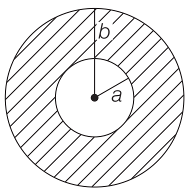
6

Rotation

Topic 1 Moment of Inertia

Objective Questions I (Only one correct option)

1. A circular disc of radius b has a hole of radius a at its centre (see figure). If the mass per unit area of the disc varies as $\left(\frac{\sigma_0}{r}\right)$, then the radius of gyration of the disc about its axis passing through the centre is **(2019 Main, 12 April I)**



(a) $\sqrt{\frac{a^2+b^2+ab}{2}}$ (b) $\frac{a+b}{2}$

(c) $\sqrt{\frac{a^2+b^2+ab}{3}}$ (d) $\frac{a+b}{3}$

2. A solid sphere of mass M and radius R is divided into two unequal parts. The first part has a mass of $\frac{7M}{8}$ and is converted into a uniform disc of radius $2R$. The second part is converted into a uniform solid sphere. Let I_1 be the moment of inertia of the disc about its axis and I_2 be the moment of inertia of the new sphere about its axis.

The ratio I_1 / I_2 is given by **(2019 Main, 10 Apri II)**

(a) 285 (b) 185 (c) 65 (d) 140

3. A thin disc of mass M and radius R has mass per unit area $\sigma(r) = kr^2$, where r is the distance from its centre. Its moment of inertia about an axis going through its centre of mass and perpendicular to its plane is **(2019 Main, 10 April I)**

(a) $\frac{MR^2}{2}$ (b) $\frac{MR^2}{6}$

(c) $\frac{MR^2}{3}$ (d) $\frac{2MR^2}{3}$

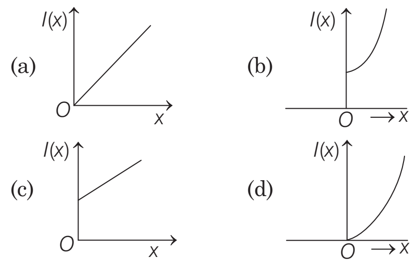
4. A stationary horizontal disc is free to rotate about its axis. When a torque is applied on it, its kinetic energy as a function of θ, where θ is the angle by which it has rotated, is given as $k\theta^2$. If its moment of inertia is I, then the angular acceleration of the disc is **(2019 Main, 9 April I)**

(a) $\frac{k}{2I}\theta$ (b) $\frac{k}{I}\theta$ (c) $\frac{k}{4I}\theta$ (d) $\frac{2k}{I}\theta$

5. A thin circular plate of mass M and radius R has its density varying as $\rho(r) = \rho_0 r$ with ρ_0 as constant and r is the distance from its centre. The moment of inertia of the circular plate about an axis perpendicular to the plate and passing through its edge is $I = aMR^2$. The value of the coefficient a is **(2019 Main, 8 April I)**

(a) $\frac{1}{2}$ (b) $\frac{3}{5}$ (c) $\frac{8}{5}$ (d) $\frac{3}{2}$

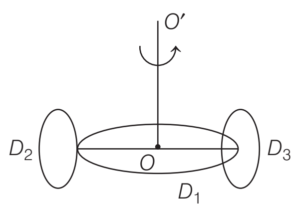
6. The moment of inertia of a solid sphere, about an axis parallel to its diameter and at a distance of x from it, is '$I(x)$'. Which one of the graphs represents the variation of $I(x)$ with x correctly? **(2019 Main, 12 Jan II)**



7. Let the moment of inertia of a hollow cylinder of length 30 cm (inner radius 10 cm and outer radius 20 cm) about its axis be I. The radius of a thin cylinder of the same mass such that its moment of inertia about its axis is also I, is **(2019 Main, 12 Jan I)**

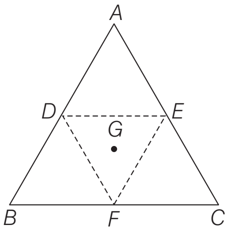
(a) 16 cm (b) 14 cm (c) 12 cm (d) 18 cm

8. A circular disc D_1 of mass M and radius R has two identical discs D_2 and D_3 of the same mass M and radius R attached rigidly at its opposite ends (see figure). The moment of inertia of the system about the axis OO' passing through the centre of D_1, as shown in the figure will be **(2019 Main, 11 Jan II)**



(a) $\frac{2}{3}MR^2$ (b) $\frac{4}{5}MR^2$ (c) $3MR^2$ (d) MR^2

9. An equilateral triangle ABC is cut from a thin solid sheet of wood. (see figure) D, E and F are the mid points of its sides as shown and G is the centre of the triangle. The moment of inertia of the triangle about an axis passing through G and perpendicular to the plane of the triangle is I_0. If the smaller triangle DEF is removed from ABC, the moment of inertia of the remaining figure about the same axis is I.

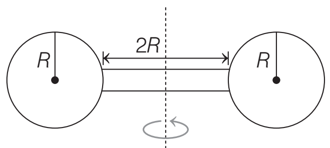


Then **(2019 Main, 11 Jan I)**

(a) $I' = \frac{3}{4}I_0$ (b) $I' = \frac{15}{16}I_0$ (c) $I' = \frac{I_0}{4}$ (d) $I' = \frac{9}{16}I_0$

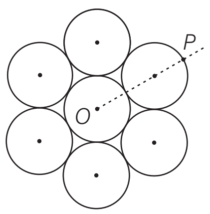
10. Two identical spherical balls of mass M and radius R each are stuck on two ends of a rod of length $2R$ and mass M (see figure).

The moment of inertia of the system about the axis passing perpendicularly through the centre of the rod is **(2019 Main, 10 Jan II)**



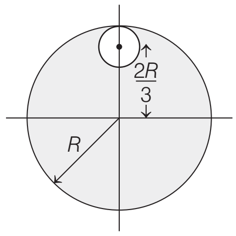
(a) $\frac{137}{15}MR^2$ (b) $\frac{209}{15}MR^2$ (c) $\frac{17}{15}MR^2$ (d) $\frac{152}{15}MR^2$

11. Seven identical circular planar discs, each of mass M and radius R are welded symmetrically as shown in the figure. The moment of inertia of the arrangement about an axis normal to the plane and passing through the point P is **(2018 Main)**



(a) $\frac{181}{2}MR^2$ (b) $\frac{19}{2}MR^2$ (c) $\frac{55}{2}MR^2$ (d) $\frac{73}{2}MR^2$

12. From a uniform circular disc of radius R and mass $9M$, a small disc of radius $\frac{R}{3}$ is removed as shown in the figure. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through centre of disc is **(2018 Main)**



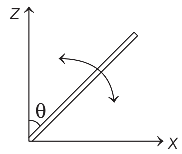
(a) $\frac{37}{9}MR^2$ (b) $4MR^2$

(c) $\frac{40}{9}MR^2$ (d) $10MR^2$

13. The moment of inertia of a uniform cylinder of length l and radius R about its perpendicular bisector is I. What is the ratio l/R such that the moment of inertia is minimum? **(2017 Main)**

(a) $\frac{\sqrt{3}}{2}$ (b) 1 (c) $\frac{3}{\sqrt{2}}$ (d) $\sqrt{\frac{3}{2}}$

14. A cylinder uniform rod of mass M and length l is pivoted at one end so that it can rotate in a vertical plane (see the figure). There is negligible friction at the pivot. The free end is held vertically above the pivot and then released. The angular acceleration of the rod when it makes an angle θ with the vertical, is **(2017 Main)**

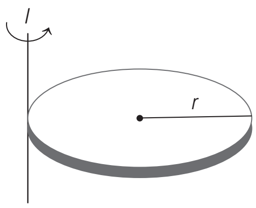


(a) $\frac{2g}{3l}\sin\theta$ (b) $\frac{3g}{2l}\cos\theta$ (c) $\frac{2g}{3l}\cos\theta$ (d) $\frac{3g}{2l}\sin\theta$

15. From a solid sphere of mass M and radius R, a cube of maximum possible volume is cut. Moment of inertia of cube about an axis passing through its centre and perpendicular to one of its faces is **(2015 Main)**

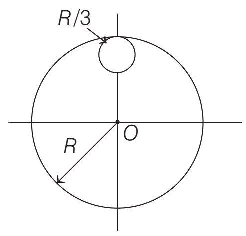
(a) $\frac{MR^2}{32\sqrt{2}\pi}$ (b) $\frac{4MR^2}{9\sqrt{3}\pi}$ (c) $\frac{MR^2}{16\sqrt{2}\pi}$ (d) $\frac{4MR^2}{3\sqrt{3}\pi}$

16. A solid sphere of radius R has moment of inertia I about its geometrical axis. It is melted into a disc of radius r and thickness t. If it's moment of inertia about the tangential axis (which is perpendicular to plane of the disc), is also equal to I, then the value of r is equal to **(2006, 3M)**



(a) $\frac{2}{\sqrt{15}}R$ (b) $\frac{2}{\sqrt{5}}R$ (c) $\frac{3}{\sqrt{15}}R$ (d) $\frac{\sqrt{3}}{\sqrt{15}}R$

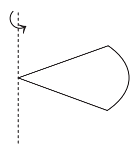
17. From a circular disc of radius R and mass $9M$, a small disc of radius $R/3$ is removed from the disc. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through O is **(2005)**



(a) $4MR^2$ (b) $\frac{40}{9}MR^2$

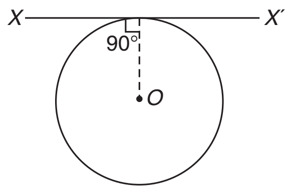
(c) $10MR^2$ (d) $\frac{37}{9}MR^2$

18. One quarter section is cut from a uniform circular disc of radius R. This section has a mass M. It is made to rotate about a line perpendicular to its plane and passing through the centre of the original disc. Its moment of inertia about the axis of rotation is **(2001)**



(a) $\frac{1}{2}MR^2$ (b) $\frac{1}{4}MR^2$ (c) $\frac{1}{8}MR^2$ (d) $\sqrt{2}MR^2$

19. A thin wire of length L and uniform linear mass density ρ is bent into a circular loop with centre at O as shown. The moment of inertia of the loop about the axis XX' is **(2000)**



(a) $\frac{\rho L^3}{8\pi^2}$ (b) $\frac{\rho L^3}{16\pi^2}$ (c) $\frac{5\rho L^3}{16\pi^2}$ (d) $\frac{3\rho L^3}{8\pi^2}$

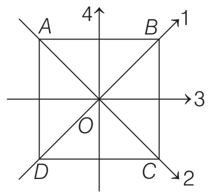
20. Let I be the moment of inertia of a uniform square plate about an axis AB that passes through its centre and is parallel to two of its sides. CD is a line in the plane of the plate that passes through the centre of the plate and makes an angle θ with AB. The moment of inertia of the plate about the axis CD is then equal to **(1998, 2M)**

(a) I (b) $I \sin^2 \theta$

(c) $I \cos^2 \theta$ (d) $I \cos^2 (\theta/2)$

Objective Question II (One or more correct option)

21. The moment of inertia of a thin square plate $ABCD$, of uniform thickness about an axis passing through the centre O and perpendicular to the plane of the plate is **(1992, 2M)**



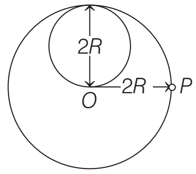
(a) $I_1 + I_2$ (b) $I_3 + I_4$

(c) $I_1 + I_3$ (d) $I_1 + I_2 + I_3 + I_4$

where, I_1, I_2, I_3 and I_4 are respectively moments of inertia about axes 1, 2, 3 and 4 which are in the plane of the plate.

Integer Answer Type Questions

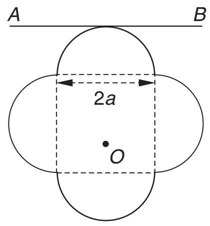
22. A lamina is made by removing a small disc of diameter $2R$ from a bigger disc of uniform mass density and radius $2R$, as shown in the figure. The moment of inertia of this lamina about axes passing through O and P is I_O and I_P, respectively. Both these axes are perpendicular to the plane of the lamina. The ratio $\frac{I_P}{I_O}$ to the nearest integer is **(2012)**



23. Four solid spheres each of diameter $\sqrt{5}$ cm and mass 0.5 kg are placed with their centres at the corners of a square of side 4 cm. The moment of inertia of the system about the diagonal of the square is $N \times 10^{-4}$ kg-m^2, then N is **(2011)**

Fill in the Blank

24. A symmetric lamina of mass M consists of a square shape with a semi-circular section over of the edge of the square as shown in figure. The side of the square is $2a$. The moment of inertia of the lamina about an axis through its centre of mass and perpendicular to the plane is $1.6\ Ma^2$. The moment of inertia of the lamina about the tangent AB in the plane of the lamina is ……… **(1997, 2M)**



Topic 2 Angular Momentum and its Conservation

Objective Questions I (Only one correct option)

1. The time dependence of the position of a particle of mass $m = 2$ is given by $\mathbf{r}(t) = 2t\hat{\mathbf{i}} - 3t^2\hat{\mathbf{j}}$. Its angular momentum, with respect to the origin, at time $t = 2$ is

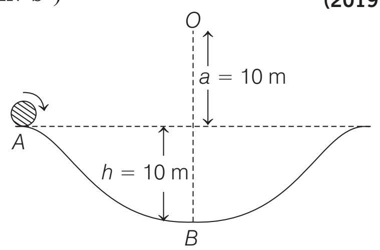
(a) $36\ \hat{\mathbf{k}}$ (b) $-34\ (\hat{\mathbf{k}} - \hat{\mathbf{i}})$

(c) $-48\ \hat{\mathbf{k}}$ (d) $48\ (\hat{\mathbf{i}} + \hat{\mathbf{j}})$

2. A thin smooth rod of length L and mass M is rotating freely with angular speed ω_0 about an axis perpendicular to the rod and passing through its centre. Two beads of mass m and negligible size are at the centre of the rod initially. The beads are free to slide along the rod. The angular speed of the system, when the beads reach the opposite ends of the rod, will be **(2019 Main, 09 April II)**

(a) $\frac{M\omega_0}{M+3m}$ (b) $\frac{M\omega_0}{M+m}$ (c) $\frac{M\omega_0}{M+2m}$ (d) $\frac{M\omega_0}{M+6m}$

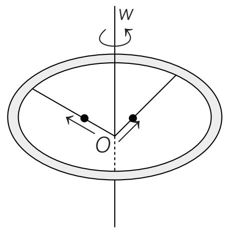
3. A particle of mass 20 g is released with an initial velocity 5 m/s along the curve from the point A, as shown in the figure. The point A is at height h from point B. The particle slides along the frictionless surface. When the particle reaches point B, its angular momentum about O will be (Take, $g = 10\ \text{m/s}^2$) **(2019 Main, 12 Jan II)**



(a) $8\ \text{kg-m}^2/\text{s}$ (b) $3\ \text{kg-m}^2/\text{s}$

(c) $2\ \text{kg-m}^2/\text{s}$ (d) $6\ \text{kg-m}^2/\text{s}$

4. A ring of mass M and radius R is rotating with angular speed ω about a fixed vertical axis passing through its centre O with two point masses each of mass $\frac{M}{8}$ at rest at O. These masses can move radially outwards along two massless rods fixed on the ring as shown in the figure.



At some instant, the angular speed of the system is $\frac{8}{9}\omega$ and one of the masses is at a distance of $\frac{3}{5}R$ from O. At this instant, the distance of the other mass from O is **(2015 Adv.)**

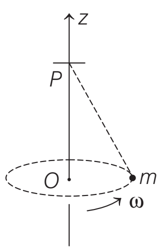
(a) $\frac{2}{3}R$ (b) $\frac{1}{3}R$

(c) $\frac{3}{5}R$ (d) $\frac{4}{5}R$

5. A bob of mass m attached to an inextensible string of length l is suspended from a vertical support. The bob rotates in a horizontal circle with an angular speed ω rad/s about the vertical support. About the point of suspension **(2014 Main)**

(a) angular momentum is conserved
(b) angular momentum changes in magnitude but not in direction
(c) angular momentum changes in direction but not in magnitude
(d) angular momentum changes both in direction and magnitude

6. A small mass m is attached to a massless string whose other end is fixed at P as shown in the figure. The mass is undergoing circular motion in the x-y plane with centre at O and constant angular speed ω. If the angular momentum of the system, calculated about O and P are denoted by $\mathbf{L}_O$ and $\mathbf{L}_P$ respectively, then **(2012)**



(a) $\mathbf{L}_O$ and $\mathbf{L}_P$ do not vary with time
(b) $\mathbf{L}_O$ varies with time while $\mathbf{L}_P$ remains constant
(c) $\mathbf{L}_O$ remains constant while $\mathbf{L}_P$ varies with time
(d) $\mathbf{L}_O$ and $\mathbf{L}_P$ both vary with time

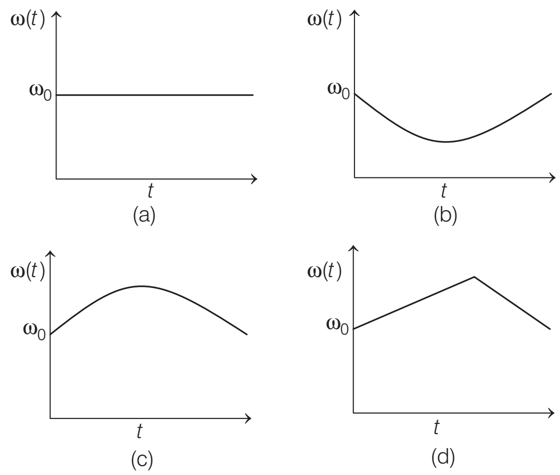
7. A child is standing with folded hands at the centre of a platform rotating about its central axis. The kinetic energy of the system is K. The child now stretches his arms so that the moment of inertia of the system doubles. The kinetic energy of the system now is **(2004)**

(a) $2K$ (b) $K/2$ (c) $K/4$ (d) $4K$

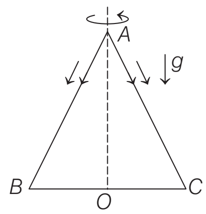
8. A particle undergoes uniform circular motion. About which point on the plane of the circle, will the angular momentum of the particle remain conserved ? **(2003)**

(a) Centre of circle
(b) On the circumference of the circle
(c) Inside the circle
(d) Outside the circle

9. A circular platform is free to rotate in a horizontal plane about a vertical axis passing through its centre. A tortoise is sitting at the edge of the platform. Now, the platform is given an angular velocity ω_0. When the tortoise move along a chord of the platform with a constant velocity (with respect to the platform). The angular velocity of the platform $\omega(t)$ will vary with time t as **(2002)**

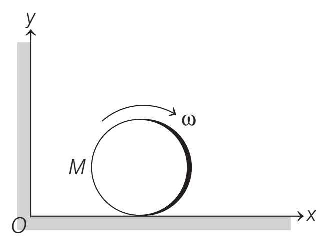


10. An equilateral triangle ABC formed from a uniform wire has two small identical beads initially located at A. The triangle is set rotating about the vertical axis AO. Then the beads are released from rest simultaneously and allowed to slide down, one along AB and other along AC as shown. Neglecting frictional effects, the quantities that are conserved as beads slides down are **(2000)**



(a) angular velocity and total energy (kinetic and potential)
(b) total angular momentum and total energy
(c) angular velocity and moment of inertia about the axis of rotation
(d) total angular momentum and moment of inertia about the axis of rotation

11. A disc of mass M and radius R is rolling with angular speed ω on a horizontal plane as shown. The magnitude of angular momentum of the disc about the origin O is **(1999, 2M)**



(a) $\left(\frac{1}{2}\right)MR^2\omega$ (b) $MR^2\omega$

(c) $\left(\frac{3}{2}\right)MR^2\omega$ (d) $2MR^2\omega$

12. A mass m is moving with a constant velocity along a line parallel to the X-axis, away from the origin. Its angular momentum with respect to the origin **(1997C, 1M)**

(a) is zero (b) remains constant
(c) goes on increasing (d) goes on decreasing

13. A particle of mass m is projected with a velocity v making an angle of 45° with the horizontal. The magnitude of the angular momentum of the projectile about the point of projection when the particle is at its maximum height h is **(1990, 2M)**

(a) zero (b) $mv^3/(4\sqrt{2}\,g)$
(c) $mv^3/(\sqrt{2}\,g)$ (d) $m\sqrt{2\,gh^3}$

14. A thin circular ring of mass M and radius r is rotating about its axis with a constant angular velocity ω. Two objects, each of mass m, are attached gently to the opposite ends of a diameter of the ring. The wheel now rotates with an angular velocity **(1983, 1M)**

(a) $\omega\, M/(M+m)$ (b) $\omega\,(M-2m)/(M+2m)$
(c) $\omega\, M/(M+2m)$ (d) $\omega\,(M+2m)/M$

Objective Question II (One or more correct option)

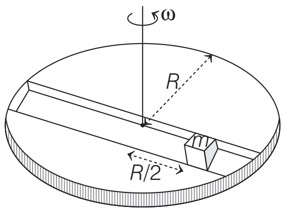
15. The torque τ on a body about a given point is found to be equal to $\mathbf{A}\times\mathbf{L}$, where $\mathbf{A}$ is a constant vector and $\mathbf{L}$ is the angular momentum of the body about that point. From this it follows that **(1998, 2M)**

(a) $\dfrac{d\mathbf{L}}{dt}$ is perpendicular to $\mathbf{L}$ at all instants of time
(b) the component of $\mathbf{L}$ in the direction of $\mathbf{A}$ does not change with time
(c) the magnitude of $\mathbf{L}$ does not change with time
(d) $\mathbf{L}$ does not change with time

Passage Based Questions

Passage 1

A frame of the reference that is accelerated with respect to an inertial frame of reference is called a non-inertial frame of reference. A coordinate system fixed on a circular disc rotating about a fixed axis with a constant angular velocity ω is an example of a non-inertial frame of reference. The relationship between the force $\mathbf{F}_{rot}$ experienced by a particle of mass m moving on the rotating disc and the force $\mathbf{F}_{in}$ experienced by the particle in an inertial frame of reference is, $\mathbf{F}_{rot}=\mathbf{F}_{in}+2m\,(\mathbf{v}_{rot}\times\vec{\omega})+m\,(\vec{\omega}\times\mathbf{r})\times\vec{\omega}$,



where, $\mathbf{v}_{rot}$ is the velocity of the particle in the rotating frame of reference and $\mathbf{r}$ is the position vector of the particle with respect to the centre of the disc. Now, consider a smooth slot along a diameter of a disc of radius R rotating counter-clockwise with a constant angular speed ω about its vertical axis through its centre. We assign a coordinate system with the origin at the centre of the disc, the X-axis along the slot, the Y-axis perpendicular to the slot and the z-axis along th rotation axis ($\omega=\omega\mathbf{k}$). A small block of mass m is gently placed in the slot at $\mathbf{r}=(R/2)\hat{\mathbf{i}}$ at $t=0$ and is constrained to move only along the slot. **(2016 Adv.)**

16. The distance r of the block at time t is

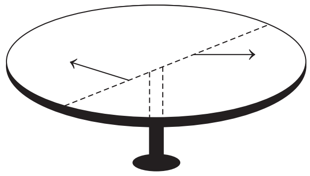
(a) $\dfrac{R}{2}\cos 2\omega t$ (b) $\dfrac{R}{2}\cos\omega t$
(c) $\dfrac{R}{4}(e^{\omega t}+e^{-\omega t})$ (d) $\dfrac{R}{4}(e^{2\omega t}+e^{-2\omega t})$

17. The net reaction of the disc on the block is

(a) $m\omega^2 R\sin\omega t\hat{\mathbf{j}}-mg\hat{\mathbf{k}}$
(b) $\dfrac{1}{2}m\omega^2 R\,(e^{\omega t}-e^{-\omega t})\hat{\mathbf{j}}+mg\hat{\mathbf{k}}$
(c) $\dfrac{1}{2}m\omega^2 R\,(e^{2\omega t}-e^{-2\omega t})\hat{\mathbf{j}}+mg\hat{\mathbf{k}}$
(d) $-m\omega^2 R\cos\omega t\hat{\mathbf{j}}-mg\hat{\mathbf{k}}$

Integer Answer Type Questions

18. A horizontal circular platform of radius 0.5 m and mass 0.45 kg is free to rotate about its axis. Two massless spring toy-guns, each carrying a steel ball of mass 0.05 kg are attached to the platform at a distance 0.25 m from the centre on its either sides along its diameter (see figure). Each gun simultaneously fires the balls horizontally and perpendicular to the diameter in opposite directions. After leaving the platform, the balls have horizontal speed of $9\,\mathrm{ms^{-1}}$ with respect to the ground. The rotational speed of the platform in rad $\mathrm{s^{-1}}$ after the balls leave the platform is **(2014 Adv.)**



19. A uniform circular disc of mass 50 kg and radius 0.4 m is rotating with an angular velocity of 10 rad/s about its own axis, which is vertical. Two uniform circular rings, each of mass 6.25 kg and radius 0.2 m, are gently placed symmetrically on the disc in such a manner that they are touching each other along the axis of the disc and are horizontal. Assume that the friction is large enough such that the rings are at rest relative to the disc and the system rotates about the original axis. The new angular velocity (in rad $\mathrm{s^{-1}}$) of the system is **(2013 Adv.)**

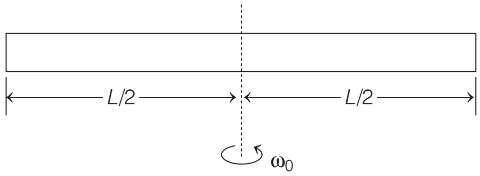
20. A binary star consists of two stars A (mass $2.2M_S$) and B (mass $11M_S$), where M_S is the mass of the sun. They are separated by distance d and are rotating about their centre of mass, which is stationary. The ratio of the total angular momentum of the binary star to the angular momentum of star B about the centre of mass is **(2010)**

Fill in the Blanks

21. A stone of mass m, tied to the end of a string, is whirled around in a horizontal circle. (Neglect the force due to gravity). The length of the string is reduced gradually keeping the angular momentum of the stone about the centre of the circle constant. Then, the tension in the string is given by $T = Ar^n$, where A is a constant, r is the instantaneous radius of the circle. Then $n = \ldots\ldots$. **(1993, 1M)**

22. A smooth uniform rod of length L and mass M has two identical beads of negligible size, each of mass m, which can slide freely along the rod. Initially, the two beads are at the centre of the rod and the system is rotating with an angular velocity ω_0 about an axis perpendicular to the rod and passing through the mid-point of the rod (see figure).

There are no external forces. When the beads reach the ends of the rod, the angular velocity of the system is…… . **(1988, 2M)**



True/False

23. A thin uniform circular disc of mass M and radius R is rotating in a horizontal plane about an axis passing through its centre and perpendicular to its plane with an angular velocity ω. Another disc of the same dimensions but of mass $M/4$ is placed gently on the first disc coaxially. The angular velocity of the system now is $2\omega/\sqrt{5}$. **(1986, 3M)**

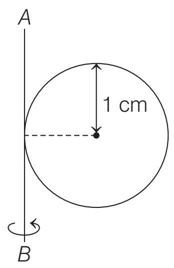
Analytical & Descriptive Questions

24. A particle is projected at time $t = 0$ from a point P on the ground with a speed v_0, at an angle of 45° to the horizontal. Find the magnitude and direction of the angular momentum of the particle about P at time $t = \dfrac{v_0}{g}$. **(1984, 6M)**

Topic 3 Pure Rolling or Rolling without Slipping

Objective Questions I (Only one correct option)

1 A metal coin of mass 5g and radius 1 cm is fixed to a thin stick AB of negligible mass as shown in the figure. The system is initially at rest. The constant torque, that will make the system rotate about AB at 25 rotations per second in 5s, is close to **(2019 Main, 10 April II)**



(a) 4.0×10^{-6} N-m (b) 2.0×10^{-5} N-m

(c) 1.6×10^{-5} N-m (d) 7.9×10^{-6} N-m

2 Moment of inertia of a body about a given axis is 1.5 kg m^2. Initially, the body is at rest. In order to produce a rotational kinetic energy of 1200 J, the angular acceleration of 20 rad/s^2 must be applied about the axis for a duration of **(2019 Main, 9 April II)**

(a) 5 s (b) 2 s (c) 3 s (d) 2.5 s

3 The following bodies are made to roll up (without slipping) the same inclined plane from a horizontal plane : (i) a ring of radius R, (ii) a solid cylinder of radius $R/2$ and (iii) a solid sphere of radius $R/4$. If in each case, the speed of the centre of mass at the bottom of the incline is same, the ratio of the maximum height they climb is **(2019 Main, 9 April I)**

(a) 10 : 15 : 7 (b) 4 : 3 : 2

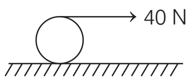
(c) 14 : 15 : 20 (d) 2 : 3 : 4

4 A solid sphere and solid cylinder of identical radii approach an incline with the same linear velocity (see figure). Both roll without slipping all throughout. The two climb maximum heights h_{sph} and h_{cyl} on the incline. The ratio $\dfrac{h_{sph}}{h_{cyl}}$ is given by

(2019 Main, 8 Ap[ril II)

(a) $\dfrac{2}{\sqrt{5}}$ (b) $\dfrac{14}{15}$ (c) 1 (d) $\dfrac{4}{5}$

5 A string is wound around a hollow cylinder of mass 5 kg and radius 0.5 m. If the string is now pulled with a horizontal force of 40 N and the cylinder is rolling without slipping on a horizontal surface (see figure), then the angular acceleration of the cylinder will be (Neglect the mass and thickness of the string)



(2019 Main, 11 Jan II)

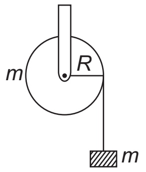
(a) 10 rad/ s^2 (b) 16 rad/ s^2

(c) 20 rad/ s^2 (d) 12 rad/ s^2

6 A homogeneous solid cylindrical roller of radius R and mass m is pulled on a cricket pitch by a horizontal force. Assuming rolling without slipping, angular acceleration of the cylinder is **(2019 Main, 10 Jan I)**

(a) $\dfrac{F}{2mR}$ (b) $\dfrac{2F}{3mR}$ (c) $\dfrac{3F}{2mR}$ (d) $\dfrac{F}{3mR}$

7. A mass m supported by a massless string wound around a uniform hollow cylinder of mass m and radius R. If the string does not slip on the cylinder, with what acceleration will the mass fall on release? **(2014 Main)**

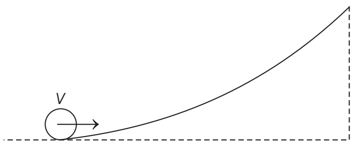


(a) $2g/3$ (b) $g/2$
(c) $5g/6$ (d) g

8. Two solid cylinders P and Q of same mass and same radius start rolling down a fixed inclined plane from the same height at the same time. Cylinder P has most of its mass concentrated near its surface, while Q has most of its mass concentrated near the axis. Which statement(s) is(are) correct? **(2012)**

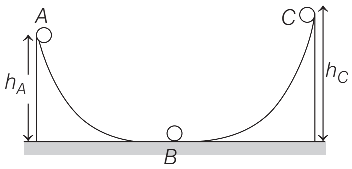
(a) Both cylinders P and Q reach the ground at the same time
(b) Cylinder P has larger linear acceleration than cylinder Q
(c) Both cylinders reach the ground with same translational kinetic energy
(d) Cylinder Q reaches the ground with larger angular speed

9. A small object of uniform density rolls up a curved surface with an initial velocity v. It reaches up to a maximum height of $\dfrac{3v^2}{4g}$ with respect to the initial position. The object is **(2007, 3M)**



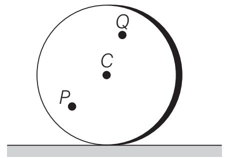
(a) ring (b) solid sphere
(c) hollow sphere (d) disc

10. A ball moves over a fixed track as shown in the figure. From A to B the ball rolls without slipping. If surface BC is frictionless and K_A, K_B and K_C are kinetic energies of the ball at A, B and C respectively, then **(2006, 5M)**



(a) $h_A > h_C$; $K_B > K_C$ (b) $h_A > h_C$; $K_C > K_A$
(c) $h_A = h_C$; $K_B = K_C$ (d) $h_A < h_C$; $K_B > K_C$

11. A disc is rolling (without slipping) on a horizontal surface. C is its centre and Q and P are two points equidistant from C. Let v_P, v_Q and v_C be the magnitude of velocities of points P, Q and C respectively, then **(2004)**



(a) $v_Q > v_C > v_P$ (b) $v_Q < v_C < v_P$
(c) $v_Q = v_P, v_C = \dfrac{1}{2}v_P$ (d) $v_Q < v_C > v_P$

12. A cylinder rolls up an inclined plane, reaches some height and then rolls down (without slipping throughout these motions). The directions of the frictional force acting on the cylinder are **(2002)**

(a) up the incline while ascending and down the incline while descending
(b) up the incline while ascending as well as descending
(c) down the incline while ascending and up the incline while descending
(d) down the incline while ascending as well as descending

Assertion and Reason

Mark your answer as

(*a*) *If Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I*
(*b*) *If Statement I is true, Statement II is true; Statement II is not a correct explanation for Statement I*
(*c*) *If Statement I is true; Statement II is false*
(*d*) *If Statement I is false; Statement II is true*

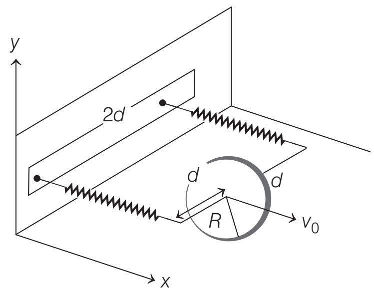
13. **Statement I** Two cylinders, one hollow (metal) and the other solid (wood) with the same mass and identical dimensions are simultaneously allowed to roll without slipping down an inclined plane from the same height. The hollow cylinder will reach the bottom of the inclined plane first.

Statement II By the principle of conservation of energy, the total kinetic energies of both the cylinders are identical when they reach the bottom of the incline. **(2008, 3M)**

Passage Based Questions

Passage 1

A uniform thin cylindrical disk of mass M and radius R is attached to two identical massless springs of spring constant k which are fixed to the wall as shown in the figure. The springs are attached to the axle of the disk diammetrically on either side at a distance d from its centre. The axle is massless and both the springs and the axle are in a horizontal plane. The unstretched length of each spring is L.



The disk is initially at its equilibrium position with its centre of mass (CM) at a distance L from the wall. The disk rolls without slipping with velocity $\mathbf{v}_0 = v_0\hat{\mathbf{i}}$. The coefficient of friction is μ.

14. The net external force acting on the disk when its centre of mass is at displacement x with respect to its equilibrium position is **(2008, 4M)**

(a) $-kx$ (b) $-2kx$ (c) $-\dfrac{2kx}{3}$ (d) $-\dfrac{4kx}{3}$

15. The centre of mass of the disk undergoes simple harmonic motion with angular frequency ω equal to **(2008 , 4M)**

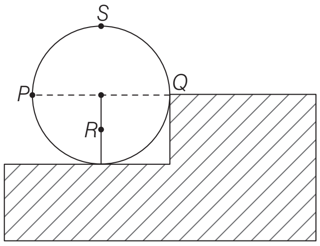
(a) $\sqrt{\dfrac{k}{M}}$ (b) $\sqrt{\dfrac{2k}{M}}$ (c) $\sqrt{\dfrac{2k}{3M}}$ (d) $\sqrt{\dfrac{4k}{3M}}$

16. The maximum value of v_0 for which the disk will roll without slipping is **(2008, 4M)**

(a) $\mu g\sqrt{\dfrac{M}{k}}$ (b) $\mu g\sqrt{\dfrac{M}{2k}}$ (c) $\mu g\sqrt{\dfrac{3M}{k}}$ (d) $\mu g\sqrt{\dfrac{5M}{2k}}$

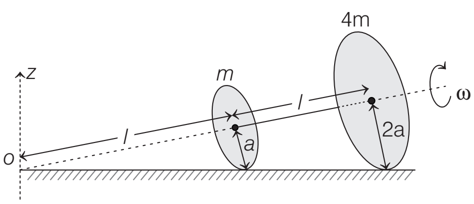
Objective Questions II (One or more correct option)

17. A wheel of radius R and mass M is placed at the bottom of a fixed step of height R as shown in the figure. A constant force is continuously applied on the surface of the wheel so that it just climbs the step without slipping. Consider the torque τ about an axis normal to the plane of the paper passing through the point Q. Which of the following options is/are correct? **(2017 Adv.)**



(a) If the force is applied normal to the circumference at point P, then τ is zero
(b) If the force is applied tangentially at point S, then $\tau \neq 0$ but the wheel never climbs the step
(c) If the force is applied at point P tangentially, then τ decreases continuously as the wheel climbs
(d) If the force is applied normal to the circumference at point X, then τ is constant

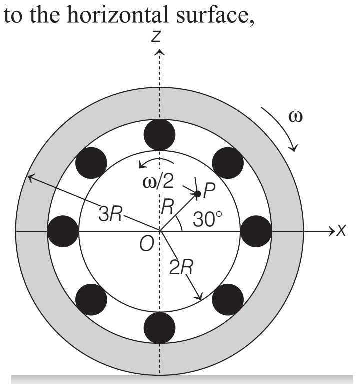
18. Two thin circular discs of mass m and $4m$, having radii of a and $2a$, respectively, are rigidly fixed by a massless, rigid rod of length $l = \sqrt{24}\,a$ through their centers. This assembly is laid on a firm and flat surface and set rolling without slipping on the surface so that the angular speed about the axis of the rod is ω. The angular momentum of the entire assembly about the point 'O' is $\mathbf{L}$ (see the figure). Which of the following statement(s) is (are) true? **(2016 Adv.)**



(a) The magnitude of the z-component of $\mathbf{L}$ is $55\,ma^2\omega$
(b) The magnitude of angular momentum of centre of mass of the assembly about the point O is $81\,ma^2\omega$
(c) The centre of mass of the assembly rotates about the Z-axis with an angular speed of $\dfrac{\omega}{5}$
(d) The magnitude of angular momentum of the assembly about its centre of mass is $17\,ma^2\dfrac{\omega}{2}$

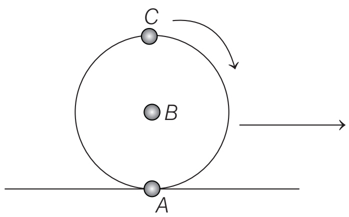
19. The figure shows a system consisting of (i) a ring of outer radius $3R$ rolling clockwise without slipping on a horizontal surface with angular speed ω and (ii) an inner disc of radius $2R$ rotating anti-clockwise with angular speed $\omega/2$.

The ring and disc are separated by frictionless ball bearings. The system is in the x-z plane. The point P on the inner disc is at a distance R from the origin, where OP makes an angle of 30° with the horizontal. Then with respect to the horizontal surface, **(2012)**



(a) the point O has a linear velocity $3R\omega\,\hat{\mathbf{i}}$
(b) the point P has a linear velocity $\dfrac{11}{4}R\omega\,\hat{\mathbf{i}} + \dfrac{\sqrt{3}}{4}R\omega\,\hat{\mathbf{k}}$
(c) the point P has a linear velocity $\dfrac{13}{4}R\omega\,\hat{\mathbf{i}} - \dfrac{\sqrt{3}}{4}R\omega\,\hat{\mathbf{k}}$
(d) the point P has a linear velocity $\left(3 - \dfrac{\sqrt{3}}{4}\right)R\omega\,\hat{\mathbf{i}} + \dfrac{1}{4}R\omega\,\hat{\mathbf{k}}$

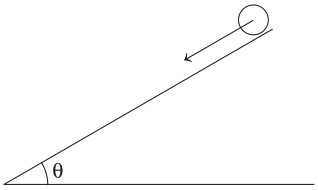
20. A sphere is rolling without slipping on a fixed horizontal plane surface.



In the figure, A is the point of contact. B is the centre of the sphere and C is its topmost point. Then, **(2009)**

(a) $\mathbf{v}_C - \mathbf{v}_A = 2(\mathbf{v}_B - \mathbf{v}_C)$ (b) $\mathbf{v}_C - \mathbf{v}_B = \mathbf{v}_B - \mathbf{v}_A$
(c) $|\mathbf{v}_C - \mathbf{v}_A| = 2|\mathbf{v}_B - \mathbf{v}_C|$ (d) $|\mathbf{v}_C - \mathbf{v}_A| = 4|\mathbf{v}_B|$

21. A solid sphere is in pure rolling motion on an inclined surface having inclination θ **(2006, 5M)**



(a) frictional force acting on sphere is $f = \mu mg \cos\theta$
(b) f is dissipative force
(c) friction will increase its angular velocity and decrease its linear velocity
(d) If θ decreases, friction will decrease

Numerical Value Based Question

22. A ring and a disc are initially at rest, side by side, at the top of an inclined plane which makes an angle 60° with the horizontal. They start to roll without slipping at the same instant of time along the shortest path. If the time difference between their reaching the ground is $(2-\sqrt{3})/\sqrt{10}$ s, then the height of the top of the inclined plane, in metres, is .... (Take, $g = 10\text{ms}^{-2}$) **(2018 Main)**

Fill in the Blanks

23. A uniform disc of mass m and radius R is rolling up a rough inclined plane which makes an angle of 30° with the horizontal. If the coefficients of static and kinetic friction are each equal to μ and the only forces acting are gravitational and frictional, then the magnitude of the frictional force acting on the disc is ……… and its direction is ……… (write up or down) the inclined plane. **(1997C, 1M)**

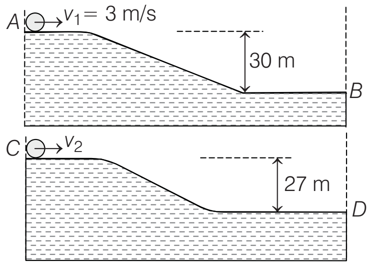
24. A cylinder of mass M and radius R is resting on a horizontal platform (which is parallel to the x-y plane) with its axis fixed along the Y-axis and free to rotate about its axis. The platform is given a motion in the x-direction given by $x = A\cos(\omega t)$. There is no slipping between the cylinder and platform. The maximum torque acting on the cylinder during its motion is ……… . **(1988, 2M)**

True / False

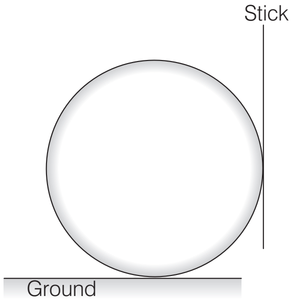
25. A ring of mass 0.3 kg and radius 0.1 m and a solid cylinder of mass 0.4 kg and of the same radius are given the same kinetic energy and released simultaneously on a flat horizontal surface such that they begin to roll as soon as released towards a wall which is at the same distance from the ring and the cylinder. The rolling friction in both cases is negligible. The cylinder will reach the wall first. **(1989, 2M)**

Integer Answer Type Question

26. Two identical uniform discs roll without slipping on two different surfaces AB and CD (see figure) starting at A and C with linear speeds v_1 and v_2, respectively, and always remain in contact with the surfaces. If they reach B and D with the same linear speed and $v_1 = 3$ m/s, then v_2 in m/s is ($g = 10\text{ m/s}^2$) **(2015 Adv.)**



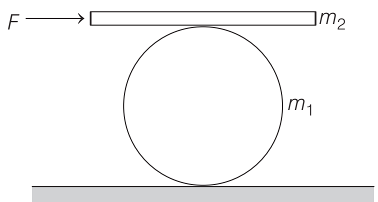
27. A boy is pushing a ring of mass 2 kg and radius 0.5 m with a stick as shown in the figure. The stick applies a force of 2 N on the ring and rolls it without slipping with an acceleration of 0.3 m/s^2. The coefficient of friction between the ground and the ring is large enough that rolling always occurs and the coefficient of friction between the stick and the ring is $\left(\frac{P}{10}\right)$. The value of P is **(2011)**



Analytical & Descriptive Questions

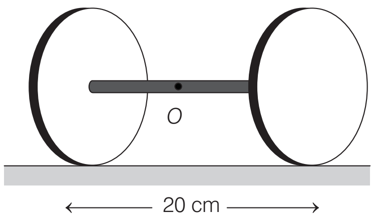
28. A solid cylinder rolls without slipping on an inclined plane inclined at an angle θ. Find the linear acceleration of the cylinder. Mass of the cylinder is M. **(2005, 4M)**

29. A man pushes a cylinder of mass m_1 with the help of a plank of mass m_2 as shown. There is no slipping at any contact. The horizontal component of the force applied by the man is F. Find **(1999, 10M)**



(a) the accelerations of the plank and the centre of mass of the cylinder and
(b) the magnitudes and directions of frictional forces at contact points.

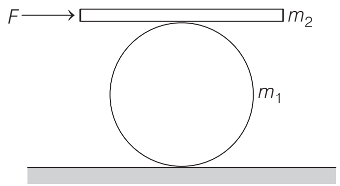
30. Two thin circular discs of mass 2 kg and radius 10 cm each are joined by a rigid massless rod of length 20 cm. The axis of the rod is along the perpendicular to the planes of the disc through their centres. This object is kept on a truck in such a way that the axis of the object is horizontal and perpendicular to the direction of motion of the truck.



Its friction with the floor of the truck is large enough, so that the object can roll on the truck without slipping. Take X-axis as the direction of motion of the truck and Z-axis as the vertically upwards direction. If the truck has an acceleration 9 m/s^2, calculate **(1997, 5M)**

(a) the force of friction on each disc and
(b) the magnitude and direction of the frictional torque acting on each disc about the centre of mass O of the object. Express the torque in the vector form in terms of unit vectors $\hat{\mathbf{i}}, \hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$ in x, y and z-directions.

31. A small sphere rolls down without slipping from the top of a track in a vertical plane. The track has an elevated section and a horizontal part. The horizontal part is 1.0 m above the ground level and the top of the track is 2.6 m above the ground. Find the distance on the ground with respect to the point B (which is vertically below the end of the track as shown in figure) where the sphere lands. During its flight as a projectile, does the sphere continue to rotate about its centre of mass ? Explain. **(1987, 7M)**



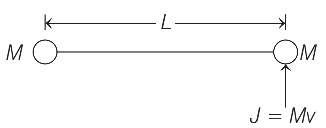
Topic 4 Collision in Rotational Motion

Objective Questions I (Only one correct option)

1 Two coaxial discs, having moments of inertia I_1 and $\frac{I_1}{2}$ are rotating with respective angular velocities ω_1 and $\frac{\omega_1}{2}$, about their common axis. They are brought in contact with each other and thereafter they rotate with a common angular velocity. If E_f and E_i are the final and initial total energies, then $(E_f - E_i)$ is **(2019 Main, 10 April I)**

(a) $-\frac{I_1\omega_1^2}{24}$ (b) $-\frac{I_1\omega_1^2}{12}$
(c) $\frac{3}{8}I_1\omega_1^2$ (d) $\frac{I_1\omega_1^2}{6}$

2. Consider a body, shown in figure, consisting of two identical balls, each of mass M connected by a light rigid rod. If an impulse $J = Mv$ is imparted to the body at one of its end, what would be its angular velocity ? **(2003)**

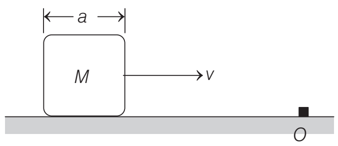


(a) v/L (b) $2v/L$ (c) $v/3L$ (d) $v/4L$

3. A smooth sphere A is moving on a frictionless horizontal plane with angular velocity ω and centre of mass velocity v. It collides elastically and head on with an identical sphere B at rest. Neglect friction everywhere. After the collision their angular speeds are ω_A and ω_B respectively. Then, **(1999, 2M)**

(a) $\omega_A < \omega_B$ (b) $\omega_A = \omega_B$ (c) $\omega_A = \omega$ (d) $\omega_B = \omega$

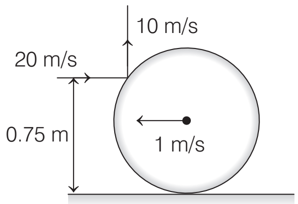
4. A cubical block of side a moving with velocity v on a horizontal smooth plane as shown. It hits a ridge at point O. The angular speed of the block after it hits O is **(1999, 2M)**



(a) $3v/4a$ (b) $3v/2a$
(c) $\sqrt{3}/\sqrt{2}\,a$ (d) zero

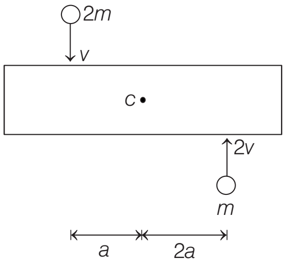
Objective Questions II (One or more correct option)

5. A thin ring of mass 2 kg and radius 0.5 m is rolling without slipping on a horizontal plane with velocity 1 m/s. A small ball of mass 0.1 kg, moving with velocity 20 m/s in the opposite direction, hits the ring at a height of 0.75 m and goes vertically up with velocity 10 m/s. Immediately after the collision, **(2011)**



(a) the ring has pure rotation about its stationary CM
(b) the ring comes to a complete stop
(c) friction between the ring and the ground is to the left
(d) there is no friction between the ring and the ground

6. A uniform bar of length 6 a and mass 8 m lies on a smooth horizontal table. Two point masses m and $2m$ moving in the same horizontal plane with speed $2v$ and v respectively, strike the bar [as shown in the figure] and stick to the bar after collision. Denoting angular velocity (about the centre of mass), total energy and centre of mass velocity by ω, E and v_c respectively, we have after collision **(1991)**



(a) $v_c = 0$ (b) $\omega = \dfrac{3v}{5a}$ (c) $\omega = \dfrac{v}{5a}$ (d) $E = \dfrac{3}{5}mv^2$

Analytical Answer Type Questions

7. A rod AB of mass M and length L is lying on a horizontal frictionless surface. A particle of mass m travelling along the surface hits the end A of the rod with a velocity v_0 in a direction perpendicular to AB. The collision is elastic. After the collision, the particle comes to rest. **(2000)**

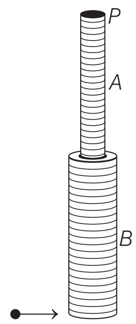
(a) Find the ratio m/M.

(b) A point P on the rod is at rest immediately after collision. Find the distance AP.

(c) Find the linear speed of the point P a time $\pi L/3v_0$ after the collision.

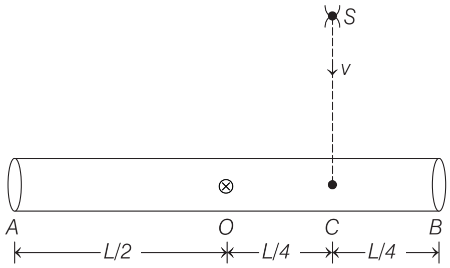
8. Two uniform rods A and B of length 0.6 m each and of masses 0.01 kg and 0.02 kg, respectively are rigidly joined end to end. The combination is pivoted at the lighter end, P as shown in figure. Such that it can freely rotate about point P in a vertical plane.

A small object of mass 0.05 kg, moving horizontally, hits the lower end of the combination and sticks to it. What should be the velocity of the object, so that the system could just be raised to the horizontal position? **(1994, 6M)**



9. A homogeneous rod AB of length $L = 1.8$ m and mass M is pivoted at the centre O in such a way that it can rotate freely in the vertical plane (figure).

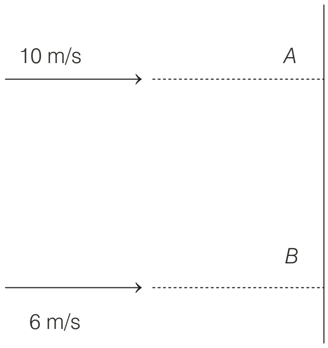
The rod is initially in the horizontal position. An insect S of the same mass M falls vertically with speed v on the point C, midway between the points O and B. Immediately after falling, the insect moves towards the end B such that the rod rotates with a constant angular velocity ω. **(1992, 8M)**



(a) Determine the angular velocity ω in terms of v and L.

(b) If the insect reaches the end B when the rod has turned through an angle of $90°$, determine v.

10. A thin uniform bar lies on a frictionless horizontal surface and is free to move in any way on the surface. Its mass is 0.16 kg and length is $\sqrt{3}$ m. Two particles, each of mass 0.08 kg are moving on the same surface and towards the bar in a direction perpendicular to the bar one with a velocity of 10 m/s, and the other with 6 m/s, as shown in figure. The first particle strikes the bar at points A and the other at point B. Points A and B are at a distance of 0.5 m from the centre of the bar. The particles strike the bar at the same instant of time and stick to the bar on collision. Calculate the loss of kinetic energy of the system in the above collision process. **(1991)**

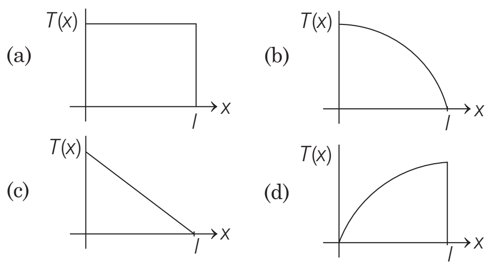


Topic 5 Miscellaneous Problems

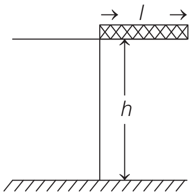
Objective Questions I (Only one correct option)

1 A uniform rod of length l is being rotated in a horizontal plane with a constant angular speed about an axis passing through one of its ends. If the tension generated in the rod due to rotation is $T(x)$ at a distance x from the axis, then which of the following graphs depicts it most closely?

(2019 Main, 12 April I)



2 A rectangular solid box of length 0.3 m is held horizontally, with one of its sides on the edge of a platform of height 5 m. When released, it slips off the table in a very short time $\tau = 0.01$ s, remaining essentially horizontal. The angle by which it would rotate when it hits the ground will be (in radians) close to **(2019 Main, 8 April II)**



(a) 0.02 (b) 0.3 (c) 0.5 (d) 0.28

3 A straight rod of length L extends from $x = a$ to $x = L + a$. The gravitational force it exerts on a point mass m at $x = 0$, if the mass per unit length of the rod is $A + Bx^2$, is given by **(2019 Main, 12 Jan I)**

(a) $Gm\left[A\left(\frac{1}{a+L} - \frac{1}{a}\right) - BL\right]$

(b) $Gm\left[A\left(\frac{1}{a+L} - \frac{1}{a}\right) + BL\right]$

(c) $Gm\left[A\left(\frac{1}{a} - \frac{1}{a+L}\right) + BL\right]$

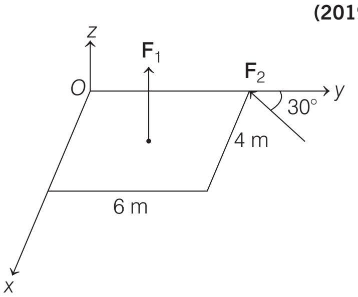
(d) $Gm\left[A\left(\frac{1}{a} - \frac{1}{a+L}\right) - BL\right]$

4 The magnitude of torque on a particle of mass 1 kg is 2.5 N-m about the origin. If the force acting on it is 1 N and the distance of the particle from the origin is 5 m, then the angle between the force and the position vector is (in radian) **(Main 2019, 11 Jan II)**

(a) $\frac{\pi}{8}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

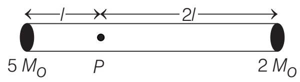
5 A slab is subjected to two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ of same magnitude F as shown in the figure. Force $\mathbf{F}_2$ is in xy-plane while force $\mathbf{F}_1$ acts along Z-axis at the point $(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}})$. The moment of these forces about point O will be **(2019 Main, 11 Jan I)**



(a) $(3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}})F$ (b) $(3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})F$

(c) $(3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}})F$ (d) $(3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})F$

6 A rigid massless rod of length $3l$ has two masses attached at each end as shown in the figure. The rod is pivoted at point P on the horizontal axis (see figure). When released from initial horizontal position, its instantaneous angular acceleration will be **(2019 Main, 10 Jan II)**

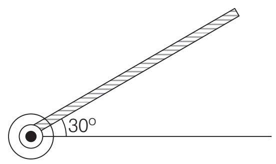


(a) $\frac{g}{13l}$ (b) $\frac{g}{2l}$ (c) $\frac{7g}{3l}$ (d) $\frac{g}{3l}$

7 To mop-clean a floor, a cleaning machine presses a circular mop of radius R vertically down with a total force F and rotates it with a constant angular speed about its axis. If the force F is distributed uniformly over the mop and if coefficient of friction between the mop and the floor is μ, the torque applied by the machine on the mop is **(2019 Main, 10 Jan I)**

(a) $\frac{2}{3}\mu FR$ (b) $\frac{\mu FR}{6}$ (c) $\frac{\mu FR}{3}$ (d) $\frac{\mu FR}{2}$

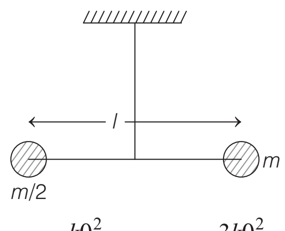
8 A rod of length 50 cm is pivoted at one end. It is raised such that if makes an angle of 30° from the horizontal as shown and released from rest. Its angular speed when it passes through the horizontal (in rad s^{-1}) will be
(Take, $g = 10\ ms^{-2}$) **(2019 Main, 09 Jan II)**



(a) $\frac{\sqrt{30}}{2}$ (b) $\sqrt{30}$

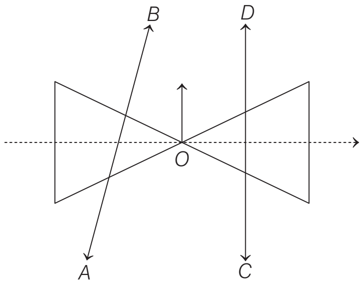
(c) $\frac{\sqrt{20}}{3}$ (d) $\sqrt{\frac{30}{2}}$

9 Two masses m and $\frac{m}{2}$ are connected at the two ends of a massless rigid rod of length l. The rod is suspended by a thin wire of torsional constant k at the centre of mass of the rod-mass system (see figure). Because of torsional constant k, the restoring torque is $\tau = k\theta$ for angular displacement θ. If the rod is rotated by θ_0 and released, the tension in it when it passes through its mean position will be **(2019 Main, 9 Jan I)**



(a) $\frac{2k\theta_0^2}{l}$ (b) $\frac{k\theta_0^2}{l}$ (c) $\frac{3k\theta_0^2}{l}$ (d) $\frac{k\theta_0^2}{2l}$

10. A roller is made by joining together two corners at their vertices O. It is kept on two rails AB and CD which are placed asymmetrically (see the figure), with its axis perpendicular to CD and its centre O at the centre of line joining AB and CD (see the figure). It is given a light push, so that it starts rolling with its centre O moving parallel to CD in the direction shown. As it moves, the roller will tend to **(2016 Main)**

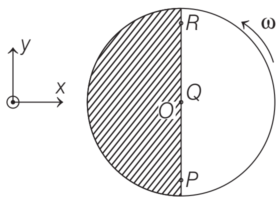


(a) turn left (b) turn right
(c) go straight (d) turn left and right alternately

11. A hoop of radius r and mass m rotating with an angular velocity ω_0 is placed on a rough horizontal surface. The initial velocity of the centre of the hoop is zero. What will be the velocity of the centre of the hoop when it ceases to slip? **(2013 Main)**

(a) $r\omega_0/4$ (b) $r\omega_0/3$ (c) $r\omega_0/2$ (d) $r\omega_0$

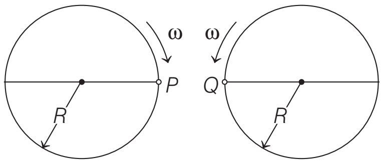
12. Consider a disc rotating in the horizontal plane with a constant angular speed ω about its centre O.



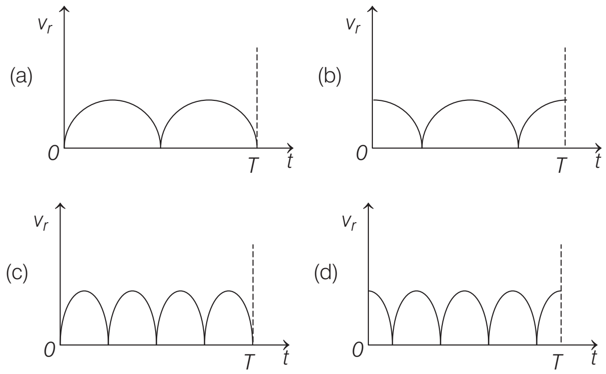
The disc has a shaded region on one side of the diameter and an unshaded region on the other side as shown in the figure. When the disc is in the orientation as shown, two pebbles P and Q are simultaneously projected at an angle towards R. The velocity of projection is in the y-z plane and is same for both pebbles with respect to the disc.
Assume that (i) they land back on the disc before the disc has completed 1/8 rotation, (ii) their range is less than half the disc radius, and (iii) ω remains constant throughout. Then **(2012)**

(a) P lands in the shaded region and Q in the unshaded region
(b) P lands in the unshaded region and Q in the shaded region
(c) both P and Q land in the unshaded region
(d) both P and Q land in the shaded region

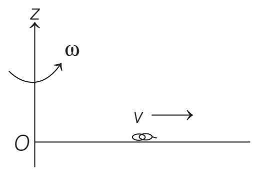
13. Two identical discs of same radius R are rotating about their axes in opposite directions with the same constant angular speed ω. The discs are in the same horizontal plane.



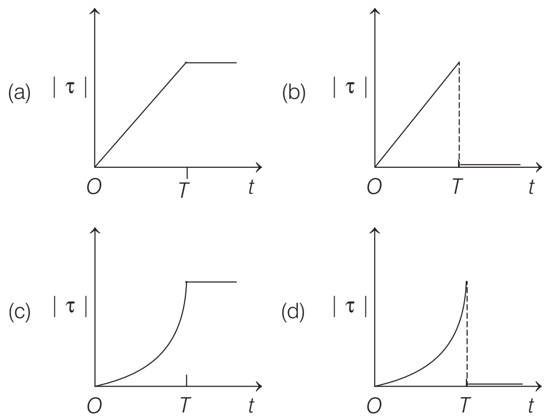
At time $t = 0$, the points P and Q are facing each other as shown in the figure. The relative speed between the two points P and Q is v_r. In one time period (T) of rotation of the discs, v_r as a function of time is best represented by **(2012)**



14. A thin uniform rod, pivoted at O, is rotating in the horizontal plane with constant angular speed ω, as shown in the figure. At time $t = 0$, a small insect starts from O and moves with constant speed v with respect to the rod towards the other end. It reaches the end of the rod at $t = T$ and stops. The angular speed of the system remains ω throughout. **(2012)**



The magnitude of the torque $|\tau|$ on the system about O, as a function of time is best represented by which plot?



15. A block of base 10 cm × 10 cm and height 15 cm is kept on an inclined plane. The coefficient of friction between them is $\sqrt{3}$. The inclination θ of this inclined plane from the horizontal plane is gradually increased from 0°. Then, **(2009)**

(a) at $\theta = 30°$, the block will start sliding down the plane
(b) the block will remain at rest on the plane up to certain θ and then it will topple
(c) at $\theta = 60°$, the block will start sliding down the plane and continue to do so at higher angles
(d) at $\theta = 60°$, the block will start sliding down the plane and on further increasing θ, it will topple at certain θ

16. A particle moves in a circular path with decreasing speed. Choose the correct statement. **(2005)**

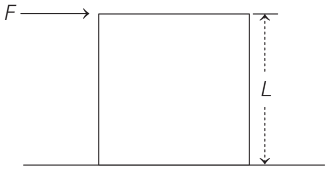
(a) Angular momentum remains constant

(b) Acceleration ($\mathbf{a}$) is towards the centre

(c) Particle moves in a spiral path with decreasing radius

(d) The direction of angular momentum remains constant

17. A cubical block of side L rests on a rough horizontal surface with coefficient of friction μ. A horizontal force F is applied on the block as shown. If the coefficient of friction is sufficiently high, so that the block does not slide before toppling, the minimum force required to topple the block is **(2000)**



(a) infinitesimal (b) $mg/4$

(c) $mg/2$ (d) $mg\ (1-\mu)$

18. Two point masses of 0.3 kg and 0.7 kg are fixed at the ends of a rod of length 1.4 m and of negligible mass. The rod is set rotating about an axis perpendicular to its length with a uniform angular speed. The point on the rod through which the axis should pass in order that the work required for rotation of the rod is minimum, is located at a distance of

(a) 0.42 m from mass of 0.3 kg **(1995, S)**

(b) 0.70 m from mass of 0.7 kg

(c) 0.98 m from mass of 0.3 kg

(d) 0.98 m from mass of 0.7 kg

19. A tube of length L is filled completely with an in-compressible liquid of mass M and closed at both the ends. The tube is then rotated in a horizontal plane about one of its ends with a uniform angular velocity ω. The force exerted by the liquid at the other end is **(1992, 2M)**

(a) $\frac{M\omega^2L}{2}$ (b) $M\omega^2L$ (c) $\frac{M\omega^2L}{4}$ (d) $\frac{M\omega^2L^2}{2}$

Objective Question II (One or more correct option)

20. The potential energy of mass m at a distance r from a fixed point O is given by $V(r)=kr^2/2$, where k is a positive constant of appropriate dimensions. This particle is moving in a circular orbit of radius R about the point O. If v is the speed of the particle and L is the magnitude of its angular momentum about O, which of the following statements is (are) true ? **(2018 Adv.)**

(a) $v=\sqrt{\frac{k}{2m}}R$ (b) $v=\sqrt{\frac{k}{m}}R$

(c) $L=\sqrt{mk}\,R^2$ (d) $L=\sqrt{\frac{mk}{2}}R^2$

21. Consider a body of mass 1.0 kg at rest at the origin at time $t=0$. A force $\mathrm{F}=(\alpha t\,\hat{\mathrm{i}}+\beta\,\hat{\mathrm{j}})$ is applied on the body, where $\alpha=1.0\mathrm{Ns}^{-1}$ and $\beta=1.0\mathrm{N}$. The torque acting on the body about the origin at time $t=1.0\mathrm{s}$ is τ. Which of the following statements is (are) true ? **(2108 Adv.)**

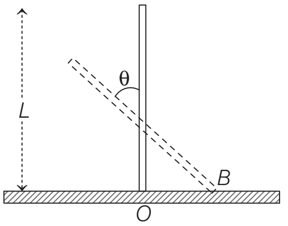
(a) $|\tau|=\frac{1}{3}\mathrm{N}\text{-}\mathrm{m}$

(b) The torque τ is in the direction of the unit vector $+\hat{k}$

(c) The velocity of the body at $t=1\mathrm{s}$ is $\mathrm{v}=\frac{1}{2}(\hat{\mathrm{i}}+2\hat{\mathrm{j}})\ \mathrm{ms}^{-1}$

(d) The magnitude of displacement of the body at $t=1\mathrm{s}$ is $\frac{1}{6}\mathrm{m}$

22. A rigid uniform bar AB of length L is slipping from its vertical position on a frictionless floor (as shown in the figure). At some instant of time, the angle made by the bar with the vertical is θ. Which of the following statements about its motion is/are correct? **(2017 Adv.)**



(a) Instantaneous torque about the point in contact with the floor is proportional to $\sin\theta$

(b) The trajectory of the point A is parabola

(c) The mid-point of the bar will fall vertically downward

(d) When the bar makes an angle θ with the vertical, the displacement of its mid-point from the initial position is proportional to $(1-\cos\theta)$

Matching Type Questions

23. In the List-I below, four different paths of a particle are given as functions of time. In these functions, α and β are positive constants of appropriate dimensions and $\alpha\neq\beta$. In each case, the force acting on the particle is either zero or conservative. In List-II, five physical quantities of the particle are mentioned: p is the linear momentum, L is the angular momentum about the origin, K is the kinetic energy, U is the potential energy and E is the total energy. Match each path in List-I with those quantities in List-II, which are conserved for that path. **(2018 Adv.)**

List-I		List-II	
P.	$\mathrm{r}(t)=\alpha\, t\,\mathrm{i}+\beta\, t\,\mathrm{j}$	1.	p
Q.	$\mathrm{r}(t)=\alpha\cos\omega t\,\mathrm{i}+\beta\sin\omega t\,\mathrm{j}$	2.	L
R.	$\mathrm{r}(t)=\alpha\,(\cos\omega t\,\mathrm{i}+\sin\omega t\,\mathrm{j})$	3.	K
S.	$\mathrm{r}(t)=a\,t\,\mathrm{i}+\frac{\beta}{2}t^2\,\mathrm{j}$	4.	U
		5.	E

(a) P $\rightarrow$ 1, 2, 3, 4, 5; Q $\rightarrow$ 2, 5; R $\rightarrow$ 2, 3, 4, 5; S $\rightarrow$ 5

(b) P $\rightarrow$ 1, 2, 3, 4, 5; Q $\rightarrow$ 3, 5; R $\rightarrow$ 2, 3, 4, 5; S $\rightarrow$ 2, 5

(c) P $\rightarrow$ 2, 3, 4; Q $\rightarrow$ 5; R $\rightarrow$ 1, 2, 4; S $\rightarrow$ 2, 5

(d) P $\rightarrow$ 1, 2, 3, 5; Q $\rightarrow$ 2, 5; R $\rightarrow$ 2, 3, 4, 5; S $\rightarrow$ 2, 5

Passage Based Questions

Passage 1

One twirls a circular ring (of mass M and radius R) near the tip of one's finger as shown in Figure 1. In the process the finger never loses contact with the inner rim of the ring. The finger traces out the surface of a cone, shown by the dotted line. The radius of the path traced out by the point where the ring and the finger is in contact is r. The finger rotates with an angular velocity ω_0. The rotating ring rolls without slipping on the outside of a smaller circle described by the point where the ring and the finger is in contact (Figure 2). The coefficient of friction between the ring and the finger is μ and the acceleration due to gravity is g.

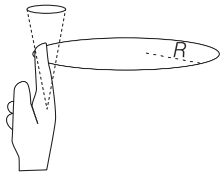


Figure 1

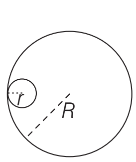


Figure 2

24. The total kinetic energy of the ring is **(2017 Adv.)**

(a) $M\omega_0^2(R-r)^2$ (b) $\frac{1}{2}M\omega_0^2(R-r)^2$

(c) $M\omega_0^2R^2$ (d) $\frac{3}{2}M\omega_0^2(R-r)^2$

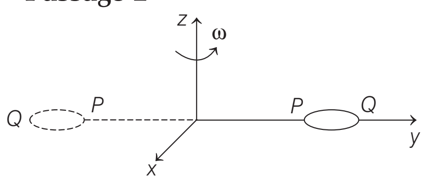
25. The minimum value of ω_0 below which the ring will drop down is **(2017 Adv.)**

(a) $\sqrt{\frac{g}{2\mu(R-r)}}$ (b) $\sqrt{\frac{3g}{2\mu(R-r)}}$

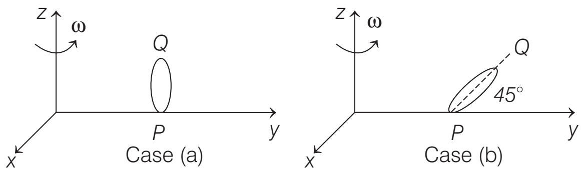
(c) $\sqrt{\frac{g}{\mu(R-r)}}$ (d) $\sqrt{\frac{2g}{\mu(R-r)}}$

Passage 2

The general motion of a rigid body can be considered to be a combination of (i) a motion of its centre of mass about an axis, and (ii) its motion about an instantaneous axis passing through the centre of mass. These axes need not be stationary. Consider, for example, a thin uniform disc welded (rigidly fixed) horizontally at its rim to a massless stick, as shown in the figure. When the disc-stick system is rotated about the origin on a horizontal frictionless plane with angular speed ω, the motion at any instant can be taken as a combination of (i) a rotation of the centre of mass of the disc about the Z-axis, and (ii) a rotation of the disc through an instantaneous vertical axis passing through its centre of mass (as is seen from the changed orientation of points P and Q). Both these motions have the same angular speed ω in this case.



Now, consider two similar systems as shown in the figure : Case (a) the disc with its face vertical and parallel to x-z plane; Case (b) the disc with its face making an angle of 45° with x-y plane and its horizontal diameter parallel to X-axis. In both the cases, the disc is welded at point P, and the systems are rotated with constant angular speed ω about the Z-axis.



26. Which of the following statements regarding the angular speed about the instantaneous axis (passing through the centre of mass) is correct? **(2012)**

(a) It is $\sqrt{2}\omega$ for both the cases

(b) It is ω for case (a); and $\frac{\omega}{\sqrt{2}}$ for case (b)

(c) It is ω for case (a); and $\sqrt{2}\omega$ for case (b)

(d) It is ω for both the cases

27. Which of the following statements about the instantaneous axis (passing through the centre of mass) is correct? **(2012)**

(a) It is vertical for both the cases (a) and (b)

(b) It is vertical for case (a); and is at 45° to the x-z plane and lies in the plane of the disc for case (b)

(c) It is horizontal for case (a); and is at 45° to the x-z plane and is normal to the plane of the disc for case (b)

(d) It is vertical for case (a); and is at 45° to the x-z plane and is normal to the plane of the disc for case (b)

Passage 3

Two discs A and B are mounted coaxially on a vertical axle. The discs have moments of inertia I and $2I$ respectively about the common axis. Disc A is imparted an initial angular velocity 2ω using the entire potential energy of a spring compressed by a distance x_1. Disc B is imparted an angular velocity ω by a spring having the same spring constant and compressed by a distance x_2. Both the discs rotate in the clockwise direction.

28. The ratio $\frac{x_1}{x_2}$ is **(2007, 4M)**

(a) 2 (b) 1/2 (c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

29. When disc B is brought in contact with disc A, they acquire a common angular velocity in time t. The average frictional torque on one disc by the other during this period is **(2007, 4M)**

(a) $\frac{2I\omega}{3t}$ (b) $\frac{9I\omega}{2t}$

(c) $\frac{9I\omega}{4t}$ (d) $\frac{3I\omega}{2t}$

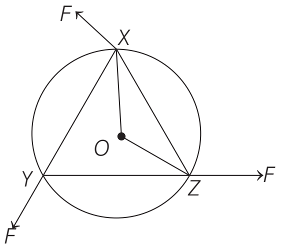
30. The loss of kinetic energy during the above process is **(2007, 4M)**

(a) $\frac{I\omega^2}{2}$ (b) $\frac{I\omega^2}{3}$

(c) $\frac{I\omega^2}{4}$ (d) $\frac{I\omega^2}{6}$

Integer Answer Type Question

31. A uniform circular disc of mass 1.5 kg and radius 0.5 m is initially at rest on a horizontal frictionless surface. Three forces of equal magnitude $F = 0.5$ N are applied simultaneously along the three sides of an equilateral triangle XYZ with its vertices on the perimeter of the disc (see figure). One second after applying the forces, the angular speed of the disc in rad s^{-1} is **(2012)**



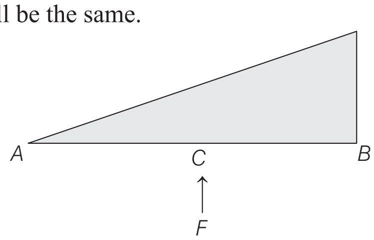
Fill in the Blanks

32. A rod of weight W is supported by two parallel knife edges A and B and is in equilibrium in a horizontal position. The knives are at a distance d from each other. The centre of mass of the rod is at distance x from A. The normal reaction on A is ……… and on B is ……… . **(1997, 2M)**

33. A uniform cube of side a and mass m rests on a rough horizontal table. A horizontal force F is applied normal to one of the faces at a point that is directly above the centre of the face, at a height $3a/4$ above the base. The minimum value of F for which the cube begins to tip about the edge is ……… (Assume that the cube does not slide). **(1984, 2M)**

True / False

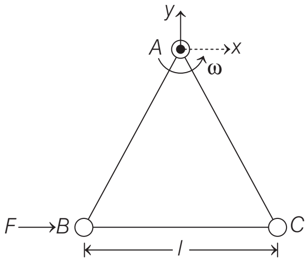
34. A triangular plate of uniform thickness and density is made to rotate about an axis perpendicular to the plane of the paper and (a) passing through A, (b) passing through B, by the application of the same force, F, at C (mid-point of AB) as shown in the figure. The angular acceleration in both the cases will be the same. **(1985, 3M)**



Analytical & Descriptive Questions

35. A rod of length L and mass M is hinged at point O. A small bullet of mass m hits the rod as shown in the figure. The bullet gets embedded in the rod. Find angular velocity of the system just after impact. **(2005, 2M)**

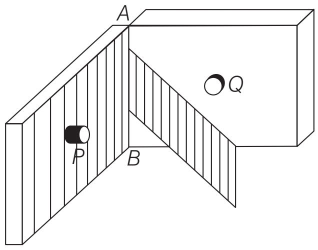
36. Three particles A, B and C, each of mass m, are connected to each other by three massless rigid rods to form a rigid, equilateral triangular body of side l. This body is placed on a horizontal frictionless table (x-y plane) and is hinged to it at the point A, so that it can move without friction about the vertical axis through A (see figure). The body is set into rotational motion on the table about A with a constant angular velocity ω. **(2002, 5M)**



(a) Find the magnitude of the horizontal force exerted by the hinge on the body.

(b) At time T, when the side BC is parallel to the X-axis, a force F is applied on B along BC (as shown). Obtain the x-component and the y-component of the force exerted by the hinge on the body, immediately after time T.

37. Two heavy metallic plates are joined together at 90° to each other. A laminar sheet of mass 30 kg is hinged at the line AB joining the two heavy metallic plates. The hinges are frictionless. The moment of inertia of the laminar sheet about an axis parallel to AB and passing through its centre of mass is 1.2 kg-m^2. Two rubber obstacles P and Q are fixed, one on each metallic plate at a distance 0.5 m from the line AB. This distance is chosen, so that the reaction due to the hinges on the laminar sheet is zero during the impact. Initially the laminar sheet hits one of the obstacles with an angular velocity 1 rad/s and turns back. If the impulse on the sheet due to each obstacle is 6 N-s. **(2001, 10M)**

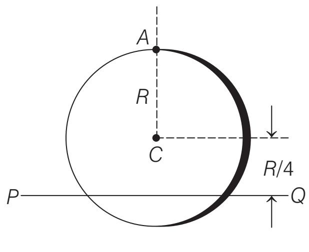


(a) Find the location of the centre of mass of the laminar sheet from AB.

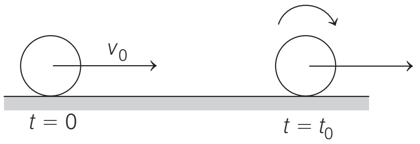
(b) At what angular velocity does the laminar sheet come back after the first impact ?

(c) After how many impacts, does the laminar sheet come to rest ?

38. A uniform circular disc has radius R and mass m. A particle, also of mass m, is fixed at a point A on the edge of the disc as shown in the figure. The disc can rotate freely about a horizontal chord PQ that is at a distance $R/4$ from the centre C of the disc. The line AC is perpendicular to PQ. Initially the disc is held vertical with the point A at its highest position. It is then allowed to fall, so that it starts rotation about PQ. Find the linear speed of the particle as it reaches its lowest position. **(1998, 8M)**



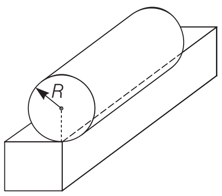
39. A uniform disc of mass m and radius R is projected horizontally with velocity v_0 on a rough horizontal floor, so that it starts off with a purely sliding motion at $t = 0$. After t_0 seconds, it acquires a purely rolling motion as shown in figure. **(1997 C, 5M)**



(a) Calculate the velocity of the centre of mass of the disc at t_0.

(b) Assuming the coefficient of friction to be μ, calculate t_0. Also calculate the work done by the frictional force as a function of time and the total work done by it over a time t much longer than t_0.

40. A rectangular rigid fixed block has a long horizontal edge. A solid homogeneous cylinder of radius R is placed horizontally at rest with its length parallel to the edge such that the axis of the cylinder and the edge of the block are in the same vertical plane as shown in figure. There is sufficient friction present at the edge, so that a very small displacement causes the cylinder to roll off the edge without slipping. Determine **(1995, 10M)**

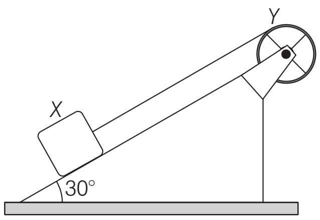


(a) the angle θ_c through which the cylinder rotates before it leaves contact with the edge,

(b) the speed of the centre of mass of the cylinder before leaving contact with the edge and

(c) the ratio of the translational to rotational kinetic energies of the cylinder when its centre of mass is in horizontal line with the edge.

41. A block X of mass 0.5 kg is held by a long massless string on a frictionless inclined plane of inclination 30° to the horizontal. The string is wound on a uniform solid cylindrical drum Y of mass 2 kg and of radius 0.2 m as shown in figure.



The drum is given an initial angular velocity such that the block X starts moving up the plane. **(1994, 6M)**

(a) Find the tension in the string during the motion.

(b) At a certain instant of time, the magnitude of the angular velocity of Y is 10 rad s^{-1}. Calculate the distance travelled by X from that instant of time until it comes to rest.

42. A carpet of mass M made of inextensible material is rolled along its length in the form of a cylinder of radius R and is kept on a rough floor. The carpet starts unrolling without sliding on the floor when a negligibly small push is given to it. Calculate the horizontal velocity of the axis of the cylindrical part of the carpet when its radius reduces to $R/2$. **(1990, 8M)**

Answers

Topic 1

1. (c) **2.** (d) **3.** (d) **4.** (d)
5. (*) **6.** (b) **7.** (a) **8.** (c)
9. (b) **10.** (a) **11.** (a) **12.** (b)
13. (d) **14.** (d) **15.** (b) **16.** (a)
17. (a) **18.** (a) **19.** (d) **20.** (a)
21. (a, b, c) **22.** (3) **23.** (9) **24.** 4.8 Ma^2

Topic 2

1. (c) **2.** (d) **3.** (d)
4. (d) **5.** (c) **6.** (c) **7.** (b)
8. (a) **9.** (c) **10.** (b) **11.** (c)
12. (b) **13.** (b) **14.** (c) **15.** (a, b, c)
16. (c) **17.** (b) **18.** 4 **19.** (8)
20. (6) **21.** –3 **22.** $\dfrac{M\omega_0}{M+6m}$
23. F **24.** $\dfrac{mv_0^3}{2\sqrt{2}g}$ in a direction perpendicular to paper inwards

Topic 3

1. (b) **2.** (b) **3.** (c) **4.** (b)
5. (b) **6.** (b)
7. (b) **8.** (d) **9.** (d) **10.** (d)
11. (a) **12.** (b) **13.** (d) **14.** (d)
15. (d) **16.** (c) **17.** (a, c) **18.** (c, d)
19. (a, b) **20.** (b, c) **21.** (c, d) **22.** (0.75)
23. $\dfrac{mg}{6}$, up
24. $\dfrac{1}{3}MRA\omega^2$ **25.** F **26.** 7 **27.** 3.6
28. $\dfrac{2}{3}g\sin\theta$ **29.** $a_{CM}=\dfrac{4F}{3m_1+8m_2}$, $a_{plank}=\dfrac{8F}{3m_1+8m_2}=2a_{CM}$

(b) $\dfrac{3Fm_1}{3m_1+8m_2}, \dfrac{Fm_1}{3m_1+8m_2}$

30. (a) $6\hat{\mathbf{i}}$ (b) $0.6(\hat{\mathbf{k}}-\hat{\mathbf{j}})$, $0.6(-\hat{\mathbf{j}}-\hat{\mathbf{k}})$, 0.85 N-m
31. 2.13 m, yes

Topic 4

1. (a)
2. (a) **3.** (c) **4.** (a) **5.** (a, c)
6. (a, c, d) **7.** (a) $\dfrac{1}{4}$ (b) $\dfrac{2}{3}L$ (c) $\dfrac{v_0}{2\sqrt{2}}$ **8.** 6.3 m/s
9. (a) $\dfrac{12v}{7L}$ (b) 3.5 ms^{-1} **10.** 2.72 J

Topic 5

1. (b) **2.** (c) **3.** (c) **4.** (d)
5. (b) **6.** (a) **7.** (a) **8.** (b)
9. (b) **10.** (a)
11. (c) **12.** (*) **13.** (a) **14.** (b)
15. (b) **16.** (d) **17.** (c) **18.** (c)
19. (a) **20.** (b,c) **21.** (a,c) **22.** (a,c,d)
23. (a) **24.** Not clear **25.** (c)
26. (d) **27.** (a) **28.** (c) **29.** (a)
30. (b) **31.** (2) **32.** $\left(\dfrac{d-x}{d}\right)W, \dfrac{xW}{d}$
33. $\dfrac{2}{3}mg$ **34.** F **35.** $\dfrac{3mv}{L(3m+M)}$
36. (a) $\sqrt{3}\,ml\omega^2$ (b) $(F_{net})_x=\dfrac{-F}{4}$, $(F_{net})_y=\sqrt{3}\,ml\omega^2$
37. (a) 0.1 m (b) 1 rad/s (c) sheet will never come to rest
38. $\sqrt{5gR}$
39. (a) $\dfrac{2}{3}v_0$ (b) $\dfrac{v_0}{3\mu g}$, For $t \le t_0, W_f=\dfrac{m\mu gt}{2}[3\mu gt-2v_0], \dfrac{-mv_0^2}{6}$
40. (a) $\theta=\cos^{-1}\dfrac{4}{7}$ (b) $\sqrt{\dfrac{4gR}{7}}$ (c) 6
41. (a) 1.63 N (b) 1.22 m **42.** $v=\sqrt{\dfrac{14Rg}{3}}$

Hints & Solutions

Topic 1 Moment of Inertia

1. **Key Idea** Radius of gyration K of any structure is given by

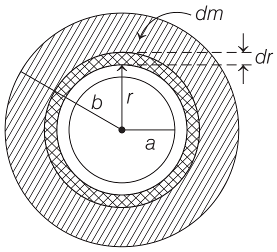
$$I = MK^2 \text{ or } K = \sqrt{\frac{I}{M}}$$

To find K, we need to find both moment of inertia I and mass M of the given structure.

Given, variation in mass per unit area (surface mass density),

$$\sigma = \frac{\sigma_0}{r} \quad \ldots\text{(i)}$$

Calculation of Mass of Disc



Let us divide whole disc in small area elements, one of them shown at r distance from the centre of the disc with its width as dr.

Mass of this element is

$$dm = \sigma \cdot dA$$

$$\Rightarrow \quad dm = \frac{\sigma_0}{r} \times 2\pi r dr \quad \text{[from Eq. (i)]} \ldots\text{(ii)}$$

Mass of the disc can be calculated by integrating it over the given limits of r,

$$\int_0^M dm = \int_a^b \sigma_0 \times 2\pi \times dr$$

$$\Rightarrow \quad M = \sigma_0 2\pi (b - a) \quad \ldots\text{(iii)}$$

Calculation of Moment of Inertia

$$I = \int_0^M r^2 dm = \int_a^b r^2 \cdot \frac{\sigma_0}{r} \times 2\pi r dr = \sigma_0 2\pi \int_a^b r^2 dr = \sigma_0 2\pi \left[\frac{r^3}{3}\right]_a^b$$

$$\Rightarrow \quad I = \frac{1}{3}\sigma_0 2\pi [b^3 - a^3] \quad \ldots\text{(iv)}$$

Now, radius of gyration,

$$K = \sqrt{\frac{I}{M}} = \sqrt{\frac{\frac{2\pi\sigma_0}{3}(b^3 - a^3)}{2\pi\sigma_0 (b - a)}}$$

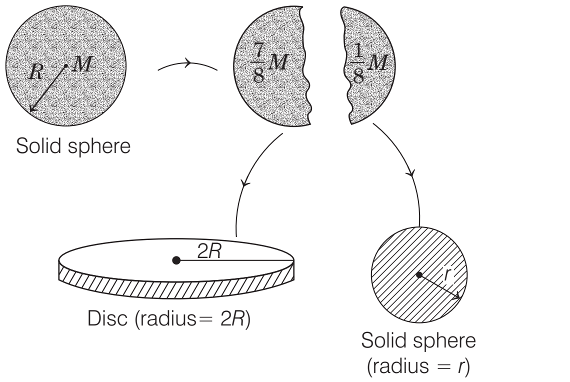
$$\Rightarrow \quad K = \sqrt{\frac{1}{3}\frac{(b^3 - a^3)}{b - a}}$$

As we know, $b^3 - a^3 = (b - a)(b^2 + a^2 + ab)$

$$\therefore \quad K = \sqrt{\frac{1}{3}(b^2 + a^2 + ab)}$$

$$\text{or} \quad K = \sqrt{\frac{(a^2 + b^2 + ab)}{3}}$$

2. The given situation is shown in the figure given below



Density of given sphere of radius R is

$$\rho = \frac{\text{Mass}}{\text{Volume}} = \frac{M}{\frac{4}{3}\pi R^3}$$

Let radius of sphere formed from second part is r, then mass of second part = volume × density

$$\frac{1}{8}M = \frac{4}{3}\pi r^3 \times \frac{M}{\frac{4}{3}\pi R^3}$$

$$\therefore \quad r^3 = \frac{R^3}{8} \Rightarrow r = \frac{R}{2}$$

Now, I_1 = moment of inertia of disc (radius $2R$ and mass $\frac{7}{8}M$) about its axis

$$= \frac{\text{Mass} \times (\text{Radius})^2}{2} = \frac{\frac{7}{8}M \times (2R)^2}{2} = \frac{7}{4}MR^2$$

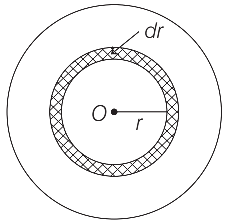
and I_2 = moment of inertia of sphere (radius $\frac{R}{2}$ and mass $\frac{1}{8}M$) about its axis

$$= \frac{2}{5} \times \text{Mass} \times (\text{Radius})^2 = \frac{2}{5} \times \frac{1}{8}M \times \left(\frac{R}{2}\right)^2 = \frac{MR^2}{80}$$

$$\therefore \text{Ratio } \frac{I_1}{I_2} = \frac{\frac{7}{4}MR^2}{\frac{1}{80}MR^2} = 140$$

3. Given, Surface mass density, $\sigma = kr^2$

So, mass of the disc can be calculated by considering small element of area $2\pi r dr$ on it and then integrating it for complete disc, i.e.



$$dm = \sigma\, dA = \sigma \times 2\pi r dr$$

$$\int dm = M = \int_0^R (kr^2)\, 2\pi r dr$$

$$\Rightarrow \quad M = 2\pi k \frac{R^4}{4} = \frac{1}{2}\pi k R^4 \quad \ldots \text{(i)}$$

Moment of inertia about the axis of the disc,

$$I = \int dI = \int dm r^2 = \int \sigma dA r^2$$

$$= \int_0^R kr^2\, (2\pi r dr)\, r^2$$

$$\Rightarrow \quad I = 2\pi k \int_0^R r^5\, dr = \frac{2\pi k R^6}{6} = \frac{\pi k R^6}{3} \quad \ldots \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$I = \frac{2}{3} MR^2$$

4. Given, kinetic energy $= k\theta^2$

We know that, kinetic energy of a rotating body about its axis $= \frac{1}{2} I\omega^2$

where, I is moment of inertia and ω is angular velocity.

So, $$\frac{1}{2} I\omega^2 = k\theta^2 \text{ or } \omega^2 = \frac{2k\theta^2}{I}$$

$$\Rightarrow \quad \omega = \sqrt{\frac{2k}{I}}\theta \quad \ldots \text{(i)}$$

Differentiating the above equation w.r.t. time on both sides, we get

$$\frac{d\omega}{dt} = \sqrt{\frac{2k}{I}} \cdot \frac{d\theta}{dt} = \sqrt{\frac{2k}{I}} \cdot \omega \qquad \left[\because \omega = \frac{d\theta}{dt}\right]$$

$\therefore$ Angular acceleration,

$$\alpha = \frac{d\omega}{dt} = \sqrt{\frac{2k}{I}} \cdot \omega = \sqrt{\frac{2k}{I}} \cdot \sqrt{\frac{2k}{I}}\theta \quad \text{[using Eq. (i)]}$$

or $$\alpha = \frac{2k}{I}\theta$$

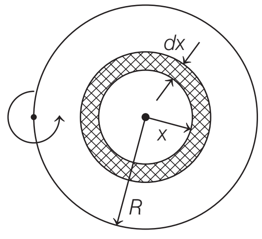
Alternate Solution

As, $$\omega^2 = \frac{2k\theta^2}{I}$$

$$\Rightarrow \quad 2\omega \frac{d\omega}{dt} = \frac{2k}{I} \cdot 2\theta \frac{d\theta}{dt} \text{ or } \omega\, d\omega = \frac{2k}{I} \theta\, d\theta$$

$$\omega \frac{d\omega}{d\theta} (= \alpha) = \frac{2k}{I} \cdot \theta \text{ or } \alpha = \frac{2k}{I}\theta$$

5. Consider an elementary ring of thickness dx and radius x.



Moment of inertia of this ring about a perpendicular axes through centre is

$$dI_c = dm \cdot x^2 = \rho_0 x (2\pi x) dx \cdot x^2 = 2\pi\rho_0 x^4 dx$$

Moment of inertia of this elementary ring about a perpendicular axes at a point through edge, (by parallel axes theorem)

$$dI = dm x^2 + dm R^2$$

$$= 2\pi\rho_0 x^4 dx + 2\pi\rho_0 R^2 x^2 dx$$

Moment of inertia of complete disc is

$$I = \int_0^R dI = \int_0^R 2\pi\rho_0 x^4 dx + \int_0^R 2\pi\rho_0 R^2 x^2 dx$$

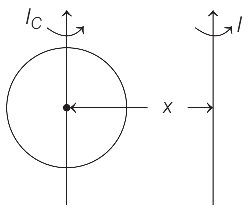
$$= \frac{2\pi\rho_0 R^5}{5} + \frac{2\pi\rho_0 R^5}{3} = \frac{16\pi\rho_0 R^5}{15}$$

$\therefore \quad a = \frac{16}{15}$ (No option matches)

6. Moment of inertia of a solid sphere about an axis through its centre of mass is

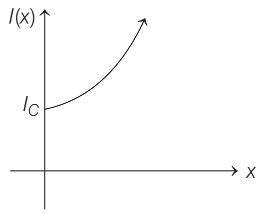
$$I_C = \frac{2}{5} MR^2$$

Moment of inertia about a parallel axis at a distance x from axis through its COM is



$I = I_C + Mx^2$ (by parallel axis theorem)

So graph of I *versus* x is parabolic are shown



7. Moment of inertia of hollow cylinder about its axis is

$$I_1 = \frac{M}{2}(R_1^2 + R_2^2)$$

where, R_1 = inner radius and
R_2 = outer radius.

Moment of inertia of thin hollow cylinder of radius R about its axis is.

$$I_2 = MR^2$$

Given, $I_1 = I_2$ and both cylinders have same mass (M).

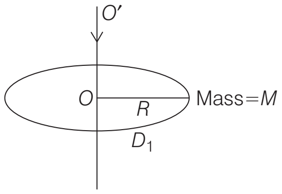
So, we have

$$\frac{M}{2}(R_1^2 + R_2^2) = MR^2$$

$$(10^2 + 20^2)/2 = R^2$$
$$R^2 = 250 = 15.8$$
$$R \approx 16\,\text{cm}$$

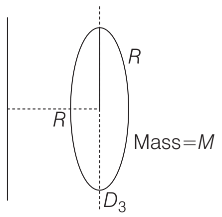
8 For disc D_1, moment of inertia across axis OO' will be

$$I_1 = \frac{1}{2}MR^2 \qquad \ldots\text{(i)}$$



For discs D_2 and D_3, OO' is an axis parallel to the diameter of disc. Using parallel axis theorem,

$$I_2 = I_3 = I_{\text{diameter}} + Md^2 \qquad \ldots\text{(ii)}$$



Here, $\quad I_{\text{diameter}} = \frac{1}{4}MR^2$

and $\quad d = R$

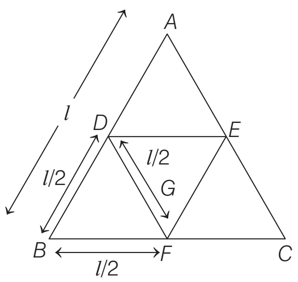
$\therefore \quad I_2 = I_3 = \frac{1}{4}MR^2 + MR^2 = \frac{5}{4}MR^2$

Now, total MI of the system

$$I = I_1 + I_2 + I_3 = \frac{1}{2}MR^2 + 2 \times \frac{5}{4}MR^2 = 3MR^2$$

9 *(b)* Suppose the mass of the ΔABC be 'M' and length of the side be 'l'.

When the ΔDEF is being removed from it, then the mass of the removed Δ will be '$M/4$' and length of its side will be '$l/2$' as shown below



Since we know, moment of inertia of the triangle about the axis, passing through its centre of gravity is,

$I = kml^2$, where k is a constant.

Then for ΔDEF, moment of inertia of the triangle about the axis,

$$I = k\left(\frac{M}{4}\right)\left(\frac{l}{2}\right)^2 = \frac{kMl^2}{16} \qquad \ldots\text{(i)}$$

Moment of inertia of ΔABC is

$$I_0 = kMl^2 \qquad \ldots\text{(ii)}$$

The moment of inertia of the remaining part will be

$$I' = I_0 - I = kMl^2 - \frac{kMl^2}{16} \; [\because \text{using Eqs. (i) and (ii)}]$$

$$= \frac{15kMl^2}{16} \text{ or } I' = \frac{15}{16}I_0$$

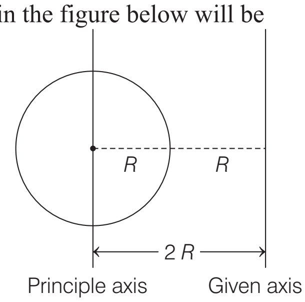
10

> **Key Idea** This problem will be solved by applying parallel axis theorem, which states that moment of inertia of a rigid body about any axis is equals to its moment of inertia about a parallel axis through its centre of mass plus the product of the mass of the body and the square of the perpendicular distance between the axis.

We know that moment of inertia (MI) about the principle axis of the sphere is given by

$$I_{\text{sphere}} = \frac{2}{5}MR^2 \qquad \ldots\text{(i)}$$

Using parallel axis theorem, moment of inertia about the given axis in the figure below will be



$$I_1 = \frac{2}{5}MR^2 + M(2R)^2$$

$$I_1 = \frac{22}{5}MR^2 \qquad \ldots\text{(ii)}$$

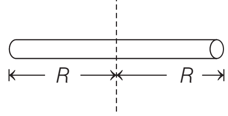
Considering both spheres at equal distance from the axis, moment of inertia due to both spheres about this axis will be

$$2I_1 = 2 \times 22/5\, MR^2$$

Now, moment of inertia of rod about its perpendicular bisector axis is given by

$$I_2 = \frac{1}{12}ML^2$$

Here, given that $\quad L = 2R$



$$\therefore \qquad I_2 = \frac{1}{12} M (2R)^2 = \frac{1}{3} MR^2 \qquad \dots \text{(iii)}$$

So, total moment of inertia of the system is

$$I = 2I_1 + I_2 = 2 \times \frac{22}{5} MR^2 + \frac{1}{3} MR^2$$

$$\Rightarrow \qquad I = \left(\frac{44}{5} + \frac{1}{3}\right) MR^2 = \frac{137}{15} MR^2$$

11. From theorem of parallel axis,

$$\mathrm{I} = \mathrm{I}_{cm} + 7M(3R)^2$$

$$= \left[\frac{MR^2}{2} + 6 \times \left\{\frac{MR^2}{2} + M(2R)^2\right\}\right] + 7M(3R)^2 = \frac{181MR^2}{2}$$

12. $I_{\text{Remaining}} = I_{\text{Total}} - I_{\text{Cavity}}$

$$\Rightarrow \qquad I = \frac{9MR^2}{2} - \left[\frac{M}{2}\left(\frac{R}{3}\right)^3 + M\left(\frac{2R}{3}\right)^2\right] = 4MR^2$$

13. MI of a solid cylinder about its perpendicular bisector of length is

$$I = M\left(\frac{l^2}{12} + \frac{R^2}{4}\right)$$

$$\Rightarrow I = \frac{mR^2}{4} + \frac{ml^2}{12} = \frac{m^2}{4\pi\rho l} + \frac{ml^2}{12} \qquad [\because \rho\pi r^2 l = m]$$

For I to be maximum,

$$\frac{dI}{dl} = -\frac{m^2}{4\pi\rho}\left(\frac{1}{l^2}\right) + \frac{ml}{6} = 0$$

$$\Rightarrow \qquad \frac{m^2}{4\mu\pi\rho} = \frac{ml^3}{6} \Rightarrow l^3 = \frac{3m}{2\pi\rho}$$

$$\Rightarrow \qquad l = \left(\frac{3}{2}\right)^{1/3} \left(\frac{m}{\pi\rho}\right)^{1/3}$$

$$\rho = \frac{m}{\pi R^2 l} \Rightarrow R^2 = \frac{m}{\pi\rho l}$$

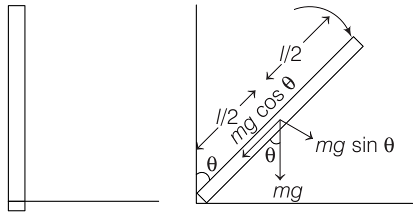
$$\Rightarrow \qquad R^2 = \frac{m}{\pi\rho}\left(\frac{2}{3}\right)^{1/3} \left(\frac{\pi\rho}{m}\right)^{1/3} = \left(\frac{m}{\pi\rho}\right)^{2/3} \left(\frac{2}{3}\right)^{1/3}$$

$$\Rightarrow \qquad R = \left(\frac{m}{\pi\rho}\right)^{1/3} \left(\frac{2}{3}\right)^{1/6}$$

$$\frac{l}{R} = \frac{\left(\frac{3}{2}\right)^{1/3}\left(\frac{m}{\pi\rho}\right)^{1/3}}{\left(\frac{m}{\pi\rho}\right)^{1/3}\left(\frac{2}{3}\right)^{1/6}} = \left(\frac{3}{2}\right)^{1/3} + \left(\frac{3}{2}\right)^{1/6}$$

$$\therefore \qquad \frac{l}{R} = \sqrt{\frac{3}{2}}$$

14. As the rod rotates in vertical plane so a torque is acting on it, which is due to the vertical component of weight of rod.



Initial condition   At any time t

Now, Torque τ = force × perpendicular distance of line of action of force from axis of rotation

$$= mg \sin\theta \times \frac{l}{2}$$

Again, Torque, $\tau = I\alpha$

Where, I = moment of inertia $= \dfrac{ml^2}{3}$

[Force and Torque frequency along axis of rotation passing through in end]

α = angular acceleration

$$\therefore \qquad mg \sin\theta \times \frac{l}{2} = \frac{ml^2}{3}\alpha$$

$$\therefore \qquad \alpha = \frac{3g \sin\theta}{2l}$$

15. Maximum possible volume of cube will occur when

$$\sqrt{3}a = 2R \qquad (a = \text{side of cube})$$

$$\therefore \qquad a = \frac{2}{\sqrt{3}} R$$

Now, density of sphere, $\rho = \dfrac{M}{\frac{4}{3}\pi R^3}$

Mass of cube, m = (volume of cube)(ρ) = $(a^3)(\rho)$

$$= \left[\frac{2}{\sqrt{3}} R\right]^3 \left[\frac{m}{\frac{4}{3}\pi R^3}\right] = \left(\frac{2}{\sqrt{3}\pi}\right) M$$

Now, moment of inertia of the cube about the said axis is

$$I = \frac{ma^2}{6} = \frac{\left(\frac{2}{\sqrt{3}\pi}\right) M \left(\frac{2}{\sqrt{3}} R\right)^2}{\sigma}$$

$$= \frac{4MR^2}{9\sqrt{3}\pi}$$

16. $$\frac{2}{5} MR^2 = \frac{1}{2} Mr^2 + Mr^2$$

or $$\frac{2}{5} MR^2 = \frac{3}{2} Mr^2$$

$$\therefore \qquad r = \frac{2}{\sqrt{15}} R$$

17. $I_{\text{remaining}} = I_{\text{whole}} - I_{\text{removed}}$

or $\quad I = \frac{1}{2}(9M)(R)^2 - \left[\frac{1}{2}m\left(\frac{R}{3}\right)^2 + \frac{1}{2}m\left(\frac{2R}{3}\right)^2\right]$ …(i)

Here, $m = \frac{9M}{\pi R^2} \times \pi\left(\frac{R}{3}\right)^2 = M$

Substituting in Eq. (i), we have

$$I = 4MR^2$$

18. Mass of the whole disc = $4M$

Moment of inertia of the disc about the given axis

$$= \frac{1}{2}(4M)R^2 = 2MR^2$$

$\therefore$ Moment of inertia of quarter section of the disc

$$= \frac{1}{4}(2MR^2) = \frac{1}{2}MR^2$$

19. Mass of the ring $M = \rho L$. Let R be the radius of the ring, then

$$L = 2\pi R$$

or $$R = \frac{L}{2\pi}$$

Moment of inertia about an axis passing through O and parallel to XX' will be

$$I_0 = \frac{1}{2}MR^2$$

Therefore, moment of inertia about XX' (from parallel axis theorem) will be given by

$$I_{XX'} = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$$

Substituting values of M and R

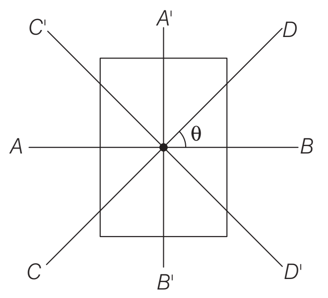
$$I_{XX'} = \frac{3}{2}(\rho L)\left(\frac{L^2}{4\pi^2}\right) = \frac{3\rho L^3}{8\pi^2}$$

20. $A'B' \perp AB$ and $C'D' \perp CD$

From symmetry $\quad I_{AB} = I_{A'B'}$

and $\quad I_{CD} = I_{C'D'}$

From theorem of perpendicular axes,



$$I_{ZZ} = I_{AB} + I_{A'B'} = I_{CD} + I_{C'D'}$$
$$= 2I_{AB} = 2I_{CD}$$
$$I_{AB} = I_{CD}$$

Alternate

The relation between I_{AB} and I_{CD} should be true for all values of θ

At $\quad \theta = 0, I_{CD} = I_{AB}$

Similarly, at $\theta = \pi/2, I_{CD} = I_{AB}$ (by symmetry)

Keeping these things in mind, only option (a) is correct.

21. Since, it is a square lamina

$$I_3 = I_4$$

and $\quad I_1 = I_2 \quad$ (by symmetry)

From perpendicular axes theorem,

Moment of inertia about an axis perpendicular to square plate and passing from O is

$$I_o = I_1 + I_2 = I_3 + I_4$$

or $\quad I_o = 2I_2 = 2I_3$

Hence, $\quad I_2 = I_3$

Rather we can say $I_1 = I_2 = I_3 = I_4$

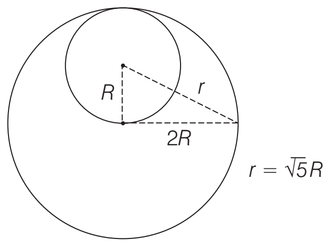
Therefore, I_o can be obtained by adding any two i.e.

$$I_o = I_1 + I_2 = I_1 + I_3$$
$$= I_1 + I_4 = I_2 + I_3$$
$$= I_2 + I_4 = I_3 + I_4$$

22. T = Total portion

R = Remaining portion and

C = Cavity and let σ = mass per unit area.



Then, $\quad m_T = \pi(2R)^2\sigma = 4\pi R^2\sigma$

$\quad m_C = \pi(R)^2\sigma = \pi R^2\sigma$

For I_P

$$I_R = I_T - I_C$$
$$= \frac{3}{2}m_T(2R)^2 - \left[\frac{1}{2}m_C R^2 + m_C r^2\right]$$
$$= \frac{3}{2}(4\pi R^2\sigma)(4R^2) - \left[\frac{1}{2}(\pi R^2\sigma) + (\pi R^2\sigma)(5R^2)\right]$$
$$= (18.5\,\pi R^4\sigma)$$

For I_O $\quad I_R = I_T - I_C$

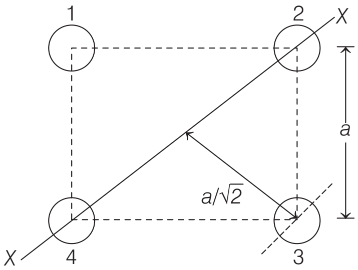
$$= \frac{1}{2}m_T(2R)^2 - \frac{3}{2}m_C R^2$$
$$= \frac{1}{2}(4\pi R^2\sigma)(4R^2) - \frac{3}{2}(\pi R^2\sigma)(R^2) = 6.5\,\pi R^4\sigma$$

$$\therefore \quad \frac{I_P}{I_O} = \frac{18.5\pi R^4\sigma}{6.5\pi R^4\sigma} = 2.846$$

Therefore, the nearest integer is 3.

23. $r = \frac{d}{2} = \frac{\sqrt{5}}{2}\text{ cm} = \frac{\sqrt{5}}{2} \times 10^{-2}\text{ m} \Rightarrow m = 0.5\text{ kg}$

$$a = 4\text{ cm} = 4 \times 10^{-2}\text{ m}$$



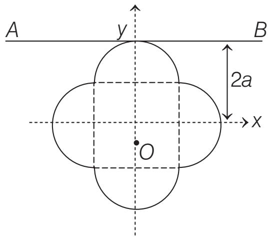
$$I_{XX} = I_1 + I_2 + I_3 + I_4$$

$$= \left[\frac{2}{5}mr^2 + m\left(\frac{a}{\sqrt{2}}\right)^2\right] + \frac{2}{5}mr^2 + \left[\frac{2}{5}mr^2 + m\left(\frac{a}{\sqrt{2}}\right)^2\right] + \frac{2}{5}mr^2$$

Substituting the values, we get

$$I_{XX} = 9 \times 10^{-4}\text{ kg-m}^2 \quad \therefore \quad N = 9$$

24. Assuming the lamina to be in x-y plane.



From, the perpendicular axis theorem,

$$I_x + I_y = I_z$$

but $I_x = I_y$ (by symmetry)

and $I_z = 1.6\,Ma^2$ (given)

$\therefore \quad I_x = \frac{I_z}{2} = 0.8\,Ma^2$

Now, from the parallel axis theorem,

$$I_{AB} = I_x + M(2a)^2 = 0.8\,Ma^2 + 4\,Ma^2 = 4.8\,Ma^2$$

Topic 2 Angular Momentum and its Conservation

1. Position of particle is, $\mathbf{r} = 2t\hat{\mathbf{i}} - 3t^2\hat{\mathbf{j}}$

where, t is instantaneous time.

Velocity of particle is

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = 2\hat{\mathbf{i}} - 6t\hat{\mathbf{j}}$$

Now, angular momentum of particle with respect to origin is given by

$$\mathbf{L} = m(\mathbf{r} \times \mathbf{v})$$

$$= m\{(2t\hat{\mathbf{i}} - 3t^2\hat{\mathbf{j}}) \times (2\hat{\mathbf{i}} - 6t\hat{\mathbf{j}})\}$$

$$= m(-12t^2(\hat{\mathbf{i}} \times \hat{\mathbf{j}}) - 6t^2(\hat{\mathbf{j}} \times \hat{\mathbf{i}}))$$

As, $\hat{\mathbf{i}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}} \times \hat{\mathbf{j}} = 0$

$\Rightarrow \quad \mathbf{L} = m(-12t^2\hat{\mathbf{k}} + 6t^2\hat{\mathbf{k}})$

As, $\hat{\mathbf{i}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}}$ and $\hat{\mathbf{j}} \times \hat{\mathbf{i}} = -\hat{\mathbf{k}}$

$\Rightarrow \quad \mathbf{L} = -m(6t^2)\hat{\mathbf{k}}$

So, angular momentum of particle of mass 2 kg at time $t = 2$ s is

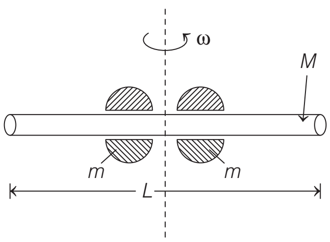
$$\mathbf{L} = (-2 \times 6 \times 2^2)\hat{\mathbf{k}} = -48\,\hat{\mathbf{k}}$$

2. As there is no external torque on system.

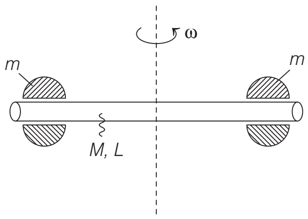
$\therefore$ Angular momentum of system is conserved.

$\Rightarrow \quad I_i\omega_i = I_f\omega_f$

Initially,



Finally,

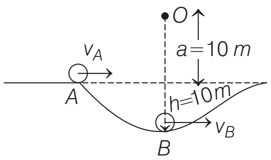


$$\Rightarrow \quad \frac{ML^2}{12}\cdot\omega_0 + 0 = \left(\frac{ML^2}{12} + 2(m)\left(\frac{L}{2}\right)^2\right)\omega$$

So, final angular speed of system is

$$\Rightarrow \quad \omega = \frac{\frac{ML^2}{12}\cdot\omega_0}{\left(\frac{ML^2 + 6mL^2}{12}\right)} = \frac{M\omega_0}{M + 6m}$$

3. The given figure is shown below as



As friction is absent, energy at A = energy at B

$$\Rightarrow \quad \frac{1}{2}mv_A^2 + mgh = \frac{1}{2}mv_B^2$$

$$\Rightarrow \quad v_A^2 + 2gh = v_B^2$$

or $\quad v_B^2 = (5)^2 + 2 \times 10 \times 10 = 225$

$$\Rightarrow \quad v_B = 15\text{ ms}^{-1}$$

Angular momentum about point 'O',

$$= mv_B r_B$$
$$= 20 \times 10^{-3} \times 15 \times 20 = 6 \text{ kg-m}^2 \text{ s}^{-1}$$

4. Let the other mass at this instant is at a distance of x from the centre O. Applying law of conservation of angular momentum, we have $\quad I_1\omega_1 = I_2\omega_2$

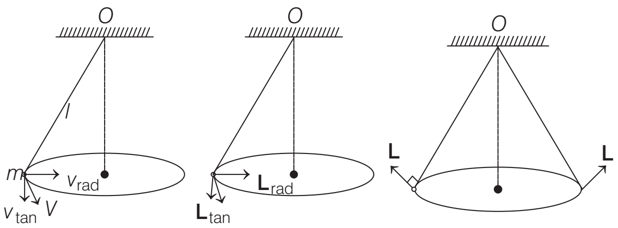
$\therefore$

$$(MR^2)(\omega) = \left[MR^2 + \frac{M}{8}\left(\frac{3}{5}R\right)^2 + \frac{M}{8}x^2\right]\left(\frac{8}{9}\omega\right)$$

Solving this equation, we get $x = \frac{4}{5}R$.

NOTE If we take identical situations with both point masses, then answer will be (c). But in that case, angular momentum is not conserved.

5. Angular momentum of the pendulum about the suspension point O is



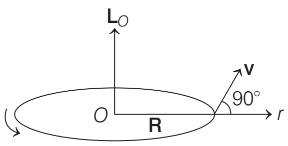
Then, v can be resolved into two components, radial component r_{rad} and tangential component r_{tan}. Due to v_{rad}, L will be tangential and due to v_{tan}, L will be radially outwards as shown. So, net angular momentum will be as shown in figure whose magnitude will be constant ($|L| = mvl$). But its direction will change as shown in the figure.

$$\mathbf{L} = m(\mathbf{r} \times \mathbf{v})$$

where, r = radius of circle.

6. Angular momentum of a particle about a point is given by $\mathbf{L} = \mathbf{r} \times \mathbf{p} = m(\mathbf{r} \times \mathbf{v})$

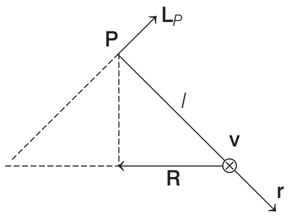
For $\mathbf{L}_O$



$|\mathbf{L}| = (mvr \sin\theta) = m(R\omega)(R)\sin 90° = \text{constant}$

Direction of $\mathbf{L}_O$ is always upwards. Therefore, complete $\mathbf{L}_O$ is constant, both in magnitude as well as direction.

For $\mathbf{L}_P$



$|\mathbf{L}_P| = (mvr\sin\theta) = (m)(R\omega)(l)\sin 90° = (mRl\omega)$

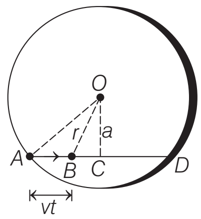
Magnitude of $\mathbf{L}_P$ will remain constant but direction of $\mathbf{L}_P$ keeps on changing.

7. From conservation of angular momentum ($I\omega = \text{constant}$), angular velocity will remain half. As, $K = \frac{1}{2}I\omega^2$

The rotational kinetic energy will become half.

8. In uniform circular motion, the only force acting on the particle is centripetal (towards centre). Torque of this force about the centre is zero. Hence, angular momentum about centre remain conserved.

9. Since, there is no external torque, angular momentum will remain conserved. The moment of inertia will first decrease till the tortoise moves from A to C and then increase as it moves from C and D. Therefore, ω will initially increase and then decrease.



Let R be the radius of platform, m the mass of disc and M is the mass of platform.

Moment of inertia when the tortoise is at A

$$I_1 = mR^2 + \frac{MR^2}{2}$$

and moment of inertia when the tortoise is at B

$$I_2 = mr^2 + \frac{MR^2}{2}$$

Here, $\quad r^2 = a^2 + [\sqrt{R^2 - a^2} - vt]^2$

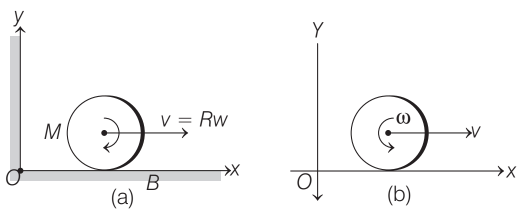
From conservation of angular momentum

$$\omega_0 I_1 = \omega(t) I_2$$

Substituting the values, we can see that variation of $\omega(t)$ is non-linear.

10. Net external torque on the system is zero. Therefore, angular momentum is conserved. Force acting on the system are only conservative. Therefore, total mechanical energy of the system is also conserved.

11. From the theorem



$$\mathbf{L}_0 = \mathbf{L}_{CM} + M(\mathbf{r} \times \mathbf{v}) \qquad \text{...(i)}$$

We may write

Angular momentum about O = Angular momentum about CM + Angular momentum of CM about origin

$$\therefore \quad L_0 = I\omega + MRv$$

$$= \frac{1}{2}MR^2\omega + MR(R\omega) = \frac{3}{2}MR^2\omega$$

NOTE That in this case [Figure (a)] both the terms in Eq. (i), i.e. L_{CM} and M $(r \times v)$ have the same direction Ä. That is why, we have used $L_0 = I\omega + MRv$. We will use $L_0 = I\omega \sim MRv$ if they are in opposite direction as shown in figure (b).

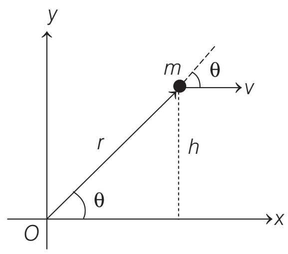
12. $|\mathbf{v}| = v$ = constant and $|\mathbf{r}| = r$ (say)

Angular momentum of the particle about origin O will be given by

$$\mathbf{L} = \mathbf{r} \times \mathbf{p} = m(\mathbf{r} \times \mathbf{v})$$

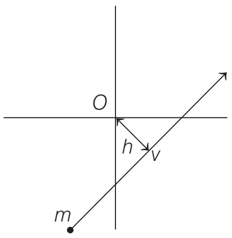
or $\quad |\mathbf{L}| = L = mrv\sin\theta = mv(r\sin\theta) = mvh$

Now, m, v and h all are constants.



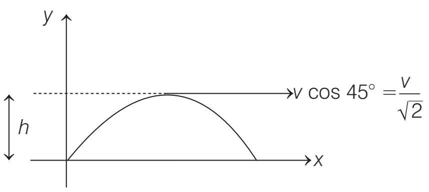
Therefore, angular momentum of particle about origin will remain constant. The direction of $\mathbf{r} \times \mathbf{v}$ also remains the same (negative z).

NOTE Angular momentum of a particle moving with constant velocity about any point is always constant. e.g. Angular momentum of the particle shown in figure about origin O will be



$L = mvh$ = constant

13. $L = m\frac{v}{\sqrt{2}}r_\perp$



Here, $r_\perp = h = \dfrac{v^2\sin^2 45°}{2g} = \dfrac{v^2}{4g}$

$$\therefore \quad L = m\left(\frac{v}{\sqrt{2}}\right)\left(\frac{v^2}{4g}\right) = \frac{mv^3}{4\sqrt{2}g}$$

14. $I_1\omega_1 = I_2\omega_2$

$$\therefore \quad \omega_2 = \frac{I_1}{I_2}\omega = \left(\frac{Mr^2}{Mr^2 + 2mr^2}\right)\omega$$

$$= \left(\frac{M}{M + 2m}\right)\omega$$

15. (a) $\tau = \mathbf{A} \times \mathbf{L}$

i.e. $\quad \dfrac{d\mathbf{L}}{dt} = \mathbf{A} \times \mathbf{L}$

This relation implies that $\dfrac{d\mathbf{L}}{dt}$ is perpendicular to both $\mathbf{A}$ and $\mathbf{L}$.

(c) Here, $\quad \mathbf{L} \cdot \mathbf{L} = L^2$

Differentiating w.r.t. time, we get

$$\mathbf{L} \cdot \frac{d\mathbf{L}}{dt} + \frac{d\mathbf{L}}{dt} \cdot \mathbf{L} = 2L\frac{dL}{dt}$$

$$\Rightarrow \quad 2\,\mathbf{L} \cdot \frac{d\mathbf{L}}{dt} = 2L\frac{dL}{dt}$$

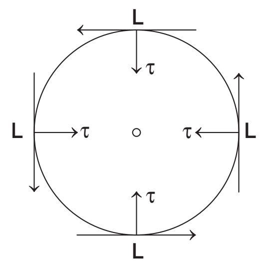
But since, $\quad \mathbf{L} \perp \dfrac{d\mathbf{L}}{dt}$

$$\therefore \quad \mathbf{L} \cdot \frac{d\mathbf{L}}{dt} = 0$$

Therefore, from Eq. (i) $\quad \dfrac{dL}{dt} = 0$

or magnitude of $\mathbf{L}$ i.e. L does not change with time.

(b) So far we are confirm about two points



(i) τ or $\dfrac{d\mathbf{L}}{dt} \perp \mathbf{L}$ and

(ii) $|\mathbf{L}| = L$ is not changing with time, therefore, it is a case when direction of $\mathbf{L}$ is changing but its magnitude is constant and τ is perpendicular to $\mathbf{L}$ at all points.

This can be written as

If $\quad \mathbf{L} = (a\cos\theta)\,\hat{\mathbf{i}} + (a\sin\theta)\,\hat{\mathbf{j}}$

Here, a = positive constant

Then $\quad \tau = (a\sin\theta)\hat{\mathbf{i}} - (a\cos\theta)\hat{\mathbf{j}}$

So, that $\mathbf{L} \cdot \tau = 0$ and $\mathbf{L} \perp \tau$

Now, $\mathbf{A}$ is constant vector and it is always perpendicular to τ. Thus, $\mathbf{A}$ can be written as $\mathbf{A} = A\hat{\mathbf{k}}$

we can see that $\mathbf{L} \cdot \mathbf{A} = 0$ i.e. $\mathbf{L} \perp \mathbf{A}$ also.

Thus, we can say that component of **L** along **A** is zero or component of **L** along **A** is always constant.

Finally, we conclude that τ, **A** and **L** are always mutually perpendicular.

16. Force on block along slot $= m\omega^2 r = ma = m\left(\dfrac{vdv}{dr}\right)$

$$\int_0^v vdv = \int_{R/2}^r \omega^2 rdr \quad \Rightarrow \quad \frac{v^2}{2} = \frac{\omega^2}{2}\left(r^2 - \frac{R^2}{4}\right)$$

$$\Rightarrow \qquad v = \omega\sqrt{r^2 - \frac{R^2}{4}} = \frac{dr}{dt} \Rightarrow \int_{R/4}^r \frac{dr}{\sqrt{r^2 - \frac{R^2}{4}}} = \int_0^t \omega\, dt$$

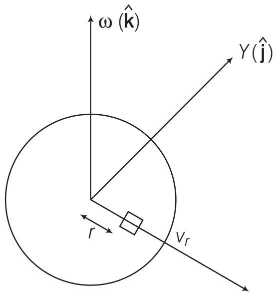
$$\ln\left(\frac{r + \sqrt{r^2 - \frac{R^2}{4}}}{\frac{R}{2}}\right) - \ln\left(\frac{R/2 + \sqrt{\frac{R^2}{4} - \frac{R^2}{4}}}{\frac{R}{4}}\right) = \omega t$$

$$\Rightarrow \quad r + \sqrt{r^2 - \frac{R^2}{4}} = \frac{R}{2}e^{\omega t}$$

$$\Rightarrow \quad r^2 - \frac{R^2}{4} = \frac{R^2}{4}e^{2\omega t} + r^2 - 2r\frac{R}{2}e^{\omega t}$$

$$\Rightarrow \qquad r = \frac{\frac{R^2}{4}e^{2\omega t} + \frac{R^2}{4}}{Re^{\omega t}} = \frac{R}{4}\,(e^{\omega t} + e^{-\omega t})$$

17.



$$\mathbf{F}_{\text{rot}} = \mathbf{F}_{\text{in}} + 2m(v_{\text{rot}}\,\hat{\mathbf{i}}) \times \omega\hat{\mathbf{k}} + m(\omega\hat{\mathbf{k}} \times r\hat{\mathbf{i}}) \times \omega\hat{\mathbf{k}}$$

$$mr\omega^2\hat{\mathbf{i}} = \mathbf{F}_{\text{in}} + 2mv_{\text{rot}}\omega(-\hat{\mathbf{j}}) + m\omega^2 r\hat{\mathbf{i}}$$

$$\mathbf{F}_{\text{in}} = 2mv_r\omega\hat{\mathbf{j}}$$

$$r = \frac{R}{4}[e^{\omega t} + e^{-\omega t}]$$

$$\frac{dr}{dt} = v_r = \frac{R}{4}[\omega e^{\omega t} - \omega e^{-\omega t}]$$

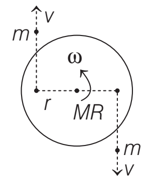
$$\mathbf{F}_{\text{in}} = 2m\frac{R\omega}{4}[e^{\omega t} - e^{-\omega t}]\omega\hat{\mathbf{j}}$$

$$\mathbf{F}_{\text{in}} = \frac{mR\omega^2}{2}[e^{\omega t} - e^{-\omega t}]\hat{\mathbf{j}}$$

Also, reaction is due to disc surface then

$$\mathbf{F}_{\text{reaction}} = \frac{mR\omega^2}{2}[e^{\omega t} - e^{-\omega t}]\hat{\mathbf{j}} + mg\hat{\mathbf{k}}$$

18. Applying conservation of angular momentum



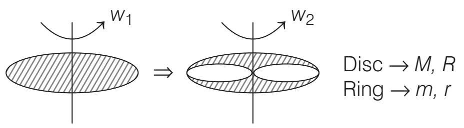
$$2mvr - \frac{MR^2}{2}\omega = 0$$

$$\omega = \frac{4mvr}{MR^2}$$

Substituting the values, we get

$$\omega = \frac{(4)(5\times10^{-2})(9)\left(\frac{1}{4}\right)}{45\times10^{-2}\times\frac{1}{4}} \Rightarrow \omega = 4 \text{ rad/s}$$

19. $I_1\omega_1 = I_2\omega_2$



$$\therefore\ \omega_2 = \left(\frac{I_1}{I_2}\right)\omega_1 = \left[\frac{\frac{1}{2}MR^2}{\frac{1}{2}MR^2 + 2(mr^2)}\right]\omega_1$$

$$= \left[\frac{50(0.4)^2}{50(0.4)^2 + 8\times(6.25)\times(0.2)^2}\right](10) = 8 \text{ rad/s}$$

20. $\dfrac{L_{\text{Total}}}{L_B} = \dfrac{(I_A + I_B)\,\omega}{I_B\cdot\omega}$ (as ω will be same in both cases)

$$= \frac{I_A}{I_B} + 1 = \frac{m_A r_A^2}{m_B\, r_B^2} + 1 = \frac{r_A}{r_B} + 1 \qquad (\text{as } m_A r_A = m_B r_B)$$

$$= \frac{11}{2.2} + 1 = 6 \qquad \left(\text{as } r \propto \frac{1}{m}\right)$$

21. $mvr = k$ (a constant) $\Rightarrow v = \dfrac{k}{mr}$

$$T = \frac{mv^2}{r} = \left(\frac{m}{r}\right)\left(\frac{k}{mr}\right)^2 = \frac{k^2}{m}\cdot\frac{1}{r^3}$$

$$= Ar^{-3} \qquad \left(\text{where, } A = \frac{k^2}{m}\right)$$

Hence, $\quad n = -3$

22. $I_1\omega_1 = I_2\omega_2$

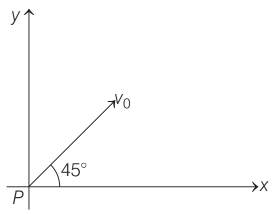
$$\therefore \quad \omega_2 = \frac{I_1}{I_2}\cdot\omega_1 = \frac{(ML^2/12)}{[(ML^2/12) + 2m(L/2)^2]}\omega_0$$

$$= \left(\frac{ML^2}{ML^2 + 6mL^2}\right)\omega_0 = \left(\frac{M}{M + 6m}\right)\omega_0$$

23. $I_1\omega_1 = I_2\omega_2$

$$\therefore\ \omega_2 = \frac{I_1}{I_2}\cdot\omega_1 = \left[\frac{\frac{MR^2}{2}}{\frac{MR^2}{2} + \frac{M}{4}\cdot\frac{R^2}{2}}\right]\omega = \frac{4}{5}\omega$$

24. In terms of $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$



$$\mathbf{u} = \frac{v_0}{\sqrt{2}}\hat{\mathbf{i}} + \frac{v_0}{\sqrt{2}}\hat{\mathbf{j}}$$

$$\Rightarrow \quad \mathbf{a} = -g\hat{\mathbf{j}}$$

$$t = \frac{v_0}{g}$$

$$\Rightarrow \quad \mathbf{v} = \mathbf{u} + \mathbf{a}t = \frac{v_0}{\sqrt{2}}\hat{\mathbf{i}} - \left(v_0 - \frac{v_0}{\sqrt{2}}\right)\hat{\mathbf{j}}$$

$$\mathbf{r} = \mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

$$= \frac{v_0^2}{\sqrt{2}g}\hat{\mathbf{i}} + \frac{v_0^2}{\sqrt{2}g}\hat{\mathbf{j}} - \frac{v_0^2}{2g}\hat{\mathbf{j}}$$

Now, angular momentum about point P at given time

$$\mathbf{L} = m(\mathbf{r} \times \mathbf{v})$$

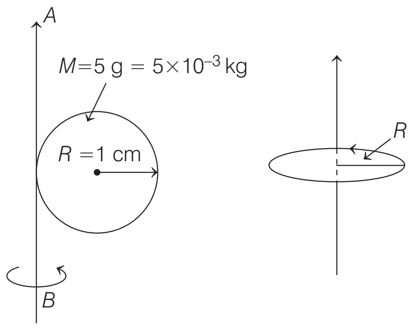
$$= m\left[-\frac{v_0^3}{\sqrt{2}g} + \frac{v_0^3}{2g} - \frac{v_0^3}{2g} + \frac{v_0^3}{2\sqrt{2}g}\right]\hat{\mathbf{k}}$$

$$= -\frac{mv_0^3}{2\sqrt{2}g}\hat{\mathbf{k}}$$

Thus, magnitude of angular momentum is $\frac{mv_0^3}{2\sqrt{2}g}$ in $-\hat{\mathbf{k}}$ direction i.e. in a direction perpendicular to paper inwards

Topic 3 Pure Rolling or Rolling without Slipping

1. Moment of inertia (MI) of a disc about a tangential axis in the plane of disc can be obtained as below.



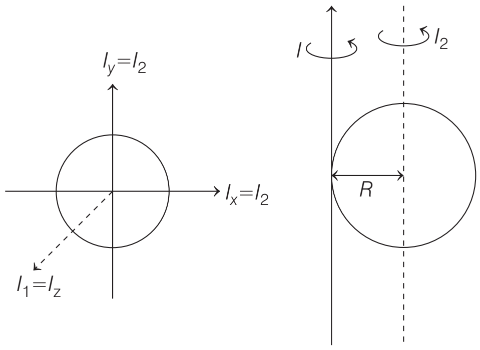
Moment of inertia of disc about it's axis,

$$I_1 = \frac{MR^2}{2}$$

From perpendicular axes theorem, moment of inertia of disc about an axis along it's diameter is

$$I_x + I_y = I_z \quad \Rightarrow \quad 2I_2 = I_1$$

$$\Rightarrow \quad I_2 = \frac{I_1}{2} = \frac{MR^2}{4}$$



So, moment of inertia about a tangential axis from parallel axes theorem is

$$I = I_2 + MR^2 = \frac{MR^2}{4} + MR^2 = \frac{5}{4}MR^2$$

Now, using torque, $\tau = I\alpha$, we have

$$\tau = I\alpha = \frac{5}{4}MR^2\left(\frac{\omega_f - \omega_i}{\Delta t}\right)$$

Here,

$$M = 5 \times 10^{-3} \text{ kg}, R = 1 \times 10^{-2} \text{ m}$$

$$\omega_f = 25 \text{ rps} = 25 \times 2\pi \frac{\text{rad}}{\text{s}} = 50\pi \frac{\text{rad}}{\text{s}},$$

$$\omega_i = 0, \ \Delta t = 5 \text{ s}$$

$$\text{So, } \tau = \frac{\frac{5}{4} \times 5 \times 10^{-3} \times (10^{-2})^2 \times 50\pi}{5}$$

$$\approx 2 \times 10^{-5} \text{ N-m}$$

2. Rotation kinetic energy of a body is given by

$$\text{KE}_{\text{rotational}} = \frac{1}{2}I\omega^2$$

where, $\omega = \omega_0 + \alpha t$

So, $$\text{KE}_{\text{rotational}} = \frac{1}{2}I(\omega_0 + \alpha t)^2 \quad \ldots \text{(i)}$$

Here, $I = 1.5 \text{ kgm}^2$,

$\text{KE} = 1200$ J and

$\alpha = 20 \text{ rad/s}^2$ and $\omega_0 = 0$

Substituting these values in Eq. (i), we get

$$1200 = \frac{1}{2}(1.5)(20 \times t)^2$$

$$\Rightarrow \quad t^2 = \frac{2 \times 1200}{1.5 \times 400} = 4$$

$$\therefore \quad t = 2 \text{ s}$$

3. From question,

let height attained by ring $= h_1$

Height attained by cylinder $= h_2$

Height attained by sphere $= h_3$

As we know that for a body which is rolling up an inclined plane (without slipping), follows the law of conservation of energy.

$\therefore$ For ring, using energy conservation law at its height h_1.

$$(\text{KE})_{\text{linear}} + (\text{KE})_{\text{rotational}} = (\text{PE})$$

$$\Rightarrow \quad \frac{1}{2}m_1 v_0^2 + \frac{1}{2}I_1\omega^2 = m_1 g h_1$$

$$\Rightarrow \quad \frac{1}{2}m_1 v_0^2 + \frac{1}{2}m_1 R^2\omega^2 = m_1 g h_1$$

$$(\because I = mR^2 \text{ for ring})$$

$$\Rightarrow \quad gh_1 = \frac{v_0^2}{2} + \frac{v_0^2}{2} \qquad (\because v_0 = \omega R)$$

$$\Rightarrow \quad h_1 = v_0^2 / g \qquad \ldots\text{(i)}$$

Similarly, for solid cylinder, applying the law of conservation of energy,

$$\frac{1}{2}m_2 v_0^2 + \frac{1}{2}I_2\omega^2 = m_2 g h_2$$

$$\Rightarrow \quad \frac{1}{2}m_2 v_0^2 + \frac{1}{2}\left(\frac{1}{2} \times m_2 \times \left(\frac{R}{2}\right)^2\right)\omega^2 = m_2 g h_2$$

$$\left[\begin{aligned} &\because I = \frac{1}{2}mR^2 \text{ for cylinder} \\ &\text{and} \quad R = \frac{R}{2} \end{aligned}\right]$$

$$\Rightarrow \quad \frac{1}{2}m_2 v_0^2 + \frac{1}{2} \times \frac{1}{8}m_2 R^2 \times \frac{v_0^2}{(R/2)^2} = m_2 g h_2$$

$$\Rightarrow \quad \frac{1}{2}v_0^2 + \frac{1}{4}v_0^2 = gh_2$$

$$\Rightarrow \quad gh_2 = \frac{3}{4}v_0^2$$

$$\Rightarrow \quad h_2 = \frac{3}{4}\left(\frac{v_0^2}{g}\right) \qquad \ldots\text{(ii)}$$

Similarly, for solid sphere,

$$\frac{1}{2}m_3 v_0^2 + \frac{1}{2}I_3\omega^2 = m_3 g h_3$$

$$\Rightarrow \frac{1}{2}m_3 v_0^2 + \frac{1}{2}\left[\frac{2}{5}m_3\left(\frac{R}{4}\right)^2\right]\omega^2 = m_3 g h_3$$

$$\left[\begin{aligned} &\because I = \frac{2}{5}mR^2 \text{ for solid sphere} \\ &\text{and} \quad R = \frac{R}{4} \end{aligned}\right]$$

$$\Rightarrow \quad \frac{1}{2}m_3 v_0^2 + \frac{1}{2} \times \frac{2}{5} \times m_3 \frac{R^2}{16} \times \frac{v_0^2}{(R/4)^2} = m_3 g h_3$$

$$\Rightarrow \quad \frac{1}{2}v_0^2 + \frac{1}{5}v_0^2 = gh_3$$

$$\Rightarrow \quad gh_3 = \frac{7}{10}v_0^2$$

or
$$h_3 = \frac{7}{10}\left(\frac{v_0^2}{g}\right) \qquad \ldots\text{(iii)}$$

$\therefore$ Taking the ratio of h_1, h_2 and h_3 by using Eqs. (i), (ii) and (iii), we get

$$h_1 : h_2 : h_3 = \frac{v_0^2}{g} : \frac{3}{4}\frac{v_0^2}{g} : \frac{7}{10}\frac{v_0^2}{g}$$

$$= 1 : \frac{3}{4} : \frac{7}{10}$$

$$\Rightarrow h_1 : h_2 : h_3 = 40 : 30 : 28 = 20 : 15 : 14$$

$\therefore$ The most appropriate option is (c).

Although, it is still not in the correct sequence.

Alternate Solution

Total kinetic energy of a rolling body is also given as

$$E_{\text{total}} = \frac{1}{2}mv^2\left[1 + \frac{K^2}{R^2}\right]$$

where, K is the radius of gyration.

Using conservation law of energy,

$$\frac{1}{2}mv^2\left[1 + \frac{K^2}{R^2}\right] = mgh$$

or
$$h = \frac{v^2}{2g}\left[1 + \frac{K^2}{R^2}\right]$$

For ring, $\dfrac{K^2}{R^2} = 1$

$$\Rightarrow h_1 = \frac{v^2}{2g}[1+1] = \frac{2v^2}{2g} = \frac{v^2}{g}$$

For solid cylinder, $\dfrac{K^2}{R^2} = \dfrac{(R/2\sqrt{2})^2}{(R/2)^2} = \dfrac{R^2}{8} \times \dfrac{4}{R^2} = \dfrac{1}{2}$

$$\Rightarrow \quad h_2 = \frac{v^2}{2g}\left[1 + \frac{1}{2}\right] = \frac{3v^2}{4g}$$

For solid sphere, $\dfrac{K^2}{R^2} = \dfrac{2}{5}$

$$\Rightarrow \quad h_3 = \frac{v^2}{2g}\left[1 + \frac{2}{5}\right] = \frac{7v^2}{10g}$$

So,the ratio of h_1, h_2 and h_3 is

$$h_1 : h_2 : h_3 = \frac{v^2}{g} : \frac{3v^2}{4g} : \frac{7}{10}\frac{v^2}{g}$$

$$= 1 : \frac{3}{4} : \frac{7}{10} = 20 : 15 : 14$$

4 When a spherical/circular body of radius r rolls without slipping, its total kinetic energy is

$$K_{\text{total}} = K_{\text{translation}} + K_{\text{rotation}}$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}I\left[\frac{v^2}{r^2}\because \omega = \frac{v}{r}\right]$$

Let v be the linear velocity and R be the radius for both solid sphere and solid cylinder.

$\therefore$ Kinetic energy of the given solid sphere will be

$$K_{\text{sph}} = \frac{1}{2}mv^2 + \frac{1}{2}I_{\text{sph}}\frac{v^2}{R^2}$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}\times\frac{2}{5}mR^2\times\frac{v^2}{R^2} = \frac{7}{10}mv^2 \quad \text{...(i)}$$

Similarly, kinetic energy of the given solid cylinder will be

$$K_{\text{cyl}} = \frac{1}{2}mv^2 + \frac{1}{2}I_{\text{cyl}}\frac{v^2}{R}$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}\times\frac{mR^2}{2}\times\frac{v^2}{R^2} = \frac{3}{4}mv^2 \quad \text{...(ii)}$$

Now, from the conservation of mechanical energy,

$$mgh = K_{\text{total}}$$

$\therefore$ For solid sphere,

$$mgh_{\text{sph}} = \frac{7}{10}mv^2 \quad \text{...(iii) [using Eq. (i)]}$$

Similarly, for solid cylinder,

$$mgh_{\text{cyl}} = \frac{3}{4}mv^2 \quad \text{...(iv) [using Eq. (ii)]}$$

Taking the ratio of Eqs. (iii) and (iv), we get

$$\frac{mgh_{\text{sph}}}{mgh_{\text{cyl}}} = \frac{\frac{7}{10}mv^2}{\frac{3}{4}mv^2} \Rightarrow \frac{h_{\text{sph}}}{h_{\text{cyl}}} = \frac{7}{10}\times\frac{4}{3} = \frac{14}{15}$$

Alternate Solution

Total kinetic energy for a rolling body without slipping can also be given as

$$K_{\text{total}} = \frac{1}{2}mv^2\left[1+\frac{K^2}{R^2}\right]$$

where, K is the radius of gyration.

$\therefore$ From law of conservation,

$$mgh = \frac{1}{2}mv^2\left[1+\frac{K^2}{R^2}\right]$$

or $$h \propto \left(1+\frac{K^2}{R^2}\right)$$

As we know that, for solid sphere,

$$K = \sqrt{\frac{2}{5}}R \Rightarrow \frac{K^2}{R^2} = \frac{2}{5}$$

Similarly, for solid cylinder,

$$K = \frac{R}{\sqrt{2}} \Rightarrow \frac{K^2}{R^2} = \frac{1}{2}$$

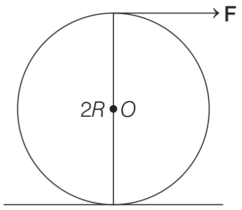
So, $$\frac{h_{\text{sph}}}{h_{\text{cyl}}} = \frac{1+\frac{2}{5}}{1+\frac{1}{2}} = \frac{\frac{7}{5}}{\frac{3}{2}} = \frac{7}{5}\times\frac{2}{3} = \frac{14}{15}$$

5 Given, $m = 5\,\text{kg}$, $R = 0.5\,\text{m}$.

Horizontal force, $F = 40\,\text{N}$

As, Cylinder is rolling without slipping.

Hence, torque is producing rotation about centre O.



So, $\tau = \mathbf{r}\times\mathbf{F}$ (Here, $r = R$)

$\theta = 90°$

So, $\tau = \mathbf{r}\times\mathbf{F} = RF$

or $\tau = 0.5\times 40 = 20\,\text{N-m}$ ...(i)

If α is acceleration of centre of mass 'O' then torque is, $\tau = I\alpha$

where, $I = MR^2$

$\therefore$ $\tau = MR^2\alpha$ ...(ii)

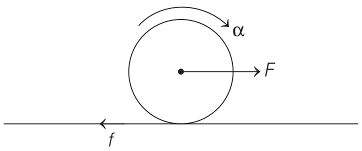
Comparing Eq. (i) with Eq. (ii),

$$MR^2\alpha = 20$$

$$\Rightarrow \quad \alpha = \frac{20}{5\times(0.5)^2}$$

or $\alpha = 16\,\text{rad / s}^2$

6 When force F is applied at the centre of roller of mass m as shown in the figure below



Its acceleration is given by

$$\frac{(F-f)}{m} = a \quad \text{... (i)}$$

where, f = force of friction and

m = mass of roller.

Torque on roller is provided by friction f and it is

$$\tau = fR = I\alpha \quad \text{...(ii)}$$

where, I = moment of inertia of solid cylindrical roller.

$= mR^2/2$

and α = angular acceleration of cylinder = a/R.

Hence, $$\tau = \frac{mR^2}{2}\cdot\frac{a}{R} = \frac{maR}{2}$$

From Eq. (ii), ($\tau = fR$)

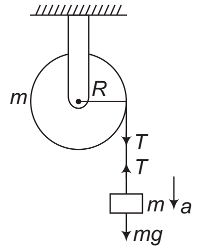
$$f = \frac{ma}{2} \quad \text{...(iii)}$$

From Eqs. (i) and (iii), we get

$$F = \frac{3}{2}ma \Rightarrow a = \frac{2F}{3m}$$

So, $$\alpha = \frac{2F}{3mR} \quad \left[\because \alpha = \frac{a}{R}\right]$$

7. For the mass m, $mg - T = ma$



As we know, $a = R\alpha$

So, $mg - T = mR\alpha$ ...(i)

Torque about centre of pully

$$T \times R = mR^2\alpha \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get, $a = g/2$

Hence, the acceleration with the mass of a body fall is $g/2$.

8. $I_P > I_Q$

In case of pure rolling, $a = \dfrac{g \sin\theta}{1 + I/mR^2}$

$a_Q > a_P$ as its moment of inertia is less. Therefore, Q reaches first with more linear speed and more translational kinetic energy.

Further, $\omega = \dfrac{v}{R}$ or $\omega \propto v$

$\therefore \quad \omega_Q > \omega_P$ as $v_P > v_Q$

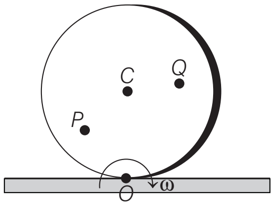
9. $$\frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v}{R}\right)^2 = mg\left(\frac{3v^2}{4g}\right)$$

$$\therefore \quad I = \frac{1}{2}mR^2$$

$\therefore$ Body is disc.

10. On smooth part BC, due to zero torque, angular velocity and hence the rotational kinetic energy remains constant. While moving from B to C translational kinetic energy converts into gravitational potential energy. So, $h_A < h_c$ and $k_B > k_C$.

11. In case of pure rolling bottom most point is the instantaneous centre of zero velocity.



Velocity of any point on the disc, $v = r\omega$, where r is the distance of point from O.

$$r_Q > r_C > r_P$$

$$\therefore \quad v_Q > v_C > v_P$$

12. $mg \sin\theta$ component is always down the plane whether it is rolling up or rolling down.Therefore, for no slipping, sense of angular acceleration should also be same in both the cases. Therefore, force of friction f always act upwards.

13. In case of pure rolling on inclined plane,

$$a = \frac{g\sin\theta}{1 + I/mR^2}$$

$$I_{\text{solid}} < I_{\text{hollow}}$$

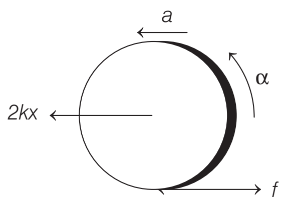
$$\therefore \quad a_{\text{solid}} > a_{\text{hollow}}$$

$\therefore$ Solid cylinder will reach the bottom first. Further, in case of pure rolling on stationary ground, work done by friction is zero. Therefore, mechanical energy of both the cylinders will remain constant.

$\therefore$ $(\text{KE})_{\text{Hollow}} = (\text{KE})_{\text{Solid}}$ = decrease in PE = mgh

$\therefore$ Correct option is (d).

14. $a = R\alpha$



$$\therefore \quad \frac{2kx - f}{M} = R\left[\frac{fR}{\frac{1}{2}MR^2}\right]$$

Solving this equation, we get

$$f = \frac{2kx}{3}$$

$$\therefore \quad |F_{\text{net}}| = 2kx - f = 2kx - \frac{2kx}{3} = \frac{4kx}{3}$$

This is opposite to displacement.

$$\therefore \quad F_{\text{net}} = -\frac{4kx}{3}$$

15. $$F_{\text{net}} = -\left(\frac{4kx}{3}\right)x$$

$$\therefore \quad a = \frac{F_{\text{net}}}{M} = -\left(\frac{4k}{3M}\right)x = -\omega^2 x$$

$$\therefore \quad \omega = \sqrt{\frac{4k}{3M}}$$

16. In case of pure rolling, mechanical energy will remain conserved.

$$\therefore \quad \frac{1}{2}Mv_0^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v_0}{R}\right)^2 = 2\left[\frac{1}{2}kx^2{}_{\max}\right]$$

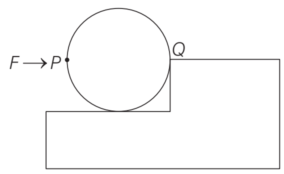
$$\therefore \quad x_{\max} = \sqrt{\frac{3M}{4k}}\,v_0$$

As, $$f = \frac{2kx}{3}$$

$$\therefore \quad F_{\max} = \mu Mg = \frac{2kx_{\max}}{3} = \frac{2k}{3}\sqrt{\frac{3M}{4k}}v_0$$

$$\therefore \quad v_0 = \mu g\sqrt{\frac{3M}{k}}$$

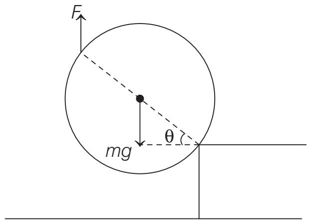
17.



(a) If force is applied normal to surface at P, then line of action of force will pass from Q and thus, $\tau = 0$.

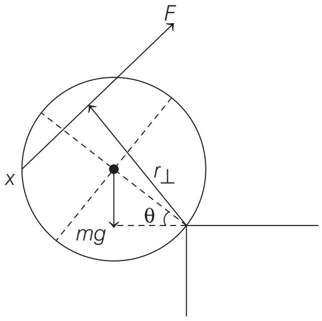
(b) Wheel can climb.

(c) $\tau = F(2R\cos\theta) - mgR\cos\theta$, $\tau \propto \cos\theta$



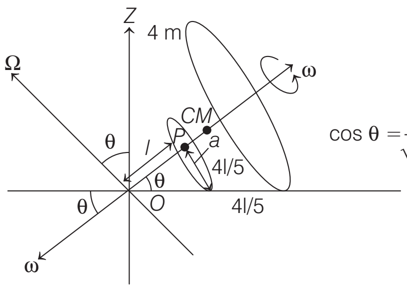
Hence, as θ increases
τ decreases. So its correct.

(d)



$\tau = Fr_\perp - mg\cos\theta$; τ increases with θ.

18.



$$\cos\theta = \frac{l}{\sqrt{l^2+a^2}} = \frac{\sqrt{24}}{5}$$

(a) $$L_z = L_{CM-O}\cos\theta - L_{D-CM}\sin\theta$$
$$= \frac{81\sqrt{24}}{5}a^2 m\omega \times \frac{\sqrt{24}}{5} - \frac{17\,ma^2\omega}{2} \times \frac{1}{\sqrt{24}}$$
$$= \frac{81\times 24 ma^2\omega}{25} - \frac{17ma^2\omega}{2\sqrt{24}}$$

(b) $$L_{CM-O} = (5m)\left[\frac{9l}{5}\omega\right]\frac{9l}{5} = \frac{81ml^2\omega}{5}$$
$$= \frac{81ml^2}{5} \times \frac{a\omega}{l}$$
$$L_{CM-O} = \frac{81mla\omega}{5} = \frac{81\sqrt{24}\,a^2 m\omega}{5}$$

(c) Velocity of point P : $a\omega = 1\omega$ then

$\omega = \frac{a\omega}{1}$ = Angular velocity of C.M. w.r.t. point O.

Angular velocity of CM w.r.t. Z-axis $= \omega_0\cos\theta$

$$\omega_{CM-z} = \frac{a\omega}{1}\,\frac{\sqrt{24}}{5} = \frac{a\omega}{\sqrt{24}\,a}\,\frac{\sqrt{24}}{5}$$

$$\omega_{CM-z} = \frac{\omega}{5}$$

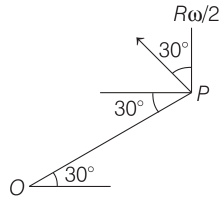
(d) $$L_{D-CM} = \frac{ma^2}{2}\omega + \frac{4m(2a)^2}{2}\omega = \frac{17ma^2\omega}{2}$$

19. Velocity of point O is

$$v_O = (3R\omega)\,\hat{\mathbf{i}}$$

$\mathbf{v}_{PO}$ is $\frac{R\cdot\omega}{2}$ in the direction shown in figure.

In vector form



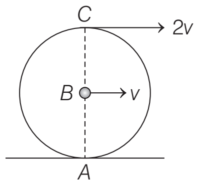
$$\mathbf{v}_{PO} = -\frac{R\omega}{2}\sin 30^\circ\,\hat{\mathbf{i}} + \frac{R\omega}{2}\cos 30^\circ\,\hat{\mathbf{k}}$$
$$= -\frac{R\omega}{4}\,\hat{\mathbf{i}} + \frac{\sqrt{3}R\omega}{4}\,\hat{\mathbf{k}}$$

But $\mathbf{v}_{PO} = \mathbf{v}_P - \mathbf{v}_O$

∴ $\mathbf{v}_P = \mathbf{v}_{PO} + \mathbf{v}_O$

$$= \left(-\frac{R\omega}{4}\,\hat{\mathbf{i}} + \frac{\sqrt{3}R\omega}{4}\,\hat{\mathbf{k}}\right) + 3R\omega\,\hat{\mathbf{i}}$$
$$= \frac{11}{4}R\omega\,\hat{\mathbf{i}} + \frac{\sqrt{3}}{4}R\omega\,\hat{\mathbf{k}}$$

20. $v_A = 0, v_B = v$ and $v_C = 2v$



21. In case of pure rolling,

$$f = \frac{mg\sin\theta}{1+\frac{mR^2}{I}} \qquad \text{(upwards)}$$

∴ $f \propto \sin\theta$

Therefore, as θ decreases force of friction will also decrease.

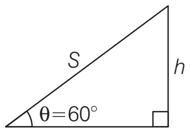
22. $$a = \frac{g\sin\theta}{1+\frac{I}{MR^2}}$$

$$a_{\text{ring}} = \frac{g\sin\theta}{2} \qquad (I = MR^2)$$

$$a_{\text{disc}} = \frac{2g\sin\theta}{3} \qquad \left(I = \frac{MR^2}{2}\right)$$

$$s = \frac{h}{\sin\theta} = \frac{1}{2}at^2$$

$$= \frac{1}{2}\left(\frac{g\sin\theta}{2}\right)t_1^2$$



$$\Rightarrow \qquad t_1 = \sqrt{\frac{4h}{g\sin^2\theta}} = \sqrt{\frac{16h}{3g}}$$

$$s = \frac{h}{\sin\theta} = \frac{1}{2}at^2 = \frac{1}{2}\left(\frac{2g\sin\theta}{3}\right)t_2^2$$

$$\Rightarrow \qquad t_2 = \sqrt{\frac{3h}{g\sin^2\theta}} = \sqrt{\frac{4h}{g}}$$

$$t_2 - t_1 = \sqrt{\frac{16h}{3g}} - \sqrt{\frac{4h}{g}} = \frac{2-\sqrt{3}}{\sqrt{10}}$$

$$\sqrt{h}\left[\frac{4}{\sqrt{3}} - 2\right] = 2 - \sqrt{3}$$

Soving this equation we get, $h = 0.75$ m.

23. The equations of motion are

$$a = \frac{mg\sin\theta - f}{m}$$

$$= \frac{mg\sin 30^\circ - f}{m} = \frac{g}{2} - \frac{f}{m} \qquad \ldots\text{(i)}$$

$$\alpha = \frac{\tau}{I} = \frac{fR}{I} = \frac{fR}{mR^2/2} = \frac{2f}{mR} \qquad \ldots\text{(ii)}$$

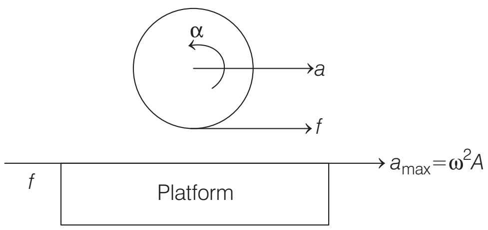
For rolling (no slipping)

$$a = R\alpha$$

or $\qquad g/2 - f/m = 2f/m$

$\therefore \qquad \dfrac{3f}{m} = g/2 \quad$ or $\quad f = mg/6$

24. Considering the motion of cylinder.



$$a = \frac{f}{M} \Rightarrow \alpha = \frac{fR}{\frac{MR^2}{2}}$$

$$= \frac{2f}{MR}$$

or $\qquad R\alpha = \dfrac{2f}{M}$

Now, $\quad a + R\alpha = a_{max} = \omega^2 A$

or $\qquad \dfrac{3f}{m} = \omega^2 A$

$\therefore \qquad f = \dfrac{M\omega^2 A}{3}$

$\therefore \qquad \tau_{max} = fR = \dfrac{M\omega^2 AR}{3}$

25. In case of ring, $\dfrac{K_R}{K_T} = 1$ (pure rolling)

or $\qquad K_R = K_T = \dfrac{K}{2}$

$\therefore \qquad \dfrac{1}{2}(0.3)\,v_1^2 = \dfrac{K}{2} \qquad$ (i)

In case of disc, $\dfrac{K_R}{K_T} = \dfrac{1}{2}$ or $K_T = \dfrac{2}{3}K$

$\therefore \qquad \dfrac{1}{2}(0.4)v_2^2 = \dfrac{2}{3}K \qquad$ …(ii)

From Eqs. (i) and (ii), $\dfrac{v_1}{v_2} = 1$ i.e. $v_1 = v_2$

or both will reach simultaneously.

NOTE In the question,

K = kinetic energy given to ring and cylinder,

K_R = rotational kinetic energy and

K_T = translational kinetic energy.

26. In case of pure rolling, mechanical energy remains constant (as work-done by friction is zero). Further in case of a disc,

$$\frac{\text{translational kinetic energy}}{\text{rotational kinetic energy}} = \frac{K_T}{K_R} = \frac{\frac{1}{2}mv^2}{\frac{1}{2}I\omega^2}$$

$$= \frac{mv^2}{\left(\frac{1}{2}mR^2\right)\left(\frac{v}{R}\right)^2} = \frac{2}{1}$$

or, $\qquad K_T = \dfrac{2}{3} \qquad$ (Total kinetic energy)

or, $\quad$ Total kinetic energy

$$K = \frac{3}{2}K_T = \frac{3}{2}\left(\frac{1}{2}mv^2\right) = \frac{3}{4}mv^2$$

Decrease in potential energy = increase in kinetic energy

or, $\quad mgh = \dfrac{3}{4}m\,(v_f^2 - v_i^2)$ or $v_f = \sqrt{\dfrac{4}{3}gh + v_i^2}$

As final velocity in both cases is same.

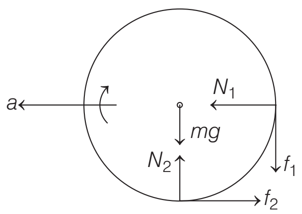
So, value of $\sqrt{\dfrac{4}{3}gh + v_i^2}$ should be same in both cases.

$\therefore \quad \sqrt{\frac{4}{3} \times 10 \times 30 + (3)^2} = \sqrt{\frac{4}{3} \times 10 \times 27 + (v_2)^2}$

Solving this equation, we get

$$v_2 = 7\,\text{m/s}$$

27.



There is no slipping between ring and ground. Hence f_2 is not maximum. But there is slipping between ring and stick. Therefore, f_1 is maximum. Now let us write the equations.

$$I = mR^2 = (2)(0.5)^2$$

$$= \frac{1}{2}\,\text{kg-m}^2$$

$$N_1 - f_2 = ma$$

or $\quad N_1 - f_2 = (2)(0.3) = 0.6\ \text{N} \quad \ldots\text{(i)}$

$$a = R\alpha = \frac{R\tau}{I} = \frac{R(f_2 - f_1)R}{I} = \frac{R^2(f_2 - f_1)}{I}$$

$\therefore \quad 0.3 = \dfrac{(0.5)^2\,(f_2 - f_1)}{(1/2)}$

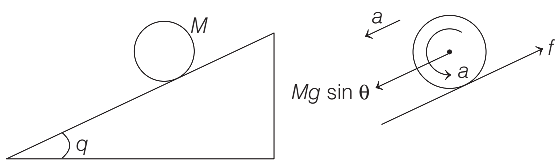
or $\quad f_2 - f_1 = 0.6\,\text{N} \quad \ldots\text{(ii)}$

$$N_1^2 + f_1^2 = (2)^2 = 4 \quad \ldots\text{(iii)}$$

Further $\quad f_1 = \mu N_1 = \left(\dfrac{P}{10}\right)N_1 \quad \ldots\text{(iv)}$

Solving above four equations we get, $P \simeq 3.6$

28. For rolling without slipping, we have



$$a = R\alpha$$

or $\quad \dfrac{Mg\sin\theta - f}{M} = R\left(\dfrac{fR}{\frac{1}{2}MR^2}\right)$

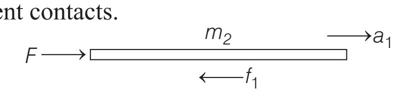
or $\quad \dfrac{Mg\sin\theta - f}{M} = \dfrac{2f}{M}$

$\therefore \quad f = \dfrac{Mg\sin\theta}{3}$

Therefore, linear acceleration of cylinder,

$$a = \frac{Mg\sin\theta - f}{M} = \frac{2}{3}g\sin\theta$$

29. We can choose any arbitrary directions of frictional forces at different contacts.



In the final answer the negative values will show the opposite directions.

Let $\quad f_1$ = friction between plank and cylinder

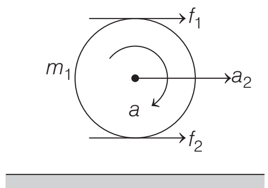
f_2 = friction between cylinder and ground

a_1 = acceleration of plank

a_2 = acceleration of centre of mass of cylinder

and $\quad \alpha$ = angular acceleration of cylinder about its CM.

Since, there is no slipping anywhere



$\therefore \quad a_1 = 2a_2 \quad \ldots\text{(i)}$

$$a_1 = \frac{F - f_1}{m_2} \quad \ldots\text{(ii)}$$

$$a_2 = \frac{f_1 + f_2}{m_1} \quad \ldots\text{(iii)}$$

$$\alpha = \frac{(f_1 - f_2)R}{I} = \frac{(f_1 - f_2)R}{\frac{1}{2}m_1 R^2}$$

$$\alpha = \frac{2(f_1 - f_2)}{m_1 R} \quad \ldots\text{(iv)}$$

$$a_2 = R\alpha = \frac{2(f_1 - f_2)}{m_1} \quad \ldots\text{(v)}$$

(a) Solving Eqs. (i) to (v), we get

$$a_1 = \frac{8F}{3m_1 + 8m_2} \quad \text{and} \quad a_2 = \frac{4F}{3m_1 + 8m_2}$$

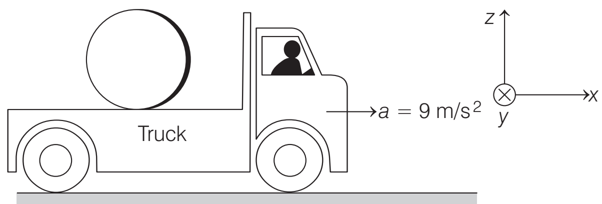
(b) $\quad f_1 = \dfrac{3m_1 F}{3m_1 + 8m_2};$

$$f_2 = \frac{m_1 F}{3m_1 + 8m_2}$$

Since, all quantities are positive, they are correctly shown in figures.

30. Given, mass of disc $m = 2\text{kg}$ and radius $R = 0.1\,\text{m}$

(a) FBD of any one disc is



Frictional force on the disc should be in forward direction.

Let a_0 be the linear acceleration of CM of disc and α the angular acceleration about its CM. Then,

$$a_0 = \frac{f}{m} = \frac{f}{2} \quad \ldots\text{(i)}$$

$$\alpha = \frac{\tau}{I} = \frac{fR}{\frac{1}{2}mR^2} = \frac{2f}{mR} = \frac{2f}{2\times 0.1} = 10f \quad \ldots\text{(ii)}$$

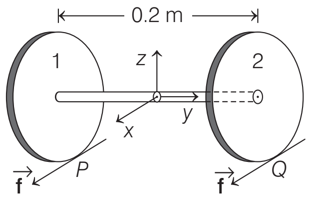
Since, there is no slipping between disc and truck. Therefore,

$$\therefore \alpha_0 + R\,\alpha = a \text{ or } \left(\frac{f}{2}\right) + (0.1)(10f) = a$$

or $\quad \frac{3}{2}f = a \Rightarrow f = \frac{2a}{3} = \frac{2\times 9.0}{3}\text{N}$

$\therefore \quad f = 6\text{N}$

Since, this force is acting in positive x-direction.



Therefore, in vector form $\mathbf{f} = (6\hat{\mathbf{i}})$ N

(b) $\tau = \mathbf{r} \times \mathbf{f}$

Here, $\mathbf{f} = (6\hat{\mathbf{i}})$ N (for both the discs)

$$\mathbf{r}_P = \mathbf{r}_1 = -0.1\,\hat{\mathbf{j}} - 0.1\hat{\mathbf{k}}$$

and $\quad \mathbf{r}_Q = \mathbf{r}_2 = 0.1\,\hat{\mathbf{j}} - 0.1\,\hat{\mathbf{k}}$

Therefore, frictional torque on disk 1 about point O (centre of mass).

$$\tau_1 = \mathbf{r}_1 \times \mathbf{f} = (-0.1\,\hat{\mathbf{j}} - 0.1\,\hat{\mathbf{k}}) \times (6\hat{\mathbf{i}})\text{ N-m}$$
$$= (0.6\,\hat{\mathbf{k}} - 0.6\,\hat{\mathbf{j}})$$

or $\quad \tau_1 = 0.6\,(\hat{\mathbf{k}} - \hat{\mathbf{j}})$ N-m

and $\quad |\tau_1| = \sqrt{(0.6)^2 + (0.6)^2}$

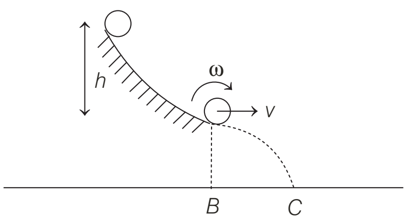
$$= 0.85 \text{ N-m}$$

Similarly, $\quad \tau_2 = \mathbf{r}_2 \times \mathbf{f} = 0.6(-\hat{\mathbf{j}} - \hat{\mathbf{k}})$

and $\quad |\tau_1| = |\tau_2| = 0.85$ N- m

31. $h = 2.6 - 1.0 = 1.6$ m

During pure rolling mechanical energy remains conserved



So, at bottom of track total kinetic energy of sphere will be mgh.

The ratio of $\quad \frac{K_R}{K_T} = \frac{2}{5}$

or $\quad K_T = \frac{5}{7}mgh = \frac{1}{2}mv^2$

$\therefore \quad v = \sqrt{\frac{10}{7}gh} = \sqrt{\frac{10}{7}\times 9.8 \times 1.6}$

$$= 4.73 \text{ m/s}$$

In projectile motion

Time to fall to ground $= \sqrt{\frac{2\times 1}{9.8}} = 0.45$ s

$\therefore$ The desired distance $BC = vt = 2.13$ m

In air, during its flight as a projectile only mg is acting on the sphere which passes through its centre of mass. Therefore, net torque about centre of mass is zero or angular velocity will remain constant.

Topic 4 Collision in Rotational Motion

1 Initial kinetic energy of the given system,

$$\text{KE}_i = \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}\left(\frac{I_1}{2}\right)\left(\frac{\omega_1}{2}\right)^2$$

$$= \left(\frac{1}{2} + \frac{1}{16}\right)I_1\omega_1^2 = \frac{9}{16}I_1\omega_1^2 \quad \ldots\text{(i)}$$

Now, using angular momentum conservation law (assuming angular velocity after contact is ω)

Initial angular momentum = Final angular momentum

$$I_1\omega_1 + \left(\frac{I_1}{2}\right)\left(\frac{\omega_1}{2}\right) = I_1\omega' + \frac{I_1}{2}\omega'$$

$$\Rightarrow \quad \frac{5}{4}\omega_1 = \frac{3}{2}\omega' \text{ or } \omega' = \frac{5}{6}\omega_1 \quad \ldots\text{(ii)}$$

Now, final kinetic energy (after contact) is

$$\text{KE}_f = \frac{1}{2}I_1\omega'^2 + \frac{1}{2}\left(\frac{I_1}{2}\right)\omega'^2$$

$$= \frac{1}{2}I_1\left(\frac{5}{6}\omega_1\right)^2 + \frac{1}{4}I_1\left(\frac{5}{6}\omega_1\right)^2 \text{ [using Eq. (ii)]}$$

$$= \left(\frac{25}{72} + \frac{25}{144}\right)I_1\omega_1^2$$

$$= \frac{25}{48}I_1\omega_1^2 \quad \ldots\text{(iii)}$$

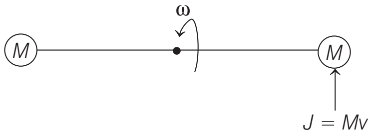
Hence, change in KE,

$$\Delta\text{KE} = \text{KE}_f - \text{KE}_i$$

$$= \frac{25}{48}I_1\omega_1^2 - \frac{9}{16}I_1\omega_1^2 \quad \text{[using Eqs. (i)]}$$

$$\Delta\text{KE} = -\frac{1}{24}I_1\omega_1^2$$

2. Let ω be the angular velocity of the rod. Applying, angular impulse = change in angular momentum about centre of mass of the system

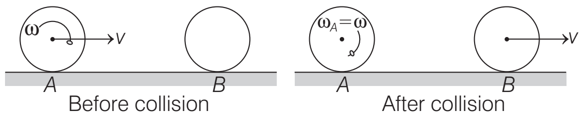


$$J.\frac{L}{2} = I_c\,\omega$$

$$\therefore \quad (Mv)\left(\frac{L}{2}\right) = (2)\left(\frac{ML^2}{4}\right)\omega$$

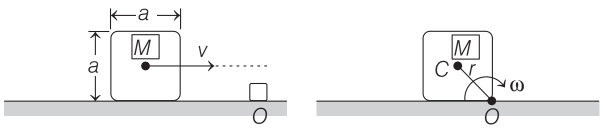
$$\therefore \quad \omega = \frac{v}{L}$$

3. Since, it is head on elastic collision between two identical spheres, they will exchange their linear velocities, i.e., A comes to rest and B starts moving with linear velocity v. As there is no friction anywhere, torque on both the spheres about their centre of mass is zero and their angular velocities remain unchanged. Therefore,



$$\omega_A = \omega \quad \text{and} \quad \omega_B = 0.$$

4. $r = \sqrt{2}\,\dfrac{a}{2}$ or $r^2 = \dfrac{a^2}{2}$



Net torque about O is zero.

Therefore, angular momentum (L) about O will be conserved, or $L_i = L_f$

$$Mv\left(\frac{a}{2}\right) = I_o\,\omega = (I_{CM} + Mr^2)\,\omega$$

$$= \left\{\left(\frac{Ma^2}{6}\right) + M\left(\frac{a^2}{2}\right)\right\}\omega$$

$$= \frac{2}{3}Ma^2\omega$$

$$\omega = \frac{3v}{4a}$$

5. The data is incomplete. Let us assume that friction from ground on ring is not impulsive during impact.

From linear momentum conservation in horizontal direction, we have

$$(-2\times 1) + (0.1\times 20) = (0.1\times 0) + (2\times v)$$

$-$ve $\longleftrightarrow$ $+$ve

Here, v is the velocity of CM of ring after impact.

Solving the above equation, we have $v = 0$

Thus, CM becomes stationary.

$\therefore$ Correct option is (a).

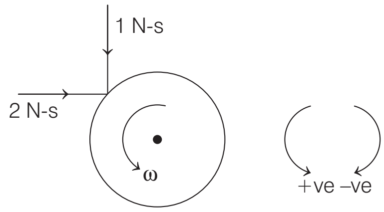
Linear impulse during impact

(i) In horizontal direction

$$J_1 = \Delta p = 0.1\times 20 = 2\ \text{N-s}$$

(ii) In vertical direction $J_2 = \Delta p = 0.1\times 10 = 1\ \text{N-s}$

Writing the equation (about CM)



Angular impulse = Change in angular momentum

We have,

$$1\times\left(\frac{\sqrt{3}}{2}\times\frac{1}{2}\right) - 2\times 0.5\times\frac{1}{2} = 2\times(0.5)^2\left[\omega - \frac{1}{0.5}\right]$$

Solving this equation ω comes out to be positive or ω anti-clockwise. So just after collision rightwards slipping is taking place.

Hence, friction is leftwards.

Therefore, option (c) is also correct.

6. $P_i = 0$

$\therefore \quad P_f = 0$ or $v_c = 0$

$L_i = L_f$ or $(2mv)\,a + (2mv)\,(2a) = I\omega$ ...(i)

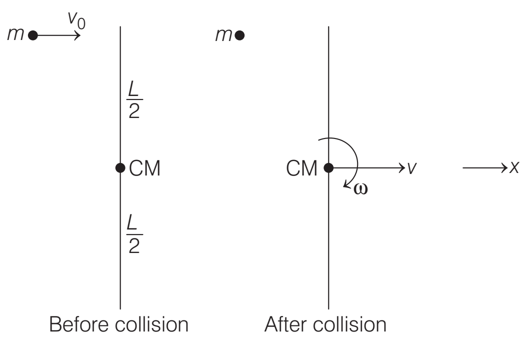
Here, $I = \dfrac{(8m)(6a)^2}{12} + m\,(2a)^2 + (2m)\,(a^2) = 30\,ma^2$

Substituting in Eq. (i), we get

$$\omega = \frac{v}{5a}$$

Further, $E = \dfrac{1}{2}I\omega^2 = \dfrac{1}{2}\times(30ma^2)\left(\dfrac{v}{5a}\right)^2 = \dfrac{3mv^2}{5}$

7. (a) Let just after collision, velocity of CM of rod is v and angular velocity about CM is ω. Applying following three laws.



(1) External force on the system (rod + mass) in horizontal plane along x-axis is zero.

$\therefore$ Applying conservation of linear momentum in x-direction.

$$mv_0 = mv \qquad \text{...(i)}$$

(2) Net torque on the system about CM of rod is zero.

$\therefore$ Applying conservation of angular momentum about CM of rod, we get $mv_0\left(\frac{L}{2}\right) = I\,\omega$

or $$mv_0\,\frac{L}{2} = \frac{ML^2}{12}\,\omega$$

or $$mv_0 = \frac{ML\omega}{6} \qquad \text{...(ii)}$$

(3) Since, the collision is elastic, kinetic energy is also conserved.

$$\therefore \quad \frac{1}{2}mv_0^2 = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

or $$mv_0^2 = Mv^2 + \frac{ML^2}{12}\omega^2 \qquad \text{...(iii)}$$

From Eqs. (i), (ii) and (iii), we get the following results

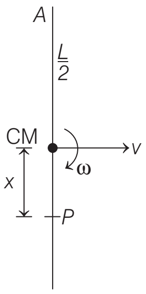
$$\frac{m}{M} = \frac{1}{4}$$

$$v = \frac{mv_0}{M} \text{ and } \omega = \frac{6\,mv_0}{ML}$$

(b) Point P will be at rest if $x\,\omega = v$

or $$x = \frac{v}{\omega} = \frac{mv_0/M}{6mv_0/ML}$$

or $$x = L/6$$



$$\therefore \quad AP = \frac{L}{2} + \frac{L}{6}$$

or $$AP = \frac{2}{3}L$$

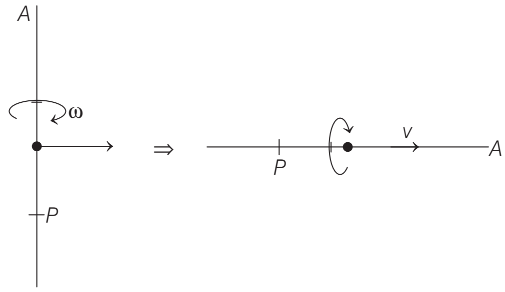
(c) After time $$t = \frac{\pi L}{3v_0}$$

angle rotated by rod, $\theta = \omega t = \dfrac{6mv_0}{ML}\cdot\dfrac{\pi L}{3v_0}$

$$= 2\pi\left(\frac{m}{M}\right)$$

$$= 2\pi\left(\frac{1}{4}\right) \quad \therefore \quad \theta = \frac{\pi}{2}$$

Therefore, situation will be as shown below

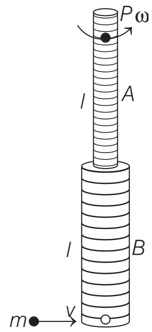


$\therefore$ Resultant velocity of point P will be

$$|\mathbf{v}_P| = \sqrt{2}v = \sqrt{2}\left(\frac{m}{M}\right)v_0$$

$$= \frac{\sqrt{2}}{4}v_0 = \frac{v_0}{2\sqrt{2}} \quad \text{or} \quad |\mathbf{v}_P| = \frac{v_0}{2\sqrt{2}}$$

8. System is free to rotate but not free to translate. During collision, net torque on the system (rod A + rod B + mass m) about point P is zero.



Therefore, angular momentum of system before collision
= angular momentum of system just after collision (about P).

Let ω be the angular velocity of system just after collision, then

$$L_i = L_f$$

$$\Rightarrow \quad mv(2l) = I\omega \qquad \text{...(i)}$$

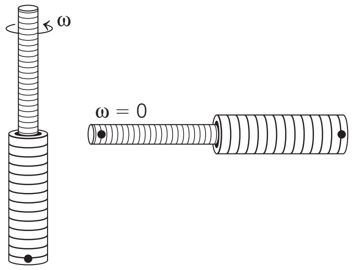
Here, I = moment of inertia of system about P

$$= m(2l)^2 + m_A(l^2/3) + m_B\left[\frac{l^2}{12} + \left(\frac{l}{2} + l\right)^2\right]$$

Given, $l = 0.6$ m, $m = 0.05$ kg, $m_A = 0.01$ kg and $m_B = 0.02$ kg.

Substituting the values, we get

$$I = 0.09 \text{ kg-m}^2$$



Therefore, from Eq. (i)

$$\omega = \frac{2mvl}{I} = \frac{(2)\,(0.05)\,(v)(0.6)}{0.09}$$

$$\omega = 0.67v \qquad \text{...(ii)}$$

Now, after collision, mechanical energy will be conserved.

Therefore, decrease in rotational KE

= increase in gravitational PE

or $\frac{1}{2} I\omega^2 = mg\,(2l) + m_A\, g\left(\frac{l}{2}\right) + m_B g\,(l + \frac{l}{2})$

or $\omega^2 = \frac{gl\,(4m + m_A + 3\,m_B)}{I}$

$= \frac{(9.8)\,(0.6)\,(4 \times 0.05 + 0.01 + 3 \times 0.02)}{0.09}$

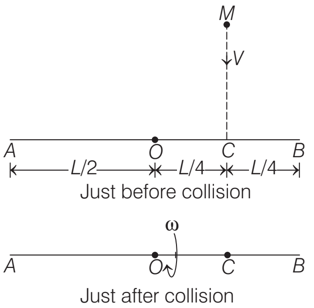
$= 17.64\ (\text{rad/s})^2$

$\therefore \quad \omega = 4.2\ \text{rad/s} \quad \dots\text{(iii)}$

Equating Eqs. (ii) and (iii), we get

$v = \frac{4.2}{0.67}$ m/s or $v = 6.3$ m/s

9. In this problem we will write K for the angular momentum because L has been used for length of the rod.



(a) Angular momentum of the system (rod + insect) about the centre of the rod O will remain conserved just before collision and after collision, i.e. $K_i = K_f$.

or $\quad Mv\,\frac{L}{4} = I\,\omega = \left[\frac{ML^2}{12} + M\left(\frac{L}{4}\right)^2\right]\omega$

or $\quad Mv\,\frac{L}{4} = \frac{7}{48}\,ML^2\,\omega$

i.e. $\quad \omega = \frac{12}{7}\,\frac{v}{L} \quad \dots\text{(i)}$

(b) Due to the torque of weight of insect about O, angular momentum of the system will not remain conserved (although angular velocity ω is constant). As the insect moves towards B, moment of inertia of the system increases, hence, the angular momentum of the system will increase.

Let at time t_1 the insect be at a distance x from O and by then the rod has rotated through an angle θ. Then, angular momentum at that moment,

$$K = \left[\frac{ML^2}{12} + Mx^2\right]\omega$$

Hence, $\frac{dK}{dt} = 2M\,\omega x\,\frac{dx}{dt} \quad (\omega = \text{constant})$

$\Rightarrow \quad \tau = 2M\omega\, x\,\frac{dx}{dt} \Rightarrow Mgx\cos\theta = 2\,M\,\omega\, x\frac{dx}{dt}$

$\Rightarrow \quad dx = \left(\frac{g}{2\omega}\right)\cos\omega t\, dt \quad (\because \theta = \omega t)$

At time $t = 0$, $x = L/4$ and at time $t = T/4$ or $\pi/2\omega$, $x = L/2$.

Substituting these limits, we get

$$\int_{L/4}^{L/2} dx = \frac{g}{2\omega}\int_0^{\pi/2\omega} (\cos\omega t)\, dt$$

$$[x]_{L/4}^{L/2} = \frac{g}{2\omega^2}\,[\sin\omega t]_0^{\pi/2\omega}$$

$\Rightarrow \quad \left(\frac{L}{2} - \frac{L}{4}\right) = \frac{g}{2\omega^2}\left[\sin\frac{\pi}{2} - \sin 0\right]$

$\frac{L}{4} = \frac{g}{2\omega^2}$ or $\omega = \sqrt{\frac{2g}{L}}$

Substituting in Eq. (i), we get

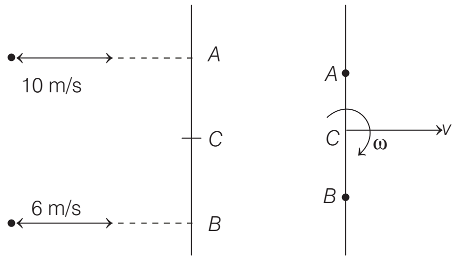
$\sqrt{\frac{2g}{L}} = \frac{12}{7}\cdot\frac{v}{L}$

or $\quad v = \frac{7}{12}\sqrt{2gL}$

$= \frac{7}{12}\sqrt{2 \times 10 \times 1.8}$

$\therefore \quad v = 3.5\,\text{m/s}$

10. Let v be the velocity of centre of mass (also at C) of rod and two particles and ω the angular velocity of the system.



From conservation of linear momentum

$$(0.08)\,(10 + 6) = [0.08 + 0.08 + 0.16]\,v$$

$\therefore \quad v = 4$ m/s

$AC = CB = 0.5$m

Similarly, conservation of angular momentum about point C.

$(0.08)\,(10)\,(0.5) - (0.08)\,(6)\,(0.5) = I\omega \quad \dots\text{(i)}$

Here, $\quad I = I_{\text{rod}} + I_{\text{two particles}}$

$= \frac{(1.6)\,(\sqrt{3})^2}{12} + 2\,(0.08)\,(0.5)^2$

$= 0.08\,\text{kg-m}^2$

Substituting in Eq. (i), we get

$\omega = 2\,\text{rad/s}$

Loss of kinetic energy

$= \frac{1}{2}(0.08)\,(10)^2 + \frac{1}{2}(0.08)\,(6)^2$

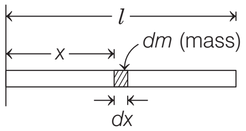
$- \frac{1}{2}\,(0.08 + 0.08 + 0.16)\,(4)^2 - \frac{1}{2}\,(0.08)\,(2)^2$

$= 4 + 1.44 - 2.56 - 0.16 = 2.72$ J

Topic 5 Miscellaneous Problems

1. **Key Idea** In a uniform rod, mass per unit length remains constant. If it is denoted by λ, then

$\lambda = \frac{m}{l}$ = constant for all segments of rod.



To find tension at x distance from fixed end, let us assume an element of dx length and dm mass. Tension on this part due to rotation is

$$dT = Kx \quad \ldots(\text{i})$$

As, $K = m\omega^2$

For this element, $K = (dm)\omega^2 \quad \ldots(\text{ii})$

$\therefore \quad dT = (dm)\omega^2 x \quad \ldots(\text{iii})$

To find complete tension in the rod, we need to integrate Eq. (iii),

$$\int_0^T dT = \int_0^m (dm)\,\omega^2 x \quad \ldots(\text{iv})$$

Using linear mass density,

$$\lambda = \frac{m}{l} = \frac{dm}{dx}$$

$\Rightarrow \quad dm = \frac{m}{l} \cdot dx \quad \ldots(\text{v})$

Putting the value of Eq. (v) in Eq. (iv), we get

$$T = \int_x^l \frac{m}{l} \cdot \omega^2 x \cdot dx = \frac{m}{l} \cdot \omega^2 \left[\frac{x^2}{2}\right]_x^l$$

$\Rightarrow \quad T = \frac{m\omega^2}{2l}[l^2 - x^2]$ or $T \propto -x^2$

2. **Key Idea** The rectangular box rotates due to torque of weight about its centre of mass.

Now, angular impulse of weight = Change in angular momentum.

$\therefore \quad mg\frac{l}{2} \times \tau = \frac{ml^2}{3}\omega$

$\Rightarrow \quad \omega = \frac{3g \times \tau}{2 \times l}$

Substituting the given values, we get

$$= \frac{3 \times 10 \times 0.01}{2 \times 0.3} = 0.5 \text{ rad s}^{-1}$$

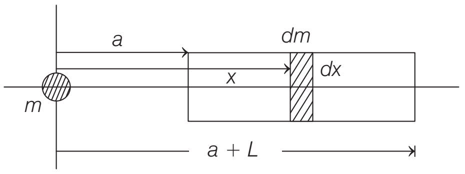
Time of fall of box,

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 5}{10}} \approx 1 \text{ s}$$

So, angle turned by box in reaching ground is

$\theta = \omega t = 0.5 \times 1 = 0.5$ rad

3. Given, situation is,



Force of attraction between mass 'm' and an elemental mass 'dm' of rod is

$$dF = \frac{Gmdm}{x^2} = \frac{Gm\,(A + Bx^2)\,dx}{x^2}$$

Total attraction force is sum of all such differential forces produced by elemental parts of rod from $x = a$ to $x = a + L$.

$$\therefore \; F = \int dF = \int_{x=a}^{x=a+L} \frac{Gm\,(A + Bx^2)}{x^2}dx$$

$$= Gm\int_{x=a}^{x=a+L}\left(\frac{A}{x^2} + B\right)dx$$

$$= Gm\left[-\frac{A}{x} + Bx\right]_{x=a}^{x=a+L}$$

$$= Gm\left(\frac{-A}{a+L} + B(a+L) + \frac{A}{a} - Ba\right)$$

$$= Gm\left(\frac{A}{a} - \frac{A}{a+L} + BL\right) = Gm\left\{A\left(\frac{1}{a} - \frac{1}{a+L}\right) + BL\right\}$$

4. Given, $m = 1$ kg

$|\tau| = 2.5$ N-m, $F = 1$ N and $r = 5$ m

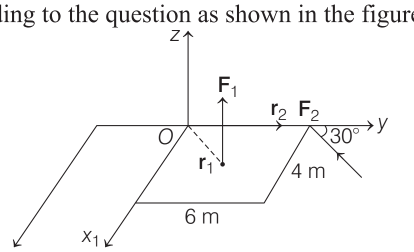
We know that, torque $|\tau| = rF\sin\theta$

$\Rightarrow \quad 2.5 = 5 \times 1 \times \sin\theta$

$\Rightarrow \quad \sin\theta = \frac{1}{2}$

or $\quad \theta = \frac{\pi}{6}$ rad

5. According to the question as shown in the figure below,



$$\mathbf{r}_1 = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} \text{ and } \mathbf{r}_2 = 6\hat{\mathbf{j}}$$

$$\mathbf{F}_1 = F\,\hat{\mathbf{k}}$$

and $\quad \mathbf{F}_2 = (-\sin 30^\circ\,\hat{\mathbf{i}} - \cos 30^\circ\,\hat{\mathbf{j}})\,F$

Moment of force is given as, $\tau = \mathbf{r} \times \mathbf{F}$

where, $\mathbf{r}$ is the perpendicular distance and $\mathbf{F}$ is the force.

$\therefore$ Moment due to $\mathbf{F}_1$

$$\tau_1 = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) \times (F\,\hat{\mathbf{k}})$$

$$= -2F\,\hat{\mathbf{j}} + 3F\,\hat{\mathbf{i}} \quad \ldots(\text{i})$$

Moment due to $\mathbf{F}_2$

$$\tau_2 = (6\hat{\mathbf{j}}) \times (-\sin 30° \hat{\mathbf{i}} - \cos 30 \hat{\mathbf{j}})\, F$$

$$= 6 \sin 30° F\, \hat{\mathbf{k}} = 3F\, \hat{\mathbf{k}} \quad \ldots\text{(ii)}$$

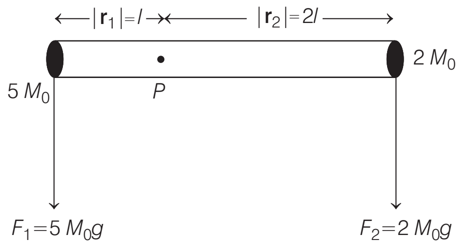
$\therefore$ Resultant torque,

$$\tau = \tau_1 + \tau_2 = 3F\, \hat{\mathbf{i}} - 2F\, \hat{\mathbf{j}} + 3F\, \hat{\mathbf{k}}$$

$$= (3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})\, F.$$

6. **Key Idea** When a rod is pivoted at any point, its angular acceleration is given by $\tau_{net} = I\alpha$.

The given condition can be drawn in the figure below



Torque (τ) about $P = \mathbf{r}_1 \times \mathbf{F}_1 + \mathbf{r}_2 \times \mathbf{F}_2$ … (i)

$\Rightarrow \quad \tau = l \times 5M_0 g$ (outwards) $- 2l \times 2M_0 g$ inwards)

$\Rightarrow \quad \tau = 5M_0 gl - 4M_0 gl$ (outwards)

$\Rightarrow \quad \tau = M_0 gl$ (outwards)

or $\quad \tau = M_0 gl$ …(ii)

Now we know that, torque is also given by

$$\tau = I\alpha \quad \ldots\text{(iii)}$$

Here, I = moment of inertia (w.r.t. point P) of rod and α = angular acceleration.

For point P, $I = (5M_0) \times l^2 + (2M_0)(2l)^2 \quad [\because I = MR^2]$

$\Rightarrow \quad I = 13M_0 l^2$ …(iv)

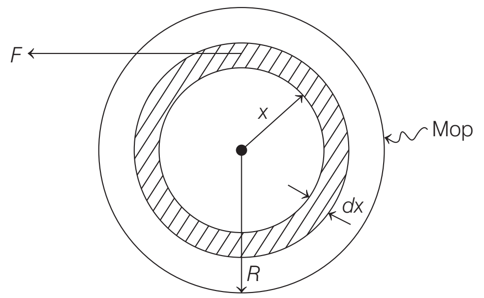
Putting value of I from Eq. (iv) in Eq. (iii), we get

$$\tau = (13M_0 l^2)\alpha \quad \ldots\text{(v)}$$

From Eqs. (ii) and (v), we get

$$M_0 gl = 13M_0 l^2 \alpha \Rightarrow \alpha = \frac{g}{13l}$$

7 Let a small strip of mop has width dx and radius x, as shown below,



Torque applied to move this strip is

$d\tau$ = Force on strip

$\times$ Perpendicular distance from the axis

$\Rightarrow d\tau$ = Force per unit area $\times$ Area of strip

$\times$ Perpendicular distance from the axis.

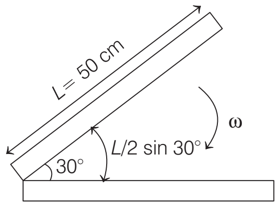
$$= \frac{\mu F}{\pi R^2} \cdot 2\pi x dx \cdot x \Rightarrow d\tau = \frac{2\mu F x^2}{R^2} \cdot dx$$

So, total torque to be applied on the mop is

$$\tau = \int_{x=0}^{x=R} d\tau = \int_0^R \frac{2\mu F x^2}{R^2} \cdot dx$$

$$= \frac{2\mu F}{R^2} \times \frac{R^3}{3} = \frac{2}{3}\mu F R \text{ (N-m)}$$

8 Lose of potential energy of rod = Gain of kinetic energy



$$\therefore \quad \Delta PE = \frac{1}{2} I\omega^2$$

(where, I = MOI of rod and ω = angular frequency of rod)

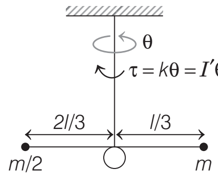
$$\Rightarrow \quad Mg \times \frac{L}{2} \sin 30° = \frac{1}{2} \times I \times \omega^2$$

$$\Rightarrow \quad Mg \frac{L}{2} \times \frac{1}{2} = \frac{1}{2} \times I \times \omega^2 \Rightarrow \frac{Mg\, L \times 2}{4 \times I} = \omega^2$$

$$\Rightarrow \quad \sqrt{\frac{MgL}{4 \times \frac{ML^2}{3}} \times \frac{2}{1}} = \omega \qquad \left[\because I = \frac{ML^2}{3} \text{ for rod}\right]$$

$$\omega = \sqrt{30} \text{ rad/s}$$

9 Since in the given question, rotational torque, $\tau \propto$ angular displacement.

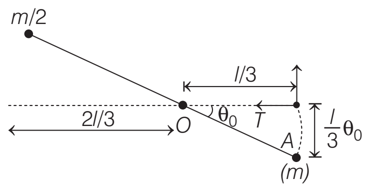


Thus, when it will be released, the system will execute SHM with a time period, $T = 2\pi \sqrt{\frac{I}{k}}$

(Where I is moment of inertia and k is torsional constant)

and the angular frequency is given as, $\omega = \sqrt{\frac{k}{I}}$.

If we know look at the top view of the above figure, we have



At some angular displacement 'θ_0', at point 'A' the maximum velocity will be

$$v_{\max} = \frac{l}{3}\theta_0\omega = \frac{l}{3}\theta_0\sqrt{\frac{k}{I}} \quad \text{...(i)}$$

Then, tension in the rod when it passes through mean position will be

$$T = \frac{m \times v_{\max}^2}{\frac{l}{3}} = \frac{ml^2\theta_0^2 k \times 3}{9 \times l \times I} \quad \text{[using Eq. (i)]}$$

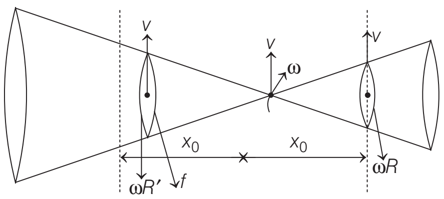
$$= \frac{ml\,\theta_0^2 k}{3I}$$

The moment of inertia I at point O,

$$= \frac{m}{2}\left(\frac{2l}{3}\right)^2 + m\left(\frac{l}{3}\right)^2 = \frac{2l^2 m}{9} + \frac{ml^2}{9} = \frac{3ml^2}{9} = \frac{ml^2}{3}$$

$$\Rightarrow \quad T = \frac{ml\,\theta_0^2\, k \times 3}{3 \times ml^2} = \frac{\theta_0^2 k}{l} = \frac{k\theta_0^2}{l}$$

10.



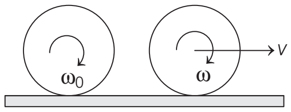
At distance x_0 from O, $v = \omega R$

Distance less than x_0, $v > \omega R$

Initially, there is pure rolling at both the contacts. As the cone moves forward, slipping at AB will start in forward direction, as radius at left contact decreases.

Thus, the cone will start turning towards left. As it moves, further slipping at CD will start in backward direction which will also turn the cone towards left.

11.



$$\omega = v/r$$

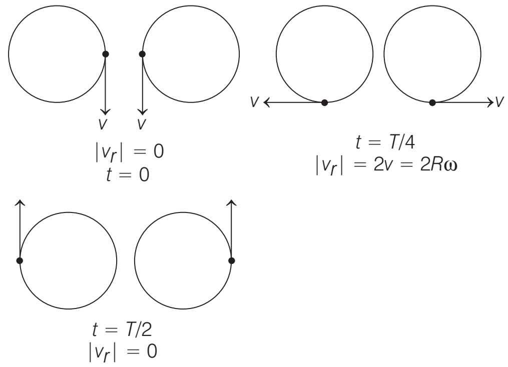
From conservation of angular momentum about bottom most point

$$mr^2\omega_0 = mvr + mr^2 \times v/r$$

$$\Rightarrow \quad v = \frac{\omega_0 r}{2}$$

12. Language of question is not very clear. For example, disc is rotating. Its different points have different velocities. Relative velocity of pebble with respect to which point, it is not clear. Further, actual initial positions of P and Q are also not given.

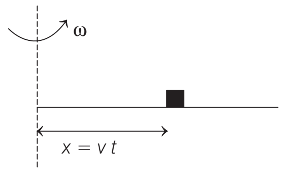
13. Language of question is wrong because relative speed is not the correct word. Relative speed between two is always zero. The correct word is magnitude of relative velocity.



Corresponding to above values, the correct graph is (a).

14. $|\mathbf{L}|$ or $L = I\omega$ (about axis of rod)

$$I = I_{\text{rod}} + mx^2 = I_{\text{rod}} + mv^2t^2$$



Here, m = mass of insect

$$\therefore \quad L = (I_{\text{rod}} + mv^2t^2)\,\omega$$

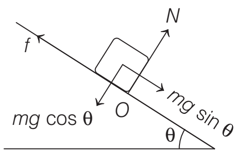
Now $|\tau| = \dfrac{dL}{dt} = (2mv^2 t\omega)$ or $|\tau| \propto t$

i.e. the graph is straight line passing through origin.

After time T, L = constant

$$\therefore \quad |\tau| \text{ or } \frac{dL}{dt} = 0$$

15. Condition of sliding is



$$mg\sin\theta > \mu mg\cos\theta$$

or $\quad \tan\theta > \mu$ or $\tan\theta > \sqrt{3}$ ...(i)

Condition of toppling is

Torque of $mg\sin\theta$ about $0 >$ torque of $mg\cos\theta$ about

$$\therefore \quad (mg\sin\theta)\left(\frac{15}{2}\right) > (mg\cos\theta)\left(\frac{10}{2}\right)$$

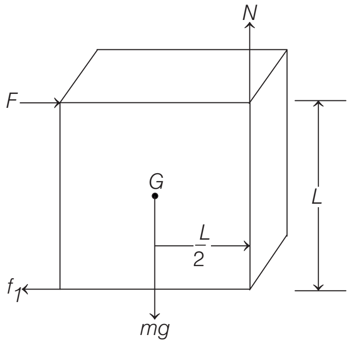
or $\quad \tan\theta > \dfrac{2}{3}$ ...(ii)

With increase in value of θ, condition of sliding is satisfied first.

16. $\mathbf{L} = m(\mathbf{r} \times \mathbf{v})$

Direction of $(\mathbf{r} \times \mathbf{v})$, hence the direction of angular momentum remains the same.

17. At the critical condition, normal reaction N will pass through point P. In this condition, $\tau_N = 0 = \tau_{fr}$ (about P) the block will topple when



$$\tau_F > \tau_{mg} \quad \text{or} \quad FL > (mg)\frac{L}{2}$$

$$\therefore \qquad F > \frac{mg}{2}$$

Therefore, the minimum force required to topple the block is

$$F = \frac{mg}{2}$$

18. Work done, $W = \frac{1}{2}I\omega^2$

If x is the distance of mass 0.3 kg from the centre of mass, we will have,

$$I = (0.3)x^2 + (0.7)(1.4 - x)^2$$

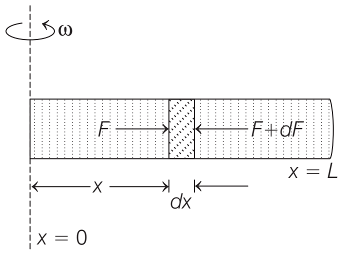
For work to be minimum, the moment of inertia (I) should be minimum, or $\frac{dI}{dx} = 0$

or $\qquad 2(0.3x) - 2(0.7)(1.4 - x) = 0$

or $\qquad (0.3)x = (0.7)(1.4 - x)$

$$\Rightarrow \qquad x = \frac{(0.7)(1.4)}{0.3 + 0.7} = 0.98\,\text{m}$$

19. Mass of the element dx is $m = \frac{M}{L}\,dx$.



This element needs centripetal force for rotation.

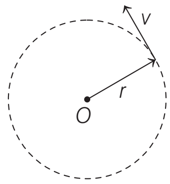
$$\therefore \qquad dF = mx\omega^2 = \left(\frac{M}{L}x\omega^2\,dx\right)$$

$$\therefore \qquad F = \int_0^L dF = \frac{M}{L}.\omega^2\int_0^L x\,dx = \frac{M\omega^2 L}{2}$$

This is the force exerted by the liquid at the other end.

20. $V = \frac{Kr^2}{2}$

$$F = -\frac{dV}{dr} = -Kr\,\text{(towards centre)}\left[F = -\frac{dV}{dr}\right]$$



$$kR = \frac{mv^2}{R}\,\text{(Centripetal force)}$$

$$v = \sqrt{\frac{kR^2}{m}} = \sqrt{\frac{k}{m}}R$$

$$L = mvR = \sqrt{mk}R^2$$

21. $\mathrm{F} = (\alpha t)\hat{\mathrm{i}} + \beta\hat{\mathrm{j}}$ $\qquad$ [at $t = 0, v = 0, \mathrm{r} = 0$]

$\alpha = 1, \beta = 1 \Rightarrow \mathrm{F} = t\hat{\mathrm{i}} + \hat{\mathrm{j}}$

$m\frac{d\mathrm{v}}{dt} = t\hat{\mathrm{i}} + \hat{\mathrm{j}}$

On integrating,

$$m\mathrm{v} = \frac{t^2}{2}\hat{\mathrm{i}} + t\hat{\mathrm{j}} \qquad [m = 1\,\text{kg}]$$

$$\frac{d\mathrm{r}}{dt} = \frac{t^2}{2}\hat{\mathrm{i}} + t\hat{\mathrm{j}} = \mathrm{v} \Rightarrow v = \frac{1}{2}(\hat{\mathbf{i}} + 2\hat{\mathbf{j}})\text{ at } t = 1\text{ s}$$

Again, on integrating,

$$\mathrm{r} = \frac{t^3}{6}\hat{\mathrm{i}} + \frac{t^2}{2}\hat{\mathrm{j}} \qquad [\mathrm{r} = 0 \text{ at } t = 0]$$

At $t = 1$ s, $\tau = (\mathrm{r} \times \mathrm{F}) = \left(\frac{1}{6}\hat{\mathrm{i}} + \frac{1}{2}\hat{\mathrm{j}}\right) \times (\hat{\mathrm{i}} + \hat{\mathrm{j}}) = -\frac{1}{3}\hat{\mathbf{k}}$

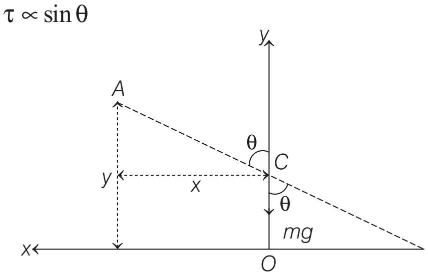
22. When the bar makes an angle θ, the height of its COM (mid-point) is $\frac{L}{2}\cos\theta$.

$\therefore$ Displacement $= L - \frac{L}{2}\cos\theta = \frac{L}{2}(1 - \cos\theta)$

Since, force on COM is only along the vertical direction, hence COM is falling vertically downward. Instantaneous torque about point of contact is

$$\tau = mg \times \frac{L}{2}\sin\theta$$

or $\qquad \tau \propto \sin\theta$



Now, $x = \frac{L}{2}\sin\theta$

$y = L\cos\theta$

$$\frac{x^2}{(L/2)^2} + \frac{y^2}{L^2} = 1$$

Path of A is an ellipse.

23. When force $F = 0 \Rightarrow$ potential energy U = constant

$F \neq 0 \Rightarrow$ force is conservative $\Rightarrow$ Total energy E = constant

List-I

(P) $\mathbf{r}(t) = \alpha t\hat{\mathbf{i}} + \beta t\hat{\mathbf{j}}$

$$\frac{d\mathbf{r}}{dt} = \mathbf{v} = \alpha\hat{\mathbf{i}} + \beta\hat{\mathbf{j}} = \text{constant} \Rightarrow \mathbf{p} = \text{constant}$$

$$|\mathbf{v}| = \sqrt{\alpha^2 + \beta^2} = \text{constant} \Rightarrow K = \text{constant}$$

$$\frac{d\mathbf{v}}{dt} = \mathbf{a} = 0 \Rightarrow F = 0 \Rightarrow U = \text{constant}$$

$E = U + K$ = constant

$\mathbf{L} = m(\mathbf{r} \times \mathbf{v}) = 0$

$\mathbf{L}$ = constant

$P \to 1, 2, 3, 4, 5$

(Q) $\mathbf{r}(t) = \alpha\cos\omega t\hat{\mathbf{i}} + \beta\sin\omega t\hat{\mathbf{j}}$

$$\frac{d\mathbf{r}}{dt} = \mathbf{v} = \alpha\omega\sin\omega t(-\hat{\mathbf{i}}) + \beta\omega\cos\omega t\hat{\mathbf{j}} \neq \text{constant}$$

$\Rightarrow$ $\mathbf{p} \neq$ constant

$$|\mathbf{v}| = \omega\sqrt{(\alpha\sin\omega t)^2 + (\beta\cos\omega t)^2} \neq \text{constant}$$

$\Rightarrow$ $K \neq$ constant

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = -\omega^2\mathbf{r} \neq 0$$

$\Rightarrow$ E = constant $= K + U$

But $K \neq$ constant $\Rightarrow U \neq$ constant

$\mathbf{L} = m(\mathbf{r} \times \mathbf{v}) = m\omega\alpha\beta\,(\hat{\mathbf{k}})$ = constant

$Q \to 2, 5$

(R) $\mathbf{r}(t) = \alpha\,(\cos\omega t\hat{\mathbf{i}} + \sin\omega t\hat{\mathbf{j}})$

$$\frac{d\mathbf{r}}{dt} = \mathbf{v} = \alpha\omega\,[\sin\omega\,t(-\hat{\mathbf{i}}) + \cos\omega t\hat{\mathbf{j}}] \neq \text{constant}$$

$\Rightarrow \mathbf{p} \neq$ constant

$|\mathbf{v}| = \alpha\omega$ = constant $\Rightarrow K$ = constant

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = -\omega^2\mathbf{r} \neq 0 \Rightarrow E = \text{constant}, U = \text{constant}$$

$\mathbf{L} = m(\mathbf{r} \times \mathbf{v}) = m\omega\alpha^2\,\hat{\mathbf{k}}$ = constant

$R \to 2, 3, 4, 5$

(S) $\mathbf{r}(t) = \alpha t\hat{\mathbf{i}} + \frac{\beta}{2}t^2\hat{\mathbf{j}}$

$$\frac{d\mathbf{r}}{dt} = \mathbf{v} = \alpha\hat{\mathbf{i}} + \beta t\hat{\mathbf{j}} \neq \text{constant} \Rightarrow \mathbf{p} \neq \text{constant}$$

$$|\mathbf{v}| = \sqrt{\alpha^2 + (\beta t)^2} \neq \text{constant} \Rightarrow K \neq \text{constant}$$

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \beta\hat{\mathbf{j}} \neq 0 \Rightarrow E = \text{constant} = K + U$$

But $K \neq$ constant

$\therefore$ $U \neq$ constant

$$\mathbf{L} = m\,(\mathbf{r} \times \mathbf{v}) = \frac{1}{2}\alpha\beta t^2\,\hat{\mathbf{k}} \neq \text{constant}$$

$S \to 5$

24. Question is not very clear.

25. If height of the cone $h >> r$

Then, $\mu N = mg$

$\mu m(R - r)\,\omega_0^2 = mg$

$$\omega_0 = \sqrt{\frac{g}{\mu(R - r)}}$$

26-27. (i) Every particle of the disc is rotating in a horizontal circle.

(ii) Actual velocity of any particle is horizontal.

(iii) Magnitude of velocity of any particle is

$$v = r\omega$$

where, r is the perpendicular distance of that particle from actual axis of rotation (Z-axis).

(iv) When it is broken into two parts then actual velocity of any particle is resultant of two velocities

$$v_1 = r_1\omega_1 \text{ and } v_2 = r_2\omega_2$$

Here,

r_1 = perpendicular distance of centre of mass from Z-axis.

ω_1 = angular speed of rotation of centre of mass from Z-axis.

r_2 = distance of particle from centre of mass and

ω_2 = angular speed of rotation of the disc about the axis passing through centre of mass.

(v) Net v will be horizontal, if v_1 and v_2 both are horizontal. Further, v_1 is already horizontal, because centre of mass is rotating about a vertical Z-axis. To make v_2 also horizontal, second axis should also be vertical.

28. $\frac{1}{2}I\,(2\omega)^2 = \frac{1}{2}kx_1^2$ ...(i)

$\frac{1}{2}(2I)\,(\omega)^2 = \frac{1}{2}kx_2^2$ ...(ii)

From Eqs. (i) and (ii), we have

$$\frac{x_1}{x_2} = \sqrt{2}$$

29. Let ω' be the common velocity. Then from conservation of angular momentum, we have

$$(I + 2I)\omega' = I(2\omega) + 2I(\omega)$$

$$\omega' = \frac{4}{3}\omega$$

From the equation,

angular impulse = change in angular momentum, for any of the disc, we have

$$\tau \cdot t = I\,(2\omega) - I\left(\frac{4}{3}\omega\right) = \frac{2I\omega}{3}$$

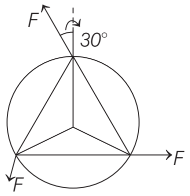
$$\therefore \quad \tau = \frac{2I\omega}{3t}$$

30. Loss of kinetic energy $= K_i - K_f$

$$= \left\{\frac{1}{2} I\,(2\omega)^2 + \frac{1}{2}(2I)(\omega)^2\right\} - \frac{1}{2}(3I)\left(\frac{4}{3}\omega\right)^2 = \frac{1}{3} I\omega^2$$

31. Angular impulse = change in angular momentum

$$\therefore \int \tau\, dt = I\omega$$



$$\Rightarrow \quad \omega = \frac{\int \tau\, dt}{I} = \frac{\int_0^t 3F \sin 30^\circ\, R\, dt}{I}$$

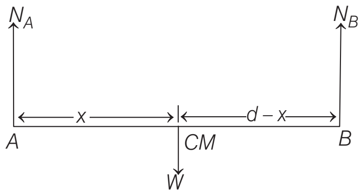
Substituting the values, we have

$$\omega = \frac{3\,(0.5)\,(0.5)\,(0.5)\,(1)}{\frac{1.5\,(0.5)^2}{2}} = 2 \text{ rad/s}$$

32. Net torque of all the forces about B should be zero.

$$\therefore \quad W\,(d - x) = N_A \cdot d$$

or
$$N_A = \left(\frac{d - x}{d}\right) W$$



For vertical equilibrium of rod

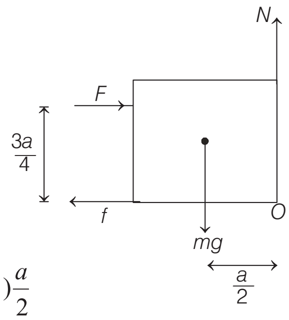
$$N_A + N_B = W$$

$$\therefore \quad N_B = \frac{x}{d} W = W - N_A$$

$$= W - \left(\frac{d - x}{d}\right) W = \frac{x}{d} W$$

33. Taking moments about point O

Moments of N (normal reaction) and f (force of friction) are zero. In critical case normal reaction will pass through O. To tip about the edge, moment of F should be greater than moment of mg. Or,



$$F\left(\frac{3a}{4}\right) > (mg)\frac{a}{2}$$

or
$$F > \frac{2}{3} mg$$

34. $\alpha = \dfrac{\tau}{I}$

$\tau = F \times r_\perp$: Torque is same in both the cases but moment of inertia depends on distribution of mass from the axis. Distribution of mass in both the cases is different. Therefore, moment of inertia will be different or the angular acceleration α will be different.

35. Angular momentum of the system about an axis perpendicular to plane of paper and passing through O will remain conserved.

$$L_i = L_f$$

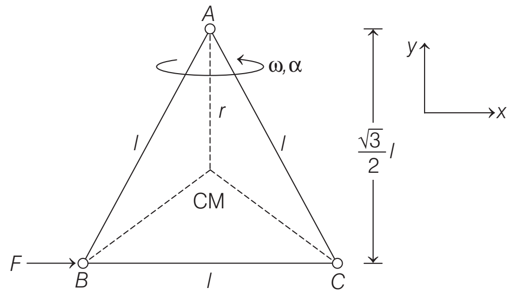
$$\therefore \quad mvL = I\omega = \left(mL^2 + \frac{ML^2}{3}\right)\omega$$

$$\therefore \quad \omega = \frac{3mv}{L\,(3m + M)}$$

36. (a) The distance of centre of mass (CM) of the system about point A will be $r = \dfrac{l}{\sqrt{3}}$

Therefore, the magnitude of horizontal force exerted by the hinge on the body is

F = centripetal force or $F = (3m)\, r\omega^2$



or
$$F = (3m)\left(\frac{l}{\sqrt{3}}\right)\omega^2$$

or
$$F = \sqrt{3}\, ml\omega^2$$

(b) Angular acceleration of system about point A is

$$\alpha = \frac{\tau_A}{I_A} = \frac{(F)\left(\frac{\sqrt{3}}{2} l\right)}{2ml^2} = \frac{\sqrt{3}\,F}{4ml}$$

Now, acceleration of CM along x-axis is

$$a_x = r\alpha = \left(\frac{l}{\sqrt{3}}\right)\left(\frac{\sqrt{3}\,F}{4ml}\right) \quad \text{or} \quad a_x = \frac{F}{4m}$$

Let F_x be the force applied by the hinge along X-axis.

Then, $\quad F_x + F = (3m) a_x$

or
$$F_x + F = (3m)\left(\frac{F}{4m}\right)$$

or
$$F_x + F = \frac{3}{4} F$$

or
$$F_x = -\frac{F}{4}$$

Further if F_y be the force applied by the hinge along Y-axis. Then,

F_y = centripetal force

or
$$F_y = \sqrt{3}\, ml\omega^2$$

37. Let r be the perpendicular distance of CM from the line AB and ω the angular velocity of the sheet just after colliding with rubber obstacle for the first time.

Obviously the linear velocity of CM before and after collision will be $v_i = (r)(1 \text{ rad/s}) = r$ and $v_f = r\omega$.

$\mathbf{v}_i$ and $\mathbf{v}_f$ will be in opposite directions.

Now, linear impulse on CM

= change in linear momentum of CM

or $\quad 6 = m(v_f + v_i) = 30(r + r\omega)$

or $\quad r(1+\omega) = \dfrac{1}{5}$ …(i)

Similarly, angular impulse about AB = change in angular momentum about AB

Angular impulse = Linear impulse

× perpendicular distance of impulse from AB

Hence, $\quad 6(0.5 \text{ m}) = I_{AB}(\omega + 1)$

(Initial angular velocity = 1 rad/s)

or $\quad 3 = [I_{CM} + Mr^2](1+\omega)$

or $\quad 3 = [1.2 + 30r^2](1+\omega)$ …(ii)

Solving Eqs. (i) and (ii) for r, we get

$r = 0.4$ m and $r = 0.1$ m

But at $r = 0.4$ m, ω comes out to be negative (-0.5 rad/s) which is not acceptable. Therefore,

(a) r = distance of CM from $AB = 0.1$ m

(b) Substituting $r = 0.1$ m in Eq. (i), we get $\omega = 1$ rad/s *ie,* the angular velocity with which sheet comes back after the first impact is 1 rad/s.

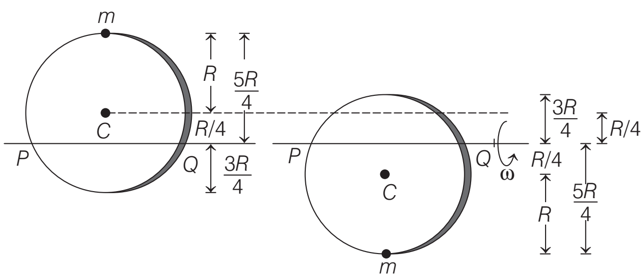
(c) Since, the sheet returns with same angular velocity of 1 rad/s, the sheet will never come to rest.

38. Initial and final positions are shown below.

Decrease in potential energy of mass

$$= mg\left\{2 \times \frac{5R}{4}\right\} = \frac{5\,mgR}{2}$$

Decrease in potential energy of disc $= mg\left\{2 \times \dfrac{R}{4}\right\} = \dfrac{mgR}{2}$



Therefore, total decrease in potential energy of system

$$= \frac{5mgR}{2} + \frac{mgR}{2} = 3mgR$$

Gain in kinetic energy of system $= \dfrac{1}{2} I\omega^2$

where, I = moment of inertia of system (disc + mass) about axis PQ

= moment of inertia of disc

+ moment of inertia of mass

$$= \left\{\frac{mR^2}{4} + m\left(\frac{R}{4}\right)^2\right\} + m\left(\frac{5R}{4}\right)^2$$

$$I = \frac{15\,mR^2}{8}$$

From conservation of mechanical energy,

Decrease in potential energy = Gain in kinetic energy

$$\therefore \quad 3\,mgR = \frac{1}{2}\left(\frac{15\,mR^2}{8}\right)\omega^2$$

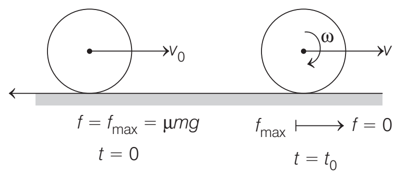
$$\Rightarrow \quad \omega = \sqrt{\frac{16g}{5R}}$$

Therefore, linear speed of particle at its lowest point

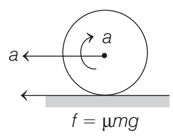
$$v = \left(\frac{5R}{4}\right)\omega = \frac{5R}{4}\sqrt{\frac{16g}{5R}}$$

$$v = \sqrt{5gR}$$

39. (a) Between the time $t = 0$ to $t = t_0$. There is forward sliding, so friction f is leftwards and maximum i.e. μmg. For time $t > t_0$, friction f will become zero, because now pure rolling has started i.e. there is no sliding (no relative motion) between the points of contact.



So, for time $t < t_0$



Linear retardation, $a = \dfrac{f}{m} = \mu g$

and angular acceleration, $\alpha = \dfrac{\tau}{I} = \dfrac{fR}{\frac{1}{2}mR^2} = \dfrac{2\mu g}{R}$

Now, let v be the linear velocity and ω, the angular velocity of the disc at time $t = t_0$, then

$$v = v_0 - at_0 = v_0 - \mu g t_0 \quad \text{…(i)}$$

and $\quad \omega = \alpha\, t_0 = \dfrac{2\mu g t_0}{R}$ …(ii)

For pure rolling to take place

$$v = R\omega$$

i.e. $\quad v_0 - \mu g t_0 = 2\mu g t_0 \Rightarrow t_0 = \dfrac{v_0}{3\mu g}$

Substituting in Eq. (i), we have

$$v = v_0 - \mu g\left(\frac{v_0}{3\mu g}\right) \Rightarrow v = \frac{2}{3}v_0$$

(b) **Work done by friction**

For $t \le t_0$, linear velocity of disc at any time t is $v = v_0 - \mu g t$ and angular velocity is $\omega = \alpha t = \dfrac{2\mu g t}{R}$. From Work-energy theorem, work done by friction upto time t = Kinetic energy of the disc at time t – Kinetic energy of the disc at time $t = 0$

$$\therefore \; W = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 - \frac{1}{2}mv_0^2$$

$$= \frac{1}{2}m[v_0 - \mu g t]^2 + \frac{1}{2}\left(\frac{1}{2}mR^2\right)\left(\frac{2\mu g t}{R}\right)^2 - \frac{1}{2}mv_0^2$$

$$= \frac{1}{2}[mv_0^2 + m\mu^2 g^2 t^2 - 2mv_0\mu g t + 2m\mu^2 g^2 t^2 - mv_0^2]$$

$$\text{or } W = \frac{m\mu g t}{2}[3\mu g t - 2v_0]$$

For $t > t_0$, friction force is zero i.e. work done in friction is zero. Hence, the energy will be conserved.

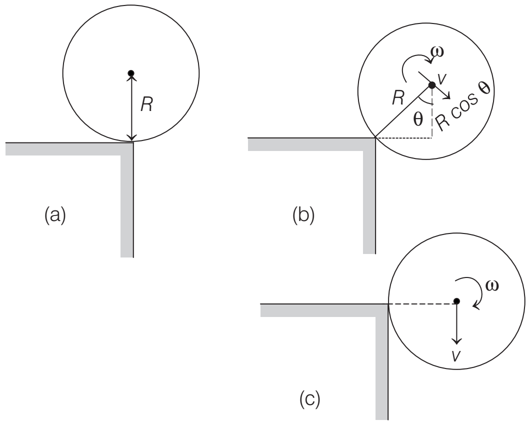
Therefore, total work done by friction over a time t much longer then t_0 is total work done upto time t_0 (because beyond this work done by friction is zero) which is equal to

$$W = \frac{m\mu g t_0}{2}[3\mu g t_0 - 2v_0]$$

Substituting $t_0 = v_0/3\mu g$, we get

$$W = \frac{mv_0}{6}[v_0 - 2v_0] \Rightarrow W = -\frac{mv_0^2}{6}$$

40. (a) The cylinder rotates about the point of contact. Hence, the mechanical energy of the cylinder will be conserved i.e.



$$\therefore \quad (\text{PE} + \text{KE})_1 = (\text{PE} + \text{KE})_2$$

$$\therefore \quad mgR + 0 = mgR\cos\theta + \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$$

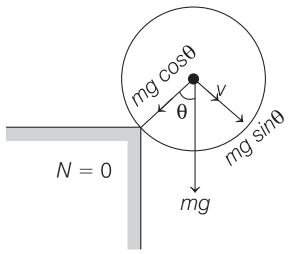
but $\omega = v/R$ (no slipping at point of contact)

and $\quad I = \dfrac{1}{2}mR^2$

Therefore,

$$mgR = mgR\cos\theta + \frac{1}{2}\left(\frac{1}{2}mR^2\right)\left(\frac{v^2}{R^2}\right) + \frac{1}{2}mv^2$$

$$\text{or } \frac{3}{4}v^2 = gR(1 - \cos\theta)$$



$$\text{or} \quad v^2 = \frac{4}{3}gR(1 - \cos\theta)$$

$$\text{or} \quad \frac{v^2}{R} = \frac{4}{3}g(1 - \cos\theta) \qquad \ldots\text{(i)}$$

At the time of leaving contact, normal reaction $N = 0$ and $\theta = \theta_c$, hence,

$$mg\cos\theta = \frac{mv^2}{R} \text{ or } \frac{v^2}{R} = g\cos\theta \qquad \ldots\text{(ii)}$$

From Eqs. (i) and (ii),

$$\frac{4}{3}g(1 - \cos\theta_c) = g\cos\theta_c$$

$$\text{or } \frac{7}{4}\cos\theta_c = 1 \text{ or } \cos\theta_c = 4/7 \text{ or } \theta_c = \cos^{-1}(4/7)$$

(b) $v = \sqrt{\dfrac{4}{3}gR(1 - \cos\theta)}$

At the time of losing contact

$\cos\theta = \cos\theta_c = 4/7$

$$\therefore \quad v = \sqrt{\frac{4}{3}gR\left(1 - \frac{4}{7}\right)} \quad \text{or} \quad v = \sqrt{\frac{4}{7}gR}$$

Therefore, speed of CM of cylinder just before losing contact is $\sqrt{\dfrac{4}{7}gR}$

(c) At the moment, when cylinder loses contact

$$v = \sqrt{\frac{4}{7}gR}$$

Therefore, rotational kinetic energy, $K_R = \dfrac{1}{2}I\omega^2$

$$\text{or} \quad K_R = \frac{1}{2}\left(\frac{1}{2}mR^2\right)\frac{v^2}{R^2} = \frac{1}{4}mv^2 = \frac{1}{4}m\left(\frac{4}{7}gR\right)$$

$$\text{or} \quad K_R = \frac{mgR}{7}$$

Now, once the cylinder loses its contact, $N = 0$, i.e the frictional force, which is responsible for its rotation, also vanishes. Hence, its rotational kinetic energy now becomes constant, while its translational kinetic energy increases. Applying conservation of energy at (a) and (c).

Decrease in gravitational PE

= Gain in rotational KE + translational KE

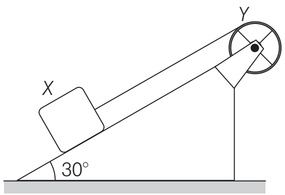
$\therefore$ Translational KE (K_T)

= Decrease in gravitational PE $- K_R$

or $$K_T = (mgR) - \frac{mgR}{7} = \frac{6}{7}mgR$$

$\therefore$ $$\frac{K_T}{K_R} = \frac{\frac{6}{7}mgR}{\frac{mgR}{7}} \text{ or } \frac{K_T}{K_R} = 6$$

41. Given, mass of block X, $m = 0.5$kg



Mass of drum Y, $M = 2$kg

Radius of drum, $R = 0.2$ m

Angle of inclined plane, $\theta = 30°$

(a) Let a be the linear retardation of block X and α be the angular retardation of drum Y. Then, $a = R\alpha$

$$mg \sin 30° - T = ma \quad \text{...(i)}$$

or $$\frac{mg}{2} - T = ma \quad \text{...(ii)}$$

$$\alpha = \frac{\tau}{I} = \frac{TR}{\frac{1}{2}MR^2}$$

or $$\alpha = \frac{2T}{MR} \quad \text{...(iii)}$$

Solving Eqs. (i), (ii) and (iii) for T, we get,

$$T = \frac{1}{2}\frac{M\,mg}{M + 2m}$$

Substituting the value, we get

$$T = \left(\frac{1}{2}\right)\left\{\frac{(2)(0.5)(9.8)}{2 + (0.5)(2)}\right\} = 1.63 \text{ N}$$

(b) From Eq. (iii), angular retardation of drum

$$\alpha = \frac{2T}{MR} = \frac{(2)(1.63)}{(2)(0.2)} = 8.15 \text{ rad/s}^2$$

or linear retardation of block

$$a = R\alpha = (0.2)(8.15) = 1.63 \text{ m/s}^2$$

At the moment when angular velocity of drum is

$$\omega_0 = 10 \text{ rad/s}$$

The linear velocity of block will be

$$v_0 = \omega_0 R = (10)(0.2) = 2 \text{m/s}$$

Now, the distance (s) travelled by the block until it comes to rest will be given by

$$s = \frac{v_0^2}{2a} \qquad (\text{Using } v^2 = v_0^2 - 2as \text{ with } v = 0)$$

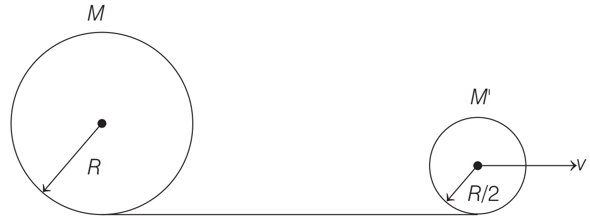
$$= \frac{(2)^2}{2(1.63)} \text{ or } s = 1.22 \text{ m}$$

42. Let M' be the mass of unwound carpet. Then,

$$M' = \left(\frac{M}{\pi R^2}\right)\pi\left(\frac{R}{2}\right)^2 = \frac{M}{4}$$

From conservation of mechanical energy :

$$MgR - M'g\frac{R}{2} = \frac{1}{2}\left(\frac{M}{4}\right)v^2 + \frac{1}{2}I\omega^2$$



or $$MgR - \left(\frac{M}{4}\right)g\left(\frac{R}{2}\right) = \frac{Mv^2}{8} + \frac{1}{2}\left(\frac{1}{2} \times \frac{M}{4} \times \frac{R^2}{4}\right)\left(\frac{v}{R/2}\right)^2$$

or $$\frac{7}{8}MgR = \frac{3Mv^2}{16}$$

$\therefore$ $$v = \sqrt{\frac{14Rg}{3}}$$