8

Simple Harmonic Motion

Topic 1 Energy in Simple Harmonic Motion

Objective Questions I (Only one correct option)

1. A pendulum is executing simple harmonic motion and its maximum kinetic energy is K_1. If the length of the pendulum is doubled and it performs simple harmonic motion with the same amplitude as in the first case, its maximum kinetic energy is K_2. Then **(2019 Main, 11 Jan III)**

(a) $K_2 = 2K_1$ (b) $K_2 = \frac{K_1}{2}$

(c) $K_2 = \frac{K_1}{4}$ (d) $K_2 = K_1$

2. A particle undergoing simple harmonic motion has time dependent displacement given by $x(t) = A\sin\frac{\pi t}{90}$. The ratio of kinetic to potential energy of this particle at $t = 210\,\text{s}$ will be **(2019 Main, 11 Jan I)**

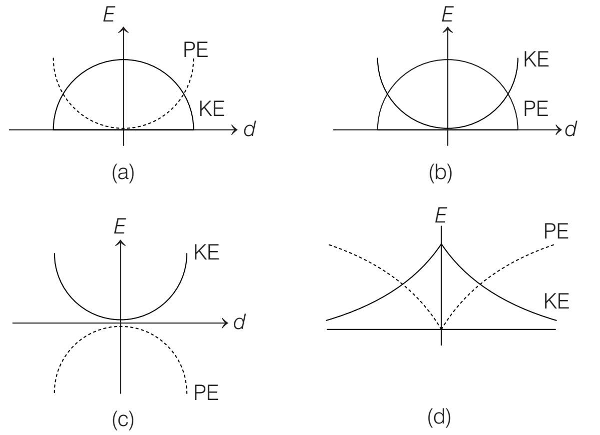
(a) 2
(b) 1
(c) $\frac{1}{9}$
(d) 3

3. A particle is executing simple harmonic motion (SHM) of amplitude A, along the X-axis, about $x = 0$. when its potential energy (PE) equals kinetic energy (KE), the position of the particle will be **(2019 Main, 9 Jan II)**

(a) A (b) $\frac{A}{2}$

(c) $\frac{A}{2\sqrt{2}}$ (d) $\frac{A}{\sqrt{2}}$

4. For a simple pendulum, a graph is plotted between its Kinetic Energy (KE) and Potential Energy (PE) against its displacement d. Which one of the following represents these correctly? (graphs are schematic and not drawn to scale) **(2015 Main)**



5. A particle executes simple harmonic motion with a frequency f. The frequency with which its kinetic energy oscillates is **(1987, 2M)**

(a) $f/2$ (b) f (c) $2f$ (d) $4f$

Objective Question II (One or more correct option)

6. A linear harmonic oscillator of force constant 2×10^6 N/m and amplitude 0.01m has a total mechanical energy of 160 J. Its **(1989, 2M)**

(a) maximum potential energy is 100 J
(b) maximum kinetic energy is 100 J
(c) maximum potential energy is 160 J
(d) maximum potential energy is zero

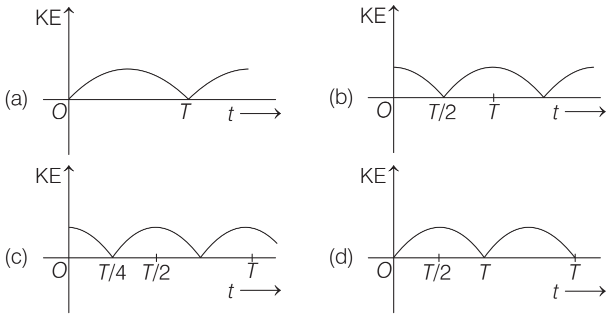
Fill in the Blank

7. An object of mass 0.2 kg executes simple harmonic oscillations along the X-axis with a frequency of $(25/\pi)$Hz. At the position $x = 0.04$, the object has kinetic energy of 0.5 J and potential energy 0.4 J. The amplitude of oscillations is ……m. **(1994, 2M)**

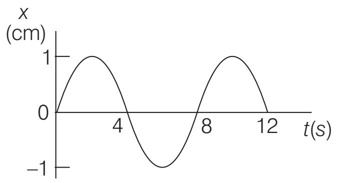
Topic 2 Graphs in Simple Harmonic Motion

Objective Questions I (Only one correct option)

1. A particle is executing simple harmonic motion with a time period T. At time $t = 0$, it is at its position of equilibrium. The kinetic energy-time graph of the particle will look, like **(2017 Main)**



2. The x-t graph of a particle undergoing simple harmonic motion is shown below. The acceleration of the particle at $t = 4/3$ s is **(2009)**



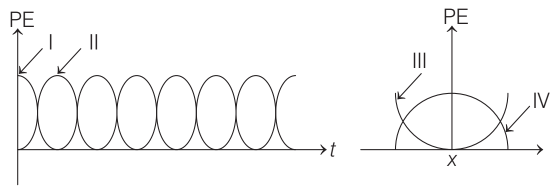
(a) $\frac{\sqrt{3}}{32}\pi^2$ cms^{-2}

(b) $\frac{-\pi^2}{32}$ cms^{-2}

(c) $\frac{\pi^2}{32}$ cms^{-2}

(d) $-\frac{\sqrt{3}}{32}\pi^2$cms^{-2}

3. For a particle executing SHM the displacement x is given by $x = A\cos\omega t$. Identify the graph which represents the variation of potential energy (PE) as a function of time t and displacement x. **(2003, 2M)**



(a) I, III (b) II, IV
(c) II, III (d) I, IV

Topic 3 Time Period of Simple Harmonic Motion

Objective Questions I (Only one correct option)

1. The displacement of a damped harmonic oscillator is given by $x(t) = e^{-0.1t}\cos(10\pi t + \phi)$.

Here, t is in seconds.

The time taken for its amplitude of vibration to drop to half of its initial value is close to **(2019 Main, 10 April I)**

(a) 27 s (b) 13 s
(c) 4 s (d) 7 s

2. A simple harmonic motion is represented by $y = 5(\sin 3\pi t + \sqrt{3}\cos 3\pi t)$ cm. The amplitude and time period of the motion are **(2019 Main, 12 Jan II)**

(a) 10 cm, $\frac{3}{2}$ s (b) 5 cm, $\frac{2}{3}$ s

(c) 5 cm, $\frac{3}{2}$ s (d) 10 cm, $\frac{2}{3}$ s

3. A particle executes simple harmonic motion with an amplitude of 5 cm. When the particle is at 4 cm from the mean position, the magnitude of its velocity in SI units is equal to that of its acceleration. Then, its periodic time (in seconds) is **(2019 Main, 10 Jan II)**

(a) $\frac{4\pi}{3}$ (b) $\frac{8\pi}{3}$ (c) $\frac{7}{3}\pi$ (d) $\frac{3}{8}\pi$

4. A student measures the time period of 100 oscillations of a simple pendulum four times. The data set is 90 s, 91 s, 92 s and 95 s. If the minimum division in the measuring clock is 1 s, then the reported mean time should be **(2016 Main)**

(a) $(92 \pm 2\text{ s})$ (b) $(92 \pm 5\text{s})$
(c) $(92 \pm 1.8\text{s})$ (d) $(92 \pm 3\text{s})$

5. A simple pendulum has time period T_1. The point of suspension is now moved upward according to the relation $y = kt^2$, $(k = 1\text{m/s}^2)$, where y is the vertical displacement. The time period now becomes T_2. The ratio of $\frac{T_1^2}{T_2^2}$ is

(Take, $g = 10\text{ m/s}^2$) **(2005, 2M)**

(a) 6/5 (b) 5/6 (c) 1 (d) 4/5

6. The period of oscillation of simple pendulum of length L suspended from the roof of the vehicle which moves without friction, down an inclined plane of inclination α, is given by **(2000, 2M)**

(a) $2\pi\sqrt{\frac{L}{g\cos\alpha}}$ (b) $2\pi\sqrt{\frac{L}{g\sin\alpha}}$

(c) $2\pi\sqrt{\frac{L}{g}}$ (d) $2\pi\sqrt{\frac{L}{g\tan\alpha}}$

7. One end of a long metallic wire of length L is tied to the ceiling. The other end is tied to a massless spring of spring constant k. A mass m hangs freely from the free end of the spring. The area of cross-section and the Young's modulus of the wire are A and Y respectively. If the mass is slightly pulled down and released, it will oscillate with a time period T equal to **(1993, 2M)**

(a) $2\pi\,(m/k)^{1/2}$ (b) $2\pi\sqrt{\dfrac{m\,(YA+kL)}{YAk}}$

(c) $2\pi\,[(mYA/kL)^{1/2}$ (d) $2\pi\,(mL/YA)^{1/2}$

8. A highly rigid cubical block A of small mass M and side L is fixed rigidly on to another cubical block B of the same dimensions and of low modulus of rigidity η such that the lower face of A completely covers the upper face of B. The lower face of B is rigidly held on a horizontal surface. A small force F is applied perpendicular to one of the side faces of A. After the force is withdrawn, block A executes small oscillations, the time period of which is given by **(1992, 2M)**

(a) $2\pi\sqrt{M\eta L}$ (b) $2\pi\sqrt{\dfrac{M\eta}{L}}$

(c) $2\pi\sqrt{\dfrac{ML}{\eta}}$ (d) $2\pi\sqrt{\dfrac{M}{\eta L}}$

9. A uniform cylinder of length L and mass M having cross-sectional area A is suspended, with its length vertical, from a fixed point by a massless spring, such that it is half-submerged in a liquid of density ρ at equilibrium position. When the cylinder is given a small downward push and released it starts oscillating vertically with a small amplitude. If the force constant of the spring is k, the frequency of oscillation of the cylinder is **(1990, 2M)**

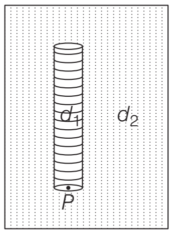
(a) $\dfrac{1}{2\pi}\left(\dfrac{k-A\rho g}{M}\right)^{1/2}$ (b) $\dfrac{1}{2\pi}\left(\dfrac{k+A\rho g}{M}\right)^{1/2}$

(c) $\dfrac{1}{2\pi}\left(\dfrac{k+\rho gL^2}{M}\right)^{1/2}$ (d) $\dfrac{1}{2\pi}\left(\dfrac{k+A\rho g}{A\rho g}\right)^{1/2}$

Analytical & Descriptive Questions

10. A solid sphere of radius R is floating in a liquid of density ρ with half of its volume submerged. If the sphere is slightly pushed and released, it starts performing simple harmonic motion. Find the frequency of these oscillations. **(2004, 4M)**

11. A thin rod of length L and uniform cross-section is pivoted at its lowest point P inside a stationary homogeneous and non-viscous liquid. The rod is free to rotate in a vertical plane about a horizontal axis passing through P.



The density d_1 of the material of the rod is smaller than the density d_2 of the liquid. The rod is displaced by small angle θ from its equilibrium position and then released. Show that the motion of the rod is simple harmonic and determine its angular frequency in terms of the given parameters. **(1996, 5M)**

12. A thin fixed ring of radius 1 m has a positive charge 1×10^{-5} C uniformly distributed over it. A particle of mass 0.9 g and having a negative charge of 1×10^{-6} C is placed on the axis at a distance of 1 cm from the centre of the ring. Show that the motion of the negatively charged particle is approximately simple harmonic. Calculate the time period of oscillations. **(1982, 5M)**

13. An ideal gas is enclosed in a vertical cylindrical container and supports a freely moving piston of mass M. The piston and the cylinder have equal cross-sectional area A. Atmospheric pressure is p_0 and when the piston is in equilibrium, the volume of the gas is V_0. The piston is now displaced slightly from its equilibrium position. Assuming that the system is completely isolated from its surroundings, show that the piston executes simple harmonic motion and find the frequency of oscillation. **(1981, 6M)**

14. A point mass m is suspended at the end of massless wire of length L and cross-sectional area A. If Y is the Young's modulus of elasticity of the material of the wire, obtain the expression for the frequency of the simple harmonic motion along the vertical line. **(1978)**

Integer Answer Type Question

15. A 0.1 kg mass is suspended from a wire of negligible mass. The length of the wire is 1 m and its cross-sectional area is 4.9×10^{-7} m^2. If the mass is pulled a little in the vertically downward direction and released, it performs simple harmonic motion of angular frequency 140 rad s^{-1}. If the Young's modulus of the material of the wire is $n\times10^{9}$ Nm^{-2}, the value of n is. **(2010)**

Topic 4 Spring Based Problems

Objective Questions I (Only one correct option)

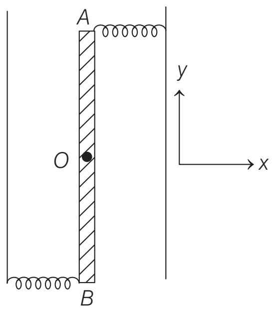
1. A spring whose unstretched length is l has a force constant k. The spring is cut into two pieces of unstretched lengths l_1 and l_2 where, $l_1 = nl_2$ and n is an integer. The ratio k_1 / k_2 of the corresponding force constants k_1 and k_2 will be **(2019 Main, 12 April II)**

(a) n (b) $\frac{1}{n^2}$ (c) $\frac{1}{n}$ (d) n^2

2. A massless spring ($k = 800$ N/m), attached with a mass (500 g) is completely immersed in 1 kg of water. The spring is stretched by 2 cm and released, so that it starts vibrating. What would be the order of magnitude of the change in the temperature of water when the vibrations stop completely? (Assume that the water container and spring receive negligible heat and specific heat of mass = 400 J/kg K, specific heat of water = 4184 J/kg K) **(2019 Main, 9 April II)**

(a) 10^{-4} K (b) 10^{-3} K (c) 10^{-1} K (d) 10^{-5} K

3. Two light identical springs of spring constant k are attached horizontally at the two ends of an uniform horizontal rod AB of length l and mass m. The rod is pivoted at its centre 'O' and can rotate freely in horizontal plane. The other ends of the two springs are fixed to rigid supports as shown in figure.

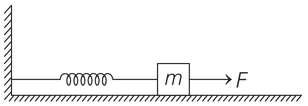


The rod is gently pushed through a small angle and released. The frequency of resulting oscillation is **(2019 Main, 12 Jan I)**

(a) $\frac{1}{2\pi}\sqrt{\frac{2k}{m}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{3k}{m}}$

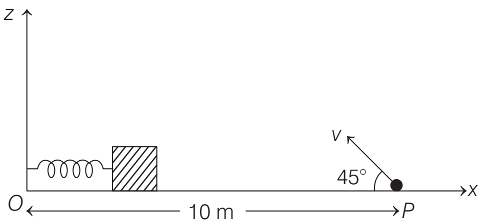
(c) $\frac{1}{2\pi}\sqrt{\frac{6k}{m}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$

4. A block of mass m lying on a smooth horizontal surface is attached to a spring (of negligible mass) of spring constant k. The other end of the spring is fixed as shown in the figure. The block is initially at rest in its equilibrium position. If now the block is pulled with a constant force F, the maximum speed of the block is **(2019 Main, 09 Jan I)**



(a) $\frac{\pi F}{\sqrt{mk}}$ (b) $\frac{F}{\sqrt{mk}}$ (c) $\frac{2F}{\sqrt{mk}}$ (d) $\frac{F}{\pi\sqrt{mk}}$

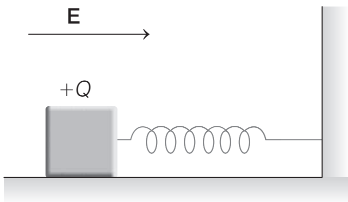
5. A small block is connected to one end of a massless spring of unstretched length 4.9 m. The other end of the spring (see the figure) is fixed. The system lies on a horizontal frictionless surface. The block is stretched by 0.2 m and released from rest at $t = 0$. It then executes simple harmonic motion with angular frequency $\omega = \frac{\pi}{3}$ rad/s. Simultaneously at $t = 0$, a small pebble is projected with speed v from point P at an angle of 45° as shown in the figure.



Point P is at a horizontal distance of 10 m from O. If the pebble hits the block at $t = 1$ s, the value of v is (take, $g = 10$ m/s^2) **(2012)**

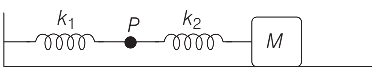
(a) $\sqrt{50}$ m/s (b) $\sqrt{51}$ m/s (c) $\sqrt{52}$ m/s (d) $\sqrt{53}$ m/s

6. A wooden block performs SHM on a frictionless surface with frequency ν_0. The block carries a charge $+Q$ on its surface. If now a uniform electric field **E** is switched-on as shown, then the SHM of the block will be **(2011)**



(a) of the same frequency and with shifted mean position
(b) of the same frequency and with the same mean position
(c) of changed frequency and with shifted mean position
(d) of changed frequency and with the same mean position

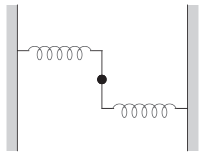
7. The mass M shown in the figure oscillates in simple harmonic motion with amplitude A. The amplitude of the point P is **(2009)**



(a) $\frac{k_1 A}{k_2}$ (b) $\frac{k_2 A}{k_1}$ (c) $\frac{k_1 A}{k_1 + k_2}$ (d) $\frac{k_2 A}{k_1 + k_2}$

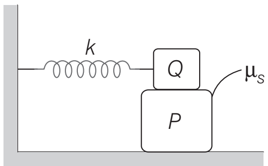
8. A uniform rod of length L and mass M is pivoted at the centre. Its two ends are attached to two springs of equal spring constants k. The springs are fixed to rigid supports as shown in the figure, and rod is free to oscillate in the horizontal plane.

The rod is gently pushed through a small angle θ in one direction and released. The frequency of oscillation is **(2009)**



(a) $\frac{1}{2\pi}\sqrt{\frac{2k}{M}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{k}{M}}$ (c) $\frac{1}{2\pi}\sqrt{\frac{6k}{M}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{24k}{M}}$

9. A block P of mass m is placed on a horizontal frictionless plane. A second block of same mass m is placed on it and is connected to a spring of spring constant k, the two blocks are pulled by a distance A. Block Q oscillates without slipping. What is the maximum value of frictional force between the two blocks? **(2004, 2M)**



(a) $kA/2$ (b) kA (c) $\mu_s mg$ (d) Zero

10. A spring of force constant k is cut into two pieces such that one piece is double the length of the other. Then, the long piece will have a force constant of **(1999, 2M)**

(a) $\frac{2}{3}k$ (b) $\frac{3}{2}k$ (c) $3k$ (d) $6k$

11. Two bodies M and N of equal masses are suspended from two separate massless springs of spring constants k_1 and k_2 respectively. If the two bodies oscillate vertically such that their maximum velocities are equal, the ratio of the one amplitude of vibration of M to that of N is **(1988, 1M)**

(a) k_1/k_2 (b) $\sqrt{k_2/k_1}$ (c) k_2/k_1 (d) $\sqrt{k_1/k_2}$

Objective Question II (One or more correct option)

12. A particle of mass m is attached to one end of a massless spring of force constant k, lying on a frictionless horizontal plane. The other end of the spring is fixed. The particle starts moving horizontally from its equilibrium position at time $t = 0$ with an initial velocity u_0. When the speed of the particle is $0.5\,u_0$, it collides elastically with a rigid wall. After this collision **(2013 Adv.)**

(a) the speed of the particle when it returns to its equilibrium position is u_0

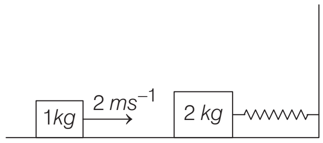
(b) the time at which the particle passes through the equilibrium position for the first time is $t = \pi\sqrt{\frac{m}{k}}$

(c) the time at which the maximum compression of the spring occurs is $t = \frac{4\pi}{3}\sqrt{\frac{m}{k}}$

(d) the time at which the particle passes through the equilibrium position for the second time is $t = \frac{5\pi}{3}\sqrt{\frac{m}{k}}$

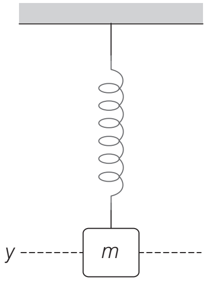
Numerical Value

13. A spring block system is resting on a frictionless floor as shown in the figure. The spring constant is $2.0\ \mathrm{Nm^{-1}}$ and the mass of the block is $2.0\,\mathrm{kg}$. Ignore the mass of the spring. Initially, the spring is in an unstretched condition. Another block of mass $1.0\ \mathrm{kg}$ moving with a speed of $2.0\ \mathrm{ms^{-1}}$ collides elastically with the first block. The collision is such that the $2.0\,\mathrm{kg}$ block does not hit the wall. The distance, in metres, between the two blocks when the spring returns to its unstretched position for the first time after the collision is .... **(2018 Adv.)**



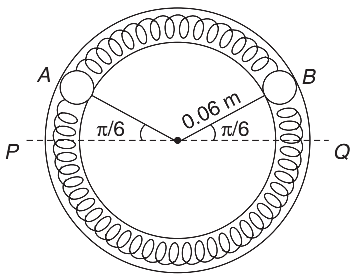
Analytical & Descriptive Questions

14. A mass m is undergoing SHM in the vertical direction about the mean position y_0 with amplitude A and angular frequency ω. At a distance y from the mean position, the mass detaches from the spring. Assume that the spring contracts and does not obstruct the motion of m. Find the distance y (measured from the mean position) such that the height h attained by the block is maximum. ($A\omega^2 > g$). **(2005)**



15. Two identical balls A and B, each of mass 0.1 kg, are attached to two identical massless springs. The spring-mass system is constrained to move inside a rigid smooth pipe bent in the form of a circle as shown in figure. The pipe is fixed in a horizontal plane. The centres of the balls can move in a circle of radius 0.06 m. Each spring has a natural length of 0.06π m and spring constant 0.1 N/m.

Initially, both the balls are displaced by an angle $\theta = \pi/6$ rad with respect to the diameter PQ of the circle (as shown in figure) and released from rest. **(1993, 6M)**



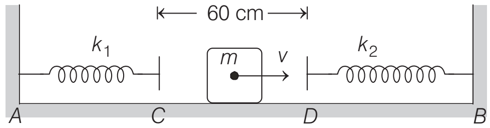
(a) Calculate the frequency of oscillation of ball B.

(b) Find the speed of ball A when A and B are at the two ends of the diameter PQ

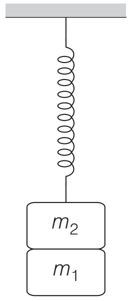
(c) What is the total energy of the system?

16. Two light springs of force constants k_1 and k_2 and a block of mass m are in one line AB on a smooth horizontal table such that one end of each spring is fixed on rigid supports and the other end is free as shown in the figure.

The distance CD between the free ends of the spring is 60 cm. If the block moves along AB with a velocity 120 cm/s in between the springs, calculate the period of oscillation of the block.
(Take, $k_1 = 1.8$ N/m, $k_2 = 3.2$ N/m, $m = 200$ g) **(1985, 6M)**



17. Two masses m_1 and m_2 are suspended together by a massless spring of spring constant k (Fig.). When the masses are in equilibrium, m_1 is removed without disturbing the system. Find the angular frequency and amplitude of oscillation of m_2. **(1981, 3M)**



18. A mass M attached to a spring oscillates with a period of 2 s. If the mass is increased by 2 kg the period increases by one sec. Find the initial mass M assuming that Hooke's law is obeyed. **(1979)**

Topic 5 Miscellaneous Problems

Objective Questions I (Only one correct option)

1. A simple pendulum oscillating in air has period T. The bob of the pendulum is completely immersed in a non-viscous liquid. The density of the liquid is $\frac{1}{16}$th of the material of the bob. If the bob is inside liquid all the time, its period of oscillation in this liquid is **(Main 2019, 9 April I)**

(a) $2T\sqrt{\frac{1}{10}}$ (b) $2T\sqrt{\frac{1}{14}}$
(c) $4T\sqrt{\frac{1}{14}}$ (d) $4T\sqrt{\frac{1}{15}}$

2. A damped harmonic oscillator has a frequency of 5 oscillations per second. The amplitude drops to half its value for every 10 oscillations. The time it will take to drop to $\frac{1}{1000}$ of the original amplitude is close to **(2019 Main, 8 April II)**

(a) 20 s (b) 50 s
(c) 100 s (d) 10 s

3. A simple pendulum of length 1 m is oscillating with an angular frequency 10 rad/s. The support of the pendulum starts oscillating up and down with a small angular frequency of 1 rad/s and an amplitude of 10^{-2} m. The relative change in the angular frequency of the pendulum is best given by **(2019 Main, 11 Jan II)**

(a) 1 rad/s (b) 10^{-5} rad/s
(c) 10^{-3} rad/s (d) 10^{-1} rad/s

4. A cylindrical plastic bottle of negligible mass is filled with 310 mL of water and left floating in a pond with still water. If pressed downward slightly and released, it starts performing simple harmonic motion at angular frequency ω. If the radius of the bottle is 2.5 cm, then ω is close to (Take, density of water $= 10^3$ kg/m^3) **(2019 Main, 10 Jan II)**

(a) 2.50 rad s^{-1} (b) 5.00 rad s^{-1}
(c) 1.25 rad s^{-1} (d) 3.75 rad s^{-1}

5. A rod of mass 'M' and length '$2L$' is suspended at its middle by a wire. It exhibits torsional oscillations. If two masses each of 'm' are attached at distance '$L/2$' from its centre on both sides, it reduces the oscillation frequency by 20%. The value of ratio m/M is close to **(2019 Main, 9 Jan II)**

(a) 0.57 (b) 0.37
(c) 0.77 (d) 0.17

6. A silver atom in a solid oscillates in simple harmonic motion in some direction with a frequency of 10^{12} per second. What is the force constant of the bonds connecting one atom with the other? (Take, molecular weight of silver = 108 and Avogadro number $= 6.02 \times 10^{23}$ g mol^{-1}) **(2018 Main)**

(a) 5.5 N/m (b) 6.4 N/m (c) 7.1 N/m (d) 2.2 N/m

7. A magnetic needle of magnetic moment 6.7×10^{-2} $\mathbf{Am}^2$ and moment of inertia 7.5×10^{-6} kg $\mathbf{m}^2$ is performing simple harmonic oscillations in a magnetic field of 0.01 T. Time taken for 10 complete oscillations is **(2017 Main)**

(a) 8.89 s (b) 6.98 s (c) 8.76 s (d) 6.65s

8. A particle performs simple harmonic motion with amplitude A. Its speed is trebled at the instant that it is at a distance $\frac{2}{3}A$ from equilibrium position. The new amplitude of the motion is **(2016 Main)**

(a) $\frac{A}{3}\sqrt{41}$ (b) $3A$ (c) $A\sqrt{3}$ (d) $\frac{7}{3}A$

9. A particle moves with simple harmonic motion in a straight line. In first τ sec, after starting from rest it travels a distance a and in next τ sec, it travels $2a$, in same direction, then **(2014 Main)**

(a) amplitude of motion is $3a$
(b) time period of oscillations is 8π
(c) amplitude of motion is $4a$
(d) time period of oscillations is 6π

10. A point mass is subjected to two simultaneous sinusoidal displacements in x-direction, $x_1(t) = A\sin\omega t$ and $x_2(t) = A\sin\left(\omega t + \frac{2\pi}{3}\right)$. Adding a third sinusoidal displacement $x_3(t) = B\sin(\omega t + \phi)$ brings the mass to a complete rest. The values of B and ϕ are **(2011)**

(a) $\sqrt{2}A, \frac{3\pi}{4}$ (b) $A, \frac{4\pi}{3}$

(c) $\sqrt{3}A, \frac{5\pi}{6}$ (d) $A, \frac{\pi}{3}$

11. A particle executes simple harmonic motion between $x = -A$ and $x = +A$. The time taken for it to go from O to $A/2$ is T_1 and to go from $A/2$ to A is T_2, then **(2001, 2M)**

(a) $T_1 < T_2$ (b) $T_1 > T_2$

(c) $T_1 = T_2$ (d) $T_1 = 2T_2$

12. A particle free to move along the X-axis has potential energy given by $U(x) = k[1 - \exp(-x^2)]$ for $-\infty \le x \le +\infty$, where k is a positive constant of appropriate dimensions. Then, **(1999, 2M)**

(a) at points away from the origin, the particle is in unstable equilibrium
(b) for any finite non-zero value of x, there is a force directed away from the origin
(c) if its total mechanical energy is $k/2$, it has its minimum kinetic energy at the origin
(d) for small displacements from $x = 0$, the motion is simple harmonic

13. A particle of mass m is executing oscillation about the origin on the X-axis. Its potential energy is $U(x) = k|x|^3$, where k is a positive constant. If the amplitude of oscillation is a, then its time period T is **(1998, 2M)**

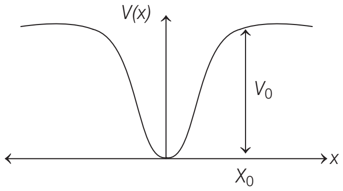
(a) proportional to $1/\sqrt{a}$ (b) independent of a

(c) proportional to $\sqrt{a}$ (d) proportional to $a^{3/2}$

Passage Based Questions

Passage

When a particle of mass m moves on the X-axis in a potential of the form $V(x) = kx^2$, it performs simple harmonic motion. The corresponding time period is proportional to $\sqrt{m/k}$, as can be seen easily using dimensional analysis. However, the motion of a particle can be periodic even when its potential energy increases on both sides of $x = 0$ in a way different from kx^2 and its total energy is such that the particle does not escape to infinity. Consider a particle of mass m moving on the X-axis. Its potential energy is $V(x) = \alpha x^4 (\alpha > 0)$ for $|x|$ near the origin and becomes a constant equal to V_0 for $|x| \ge X_0$ (see figure). **(2010)**



14. If the total energy of the particle is E, it will perform periodic motion only if

(a) $E < 0$ (b) $E > 0$ (c) $V_0 > E > 0$ (d) $E > V_0$

15. For periodic motion of small amplitude A, the time period T of this particle is proportional to

(a) $A\sqrt{\frac{m}{\alpha}}$ (b) $\frac{1}{A}\sqrt{\frac{m}{\alpha}}$

(c) $A\sqrt{\frac{\alpha}{m}}$ (d) $\frac{1}{A}\sqrt{\frac{\alpha}{m}}$

16. The acceleration of this particle for $|x| > X_0$ is

(a) proportional to V_0 (b) proportional to $\frac{V_0}{mX_0}$

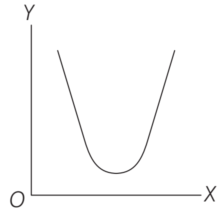
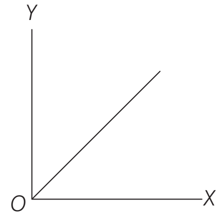
(c) proportional to $\sqrt{\frac{V_0}{mX_0}}$ (d) zero

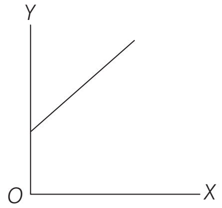
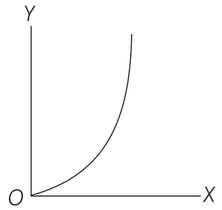
Fill in the Blank

17. Two simple harmonic motions are represented by the equations $y_1 = 10\sin(3\pi t + \pi/4)$ and $y_2 = 5(\sin 3\pi t + \sqrt{3}\cos 3\pi t)$. Their amplitudes are in the ratio of .... **(1986, 2M)**

Match the Columns

18. Column I gives a list of possible set of parameters measured in some experiments. The variations of the parameters in the form of graphs are shown in Column II. Match the set of parameters given in Column I with the graphs given in Column II. **(2008, 7M)**

	Column I		Column II
(A)	Potential energy of a simple pendulum (Y-axis) as a function of displacement (X-axis).	(p)	
(B)	Displacement (Y-axis) as a function of time (X-axis) for a one dimensional motion at zero or constant acceleration when the body is moving along the positive x-direction.	(q)	

Column I	Column II
(C) Range of a projectile (Y-axis) as a function of its velocity (X-axis) when projected at a fixed angle.	(r) 
(D) The square of the time period (Y-axis) of a simple pendulum as a function of its length (X-axis).	(s) 

19. Column I describes some situations in which a small object moves. Column II describes some characteristics of these motions. Match the situations in Column I with the characteristics in Column II. **(2007, 6M)**

Column I	Column II
(A) The object moves on the x-axis under a conservative force in such a way that its speed and position satisfy $v = c_1\sqrt{c_2 - x^2}$, where c_1 and c_2 are positive constants.	(p) The object executes a simple harmonic motion.
(B) The object moves on the x-axis in such a way that its velocity and its displacement from the origin satisfy $v = -kx$, where k is a positive constant.	(q) The object does not change its direction.
(C) The object is attached to one end of a mass-less spring of a given spring constant. The other end of the spring is attached to the ceiling of an elevator. Initially everything is at rest. The elevator starts going upwards with a constant acceleration a. The motion of the object is observed from the elevator during the period it maintains this acceleration.	(r) The kinetic energy of the object keeps on decreasing.
(D) The object is projected from the earth's surface vertically upwards with a speed $2\sqrt{\frac{GM_e}{R_e}}$, where M_e is the mass of the earth and R_e is the radius of the earth. Neglect forces from objects other than the earth.	(s) The object can change its direction only once.

Objective Questions II (One or more correct option)

20. A block with mass M is connected by a massless spring with stiffness constant k to a rigid wall and moves without friction on a horizontal surface. The block oscillates with small amplitude A about an equilibrium position x_0. Consider two cases : (i) when the block is at x_0 and (ii) when the block is at $x = x_0 + A$. In both the cases, a particle with mass m ($< M$) is softly placed on the block after which they stick to each other. Which of the following statement(s) is (are) true about the motion after the mass m is placed on the mass M? **(2016 Adv.)**

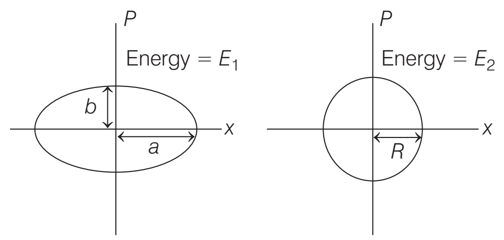
(a) The amplitude of oscillation in the first case changes by a factor of $\sqrt{\frac{M}{m+M}}$, whereas in the second case it remains unchanged

(b) The final time period of oscillation in both the cases is same

(c) The total energy decreases in both the cases

(d) The instantaneous speed at x_0 of the combined masses decreases in both the cases

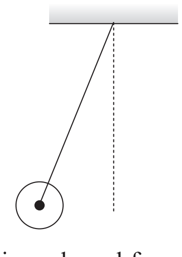
21. Two independent harmonic oscillators of equal masses are oscillating about the origin with angular frequencies ω_1 and ω_2 and have total energies E_1 and E_2, respectively. The variations of their momenta p with positions x are shown in the figures. If $\frac{a}{b} = n^2$ and $\frac{a}{R} = n$, then the correct equations is/are **(2015 Adv.)**



(a) $E_1\omega_1 = E_2\omega_2$ (b) $\frac{\omega_2}{\omega_1} = n^2$

(c) $\omega_1\omega_2 = n^2$ (d) $\frac{E_1}{\omega_1} = \frac{E_2}{\omega_2}$

22. A metal rod of length L and mass m is pivoted at one end. A thin disc of mass M and radius R ($< L$) is attached at its centre to the free end of the rod. Consider two ways the disc is attached. **case *A***—the disc is not free to rotate about its centre and **case *B***—the disc is free to rotate about its centre. The rod-disc system performs SHM in vertical plane after being released from the same displaced position. Which of the following statement(s) is/are true? **(2011)**



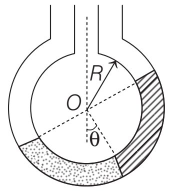
(a) Restoring torque in case A = Restoring torque in case B
(b) Restoring torque in case A < Restoring torque in case B
(c) Angular frequency for case A > Angular frequency for case$B$
(d) Angular frequency for case A < Angular frequency for case B

23. Function $x = A\sin^2\omega t + B\cos^2\omega t + C\sin\omega t\cos\omega t$ represents SHM **(2006, 5M)]**
(a) For any value of A, B and C (except $C = 0$)
(b) If $A = -B$, $C = 2B$, amplitude = $|B\sqrt{2}|$
(c) If $A = B$; $C = 0$
(d) If $A = B$; $C = 2B$, amplitude = $|B|$

24. Three simple harmonic motions in the same direction having the same amplitude and same period are superposed. If each differ in phase from the next by 45°, then **(1999, 3M)**
(a) the resultant amplitude is $(1+\sqrt{2})a$
(b) the phase of the resultant motion relative to the first is 90°
(c) the energy associated with the resulting motion is $(3+2\sqrt{2})$ times the energy associated with any single motion
(d) the resulting motion is not simple harmonic

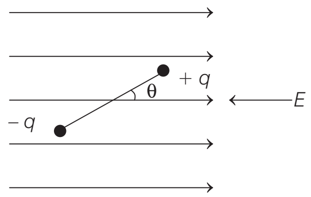
Analytical & Descriptive Questions

25. Two non-viscous, incompressible and immiscible liquids of densities ρ and $1.5\,\rho$ are poured into the two limbs of a circular tube of radius R and small cross-section kept fixed in a vertical plane as shown in figure. Each liquid occupies one-fourth the circumference of the tube. **(1991, 4 + 4 M)**



(a) Find the angle θ that the radius to the interface makes with the vertical in equilibrium position.
(b) If the whole liquid column is given a small displacement from its equilibrium position, show that the resulting oscillations are simple harmonic. Find the time period of these oscillations.

26. A point particle of mass M attached to one end of a massless rigid non- conducting rod of length L. Another point particle of the same mass is attached to the other end of the rod. The two particles carry charges $+q$ and $-q$ respectively. This arrangement is held in a region of a uniform electric field E such that the rod makes a small angle θ (say of about 5 degrees) with the field direction. Find an expression for the minimum time needed for the rod to become parallel to the field after it is set free. **(1989, 8M)**



Answers

Topic 1

1. (b) **2.** (*) **3.** (d)
4. (a) **5.** (c) **6.** (b, c) **7.** 0.06

Topic 2

1. (c) **2.** (d) **3.** (a)

Topic 3

1. (d) **2.** (d) **3.** (b)
4. (a) **5.** (a) **6.** (a) **7.** (b)
8. (d) **9.** (b) **10.** $\dfrac{1}{2\pi}\sqrt{\dfrac{3g}{2R}}$
11. $\omega = \sqrt{\dfrac{3g(d_2 - d_1)}{2d_1L}}$ **12.** 0.628 s
13. $\dfrac{1}{2\pi}\sqrt{\dfrac{\gamma\,(p_0A^2 + MgA)}{V_0M}}$ **14.** $f = \dfrac{1}{2\pi}\sqrt{\dfrac{YA}{mL}}$ **15.** 4

Topic 4

1. (c) **2.** (d) **3.** (c) **4.** (b)
5. (a) **6.** (a) **7.** (d)
8. (c) **9.** (a) **10.** (b) **11.** (b)
12. (a, d) **13.** (2.09) **14.** $\dfrac{g}{\omega^2}$
15. (a) $\dfrac{1}{\pi}$ Hz (b) 0.0628 m/s (c) 3.9×10^{-4} J
16. 2.82 s **17.** $\omega = \sqrt{\dfrac{k}{m_2}}$, $A = \dfrac{m_1g}{k}$ **18.** $M = 1.6$ kg

Topic 5

1. (d) **2.** (a) **3.** (c) **4.** (*)
5. (b) **6.** (c) **7.** (d) **8.** (d)
9. (d) **10.** (b) **11.** (a) **12.** (d)
13. (a) **14.** (c) **15.** (b) **16.** (d)
17. 1 : 1
18. A $\to$ p or p, s B $\to$ q, s or q, r, s C $\to$ s D $\to$ q
19. A $\to$ p B $\to$ q,r C $\to$ p D $\to$ q, r
20. (a, b, d) **21.** (b, d) **22.** (a, d) **23.** (b, d)
24. (a, c) **25.** (a) $\theta = \tan^{-1}\left(\dfrac{1}{5}\right)$ (b) $2\pi\sqrt{\dfrac{R}{6.11}}$
26. $\dfrac{\pi}{2}\sqrt{\dfrac{ML}{2qE}}$

Hints & Solutions

Topic 1 Energy in Simple Harmonic Motion

1. Kinetic energy of a pendulum is maximum at its mean position. Also, maximum kinetic energy of pendulum

$$K_{\max} = \frac{1}{2} m\omega^2 a^2$$

where,angular frequency

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{2\pi\sqrt{\frac{l}{g}}}$$

or $\omega = \sqrt{\frac{g}{l}}$ or $\omega^2 = \frac{g}{l}$

and a = amplitude.

As amplitude is same in both cases so;

$$K_{\max} \propto \omega^2$$

or $K_{\max} \propto \frac{1}{l}$ [$\because g$ is constant]

According to given data, $K_1 \propto \frac{1}{l}$

and $K_2 \propto \frac{1}{2l}$

$\therefore$ $\frac{K_1}{K_2} = \left(\frac{1/l}{1/2l}\right) = 2$

or $K_1 = 2K_2 \Rightarrow K_2 = \frac{K_1}{2}$

2. Here given, displacement, $x(t) = A \sin \frac{\pi t}{90}$

where A is amplitude of S.H.M., t is time taken by particle to reach a point where its potential energy $U = \frac{1}{2}kx^2$ and kinetic energy $= \frac{1}{2}k(A^2 - x^2)$ here k is force constant and x is position of the particle.

Potential energy (U) at $t = 210$ s is

$$U = \frac{1}{2}kx^2 = \frac{1}{2}kA^2 \sin^2\left(\frac{210}{90}\pi\right)$$

$$= \frac{1}{2}kA^2 \sin^2\left(2\pi + \frac{3}{9}\pi\right) = \frac{1}{2}kA^2 \sin^2\left(\frac{\pi}{3}\right)$$

Kinetic energy at $t = 210$ s, is

$$K = \frac{1}{2}k(A^2 - x^2)$$

$$= \frac{1}{2}kA^2\left[1 - \sin^2\left(\frac{210\pi}{90}\right)\right]$$

$$= \frac{1}{2}kA^2 \cos^2(210\,\pi/90)$$

$$\Rightarrow \quad K = \frac{1}{2}kA^2 \cos^2(\pi/3)$$

So, ratio of kinetic energy to potential energy is

$$\frac{K}{U} = \frac{\frac{1}{2}kA^2\cos^2(\pi/3)}{\frac{1}{2}kA^2\sin^2(\pi/3)} = \cot^2(\pi/3) = \frac{1}{3}$$

$\therefore$ No option given is correct.

3. Here, A = amplitude of particle in SHM

We know that in SHM potential energy (U) of a particle is given by the relation at distance x from the mean position is

$$U = \frac{1}{2}kx^2$$

and at the same point kinetic energy (K)

= Total energy – Potential energy

$$= \frac{1}{2}kA^2 - \frac{1}{2}kx^2$$

According to the question,

potential energy = kinetic energy

$\therefore$ $\frac{1}{2}kx^2 = \frac{1}{2}kA^2 - \frac{1}{2}kx^2$

or $kx^2 = \frac{kA^2}{2}$ or $x = \pm \frac{A}{\sqrt{2}}$

4. Taking minimum potential energy at mean position to be zero, the expression of KE and PE are

$\text{KE} = \frac{1}{2}m\omega^2(A^2 - d^2)$ and $\text{PE} = \frac{1}{2}m\omega^2 d^2$

Both graphs are parabola. At $d = 0$, the mean position,

$\text{PE} = 0$ and $\text{KE} = \frac{1}{2}m\omega^2 A^2$ = maximum

At $d = \pm A$, the extreme positions,

$\text{KE} = 0$ and $\text{PE} = \frac{1}{2}m\omega^2 A^2$ = maximum

Therefore, the correct graph is (a).

5. In SHM, frequency with which kinetic energy oscillates is two times the frequency of oscillation of displacement.

$\therefore$ Correct answer is (c).

6. $\frac{1}{2}kA^2 = \frac{1}{2} \times 2 \times 10^6 \times (10^{-2})^2 = 100$ J

This is basically the energy of oscillation of the particle.

K, U and E at mean position ($x = 0$) and extreme position ($x = \pm A$) are shown in figure.

$x = 0$	$x = A$
$K = 100$ J = Maximum	$K = 0$ J
$U = 60$ J = Minimum	$U = 160$ J = Maximum
$E = 160$ J = Constant	$E = 160$ J = Constant

$\therefore$ Correct options are (b) and (c).

7. Since, $\nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$\Rightarrow \quad \frac{25}{\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{0.2}}$

or $\quad k = 50 \times 50 \times 0.2 = 500\,\text{N/m}$

If A is the amplitude of oscillation,

Total energy = KE + PE

$$\frac{1}{2}kA^2 = 0.5 + 0.4$$

$$A = \sqrt{\left(\frac{2 \times 0.9}{500}\right)} = 0.06\,\text{m}$$

Topic 2 Graphs in Simple Harmonic Motion

1. KE is maximum at mean position and minimum at extreme position $\left(\textbf{at } t = \frac{T}{4}\right)$.

2. $T = 8\,\text{s},\ \omega = \frac{2\pi}{T} = \left(\frac{\pi}{4}\right)\text{rads}^{-1}$

$\Rightarrow \quad x = A\sin\omega t$

$\therefore \quad a = -\omega^2 x = -\left(\frac{\pi^2}{16}\right)\sin\left(\frac{\pi}{4}t\right)$

Substituting $t = \frac{4}{3}$ s, we get

$$a = -\left(\frac{\sqrt{3}}{32}\pi^2\right)\text{cm-s}^{-2}$$

3. Potential energy is minimum (in this case zero) at mean position $(x = 0)$ and maximum at extreme positions $(x = \pm A)$.

At time $t = 0, x = A$. Hence, PE should be maximum. Therefore, graph I is correct. Further in graph III, PE is minimum at $x = 0$. Hence, this is also correct .

Topic 3 Time Period of Simple Harmonic Motion

1. Given, displacement is

$$x(t) = e^{-0.1t}\cos(10\pi t + \phi)$$

Here, amplitude of the oscillator is

$$A = e^{-0.1t} \qquad \ldots \text{(i)}$$

Let it takes t seconds for amplitude to be dropped by half.

At $\quad t = 0 \Rightarrow A = 1 \quad$ [from Eq. (i)]

At $\quad t = t \Rightarrow A' = \frac{A}{2} = \frac{1}{2}$

So, Eq. (i) can be written as

$$e^{-0.1t} = \frac{1}{2}$$

or $\quad e^{0.1t} = 2$

or $\quad 0.1t = \ln(2)$

or $\quad t = \frac{1}{0.1}\ln(2) = 10\ln(2)$

Now, $\ln(2) = 0.693$

$\therefore \quad t = 10 \times 0.693 = 6.93\,\text{s}$

or $\quad t \approx 7\,\text{s}$

2. Equation for SHM is given as

$$y = 5(\sin 3\pi t + \sqrt{3}\cos 3\pi t)$$

$$= 5 \times 2\left(\frac{1}{2} \times \sin 3\pi t + \frac{\sqrt{3}}{2}\cos 3\pi t\right)$$

$$= 5 \times 2\left(\cos\frac{\pi}{3}.\sin 3\pi t + \sin\frac{\pi}{3}.\,3\pi t\right)$$

$$= 5 \times 2\sin\left(3\pi t + \frac{\pi}{3}\right)$$

[using, $\sin(a + b) = \sin a\cos b + \cos a\sin b$]

or $\quad y = 10\sin\left(3\pi t + \frac{\pi}{3}\right)$

Comparing this equation with the general equation of SHM, i.e.

$$y = A\sin\left(\frac{2\pi t}{T} + \phi\right),$$

We get, amplitude, $A = 10\,\text{cm}$

and $\quad 3\pi = \frac{2\pi}{T}$

or Time period, $\quad T = \frac{2}{3}\,\text{s}$

3. In simple harmonic motion, position (x), velocity (v) and acceleration (a) of the particle are given by

$$x = A\sin\omega t$$

$$v = \omega\sqrt{A^2 - x^2} \text{ or } v = A\omega\cos\omega t$$

and $\quad a = -\omega^2 x \quad$ or $a = -\omega^2 A\sin\omega t$

Given, amplitude $A = 5$ cm and displacement $x = 4$ cm.

At this time (when $x = 4$ cm), velocity and acceleration have same magnitude.

$\Rightarrow \quad |v_{x=4}| = |a_{x=4}|$ or $|\omega\sqrt{5^2 - 4^2}| = |-4\omega^2|$

$\Rightarrow \quad 3\omega = +4\omega^2 \Rightarrow \omega = (3/4)\,\text{rad/s}$

So, time period, $T = \frac{2\pi}{\omega} \Rightarrow T = \frac{2\pi}{3} \times 4 = \frac{8\pi}{3}$ s

4. True value $= \frac{90 + 91 + 95 + 92}{4} = 92$

Mean absolute error

$$= \frac{|92 - 90| + |92 - 91| + |92 - 95| + |92 - 92|}{4}$$

$$= \frac{2 + 1 + 3 + 0}{4} = 1.5$$

Value $= (92 \pm 1.5)$

Since, least count is 1 sec

$\therefore \qquad \text{Value} = (92 \pm 2\text{s})$

5. $y = kt^2 \quad \Rightarrow \quad \dfrac{d^2y}{dt^2} = 2k$

or $\quad a_y = 2\text{m/s}^2 \qquad$ (as $k = 1\text{m/s}^2$)

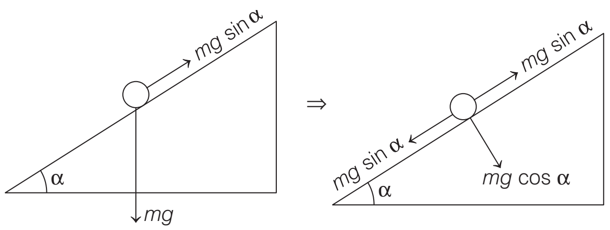
$$T_1 = 2\pi\sqrt{\frac{l}{g}}$$

and
$$T_2 = 2\pi\sqrt{\frac{l}{g + a_y}}$$

$$\therefore \quad \frac{T_1^2}{T_2^2} = \frac{g + a_y}{g} = \frac{10 + 2}{10} = \frac{6}{5}$$

$\therefore$ Correct answer is (a).

6. Free body diagram of bob of the pendulum with respect to the accelerating frame of reference is as follows



$\therefore$ Net force on the bob is $F_{\text{net}} = mg\cos\alpha$

or net acceleration of the bob is $g_{\text{eff}} = g\cos\alpha$

$$T = 2\pi\sqrt{\frac{L}{g_{\text{eff}}}}$$

or
$$T = 2\pi\sqrt{\frac{L}{g\cos\alpha}}$$

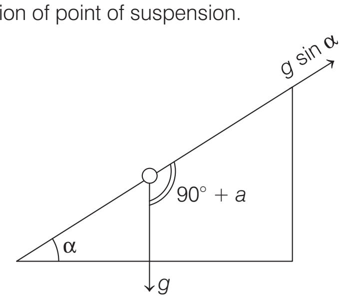
NOTE

- Whenever point of suspension is accelerating

Take, $\quad T = 2\pi\sqrt{\dfrac{L}{g_{\text{eff}}}}$

where, $\quad g_{\text{eff}} = g - a$

a = acceleration of point of suspension.



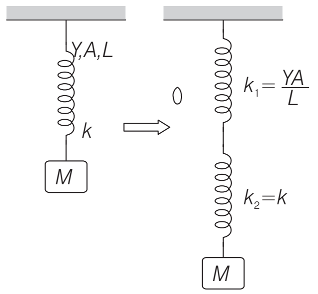
In this question, $a = g\sin\alpha$ (down the plane)

$\therefore \qquad |\,g - a\,| = g_{\text{eff}}$

$= \sqrt{g^2 + (g\sin\alpha)^2 + 2(g)(g\sin\alpha)\cos(90° + \alpha)}$

$= g\cos\alpha$

7. $k_{\text{eq}} = \dfrac{k_1k_2}{k_1 + k_2} = \dfrac{\frac{YA}{L}}{\frac{YA}{L} + k} = \dfrac{YAk}{YA + Lk}$



$$\therefore \qquad T = 2\pi\sqrt{\frac{m}{k_{\text{eq}}}}$$

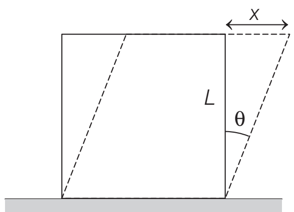
$$= 2\pi\sqrt{\frac{m(YA + Lk)}{YAk}}$$

NOTE Equivalent force constant for a wire is given by $k = \dfrac{YA}{L}$. Because in case of a wire, $F = \left(\dfrac{YA}{L}\right)\Delta L$ and in case of spring, $F = k.\Delta x$. Comparing these two, we find k of wire $= \dfrac{YA}{L}$.

8. Modulus of rigidity, $\eta = F/A\,\theta$

Here, $\quad A = L^2$

and $\quad \theta = \dfrac{x}{L}$



Therefore, restoring force is

$$F = -\eta A\theta = -\eta Lx$$

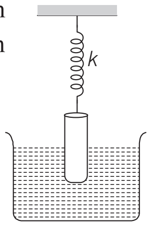
or acceleration, $a = \dfrac{F}{M} = -\dfrac{\eta L}{M}x$

Since, $a \propto -x$, oscillations are simple harmonic in nature, time period of which is given by

$$T = 2\pi\sqrt{\left|\frac{\text{displacement}}{\text{acceleration}}\right|}$$

$$= 2\pi\sqrt{\left|\frac{x}{a}\right|} = 2\pi\sqrt{\frac{M}{\eta L}}$$

9. When cylinder is displaced by an amount x from its mean position, spring force and upthrust both will increase. Hence,



Net restoring force = extra spring force + extra upthrust

or $\quad F = -(kx + Ax\rho g)$

or $\quad a = -\left(\dfrac{k + \rho A g}{M}\right)x$

Now, $\quad f = \dfrac{1}{2\pi}\sqrt{\left|\dfrac{a}{x}\right|} = \dfrac{1}{2\pi}\sqrt{\dfrac{k + \rho A g}{M}}$

$\therefore$ Correct option is (b).

10. Half of the volume of sphere is submerged.

For equilibrium of sphere, weight = upthrust

$\therefore \quad V\rho_s g = \dfrac{V}{2}(\rho_L)(g)$

$$\rho_s = \frac{\rho_L}{2} \quad \ldots\text{(i)}$$

When slightly pushed downwards by x, weight will remain as it is while upthrust will increase. The increased upthrust will become the net restoring force (upwards).

$F = -$ (extra upthrust)

$= -$ (extra volume immersed) $(\rho_L)(g)$

or $\quad ma = -(\pi R^2)x\rho_L g \quad$ (a = acceleration)

$\therefore \quad \dfrac{4}{3}\pi R^3\left(\dfrac{\rho}{2}\right)a = -(\pi R^2 \rho g)x$

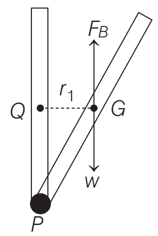
$\therefore \quad a = -\left(\dfrac{3g}{2R}\right)x$

as $a \propto -x$, motion is simple harmonic.

Frequency of oscillation, $f = \dfrac{1}{2\pi}\sqrt{\left|\dfrac{a}{x}\right|}$

$$= \frac{1}{2\pi}\sqrt{\frac{3g}{2R}}$$

11. Let S be the area of cross-section of the rod.



In the displaced position, as shown in figure, weight (w) and upthrust (F_B) both pass through its centre of gravity G.

Here, w = (volume) (density of rod)

$g = (SL)(d_1)g$

F_B = (Volume) (density of liquid) g

$= (SL)(d_2)g$

Given that, $d_1 < d_2$. Therefore, $w < F_B$

Therefore, net force acting at G will be

$F = F_B - w = (SLg)(d_2 - d_1)$ upwards. Restoring torque of this force about point P is

$$\tau = F \times r_\perp = (SLg)(d_2 - d_1)(QG)$$

or $\quad \tau = -(SLg)(d_2 - d_1)\left(\dfrac{L}{2}\sin\theta\right)$

Here, negative sign shows the restoring nature of torque.

or $\quad \tau = -\left\{\dfrac{SL^2 g\,(d_2 - d_1)}{2}\right\}\theta \quad \ldots\text{(i)}$

$\sin\theta \approx \theta$ for small values of θ

From Eq. (i), we see that $\tau \propto -\theta$

Hence, motion of the rod will be simple harmonic.

Rewriting Eq. (i) as

$$I\frac{d^2\theta}{dt^2} = -\left\{\frac{SL^2 g\,(d_2 - d_1)}{2}\right\}\theta \quad \ldots\text{(ii)}$$

Here, I = moment of inertia of rod about an axis passing through P.

$$I = \frac{ML^2}{3} = \frac{(SLd_1)L^2}{3}$$

Substituting this value of I in Eq. (ii), we have

$$\frac{d^2\theta}{dt^2} = -\left\{\frac{3}{2}\,\frac{g\,(d_2 - d_1)}{d_1\,L}\right\}\theta$$

Comparing this equation with standard differential equation of SHM, i.e. $\dfrac{d^2\theta}{dt^2} = -\omega^2\theta$

The angular frequency of oscillation is

$$\omega = \sqrt{\frac{3g\,(d_2 - d_1)}{2\,d_1\,L}}$$

12. Given, $Q = 10^{-5}$ C

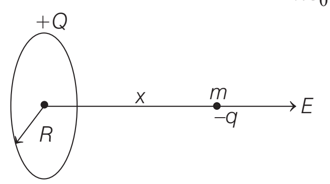
$q = 10^{-6}$ C, $\quad R = 1$ m

and $\quad m = 9 \times 10^{-4}$ kg

Electric field at a distance x from the centre on the axis of a ring is given by

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Qx}{(R^2 + x^2)^{3/2}}$$

If $\; x << R, \quad R^2 + x^2 \approx R^2 \;$ and $\; E \approx \dfrac{Qx}{4\pi\varepsilon_0 R^3}$



Net force on negatively charged particle would be qE and towards the centre of ring. Hence, we can write

$$F = -\frac{Q\,q\,x}{(4\pi\varepsilon_0)R^3}$$

or acceleration $a = \frac{F}{m} = -\frac{Qqx}{(4\pi\varepsilon_0)\,mR^3}$

as $a \propto -x$, motion of the particle is simple harmonic in nature. Time period of which will be given by

$$T = 2\pi\sqrt{\left|\frac{x}{a}\right|} \text{ or } T = 2\pi\sqrt{\frac{(4\pi\varepsilon_0)mR^3}{Qq}}$$

Substituting the values, we get

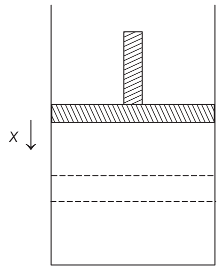
$$T = 2\pi\sqrt{\frac{(9\times 10^{-4})(1)^3}{(9\times 10^{9})(10^{-5})(10^{-6})}}$$

$$= 0.628\text{ s}$$

13. In equilibrium pressure inside the cylinder

= pressure just outside it

or $$p = p_0 + \frac{Mg}{A} \quad \text{...(i)}$$



When piston is displaced slightly by an amount x, change in volume, $dV = -Ax$

Since, the cylinder is isolated from the surroundings, process is adiabatic in nature. In adiabatic process,

$$\frac{dp}{dV} = -\gamma\frac{p}{V}$$

or increase in pressure inside the cylinder,

$$dp = -\frac{(\gamma p)}{V}(dV) = \gamma\left(\frac{p_0 + \frac{Mg}{A}}{V_0}\right)(Ax)$$

This increase in pressure when multiplied with area of cross-section A will give a net upward force (or the restoring force). Hence,

$$F = -(dp)\,A = -\gamma\left(\frac{p_0A^2 + MgA}{V_0}\right)x$$

or $$a = \frac{F}{M} = -\gamma\left(\frac{p_0A^2 + MgA}{V_0M}\right)x$$

Since, $a \propto -x$, motion of the piston is simple harmonic in nature. Frequency of this oscillation is given by

$$f = \frac{1}{2\pi}\sqrt{\left|\frac{a}{x}\right|} = \frac{1}{2\pi}\sqrt{\frac{\gamma\,(p_0A^2 + MgA)}{V_0M}}$$

14. Force constant of a wire $k = \frac{YA}{L}$

Frequency of oscillation $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

or $$f = \frac{1}{2\pi}\sqrt{\frac{(YA/L)}{m}} = \frac{1}{2\pi}\sqrt{\frac{YA}{mL}}$$

15. $$\omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{YA}{lm}} = \sqrt{\frac{(n\times 10^{9})(4.9\times 10^{-7})}{1\times 0.1}}$$

Putting, $\omega = 140\text{ rad s}^{-1}$ in above equation we get, $n = 4$

$\therefore$ Answer is 4.

Topic 4 Spring Based Problems

1. If parameters like material, number of loops per unit length, area of cross-section, etc., are kept same, then force constant of spring is inversely proportional to its length.

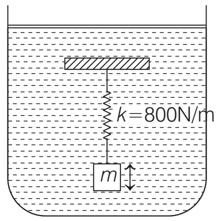
In given case, all other parameters are same for both parts of spring.

So, $$k_1 \propto \frac{1}{l_1} \text{ and } k_2 \propto \frac{1}{l_2}$$

$\therefore$ $$\frac{k_1}{k_2} = \frac{l_2}{l_1}$$

$$= \frac{l_2}{nl_2} = \frac{1}{n} \quad [\because l_1 = nl_2]$$

2. The given situation is shown in the figure given below



When vibrations of mass are suddenly stopped, oscillation energy (or stored energy of spring) is dissipated as heat, causing rise of temperature.

So, conversation of energy gives

$$\frac{1}{2}kx_m^2 = (m_1s_1 + m_2s_2)\Delta T$$

where, x_m = amplitude of oscillation,

s_1 = specific heat of mass,

s_2 = specific heat of water

and ΔT = rise in temperature.

Substituting values given in question, we have

$$\frac{1}{2}\times 800\times(2\times 10^{-2})^2$$

$$= \left(\left(\frac{500}{1000}\right)\times 400 + 1\times 4184\right)\Delta T$$

$$\Rightarrow \Delta T = \frac{16\times 10^{-2}}{4384} = 3.65\times 10^{-5}\text{ K}$$

3. When a system oscillates, the magnitude of restoring torque of system is given by

$$\tau = C\theta \quad \ldots\text{(i)}$$

where, C = constant that depends on system.

Also, $$\tau = I\alpha \quad \ldots\text{(ii)}$$

where, I = moment of inertia

and α = angular acceleration

From Eqs. (i) and (ii),

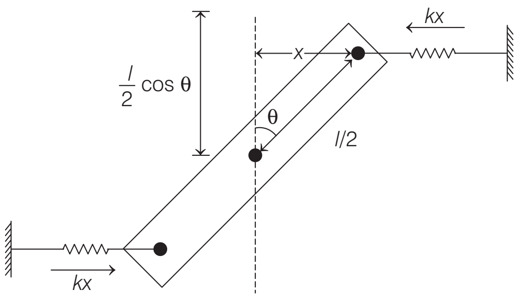
$$\alpha = \frac{C}{I}\cdot\theta \quad \ldots\text{(iii)}$$

and time period of oscillation of system will be

$$T = 2\pi\sqrt{\frac{I}{C}}$$

In given case, magnitude of torque is

τ = Force $\times$ perpendicular distance



$$\tau = 2kx \times \frac{l}{2}\cos\theta$$

For small deflection,

$$\tau = \left(\frac{kl^2}{2}\right)\theta \quad \ldots\text{(iv)}$$

$\because$ For small deflections, $\sin\theta = \dfrac{x}{(l/2)} \approx \theta$

$$\Rightarrow \quad x = \frac{l\theta}{2}$$

Also, $\cos\theta \approx 1$

comparing Eqs. (iv) and (i), we get

$$C = \frac{kl^2}{2} \Rightarrow \alpha = \frac{(kl^2/2)}{\left(\frac{1}{12}ml^2\right)}\cdot\theta$$

$$\Rightarrow \quad \alpha = \frac{6k}{m}\cdot\theta$$

Hence, time period of oscillation is

$$T = 2\pi\sqrt{\frac{m}{6k}}$$

Frequency of oscillation is given by

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{6k}{m}}$$

4. In a spring-block system, when a block is pulled with a constant force F, then its speed is maximum at the mean position. Also, it's acceleration will be zero.

In that case, force on the system is given as,

$$F = kx \quad \ldots\text{(i)}$$

where, x is the extension produced in the spring.

or $$x = \frac{F}{k}$$

Now we know that, for a system vibrating at its mean position, its maximum velocity is given as,

$$v_{\max} = A\omega$$

where, A is the amplitude and ω is the angular velocity.

Since, the block is at its mean position.

So, $$A = x = \frac{F}{k}$$

$$v_{\max} = \frac{F}{k}\sqrt{\frac{k}{m}} \quad \left[\because \omega = \sqrt{\frac{k}{m}}\right] = \frac{F}{\sqrt{km}}$$

Alternate Method

According to the work-energy theorem,

net work done = change in the kinetic energy

Here, net work done = work done due to external force (W_{ext})+ work done due to the spring (W_{spr}).

As, $$W_{\text{ext}} = F\cdot x$$

and $$W_{\text{spr}} = \frac{-1}{2}kx^2$$

$$\Rightarrow \quad \Delta\text{KE} = F\cdot x + \left(-\frac{1}{2}kx^2\right)$$

$$(\Delta\text{KE})_f - (\Delta\text{KE})_i = F\cdot x - \frac{1}{2}kx^2$$

$$\Rightarrow \quad \frac{1}{2}mv_{\max}^2 - \frac{1}{2}m(0)^2 = F\cdot\left(\frac{F}{k}\right) - \frac{1}{2}k\left(\frac{F}{k}\right)^2$$

[using Eq. (i)]

$$\Rightarrow \quad \frac{1}{2}mv_{\max}^2 = \frac{F^2}{k} - \frac{F^2}{2k} = \frac{F^2}{2k}$$

or $$v_{\max}^2 = \frac{F^2}{km}$$

$$\Rightarrow \quad v_{\max} = F/\sqrt{km}$$

5. t = time of flight of projectile

$$= \frac{2v\sin\theta}{g} \quad (\theta = 45^\circ)$$

$$\therefore \quad v = \frac{gt}{2\sin\theta} = \frac{10\times 1}{2\times 1/\sqrt{2}}$$

$$= \sqrt{50} \text{ m/s}$$

6. Frequency or time period of SHM depends on variable forces. It does not depend on constant external force. Constant external force can only change the mean position. For example, in the given question mean position is at natural length of spring in the absence of electric field.

Whereas in the presence of electric field mean position will be obtained after a compression of x_0. Where x_0 is given by

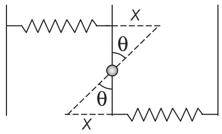
$$Kx_0 = QE \text{ or } x_0 = \frac{QE}{K}$$

$\therefore$ Correct answer is (a).

7. $x_1 + x_2 = A$ and $k_1 x_1 = k_2 x_2$ or $\frac{x_1}{x_2} = \frac{k_2}{k_1}$

Solving these equations, we get $x_1 = \left(\frac{k_2}{k_1 + k_2}\right) A$

8. $x = \frac{L}{2}\theta,$



Restoring torque $= -(2kx)\cdot\frac{L}{2}$

$$\alpha = -\frac{kL(L/2\theta)}{I} = -\left[\frac{kL^2/2}{ML^2/12}\right]\cdot\theta = -\left(\frac{6k}{M}\right)\theta$$

$$\therefore \quad f = \frac{1}{2\pi}\sqrt{\left|\frac{\alpha}{\theta}\right|} = \frac{1}{2\pi}\sqrt{\frac{6k}{M}}$$

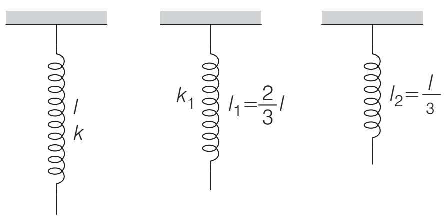
9. Angular frequency of the system, $\omega = \sqrt{\frac{k}{m+m}} = \sqrt{\frac{k}{2m}}$

Maximum acceleration of the system will be, $\omega^2 A$ or $\frac{kA}{2m}$.

This acceleration to the lower block is provided by friction.

Hence, $f_{\max} = ma_{\max} = m\omega^2 A = m\left(\frac{kA}{2m}\right) = \frac{kA}{2}$

10. $l_1 = 2l_2 \quad \therefore \; l_1 = \frac{2}{3}l$



$$\therefore \quad k_1 = \frac{3}{2}k$$

Force constant $k \propto \frac{1}{\text{length of spring}}$

11. $(v_M)_{\max} = (v_N)_{\max}$

$\therefore \quad \omega_M A_M = \omega_N A_N$

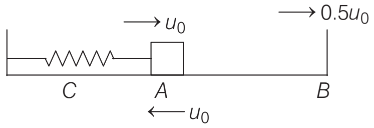
or $\quad \frac{A_M}{A_N} = \frac{\omega_N}{\omega_M} = \sqrt{\frac{k_2}{k_1}} \qquad \left(\because \omega = \sqrt{\frac{k}{m}}\right)$

$\therefore$ Correct answer is (b).

12. At equilibrium $(t = 0)$ particle has maximum velocity u_0. Therefore velocity at time t can be written as

$$u = u_{\max}\cos\omega t + u_0\cos\omega t$$

writing, $\quad u = 0.5u_0 = u_0 \cos\omega t$



$\therefore \quad \omega t = \frac{\pi}{3}$

$\therefore \quad \frac{2\pi}{T}t = \frac{\pi}{3}$

$\therefore \quad t = \frac{T}{6}$

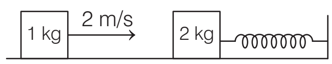
(b) $t = t_{AB} + t_{BA} = \frac{T}{6} + \frac{T}{6} = \frac{T}{3} = \frac{2\pi}{3}\sqrt{\frac{m}{k}}$

(c) $t = t_{AB} + t_{BA} + t_{AC} = \frac{T}{6} + \frac{T}{6} + \frac{T}{4} = \frac{7}{12}T = \frac{7\pi}{6}\sqrt{\frac{m}{k}}$

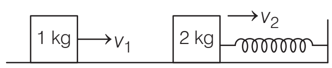
(d) $t = t_{AB} + t_{BA} + t_{AC} + t_{CA} = \frac{t}{6} + \frac{T}{6} + \frac{T}{4} + \frac{T}{4} = \frac{5}{6}T$

$$= \frac{5\pi}{3}\sqrt{\frac{m}{k}}$$

13. Just Before Collision,



Just After Collision



Let velocities of 1 kg and 2 kg blocks just after collision be v_1 and v_2 respectively.

From momentum conservation principle,

$$1 \times 2 = 1v_1 + 2v_2 \qquad \ldots\text{(i)}$$

Collision is elastic. Hence $e = 1$ or relative velocity of separation = relation velocity of approach.

$$v_2 - v_1 = 2 \qquad \ldots\text{(ii)}$$

From Eqs. (i) and (ii),

$$v_2 = \frac{4}{3}\text{ m/s}, \; v_1 = \frac{-2}{3}\text{ m/s}$$

2 kg block will perform SHM after collision,

$$t = \frac{T}{2} = \pi\sqrt{\frac{m}{k}} = 3.14\text{ s}$$

$$\text{Distance} = |v_1|t = \frac{2}{3} \times 3.14 = 2.093 = 2.09\text{ m}$$

14. At distance y above the mean position velocity of the block.

$$v = \omega\sqrt{A^2 - y^2}$$

After detaching from the spring net downward acceleration of the block will be g.

Therefore, total height attained by the block above the mean position,

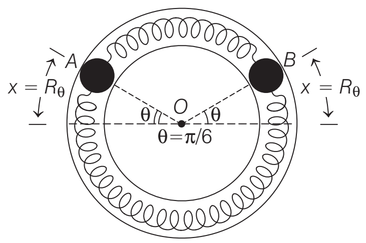
$$h = y + \frac{v^2}{2g} = y + \frac{\omega^2 (A^2 - y^2)}{2g}$$

For h to be maximum $dh/dy = 0$

Putting $\frac{dh}{dy} = 0$, we get $y = \frac{g}{\omega^2} = y_{\max}$

15. Given, mass of each block A and B, $m = 0.1$ kg

Radius of circle, $R = 0.06$ m



Natural length of spring $l_0 = 0.06\,\pi = \pi R$ (Half circle)

and spring constant, $k = 0.1$ N/m

In the stretched position elongation in each spring

$$x = R\,\theta.$$

Let us draw FBD of A.

Spring in lower side is stretched by $2x$ and on upper side compressed by $2x$. Therefore, each spring will exert a force $2kx$ on each block. Hence, a restoring force, $F = 4kx$ will act on A in the direction shown in figure below .

Restoring torque of this force about origin

$$\tau = -F \cdot R = -(4kx)R = -(4kR\theta)\,R$$

or $\quad \tau = -4kR^2 \cdot \theta \quad$ ...(i)

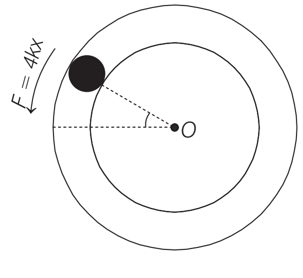
Since, $\tau \propto -\theta$, each ball executes angular SHM about origin O.

Eq. (i) can be rewritten as

$$I\alpha = -4kR^2\,\theta$$

or $\quad (mR^2)\,\alpha = -4kR^2\,\theta$

or $\quad \alpha = -\left(\frac{4k}{m}\right)\theta$



(a) Frequency of oscillation,

$$f = \frac{1}{2\pi}\sqrt{\left|\frac{\text{acceleration}}{\text{displacement}}\right|}$$

$$= \frac{1}{2\pi}\sqrt{\left|\frac{\alpha}{\theta}\right|}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{4k}{m}}$$

Substituting the values, we have

$$f = \frac{1}{2\pi}\sqrt{\frac{4 \times 0.1}{0.1}} = \frac{1}{\pi}\text{ Hz}$$

(b) In stretched position, potential energy of the system is

$$\text{PE} = 2\left\{\frac{1}{2}k\right\}\{2x\}^2 = 4kx^2$$

and in mean position, both the blocks have kinetic energy only. Hence, KE $= 2\left\{\frac{1}{2}mv^2\right\} = mv^2$

From energy conservation PE = KE

$\therefore \quad 4kx^2 = mv^2$

$\therefore \quad v = 2x\sqrt{\frac{k}{m}} = 2R\theta\sqrt{\frac{k}{m}}$

Substituting the values $v = 2\,(0.06)\,(\pi/6)\sqrt{\frac{0.1}{0.1}}$

or $\quad v = 0.0628$ m/s

(c) Total energy of the system, E

= PE in stretched position

or = KE in mean position

$$E = mv^2 = (0.1)\,(0.0628)^2 \text{ J}$$

or $E = 3.9 \times 10^{-4}$ J

16. Between C and D block will move with constant speed of 120 cm/s. Therefore, period of oscillation will be (starting from C).

$$T = t_{CD} + \frac{T_2}{2} + t_{DC} + \frac{T_1}{2}$$

Here, $\quad T_1 = 2\pi\sqrt{\frac{m}{k_1}}$ and $T_2 = 2\pi\sqrt{\frac{m}{k_2}}$

and $\quad t_{CD} = t_{DC} = \frac{60}{120} = 0.5$ s

$\therefore \quad T = 0.5 + \frac{2\pi}{2}\sqrt{\frac{0.2}{3.2}} + 0.5 + \frac{2\pi}{2}\sqrt{\frac{0.2}{1.8}}$

$(m = 200\text{ g} = 0.2\text{ kg})$

$$T = 2.82\text{ s}$$

17. When m_1 is removed only m_2 is left. Therefore, angular frequency $\omega = \sqrt{k/m_2}$

Let x_1 be the extension when only m_2 is left. Then,

$$kx_1 = m_2 g \text{ or } x_1 = \frac{m_2 g}{k} \quad \text{...(i)}$$

Similarly, let x_2 be the extension in equilibrium when both m_1 and m_2 are suspended. Then,

$$(m_1 + m_2)g = kx_2$$

$\Rightarrow \quad x_2 = \frac{(m_1 + m_2)g}{k} \quad$ ...(ii)

From Eqs. (i) and (ii), amplitude of oscillation

$$A = x_2 - x_1 = \frac{m_1 g}{k}$$

18. $\quad T = 2\pi\sqrt{\frac{M}{K}} \quad$ or $\quad T \propto \sqrt{M}, \frac{T_2}{T_1} = \sqrt{\frac{M_2}{M_1}}$

$\therefore \quad \frac{3}{2} = \sqrt{\frac{M+2}{M}} \quad$ ...(I)

Solving the equation (i), we get, $M = 1.6$ kg.

Topic 5 Miscellaneous Problems

1. We know that,
Time period of a pendulum is given by

$$T = 2\pi\sqrt{L/g_{\text{eff}}} \quad \ldots(\text{i})$$

Here, L is the length of the pendulum and g_{eff} is the effective acceleration due to gravity in the respective medium in which bob is oscillating.

Initially, when bob is oscillating in air, $g_{\text{eff}} = g$.

So, initial time period, $T = 2\pi\sqrt{\dfrac{L}{g}}$ $\quad\ldots$(ii)

Let ρ_{bob} be the density of the bob.

When this bob is dipped into a liquid whose density is given as

$$\rho_{\text{liquid}} = \frac{\rho_{\text{bob}}}{16} = \frac{\rho}{16} \quad (\text{given})$$

$\therefore$ Net force on the bob is

$$F_{\text{net}} = V\rho g - V\cdot\frac{\rho}{16}\cdot g \quad \ldots(\text{iii})$$

(where, V = volume of the bob = volume of displaced liquid by the bob when immersed in it). If effective value of gravitational acceleration on the bob in this liquid is g_{eff}, then net force on the bob can also be written as

$$F_{\text{net}} = V\rho g_{\text{eff}} \quad \ldots(\text{iv})$$

Equating Eqs. (iii) and (iv), we have

$$V\rho g_{\text{eff}} = V\rho g - V\rho g/16$$

$$\Rightarrow \quad g_{\text{eff}} = g - g/16 = \frac{15}{16}g \quad \ldots(\text{v})$$

Substituting the value of g_{eff} from Eq. (v) in Eq. (i), the new time period of the bob will be

$$T' = 2\pi\sqrt{\frac{L}{g_{\text{eff}}}} = 2\pi\sqrt{\frac{16}{15}\frac{L}{g}} \Rightarrow T' = \sqrt{\frac{16}{15}} \times 2\pi\sqrt{\frac{L}{g}}$$

$$= \frac{4}{\sqrt{15}} \times T \quad [\text{using Eq. (ii)}]$$

2. Given, frequency of oscillations is $f = 5\,\text{osc s}^{-1}$

$\Rightarrow$ Time period of oscillations is $T = \dfrac{1}{f} = \dfrac{1}{5}\text{s}$

So, time for 10 oscillations is $= \dfrac{10}{5} = 2\text{s}$

Now, if A_0 = initial amplitude at $t = 0$ and γ = damping factor, then for damped oscillations, amplitude after t second is given as

$$A = A_0 e^{-\gamma t}$$

$\therefore$ After 2 s,

$$\frac{A_0}{2} = A_0 e^{-\gamma(2)} \quad \Rightarrow \quad 2 = e^{2\gamma}$$

$$\Rightarrow \quad \gamma = \frac{\log 2}{2} \quad \ldots(\text{i})$$

Now, when amplitude is $\dfrac{1}{1000}$ of initial amplitude, i.e.

$$\frac{A_0}{1000} = A_0 e^{-\gamma t}$$

$$\Rightarrow \quad \log(1000) = \gamma t$$

$$\Rightarrow \quad \log(10^3) = \gamma t$$

$$3\log 10 = \gamma t$$

$$\Rightarrow \quad t = \frac{2\times 3\log 10}{\log 2} \quad [\text{using Eq. (i)}]$$

$$\Rightarrow \quad t = 19.93\,\text{s}$$

or $\quad t \approx 20\,\text{s}$

3. We know that time period of a pendulum is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

So, angular frequency $\omega = \dfrac{2\pi}{T} = \sqrt{\dfrac{g}{l}}$ $\quad$..(i)

Now, differentiate both side w.r.t g

$$\therefore \quad \frac{d\omega}{dg} = \frac{1}{2\sqrt{g}\sqrt{l}}$$

$$d\omega = \frac{dg}{2\sqrt{g}\sqrt{l}} \quad \ldots(\text{ii})$$

By dividing Eq. (ii) by Eq. (i), we get

$$\frac{d\omega}{\omega} = \frac{dg}{2g}$$

Or we can write

$$\frac{\Delta\omega}{\omega} = \frac{\Delta g}{2g} \quad \ldots(\text{iii})$$

As Δg is due to oscillation of support.

$$\therefore \quad \Delta g = 2\omega^2 A$$

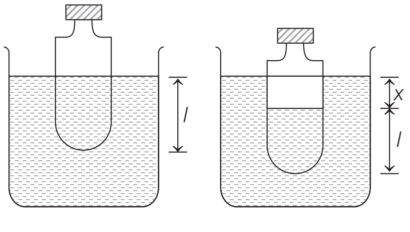
($\omega_1 \to 1\,\text{rad/s}$, support)

Putting value of Δg in Eq. (iii) we get

$$\frac{\Delta\omega}{\omega} = \frac{1}{2}\cdot\frac{2\omega_1^2 A}{g} = \frac{\omega_1^2 A}{g}; \ (A = 10^{-2}\,\text{m}^2)$$

$$\Rightarrow \quad \frac{\Delta\omega}{\omega} = \frac{1\times 10^{-2}}{10} = 10^{-3}\ \text{rad/s}$$

4. In equilibrium condition bottle floats in water and its length 'l' inside water is same as the height of water upto which bottle is filled.



So, l = Volume of water in bottle/Area

$$= \frac{310}{\pi\times(2.5)^2} = 15.8\,\text{cm} = 0.158\,\text{m}$$

When bottle is slightly pushed inside by an amount x then, restoring force acting on the bottle is the upthrust of fluid displaced when bottle goes into liquid by amount x.

So, restoring force is;

$$F = -(\rho Ax)g \quad \ldots\text{(i)}$$

where ρ = density of water,

A = area of cross-section of bottle and

x = displacement from equilibriumposition

But $\quad F = ma \quad \ldots$(ii)

where, $\quad m$ = mass of water and bottle system

$= Al\rho$

From (i) and (ii) we have,

$$Al\rho\alpha = -\rho Axg \text{ or } a = -\frac{g}{l}x$$

As for SHM, $a = -\omega^2 x$

We have $\omega = \sqrt{\frac{g}{l}} = \sqrt{\frac{10}{0.158}} = \sqrt{63.29} \approx 8 \text{ rad s}^{-1}$

$\therefore$ No option is correct.

5 We know that in case of torsonal oscillation frequency

$$\nu = \frac{k}{\sqrt{I}}$$

where, I is moment of inertia and k is torsional constant.

$\therefore$ According to question, $\nu_1 = \dfrac{k}{\sqrt{\dfrac{M(2L)^2}{12}}}$

(As, MOI of a bar is $I = \dfrac{ML^2}{12}$)

or $$\nu_1 = \frac{k}{\sqrt{\dfrac{ML^2}{3}}} \quad \ldots\text{(i)}$$

When two masses are attached at ends of rod. Then its moment of inertia is

$$\frac{M(2L)^2}{12} + 2m\left(\frac{L}{2}\right)^2$$

So, new frequency of oscillations is,

$$\nu_2 = \frac{k}{\sqrt{\dfrac{M(2L)^2}{12} + 2m\left(\dfrac{L}{2}\right)^2}}$$

$$\nu_2 = \frac{k}{\sqrt{\dfrac{ML^2}{3} + \dfrac{mL^2}{2}}} \quad \ldots\text{(ii)}$$

As, $\quad \nu_2 = 80\%$ of $\nu_1 = 0.8\nu_1$

So, $$\frac{k}{\sqrt{\dfrac{ML^2}{3} + \dfrac{mL^2}{2}}} = \frac{0.8 \times k}{\sqrt{\dfrac{ML^2}{3}}}$$

After solving it, we get,

$$\frac{m}{M} = 0.37$$

6. Given, frequency, $f = 10^{12}$ /sec

Angular frequency, $\omega = 2\pi f = 2\pi \times 10^{12}$ /sec

Force constant, $k = m\omega^2 = \dfrac{108 \times 10^{-3}}{6.02 \times 10^{23}} \times 4\pi^2 \times 10^{24}$

$$k = 7.1 \text{ N/m}$$

7. Time period of oscillation is

$$T = 2\pi\sqrt{\frac{I}{MB}}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{7.5 \times 10^{-6}}{6.7 \times 10^{-2} \times 0.01}} = 0.665 \text{ s}$$

Hence, time for 10 oscillations is $t = 6.65$ s.

8. $v = \omega\sqrt{A^2 - x^2}$ At, $x = \dfrac{2A}{3}$

$$v = \omega\sqrt{A^2 - \left(\frac{2A}{3}\right)^2} = \frac{\sqrt{5}}{3}\omega A$$

As, velocity is trebled, hence $v' = \sqrt{5}A\omega$

This leads to new amplitude A'

$$\therefore \quad \omega\sqrt{A'^2 - \left(\frac{2A}{3}\right)^2} = \sqrt{5}A\omega$$

$$\Rightarrow \quad \omega^2\left[A'^2 - \frac{4A^2}{9}\right] = 5A^2\omega^2$$

$$\Rightarrow \quad A'^2 = 5A^2 + \frac{4}{9}A^2 = \frac{49}{9}A^2$$

$$A' = \frac{7}{3}A$$

9. In SHM, a particle starts from rest, we have

i.e. $\quad x = A\cos\omega t$, at $t = 0$, $x = A$

When $\quad t = \tau$, then $x = A - a \quad$ ...(i)

When $\quad t = 2\tau$, then

$$x = A - 3a \quad \ldots\text{(ii)}$$

On comparing Eqs. (i) and (ii), we get

$$A - a = A\cos\omega\tau$$

$$A - 3a = A\cos 2\omega\tau$$

As $\cos 2\omega\tau = 2\cos^2\omega\tau - 1$

$$\Rightarrow \quad \frac{A - 3a}{A} = 2\left(\frac{A - a}{A}\right)^2 - 1$$

$$\Rightarrow \quad \frac{A - 3a}{A} = \frac{2A^2 + 2a^2 - 4Aa - A^2}{A^2}$$

$$A^2 - 3aA = A^2 + 2a^2 - 4Aa$$

$$a^2 = 2aA$$

$$A = 2a$$

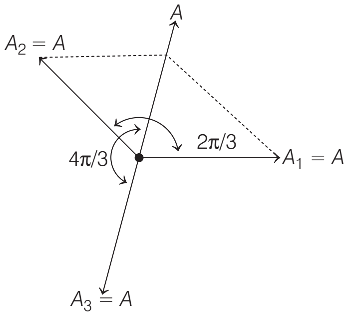
Now, $\quad A - a = A\cos\omega\tau$

$\Rightarrow \quad \cos\omega\tau = 1/2$

$$\Rightarrow \qquad \frac{2\pi}{T}\tau = \frac{\pi}{3}$$

$$\Rightarrow \qquad T = 6\pi$$

10.



Resultant amplitude of x_1 and x_2 is A at angle $\left(\frac{\pi}{3}\right)$ from A_1. To make resultant of x_1, x_2 and x_3 to be zero. A_3 should be equal to A at angle $\phi = \frac{4\pi}{3}$ as shown in figure.

$\therefore$ Correct answer is (b).

Alternate Solution.

It we substitute, $x_1 + x_2 + x_3 = 0$

or $A \sin \omega t + A \sin\left(\omega t + \frac{2\pi}{3}\right) + B \sin(\omega t + \phi) = 0$

Then by applying simple mathematics we can prove that

$$B = A \quad \text{and} \quad \phi = \frac{4\pi}{3}.$$

11. In SHM, velocity of particle also oscillates simple harmonically. Speed is more near the mean position and less near the extreme positions. Therefore, the time taken for the particle to go from O to $A/2$ will be less than the time taken to go it from A/2 to A, or $T_1 < T_2$.

NOTE

From the equations of SHM we can show that

$$t_1 = T_{o-A/2} = T/12 \quad \text{and} \quad t_2 = T_{A/2-A} = T/6$$

So, that $t_1 + t_2 = T_{o-A} = T/4$

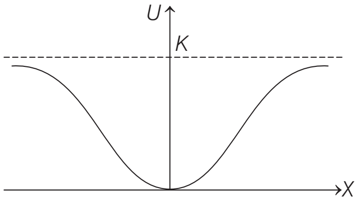
12. $U(x) = k(1 - e^{-x^2})$

It is an exponentially increasing graph of potential energy (U) with x^2. Therefore, U *versus* x graph will be as shown.

At origin.

Potential energy U is minimum (therefore, kinetic energy will be maximum) and force acting on the particle is zero because

$$F = \frac{-dU}{dx} = -(\text{slope of } U\text{-}x \text{ graph}) = 0.$$



Therefore, origin is the stable equilibrium position. Hence, particle will oscillate simple harmonically about $x = 0$ for small displacements. Therefore, correct option is (d).

(a), (b) and (c) options are wrong due to following reasons.

(a) At equilibrium position $F = \frac{-dU}{dx} = 0$ i.e. slope of U-x graph should be zero and from the graph we can see that slope is zero at $x = 0$ and $x = \pm\infty$.

Now, among these equilibriums stable equilibrium position is that where U is minimum (Here $x = 0$). Unstable equilibrium position is that where U is maximum (Here none).

Neutral equilibrium position is that where U is constant (Here $x = \pm\infty$).

Therefore, option (a) is wrong.

(b) For any finite non-zero value of x, force is directed towards the origin because origin is in stable equilibrium position. Therefore, option (b) is incorrect.

(c) At origin, potential energy is minimum, hence kinetic energy will be maximum. Therefore, option (c) is also wrong.

13. $U(x) = k|x|^3$

$$\therefore \quad [k] = \frac{[U]}{[x^3]} = \frac{[ML^2\ T^{-2}]}{[L^3]} = [ML^{-1}\ T^{-2}]$$

Now, time period may depend on

$$T \propto (\text{mass})^x\ (\text{amplitude})^y\ (k)^z$$

$$[M^0L^0T] = [M]^x\ [L]^y\ [ML^{-1}\ T^{-2}]^z$$

$$= [M^{x+z} L^{y-z} T^{-2z}]$$

Equating the powers, we get

$$-2z = 1 \quad \text{or} \quad z = -1/2$$

$$y - z = 0 \quad \text{or} \quad y = z = -1/2$$

Hence, $T \propto (\text{amplitude})^{-1/2}) \propto (a)^{-1/2}$

or $\qquad T \propto \frac{1}{\sqrt{a}}$

14. If $E > V_0$, particle will escape. But simultaneously for oscillations, $E > 0$.

Hence, the correct answer is $V_0 > E > 0$

or the correct option is (c).

15. $[\alpha] = \left[\frac{PE}{x^4}\right] = \left[\frac{ML^2T^{-2}}{L^4}\right] = [ML^{-2}T^{-2}]$

$$\therefore \qquad \left[\frac{m}{\alpha}\right] = [L^2T^2]$$

$$\Rightarrow \qquad \left[\frac{1}{A}\sqrt{\frac{m}{\alpha}}\right] = [T]$$

As dimensions of amplitude A is [L].

Hence, the correct option is (b).

16. For $|x| > X_0$, potential energy is constant. Hence, kinetic energy, speed or velocity will also remain constant.

$\therefore$ Acceleration will be zero.

Hence, the correct option is (d).

17. $A_1 = 10$ (directly)

For A_2 : $y_2 = 5\sin 3\pi t + 5\sqrt{3}\cos 3\pi t$

$$= 5\sin 3\pi t + 5\sqrt{3}\sin\left(3\pi t + \frac{\pi}{2}\right)$$

i.e. phase difference between two functions is $\frac{\pi}{2}$, so the resultant amplitude A_2 can be obtained by the vector method as under

$$A_2 = \sqrt{(5)^2 + (5\sqrt{3})^2} = 10$$

$$\therefore \quad \frac{A_1}{A_2} = \frac{10}{10} = 1$$

18. (A) Potential energy is minimum at mean position.

(B) For $a = 0$, $s = vt \rightarrow$ option (q)

or $s = s_0 + vt \rightarrow$ option (r)

For a = constant, $s = ut + \frac{1}{2}at^2 \rightarrow$ option (s)

(C) $R = \dfrac{v^2 \sin 2\theta}{g}$

$\therefore$ $R \propto v^2 \rightarrow$ option (s)

(D) $T = 2\pi\sqrt{\dfrac{l}{g}}$

$\therefore$ $T^2 \propto l \rightarrow$ option (q)

19. (A) Compare with the standard equation of SHM

$$v = \omega\sqrt{A^2 - x^2}$$

we see that the given motion is SHM with,

$$\omega = C_1 \text{ and } A^2 = C_2$$

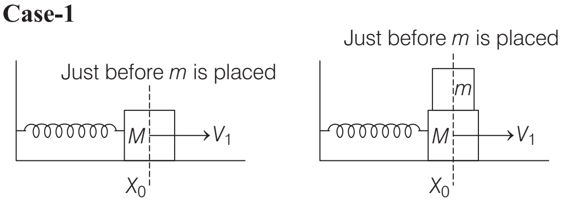
(B) The equation shows that the object does not change its direction and kinetic energy of the object keeps on decreasing.

(C) A pseudo force (with respect to elevator) will start acting on the object. Its means position is now changed and it starts SHM.

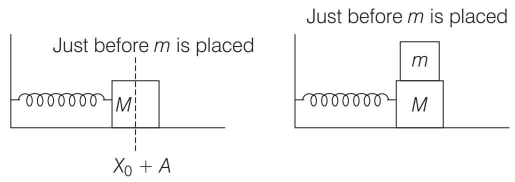
(D) The given velocity is greater than the escapevelocity

$= \left(\sqrt{\dfrac{2GM_e}{R_e}}\right)$. Therefore it keeps on moving towards infinity with decreasing speed.

20. **Case-1**



Case-2



In case-1,

$$Mv_1 = (M + m)v_2$$

$$v_2 = \left(\frac{M}{M + m}\right)v_1$$

$$\sqrt{\frac{k}{M + m}}\,A_2 = \left(\frac{M}{M + m}\right)\sqrt{\frac{k}{M}}\,A_1$$

$$A_2 = \sqrt{\frac{k}{M + m}}\,A_1$$

In case-2 $\quad A_2 - A_1$

$$T = 2\pi\sqrt{\frac{M + m}{k}} \text{ in both cases.}$$

Total energy decreases in first case whereas remain same in 2nd case. Instantaneous speed at x_0 decreases in both cases.

21. **Ist Particle**

$$P = 0 \text{ at } x = a$$

$\Rightarrow$ 'a' is the amplitude of oscillation 'A_1'.

At $x = 0$, $P = b$ (at mean position)

$$\Rightarrow \quad mv_{\max} = b$$

$$v_{\max} = \frac{b}{m}$$

$$E_1 = \frac{1}{2}mv_{\max}^2 = \frac{m}{2}\left[\frac{b}{m}\right]^2 = \frac{b^2}{2m}$$

$$A_1\omega_1 = v_{\max} = \frac{b}{m}$$

$$\Rightarrow \quad \omega_1 = \frac{b}{ma} = \frac{1}{mn^2} \quad \left(A_1 = a, \frac{a}{b} = n^2\right)$$

IInd Particle

$$P = 0 \text{ at } x = R$$

$$\Rightarrow \quad A_2 = R$$

At $\quad x = 0, P = R$

$$\Rightarrow \quad v_{\max} = \frac{R}{m}$$

$$E_2 = \frac{1}{2}mv_{\max}^2 = \frac{m}{2}\left[\frac{R}{m}\right]^2 = \frac{R^2}{2m}$$

$$A_2\omega_2 = \frac{R}{m}$$

$$\Rightarrow \quad \omega_2 = \frac{R}{mR} = \frac{1}{m}$$

(b) $\dfrac{\omega_2}{\omega_1} = \dfrac{1/m}{1/mn^2} = n^2$

(c) $\omega_1\omega_2 = \dfrac{1}{mn^2} \times \dfrac{1}{m} = \dfrac{1}{m^2n^2}$

(d) $\dfrac{E_1}{\omega_1} = \dfrac{b^2/2m}{1/mn^2} = \dfrac{b^2n^2}{2} = \dfrac{a^2}{2n^2} = \dfrac{R^2}{2}$

$$\frac{E_2}{\omega_2} = \frac{R^2/2m}{1/m} = \frac{R^2}{2}$$

$$\Rightarrow \quad \frac{E_1}{\omega_1} = \frac{E_2}{\omega_2}$$

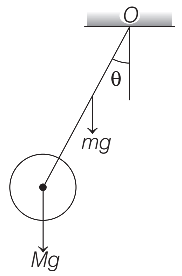
NOTE

It is not given that the second figure is a circle. But from the figure and as per the requirement of question, we consider it is a circle.

22. $\tau_A = \tau_B = \left(mg\dfrac{L}{2}\sin\theta + MgL\sin\theta\right)$

= Restoring torque about point O.

In case A, moment of inertia will be more. Hence, angular acceleration $(\alpha = \tau/I)$ will be less. Therefore, angular frequency will be less.



23. For $A = -B$ and $C = 2B$

$$X = B\cos 2\omega t + B\sin 2\omega t$$
$$= \sqrt{2}B\sin\left(2\omega t + \frac{\pi}{4}\right)$$

This is equation of SHM of amplitude $\sqrt{2}B$.

If $A = B$ and $C = 2B$, then $X = B + B\sin 2\omega t$

This is also equation of SHM about the point $X = B$. Function oscillates between $X = 0$ and $X = 2B$ with amplitude B.

24. From superposition principle

$$y = y_1 + y_2 + y_3$$
$$= a\sin\omega t + a\sin(\omega t + 45°) + a\sin(\omega t + 90°)$$
$$= a[\sin\omega t + \sin(\omega t + 90°)] + a\sin(\omega t + 45°)$$
$$= 2a\sin(\omega t + 45°)\cos 45° + a\sin(\omega t + 45°)$$
$$= (\sqrt{2} + 1)a\sin(\omega t + 45°) = A\sin(\omega t + 45°)$$

Therefore, resultant motion is simple harmonic of amplitude

$$A = (\sqrt{2} + 1)a$$

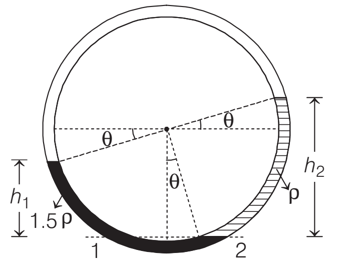
and which differ in phase by 45° relative to the first.

Energy in SHM $\propto$ (amplitude)^2 $\left[E = \dfrac{1}{2}mA^2\omega^2\right]$

$$\therefore \quad \frac{E_{\text{resultant}}}{E_{\text{single}}} = \left(\frac{A}{a}\right)^2 = (\sqrt{2}+1)^2 = (3 + 2\sqrt{2})$$

$$\therefore \quad E_{\text{resultant}} = (3 + 2\sqrt{2})E_{\text{single}}$$

25. (a) In equilibrium, pressure of same liquid at same level will be same.



Therefore, $p_1 = p_2$ or $p + (1.5\rho g h_1) = p + (\rho g h_2)$

(p = pressure of gas in empty part of the tube)

$$\therefore \quad 1.5h_1 = h_2$$

$$1.5[R\cos\theta - R\sin\theta] = \rho(R\cos\theta + R\sin\theta)$$

or $\quad 3\cos\theta - 3\sin\theta = 2\cos\theta + 2\sin\theta$

or $\quad 5\tan\theta = 1$

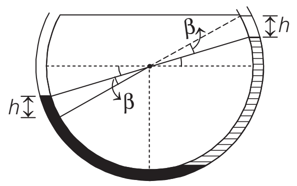
$$\theta = \tan^{-1}\left(\frac{1}{5}\right)$$

(b) When liquids are slightly disturbed by an angle β.

Net restoring pressure

$\Delta p = 1.5\rho g h + \rho g h = 2.5\rho g h$

This pressure will be equal at all sections of the liquid. Therefore, net restoring torque on the whole liquid.



$$\tau = -(\Delta p)(A)(R)$$

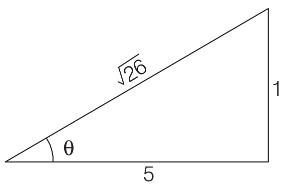
or $\tau = -2.5\rho g h AR$

$$= -2.5\rho g AR[R\sin(\theta + \beta) - R\sin\theta]$$
$$= -2.5\rho g AR^2[\sin\theta\cos\beta + \sin\beta\cos\theta - \sin\theta]$$

Assuming $\cos\beta \approx 1$ and $\sin\beta \approx \beta$ (as β is small)

$$\therefore \quad \tau = -(2.5\rho A g R^2\cos\theta)\beta$$

or $\quad I\alpha = -(2.5\rho A g R^2\cos\theta)\beta \quad$ ...(i)



Here, $\quad I = (m_1 + m_2)R^2$

$$= \left[\left(\frac{\pi R}{2}\cdot A\right)\rho + \left(\frac{\pi R}{2}\right)\cdot A(1.5\rho)\right]R^2$$

$$= (1.25\pi R^3\rho)A$$

and $$\cos\theta = \frac{5}{\sqrt{26}} = 0.98$$

Substituting in Eq. (i), we have $\alpha = -\dfrac{(6.11)\beta}{R}$

As angular acceleration is proportional to $-\beta$, motion is simple harmonic in nature.

$$T = 2\pi\sqrt{\left|\frac{\beta}{\alpha}\right|} = 2\pi\sqrt{\frac{R}{6.11}}$$

26. A torque will act on the rod, which tries to align the rod in the direction of electric field. This torque will be of restoring nature and has a magnitude $PE\sin\theta$. Therefore, we can write

$$\tau = -PE\sin\theta \quad \text{or} \quad I\alpha = -PE\sin\theta \qquad \text{...(i)}$$

Here, $I = 2M\left(\dfrac{L}{2}\right)^2 = \dfrac{ML^2}{2}$ and $P = qL$

Further, since θ is small so, we can write, $\sin\theta \approx \theta$.

Substituting these values in Eq. (i), we have

$$\left(\frac{ML^2}{2}\right)\alpha = -(qL)(E)\theta$$

or $$\alpha = -\left(\frac{2qE}{ML}\right)\theta$$

As α is proportional to $-\theta$, motion of the rod is simple harmonic in nature, time period of which is given by

$$T = 2\pi\sqrt{\left|\frac{\theta}{\alpha}\right|} = 2\pi\sqrt{\frac{ML}{2qE}}$$

The desired time will be,

$$t = \frac{T}{4} = \frac{\pi}{2}\sqrt{\frac{ML}{2qE}}$$