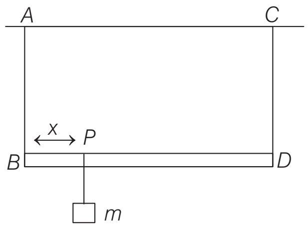
13. A student is performing the experiment of resonance column. The diameter of the column tube is 4 cm. The frequency of the tuning fork is 512 Hz. The air temperature is 38° C in which the speed of sound is 336 m/s. The zero of the meter scale coincides with the top end of the resonance column tube. When the first resonance occurs, the reading of the water level in the column is **(2012)**

(a) 14.0 cm (b) 15.2 cm (c) 16.4 cm (d) 17.6 cm

14. A hollow pipe of length 0.8 m is closed at one end. At its open end a 0.5 m long uniform string is vibrating in its second harmonic and it resonates with the fundamental frequency of the pipe. If the tension in the wire is 50 N and the speed of sound is $320\ ms^{-1}$, the mass of the string is

(a) 5 g (b) 10 g (c) 20 g (d) 40 g **(2010)**

15. A massless rod BD is suspended by two identical massless strings AB and CD of equal lengths. A block of mass m is suspended from point P such that BP is equal to x. If the fundamental frequency of the left wire is twice the fundamental frequency of right wire, then the value of x is **(2006)**



(a) $l/5$ (b) $l/4$ (c) $4l/5$ (d) $3l/4$

16. A tuning fork of 512 Hz is used to produce resonance in a resonance tube experiment. The level of water at first resonance is 30.7 cm and at second resonance is 63.2 cm. The error in calculating velocity of sound is **(2005, 2M)**

(a) 204.1 cm/s (b) 110 cm/s
(c) 58 cm/s (d) 280 cm/s

17. An open pipe is in resonance in 2nd harmonic with frequency f_1. Now one end of the tube is closed and frequency is increased to f_2 such that the resonance again occurs in nth harmonic. Choose the correct option. **(2005, 2M)**

(a) $n = 3,\ f_2 = (3/4) f_1$ (b) $n = 3,\ f_2 = (5/4) f_1$
(c) $n = 5,\ f_2 = \frac{5}{4} f_1$ (d) $n = 5,\ f_2 = \frac{3}{4} f_1$

18. A closed organ pipe of length L and an open organ pipe contain gases of densities ρ_1 and ρ_2 respectively. The compressibility of gases are equal in both the pipes. Both the pipes are vibrating in their first overtone with same frequency. The length of the open organ pipe is **(2004, 2M)**

(a) $\frac{L}{3}$ (b) $\frac{4L}{3}$ (c) $\frac{4L}{3}\sqrt{\frac{\rho_1}{\rho_2}}$ (d) $\frac{4L}{3}\sqrt{\frac{\rho_2}{\rho_1}}$

19. A sonometer wire resonates with a given tuning fork forming standing waves with five antinodes between the two bridges when a mass of 9 kg is suspended from the wire. When this mass is replaced by mass M. The wire resonates with the same tuning fork forming three antinodes for the same positions of the bridges. The value of M is **(2002, 2M)**

(a) 25 kg (b) 5 kg (c) 12.5 kg (d) 1/25 kg

20. Two vibrating strings of the same material but of lengths L and $2L$ have radii $2r$ and r respectively. They are stretched under the same tension. Both the strings vibrate in their fundamental modes. The one of length L with frequency ν_1 and the other with frequency ν_2. The ratio ν_1/ν_2 is given by **(2000, 2M)**

(a) 2 (b) 4 (c) 8 (d) 1

21. An open pipe is suddenly closed at one end with the result that the frequency of third harmonic of the closed pipe is found to be higher by 100 Hz then the fundamental frequency of the open pipe. The fundamental frequency of the open pipe is **(1996, 2M)**

(a) 200 Hz (b) 300 Hz (c) 240 Hz (d) 480 Hz

22. An object of specific gravity ρ is hung from a thin steel wire. The fundamental frequency for transverse standing waves in the wire is 300 Hz. The object is immersed in water, so that one half of its volume is submerged. The new fundamental frequency (in Hz) is **(1995, 2M)**

(a) $300\left(\dfrac{2\rho-1}{2\rho}\right)^{1/2}$ (b) $300\left(\dfrac{2\rho}{2\rho-1}\right)^{1/2}$

(c) $300\left(\dfrac{2\rho}{2\rho-1}\right)$ (d) $300\left(\dfrac{2\rho-1}{2\rho}\right)$

23. An organ pipe P_1 closed at one end vibrating in its first harmonic and another pipe P_2 open at both ends vibrating in its third harmonic are in resonance with a given tuning fork. The ratio of the length of P_1 and P_2 is **(1988, 2M)**

(a) 8/3 (b) 3/8 (c) 1/6 (d) 1/3

24. A wave represented by the equation $y = a\cos(kx - \omega t)$ is superimposed with another wave to form a stationary wave such that point $x = 0$ is a node. The equation for the other wave is **(1988, 1M)**

(a) $a\sin(kx + \omega t)$ (b) $-a\cos(kx - \omega t)$

(c) $-a\cos(kx + \omega t)$ (d) $-a\sin(kx - \omega t)$

25. A tube, closed at one end and containing air, produces, when excited, the fundamental note of frequency 512 Hz. If the tube is opened at both ends the fundamental frequency that can be excited is (in Hz) **(1986, 2M)**

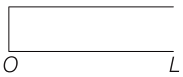
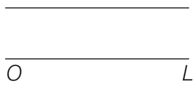
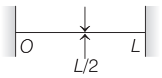
(a) 1024 (b) 512 (c) 256 (d) 128

26. A cylindrical tube, open at both ends, has a fundamental frequency f in air. The tube is dipped vertically in water so that half of its length is in water. The fundamental frequency of the air column is now **(1981, 2M)**

(a) $f/2$ (b) $3f/4$ (c) f (d) $2f$

Match the Columns

27. **Column I** shows four systems, each of the same length L, for producing standing waves. The lowest possible natural frequency of a system is called its fundamental frequency, whose wavelength is denoted as λ_f. Match each system with statements given in **Column II** describing the nature and wavelength of the standing waves. **(2011)**

	Column I		Column II
(A)	Pipe closed at one end 	(p)	Longitudinal waves
(B)	Pipe open at both ends 	(q)	Transverse waves
(C)	Stretched wire clamped at both ends 	(r)	$\lambda_f = L$
(D)	Stretched wire clamped at both ends and at mid-point 	(s)	$\lambda_f = 2L$
		(t)	$\lambda_f = 4L$

Objective Questions II (One or more correct option)

28. In an experiment to measure the speed of sound by a resonating air column, a tuning fork of frequency 500 Hz is used. The length of the air column is varied by changing the level of water in the resonance tube. Two successive resonances are heard at air columns of length 50.7 cm and 83.9 cm. Which of the following statements is (are) true?

(a) The speed of sound determined from this experiment is 332 m s^{-1}.

(b) The end correction in this experiment is 0.9 cm.

(c) The wavelength of the sound wave is 66.4 cm.

(d) The resonance at 50.7 cm corresponds to the fundamental harmonic.

29. One end of a taut string of length 3 m along the X-axis is fixed at $x = 0$. The speed of the waves in the string is 100 ms^{-1}. The other end of the string is vibrating in the y-direction so that stationary waves are set up in the string. The possible waveform(s) of these stationary wave is (are) **(2014 Adv.)**

(a) $y(t) = A\sin\dfrac{\pi x}{6}\cos\dfrac{50\pi t}{3}$

(b) $y(t) = A\sin\dfrac{\pi x}{3}\cos\dfrac{100\pi t}{3}$

(c) $y(t) = A\sin\dfrac{5\pi x}{6}\cos\dfrac{250\pi t}{3}$

(d) $y(t) = A\sin\dfrac{5\pi x}{2}\cos 250\pi t$

30. A horizontal stretched string, fixed at two ends, is vibrating in its fifth harmonic according to the equation, $y(x, t) = (0.01\,\text{m})\,[\sin(62.8\,\text{m}^{-1})x]\cos[(628\,\text{s}^{-1})t]$.

Assuming $\pi = 3.14$, the correct statement(s) is (are) **(2012)**

(a) the number of nodes is 5
(b) the length of the string is 0.25 m
(c) the maximum displacement of the mid-point of the string from its equilibrium position is 0.01 m
(d) the fundamental frequency is 100 Hz

31. Standing waves can be produced **(1999, 3M)**
(a) on a string clamped at both ends
(b) on a string clamped at one end and free at the other
(c) when incident wave gets reflected from a wall
(d) when two identical waves with a phase difference of π are moving in the same direction

32. The (x, y) coordinates of the corners of a square plate are $(0,0),(L,0,),(L,L)$ and $(0,L)$. The edges of the plate are clamped and transverse standing waves are set-up in it. If $u(x,y)$ denotes the displacement of the plate at the point (x,y) at some instant of time, the possible expression (s) for u is (are) (a = positive constant) **(1998, 2M)**
(a) $a\cos(\pi x/2L)\cos(\pi y/2L)$ (b) $a\sin(\pi x/L)\sin(\pi y/L)$
(c) $a\sin(\pi x/L)\sin(2\pi y/L)$ (d) $a\cos(2\pi x/L)\sin(\pi y/L)$

33. A wave disturbance in a medium is described by $y(x,t) = 0.02\cos\left(50\pi t + \frac{\pi}{2}\right)\cos(10\pi x)$, where x and y are in metre and t is in second. **(1995, 2M)**
(a) A node occurs at $x = 0.15$ m
(b) An antinode occurs at $x = 0.3$ m
(c) The speed of wave is 5 ms^{-1}
(d) The wavelength of wave is 0.2 m

34. Velocity of sound in air is 320m/s. A pipe closed at one end has a length of 1 m. Neglecting end corrections, the air column in the pipe can resonate for sound of frequency **(1989, 2M)**
(a) 80 Hz (b) 240 Hz (c) 320 Hz (d) 400 Hz

35. An air column in a pipe, which is closed at one end, will be in resonance with a vibrating tuning fork of frequency 264 Hz, if the length of the column in cm is (Speed of sound in air = 330 m/s) **(1985, 2M)**
(a) 31.25 (b) 62.50 (c) 93.75 (d) 125

Fill in the Blanks

36. A cylinder resonance tube open at both ends has fundamental frequency f in air. Half of the length of the tube is dipped vertically in water. The fundamental frequency to the air column now is …… . **(1992, 2M)**

37. In a sonometer wire, the tension is maintained by suspending a 50.7 kg mass from the free end of the wire. The suspended mass has a volume of 0.0075 m^3. The fundamental frequency of vibration of the wire is 260 Hz. If the suspended mass is completely submerged in water, the fundamental frequency will become …… Hz. **(1987, 2M)**

38. Sound waves of frequency 660 Hz fall normally on a perfectly reflecting wall. The shortest distance from the wall at which the air particles have maximum amplitude of vibration is …… m. Speed of sound = 330 m/s. **(1984, 2M)**

Integer Answer Type Questions

39. When two progressive waves $y_1 = 4\sin(2x - 6t)$ and $y_2 = 3\sin\left(2x - 6t - \frac{\pi}{2}\right)$ are superimposed, the amplitude of the resultant wave is **(2010)**

40. A 20 cm long string, having a mass of 1.0 g, is fixed at both the ends. The tension in the string is 0.5 N. The string is set into vibration using an external vibrator of frequency 100 Hz. Find the separation (in cm) between the successive nodes on the string. **(2009)**

Analytical & Descriptive Questions

41. A string of mass per unit length μ is clamped at both ends such that one end of the string is at $x = 0$ and the other is at $x = l$. When string vibrates in fundamental mode amplitude of the mid-point O of the string is a, and tension in the string is T. Find the total oscillation energy stored in the string. **(2003, 4M)**

42. Two narrow cylindrical pipes A and B have the same length. Pipe A is open at both ends and is filled with a monoatomic gas of molar mass M_A. Pipe B is open at one end and closed at the other end, and is filled with a diatomic gas of molar mass M_B. Both gases are at the same temperature. **(2002, 5M)**
(a) If the frequency to the second harmonic of pipe A is equal to the frequency of the third harmonic of the fundamental mode in pipe B, determine the value of M_A/M_B.
(b) Now the open end of the pipe B is closed (so that the pipe is closed at both ends). Find the ratio of the fundamental frequency in pipe A to that in pipe B.

43. A 3.6 m long pipe resonates with a source of frequency 212.5 Hz when water level is at certain heights in the pipe. Find the heights of water level (from the bottom of the pipe) at which resonances occur. Neglect end correction . Now the pipe is filled to a height H ($\approx$ 3.6m). A small hole is drilled very close to its bottom and water is allowed to leak. Obtain an expression for the rate of fall of water level in the pipe as a function of H. If the radii of the pipe and the hole are 2×10^{-2} m and 1×10^{-3} m respectively. Calculate the time interval between the occurrence of first two resonances. Speed of sound in air is 340 m/s and $g = 10 m/s^2$. **(2000, 10M)**

44. The air column in a pipe closed at one end is made to vibrate in its second overtone by tuning fork of frequency 440 Hz. The speed of sound in air is 330 m/s. End corrections may be neglected. Let p_0 denote the mean pressure at any point in the pipe and Δp_0 the maximum amplitude of pressure variation. **(1998, 8M)**
(a) Find the length L of the air column.
(b) What is the amplitude of pressure variation at the middle of the column?
(c) What are the maximum and minimum pressures at the open end of the pipe?
(d) What are the maximum and minimum pressures at the closed end of the pipe?

45. A metallic rod of length 1 m is rigidly clamped at its mid-point. Longitudinal stationary waves are set-up in the rod in such a way that there are two nodes on either side of the mid-point. The amplitude of an antinode is 2×10^{-6} m. Write the equation of motion at a point 2 cm from the mid-point and those of the constituent waves in the rod. (Young's modulus of the material of the rod $= 2\times10^{11}\ \text{Nm}^{-2}$; density $= 8000\ \text{kg m}^{-3}$) **(1994, 6M)**

46. The vibrations of a string of length 60 cm fixed at both ends are represented by the equation

$$y = 4\sin\left(\frac{\pi x}{15}\right)\cos(96\pi t)$$

where, x and y are in cm and t is in second. **(1985,6M)**

(a) What is the maximum displacement of a point at $x = 5$ cm?

(b) Where are the nodes located along the string?

(c) What is the velocity of the particle at $x = 7.5$ cm at $t = 0.25$ s?

(d) Write down the equations of the component waves whose superposition gives the above wave.

Topic 3 Wave Speed

Objective Questions I (Only one correct option)

1. The pressure wave $p = 0.01\sin[1000t - 3x]\text{Nm}^{-2}$, corresponds to the sound produced by a vibrating blade on a day when atmospheric temperature is 0°C. On some other day when temperature is T, the speed of sound produced by the same blade and at the same frequency is found to be $336\ \text{ms}^{-1}$. Approximate value of T is **(2019 Main, 9 April I)**

(a) 15°C (b) 11°C

(c) 12°C (d) 4°C

2. A travelling harmonic wave is represented by the equation $y(x,t) = 10^{-3}\sin(50t + 2x)$, where x and y are in metre and t is in second. Which of the following is a correct statement about the wave? **(2019 Main, 12 Jan I)**

(a) The wave is propagating along the negative X-axis with speed $25\ \text{ms}^{-1}$.

(b) The wave is propagating along the positive X-axis with speed $25\ \text{ms}^{-1}$.

(c) The wave is propagating along the positive X-axis with speed $100\ \text{ms}^{-1}$.

(d) The wave is propagating along the negative X-axis with speed $100\ \text{ms}^{-1}$.

3. An observer is moving with half the speed of light towards a stationary microwave source emitting waves at frequency 10 GHz. What is the frequency of the microwave measured by the observer? (speed of light $= 3\times10^8\ \text{ms}^{-1}$) **(2017 Main)**

(a) 12.1 GHz (b) 17.3 GHz

(c) 15.3 GHz (d) 10.1 GHz

4. A source of sound of frequency 600 Hz is placed inside water. The speed of sound in water is 1500 m/s and in air it is 300 m/s. The frequency of sound recorded by an observer who is standing in air is **(2004, 2M)**

(a) 200 Hz (b) 3000 Hz

(c) 120 Hz (d) 600 Hz

5. Two monoatomic ideal gases 1 and 2 of molecular masses m_1 and m_2 respectively are enclosed in separate containers kept at the same temperature. The ratio of the speed of sound in gas 1 to the gas 2 is given by **(2000, 2M)**

(a) $\sqrt{\frac{m_1}{m_2}}$ (b) $\sqrt{\frac{m_2}{m_1}}$ (c) $\frac{m_1}{m_2}$ (d) $\frac{m_2}{m_1}$

6. The ratio of the speed of sound in nitrogen gas to that in helium gas, at 300 K is **(1999, 2M)**

(a) $\sqrt{(2/7)}$ (b) $\sqrt{(1/7)}$ (c) $(\sqrt{3})/5$ (d) $(\sqrt{6})/5$

True / False

7. The ratio of the velocity of sound in hydrogen gas $\left(\gamma = \frac{7}{5}\right)$ to that in helium gas $\left(\gamma = \frac{5}{3}\right)$ at the same temperature is $\sqrt{21/5}$. **(1983, 2M)**

Analytical & Descriptive Questions

8. In a resonance tube experiment to determine the speed of sound in air, a pipe of diameter 5 cm is used. The air column in pipe resonates with a tuning fork of frequency 480 Hz when the minimum length of the air column is 16 cm. Find the speed of sound in air at room temperature. **(2003, 2M)**

9. A long wire PQR is made by joining two wires PQ and QR of equal radii. PQ has length 4.8 m and mass 0.06 kg. QR has length 2.56 m and mass 0.2 kg. The wire PQR is under a tension of 80 N. A sinusoidal wave pulse of amplitude 3.5 cm is sent along the wire PQ from the end P. No power is dissipated during the propagation of the wave pulse. Calculate **(1999, 10M)**

(a) the time taken by the wave pulse to reach the other end R and

(b) the amplitude of the reflected and transmitted wave pulse after the incident wave pulse crosses the joint Q.

10. A steel wire of length 1 m, mass 0.1 kg and uniform cross-sectional area $10^{-6}\,m^2$ is rigidly fixed at both ends. The temperature of the wire is lowered by 20°C. If transverse waves are set-up by plucking the string in the middle, calculate the frequency of the fundamental mode of vibration. **(1984, 6M)**

Given : $Y_{steel} = 2 \times 10^{11}\,N/m^2$,

$\alpha_{steel} = 1.21 \times 10^{-5}/°C$.

11. A uniform rope of length 12 m and mass 6 kg hangs vertically from a rigid support. A block of mass 2 kg is attached to the free end of the rope. A transverse pulse of wavelength 0.06 m is produced at the lower end of the rope. What is the wavelength of the pulse when it reaches the top of the rope ? **(1984, 6M)**

12. A copper wire is held at the two ends by rigid supports. At 30°C, the wire is just taut, with negligible tension. Find the speed of transverse waves in this wire at 10°C.

Given, Young modulus of copper $= 1.3 \times 10^{11}$ N/m^2.

Coefficient of linear expansion of copper $= 1.7 \times 10^{-5}\,°C^{-1}$.

Density of copper $= 9 \times 10^3$ kg/m^3. **(1979, 4M)**

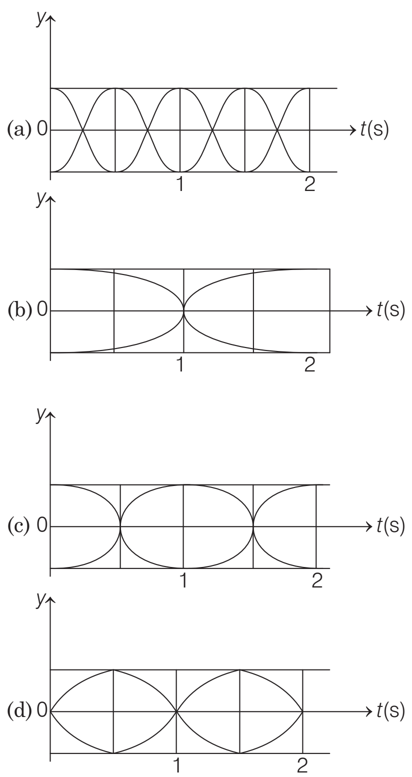
Topic 4 Beats and Doppler Effect

Objective Questions I (Only one correct option)

1. Two sources of sound S_1 and S_2 produce sound waves of same frequency 660 Hz. A listener is moving from source S_1 towards S_2 with a constant speed u m/s and he hears 10 beats/s. The velocity of sound is 330 m/s. Then, u equal to **(2019 Main, 12 April II)**

(a) 5.5 m/s (b) 15.0 m/s (c) 2.5 m/s (d) 10.0 m/s

2. The correct figure that shows schematically, the wave pattern produced by superposition of two waves of frequencies 9Hz and 11 Hz, is **(2019 Main, 10 April II)**



3. A source of sound S is moving with a velocity of 50 m/s towards a stationary observer. The observer measures the frequency of the source as 1000 Hz. What will be the apparent frequency of the source when it is moving away from the observer after crossing him? (Take, velocity of sound in air is 350 m/s) **(2019 Main, 10 April II)**

(a) 807 Hz (b) 1143 Hz (c) 750 Hz (d) 857 Hz

4. A stationary source emits sound waves of frequency 500 Hz. Two observers moving along a line passing through the source detect sound to be of frequencies 480 Hz and 530 Hz. Their respective speeds are in ms^{-1},

(Take, speed of sound = 300 m/s) **(2019 Main, 10 April I)**

(a) 12, 16 (b) 12, 18 (c) 16, 14 (d) 8, 18

5. Two cars A and B are moving away from each other in opposite directions. Both the cars are moving with a speed of 20 ms^{-1} with respect to the ground. If an observer in car A detects a frequency 2000 Hz of the sound coming from car B, what is the natural frequency of the sound source in car B ?

(speed of sound in air = 340 ms^{-1}) **(2019 Main, 9 April II)**

(a) 2060 Hz (b) 2250 Hz (c) 2300 Hz (d) 2150 Hz

6. A train moves towards a stationary observer with speed 34 m/s. The train sounds a whistle and its frequency registered by the observer is f_1. If the speed of the train is reduced to 17 m/s, the frequency registered is f_2. If speed of sound is 340 m/s, then the ratio $\frac{f_1}{f_2}$ is **(2019 Main, 10 Jan I)**

(a) $\frac{19}{18}$ (b) $\frac{21}{20}$ (c) $\frac{20}{19}$ (d) $\frac{18}{17}$

7. A musician produce the sound of second harmonics from open end flute of 50 cm. The other person moves toward the musician with speed 10 km/h from the second end of room. If the speed of sound 330 m/s, the frequency heard by running person will be **(2019 Main, 9 Jan II)**

(a) 666 Hz (b) 500 Hz (c) 753 Hz (d) 333 Hz

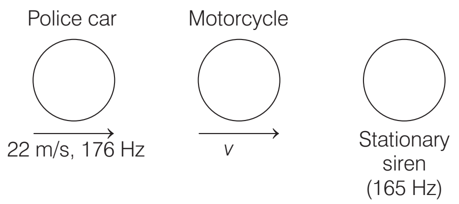
8. A train is moving on a straight track with speed 20 ms^{-1}. It is blowing its whistle at the frequency of 1000 Hz. The percentage change in the frequency heard by a person standing near the track as the train passes him is close to (speed of sound = 320 ms^{-1}) **(2015 Main)**

(a) 12% (b) 6% (c) 18% (d) 24%

9. A police car with a siren of frequency 8 kHz is moving with uniform velocity 36 km/h towards a tall building which reflects the sound waves. The speed of sound in air is 320 m/s. The frequency of the siren heard by the car driver is
(a) 8.50 kHz (b) 8.25 kHz **(2011)**
(c) 7.75 kHz (d) 7.50 kHz

10. A vibrating string of certain length l under a tension T resonates with a mode corresponding to the first overtone (third harmonic) of an air column of length 75 cm inside a tube closed at one end. The string also generates 4 beats/s when excited along with a tuning fork of frequency n. Now when the tension of the string is slightly increased the number of beats reduces to 2 per second. Assuming the velocity of sound in air to be 340 m/s, the frequency n of the tuning fork in Hz is **(2008, 3M)**
(a) 344 (b) 336
(c) 117.3 (d) 109.3

11. A police car moving at 22 m/s chases a motorcyclist. The police man sounds his horn at 176 Hz, while both of them move towards a stationary siren of frequency 165 Hz. Calculate the speed of the motorcycle, if it is given that the motorcyclist does not observe any beats (speed of sound = 330 m/s). **(2003, 2M)**



(a) 33 m/s (b) 22 m/s (c) Zero (d) 11 m/s

12. A siren placed at a railway platform is emitting sound of frequency 5 kHz. A passenger sitting in a moving train A records a frequency of 5.5 kHz, while the train approaches the siren. During his return journey in a different train B he records a frequency of 6.0 kHz while approaching the same siren. The ratio of the velocity of train B to that of train A is
(a) 242/252 (b) 2 **(2002, 2M)**
(c) 5/6 (d) 11/6

13. A train moves towards a stationary observer with speed 34 m/s. The train sounds a whistle and its frequency registered by the observer is f_1. If the train's speed is reduced to 17 m/s, the frequency registered is f_2. If the speed of sound is 340 m/s, then the ratio f_1/f_2 is **(2000, 2M)**
(a) 18/19 (b) 1/2 (c) 2 (d) 19/18

14. A whistle giving out 450 Hz approaches a stationary observer at a speed of 33 m/s. The frequency heard by the observer (in Hz) is (Speed of sound = 330 m/s) **(1997, 1M)**
(a) 409 (b) 429 (c) 517 (d) 500

Objective Questions II (One or more correct option)

15. Two vehicles, each moving with speed u on the same horizontal straight road, are approaching each other. Wind blows along the road with velocity w. One of these vehicles blows a whistle of frequency f_1. An observer in the other vehicle hears the frequency of the whistle to be f_2. The speed of sound in still air is v. The correct statement(s) is (are)
(a) If the wind blows from the observer to the source, $f_2 > f_1$ **(2013 Adv.)**
(b) If the wind blows from the source to the observer, $f_2 > f_1$
(c) If the wind blows from the observer to the source, $f_2 < f_1$
(d) If the wind blows from the source to the observer, $f_2 < f_1$

16. A sound wave of frequency f travels horizontally to the right. It is reflected from a large vertical plane surface moving to left with a speed v. The speed of sound in medium is c **(1995, 2M)**
(a) The number of waves striking the surface per second is $f\dfrac{(c+v)}{c}$
(b) The wavelength of reflected wave is $\dfrac{c\,(c-v)}{f\,(c+v)}$
(c) The frequency of the reflected wave is $f\dfrac{(c+v)}{(c-v)}$
(d) The number of beats heard by a stationary listener to the left of the reflecting surface is $\dfrac{vf}{c-v}$

17. Two identical straight wires are stretched so as to produce 6 beats/s when vibrating simultaneously. On changing the tension slightly in one of them, the beat frequency remains unchanged. Denoting by T_1, T_2 the higher and the lower initial tension in the strings, then it could be said that while making the above changes in tension **(1991, 2M)**
(a) T_2 was decreased (b) T_2 was increased
(c) T_1 was decreased (d) T_1 was increased

Numerical Value

18. Two men are walking along a horizontal straight line in the same direction. The main in front walks at a speed $1.0\,\text{ms}^{-1}$ and the man behind walks at a speed $2.0\,\text{ms}^{-1}$. A third man is standing at a height 12 m above the same horizontal line such that all three men are in a vertical plane. The two walking men are blowing identical whistles which emit a sound of frequency 1430 Hz. The speed of sound in air $330\,\text{ms}^{-1}$. At the instant, when the moving men are 10 m apart, the stationary man is equidistant from them. The frequency of beats in Hz, heard by the stationary man at this instant, is .... **(2018 Adv.)**

Fill in the Blank

19. A bus is moving towards a huge wall with a velocity of $5\ ms^{-1}$. The driver sounds a horn of frequency 200 Hz. The frequency of the beats heard by a passenger of the bus will be …… Hz. (Speed of sound in air = $342\ ms^{-1}$) **(1992, 2M)**

True / False

20. A source of sound with frequency 256 Hz is moving with a velocity v towards a wall and an observer is stationary between the source and the wall. When the observer is between the source and the wall he will hear beats. **(1985, 3M)**

Integer Answer Type Questions

21. A stationary source emits sound of frequency $f_0 = 492$ Hz. The sound is reflected by a large car approaching the source with a speed of $2\,ms^{-1}$. The reflected signal is received by the source and superposed with the original. What will be the beat frequency of the resulting signal in Hz? (Given that the speed of sound in air is 330 ms^{-1} and the car reflects the sound at the frequency it has received). **(2017 Adv.)**

22. Four harmonic waves of equal frequencies and equal intensities I_0 have phase angles 0, $\frac{\pi}{3}$, $\frac{2\pi}{3}$ and π. When they are superposed, the intensity of the resulting wave is nI_0. The value of n is **(2015 Adv.)**

Analytical & Descriptive Questions

23. An observer standing on a railway crossing receives frequency of 2.2 kHz and 1.8 kHz when the train approaches and recedes from the observer. Find the velocity of the train. **(2005, 2M)**

(The speed of the sound in air is 300 m/s.)

24. A boat is travelling in a river with a speed 10 m/s along the stream flowing with a speed 2 m/s. From this boat a sound transmitter is lowered into the river through a rigid support. The wavelength of the sound emitted from the transmitter inside the water is 14.45 mm. Assume that attenuation of sound in water and air is negligible. **(2001, 10M)**

(a) What will be the frequency detected by a receiver kept inside the river downstream?

(b) The transmitter and the receiver are now pulled up into air. The air is blowing with a speed 5 m/s in the direction opposite to the river stream. Determine the frequency of the sound detected by the receiver.

(Temperature of the air and water $= 20^\circ$ C ; Density of river water $= 10^3$ kg/m^3 ;

Bulk modulus of the water $= 2.088 \times 10^9$ Pa ;

Gas constant $R = 8.31$ J/mol-K ;

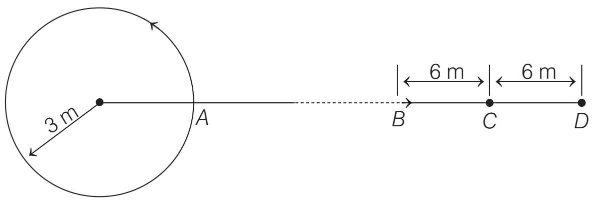
Mean molecular mass of air $= 28.8 \times 10^{-3}$ kg/mol ;

C_p/C_V for air = 1.4)

25. A band playing music at a frequency f is moving towards a wall at a speed v_b. A motorist is following the band with a speed v_m. If v is the speed of sound. Obtain an expression for the beat frequency heard by the motorist. **(1997, 5M)**

26. A whistle emitting a sound of frequency 440 Hz is tied to a string of 1.5 m length and rotated with an angular velocity of 20 rad/s in the horizontal plane. Calculate the range of frequencies heard by an observer stationed at a large distance from the whistle. (Speed of sound = 330 m/s). **(1996, 3M)**

27. A source of sound is moving along a circular path of radius 3 m with an angular velocity of 10 rad/s. A sound detector located far away from the source is executing linear simple harmonic motion along the line BD (see figure) with an amplitude $BC = CD = 6$ m. The frequency of oscillation of the detector is $5/\pi$ per second. The source is at the point A when the detector is at the point B. If the source emits a continuous sound wave of frequency 340 Hz, find the maximum and the minimum frequencies recorded by the detector. (Speed of sound = 340 m/s) **(1990, 7M)**



28. Two tuning forks with natural frequencies of 340 Hz each move relative to a stationary observer. One fork moves away from the observer, while the other moves towards him at the same speed. The observer hears beats of frequency 3 Hz. Find the speed of the tuning fork. Speed of sound = 340 m/s. **(1986, 8M)**

29. A sonometer wire under tension of 64 N vibrating in its fundamental mode is in resonance with a vibrating tuning fork. The vibrating portion of the sonometer wire has a length of 10 cm and mass of 1 g. The vibrating tuning fork is now moved away from the vibrating wire with a constant speed and an observer standing near the sonometer hears one beat per second. Calculate the speed with which the tuning fork is moved, if the speed of sound in air is 300 m/s. **(1983, 6M)**

30. A string 25 cm long and having a mass of 2.5 g is under tension . A pipe closed at one end is 40 cm long. When the string is set vibrating in its first overtone and the air in the pipe in its fundamental frequency, 8 beats/s are heard. It is observed that decreasing the tension in the string decreases the beat frequency. If the speed of sound in air is 320 m/s find the tension in the string. **(1982, 7M)**

31. A source of sound of frequency 256 Hz is moving rapidly towards a wall with a velocity of 5 m/ s. How many beats per second will be heard by the observer on source itself if sound travels at a speed of 330 m/s ? **(1981, 4M)**

Topic 5 Miscellaneous Problems

Objective Questions I (Only one correct option)

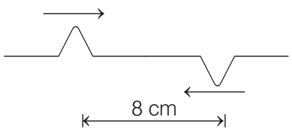
1. A small speaker delivers 2 W of audio output. At what distance from the speaker will one detect 120 dB intensity sound? [Take, reference intensity of sound as 10^{-12} W/m^2 **(2019 Main, 12 April II)]**

(a) 40 cm (b) 20 cm (c) 10 cm (d) 30 cm

2. In the experiment for the determination of the speed of sound in air using the resonance column method, the length of the air column that resonates in the fundamental mode, with a tuning fork is 0.1 m. When this length is changed to 0.35 m, the same tuning fork resonates with the first overtone. Calculate the end correction. **(2003, 2M)**

(a) 0.012 m (b) 0.025 m (c) 0.05 m (d) 0.024 m

3. Two pulses in a stretched string, whose centres are initially 8 cm apart, are moving towards each other as shown in the figure. The speed of each pulse is 2 cm/s. After 2 s the total energy of the pulses will be **(2001, 2M)**



(a) zero
(b) purely kinetic
(c) purely potential
(d) partly kinetic and partly potential

4. The ends of a stretched wire of length L are fixed at $x = 0$ and $x = L$. In one experiment the displacement of the wire is $y_1 = A \sin\left(\frac{\pi x}{L}\right) \sin \omega t$ and energy is E_1 and in other experiment its displacement is $y_2 = A \sin\left(\frac{2\pi x}{L}\right) \sin 2\omega t$ and energy is E_2. Then **(2001, 1M)**

(a) $E_2 = E_1$ (b) $E_2 = 2E_1$
(c) $E_2 = 4E_1$ (d) $E_2 = 16E_1$

5. A string of length 0.4 m and mass 10^{-2} kg is tightly clamped at its ends. The tension in the string is 1.6 N. Identical wave pulses are produced at one end at equal intervals of time Δt. The minimum value of Δt, which allows constructive interference between successive pulses, is **(1998, 2M)**

(a) 0.05 s (b) 0.10 s (c) 0.20 s (d) 0. 40 s

6. The extension in a string, obeying Hooke's law, is x. The speed of transverse wave in the stretched string is v. If the extension in the string is increased to 1.5 x, the speed of transverse wave will be **(1996, 2M)**

(a) 1.22 v (b) 0.61 v (c) 1.50 v (d) 0.75 v

7. The displacement y of a particle executing periodic motion is given by

$$y = 4\cos^2\left(\frac{1}{2}t\right)\sin(1000t)$$

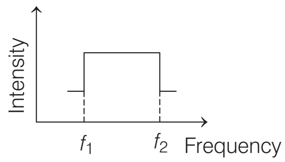
This expression may be considered to be a result of the superposition of .... independent harmonic motions. **(1992, 2M)**

(a) two (b) three
(c) four (d) five

Passage Based Questions

Passage 1

Two trains A and B are moving with speeds 20 m/s and 30 m/s respectively in the same direction on the same straight track, with B ahead of A. The engines are at the front ends. The engine of train A blows a long whistle.

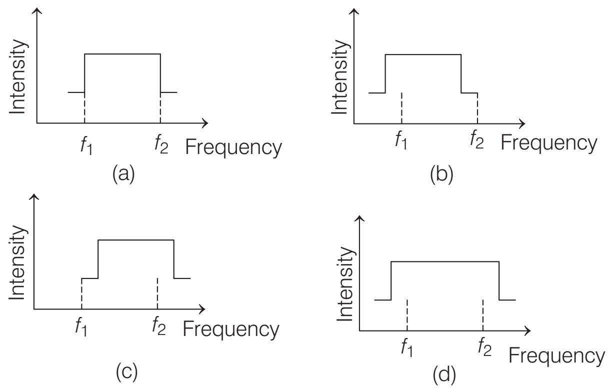


Assume that the sound of the whistle is composed of components varying in frequency from $f_1 = 800$ Hz to $f_2 = 1120$ Hz, as shown in the figure. The spread in the frequency (highest frequency–lowest frequency) is thus 320 Hz. The speed of sound in air is 340 m/s.

8. The speed of sound of the whistle is **(2007, 4M)**

(a) 340 m/s for passengers in A and 310 m/s for passengers in B
(b) 360 m/s for passengers in A and 310 m/s for passengers in B
(c) 310 m/s for passengers in A and 360 m/s for passengers in B
(d) 340 m/s for passengers in both the trains

9. The distribution of the sound intensity of the whistle as observed by the passengers in train A is best represented by **(2007, 4M)**



10. The spread of frequency as observed by the passengers in train B is **(2007, 4M)**

(a) 310 Hz (b) 330 Hz
(c) 350 Hz (d) 290 Hz

Passage 2

Two plane harmonic sound waves are expressed by the equations.

$$y_1(x, t) = A\cos(0.5\pi x - 100\pi t)$$
$$y_2(x, t) = A\cos(0.46\pi x - 92\pi t)$$

(All parameters are in MKS)

11. How many times does an observer hear maximum intensity in one second ? **(2006, 5M)**
(a) 4 (b) 10 (c) 6 (d) 8

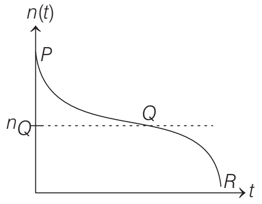
12. What is the speed of the sound ? **(2006, 5M)**
(a) 200 m/s (b) 180 m/s (c) 192 m/s (d) 96 m/s

13. At $x = 0$ how many times the amplitude of $y_1 + y_2$ is zero in one second ? **(2006, 5M)**
(a) 192 (b) 48 (c) 100 (d) 96

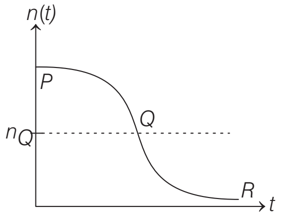
Objective Questions II (One or more correct option)

14. Two loudspeakers M and N are located 20m apart and emit sound at frequencies 118 Hz and 121 Hz, respectively. A car in initially at a point P, 1800 m away from the mid-point Q of the line MN and moves towards Q constantly at 60 km/h along the perpendicular bisector of MN. It crosses Q and eventually reaches a point R, 1800 m away from Q. Let $v(t)$ represent the beat frequency measured by a person sitting in the car at time t. Let ν_P, ν_Q and ν_R be the beat frequencies measured at locations P, Q and R respectively. The speed of sound in air is 330 ms^{-1}. Which of the following statement(s) is (are) true regarding the sound heard by the person? **(2016 Adv.)**

(a) The plot below represents schematically the variation of beat frequency with time



(b) The rate of change in beat frequency is maximum when the car passes through Q
(c) $\nu_P + \nu_R = 2\nu_Q$
(d) The plot below represents schematically the variations of beat frequency with time



15. A student is performing an experiment using a resonance column and a tuning fork of frequency 244 s^{-1}. He is told that the air in the tube has been replaced by another gas (assume that the column remains filled with the gas). If the minimum height at which resonance occurs is (0.350 ± 0.005) m, the gas in the tube is **(2014 Adv.)**

(**Useful information** : $\sqrt{167RT} = 640\ J^{1/2}\ mole^{-1/2}$; $\sqrt{140RT} = 590\ J^{1/2}\ mole^{-1/2}$. The molar masses M in grams are given in the options. Take the value of $\sqrt{10/M}$ for each gas as given there.)
(a) Neon ($M = 20$, $\sqrt{10/20} = 7/10$)
(b) Nitrogen ($M = 28$, $\sqrt{10/28} = 3/5$)
(c) Oxygen ($M = 32$, $\sqrt{10/32} = 9/16$)
(d) Argon ($M = 36$, $\sqrt{10/36} = 17/32$)

16. A person blows into open-end of a long pipe. As a result, a high-pressure pulse of air travels down the pipe. When this pulse reaches the other end of the pipe, **(2012)**
(a) a high-pressure pulse starts travelling up the pipe, if the other end of the pipe is open.
(b) a low-pressure pulse starts travelling up the pipe, if the other end of the pipe is open.
(c) a low-pressure pulse starts travelling up the pipe, if the other end of the pipe is closed.
(d) a high-pressure pulse starts travelling up the pipe, if the other end of the pipe is closed.

17. A student performed the experiment to measure the speed of sound in air using resonance air-column method. Two resonances in the air-column were obtained by lowering the water level. The resonance with the shorter air-column is the first resonance and that with the longer air column is the second resonance. Then, **(2009)**
(a) the intensity of the sound heard at the first resonance was more than that at the second resonance.
(b) the prongs of the tuning fork were kept in a horizontal plane above the resonance tube.
(c) the amplitude of vibration of the ends of the prongs is typically around 1 cm.
(d) the length of the air-column at the first resonance was somewhat shorter than 1/4th of the wavelength of the sound in air.

18. In a wave motion $y = a\sin(kx - \omega t)$, y can represent **(1999, 3M)**
(a) electric field (b) magnetic field
(c) displacement (d) pressure

19. As a wave propagates **(1999, 3M)**
(a) the wave intensity remains constant for a plane wave
(b) the wave intensity decreases as the inverse of the distance from the source for a spherical wave
(c) the wave intensity decreases as the inverse square of the distance from the source for a spherical wave
(d) total intensity of the spherical wave over the spherical surface centred at the source remains constant at all times

20. The displacement of particles in a string stretched in the x-direction is represented by y. Among the following expressions for y, those describing wave motion is (are) **(1987, 2M)**
(a) $\cos kx \sin \omega t$ (b) $k^2x^2 - \omega^2t^2$
(c) $\cos^2(kx + \omega t)$ (d) $\cos(k^2x^2 - \omega^2t^2)$

Fill in the Blank

21. A plane progressive wave of frequency 25 Hz, amplitude 2.5×10^{-5} m and initial phase zero propagates along the negative x-direction with a velocity of 300 m/s. At any instant, the phase difference between the oscillations at two points 6 m apart along the line of propagation is …… and the corresponding amplitude difference is …… m. **(1997, 1M)**

True / False

22. A plane wave of sound travelling in air is incident upon a plane water surface. The angle of incidence is 60°. Assuming Snell's law to be valid for sound waves, it follows that the sound wave will be refracted into water away from the normal. **(1984, 2M)**

Integer Answer Type Question

23. A stationary source is emitting sound at a fixed frequency f_0, which is reflected by two cars approaching the source. The difference between the frequencies of sound reflected from the cars is 1.2% of f_0. What is the difference in the speeds of the cars (in km per hour) to the nearest integer ? The cars are moving at constant speeds much smaller than the speed of sound which is 330 ms^{-1}.

(2010)

Analytical & Descriptive Questions

24. Two radio stations broadcast their programmes at the same amplitude A and at slightly different frequencies ω_1 and ω_2 respectively, where $\omega_1 - \omega_2 = 10^3$ Hz. A detector receives the signals from the two stations simultaneously. It can only detect signals of intensity $\geq 2A^2$. **(1993, 4M)**

(a) Find the time interval between successive maxima of the intensity of the signal received by the detector.

(b) Find the time for which the detector remains idle in each cycle of the intensity of the signal.

25. The displacement of the medium in a sound wave is given by the equation $y_i = A \cos(ax + bt)$ where A, a and b are positive constants. The wave is reflected by an obstacle situated a $x = 0$. The intensity of the reflected wave is 0.64 times that of the incident wave. **(1991, 8M)**

(a) What are the wavelength and frequency of incident wave ?

(b) Write the equation for the reflected wave.

(c) In the resultant wave formed after reflection, find the maximum and minimum values of the particle speeds in the medium.

(d) Express the resultant wave as a superposition of a standing wave and a travelling wave. What are the positions of the antinodes of the standing wave? What is the direction of propagation of travelling wave ?

26. A train approaching a hill at a speed of 40 km/h sounds a whistle of frequency 580 Hz when it is at a distance of 1 km from a hill. A wind with a speed of 40 km/h is blowing in the direction of motion of the train . Find **(1988, 5M)**

(a) the frequency of the whistle as heard by an observer on the hill,

(b) the distance from the hill at which the echo from the hill is heard by the driver and its frequency.

(Velocity of sound in air = 1200 km/h)

27. The following equations represent transverse waves;

$z_1 = A\cos(kx - \omega t)$; $z_2 = A\cos(kx + \omega t)$:

$z_3 = A\cos(ky - \omega t)$

Identify the combination (s) of the waves which will produce (a) standing wave (s), (b) a wave travelling in the direction making an angle of 45 degrees with the positive X and positive Y-axes. In each case, find the position at which the resultant intensity is always zero. **(1987, 7M)**

Answers

Topic 1

1. (b)	**2.** (d)	**3.** (d)	**4.** (b)
5. (c)	**6.** (a)	**7.** (a)	**8.** (b)
9. (a, c, d)	**10.** (b, d)	**11.** (a, c)	**12.** (b, c)
13. (a,b,c,d)	**14.** (*)	**15.** $2\pi\nu A$, $4\pi^2\nu^2 A$	

16. $y = (0.1\text{ m})\sin[(30\text{ rad/s})t \pm (1.5\text{ m}^{-1})x + \phi]$

Topic 2

1. (b)	**2.** (b)	**3.** (c)	**4.** (c)
5. (a)	**6.** (b)	**7.** (d)	**8.** (d)
9. (b)	**10.** (d)	**11.** (c)	**12.** (b)
13. (b)	**14.** (b)	**15.** (a)	**16.** (d)
17. (c)	**18.** (c)	**19.** (a)	**20.** (d)
21. (a)	**22.** (a)	**23.** (c)	**24.** (c)
25. (a)	**26.** (c)		

27. A $\to$ p, t; B $\to$ p, s; C $\to$ q, s; D $\to$ q, r

28. (a,c)	**29.** (a, c, d)	**30.** (b, c)	**31.** (a, b, c)
32. (b, c)	**33.** (a, b, c, d)	**34.** (a, b, d)	**35.** (a, c)
36. f	**37.** 240	**38.** 0.125	**39.** (5)

40. (5) **41.** $\dfrac{\pi^2 a^2 T}{4l}$ **42.** (a) $\dfrac{400}{189}$ (b) $\dfrac{3}{4}$

43. 3.2 m, 2.4 m, 1.6 m, 0.8 m, $-\dfrac{dH}{dt} = (1.11 \times 10^{-2})\sqrt{H}$, 43 s

44. (a) $\dfrac{15}{16}$ m (b) $\pm\dfrac{\Delta p_0}{\sqrt{2}}$ (c) equal to mean pressure

(d) $p_0 + \Delta p_0$, $p_0 - \Delta p_0$

45. $y = 2 \times 10^{-6}\sin(0.1\,\pi)\sin(25000\,\pi t)$,

$y_1 = 10^{-6}\sin(25000\,\pi t - 5\pi x)$, $y_2 = 10^{-6}\sin(25000\,\pi t + 5\pi x)$

46. (a) $2\sqrt{3}$ cm (b) $x = 0$, 15 cm, 30 cm… etc

(c) Zero (d) $y_1 = 2\sin\left(\dfrac{\pi x}{15} - 96\pi t\right)$

and $y_2 = 2\sin\left(\dfrac{\pi x}{15} + 96\pi t\right)$

Topic 3

1. (d)	**2.** (a)	**3.** (b)	**4.** (d)
5. (b)	**6.** (c)	**7.** F	**8.** 336 m/s

9. (a) 0.14 s (b) $A_r = 1.5$ cm and $A_t = 2.0$ cm

10. 11 Hz **11.** 0.12 m **12.** 70.1 m/s

Topic 4

1. (c) **2.** (a) **3.** (c) **4.** (b)
5. (b) **6.** (a) **7.** (a) **8.** (a)
9. (a) **10.** (a) **11.** (b) **12.** (b)
13. (d) **14.** (d) **15.** (a, b) **16.** (a, b, c)
17. (b, c) **18.** 5 **19.** 6 **20.** F
21. 6 **22.** 3 **23.** $v_T = 30\,\text{m/s}$

24. (a) 1.0069×10^5 Hz (b) 1.0304×10^5 Hz

25. $\dfrac{2v_b(v + v_m)}{(v^2 - v_b^2)}$ **26.** 403.3 Hz, 484 Hz

27. 438.7 Hz to 257.3 Hz **28.** 1.5 m/s

29. 0.75 m/s **30.** 27.04 N **31.** 7.87 Hz

Topic 5

1. (a) **2.** (b) **3.** (b) **4.** (c) **5.** (b)
6. (a) **7.** (b) **8.** (b) **9.** (a)
10. (a) **11.** (a) **12.** (a) **13.** (c)
14. (b, c, d) **15.** (d) **16.** (a, b, d) **17.** (a, d)
18. (a, b, c, d) **19.** (a, c, d) **20.** (a, c) **21.** π, zero
22. T **23.** 7

24. (a) 6.28×10^{-3}s (b) 1.57×10^{-3}s

25. (a) $\dfrac{2\pi}{a}, \dfrac{b}{2\pi}$ (b) $y_r = -0.8\,A\cos(ax - bt)$ (c) $1.8\,Ab$, zero

(d) $y = -1.6\,A\sin ax\sin bt + 0.2A\cos(ax + bt)$. Antinodes are at $x = \left[n + \dfrac{(-1)^n}{2}\right]\dfrac{\pi}{a}$. Travelling wave is propagating in negative x-direction.

26. (a) 599.33 Hz (b) 0.935 km, 621.43 Hz

27. (a) z_1 and z_2; $x = (2n+1)\dfrac{\pi}{2k}$ where $n = 0, \pm 1, \pm 2 \ldots$ etc. (b) z_1 and z_3, $x - y = (2n+1)\dfrac{\pi}{k}$ where $n = 0, \pm 1, \pm 2 \ldots$ etc.

Hints & Solutions

Topic 1 Wave Pulse and Travelling Wave

1. From the given snapshot at $t = 0$,

$y = 0$ at $x = 0$

and $y = -$ ve when x increases from zero.

Standard expression of any progressive wave is given by $y = A\sin(kx - \omega t + \phi)$

Here, ϕ is the phase difference, we need to get

at $t = 0$ $y = A\sin(kx + \phi)$

Clearly $\phi = \pi$, so that

$$y = A\sin(kx + \pi)$$

$$y = -A\sin(kx)$$

$\Rightarrow$ $y = 0$ at $x = 0$

and $y = -$ve at $x > 0$

2. Given, velocity of submarine (A),

$$v_A = 18\,\text{km/h} = \frac{18000}{3600}\,\text{m/s}$$

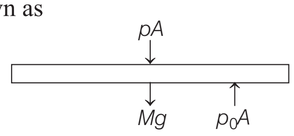
or $v_A = 5\,\text{m/s}$ …(i)

and velocity of submarine (B),

$$v_B = 27\,\text{km/h} = \frac{27000}{3600}\,\text{m/s}$$

or $v_B = 7.5\,\text{m/s}$ …(ii)

Signal sent by submarine (B) is detected by submarine (A) can be shown as



Frequency of the signal, $f_o = 500\,\text{Hz}$

So, in this relative motion, frequency received by submarine (A) is

$$f_1 = \left(\frac{v_S - v_A}{v_S - v_B}\right)f_o = \left(\frac{1500 - 5}{1500 - 7.5}\right)500\,\text{Hz}$$

$$\Rightarrow \quad f_1 = \frac{1495}{1492.5} \times 500\,\text{Hz}$$

The reflected frequency f_1 is now received back by submarine (B).

So, frequency received at submarine (B) is

$$f_2 = \left(\frac{v_S + v_B}{v_S + v_A}\right)f_1 = \left(\frac{1500 + 7.5}{1500 + 5}\right)\left(\frac{1495}{1492.5}\right)500\,\text{Hz}$$

$$\Rightarrow \quad f_2 = \left(\frac{1507.5}{1505}\right)\left(\frac{1495}{1492.5}\right)500\,\text{Hz}$$

$$\Rightarrow \quad f_2 = 1.00166 \times 1.00167 \times 500$$

$$\Rightarrow \quad f_2 = 501.67\,\text{Hz}$$

$$\approx 502\,\text{Hz}$$

3. Velocity 'v' of the wave on the string $= \sqrt{\dfrac{T}{\mu}}$

where, T = tension and μ = mass per unit length. Substituting the given values, we get

$$v = \sqrt{\frac{8}{5} \times 1000} = 40\,\text{ms}^{-1}$$

Wavelength of the wave on the string,

$$\lambda = \frac{v}{f}$$

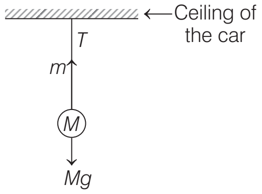
where, f = frequency of wave.

$$\Rightarrow \quad \lambda = \frac{40}{100}\text{ m} = 40\text{ cm}$$

$\therefore$ Separation between two successive nodes is,

$$d = \frac{\lambda}{2} = \frac{40}{2} = 20.0\text{ cm}$$

4. When the car is at rest, then the situation can be shown in the figure below.



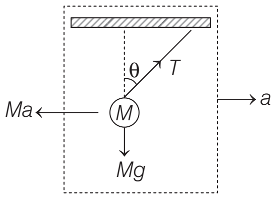
Since, $m << M$, then we can neglect the mass of the string.

So, the initial velocity of the wave in the string can be given as,

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{Mg}{\frac{M}{l}}} = 60\text{ m/s} \quad \text{...(i)}$$

where, T is the tension in the string and μ is the mass of the block per unit length (l).

Now, when the car has acceleration 'a', then the situation can be shown in the figure given below,



Resolving the components of 'T' along X-and Y-axis, we get

$$T\cos\theta = Mg \quad \text{...(ii)}$$

$$T\sin\theta = Ma \quad \text{...(iii)}$$

Squaring both sides of Eqs. (ii) and (iii) and adding them, we get

$$T^2(\sin^2\theta + \cos^2\theta) = M^2(g^2 + a^2)$$

$$\Rightarrow \quad T = M(g^2 + a^2)^{\frac{1}{2}} \quad \text{...(iv)}$$

Now, the velocity of the wave in the string would be given as,

$$v' = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{M(g^2 + a^2)^{\frac{1}{2}}}{\frac{M}{l}}} = 60.5\text{ m/s} \quad \text{...(v)}$$

Dividing Eq. (i) and Eq. (v), we get

$$\frac{v}{v'} = \frac{\sqrt{\frac{Mg}{\frac{M}{l}}}}{\sqrt{\frac{M(g^2 + a^2)^{\frac{1}{2}}}{\frac{M}{l}}}} = \sqrt{\frac{g}{(g^2 + a^2)^{\frac{1}{2}}}} = \frac{60}{60.5}$$

or $$\frac{60.5}{60} = \sqrt{\frac{(g^2 + a^2)^{\frac{1}{2}}}{g}}$$

Squaring both the sides, we get

$$\frac{(60.5)^2}{(60)^2} = \frac{(g^2 + a^2)^{\frac{1}{2}}}{g}$$

or $$\left(1 + \frac{0.5}{60}\right)^2 = \sqrt{1 + \left(\frac{a}{g}\right)^2}$$

Using binomial expansion,

$$(1 + x)^n = 1 + \frac{nx}{1!} + \dots$$

On both sides, we get

$$1 + 2 \times \frac{0.5}{60} = 1 + \frac{1}{2}\cdot\frac{a^2}{g^2}$$

$$\Rightarrow 1 + \frac{1}{60} = 1 + \frac{1}{2}\cdot\frac{a^2}{g^2} \quad \text{or} \quad \frac{a}{g} = \frac{1}{\sqrt{30}}$$

or $$a = \frac{g}{\sqrt{30}} = \frac{g}{5.4} \simeq \frac{g}{5}$$

5. At distance x from the bottom

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{\left(\frac{mgx}{L}\right)}{\left(\frac{m}{L}\right)}} = \sqrt{gx}$$

$$\therefore \quad \frac{dx}{dt} = \sqrt{x}\sqrt{g}$$

$$\Rightarrow \quad \int_0^L x^{-1/2}dx = \sqrt{g}\int_0^t dt$$

$$\Rightarrow \quad \left[\left.\frac{x^{1/2}}{(1/2)}\right|_0^L\right] = \sqrt{g}\cdot t$$

$$\Rightarrow \quad t = \frac{2\sqrt{L}}{\sqrt{g}}$$

$$\Rightarrow \quad t = 2\sqrt{\frac{20}{10}} = 2\sqrt{2}\text{s}$$

6. Particle velocity $v_p = -v$ (slope of y-x graph)

Here, $v = +$ ve, as the wave is travelling in positive x-direction.

Slope at P is negative.

$\therefore$ Velocity of particle is in positive y (or $\hat{\mathbf{j}}$) direction.

$\therefore$ Correct option is (a).

7. This is an equation of a travelling wave in which particles of the medium are in SHM and maximum particle velocity in SHM is $A\omega$, where A is the amplitude and ω the angular velocity.

8. Wave velocity $v = \dfrac{\text{coefficient of } t}{\text{coefficient of } x} = \dfrac{2\pi f}{2\pi/\lambda} = \lambda f$

Maximum particle velocity $v_{pm} = \omega A = 2\pi f y_0$

Given, $\quad v_{pm} = 4v$

or $$2\pi f\, y_0 = 4\lambda f$$

$\therefore$ $$\lambda = \frac{\pi y_0}{2}$$

9. $v = \sqrt{\dfrac{T}{\mu}}$, so speed at any position will be same for both pulses, therefore time taken by both pulses will be same.

$$\lambda f = v$$

$\Rightarrow$ $$\lambda = \frac{v}{f}$$

$\Rightarrow$ $$\lambda \propto v \propto T$$

since when pulse 1 reaches at A tension and hence speed decreases therefore λ decreases.

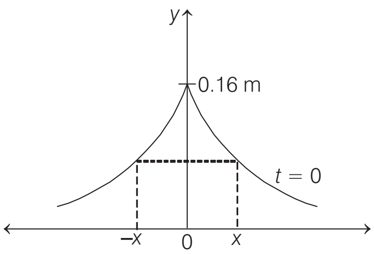
NOTE

If we refer velocity by magnitude only, then option (a, c, d) will be correct, else only (a, c) will be correct.

10. The shape of pulse at $x = 0$ and $t = 0$ would be as shown, in Fig. (a).

$$y(0,0) = \frac{0.8}{5} = 0.16\text{ m}$$

From the figure it is clear that $y_{max} = 0.16\text{ m}$



(a)

Pulse will be symmetric (Symmetry is checked about y_{max}) if at $t = 0$

$$y(x) = y(-x)$$

From the given equation,

and $$\left.\begin{aligned} y(x) &= \frac{0.8}{16x^2 + 5} \\ y(-x) &= \frac{0.8}{16x^2 + 5} \end{aligned}\right\} \text{ at } t = 0$$

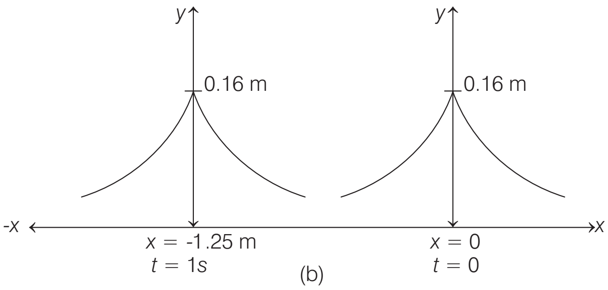
or $$y(x) = y(-x)$$

Therefore, pulse is symmetric.

Speed of pulse

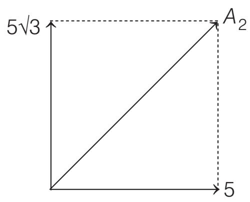
At $t = 1\text{ s}$ and $x = -1.25\text{ m}$

value of y is again 0.16 m, i.e. pulse has travelled a distance of 1.25 m in 1 s in negative x-direction or we can say that the speed of pulse is 1.25 m/s and it is travelling in negative x-direction. Therefore, it will travel a distance of 2.5 m in 2 s. The above statement can be better understood from Fig. (b).



(b)

11. Maximum speed of any point on the string $= a\omega = a(2\pi f)$



$\therefore$ $$= \frac{v}{10} = \frac{10}{10} = 1 \qquad (\text{Given, } v = 10\text{ m/s})$$

$\therefore$ $$2\pi a f = 1$$

$\Rightarrow$ $$f = \frac{1}{2\pi a}$$

$$a = 10^{-3}\text{ m} \qquad (\text{Given})$$

$\therefore$ $$f = \frac{1}{2\pi \times 10^{-3}} = \frac{10^3}{2\pi}\text{ Hz}$$

Speed of wave $v = f\lambda$

$\therefore$ $$(10\text{ m/s}) = \left(\frac{10^3}{2\pi}\text{s}^{-1}\right)\lambda$$

$\Rightarrow$ $\therefore$ $$\lambda = 2\pi \times 10^{-2}\text{ m}$$

12. $\omega = 15\pi$, $k = 10\pi$

Speed of wave, $v = \dfrac{\omega}{k} = 1.5\text{ m/s}$

Wavelength of wave $\lambda = \dfrac{2\pi}{k} = \dfrac{2\pi}{10\pi} = 0.2\text{ m}$

$10\pi x$ and $15\pi t$ have the same sign. Therefore, wave is travelling in negative x-direction.

$\therefore$ Correct options are (b) and (c).

13. $y = 10^{-4}\sin(60t + 2x)$

$A = 10^{-4}\text{ m}$, $\omega = 60\text{ rad/s}$, $k = 2\text{ m}^{-1}$

Speed of wave, $v = \dfrac{\omega}{k} = 30\text{ m/s}$

Frequency, $f = \dfrac{\omega}{2\pi} = \dfrac{30}{\pi}\text{ Hz.}$

Wavelength $\lambda = \dfrac{2\pi}{k} = \pi\text{ m}$

Further, 60 t and $2x$ are of same sign. Therefore, the wave should travel in negative x-direction.

$\therefore$ All the options are correct.

14. In the question, it is given that the shape of the wave disturbance does not change, while in the opinion of author this is not true.

From the function, $y = \dfrac{1}{(1+x)^2}$, we can see that, at $x = -1$, $y = \infty$.

Whereas from the second function, $y = \dfrac{1}{[1+(x-1)^2]}$ we don't get any point where $y = \infty$.

So, how can we say that shape has not changed.

15. (a) Particle velocity amplitude means maximum speed

$$= \omega A = 2\pi \nu A$$

(b) Particle acceleration amplitude

$$= \omega^2 A = 4\pi^2\nu^2 A$$

16. Maximum particle velocity,

$$\omega A = 3\text{ m/s} \qquad \ldots\text{(i)}$$

Maximum particle acceleration,

$$\omega^2 A = 90\text{ m/s}^2 \qquad \ldots\text{(ii)}$$

Velocity of wave, $\dfrac{\omega}{k} = 20\text{ m/s}$ ...(iii)

From Eqs. (i), (ii) and (iii), we get

$$\omega = 30\text{ rad/s} \Rightarrow A = 0.1\text{m} \quad \text{and} \quad k = 1.5\text{ m}^{-1}$$

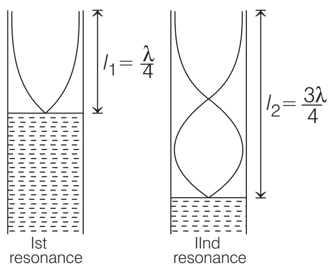
$\therefore$ Equation of waveform should be

$$y = A\sin(\omega t + kx + \phi)$$

$$y = (0.1\text{ m})\sin[(30\text{ rad/s})\,t \pm (1.5\text{m}^{-1})x + \phi]$$

Topic 2 Standing Waves, Stretched Wire and Organ Pipes

1. In a resonance tube apparatus, first and second resonance occur as shown



As in a stationary wave, distance between two successive nodes is $\dfrac{\lambda}{2}$ and distance of a node and an antinode is $\dfrac{\lambda}{4}$.

$$l_2 - l_1 = \frac{3\lambda}{4} - \frac{\lambda}{4} = \frac{\lambda}{2}$$

So, speed of sound, $v = f\lambda = f \times 2(l_2 - l_1)$

$$= 480 \times 2 \times (70 - 30) \times 10^{-2} = 384\text{ ms}^{-1}$$

2. Frequency of vibration of a string in nth harmonic is given by

$$f_n = n \cdot \frac{v}{2l} \qquad \ldots\text{(i)}$$

where, v = speed of sound and l = length of string.

Here, $f_3 = 240\text{ Hz}$, $l = 2$ m and $n = 3$

Substituting these values in Eq. (i), we get

$$\therefore \quad 240 = 3 \times \frac{v}{2\times 2} \Rightarrow v = \frac{4\times 240}{3} = 320\text{ ms}^{-1}$$

Also, fundamental frequency is

$$f = \frac{f_n}{n} = \frac{f_3}{3} = \frac{240}{3} = 80\text{ Hz}$$

3. Given equation of stationary wave is

$$Y = 0.3\sin(0.157x)\cos(200\pi t)$$

Comparing it with general equation of stationary wave, i.e. $Y = a\sin kx\cos\omega t$, we get

$$k = \left(\frac{2\pi}{\lambda}\right) = 0.157$$

$$\Rightarrow \quad \lambda = \frac{2\pi}{0.157} = 4\pi^2 \quad \left(\because \frac{1}{2\pi} \approx 0.157\right) \ldots\text{(i)}$$

$$\text{and} \quad \omega = 200\pi = \frac{2\pi}{T} \Rightarrow T = \frac{1}{20}\text{ s}$$

As the possible wavelength associated with nth harmonic of a vibrating string, i.e. fixed at both ends is given as

$$\lambda = \frac{2l}{n} \quad \text{or} \quad l = n\left(\frac{\lambda}{2}\right)$$

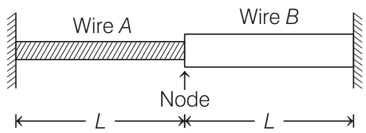
Now, according to question, string is fixed from both ends and oscillates in 4th harmonic, so

$$4\left(\frac{\lambda}{2}\right) = l \Rightarrow 2\lambda = l$$

or $l = 2 \times 4\pi^2 = 8\pi^2$ [using Eq. (i)]

Now, $\pi^2 \approx 10 \Rightarrow l \approx 80\text{ m}$

4.



Let mass per unit length of wires are μ_A and μ_B, respectively.

$\because$ For same material, density is also same.

So, $\mu_A = \dfrac{\rho\pi r^2 L}{L} = \mu$ and $\mu_B = \dfrac{\rho 4\pi r^2 L}{L} = 4\mu$

Tension (T) in both connected wires are same.

So, speed of wave in wires are

$$v_A = \sqrt{\frac{T}{\mu_A}} = \sqrt{\frac{T}{\mu}} \quad [\because \mu_A = \mu \text{ and } \mu_B = 4\mu]$$

$$\text{and} \quad v_B = \sqrt{\frac{T}{\mu_B}} = \sqrt{\frac{T}{4\mu}}$$

So, nth harmonic in such wires system is

$$f_{nth} = \frac{pv}{2L}$$

$$\Rightarrow f_A = \frac{pv_A}{2L} = \frac{p}{2L}\sqrt{\frac{T}{\mu}} \quad \text{(for } p \text{ antinodes)}$$

Similarly, $f_B = \frac{qv_B}{2L} = \frac{q}{2L}\sqrt{\frac{T}{4\mu}} = \frac{1}{2}\left(\frac{q}{2L}\sqrt{\frac{T}{\mu}}\right)$

(for q antinodes)

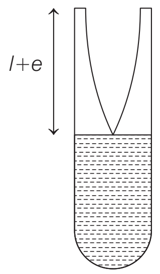
As frequencies f_A and f_B are given equal.

So, $f_A = f_B \Rightarrow \frac{p}{2L}\sqrt{\frac{T}{\mu}} = \frac{q}{2}\left[\frac{1}{2L}\sqrt{\frac{T}{\mu}}\right]$

$$\frac{p}{q} = \frac{1}{2} \Rightarrow p : q = 1 : 2$$

5. **Key idea** To overcome the error occured in measurement of resonant length.We introduce end correction factor e in length.

In first resonance, length of air coloumn $= \frac{\lambda}{4}$.



So, $l_1 + e = \frac{\lambda}{4}$ or $11 \times 4 + 4e = \lambda$

So, speed of sound is

$$\Rightarrow v = f_1\lambda = 512(44 + 4e) \quad \text{...(i)}$$

And in second case,

$$l_1' + e = \frac{\lambda'}{4} \text{ or } 27 \times 4 + 4e = \lambda'$$

$$\Rightarrow v = f_2\lambda' = 256\,(108 + 4e) \quad \text{...(ii)}$$

Dividing both Eqs. (i) and (ii), we get

$$1 = \frac{512\,(44 + 4e)}{256\,(108 + 4e)} \Rightarrow e = 5\,\text{cm}$$

Substituting value of e in Eq. (i), we get

Speed of sound $v = 512\,(44 + 4e)$

$= 512\,(44 + 4 \times 5)$

$= 512 \times 64\,\text{cm s}^{-1} = 327.68\,\text{ms}^{-1} \approx 328\,\text{ms}^{-1}$

6. Given, equation can be rewritten as,

$$y = 0.03\sin 450\left(t - \frac{9x}{450}\right) \quad \text{...(i)}$$

We know that the general equation of a travelling wave is given as,

$$y = A\sin\omega\,(t - x/v) \quad \text{...(ii)}$$

Comparing Eqs. (i) and (ii), we get

$$\text{velocity, } v = \frac{450}{9} = 50\,\text{m/s}$$

and angular velocity, $\omega = 450$ rad/s

As, the velocity of wave on stretched string with tension (T) is given as $v = \sqrt{T/\mu}$

where, μ is linear density

$\therefore \quad T = \mu v^2 = 5 \times 10^{-3} \times 50 \times 50 = 12.5\,\text{N}$

($\because$ given, $\mu = 5\,\text{g/m} = 5 \times 10^{-3}\,\text{kg/m}$)

7. Fundamental frequency of closed organ pipe is given by $f_0 = v/4L$, where v is the velocity of sound in it and L is the length of the pipe.

Also, overtone frequencies are given by

$$f = (2n+1)\frac{v}{4L} \quad \text{or} \quad f = (2n+1)\,f_0$$

Given that, $f_0 = 1500$ Hz and $f_{max} = 20000$ Hz

This means, $f_{max} > f$

So, $\quad f_{max} > (2n+1)\,f_0$

$\Rightarrow \quad 20000 > (2n+1)\,1500$

$\Rightarrow \quad 2n + 1 < 13.33 \Rightarrow 2n < 13.33 - 1$

$\Rightarrow \quad 2n < 12.33 \quad$ or $\quad n < 6.16$

or $n = 6$ (integer number)

Hence, total six overtones will be heard.

8. Velocity 'v' of the wave on the string $= \sqrt{\frac{T}{\mu}}$

where, T = tension and μ = mass per unit length.

Substituting the given values, we get

$$v = \sqrt{\frac{8}{5} \times 1000} = 40\,\text{ms}^{-1}$$

Wavelength of the wave on the string, $\lambda = \frac{v}{f}$

where, $\quad f$ = frequency of wave.

$\Rightarrow \quad \lambda = \frac{40}{100}\,\text{m} = 40\,\text{cm}$

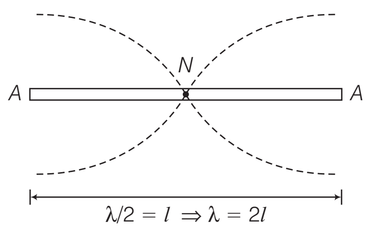
$\therefore$ Separation between two successive nodes is,

$$d = \frac{\lambda}{2} = \frac{40}{2} = 20.0\,\text{cm}.$$

9. Wave velocity $(v) = \sqrt{\frac{Y}{\rho}} = 5.86 \times 10^3$ m/s

For fundamental mode, $\lambda = 2l = 1.2$ m

$\therefore \quad$ Fundamental frequency $= \frac{v}{\lambda} = 4.88$ kHz ≈ 5 kHz



10. Fundamental frequency of open pipe.

$$f = \frac{v}{2l}$$

Now, after half filled with water it becomes a closed pipe of length $\frac{l}{2}$.

Fundamental frequency of this closed pipe,

$$f' = \frac{v}{4(l/2)} = \frac{v}{2l} = f$$

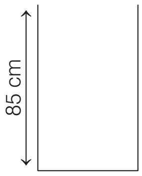
11. For closed organ pipe $= \frac{(2n+1)v}{4l}$ $[n = 0, 1, 2 ......]$

$$\frac{(2n+1)v}{4l} < 1250$$

$$(2n+1) < 1250 \times \frac{4 \times 0.85}{340}$$

$$(2n+1) < 12.5\ 2n < 11.50$$

$$n < 5.25$$

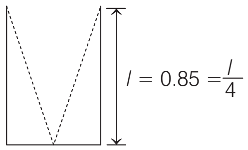


So, $\quad n = 0, 1, 2, 3, ..., 5$

So, we have 6 possibilities.

Alternate method

In closed organ pipe, fundamental node



i.e. $\quad \frac{\lambda}{4} = 0.85 \quad \Rightarrow \quad \lambda = 4 \times 0.85$

As we know, $\nu = \frac{c}{\lambda} \quad \Rightarrow \quad \frac{340}{4 \times 0.85} = 100\ \text{Hz}$

$\therefore$ Possible frequencies = 100 Hz, 300 Hz, 500 Hz, 700 Hz, 900 Hz, 1100 Hz below 1250 Hz.

12. Fundamental frequency of sonometer wire

$$f = \frac{v}{2l} = \frac{1}{2l}\sqrt{\frac{T}{\mu}} = \frac{1}{2l}\sqrt{\frac{T}{Ad}}$$

Here, μ = mass per unit length of wire.

Also, Young's modulus of elasticity $Y = \frac{Tl}{A\Delta l}$

$$\Rightarrow \frac{T}{A} = \frac{Y\Delta l}{l} \Rightarrow f = \frac{1}{2l}\sqrt{\frac{Y\Delta l}{ld}} \Rightarrow l = 1.5\ \text{m}, \frac{\Delta l}{l} = 0.01$$

$$d = 7.7 \times 10^3\ \text{kg/m}^3 \quad \Rightarrow \quad Y = 2.2 \times 10^{11}\ \text{N/m}^2$$

After substituting the values we get,

$$f \approx 178.2\ \text{Hz}$$

13. With end correction,

$$f = n\left[\frac{v}{4(l+e)}\right], \quad (\text{where, } n = 1, 3, \ldots)$$

$$= n\left[\frac{v}{4(l+0.6r)}\right]$$

Because, $e = 0.6\,r$, where r is radius of pipe.

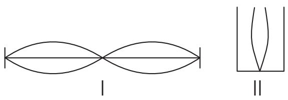
For first resonance, $n = 1$

$$\therefore \quad f = \frac{v}{4(l+0.6r)}$$

or $\quad l = \frac{v}{4f} - 0.6r = \left[\left(\frac{336 \times 100}{4 \times 512}\right) - 0.6 \times 2\right]\text{cm}$

$$= 15.2\ \text{cm}$$

14.



Given, 2nd harmonic of I = Fundamental of II

$$\therefore \quad 2\left(\frac{v_1}{2l_1}\right) = \frac{v_2}{4l_2} \quad \Rightarrow \quad \frac{T/\mu}{l_1} = \frac{v_2}{4l_2}$$

$$\Rightarrow \quad \mu = \frac{16\,T\,l_2^2}{v_2^2\,l_1^2} = \frac{16 \times 50 \times (0.8)^2}{(320)^2 \times (0.5)^2}$$

$$= 0.02\ \text{kg/m}$$

$$\therefore \quad m_1 = \mu\,l_1 = (0.02)(0.2) = 0.01\ \text{kg} = 10\,\text{g}$$

15. $f \propto v \propto \sqrt{T} \quad \Rightarrow \quad f_{AB} = 2f_{CD}$

$\therefore \quad T_{AB} = 4T_{CD}$ ...(i)

Further $\quad \Sigma\tau_p = 0$

$\therefore \quad T_{AB}(x) = T_{CD}\,(l - x)$

or $\quad 4x = l - x \quad$ (as $T_{AB} = 4T_{CD}$)

or $\quad x = l/5$

16. The question is incomplete, as speed of sound is not given. Let us assume speed of sound as 330 m/s. Then, method will be as under.

$$\frac{\lambda}{2} = (63.2 - 30.7)\ \text{cm} \quad \text{or} \quad \lambda = 0.65\,\text{m}$$

$\therefore$ Speed of sound observed,

$$v_0 = f\lambda = 512 \times 0.65 = 332.8\ \text{m/s}$$

$\therefore$ Error in calculating velocity of sound = 2.8 m/s

$$= 280\ \text{cm/s}$$

17. $\quad f_1 = \frac{v}{l} \quad$ (2nd harmonic of open pipe)

$f_2 = n\left(\frac{v}{4l}\right)$ (nth harmonic of closed pipe)

Here, n is odd and $f_2 > f_1$. It is possible when $n = 5$ because with $n = 5$

$$f_2 = \frac{5}{4}\left(\frac{v}{l}\right) = \frac{5}{4}\,f_1$$

18. $f_c = f_o$ (both first overtone)

or $\quad 3\left(\frac{v_c}{4L}\right) = 2\left(\frac{v_o}{2l_o}\right)$

$$\therefore \quad l_o = \frac{4}{3}\left(\frac{v_o}{v_c}\right)L = \frac{4}{3}\sqrt{\frac{\rho_1}{\rho_2}}\,L \qquad \text{as } v \propto \frac{1}{\sqrt{\rho}}$$

19. Let $f_0 =$ frequency of tuning fork

Then, $f_0 = \dfrac{5}{2l}\sqrt{\dfrac{9g}{\mu}}$ (μ = mass per unit length of wire)

$= \dfrac{3}{2l}\sqrt{\dfrac{Mg}{\mu}}$

Solving this, we get $M = 25$ kg

In the first case, frequency corresponds to fifth harmonic while in the second case it corresponds to third harmonic.

20. Fundamental frequency is given by

$$\nu = \frac{1}{2l}\sqrt{\frac{T}{\mu}} \quad \text{(with both the ends fixed)}$$

$\therefore$ Fundamental frequency

$$\nu \propto \frac{1}{l\sqrt{\mu}} \quad \text{(for same tension in both strings)}$$

where, $\mu =$ mass per unit length of wire

$= \rho . A$ (ρ = density)

$= \rho(\pi r^2)$ or $\sqrt{\mu} \propto r$

$$\therefore \quad \nu \propto \frac{1}{rl}$$

$$\therefore \quad \frac{\nu_1}{\nu_2} = \left(\frac{r_2}{r_1}\right)\left(\frac{l_2}{l_1}\right) = \left(\frac{r}{2r}\right)\left(\frac{2L}{L}\right) = 1$$

21. Length of the organ pipe is same in both the cases.

Fundamental frequency of open pipe is $f_1 = \dfrac{v}{2l}$ and

Frequency of third harmonic of closed pipe will be

$$f_2 = 3\left(\frac{v}{4l}\right)$$

Given that, $f_2 = f_1 + 100$

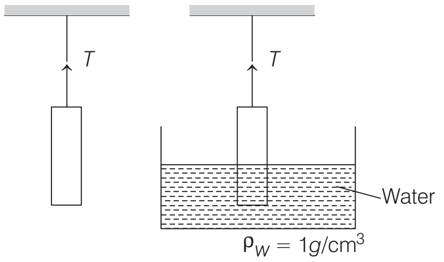
or $f_2 - f_1 = 100$ or $\dfrac{3}{4}\left(\dfrac{v}{l}\right) - \left(\dfrac{1}{2}\right)\left(\dfrac{v}{l}\right) = 100$

$\Rightarrow \dfrac{v}{4l} = 100\,\text{Hz} \quad \therefore \quad \dfrac{v}{2l}$ or $f_1 = 200\,\text{Hz}$

Therefore, fundamental frequency of the open pipe is 200 Hz.

22. The diagramatic representation of the given problem is shown in figure. The expression of fundamental frequency is

$$\nu = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$$



In air $T = mg = (V\rho)\,g$

$$\therefore \quad \nu = \frac{1}{2l}\sqrt{\frac{A\rho g}{\mu}} \quad \ldots(i)$$

When the object is half immersed in water

$$T' = mg - \text{upthrust} = V\rho g - \left(\frac{V}{2}\right)\rho_w g$$

$$= \left(\frac{V}{2}\right) g\,(2\rho - \rho_w)$$

The new fundamental frequency is

$$\nu' = \frac{1}{2l} \times \sqrt{\frac{T'}{\mu}} = \frac{1}{2l}\sqrt{\frac{(Vg/2)(2\rho - \rho_w)}{\mu}} \quad \ldots(ii)$$

$$\therefore \quad \frac{\nu'}{\nu} = \left(\sqrt{\frac{2\rho - \rho_w}{2\rho}}\right)$$

or $\nu' = \nu\left(\dfrac{2\rho - \rho_w}{2\rho}\right)^{1/2}$

$$= 300\left(\frac{2\rho - 1}{2\rho}\right)^{1/2}\ \text{Hz}$$

23. First harmonic of closed = Third harmonic of open

$$\therefore \quad \frac{v}{4l_1} = 3\left(\frac{v}{2l_2}\right) \Rightarrow \therefore \quad \frac{l_1}{l_2} = \frac{1}{6}$$

24. For a stationary wave to form, two identical waves should travel in opposite direction. Further at $x = 0$, resultant y (from both the waves) should be zero at all instants.

25. $f_c = \dfrac{v}{4l} = 512\,\text{Hz}$

$$f_o = \frac{v}{2l} = 2f_c = 1024\,\text{Hz}$$

26. Initially the tube was open at both ends and then it is closed.

$$f_o = \frac{v}{2l_o} \text{ and } f_c = \frac{v}{4l_c}$$

Since, tube is half dipped in water, $l_c = \dfrac{l_o}{2}$

$$\therefore \quad f_c = \frac{v}{4\left(\frac{l_o}{2}\right)} = \frac{v}{2l_o} = f_o = f$$

27. In organ pipes, longitudinal waves are formed.

In string, transverse waves are formed. Open end of pipe is displacement antinode and closed end is displacement node. In case of string fixed end of a string is node.

Further, least distance between a node and an antinode is $\dfrac{\lambda}{4}$ and between two nodes is $\dfrac{\lambda}{2}$. Keeping these points in mind answer to this question is as under ; (A) $\rightarrow$ (p, t); (B) $\rightarrow$ (p, s); (C) $\rightarrow$ (q, s); (D) $\rightarrow$ (q, r)

28. Let nth harmonic is corresponding to 50.7 cm and $(n+1)$th harmonic is corresponding 83.9 cm.

$\therefore$ Their difference is $\frac{\lambda}{2}$.

$$\therefore \quad \frac{\lambda}{2} = (83.9 - 50.7)\text{ cm}$$

$$\text{or} \quad \lambda = 66.4 \text{ cm}$$

$$\therefore \quad \frac{\lambda}{4} = 16.6\text{ cm}$$

Length corresponding to fundamental mode must be close to $\frac{\lambda}{4}$ and 50.7 cm must be an odd multiple of this length.

$16.6 \times 3 = 49.8$ cm. Therefore, 50.7 is 3rd harmonic.

If end correction is e, then

$$e + 50.7 = \frac{3\lambda}{4}$$

$$e = 49.8 - 50.7 = -0.9\text{ cm}$$

$\therefore$ Speed of sound, $v = f\lambda$

$$\Rightarrow \quad v = 500 \times 66.4 \text{ cm/s} = 332 \text{ m/s}$$

29. There should be a node at $x = 0$ and antinode at $x = 3$ m.

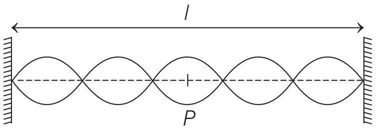
$$\text{Also,} \quad v = \frac{\omega}{k} = 100 \text{ m/s.}$$

$$\therefore \quad y = 0 \text{ at } x = 0$$

$$\text{and} \quad y = \pm A \text{ at } x = 3 \text{ m.}$$

Only (a), (c) and (d) satisfy the condition.

30.



Number of nodes = 6

From the given equation, we can see that

$$k = \frac{2\pi}{\lambda} = 62.8 \text{ m}^{-1}$$

$$\therefore \quad \lambda = \frac{2\pi}{62.8}\text{ m} = 0.1 \text{ m}$$

$$l = \frac{5\lambda}{2} = 0.25 \text{ m}$$

The mid-point of the string is P, an antinode

$\therefore$ maximum displacement = 0.01 m

$$\omega = 2\pi f = 628 \text{ s}^{-1}$$

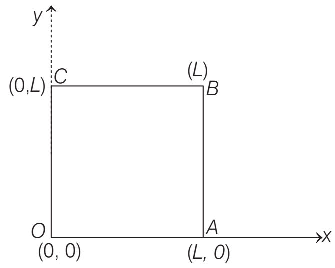
$$\therefore \quad f = \frac{628}{2\pi} = 100 \text{ Hz}$$

But this is fifth harmonic frequency.

$\therefore$ Fundamental frequency $f_0 = \frac{f}{5} = 20$ Hz

31. Standing waves can be produced only when two similar type of waves (same frequency and speed, but amplitude may be different) travel in opposite directions.

32. Since, the edges are clamped, displacement of the edges $u(x, y) = 0$ for



Line,	OA i.e. $y = 0$		$0 \le x \le L$
	AB i.e. $x = L$		$0 \le y \le L$
	BC i.e. $y = L$		$0 \le x \le L$
	OC i.e. $x = 0$		$0 \le y \le L$

The above conditions are satisfied only in alternatives (b) and (c).

Note that $u(x, y) = 0$, for all four values *eg,* in alternative (d), $u(x, y) = 0$ for $y = 0$, $y = L$ but it is not zero for $x = 0$ or $x = L$. Similarly, in option (a) $u(x, y) = 0$ at $x = L$, $y = L$ but it is not zero for $x = 0$ or $y = 0$ while in options (b) and (c), $u(x, y) = 0$ for $x = 0, y = 0$ $x = L$ and $y = L$.

33. It is given that $y(x, t) = 0.02\cos(50\pi t + \pi/2)\cos(10\pi x)$

$$\cong A\cos(\omega t + \pi/2)\cos kx$$

Node occurs when $kx = \frac{\pi}{2}, \frac{3\pi}{2}$ etc.

$$10\pi x = \frac{\pi}{2}, \frac{3\pi}{2} \Rightarrow x = 0.05\text{ m}, 0.15\text{ m} \qquad \text{option (a)}$$

Antinode occurs when $kx = \pi, 2\pi, 3\pi$ etc.

$$10\pi x = \pi, 2\pi, 3\pi \text{ etc.}$$

$$\Rightarrow \quad x = 0.1\text{ m}, 0.2\text{ m}, 0.3\text{ m} \qquad \text{option (b)}$$

Speed of the wave is given by,

$$v = \frac{\omega}{k} = \frac{50\pi}{10\pi} = 5 \text{ m/s} \qquad \text{option (c)}$$

Wavelength is given by,

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{10\pi} = \left(\frac{1}{5}\right)\text{m} = 0.2\text{m} \qquad \text{option (d)}$$

34. For closed pipe, $f = n\left(\frac{v}{4l}\right)$ $n = 1, 3, 5\ldots$

$$\text{For } n = 1, \quad f_1 = \frac{v}{4l} = \frac{320}{4 \times 1} = 80 \text{ Hz}$$

$$\text{For } n = 3, \quad f_3 = 3f_1 = 240 \text{ Hz}$$

$$\text{For } n = 5, \quad f_5 = 5f_1 = 400 \text{ Hz}$$

$\therefore$ Correct options are (a), (b) and (d).

35. For closed organ pipe,

$$f = n\left(\frac{v}{4l}\right) \text{ where, } n = 1, 3, 5\ldots\ldots$$

$$\therefore \quad l = \frac{nv}{4f}$$

$$\text{For } n = 1, \quad l_1 = \frac{(1)(330)}{4 \times 264} \times 100 \text{ cm} = 31.25 \text{ cm}$$

$$\text{For } n = 3, \quad l_3 = 3l_1 = 93.75 \text{ cm}$$

$$\text{For } n = 5, \quad l_5 = 5l_1 = 156.25 \text{cm}$$

$\therefore$ Correct options are (a) and (c).

36. Fundamental frequency of open pipe $f = \dfrac{v}{2l}$

and fundamental frequency of closed pipe,

$$f' = \frac{v}{4(l/2)} = \frac{v}{2\,l} \quad \text{or} \quad f' = f$$

37. Fundamental frequency $f = \dfrac{v}{2l} = \dfrac{\sqrt{T/\mu}}{2l}$ or $f \propto \sqrt{T}$

$$\therefore \qquad \frac{f'}{f} = \sqrt{\frac{w-F}{w}}$$

Here, w = weight of mass and F = upthrust

$$\therefore \qquad f' = f\sqrt{\frac{w-F}{w}}$$

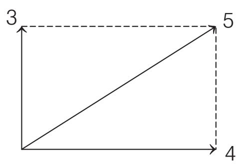
Substituting the values, we have

$$f' = 260\sqrt{\frac{(50.7)g-(0.0075)(10^3)g}{(50.7)g}} = 240\,\text{Hz}$$

38. Wall will be a node (displacement). Therefore, shortest distance from the wall at which air particles have maximum amplitude of vibration (displacement antinode) should be $\lambda/4$. Here, $\lambda = \dfrac{v}{f} = \dfrac{330}{660} = 0.5\,\text{m}$

$\therefore$ Desired distance is $\dfrac{0.5}{4} = 0.125\,\text{m}$

39. Phase difference between the two waves is 90°. Amplitudes are added by vector method.



$$A = \sqrt{3^2+4^2}$$

$\therefore$ Answer is 5.

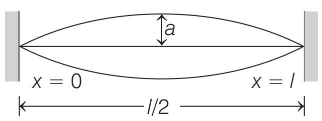
40. Distance between the successive nodes,

$$d = \frac{\lambda}{2} = \frac{v}{2f} = \frac{\sqrt{T/\mu}}{2f}$$

Substituting the values we get, $d = 5\,\text{cm}$

41. $l = \dfrac{\lambda}{2}$ or $\lambda = 2l$, $k = \dfrac{2\pi}{\lambda} = \dfrac{\pi}{l}$

The amplitude at a distance x from



$x = 0$ is given by $A = a\sin kx$

Total mechanical energy at x of length dx is

$$dE = \frac{1}{2}(dm)\,A^2\,\omega^2$$

$$= \frac{1}{2}(\mu dx)\,(a\sin kx)^2\,(2\pi f)^2$$

or $\qquad dE = 2\pi^2\,\mu f^2\,a^2\sin^2 kx\,dx \qquad$ ...(i)

Here, $\qquad f^2 = \dfrac{v^2}{\lambda^2} = \dfrac{\left(\frac{T}{\mu}\right)}{(4l^2)}$ and $k = \dfrac{\pi}{l}$

Substituting these values in Eq. (i) and integrating it from $x = 0$ to $x = l$, we get total energy of string

$$E = \frac{\pi^2\,a^2T}{4l}.$$

42. (a) Frequency of second harmonic in pipe A

= frequency of third harmonic in pipe B

$$\therefore \qquad 2\left(\frac{v_A}{2l_A}\right) = 3\left(\frac{v_B}{4l_B}\right)$$

or $\qquad \dfrac{v_A}{v_B} = \dfrac{3}{4}$ or $\dfrac{\sqrt{\dfrac{\gamma_A RT_A}{M_A}}}{\sqrt{\dfrac{\gamma_B RT_B}{M_B}}} = \dfrac{3}{4} \qquad (\text{as } l_A = l_B)$

or $\quad \sqrt{\dfrac{\gamma_A}{\gamma_B}}\sqrt{\dfrac{M_B}{M_A}} = \dfrac{3}{4}$ (as $T_A = T_B$) $\Rightarrow \dfrac{M_A}{M_B} = \dfrac{\gamma_A}{\gamma_B}\left(\dfrac{16}{9}\right)$

$$= \left(\frac{5/3}{7/5}\right)\left(\frac{16}{9}\right) \qquad \left(\gamma_A = \frac{5}{3} \text{ and } \gamma_B = \frac{7}{5}\right)$$

or $\quad \dfrac{M_A}{M_B} = \left(\dfrac{25}{21}\right)\left(\dfrac{16}{9}\right) = \dfrac{400}{189}$

(b) Ratio of fundamental frequency in pipe A and in pipe B is

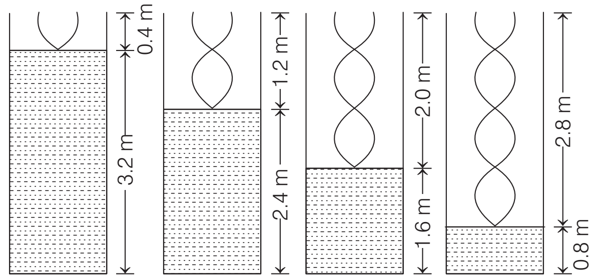
$$\frac{f_A}{f_B} = \frac{v_A/2l_A}{v_B/2l_B} = \frac{v_A}{v_B} \qquad (\text{as } l_A = l_B)$$

$$= \frac{\sqrt{\dfrac{\gamma_A\,RT_A}{M_A}}}{\sqrt{\dfrac{\gamma_B RT_B}{M_B}}} = \sqrt{\frac{\gamma_A}{\gamma_B}\cdot\frac{M_B}{M_A}} \qquad (\text{as } T_A = T_B)$$

Substituting $\dfrac{M_B}{M_A} = \dfrac{189}{400}$ from part (a), we get

$$\frac{f_A}{f_B} = \sqrt{\frac{25}{21}\times\frac{189}{400}} = \frac{3}{4}$$

43. Speed of sound $v = 340\,\text{m/s}$



Let l_0 be the length of air column corresponding to the fundamental frequency. Then,

$$\frac{v}{4l_0} = 212.5 \quad \text{or} \quad l_0 = \frac{v}{4(212.5)} = \frac{340}{4(212.5)} = 0.4 \text{ m}$$

In closed pipe only odd harmonics are obtained. Now let l_1, l_2, l_3, l_4, etc., be the lengths corresponding to the 3rd harmonic, 4th harmonic, 7th harmonic etc. Then,

$$3\left(\frac{v}{4l_1}\right) = 212.5 \quad \Rightarrow \quad l_1 = 1.2 \text{ m}$$

$$5\left(\frac{v}{4l_2}\right) = 212.5 \quad \Rightarrow \quad l_2 = 2.0 \text{ m}$$

and
$$7\left(\frac{v}{4l_3}\right) = 212.5 \quad \Rightarrow \quad l_3 = 2.8 \text{ m}$$

$$9\left(\frac{v}{4l_4}\right) = 212.5 \quad \Rightarrow \quad l_4 = 3.6 \text{ m}$$

or heights of water level are $(3.6 - 0.4)$ m, $(3.6 - 1.2)$ m, $(3.6 - 2.0)$ m and $(3.6 - 2.8)$ m.

$\therefore$ Heights of water level are 3.2 m, 2.4m, 1.6m and 0.8 m.

Let A and a be the area of cross-sections of the pipe and hole respectively. Then,

$$A = \pi\,(2 \times 10^{-2})^2 = 1.26 \times 10^{-3} \text{ m}^2$$

and $a = \pi\,(10^{-3})^2 = 3.14 \times 10^{-6} \text{ m}^2$

Velocity of efflux, $v = \sqrt{2gH}$

Continuity equation at 1 and 2 gives

$$a\sqrt{2gH} = A\left(\frac{-dH}{dt}\right)$$

$\therefore$ Rate of fall of water level in the pipe,

$$\left(\frac{-dH}{dt}\right) = \frac{a}{A}\sqrt{2gH}$$

Substituting the values, we get

$$\frac{-dH}{dt} = \frac{3.14 \times 10^{-6}}{1.26 \times 10^{-3}}\sqrt{2 \times 10 \times H}$$

or
$$-\frac{dH}{dt} = (1.11 \times 10^{-2})\sqrt{H}$$

Between first two resonances, the water level falls from 3.2 m to 2.4 m.

$\therefore$
$$\frac{dH}{\sqrt{H}} = -(1.11 \times 10^{-2})\,dt$$

or
$$\int_{3.2}^{2.4} \frac{dH}{\sqrt{H}} = -(1.11 \times 10^{-2})\int_0^t dt$$

or $2[\sqrt{2.4} - \sqrt{3.2}] = -(1.11 \times 10^{-2}) \cdot t$ or $t \approx 43$ s

NOTE

- Rate of fall of level at a height h is

$$\left(\frac{-dh}{dt}\right) = \frac{a}{A}\sqrt{2gh} \propto \sqrt{H}$$

i.e., rate decreases as the height of water (or any other liquid) decreases in the tank. That is why, the time required to empty the first half of the tank is less than the time required to empty the rest half of the tank.

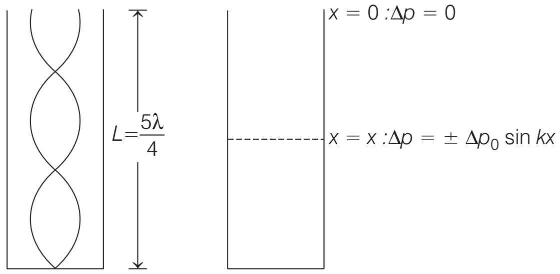
44. (a) Frequency of second overtone of the closed pipe

$$= 5\left(\frac{v}{4L}\right) = 440 \Rightarrow L = \frac{5v}{4 \times 440} \text{ m}$$

Substituting v = speed of sound in air = 330 m/s

$$L = \frac{5 \times 330}{4 \times 440} = \frac{15}{16} \text{ m}$$

$$\lambda = \frac{4L}{5} = \frac{4\left(\frac{15}{16}\right)}{5} = \frac{3}{4} \text{ m}$$



(b) Open end is displacement antinode. Therefore, it would be a pressure node.

or at $x = 0;\ \Delta p = 0$

Pressure amplitude at $x = x$, can be written as,

$$\Delta p = \pm \Delta p_0 \sin kx$$

where,
$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{3/4} = \frac{8\pi}{3} \text{ m}^{-1}$$

Therefore, pressure amplitude at $x = \frac{L}{2} = \frac{15/16}{2}$ m or (15/32) m will be

$$\Delta p = \pm \Delta p_0 \sin\left(\frac{8\pi}{3}\right)\left(\frac{15}{32}\right)$$

$$= \pm \Delta p_0 \sin\left(\frac{5\pi}{4}\right)$$

$$\Delta p = \pm \frac{\Delta p_0}{\sqrt{2}}$$

(c) Open end is a pressure node i.e. $\Delta p = 0$

Hence, $p_{\max} = p_{\min}$ = Mean pressure (p_0)

(d) Closed end is a displacement node or pressure antinode.

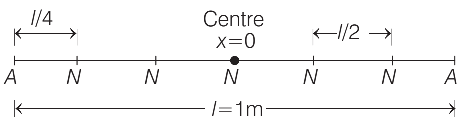
Therefore, $p_{\max} = p_0 + \Delta p_0$

and $p_{\min} = p_0 - \Delta p_0$

45. Speed of longitudinal travelling wave in the rod will be

$$v = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{2 \times 10^{11}}{8000}} = 5000 \text{ m/s}$$

Amplitude at antinode = $2A$ (Here, A is the amplitude of constituent waves)



$$= 2\times10^{-6}\text{m}$$

$\therefore$ $A = 10^{-6}\text{ m} \Rightarrow l = \dfrac{5\lambda}{2}$

$\Rightarrow$ $\lambda = \dfrac{2l}{5} = \dfrac{(2)(1.0)}{5}\text{m} = 0.4\text{ m}$

Hence, the equation of motion at a distance x from the mid-point will be given by,

$$y = 2A\sin kx\sin\omega t$$

Here, $k = \dfrac{2\pi}{0.4} = 5\pi$

$$\omega = 2\pi f = 2\pi\frac{v}{\lambda}$$

$$= 2\pi\left(\frac{5000}{0.4}\right)\text{rad/s} = 25000\pi$$

$\therefore$ $y = (2\times10^{-6})\sin(5\pi x)\sin(25000\pi t)$

Therefore, y at a distance $x = 2\text{cm} = 2\times10^{-2}\text{m}$

is $y = 2\times10^{-6}\sin(5\pi\times2\times10^{-2})\sin(25000\pi t)$

or $y = 2\times10^{-6}\sin(0.1\pi)\sin(25000\pi t)$

The equations of constituent waves are

$$y_1 = A\sin(\omega t - kx)\text{ and } y_2 = A\sin(\omega t + kx)$$

or $y_1 = 10^{-6}\sin(25000\pi t - 5\pi x)$

and $y_2 = 10^{-6}\sin(25000\pi t + 5\pi x)$

46. (a) $y = 4\sin\dfrac{\pi x}{15}\cos(96\pi t) = A_x\cos(96\pi t)$

Here, $A_x = 4\sin\dfrac{\pi x}{15}$

at $x = 5\text{ cm}$, $A_x = 4\sin\dfrac{5\pi}{15} = 4\sin\dfrac{\pi}{3} = 2\sqrt{3}\text{cm}$

This is the amplitude or maximum displacement at $x = 5\text{ cm}$.

(b) Nodes are located where $A_x = 0$

or $\dfrac{\pi x}{15} = 0, \pi, 2\pi\ldots\ldots$ or $x = 0, 15\text{ cm}, 30\text{ cm}$ etc.

(c) Velocity of particle,

$$v_p = \left|\frac{\partial y}{\partial t}\right|_{x\,=\,\text{constant}} = -384\pi\sin\left(\frac{\pi x}{15}\right)\sin(96\pi t)$$

At $x = 7.5\text{cm}$ and $t = 0.25\text{ s}$

$$v_p = -384\pi\sin\left(\frac{\pi}{2}\right)\sin(24\pi) = 0$$

(d) Amplitude of components waves is $A = \dfrac{4}{2} = 2\text{ cm}$

$\omega = 96\pi$ and $k = \dfrac{\pi}{15}$

$\therefore$ Component waves are,

$$y_1 = 2\sin\left(\frac{\pi}{15}x - 96\pi t\right)$$

and $y_2 = 2\sin\left(\dfrac{\pi}{15}x + 96\pi t\right)$

Topic 3 Wave Speed

1. Given, $p = 0.01\sin(1000t - 3x)\text{ N/m}^2$

Comparing with the general equation of pressure wave of sound, i.e. $p_0\sin(\omega t - kx)$,

we get

$$\omega = 1000\text{ and }k = 3$$

Also, $k = \dfrac{\omega}{v} \Rightarrow v = \omega/k$

$\therefore$ Velocity of sound is $|v_1| = \dfrac{1000}{3}$

Or

Speed of sound wave can also be calculated as

$$v = -\frac{(\text{coefficient of }t)}{(\text{coefficient of }x)} = -\frac{1000}{(-3)} = \frac{1000}{3}\text{ m/s}$$

Now, relation between velocity of sound and temperature is

$$v = \sqrt{\frac{\gamma RT}{m}} \Rightarrow v \propto \sqrt{T}$$

or $\dfrac{v_2}{v_1} = \sqrt{\dfrac{T_2}{T_1}} \Rightarrow T_2 = \dfrac{v_2^2}{v_1^2}\cdot T_1$

Here, $v_2 = 336\text{ m/s}$, $v_1 = 1000/3\text{ m/s}$, $T_1 = 0°\text{C} = 273\text{ K}$

$\therefore$ $T_2 = \dfrac{(336)^2}{(1000/3)^2}\times273 = 277.38\text{ K}$

$\therefore$ $T_2 = 4.38°\text{C} \simeq 4°\text{C}$

2. Wave equation is given by,

$$y = 10^{-3}\sin(50t + 2x)$$

Speed of wave is obtained by differentiating phase of wave. Now, phase of wave from given equation is

$$\phi = 50t + 2x = \text{constant}$$

Differentiating 'ϕ' w.r.t 't', we get

$$\frac{d}{dx}(50t + 2x) = \frac{d}{dt}(\text{constant})$$

$\Rightarrow$ $50 + 2\left(\dfrac{dx}{dt}\right) = 0$

$\Rightarrow$ $\dfrac{dx}{dt} = \dfrac{-50}{2} = -25\text{ ms}^{-1}$

So, wave is propagating in negative x–direction with a speed of 25 ms^{-1}.

Alternate Method

The general equation of a wave travelling in negative x direction is given as

$$y = a\sin(\omega t + kx) \quad \ldots\text{(i)}$$

Given equation of wave is

$$y = 10^{-3}\sin(50 + 2x) \quad \ldots\text{(ii)}$$

Comparing Eqs. (i) and (ii), we get

$$\omega = 50\text{ and }k = 2$$

Velocity of the wave, $v = \dfrac{\omega}{k} = \dfrac{50}{2} = 25\,\text{m/s}$

3. As observer is moving with relativistic speed; formula $\dfrac{\Delta f}{f} = \dfrac{v_{\textbf{radial}}}{c}$, does not apply here.

Relativistic doppler's formula is

$$f_{\textbf{observed}} = f_{\textbf{actual}} \cdot \left(\frac{1 + v/c}{1 - v/c}\right)^{1/2}$$

Here, $\dfrac{v}{c} = \dfrac{1}{2}$

So, $f_{\textbf{observed}} = f_{\textbf{actual}} \left(\dfrac{3/2}{1/2}\right)^{1/2}$

$\therefore \quad f_{\textbf{observed}} = 10 \times \sqrt{3} = 17.3\,\text{GHz}$

4. The frequency is a characteristic of source. It is independent of the medium. Hence, the correct option is (d).

5. Speed of sound in a gas is given by

$$v = \sqrt{\frac{\gamma RT}{M}}, \; v \propto \frac{1}{\sqrt{M}}$$

$$\therefore \quad \frac{v_1}{v_2} = \sqrt{\frac{M_2}{M_1}} = \sqrt{\frac{m_2}{m_1}}$$

Here, $\gamma = \dfrac{C_p}{C_V} = \dfrac{5}{3}$ for both the gases $\left(\gamma_{\text{monoatomic}} = \dfrac{5}{3}\right)$

6. Speed of sound in an ideal gas is given by

$v = \sqrt{\dfrac{\gamma RT}{M}} \Rightarrow \therefore \; v \propto \sqrt{\dfrac{\gamma}{M}}$ (T is same for both the gases)

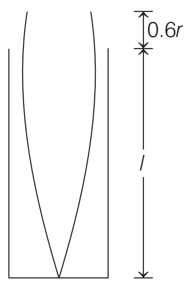
$$\therefore \quad \frac{v_{N_2}}{v_{He}} = \sqrt{\frac{\gamma_{N_2}}{\gamma_{He}} \cdot \frac{M_{He}}{M_{N_2}}} = \sqrt{\frac{7/5}{5/3}\left(\frac{4}{28}\right)} = \sqrt{3}/5$$

$$\gamma_{N_2} = \frac{7}{5} \qquad \text{(Diatomic)}$$

$$\gamma_{He} = \frac{5}{3} \qquad \text{(Monoatomic)}$$

7. $v_{\text{sound}} = \sqrt{\dfrac{\gamma RT}{M}} \Rightarrow \dfrac{v_{H_2}}{v_{He}} = \dfrac{\sqrt{\gamma_{H_2}/M_{H_2}}}{\sqrt{\gamma_{He}/M_{He}}} = \dfrac{\sqrt{(7/5)/2}}{\sqrt{(5/3)/4}} = \sqrt{\dfrac{42}{25}}$

8. Fundamental frequency, $f = \dfrac{v}{4(l + 0.6r)}$



$\therefore$ Speed of sound $v = 4f(l + 0.6r)$.

or $\quad v = (4)(480)[(0.16) + (0.6)(0.025)]$

$= 336\,\text{m/s}$

9. Amplitude of incident wave $A_i = 3.5\,\text{cm}$

Tension $T = 80\text{N}$

Amplitude of incident wave $A_i = 3.5\text{cm}$

Mass per unit length of wire PQ is

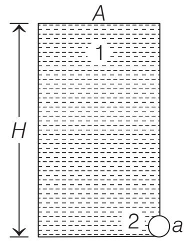
$$m_1 = \frac{0.06}{4.8} = \frac{1}{80}\,\text{kg/m}$$

and mass per unit length of wire QR is

$$m_2 = \frac{0.2}{2.56} = \frac{1}{12.8}\,\text{kg/m}$$

(a) Speed of wave in wire PQ is

$$v_1 = \sqrt{\frac{T}{m_1}} = \sqrt{\frac{80}{1/80}} = 80\,\text{m/s}$$



and speed of wave in wire

$$QR \text{ is } v_2 = \sqrt{\frac{T}{m_2}} = \sqrt{\frac{80}{1/12.8}}$$

$$= 32\,\text{m/s}$$

$\therefore$ Time taken by the wave pulse to reach from P to R is

$$t = \frac{4.8}{v_1} + \frac{2.56}{v_2} = \left(\frac{4.8}{80} + \frac{2.56}{32}\right)\text{s}$$

$$t = 0.14\,\text{s}$$

(b) The expressions for reflected and transmitted amplitudes (A_r and A_t) in terms of v_1, v_2 and A_i are as follows :

$$A_r = \frac{v_2 - v_1}{v_2 + v_1} A_i \quad \text{and} \quad A_t = \frac{2v_2}{v_1 + v_2} A_i$$

Substituting the values, we get

$$A_r = \left(\frac{32 - 80}{32 + 80}\right)(3.5) = -1.5\,\text{cm}$$

i.e. the amplitude of reflected wave will be 1.5 cm. Negative sign of A_r indicates that there will be a phase change of π in reflected wave. Similarly,

$$A_t = \left(\frac{2 \times 32}{32 + 80}\right)(3.5) = 2.0\,\text{cm}$$

i.e. the amplitude of transmitted wave will be 2.0 cm.

NOTE The expressions of A_r *and* A_t are derived as below.

Derivation

Suppose the incident wave of amplitude A_i and angular frequency ω is travelling in positive x-direction with velocity v_1 then, we can write

$$y_i = A_i \sin w\left[t - \frac{x}{v_1}\right] \qquad \ldots\text{(i)}$$

In reflected as well as transmitted wave, ω will not change, therefore, we can write,

$$y_r = A_r \sin\omega\left[t + \frac{x}{v_1}\right] \quad \ldots\text{(ii)}$$

and $$y_t = A_t \sin\omega\left[t - \frac{x}{v_2}\right] \quad \ldots\text{(iii)}$$

Now, as wave is continuous, so at the boundary $(x = 0)$.

Continuity of displacement requires

$$y_i + y_r = y_t \text{ for } (x = 0)$$

Substituting, we get

$$A_i + A_r = A_t \quad \ldots\text{(iv)}$$

Also at the boundary, slope of wave will be continuous *i.e.,*

$$\frac{\partial y_i}{\partial x} + \frac{\partial y_r}{\partial x} = \frac{\partial y_t}{\partial x}$$

for $x = 0$

Which gives,

$$A_i - A_r = \left(\frac{v_1}{v_2}\right) A_t \quad \ldots\text{(v)}$$

Solving Eqs. (iv) and (v) for A_r and A_t, we get the required equations i.e.

$$A_r = \frac{v_2 - v_1}{v_2 + v_1} A_i$$

and $$A_t = \frac{2v_2}{v_2 + v_1} A_i$$

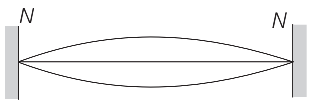
10. The temperature stress is $\sigma = Y\alpha\Delta\theta$

or tension in the steel wire $T = \sigma A = YA\alpha\Delta\theta$

Substituting the values, we have

$$T = (2 \times 10^{11})(10^{-6})(1.21 \times 10^{-5})(20) = 48.4 \text{ N}$$

Speed of transverse wave on the wire, $v = \sqrt{\frac{T}{\mu}}$



$l/2 = l$
or $l = 2l$

Here, μ = mass per unit length of wire $= 0.1 \text{ kg/m}$

$$\therefore \quad v = \sqrt{\frac{48.4}{0.1}} = 22 \text{ m/s}$$

Fundamental frequency

$$f_o = \frac{v}{2l} = \frac{22}{2 \times 1} = 11 \text{ Hz}$$

11. $v = \sqrt{T/\mu}$

$$\frac{v_{\text{top}}}{v_{\text{bottom}}} = \sqrt{\frac{T_{\text{top}}}{T_{\text{bottom}}}} = \sqrt{\frac{6+2}{2}} = 2 \quad \ldots\text{(i)}$$

Frequency will remain unchanged. Therefore, Eq. (i) can be written as,

$$\frac{f\lambda_{\text{top}}}{f\lambda_{\text{bottom}}} = 2$$

or $$\lambda_{\text{top}} = 2\,(\lambda_{\text{bottom}}) = 2 \times 0.06 = 0.12 \text{ m}$$

12. Tension due to thermal stresses,

$$T = YA\,\alpha \cdot \Delta\theta \quad \Rightarrow \quad v = \sqrt{\frac{T}{\mu}}$$

Here, μ = mass per unit length. $= \rho A$

$$\therefore \quad v = \sqrt{\frac{T}{\rho A}} = \sqrt{\frac{YA\alpha \cdot \Delta\theta}{\rho A}} = \sqrt{\frac{Y\alpha\,\Delta\theta}{\rho}}$$

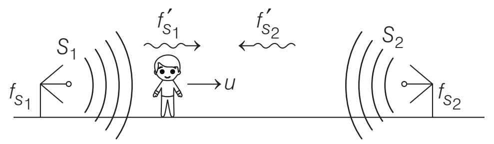
Substituting the values we have,

$$v = \sqrt{\frac{1.3 \times 10^{11} \times 1.7 \times 10^{-5} \times 20}{9 \times 10^3}}$$

$$= 70.1 \text{ m/s}$$

Topic 4 Beats and Doppler Effect

1. When observer moves away from S_1 and towards S_2,



then due to Doppler's effect observed frequencies of sources by observer are

$$f'_{S_1} = \frac{v - v_o}{v} \cdot f_{S_1}$$

(observer moving away from source)

and $$f'_{S_2} = \left(\frac{v + v_o}{v}\right) \cdot f_{S_2}$$

(observer moving towards source)

(where, v = speed to sound, v_o = speed of observer)

So, beat frequency heard by observer is

$$f_b = f'_{S_2} - f'_{S_1}$$

Here, $v_o = u,\ v = 330 \text{ ms}^{-1}$

$$f_b = 10 \text{ Hz},\ f_{S_1} = f_{S_2} = 660 \text{ Hz}$$

On putting the values, we get

$$f_b = f'_{S_2} - f'_{S_1}$$

$$= \left(\frac{v + v_o}{v}\right) \cdot f_{S_2} - \left(\frac{v - v_o}{v}\right) f_{S_1}$$

$$= f_{S_1}\left\{\frac{v + v_o}{v} - \frac{v - v_o}{v}\right\} = f_{S_1} \cdot \frac{2v_o}{v}$$

$$\Rightarrow \quad 10 = \frac{660 \times 2u}{330} \quad [\because v_o = u]$$

$$\Rightarrow \quad u = \frac{330 \times 10}{2 \times 660} \Rightarrow u = 2.5 \text{ ms}^{-1}$$

2. When waves of nearby frequencies overlaps, beats are produced.

Beat frequency is given by, $f_{\text{beat}} = f_1 - f_2$

Here, $f_1 = 11 \text{ Hz}$ and $f_2 = 9 \text{ Hz}$

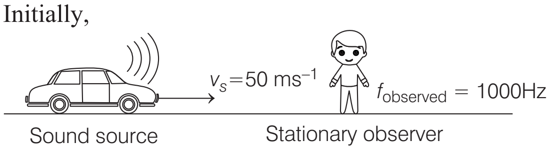
$\Rightarrow$ Beat frequency is, $f_{\text{beat}} = 11 - 9 = 2\,\text{Hz}$

Hence, time period of beats or time interval between beats,

$$T = \frac{1}{f_{\text{beat}}} \Rightarrow T = \frac{1}{2} = 0.5\,\text{s}$$

So, resultant wave has a time period of 0.5 s which is correctly depicted in option (a) only.

3. Initially,



After sometime,

When source is moving towards stationary observer, frequency observed is more than source frequency due to Doppler's effect, it is given by

$$f_{\text{observed}} = f\left(\frac{v}{v - v_s}\right)$$

where, f = source frequency,

f_o = observed frequency = 1000 Hz,

v = speed of sound in air = 350 ms^{-1}

and v_s = speed of source = 50 ms^{-1}

So, $$f = \frac{f_{\text{obs}}(v - v_s)}{v} = \frac{1000(350 - 50)}{350} = \frac{6000}{7}\,\text{Hz}$$

When source moves away from stationary observer, observed frequency will be lower due to Doppler's effect and it is given by

$$f_0 = f\left(\frac{v}{v + v_s}\right) = \frac{6000 \times 350}{7 \times (350 + 50)}$$

$$= \frac{6000 \times 350}{7 \times 400} = 750\,\text{Hz}$$

4. Given,

Frequency of sound source $(f_0) = 500$ Hz Apparent frequency heard by observer 1, $f_1 = 480$ Hz and apparent frequency heard by observer 2, $f_2 = 530$ Hz.

Let v_0 be the speed of sound.

When observer moves away from the source, Apparent

frequency, $$f_1 = f_0\left(\frac{v - v_0'}{v}\right) \quad \ldots \text{(i)}$$

When observer moves towards the source,

Apparent frequency, $$f_2 = f_0\left(\frac{v + v_0''}{v}\right) \quad \ldots \text{(ii)}$$

Substituting values in Eq. (i), we get

$$480 = 500\left(\frac{300 - v_0'}{300}\right)$$

$$\Rightarrow \quad 96 \times 3 = 300 - v_0'$$

$$\Rightarrow \quad v_0' = 12\,\text{m/s}$$

Substituting values in Eq. (ii), we get

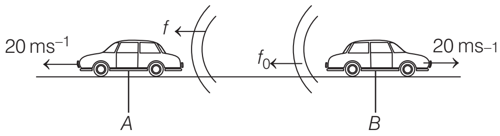
$$530 = 500\left(\frac{330 + v''}{300}\right)$$

$$\Rightarrow \quad 106 \times 3 = 300 + v''$$

$$\Rightarrow \quad v_0'' = 18\,\text{m/s}$$

Thus, their respective speeds (in m/s) is 12 and 18.

5. The given condition can be shown below as,



Here, source and observer both are moving away from each other. So, by Doppler's effect, observed frequency is given by

$$f = f_0\left(\frac{v + v_o}{v - v_s}\right) \quad \ldots \text{(i)}$$

where, v = speed of sound = 340 ms^{-1},

v_o = speed of observer = $-$ 20 ms^{-1},

v_s = speed of source = $-$ 20 ms^{-1},

f_0 = true frequency

and f = apparent frequency = 2000 Hz

Substituting the given values in Eq. (i), we get

$$2000 = \left(\frac{340 - 20}{340 + 20}\right) \times f_0$$

$$\Rightarrow \quad f_0 = \frac{2000 \times 360}{320} = 2250\,\text{Hz}$$

6. When a source is moving towards an stationary observer, observed frequency is given by

$$f_{\text{observed}} = f\left(\frac{v}{v + v_s}\right)$$

where, f = frequency of sound from the source,

v = speed of sound and v_s = speed of source.

Now applying above formula to two different conditions given in problem, we get

$$f_1 = \text{Observed frequency} = f\left(\frac{340}{340 - 34}\right)$$

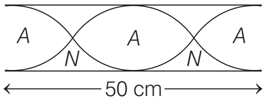
$$= f\left(\frac{340}{306}\right)$$

and f_2 = Observed frequency when speed of source is reduced

$$= f\left(\frac{340}{340 - 17}\right) = \frac{340}{323}$$

So, the ratio $f_1 : f_2$ is $\dfrac{f_1}{f_2} = \dfrac{323}{306} = \dfrac{19}{18}$.

7. According to the question, the musician uses a open flute of length 50 cm and produce second harmonic sound waves



When the flute is open from both ends and produce second harmonic, then

$$L = \lambda_2$$

$$\Rightarrow \qquad f_2 = \frac{v}{L}$$

where, λ_2 = wavelength for second harmonic,

f_2 = frequency for second harmonic

and v = speed of wave.

For given question $\Rightarrow f_2 = \frac{v}{L} = \frac{330}{50 \times 10^{-2}}$

$f_2 = 660\,\text{Hz}$ (frequency produce by source)

Now, a person runs towards the musician from another end of a hall

$v_{\text{observer}} = 10\,\text{km/h}$ (towards source)

There is apparant change in frequency, which heard by person and given by Doppler's effect formula

$$\Rightarrow \qquad \nu' = f' = \nu\left[\frac{v_{\text{sound}} + v_{\text{observer}}}{v_{\text{sound}}}\right]$$

$$f' = f_2\left[\frac{v_s + v_o}{v_{\text{sound}}}\right]$$

$$f' = 660\left[\frac{330 + \frac{50}{18}}{330}\right]$$

$$f' = 666\,\text{Hz}$$

Hence, option (a) is correct.

8. Observer is stationary and source is moving.

During approach, $f_1 = f\dfrac{v}{v - v_s}$

$$= 1000\left(\frac{320}{320 - 20}\right) = 1066.67\text{ Hz}$$

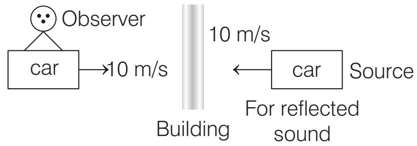
During recede, $f_2 = f\left(\dfrac{v}{v + v_s}\right)$

$$= 1000\left(\frac{320}{320 + 20}\right) = 941.18\text{ Hz}$$

$$|\%\text{ change in frequency}| = \left(\frac{f_1 - f_2}{f_1}\right) \times 100 \approx 12\%$$

9. $36\text{ km/h} = 36 \times \dfrac{5}{18} = 10\text{ m/s}$

Apparent frequency of sound heard by car driver (observer) reflected from the building will be



$$f' = f\left(\frac{v + v_o}{v - v_s}\right) = 8\left(\frac{320 + 10}{320 - 10}\right) = 8.5\,\text{kHz}$$

$\therefore$ Correct option is (a).

10. With increase in tension, frequency of vibrating string will increase. Since number of beats are decreasing. Therefore, frequency of vibrating string or third harmonic frequency of closed pipe should be less than the frequency of tuning fork by 4.

$\therefore$ Frequency of tuning fork

= third harmonic frequency of closed pipe + 4

$$= 3\left(\frac{v}{4l}\right) + 4 = 3\left(\frac{340}{4 \times 0.75}\right) + 4$$

$$= 344\text{ Hz}$$

$\therefore$ Correct option is (a).

11. The motorcyclist observes no beats. So, the apparent frequency observed by him from the two sources must be equal.

$$f_1 = f_2$$

$$\therefore \qquad 176\left(\frac{330 - v}{330 - 22}\right) = 165\left(\frac{330 + v}{330}\right)$$

Solving this equation, we get $v = 22\,\text{m/s}$

12. Using the formula $f' = f\left(\dfrac{v + v_0}{v}\right)$

we get, $$5.5 = 5\left(\frac{v + v_A}{v}\right) \qquad \ldots\text{(i)}$$

and $$6.0 = 5\left(\frac{v + v_B}{v}\right) \qquad \ldots\text{(ii)}$$

Here, v = speed of sound

v_A = speed of train A

v_B = speed of train B

Solving Eqs. (i) and (ii), we get $\dfrac{v_B}{v_A} = 2$

13. $f_1 = f\left(\dfrac{v}{v - v_s}\right) \Rightarrow f_1 = f\left(\dfrac{340}{340 - 34}\right) = f\left(\dfrac{340}{306}\right)$

and $$f_2 = f\left(\frac{340}{340 - 17}\right) = f\left(\frac{340}{323}\right)$$

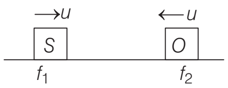
$$\therefore \qquad \frac{f_1}{f_2} = \frac{323}{306} = \frac{19}{18}$$

14. Source is moving towards the observer

$$f' = f\left(\frac{v}{v - v_s}\right) = 450\left(\frac{330}{330 - 33}\right)$$

$$f' = 500\text{ Hz}$$

15. When wind blows from S to O



$$f_2 = f_1\left(\frac{v + w + u}{v + w - u}\right)$$

or $\quad f_2 > f_1$

when wind blows from O to S

$$f_2 = f_1\left(\frac{v - w + u}{v - w - u}\right)$$

$\therefore \quad f_2 > f_1$

16. Moving plane is like a moving observer. Therefore, number of waves encountered by moving plane,

$$f_1 = f\left(\frac{v + v_0}{v}\right) = f\left(\frac{c + v}{c}\right)$$

Frequency of reflected wave,

$$f_2 = f_1\left(\frac{v}{v - v_s}\right) = f\left(\frac{c + v}{c - v}\right)$$

Wavelength of reflected wave

$$\lambda_2 = \frac{v}{f_2} = \frac{c}{f_2} = \frac{c}{f}\left(\frac{c - v}{c + v}\right)$$

17. $\quad T_1 > T_2$

$\therefore \quad v_1 > v_2$

or $\quad f_1 > f_2$

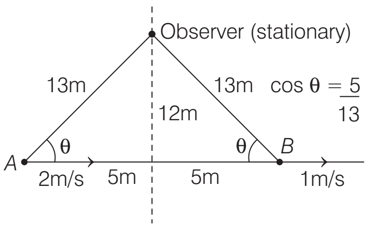
and $\quad f_1 - f_2 = 6\,\text{Hz}$

Now, if T_1 is increased, f_1 will increase or $f_1 - f_2$ will increase. Therefore, (d) option is wrong.

If T_1 is decreased, f_1 will decrease and it may be possible that now $f_2 - f_1$ become 6 Hz. Therefore, (c) option is correct.

Similarly, when T_2 is increased, f_2 will increase and again $f_2 - f_1$ may become equal to 6 Hz. So, (b) is also correct. But (a) is wrong.

18.



$$f_A = 1430\left[\frac{330}{330 - 2\cos\theta}\right]$$

$$= 1430\left[\frac{1}{1 - \frac{2\cos\theta}{330}}\right] \approx 1430\left[1 + \frac{2\cos\theta}{330}\right]$$

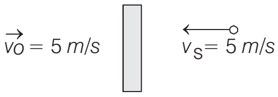
[from binomial expansion]

$$f_B = 1430\left[\frac{330}{330 + 1\cos\theta}\right] \approx 1430\left[1 - \frac{\cos\theta}{330}\right]$$

Beat frequency $= f_A - f_B = 1430\left[\frac{3\cos\theta}{330}\right] = 13\cos\theta$

$$= 13\left(\frac{5}{13}\right) = 5.00\,\text{Hz}$$

19. Frequency of refracted sound

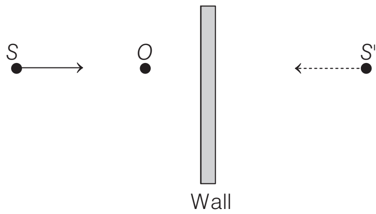


$$f' = f\left(\frac{v + v_o}{v - v_s}\right) = 200\left(\frac{342 + 5}{342 - 5}\right)\text{Hz} = 205.93\,\text{Hz}$$

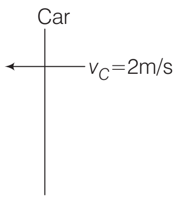
$\therefore$ Beat frequency $= f' - f = (205.93 - 200)$

$= 5.93\,\text{Hz} \simeq 6\,\text{Hz}$

20. For reflected wave an image of source S' can assumed as shown. Since, both S and S' are approaching towards observer, no beats will be heard.



21.



Frequency observed at car

$$f_1 = f_0\left(\frac{v + v_C}{v}\right) \qquad (v = \text{speed of sound})$$

Frequency of reflected sound as observed at the source

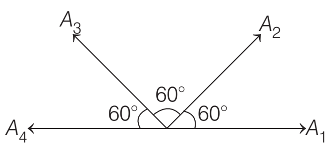
$$f_2 = f_1\left(\frac{v}{v - v_C}\right) = f_0\left(\frac{v + v_C}{v - v_C}\right)$$

Beat frequency $= f_2 - f_0$

$$= f_0\left[\frac{v + v_C}{v - v_C} - 1\right] = f_0\left[\frac{2v_C}{v - v_C}\right]$$

$$= 492 \times \frac{2 \times 2}{328} = 6\,\text{Hz}$$

22. Let individual amplitudes are A_0 each. Amplitudes can be added by vector method.



$$A_1 = A_2 = A_3 = A_4 = A_0$$

Resultant of A_1 and A_4 is zero. Resultant of A_2 and A_3 is

$$A = \sqrt{A_0^2 + A_0^2 + 2A_0A_0\cos 60^\circ} = \sqrt{3}A_0$$

This is also the net resultant.

Now, $\quad I \propto A^2$

$\therefore$ Net intensity will become $3I_0$.

$\therefore$ Answer is 3.

23. From the relation, $f' = f\left(\dfrac{v}{v \pm v_s}\right)$, we have

$$2.2 = f\left[\frac{300}{300 - v_T}\right] \quad \text{...(i)}$$

and
$$1.8 = f\left[\frac{300}{300 + v_T}\right] \quad \text{...(ii)}$$

Here, $v_T = v_s$ = velocity of source/train

Solving Eqs. (i) and (ii), we get

$$v_T = 30\,\text{m/s}$$

24. Velocity of sound in water is

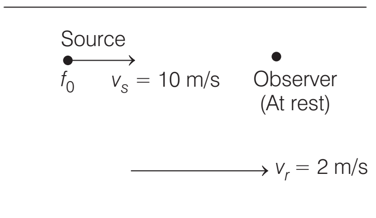
$$v_w = \sqrt{\frac{\beta}{\rho}} = \sqrt{\frac{2.088 \times 10^9}{10^3}} = 1445 \text{ m/s}$$

Frequency of sound in water will be

$$f_0 = \frac{v_w}{\lambda_w} = \frac{1445}{14.45 \times 10^{-3}} \text{ Hz}$$

$$f_0 = 10^5 \text{ Hz}$$

(a) Frequency of sound detected by receiver (observer) at rest would be



$$f_1 = f_0\left(\frac{v_w + v_r}{v_w + v_r - v_s}\right) = (10^5)\left(\frac{1445 + 2}{1445 + 2 - 10}\right) \text{Hz}$$

$$f_1 = 1.0069 \times 10^5 \text{ Hz}$$

(b) Velocity of sound in air is

$$v_a = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{(1.4)(8.31)(20 + 273)}{28.8 \times 10^{-3}}}$$

$$= 344 \text{ m/s}$$

$\therefore$ Frequency does not depend on the medium. Therefore, frequency in air is also $f_0 = 10^5$ Hz.

$\therefore$ Frequency of sound detected by receiver (observer) in air would be

$$f_2 = f_0\left(\frac{v_a - w}{v_a - w - v_s}\right)$$

$$= 10^5\left[\frac{344 - 5}{344 - 5 - 10}\right] \text{Hz}$$

$$f_2 = 1.0304 \times 10^5 \text{ Hz}$$

25. Frequency received in case 1.

$$f_1 = f\left(\frac{v + v_m}{v + v_b}\right)$$

and in case 2, $f_2 = f\left(\dfrac{v + v_m}{v - v_b}\right)$

Obviously, $f_2 > f_1$

$\therefore$ Beat frequency,

$$f_b = f_2 - f_1 = f\left(\frac{v + v_m}{v - v_b}\right) - f\left(\frac{v + v_m}{v + v_b}\right)$$

or
$$f_b = \frac{2\,v_b(v + v_m)}{v^2 - v_b^2}$$

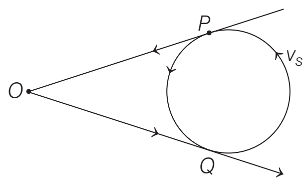
26. v_s = Speed of source (whistle) = $R\omega$ = (1.5) (20) m/s

$v_s = 30$ m/s

Maximum frequency will be heard by the observer in position P and minimum in position Q. Now,

$$f_{\max} = f\left(\frac{v}{v - v_s}\right)$$

where, $\;v$ = speed of sound in air = 330 m/s



$$= (440)\left(\frac{330}{330 - 30}\right) \text{Hz}$$

$$f_{\max} = 484 \text{ Hz}$$

and
$$f_{\min} = f\left(\frac{v}{v + v_s}\right) = (440)\left(\frac{330}{330 + 30}\right)$$

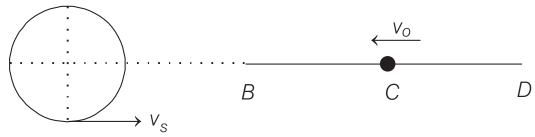
$$f_{\min} = 403.33 \text{ Hz}$$

Therefore, range of frequencies heard by observer is from 484 Hz to 403.33 Hz.

27. Angular frequency of detector

$$\omega = 2\pi f = 2\pi\left(\frac{5}{\pi}\right) = 10 \text{ rad/s}$$

Since, angular frequency of source of sound and of detector are equal, their time periods will also be equal.



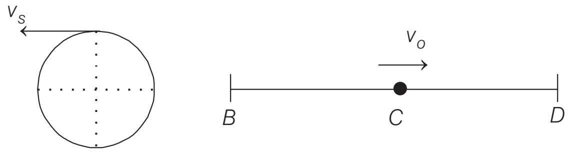
Maximum frequency will be heard in the position shown in figure. Since, the detector is far away from the source, we can use,

$$f_{max} = f\left(\frac{v + v_o}{v - v_s}\right)$$

Here, v = speed of sound = 340 m/s (given)

$$v_s = R\omega = 30\,\text{m/s}$$
$$v_o = \omega A = 60\,\text{m/s}$$

$$\therefore \quad f_{max} = 340\left[\frac{340 + 60}{340 - 30}\right]$$
$$= 438.7\,\text{Hz}$$



Minimum frequency will be heard in the condition shown in figure. The minimum frequency will be

$$f_{min} = f\left[\frac{v - v_o}{v + v_s}\right] = 340\left[\frac{340 - 60}{340 + 30}\right]$$
$$= 257.3\,\text{Hz}$$

28. Given, $f_1 - f_2 = 3\text{Hz}$

or $$f\left(\frac{v}{v - v_s}\right) - f\left(\frac{v}{v + v_s}\right) = 3$$

or $$340\left[\frac{340}{340 - v_s}\right] - 340\left[\frac{340}{340 + v_s}\right] = 3$$

or $$340\left[\left(1 - \frac{v_s}{340}\right)^{-1}\right] - 340\left[\left(1 + \frac{v_s}{340}\right)^{-1}\right] = 3$$

As $v_s << 340\,\text{m/s}$

Using binomial expansion, we have

$$340\left(1 + \frac{v_s}{340}\right) - 340\left(1 - \frac{v_s}{340}\right) = 3$$

$$\therefore \quad \frac{2 \times 340 \times v_s}{340} = 3$$

$$\therefore \quad v_s = 1.5\,\text{m/s}$$

29. Fundamental frequency of sonometer wire,

$$f = \frac{v}{2l} = \frac{\sqrt{T/\mu}}{2l} = \frac{1}{2 \times 0.1}\sqrt{\frac{64 \times 0.1}{10^{-3}}}$$
$$= 400\,\text{Hz}$$

Given beat frequency, $f_b = f - f' = 1\text{Hz}$

$$\therefore \quad f' = 399\text{Hz}$$

Using, $$f' = f\left(\frac{v}{v + v_s}\right)$$

or $$399 = 400\left(\frac{300}{300 + v_s}\right)$$

$$\therefore \quad v_s = 0.75\,\text{m/s}$$

30. By decreasing the tension in the string beat frequency is decreasing, it means frequency of string was greater than frequency of pipe. Thus,

First overtone frequency of string – Fundamental frequency of closed pipe = 8

$$\therefore \quad 2\left(\frac{v_1}{2l_1}\right) - \left(\frac{v_2}{4l_2}\right) = 8 \text{ or } v_1 = l_1\left[8 + \frac{v_2}{4l_2}\right]$$

Substituting the value, we have

$$v_1 = 0.25\left[8 + \frac{320}{4 \times 0.4}\right] = 52\,\text{m/s}$$

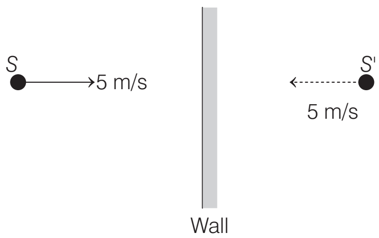
Now, $$v_1 = \sqrt{\frac{T}{\mu}}$$

$$\therefore \quad T = \mu v_1^2 = \left(\frac{m}{l}\right)v_1^2 = \left(\frac{2.5 \times 10^{-3}}{0.25}\right)(52)^2 = 27.04\,\text{N}$$

31. Frequency heard by the observer due to S' (reflected wave)

$$f' = f\left(\frac{v + v_o}{v - v_s}\right) = 256\left(\frac{330 + 5}{330 - 5}\right)$$
$$= 263.87\,\text{Hz}$$

$\therefore$ Beat frequency $f_b = f' - f = 7.87\,\text{Hz}$



Topic 5 Miscellaneous Problems

1. Loudness of sound in decible is given by

$$\beta = 10\log_{10}\left(\frac{I}{I_0}\right)$$

where, I = intensity of sound in W/m^2,

I_0 = reference intensity (= 10^{-12} W/m^2), chosen because it is near the lower limit of the human hearing range.

Here, $\beta = 120\,\text{dB}$

So, we have $120 = 10\log_{10}\left(\frac{I}{10^{-12}}\right)$

$$\Rightarrow \quad 12 = \log_{10}\left(\frac{I}{10^{-12}}\right)$$

Taking antilog, we have

$$\Rightarrow \quad 10^{12} = \frac{I}{10^{-12}}$$

$$\Rightarrow \quad I = 1\,\text{W/m}^2$$

This is the intensity of sound reaching the observer.

Now, intensity, $I = \dfrac{P}{4\pi r^2}$

where, r = distance from source,

P = power of output source.

Here, $P = 2$ W, we have

$$1 = \frac{2}{4\pi r^2} \Rightarrow r^2 = \frac{1}{2\pi} \Rightarrow r = \sqrt{\frac{1}{2\pi}}\ \text{m} = 0.398\ \text{m} \approx 40\,\text{cm}$$

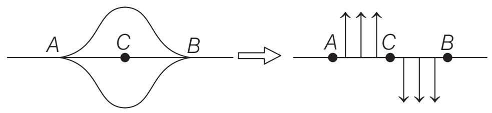
2. Let Δl be the end correction.

Given that, fundamental tone for a length 0.1m
= first overtone for the length 0.35 m.

$$\frac{v}{4(0.1+\Delta l)} = \frac{3v}{4(0.35+\Delta l)}$$

Solving this equation, we get $\Delta l = 0.025\,\text{m} = 2.5\,\text{cm}$

3. After two seconds both the pulses will move 4 cm towards each other. So, by their superposition, the resultant displacement at every point will be zero. Therefore, total energy will be purely in the form of kinetic. Half of the particles will be moving upwards and half downwards.



4. Energy $E \propto (\text{amplitude})^2\ (\text{frequency})^2$

Amplitude (A) is same in both the cases, but frequency 2ω in the second case is two times the frequency (ω) in the first case.

Therefore, $E_2 = 4E_1$

5. Mass per unit length of the string,

$$m = \frac{10^{-2}}{0.4} = 2.5 \times 10^{-2}\,\text{kg/m}$$

$\therefore$ Velocity of wave in the string,

$$v = \sqrt{\frac{T}{m}} = \sqrt{\frac{1.6}{2.5\times 10^{-2}}}$$

$$v = 8\,\text{m/s}$$

For constructive interference between successive pulses

$$\Delta t_{\min} = \frac{2l}{v} = \frac{(2)(0.4)}{8} = 0.10\,\text{s}$$

(After two reflections, the wave pulse is in same phase as it was produced, since in one reflection its phase changes by π, and if at this moment next identical pulse is produced, then constructive interference will be obtained.)

6. From Hooke's law
Tension in a string $(T) \propto$ extension (x)

and speed of sound in string

$$v = \sqrt{T/\mu}$$

or $\quad v \propto \sqrt{T}$

Therefore, $\quad v \propto \sqrt{x}$

x is increased to 1.5 times i.e. speed will increase by $\sqrt{1.5}$ times or 1.22 times. Therefore, speed of sound in new position will be 1.22 v.

7. The given equation can be written as

$$y = 2\left(2\cos^2 \frac{t}{2}\right)\sin(1000\,t)$$

$$y = 2(\cos t + 1)\sin(1000\,t)$$

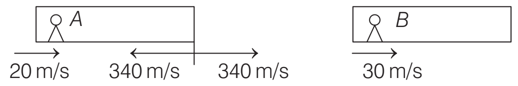
$$= 2\cos t \sin 1000\,t + 2\sin(1000\,t)$$

$$= \sin(1001\,t) + \sin(999\,t) + 2\sin(1000\,t)$$

i.e. the given expression is a result of superposition of three independent harmonic motions of angular frequencies 999, 1000 and 1001 rad/s.

8. $v_{SA} = 340 + 20 = 360\,\text{m/s}$

$v_{SB} = 340 - 30 = 310\,\text{m/s}$



9. For the passengers in train A, there is no relative motion between source and observer, as both are moving with velocity 20 m/s. Therefore, there is no change in observed frequencies and correspondingly there is no change in their intensities. Therefore, the correct option is (a).

10. For the passengers in train B, observer is receding with velocity 30 m/s and source is approaching with velocity 20 m/s.

$$\therefore \quad f'_1 = 800\left(\frac{340-30}{340-20}\right) = 775\,\text{Hz}$$

and $\quad f'_2 = 1120\left(\dfrac{340-30}{340-20}\right) = 1085\,\text{Hz}$

$\therefore$ Spread of frequency $= f'_2 - f'_1$
$= 310\,\text{Hz}$

11. In one second number of maximas is called the beat frequency. Hence,

$$f_b = f_1 - f_2 = \frac{100\pi}{2\pi} - \frac{92\pi}{2\pi} = 4\text{Hz}$$

12. Speed of wave $v = \dfrac{\omega}{k}$

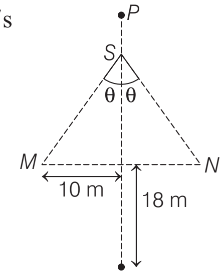
or $\quad v = \dfrac{100\pi}{0.5\,\pi}$ or $\dfrac{92\pi}{0.46\,\pi} = 200\,\text{m/s}$

13. At $\quad x = 0,\ y = y_1 + y_2 = 2A\cos 96\pi t \cos 4\pi t$

Frequency of $\cos(96\pi t)$ function is 48 Hz and that of $\cos(4\pi t)$ function is 2 Hz.

In one second, cos function becomes zero at $2f$ times, where f is the frequency. Therefore, first function will become zero at 96 times and the second at 4 times. But second will not overlap with first. Hence, net y will become zero 100 times in 1 s.

14. Speed of car, $v = 60\text{km/h} = \frac{500}{3}\text{m/s}$



At a point S, between P and Q

$$\nu'_M = \nu_M\left(\frac{C + v\cos\theta}{C}\right);$$

$$\nu'_N = \nu_N\left(\frac{C + v\cos\theta}{C}\right)$$

$$\Rightarrow \Delta v = (v_n - v_M)\left(1 + \frac{v\cos\theta}{C}\right)$$

Similarly, between Q and R

$$\Delta\nu = (\nu_N - \nu_M)\left(1 - \frac{v\cos\theta}{C}\right)$$

$$\frac{d(\Delta\nu)}{dt} = \pm(\nu_N - \nu_M)\frac{v}{C}\sin\theta\frac{d\theta}{dt}$$

$\theta = 0°$ at the P and R as they are large distance apart.

$\Rightarrow$ Slope of graph is zero.

at $Q, \theta = 90°$ $\sin\theta$ is maximum also value of $\frac{d\theta}{dt}$ is maximum

as $\frac{d\theta}{dt} = \frac{v}{r}$, where v is its velocity and r is the length of the line joining P and S and r is minimum at Q.

$\Rightarrow$ Slope is maximum at Q.

At P, $\nu_P = \Delta\nu = (\nu_N - \nu_M)\left(1 + \frac{v}{C}\right)(\theta \approx 0°)$

At R, $\nu_R = \Delta\nu = (\nu_N - \nu_M)\left(1 - \frac{v}{C}\right)(\theta \approx 0°)$

At Q, $\nu_Q = \Delta\nu = (\nu_N - \nu_M)\ (\theta = 90°)$

From these equations, we can see that

$$\nu_P + \nu_R = 2\nu_Q$$

15. Minimum length $= \frac{\lambda}{4}$

$$\lambda = 4l$$

Now, $\quad v = f\lambda = (244) \times 4 \times l$

as $\quad l = 0.350 \pm 0.005$

$\Rightarrow v$ lies between 336.7 m/s to 346.5 m/s

Now, $v = \sqrt{\frac{\gamma RT}{M \times 10^{-3}}}$, here M is molecular mass in gram

$$= \sqrt{100\gamma RT} \times \sqrt{\frac{10}{M}}.$$

For monoatomic gas, $\gamma = 1.67$

$$\Rightarrow \qquad v = 640 \times \sqrt{\frac{10}{M}}$$

For diatomic gas, $\gamma = 1.4$

$$\Rightarrow v = 590 \times \sqrt{\frac{10}{M}} \Rightarrow v_{Ne} = 640 \times \frac{7}{10} = 448\text{ m/s}$$

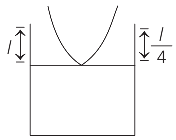
$$\Rightarrow v_{Ar} = 640 \times \frac{17}{32} = 340\text{ m/s} \Rightarrow v_{O_2} = 590 \times \frac{9}{16} = 331.8\text{ m/s}$$

$$\Rightarrow v_{N_2} = 590 \times \frac{3}{5} = 354\text{ m/s}$$

$\therefore$ Only possible answer is Argon.

16. At open end phase of pressure wave changes by 180°. So, compression returns as rarefaction. At closed end there is no phase change. So, compression returns as compression and rarefaction as rarefaction.

17. $l < \frac{\lambda}{4}$

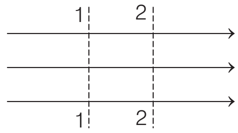


Further, larger the length of air column, feebler is the intensity.

18. In case of sound wave, y can represent pressure and displacement, while in case of an electromagnetic wave it represents electric and magnetic fields.

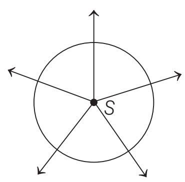
NOTE In general, y is any general physical quantity which is made to oscillate at one place and these oscillations are propagated to other places also.

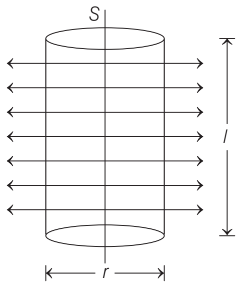
19. For a plane wave intensity (energy crossing per unit area per unit time) is constant at all points.



But for a spherical wave, intensity at a distance r from a point source of power P (energy transmitted per unit time) is given by

$$I = \frac{P}{4\pi r^2} \quad \text{or} \quad I \propto \frac{1}{r^2}$$





NOTE For a line source $I \propto \frac{1}{r}$ because, $I = \frac{P}{\pi r l}$

20. (a) and (c) options satisfy the condition;

$$\frac{\partial^2 y}{\partial x^2} = (\text{constant})\frac{\partial^2 y}{\partial t^2}$$

21. $v = 300\text{ m/s}, \quad f = 25\text{Hz}$

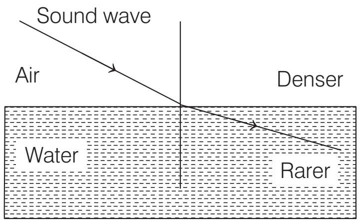
$\therefore$ Wavelength $\lambda = \frac{v}{f} = \frac{300}{25} = 12\text{ m}$

(a) Phase difference $\Delta\phi = \frac{2\pi}{\lambda}$ (path difference)

$$= \frac{2\pi}{12} \times 6 = \pi$$

(b) In plane progressive wave amplitude does not change.

22. For sound wave water is rarer medium because speed of sound wave in water is more. When a wave travels from a denser medium to rarer medium it refracts away from the normal.



23. Firstly, car will be treated as an observer which is approaching the source. Then, it will be treated as a source, which is moving in the direction of sound.

Hence, $f_1 = f_0\left(\dfrac{v+v_1}{v-v_1}\right)$

$$f_2 = f_0\left(\frac{v+v_2}{v-v_2}\right)$$

$$\therefore \quad f_1 - f_2 = \left(\frac{1.2}{100}\right) f_0 = f_0\left[\frac{v+v_1}{v-v_1} - \frac{v+v_2}{v-v_2}\right]$$

or $\left(\dfrac{1.2}{100}\right) f_0 = \dfrac{2v\,(v_1 - v_2)}{(v-v_1)(v-v_2)} f_0$

As v_1 and v_2 are very very less than v.

We can write, $(v - v_1)$ or $(v - v_2) \approx v$

$$\therefore \quad \left(\frac{1.2}{100}\right) f_0 = \frac{2\,(v_1 - v_2)}{v} f_0$$

or $(v_1 - v_2) = \dfrac{v \times 1.2}{200} = \dfrac{330 \times 1.2}{200} = 1.98 \text{ ms}^{-1}$

$$= 7.128 \text{ kmh}^{-1}$$

$\therefore$ The nearest integer is 7.

24. (a) If the detector is at $x = 0$, the two radiowaves can be represented as

$$y_1 = A \sin \omega_1 t \quad \text{and} \quad y_2 = A \sin \omega_2 t$$

(Given, $A_1 = A_2 = A$)

By the principle of superposition

$$y = y_1 + y_2 = A \sin \omega_1 t + A \sin \omega_2 t$$

$$y = 2A \cos\left(\frac{\omega_1 - \omega_2}{2} t\right) \sin\left(\frac{\omega_1 + \omega_2}{2} t\right)$$

$$= A_0 \sin\left(\frac{\omega_1 + \omega_2}{2} t\right)$$

Here, $A_0 = 2A \cos\left(\dfrac{\omega_1 - \omega_2}{2} t\right)$

Since, $I \propto (A_0)^2 \propto 4A^2 \cos^2\left(\dfrac{\omega_1 - \omega_2}{2} t\right)$

So, intensity will be maximum when

$$\cos^2\left(\frac{\omega_1 - \omega_2}{2} t\right) = \text{maximum} = 1$$

or $\cos\left(\dfrac{\omega_1 - \omega_2}{2} t\right) = \pm 1$

or $\dfrac{\omega_1 - \omega_2}{2} t = 0, \pi, 2\pi \ldots$

i.e., $t = 0, \dfrac{2\pi}{\omega_1 - \omega_2}, \dfrac{4\pi}{\omega_1 - \omega_2}, \ldots \dfrac{2n\pi}{\omega_1 - \omega_2} \quad n = 0, 1, 2 \ldots$

Therefore, time interval between any two successive maxima is $\dfrac{2\pi}{\omega_1 - \omega_2} = \dfrac{2\pi}{10^3}$ s or 6.28×10^{-3} s.

(b) The detector can detect if resultant intensity $\geq 2A^2$, or the resultant amplitude $\geq \sqrt{2}A$.

Hence, $2A \cos\left(\dfrac{\omega_1 - \omega_2}{2} t\right) \geq \sqrt{2}A$

$$\cos\left(\frac{\omega_1 - \omega_2}{2} t\right) \geq \frac{1}{\sqrt{2}}$$

Therefore, the detector lies idle. When value of $\cos\left(\dfrac{\omega_1 - \omega_2}{2} t\right)$ is between 0 and $1/\sqrt{2}$]

or when $\dfrac{\omega_1 - \omega_2}{2} t$ is between $\dfrac{\pi}{2}$ and $\dfrac{\pi}{4}$

or t lies between

$$\frac{\pi}{\omega_1 - \omega_2} \text{ and } \frac{\pi}{2(\omega_1 - \omega_2)}$$

$$\therefore \quad t = \frac{\pi}{\omega_1 - \omega_2} - \frac{\pi}{2(\omega_1 - \omega_2)}$$

$$= \frac{\pi}{2(\omega_1 - \omega_2)} = \frac{\pi}{2 \times 10^3}$$

$$t = 1.57 \times 10^{-3} \text{ s}$$

Hence, the detector lies idle for a time of 1.57×10^{-3} s in each cycle.

25. (a) Wavelength of incident wave $= \dfrac{2\pi}{a}$

and frequency of incident wave $= \dfrac{b}{2\pi}$

(b) Intensity of reflected wave has become 0.64 times. But since $I \propto A^2$ amplitude of reflected wave will become 0.8 times.

a and b will remain as it is. But direction of velocity of wave will become opposite. Further there will be a phase change of π, as it is reflected by an obstacle (denser medium). Therefore, equation of reflected wave would be

$$y_r = 0.8A \cos[ax - bt + \pi] = -0.8A \cos(ax - bt)$$

(c) The equation of resultant wave will be,

$$y = y_i + y_r = A\cos(ax + bt) - 0.8A\cos(ax - bt)$$

Particle velocity

$$v_p = \frac{\partial y}{\partial t} = -Ab\sin(ax + bt) - 0.8Ab\sin(ax - bt)$$

Maximum particle speed can be $1.8\,Ab$, where

$\sin(ax+bt) = \pm 1$ and $\sin(ax-bt) = \pm 1$

and minimum particle speed can be zero, where

$\sin(ax+bt)$ and $\sin(ax-bt)$ both are zero.

(d) The resultant wave can be written as,

$$y = [0.8\,A\cos(ax+bt) - 0.8A\cos(ax-bt)] + 0.2A\cos(ax+bt)$$

$$= -1.6A\sin ax\sin bt + 0.2A\cos(ax+bt)$$

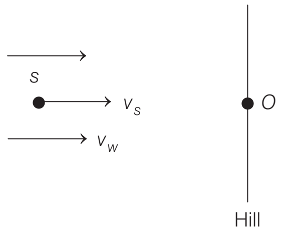
In this equation, $(-1.6A\sin ax\sin bt)$ is the equation of standing wave and $0.2\,A\cos(ax+bt)$ is the equation of travelling wave. The travelling wave is travelling in negative x-direction.

Antinodes are the points where,

$\sin ax = \pm 1$

or $\quad ax = \left[n\pi + (-1)^n\dfrac{\pi}{2}\right]$ or $x = \left[n + \dfrac{(-1)^n}{2}\right]\dfrac{\pi}{a}$

26. Given $v_s = v_w = 40$ km/h and $v = 1200$ km/h = speed of sound.

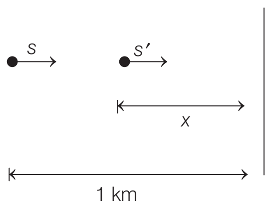


(a) Frequency observed by observer $f' = f\left(\dfrac{v+v_w}{v+v_w-v_s}\right)$

Substituting the values, we have

$$f' = 580\left[\frac{1200+40}{1200+40-40}\right] = 599.33\text{ Hz}$$

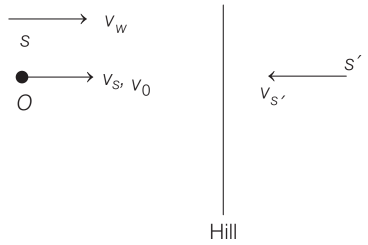
(b) Let x be the distance of the source from the hill at which echo is heard of the sound which was produced when source was at a distance 1 km from the hill. Then, time taken by the source to reach from s to s' = time taken by the sound to reach from s to hill and then from hill to s'. Thus,



$$\frac{1-x}{40} = \frac{1}{1200+40} + \frac{x}{1200-40}$$

Solving this equation, we get $x = 0.935$ km

Frequency heard by the driver of the reflected wave



$$f'' = f\left[\frac{v - v_w + v_o}{v - v_w - v_{s'}}\right] = 580\left[\frac{1200-40+40}{1200-40-40}\right] = 621.43\text{ Hz}$$

27. (a) For two waves to form a standing wave, they must be identical and should move in opposite directions. Therefore, z_1 and z_2 will produce a standing wave. The equation of standing wave in this case would be,

$z = z_1 + z_2 = 2A\cos kx\cos\omega t = A_x\cos\omega t$

Here, $\quad A_x = 2A\cos kx$

Resultant intensity will be zero, at the positions

where, $A_x = 0$

or $\quad kx = (2n+1)\dfrac{\pi}{2} \qquad$ where $n = 0, \pm 1, \pm 2$...... etc.

or $\quad x = (2n+1)\dfrac{\pi}{2k} \qquad$ where $n = 0, \pm 1, \pm 2$......etc.

(b) z_1 is a wave travelling in positive X-axis and z_3 is a wave travelling in positive Y-axis.

So, by their superposition a wave will be formed which will travel in positive x and positive y-axis. The equation of wave would be

$$z = z_1 + z_3 = 2A\cos\left[\frac{kx+ky}{2} - \omega t\right]\cos\left(\frac{kx-ky}{2}\right)$$

The resultant intensity is zero, where,

$$\cos k\left(\frac{x-y}{2}\right) = 0$$

or $$\frac{k(x-y)}{2} = (2n+1)\frac{\pi}{2}$$

or $$(x-y) = (2n+1)\frac{\pi}{k}$$

where $n = 0, \pm 1, \pm 2, \ldots$ etc.