9

Properties of Matter

Topic 1 Elasticity

Objective Questions I (Only one correct option)

1. In an experiment, brass and steel wires of length 1 m each with areas of cross-section 1 mm^2 are used. The wires are connected in series and one end of the combined wire is connected to a rigid support and other end is subjected to elongation. The stress requires to produce a net elongation of 0.2 mm is

[Take, the Young's modulus for steel and brass are respectively 120×10^9 N/ m^2 and 60×10^9 N/ m^2] **(2019 Main, 10 April II)**

(a) 1.2×10^6 N/ m^2 (b) 0.2×10^6 N/ m^2

(c) 1.8×10^6 N/ m^2 (d) 4.0×10^6 N/ m^2

2. The elastic limit of brass is 379 MPa. What should be the minimum diameter of a brass rod, if it is to support a 400 N load without exceeding its elastic limit? **(2019 Main, 10 April II)**

(a) 0.90 mm (b) 1.00 mm

(c) 1.16 mm (d) 1.36 mm

3. Young's moduli of two wires A and B are in the ratio 7 : 4. Wire A is 2 m long and has radius R. Wire B is 1.5 m long and has radius 2 mm. If the two wires stretch by the same length for a given load, then the value of R is close to **(2019 Main, 8 April II)**

(a) 1.3 mm (b) 1.5 mm

(c) 1.9 mm (d) 1.7 mm

4. A steel wire having a radius of 2.0 mm, carrying a load of 4 kg, is hanging from a ceiling. Given that $g = 3.1\pi$ ms^{-2}, what will be the tensile stress that would be developed in the wire? **(2019 Main, 8 April I)**

(a) $6.2 \times 10^6 Nm^{-2}$ (b) $5.2 \times 10^6 Nm^{-2}$

(c) $3.1 \times 10^6 Nm^{-2}$ (d) $4.8 \times 10^6 Nm^{-2}$

5. A boy's catapult is made of rubber cord which is 42 cm long, with 6 mm diameter of cross-section and of negligible mass. The boy keeps a stone weighing 0.02 kg on it and stretches the cord by 20 cm by applying a constant force. When released the stone flies off with a velocity of 20 ms^{-1}. Neglect the change in the area of cross-section of the cord while stretched. The Young's modulus of rubber is closest to **(Main 2019, 8 April I)**

(a) $10^6 Nm^{-2}$ (b) $10^4 Nm^{-2}$

(c) $10^8 Nm^{-2}$ (d) $10^3 Nm^{-2}$

6. A load of mass M kg is suspended from a steel wire of length 2 m and radius 1.0 mm in Searle's apparatus experiment. The increase in length produced in the wire is 4.0 mm. Now, the load is fully immersed in a liquid of relative density 2. The relative density of the material of load is 8. The new value of increase in length of the steel wire is **(2019 Main, 12 Jan II)**

(a) zero (b) 5.0 mm

(c) 4.0 mm (d) 3.0 mm

7. A rod of length L at room temperature and uniform area of cross-section A, is made of a metal having coefficient of linear expansion α /°C. It is observed that an external compressive force F, is applied on each of its ends, prevents any change in the length of the rod, when its temperature rises by ΔT K. Young's modulus, Y for this metal is **(2019 Main, 9 Jan I)**

(a) $\dfrac{F}{2A\alpha\ \Delta T}$ (b) $\dfrac{F}{A\alpha(\Delta T - 273)}$

(c) $\dfrac{2F}{A\alpha\Delta T}$ (d) $\dfrac{F}{A\alpha\Delta T}$

8. A solid sphere of radius r made of a soft material of bulk modulus K is surrounded by a liquid in a cylindrical container. A massless piston of area a floats on the surface of the liquid, covering entire cross-section of cylindrical container. When a mass m is placed on the surface of the piston to compress the liquid, the fractional decrement in the radius of the sphere, $\left(\dfrac{dr}{r}\right)$ is **(2018 Main)**

(a) $\dfrac{mg}{Ka}$ (b) $\dfrac{Ka}{mg}$

(c) $\dfrac{Ka}{3mg}$ (d) $\dfrac{mg}{3Ka}$

9. A pendulum made of a uniform wire of cross-sectional area A has time period T. When an additional mass M is added to its bob, the time period changes to T_M. If the Young's modulus of the material of the wire is Y, then $\dfrac{1}{Y}$ is equal to (g = gravitational acceleration) **(2015 Main)**

(a) $\left[1-\left(\frac{T}{T_M}\right)^2\right]\frac{A}{Mg}$ (b) $\left[\left(\frac{T_M}{T}\right)^2-1\right]\frac{Mg}{A}$

(c) $\left[1-\left(\frac{T_M}{T}\right)^2\right]\frac{A}{Mg}$ (d) $\left[\left(\frac{T_M}{T}\right)^2-1\right]\frac{A}{Mg}$

10. One end of a horizontal thick copper wire of length $2L$ and radius $2R$ is welded to an end of another horizontal thin copper wire of length L and radius R. When the arrangement is stretched by applying forces at two ends, the ratio of the elongation in the thin wire to that in the thick wire is **(2013 Adv.)**

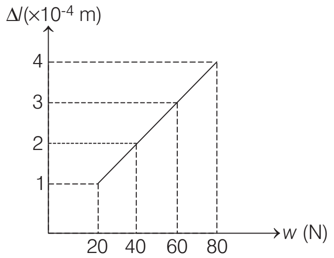
(a) 0.25 (b) 0.50 (c) 2.00 (d) 4.00

11. The pressure of a medium is changed from 1.01×10^5 Pa to 1.165×10^5 Pa and change in volume is 10% keeping temperature constant. The bulk modulus of the medium is **(2005, 2M)**

(a) 204.8×10^5 Pa (b) 102.4×10^5 Pa

(c) 51.2×10^5 Pa (d) 1.55×10^5 Pa

12. The adjacent graph shows the extension (Δl) of a wire of length 1 m suspended from the top of a roof at one end and with a load w connected to the other end. If the cross-sectional area of the wire is 10^{-6} m^2, calculate from the graph the Young's modulus of the material of the wire. **(2003, 2M)**



(a) 2×10^{11} N/m^2 (b) 2×10^{-11} N/m^2

(c) 3×10^{12} N/m^2 (d) 2×10^{13} N/m^2

13. A given quantity of an ideal gas is at pressure p and absolute temperature T. The isothermal bulk modulus of the gas is **(1998, 2M)**

(a) $\frac{2}{3}p$ (b) p (c) $\frac{3}{2}p$ (d) $2p$

14. Two rods of different materials having coefficients of thermal expansion α_1, α_2 and Young's moduli Y_1, Y_2 respectively are fixed between two rigid massive walls. The rods are heated such that they undergo the same increase in temperature. There is no bending of the rods. If $\alpha_1 : \alpha_2 = 2 : 3$, the thermal stresses developed in the two rods are equal provided $Y_1 : Y_2$ is equal to **(1989, 2M)**

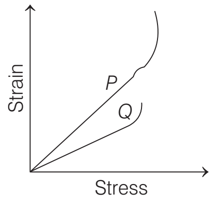
(a) 2 : 3 (b) 1 : 1 (c) 3 : 2 (d) 4 : 9

15. The following four wires are made of the same material. Which of these will have the largest extension when the same tension is applied ? **(1981, 2M)**

(a) Length = 50 cm, diameter = 0.5 mm
(b) Length = 100 cm, diameter = 1 mm
(c) Length = 200 cm, diameter = 2 mm
(d) Length = 300 cm, diameter = 3 mm

Objective Question II (One or more correct option)

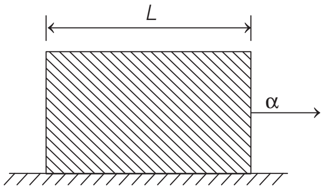
16. In plotting stress *versus* strain curves for two materials P and Q, a student by mistake puts strain on the Y-axis and stress on the X-axis as shown in the figure. Then the correct statements is/are **(2015 Adv.)**



(a) P has more tensile strength than Q
(b) P is more ductile than Q
(c) P is more brittle than Q
(d) The Young's modulus of P is more than that of Q

Fill in the Blanks

17. A uniform rod of length L and density ρ is being pulled along a smooth floor with a horizontal acceleration α (see figure). The magnitude of the stress at the transverse cross-section through the mid-point of the rod is ……… . **(1993, 1M)**



18. A solid sphere of radius R made of a material of bulk modulus k is surrounded by a liquid in a cylindrical container. A massless piston of area A floats on the surface of the liquid. When a mass M is placed on the piston to compress the liquid the fractional change in the radius of the sphere, $\delta R/R$, is …… . **(1988, 2M)**

19. A wire of length L and cross-sectional area A is made of a material of Young's modulus Y. If the wire is stretched by an amount x, the work done is …… . **(1987, 2M)**

Analytical & Descriptive Questions

20. In Searle's experiment, which is used to find Young's modulus of elasticity, the diameter of experimental wire is $D = 0.05$ cm (measured by a scale of least count 0.001 cm) and length is $L = 110$ cm (measured by a scale of least count 0.1 cm). A weight of 50 N causes an extension of $l = 0.125$ cm (measured by a micrometer of least count 0.001 cm). Find maximum possible error in the values of Young's modulus. Screw gauge and meter scale are free from error. **(2004, 2M)**

21. A thin rod of negligible mass and area of cross-section 4×10^{-6} m^2, suspended vertically from one end, has a length of 0.5 m at 100°C. The rod is cooled to 0°C, but prevented from contracting by attaching a mass at the lower end. Find

(a) this mass and

(b) the energy stored in the rod, given for the rod. Young's modulus $= 10^{11}$ N/m^2, coefficient of linear expansion $= 10^{-5}$ K^{-1} and $g = 10$ m/s^2. **(1997 C, 5M)**

Topic 2 Ideal Fluids at Rest

Objective Questions I (Only one correct option)

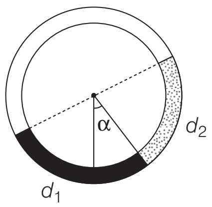
1. A submarine experience a pressure of 5.05×10^6 Pa at a depth of d_1 in a sea. When it goes further to a depth of d_2, it experiences a pressure of 8.08×10^6 Pa, then $d_2 - d_1$ is approximately (density of water $= 10^3$ kg/ m^3 and acceleration due to gravity = 10 ms^{-2}) **(2019 Main, 10 Apr II)**

(a) 500 m (b) 400 m (c) 600 m (d) 300 m

2. A wooden block floating in a bucket of water has $\frac{4}{5}$ of its volume submerged. When certain amount of an oil is poured into the bucket, it is found that the block is just under the oil surface with half of its volume under water and half in oil. The density of oil relative to that of water is **(2019 Main, 9 April II)**

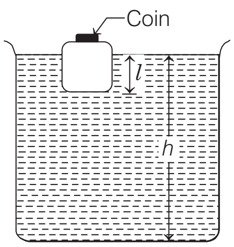
(a) 0.6 (b) 0.8 (c) 0.7 (d) 0.5

3. There is a circular tube in a vertical plane. Two liquids which do not mix and of densities d_1 and d_2 are filled in the tube. Each liquid subtends 90° angle at centre. Radius joining their interface makes an angle α with vertical. Ratio d_1 / d_2 is **(2014 Main)**



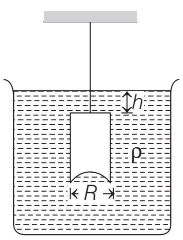
(a) $\frac{1+\sin\alpha}{1-\sin\alpha}$ (b) $\frac{1+\cos\alpha}{1-\cos\alpha}$ (c) $\frac{1+\tan\alpha}{1-\tan\alpha}$ (d) $\frac{1+\sin\alpha}{1-\cos\alpha}$

4. A wooden block, with a coin placed on its top, floats in water as shown in figure. The distance l and h are shown there. After sometime the coin falls into the water. Then **(2002,2M)**



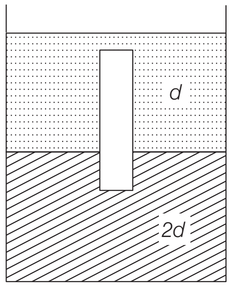
(a) l decreases and h increases
(b) l increases and h decreases
(c) Both l and h increase
(d) Both l and h decrease

5. A hemispherical portion of radius R is removed from the bottom of a cylinder of radius R. The volume of the remaining cylinder is V and mass M. It is suspended by a string in a liquid of density ρ, where it stays vertical. The upper surface of the cylinder is at a depth h below the liquid surface. The force on the bottom of the cylinder by the liquid is **(2001, 2M)**



(a) Mg (b) $Mg - V\rho g$
(c) $Mg + \pi R^2 h\rho g$ (d) $\rho g\,(V + \pi R^2 h)$

6. A homogeneous solid cylinder of length L and cross-sectional area $A/5$ is immersed such that it floats with its axis vertical at the liquid-liquid interface with length $L/4$ in the denser liquid as shown in the figure. The lower density liquid is open to atmosphere having pressure p_0.



Then, density D of solid is given by **(1995, 2M)**

(a) $\frac{5}{4}d$ (b) $\frac{4}{5}d$ (c) $4d$ (d) $\frac{d}{5}$

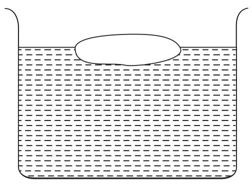
7. A vessel contains oil (density = 0.8 g/cm^3) over mercury (density = 13.6 g/cm^3). A homogeneous sphere floats with half its volume immersed in mercury and the other half in oil. The density of the material of the sphere in g/cm^3 is **(1988, 2M)**

(a) 3.3 (b) 6.4 (c) 7.2 (d) 12. 8

8. A U-tube of uniform cross-section is partially filled with a liquid I. Another liquid II which does not mix with liquid I is poured into one side. It is found that the liquid levels of the two sides of the tube are the same, while the level of liquid I has risen by 2 cm. If the specific gravity of liquid I is 1.1, the specific gravity of liquid II must be **(1983, 1M)**

(a) 1.12 (b) 1.1 (c) 1.05 (d) 1.0

9. A body floats in a liquid contained in a beaker. The whole system as shown in figure falls freely under gravity. The upthrust on the body is **(1982, 3M)**



(a) zero
(b) equal to the weight of liquid displaced
(c) equal to the weight of the body in air
(d) equal to the weight of the immersed portion of the body

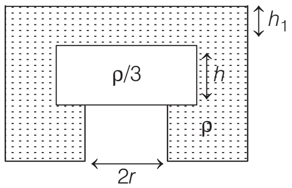
10. A metal ball immersed in alcohol weighs w_1 at 0°C and w_2 at 50°C. The coefficient of cubical expansion of the metal is less than that of the alcohol. Assuming that the density of the metal is large compared to that of alcohol, it can be shown that **(1980, 2M)**

(a) $w_1 > w_2$ (b) $w_1 = w_2$
(c) $w_1 < w_2$ (d) All of these

Passage Based Questions

Passage

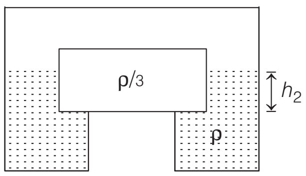
A wooden cylinder of diameter $4r$, height h and density $\rho/3$ is kept on a hole of diameter $2r$ of tank, filled with liquid of density ρ as shown in the figure.



11. Now level of the liquid starts decreasing slowly. When the level of liquid is at a height h_1 above the cylinder the block starts moving up. At what value of h_1, will the block rise ? **(2006, 5M)**

(a) $4h/9$ (b) $5h/9$
(c) $5h/3$ (d) Remains same

12. The block in the above question is maintained at the position by external means and the level of liquid is lowered. The height h_2 when this external force reduces to zero is **(2006, 5M)**



(a) $\frac{4h}{9}$ (b) $\frac{5h}{9}$
(c) Remains same (d) $2h/3$

13. If height h_2 of water level is further decreased, then **(2006, 5M)**

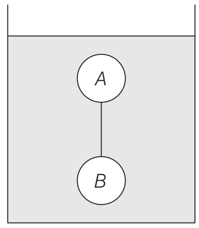
(a) cylinder will not move up and remains at its original position
(b) for $h_2 = h/3$, cylinder again starts moving up
(c) for $h_2 = h/4$, cylinder again starts moving up
(d) for $h_2 = h/5$, cylinder again starts moving up

Objective Questions II (One or more correct option)

14. A solid sphere of radius R and density ρ is attached to one end of a massless spring of force constant k. The other end of the spring is connected to another solid sphere of radius R and density 3ρ. The complete arrangement is placed in a liquid of density 2ρ and is allowed to reach equilibrium. The correct statement(s) is (are) **(2013 Adv.)**

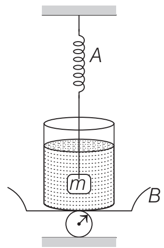
(a) the net elongation of the spring is $\frac{4\pi R^3 \rho g}{3k}$
(b) the net elongation of the spring is $\frac{8\pi R^3 \rho g}{3k}$
(c) the light sphere is partially submerged
(d) the light sphere is completely submerged

15. Two solid spheres A and B of equal volumes but of different densities d_A and d_B are connected by a string. They are fully immersed in a fluid of density d_F. They get arranged into an equilibrium state as shown in the figure with a tension in the string. The arrangement is possible only if **(2011)**



(a) $d_A < d_F$
(b) $d_B > d_F$
(c) $d_A > d_F$
(d) $d_A + d_B = 2d_F$

16. The spring balance A reads 2 kg with a block m suspended from it. A balance B reads 5 kg when a beaker with liquid is put on the pan of the balance. The two balances are now so arranged that the hanging mass is inside the liquid in the beaker as shown in the figure. In this situation **(1985, 2M)**

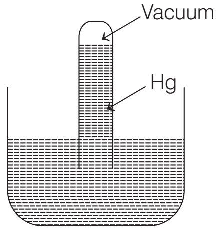


(a) the balance A will read more than 2 kg
(b) the balance B will read more than 5 kg
(c) the balance A will read less than 2 kg and B will read more than 5 kg
(d) the balances A and B will read 2 kg and 5 kg respectively

True / False

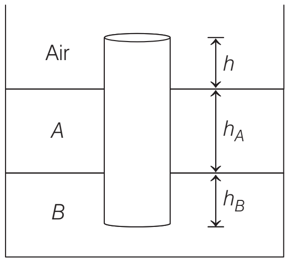
17. A block of ice with a lead shot embedded in it is floating on water contained in a vessel. The temperature of the system is maintained at 0°C as the ice melts. When the ice melts completely the level of water in the vessel rises. **(1986, 3M)**

18. A barometer made of a very narrow tube (see figure) is placed at normal temperature and pressure. The coefficient of volume expansion of mercury is 0.00018/°C and that of the tube is negligible. The temperature of mercury in the barometer is now raised by 1°C but the temperature of the atmosphere does not change. Then, the mercury height in the tube remains unchanged. **(1983, 2M)**



Analytical & Descriptive Questions

19. A uniform solid cylinder of density 0.8 g/cm^3 floats in equilibrium in a combination of two non-mixing liquids A and B with its axis vertical. The densities of the liquids A and B are 0.7 g/cm^3 and 1.2 g/cm^3, respectively. The height of liquid A is $h_A = 1.2$ cm. The length of the part of the cylinder immersed in liquid B is $h_B = 0.8$ cm. **(2002, 5M)**



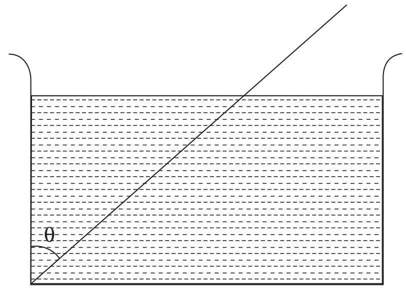
(a) Find the total force exerted by liquid A on the cylinder.

(b) Find h, the length of the part of the cylinder in air.

(c) The cylinder is depressed in such a way that its top surface is just below the upper surface of liquid A and is then released. Find the acceleration of the cylinder immediately after it is released.

20. A wooden stick of length L, radius R and density ρ has a small metal piece of mass m (of negligible volume) attached to its one end. Find the minimum value for the mass m (in terms of given parameters) that would make the stick float vertically in equilibrium in a liquid of density $\sigma\ (>\rho)$. **(1999, 10M)**

21. A wooden plank of length 1 m and uniform cross-section is hinged at one end to the bottom of a tank as shown in figure. The tank is filled with water upto a height 0.5 m. The specific gravity of the plank is 0.5. Find the angle θ that the plank makes with the vertical in the equilibrium position (exclude the case $\theta = 0°$). **(1984, 8M)**



22. A boat floating in a water tank is carrying a number of large stones. If the stones are unloaded into water, what will happen to the water level? **(1979)**

23. A column of mercury of length 10 cm is contained in the middle of a horizontal tube of length 1 m which is closed at both the ends. The two equal lengths contain air at standard atmospheric pressure of 0.76 m of mercury. The tube is now turned to vertical position. By what distance will the column of mercury be displaced ? Assume temperature to be constant. **(1978)**

Topic 3 Ideal Fluids in Motion

Objective Questions I (Only one correct option)

1 A solid sphere of radius R acquires a terminal velocity v_1 when falling (due to gravity) through a viscous fluid having a coefficient of viscosity η. The sphere is broken into 27 identical solid spheres. If each of these spheres acquires a terminal velocity, v_2 when falling through the same fluid, the ratio (v_1 / v_2) equals **(2019 Main, 12 April II)**

(a) 9 (b) 1/27 (c) 1/9 (d) 27

2 Water from a tap emerges vertically downwards with an initial speed of $1.0\ \text{ms}^{-1}$. The cross-sectional area of the tap is $10^{-4}\ \text{m}^2$. Assume that the pressure is constant throughout the stream of water and that the flow is streamlined. The cross-sectional area of the stream, 0.15 m below the tap would be [Take, $g = 10\ \text{ms}^{-2}$] **(2019 Main, 10 April II)**

(a) $2 \times 10^{-5}\ \text{m}^2$ (b) $1 \times 10^{-5}\ \text{m}^2$

(c) $5 \times 10^{-4}\ \text{m}^2$ (d) $5 \times 10^{-5}\ \text{m}^2$

3 A long cylindrical vessel is half-filled with a liquid. When the vessel is rotated about its own vertical axis, the liquid rises up near the wall. If the radius of vessel is 5 cm and its rotational speed is 2 rotations per second, then the difference in the heights between the centre and the sides (in cm) will be **(2019 Main, 12 Jan II)**

(a) 0.1 (b) 1.2 (c) 0.4 (d) 2.0

4 A liquid of density ρ is coming out of a hose pipe of radius a with horizontal speed v and hits a mesh. 50% of the liquid passes through the mesh unaffected 25% losses all of its momentum and, 25% comes back with the same speed. The resultant pressure on the mesh will be **(2019 Main, 11 Jan I)**

(a) ρv^2 (b) $\frac{1}{2}\rho v^2$ (c) $\frac{1}{4}\rho v^2$ (d) $\frac{3}{4}\rho v^2$

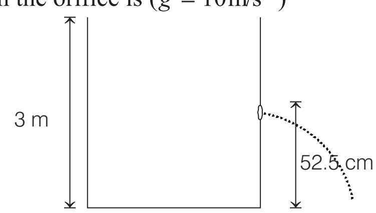
5 Water flows into a large tank with flat bottom at the rate of $10^{-4}\ \text{m}^3\text{s}^{-1}$. Water is also leaking out of a hole of area 1 cm^2 at its bottom. If the height of the water in the tank remains steady, then this height is **(2019 Main, 10 Jan I)**

(a) 4 cm (b) 2.9 cm (c) 5.1 cm (d) 1.7 cm

6 The top of a water tank is open to air and its water level is maintained. It is giving out $0.74\ \text{m}^3$ water per minute through a circular opening of 2 cm radius is its wall. The depth of the centre of the opening from the level of water in the tank is close to **(2019 Main, 9 Jan II)**

(a) 4.8 m (b) 6.0 m (c) 2.9 m (d) 9.6 m

7. Water is filled in a cylindrical container to a height of 3 m. The ratio of the cross-sectional area of the orifice and the beaker is 0.1. The square of the speed of the liquid coming out from the orifice is ($g = 10\ \text{m/s}^2$) **(2005, 2M)**



(a) $50\ \text{m}^2/\text{s}^2$ (b) $50.5\ \text{m}^2/\text{s}^2$

(c) $51\ \text{m}^2/\text{s}^2$ (d) $52\ \text{m}^2/\text{s}^2$

8. A large open tank has two holes in the wall. One is a square hole of side L at a depth y from the top and the other is a circular hole of radius R at a depth $4y$ from the top. When the tank is completely filled with water, the quantities of water flowing out per second from both holes are the same. Then, R is equal to **(2000, 2M)**

(a) $L/\sqrt{2\pi}$ (b) $2\pi L$ (c) L (d) $L/2\pi$

9. Water from a tap emerges vertically downwards with an initial speed of 1.0 m/s. The cross-sectional area of tap is 10^{-4} m^2. Assume that the pressure is constant throughout the steam of water and that the flow is steady, the cross-sectional area of stream 0.15 m below the tap is **(1998, 2M)**

(a) 5.0×10^{-4} m^2 (b) 1.0×10^{-4} m^2

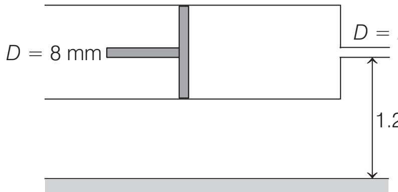
(c) 5.0×10^{-5} m^2 (d) 2.0×10^{-5} m^2

Fill in the Blank

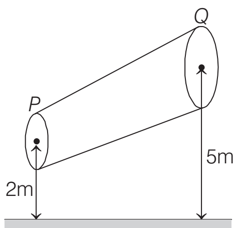
10. A horizontal pipeline carries water in a streamline flow. At a point along the pipe, where the cross-sectional area is 10 cm^2, the water velocity is 1ms^{-1} and the pressure is 2000 Pa. The pressure of water at another point where the cross-sectional area is 5 cm^2, is ......Pa. (Density of water = 10^3 kg-m^3) **(1994, 2M)**

Analytical & Descriptive Questions

11. Consider a horizontally oriented syringe containing water located at a height of 1.25 m above the ground. The diameter of the plunger is 8 mm and the diameter of the nozzle is 2 mm. The plunger is pushed with a constant speed of 0.25 m/s. Find the horizontal range of water stream on the ground. (Take $g = 10$ m/s^2). **(2004, 2M)**



12. A non-viscous liquid of constant density 1000 kg/m^3 flows in streamline motion along a tube of variable cross-section. The tube is kept inclined in the vertical plane as shown in the figure. The area of cross-section of the tube at two points P and Q at heights of 2 m and 5 m are respectively 4×10^{-3} m^2 and 8×10^{-3} m^2. The velocity of the liquid at point P is 1 m/s. Find the work done per unit volume by the pressure and the gravity forces as the fluid flows from point P to Q. **(1997, 5M)**

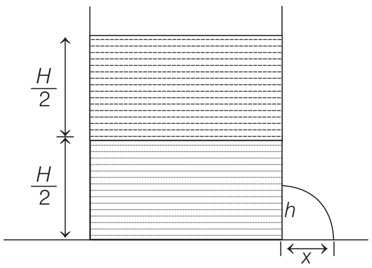


13. A large open top container of negligible mass and uniform cross-sectional area A has a small hole of cross-sectional area $A/100$ in its side wall near the bottom. The container is kept on a smooth horizontal floor and contains a liquid of density ρ and mass m_0. Assuming that the liquid starts flowing out horizontally through the hole at $t = 0$. Calculate **(1997 C, 5M)**

(a) the acceleration of the container and

(b) velocity of efflux when 75% of the liquid has drained out.

14. A container of large uniform cross-sectional area A resting on a horizontal surface, holds two immiscible, non-viscous and incompressible liquids of densities d and $2d$, each of height $H/2$ as shown in figure. The lower density liquid is open to the atmosphere having pressure p_0. **(1995, 5+5 M)**



(a) A homogeneous solid cylinder of length $L(L < H/2)$, cross-sectional area $A/5$ is immersed such that it floats with its axis vertical at the liquid-liquid interface with length $L/4$ in the denser liquid. Determine

(i) the density D of the solid,

(ii) the total pressure at the bottom of the container.

(b) The cylinder is removed and the original arrangement is restored. A tiny hole of area s ($s << A$) is punched on the vertical side of the container at a height h ($h < H/2$).

Determine :

(i) the initial speed of efflux of the liquid at the hole,

(ii) the horizontal distance x travelled by the liquid initially, and

(iii) the height h_m at which the hole should be punched so that the liquid travels the maximum distance x_m initially. Also calculate x_m.

(Neglect the air resistance in these calculations)

Topic 4 Surface Tension and Viscosity

Objective Questions I (Only one correct option)

1 The ratio of surface tensions of mercury and water is given to be 7.5 while the ratio of their densities is 13.6. Their contact angles with glass are close to 135° and 0°, respectively. It is observed that mercury gets depressed by an amount h in a capillary tube of radius r_1, while water rises by the same amount h in a capillary tube of radius r_2. The ratio (r_1 / r_2), is then close to **(2019 Main, 10 April I)**

(a) 3/5 (b) 2/3 (c) 2/5 (d) 4/5

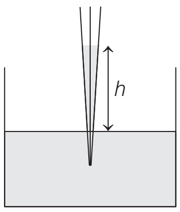
2 If M is the mass of water that rises in a capillary tube of radius r, then mass of water which will rise in a capillary tube of radius $2r$ is **(2019 Main, 9 April I)**

(a) $2M$ (b) $4M$ (c) $\frac{M}{2}$ (d) M

3. The following observations were taken for determining surface tension T of water by capillary method. Diameter of capillary, $d = 1.25 \times 10^{-2}$ m rise of water, $h = 1.45 \times 10^{-2}$m. Using $g = 9.80$ $\mathbf{m/s}^2$ and the simplified relation $T = \frac{rhg}{2} \times 10^3 \mathbf{N/m}$, the possible error in surface tension is closest to **(2017 Main)**

(a) 1.5% (b) 2.4% (c) 10% (d) 0.15%

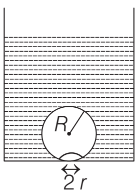
4. A glass capillary tube is of the shape of truncated cone with an apex angle α so that its two ends have cross-sections of different radii. When dipped in water vertically, water rises in it to a height h, where the radius of its cross-section is b. If the surface tension of water is S, its density is ρ, and its contact angle with glass is θ, the value of h will be (g is the acceleration due to gravity) **(2014 Adv.)**



(a) $\frac{2S}{b\rho g} \cos(\theta - \alpha)$ (b) $\frac{2S}{b\rho g} \cos(\theta + \alpha)$

(c) $\frac{2S}{b\rho g} \cos(\theta - \alpha / 2)$ (d) $\frac{2S}{b\rho g} \cos(\theta + \alpha / 2)$

5. On heating water, bubbles beings formed at the bottom of the vessel detach and rise. Take the bubbles to be spheres of radius R and making a circular contact of radius r with the bottom of the vessel. If $r << R$ and the surface tension of water is T, value of r just before bubbles detach is (density of water is ρ_w) **(2014 Main)**

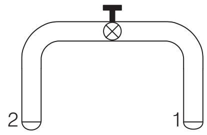


(a) $R^2\sqrt{\frac{\rho_w g}{3T}}$ (b) $R^2\sqrt{\frac{\rho_w g}{6T}}$ (c) $R^2\sqrt{\frac{\rho_w g}{T}}$ (d) $R^2\sqrt{\frac{3\rho_w g}{T}}$

6. Assume that a drop of liquid evaporates by decrease in its surface energy, so that its temperature remains unchanged. What should be the minimum radius of the drop for this to be possible? The surface tension is T, denstiy of liquid is ρ and L is its latent heat of vaporisation **(2013 Main)**

(a) $\frac{\rho L}{T}$ (b) $\sqrt{\frac{T}{\rho L}}$ (c) $\frac{T}{\rho L}$ (d) $\frac{2T}{\rho L}$

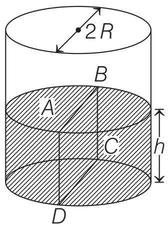
7. A glass tube of uniform internal radius (r) has a valve separating the two identical ends. Initially, the valve is in a tightly closed position. End 1 has a hemispherial soap bubble of radius r. End 2 has sub-hemispherical soap bubble as shown in figure.



Just after opening the valve. **(2008, 3M)**

(a) air from end 1 flows towards end 2. No change in the volume of the soap bubbles
(b) air from end 1 flows towards end 2. Volume of the soap bubble at end 1 decreases
(c) no change occurs
(d) air from end 2 flows towards end 1. Volume of the soap bubble at end 1 increases

8. Water is filled up to a height h in a beaker of radius R as shown in the figure. The density of water is ρ, the surface tension of water is T and the atmospheric pressure is p_0. Consider a vertical section $ABCD$ of the water column through a diameter of the beaker. The force on water on one side of this section by water on the other side of this section has magnitude **(2007, 3M)**



(a) $|2p_0Rh + \pi R^2 \rho gh - 2RT|$
(b) $|2p_0Rh + R\rho gh^2 - 2RT|$
(c) $|p_0\pi R^2 + R\rho gh^2 - 2RT|$
(d) $|p_0\pi R^2 + R\rho gh^2 + 2RT|$

Objective Questions II (Only one correct option)

9. A uniform capillary tube of inner radius r is dipped vertically into a beaker filled with water. The water rises to a height h in the capillary tube above the water surface in the beaker. The surface tension of water is σ. The angle of contact between water and the wall of the capillary tube is θ. Ignore the mass of water in the meniscus. Which of the following statements is (are) true ? **(2018 Adv.)**

(a) For a given material of the capillary tube, h decreases with increase in r
(b) For a given material of the capillary tube, h is independent of σ
(c) If this experiment is performed in a lift going up with a constant acceleration, then h decreases
(d) h is proportional to contact angle θ

Passage Based Question

Passage 1

When liquid medicine of density ρ is to be put in the eye, it is done with the help of a dropper. As the bulb on the top of the dropper is pressed, a drop forms at the opening of the dropper. We wish to estimate the size of the drop. We first assume that the drop formed at the opening is spherical because that requires a minimum increase in its surface energy. To determine the size, we calculate the net vertical force due to the surface tension T when the radius of the drop is R. When this force becomes smaller than the weight of the drop, the drop gets detached from the dropper. **(2010)**

10. If the radius of the opening of the dropper is r, the vertical force due to the surface tension on the drop of radius R (assuming $r << R$) is

(a) $2\pi rT$ (b) $2\pi RT$ (c) $\frac{2\pi r^2 T}{R}$ (d) $\frac{2\pi R^2 T}{r}$

11. If $r = 5 \times 10^{-4}$ m, $\rho = 10^3$ kg m^{-3}, $g = 10\,\text{ms}^{-2}$, $T = 0.11$ Nm^{-1}, the radius of the drop when it detaches from the dropper is approximately

(a) 1.4×10^{-3} m (b) 3.3×10^{-3} m
(c) 2.0×10^{-3} m (d) 4.1×10^{-3} m

12. After the drop detaches, its surface energy is

(a) 1.4×10^{-6} J (b) 2.7×10^{-6} J
(c) 5.4×10^{-6} J (d) 8.1×10^{-9} J

Integer Answer Type Questions

13. A drop of liquid of radius $R = 10^{-2}$ m having surface tension $S = \frac{0.1}{4\pi}$ Nm^{-1} divides itself into K identical drops. In this process the total change in the surface energy $\Delta U = 10^{-3}$ J. If $K = 10^{\alpha}$, then the value of α is **(2017 Adv.)**

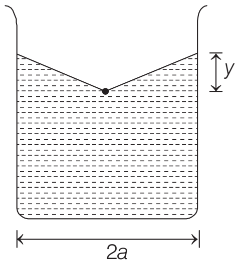
14. Consider two solid spheres P and Q each of density 8 gm cm^{-3} and diameters 1 cm and 0.5 cm, respectively. Sphere P is dropped into a liquid of density 0.8 gm cm^{-3} and viscosity $\eta = 3$ poiseulles. Sphere Q is dropped into a liquid of density 1.6 gm cm^{-3} and viscosity $\eta = 2$ poiseulles. The ratio of the terminal velocities of P and Q is **(2016 Adv.)**

15. Two soap bubbles A and B are kept in a closed chamber where the air is maintained at pressure 8 Nm^{-2}. The radii of bubbles A and B are 2 cm, respectively. Surface tension of the soap-water used to make bubbles is 0.04 Nm^{-1}. Find the ratio $\frac{n_B}{n_A}$, where n_A and n_B are the number of moles of air in bubbles A and B, respectively. [Neglect the effect of gravity] **(2009)**

Analytical & Descriptive Questions

16. A small sphere falls from rest in a viscous liquid. Due to friction, heat is produced. Find the relation between the rate of production of heat and the radius of the sphere at terminal velocity. **(2004, 2M)**

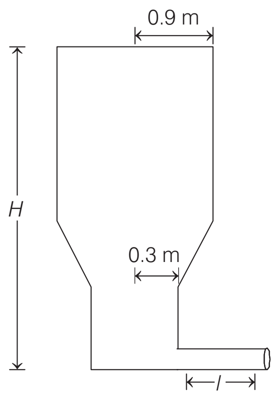
17. A container of width $2a$ is filled with a liquid. A thin wire of weight per unit length λ is gently placed over the liquid surface in the middle of the surface as shown in the figure. As a result, the liquid surface is depressed by a distance y ($y << a$). Determine the surface tension of the liquid. **(2004, 2M)**



18. A soap bubble is being blown at the end of very narrow tube of radius b. Air (density ρ) moves with a velocity v inside the tube and comes to rest inside the bubble. The surface tension of the soap solution is T. After sometime the bubble, having grown to radius r separates from the tube. Find the value of r. Assume that $r >> b$ so, that you can consider the air to be falling normally on the bubble's surface. **(2003, 4M)**

19. A liquid of density 900 kg/m^3 is filled in a cylindrical tank of upper radius 0.9 m and lower radius 0.3 m. A capillary tube of length l is attached at the bottom of the tank as shown in the figure. The capillary has outer radius 0.002 m and inner radius a. When pressure p is applied at the top of the tank volume flow rate of the liquid is 8×10^{-6} m^3/s and if capillary tube is detached, the liquid comes out from the tank with a velocity 10 m/s.

Determine the coefficient of viscosity of the liquid.
[Given, $\pi a^2 = 10^{-6}$ m^2 and $a^2/l = 2 \times 10^{-6}$ m] **(2003, 4M)**



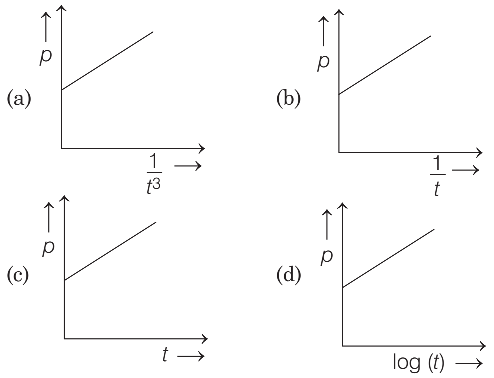
Topic 5 Miscellaneous Problems

Objective Questions I (Only one correct option)

1 Water from a pipe is coming at a rate of 100 liters per minute. If the radius of the pipe is 5 cm, the Reynolds number for the flow is of the order of (density of water = 1000kg/m^3, coefficient of viscosity of water = 1mPa s) **(2109 Main, 8 April I)**

(a) 10^3 (b) 10^4 (c) 10^2 (d) 10^6

2 A soap bubble, blown by a mechanical pump at the mouth of a tube, increases in volume, with time, at a constant rate. The graph that correctly depicts the time dependence of pressure inside the bubble is given by **(2019 Main, 12 Jan II)**



3. A man grows into a giant such that his linear dimensions increase by a factor of 9. Assuming that his density remains same, the stress in the leg will change by a factor of **(2017 Main)**

(a) $\frac{1}{9}$ (b) 81 (c) $\frac{1}{81}$ (d) 9

4. A person in a lift is holding a water jar, which has a small hole at the lower end of its side. When the lift is at rest, the water jet coming out of the hole hits the floor of the lift at a distance d of 1.2 m from the person. In the following, state of the lift's motion is given in List I and the distance where the water jet hits the floor of the lift is given in List II. Match the statements from List I with those in List II and select the correct answer using the code given below the lists. **(2014 Adv.)**

	List I		List II
A	Lift is accelerating vertically up.	p	$d = 1.2$ m
B	Lift is accelerating vertically down with an acceleration less than the gravitational acceleration.	q	$d > 1.2$ m
C	Lift is moving vertically up with constant speed.	r	$d < 1.2$ m
D	Lift is falling freely.	s	No water leaks out of the jar

Codes

(a) A-q, B-r, C-q, D-s (b) A-q, B-r, C-p, D-s
(c) A-p, B-p, C-p, D-s (d) A-q, B-r, C-p, D-p

5. The pressure that has to be applied to the ends of a steel wire of length 10 cm to keep its length constant when its temperature is raised by 100°C is (For steel, Young's modulus is $2 \times 10^{11} \text{Nm}^{-2}$ and coefficient of thermal expansion is $1.1 \times 10^{-5} \text{ K}^{-1}$) **(2014 Main)**

(a) 2.2×10^8 Pa (b) 2.2×10^9 Pa
(c) 2.2×10^7 Pa (d) 2.2×10^6 Pa

6. An open glass tube is immersed in mercury in such a way that a length of 8 cm extends above the mercury level. The open end of the tube is then closed and sealed and the tube is raised vertically up by additional 46 cm. What will be length of the air column above mercury in the tube now? (Atmospheric pressure = 76 cm of Hg) **(2014 Main)**

(a) 16 cm (b) 22 cm (c) 38 cm (d) 6 cm

7. A uniform cylinder of length L and mass M having cross-sectional area A is suspended, with its length vertical from a fixed point by a massless spring such that it is half submerged in a liquid of density σ at equilibrium position. The extension x_0 of the spring when it is in equilibrium is **(2013 Main)**

(a) $\frac{Mg}{k}$ (b) $\frac{Mg}{k}\left(1 - \frac{LA\sigma}{M}\right)$
(c) $\frac{Mg}{k}\left(1 - \frac{LA\sigma}{2M}\right)$ (d) $\frac{Mg}{k}\left(1 + \frac{LA\sigma}{M}\right)$

8. A thin uniform cylindrical shell, closed at both ends, is partially filled with water. It is floating vertically in water in half-submerged state. If ρ_c is the relative density of the material of the shell with respect to water, then the correct statement is that the shell is **(2012)**

(a) more than half-filled if ρ_c is less than 0.5
(b) more than half-filled if ρ_c is more than 1.0
(c) half-filled if ρ_c is more than 0.5
(d) less than half-filled if ρ_c is less than 0.5

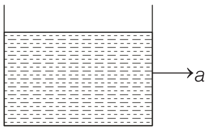
9. When a block of iron floats in mercury at 0°C, fraction k_1 of its volume is submerged, while at the temperature 60°C, a fraction k_2 is seen to be submerged. If the coefficient of volume expansion of iron is γ_{Fe} and that of mercury is γ_{Hg}, then the ratio k_1/k_2 can be expressed as **(2001, 2M)**

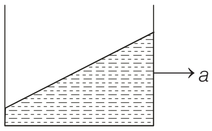
(a) $\frac{1 + 60\gamma_{\text{Fe}}}{1 + 60\gamma_{\text{Hg}}}$ (b) $\frac{1 - 60\gamma_{\text{Fe}}}{1 + 60\gamma_{\text{Hg}}}$ (c) $\frac{1 + 60\gamma_{\text{Fe}}}{1 - 60\gamma_{\text{Hg}}}$ (d) $\frac{1 + 60\gamma_{\text{Hg}}}{1 + 60\gamma_{\text{Fe}}}$

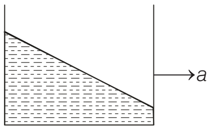
10. A closed compartment containing gas is moving with some acceleration in horizontal direction. Neglect effect of gravity. Then, the pressure in the compartment is **(1999, 2M)**

(a) same everywhere (b) lower in front side
(c) lower in rear side (d) lower in upper side

11. A vessel containing water is given a constant acceleration a towards the right along a straight horizontal path. Which of the following diagrams represents the surface of the liquid ? **(1981, 2M)**

(a)


(b)


(c)


(d) None of these

Objective Questions II (One or more correct options)

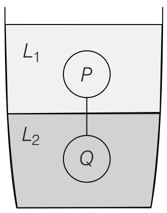
12. Consider a thin square plate floating on a viscous liquid in a large tank. The height h of the liquid in the tank is much less than the width of the tank. The floating plate is pulled horizontally with a constant velocity u_0. Which of the following statements is (are) true? **(2018 Adv.)**

(a) The resistive force of liquid on the plate is inversely proportional to h
(b) The resistive force of liquid on the plate is independent of the area of the plate
(c) The tangential (shear) stress on the floor of the tank increases with u_0
(d) The tangential (shear) stress on the plate varies linearly with the viscosity η of the liquid

13. A flat plane is moving normal to its plane through a gas under the action of a constant force F. The gas is kept at a very low pressure. The speed of the plate v is much less than the average speed u of the gas molecules. Which of the following options is/are true? **(2017 Adv.)**

(a) At a later time the external force F balances the resistive force
(b) The plate will continue to move with constant non-zero acceleration, at all times
(c) The resistive force experienced by the plate is proportioal to v
(d) The pressure difference between the leading and trailing faces of the plate is proportional to uv

14. Two spheres P and Q for equal radii have densities ρ_1 and ρ_2, respectively. The spheres are connected by a massless string and placed in liquids L_1 and L_2 of densities σ_1 and σ_2 and viscosities η_1 and η_2, respectively. They float in equilibrium with the sphere P in L_1 and sphere Q in L_2 and the string being taut (see figure). If sphere P alone in L_2 has terminal velocity $\mathbf{v}_P$ and Q alone in L_1 has terminal velocity $\mathbf{v}_Q$, then **(2015 Adv.)**



(a) $\dfrac{|\mathbf{v}_P|}{|\mathbf{v}_Q|} = \dfrac{\eta_1}{\eta_2}$
(b) $\dfrac{|\mathbf{v}_P|}{|\mathbf{v}_Q|} = \dfrac{\eta_2}{\eta_1}$
(c) $\mathbf{v}_P \cdot \mathbf{v}_Q > 0$
(d) $\mathbf{v}_P \cdot \mathbf{v}_Q < 0$

Assertion and Reason

Mark your answer as

(*a*) *If Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I*
(*b*) *If Statement I is true, Statement II is true; Statement II is not a correct explanation for Statement I*
(*c*) *If Statement I is true; Statement II is false*
(*d*) *If Statement I is false; Statement II is true*

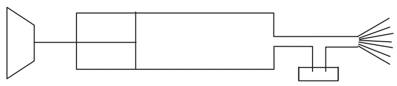
15. **Statement I** The stream of water flowing at high speed from a garden hose pipe tends to spread like a fountain when held vertically up, but tends to narrow down when held vertically down.

Statement II In any steady flow of an incompressible fluid, the volume flow rate of the fluid remains constant. **(2008, 3M)**

Passage Based Questions

Passage 1

A spray gun is shown in the figure where a piston pushes air out of nozzle. A thin tube of uniform cross-section is connected to the nozzle. The other end of the tube is in a small liquid container. As the piston pushes air through the nozzle, the liquid from the container rises into the nozzle and is sprayed out.



For the spray gun shown, the radii of the piston and the nozzle are 20 mm and 1 mm respectively. The upper end of the container is open to the atmosphere. **(2014 Adv.)**

16. If the piston is pushed at a speed of 5 mms^{-1}, the air comes out of the nozzle with a speed of

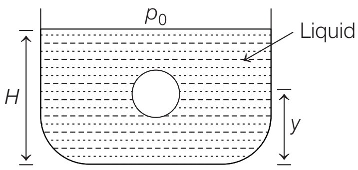
(a) 0.1 ms^{-1} (b) 1 ms^{-1} (c) 2 ms^{-1} (d) 8 ms^{-1}

17. If the density of air is ρ_a and that of the liquid ρ_l, then for a given piston speed the rate (volume per unit time) at which the liquid is sprayed will be proportional to

(a) $\sqrt{\dfrac{\rho_a}{\rho_l}}$ (b) $\sqrt{\rho_a \rho_l}$ (c) $\sqrt{\dfrac{\rho_l}{\rho_a}}$ (d) ρ_l

Passage 2

A small spherical monoatomic ideal gas bubble ($\gamma = 5/3$) is trapped inside a liquid of density ρ_l (see figure). Assume that the bubble does not exchange any heat with the liquid. The bubble contains n moles of gas. The temperature of the gas when the bubble is at the bottom is T_0, the height of the liquid is H and the atmospheric pressure is p_0 (Neglect surface tension) **(2008, 4M)**



18. As the bubble moves upwards, besides the buoyancy force the following forces are acting on it. **(2008, 4M)**

(a) Only the force of gravity

(b) The force due to gravity and the force due to the pressure of the liquid

(c) The force due to gravity, the force due to the pressure of the liquid and the force due to viscosity of the liquid

(d) The force due to gravity and the force due to viscosity of the liquid

19. When the gas bubble is at a height y from the bottom, its temperature is **(2008, 4M)**

(a) $T_0\left(\frac{p_0+\rho_l gH}{p_0+\rho_l gy}\right)^{2/5}$ (b) $T_0\left(\frac{p_0+\rho_l g(H-y)}{p_0+\rho_l gH}\right)^{2/5}$

(c) $T_0\left(\frac{p_0+\rho_l gH}{p_0+\rho_l gy}\right)^{3/5}$ (d) $T_0\left(\frac{p_0+\rho_l g(H-y)}{p_0+\rho_l gH}\right)^{3/5}$

20. The buoyancy force acting on the gas bubble is (Assume R is the universal gas constant) **(2008, 4M)**

(a) $\rho_l nRgT_0\frac{(p_0+\rho_l gH)^{2/5}}{(p_0+\rho_l gy)^{2/5}}$

(b) $\frac{\rho_l nRgT_0}{(p_0+\rho_l gH)^{2/5}[p_0+\rho_l g(H-y)]^{3/5}}$

(c) $\rho_l nRgT_0\frac{(p_0+\rho_l gH)^{3/5}}{(p_0+\rho_l gy)^{8/5}}$

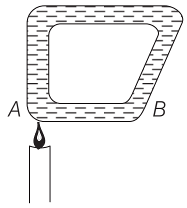
(d) $\frac{\rho_l nRgT_0}{(p_0+\rho_l gH)^{3/5}[p_0+\rho_l g(H-y)]^{2/5}}$

Fill in the Blank

21. A piece of metal floats on mercury. The coefficients of volume expansion of the metal and mercury are γ_1 and γ_2 respectively. If the temperatures of both mercury and the metal are increased by an amount ΔT, the fraction of the volume of the metal submerged in mercury changes by the factor …… . **(1991, 2M)**

True / False

22. Water in a closed tube (see figure) is heated with one arm vertically placed above a lamp. Water will begin to circulate along the tube in counter-clockwise direction. **(1983, 2M)**



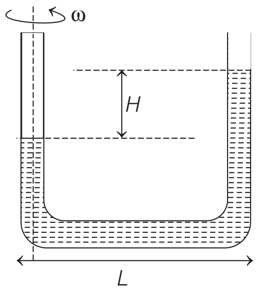
Integer Answer Type Question

23. A cylindrical vessel of height 500 mm has an orifice (small hole) at its bottom. The orifice is initially closed and water is filled in it upto height H. Now, the top is completely sealed with a cap and the orifice at the bottom is opened. Some water comes out from the orifice and the water level in the vessel becomes steady with height of water column being 200 mm. Find the fall in height (in mm) of water level due to opening of the orifice. **(2009)**

[Take atmospheric pressure $= 1.0\times 10^5\ \text{Nm}^{-2}$, density of water $= 1000\ \text{kg m}^{-3}$ and $g = 10\ \text{ms}^{-2}$. Neglect any effect of surface tension.]

Analytical & Descriptive Questions

24. A U-shaped tube contains a liquid of density ρ and it is rotated about the line as shown in the figure. Find the difference in the levels of liquid column. **(2005, 2M)**



25. A ball of density d is dropped on to a horizontal solid surface. It bounces elastically from the surface and returns to its original position in a time t_1. Next, the ball is released and it falls through the same height before striking the surface of a liquid of density d_L. **(1992, 8 M)**

(a) If $d < d_L$, obtain an expression (in terms of d, t_1 and d_L) for the time t_2 the ball takes to come back to the position from which it was released.

(b) Is the motion of the ball simple harmonic ?

(c) If $d = d_L$, how does the speed of the ball depend on its depth inside the liquid? Neglect all frictional and other dissipative forces. Assume the depth of the liquid to be large.

26. Two identical cylindrical vessels with their bases at the same level each contain a liquid of density ρ . The height of the liquid in one vessel is h_1 and in the other is h_2. The area of either base is A. What is the work done by gravity in equalising the levels when the two vessels are connected ? **(1981, 4 M)**

Answers

Topic 1

1. (*) **2.** (c) **3.** (d) **4.** (c)
5. (a) **6.** (d) **7.** (a) **8.** (d)
9. (d) **10.** (c) **11.** (d) **12.** (a)
13. (b) **14.** (c) **15.** (a) **16.** (a, b)
17. $\frac{\rho L \alpha}{2}$ **18.** $\frac{Mg}{3AK}$ **19.** $\frac{1}{2}\left(\frac{YA}{L}\right)x^2$
20. 1.09×10^{10} N/m^2 **21.** (a) 40 kg (b) 0.1 J

Topic 2

1. (d) **2.** (a)
3. (c) **4.** (d) **5.** Most appropriate option is (d)
6. (a) **7.** (c) **8.** (b) **9.** (a)
10. (c) **11.** (c) **12.** (a) **13.** (a)
14. (a, d) **15.** (a, b, d) **16.** (b, c) **17.** F
18. F **19.** (a) Zero (b) 0.25 cm (c) $g/6$
20. $\pi R^2 L(\sqrt{\rho\sigma} - \rho)$ **21.** 45°
22. Level will come down. **23.** 2.95 cm

Topic 3

1. (a) **2.** (d) **3.** (d) **4.** (d)
5. (c) **6.** (a)
7. (a) **8.** (a) **9.** (c) **10.** 500
11. 2m **12.** 29025 J/m^3, – 29400 J/m^3
13. (a) $\frac{g}{50}$ (b) $\sqrt{\frac{gm_0}{2A\rho}}$ **14.** (a) (i) $\frac{5d}{4}$ (ii) $p = p_0 + \frac{dg(6H + L)}{4}$
(b) (i) $\sqrt{(3H - 4h)\frac{g}{2}}$ (ii) $\sqrt{h(3H - 4h)}$ (iii) At $h = \frac{3H}{8}$; $\frac{3}{4}H$

Topic 4

1. (c) **2.** (a)
3. (a)
4. (d) **5.** (*) **6.** (d) **7.** (b)
8. (b) **9.** (a, c)
10. (c) **11.** (a) **12.** (b) **13.** 6
14. 3 **15.** 6 **16.** $\frac{dQ}{dt} \propto r^5$
17. $\frac{\lambda a}{2y}$ **18.** $\frac{4T}{\rho v^2}$ **19.** $\frac{1}{720}$ N-s/m^2

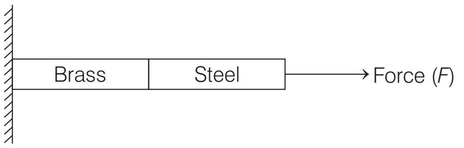
Topic 5

1. (b) **2.** (b)
3. (d) **4.** (c) **5.** (a) **6.** (a)
7. (c) **8.** (a) **9.** (a) **10.** (b)
11. (c) **12.** (a,c,d) **13.** (a, c, d) **14.** (a, d)
15. (a) **16.** (c) **17.** (a) **18.** (d)
19. (b) **20.** (b) **21.** $\frac{1 + \gamma_2 \Delta T}{1 + \gamma_1 \Delta T}$ **22.** F
23. 6 **24.** $H = \frac{\omega^2 L^2}{2g}$
25. (a) $\frac{t_1 d_L}{d_L - d}$ (b) no (c) remains same
26. $\frac{\rho A g}{4}(h_1 - h_2)^2$

Hints & Solutions

Topic 1 Elasticity

1. In given experiment, a composite wire is stretched by a force F.



Net elongation in the wire = elongation in brass wire + elongation in steel wire …(i)

Now, Young's modulus of a wire of cross-section (A) when some force (F) is applied, $Y = \frac{Fl}{A\Delta l}$

We have,

$$\Delta l = \text{elongation} = \frac{Fl}{AY}$$

So, from relation (i), we have

$$\Delta l_{\text{net}} = \Delta l_{\text{brass}} + \Delta l_{\text{steel}}$$

$$\Rightarrow \quad \Delta l_{\text{net}} = \left(\frac{Fl}{AY}\right)_{\text{brass}} + \left(\frac{Fl}{AY}\right)_{\text{steel}}$$

As wires are connected in series and they are of same area of cross-section, length and subjected to same force, so

$$\Delta l_{\text{net}} = \frac{F}{A}\left(\frac{l}{Y_{\text{brass}}} + \frac{l}{Y_{\text{steel}}}\right)$$

Here,

$$\Delta l_{\text{net}} = 0.2\,\text{mm} = 0.2 \times 10^{-3}\ \text{m}$$

and $$l = 1\,\text{m}$$

$$Y_{\text{brass}} = 60 \times 10^9\,\text{Nm}^{-2},\ Y_{\text{steel}} = 120 \times 10^9\,\text{Nm}^{-2}$$

On putting the values, we have

$$0.2 \times 10^{-3} = \frac{F}{A}\left(\frac{1}{60 \times 10^9} + \frac{1}{120 \times 10^9}\right)$$

$$\Rightarrow \quad \text{Stress} = \frac{F}{A} = 8\times 10^6 \text{Nm}^{-2}$$

No options matches.

2. Let d_{min} = minimum diameter of brass.

Then, stress in brass rod is given by

$$\sigma = \frac{F}{A} = \frac{4F}{\pi d^2_{\text{min}}} \qquad \left[\because A = \frac{\pi d^2}{4}\right]$$

For stress not to exceed elastic limit, we have

$\sigma \le 379$ MPa

$$\Rightarrow \quad \frac{4F}{\pi d^2_{\text{min}}} \le 379\times 10^6$$

Here, $F = 400$ N

$$\therefore \quad d^2_{\text{min}} = \frac{1600}{\pi\times 379\times 10^6}$$

$$\Rightarrow \quad d_{\text{min}} = 1.16\times 10^{-3} \text{ m} = 1.16 \text{ mm}$$

3. When a wire is stretched, then change in length of wire is

$\Delta l = \dfrac{Fl}{\pi r^2 Y}$, where Y is its Young's modulus.

Here, for wires A and B,

$$l_A = 2 \text{ m}, l_B = 1.5 \text{ m},$$

$$\frac{Y_A}{Y_B} = \frac{7}{4}, r_B = 2 \text{ mm} = 2\times 10^{-3} \text{ m and } \frac{F_A}{F_B} = 1$$

As, it is given that $\Delta l_A = \Delta l_B$

$$\Rightarrow \frac{F_A l_A}{\pi r_A^2 Y_A} = \frac{F_B l_B}{\pi r_B^2 Y_B}$$

$$\Rightarrow \quad r_A^2 = \frac{F_A}{F_B}\cdot\frac{l_A}{l_B}\cdot\frac{Y_B}{Y_A}\cdot r_B^2$$

$$= 1\times\frac{2}{1.5}\times\frac{4}{7}\times 4\times 10^{-6} \text{ m} = 3.04\times 10^{-6} \text{ m}$$

or $\quad r_A = 1.7\times 10^{-3}$ m

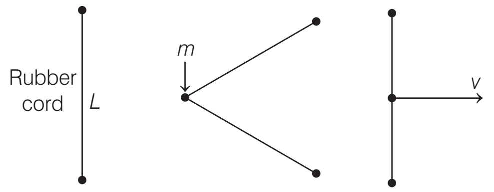
or $\quad r_A = 1.7$ mm

4. Given, radius of wire, $r = 2\text{ mm} = 2\times 10^{-3}$ m Weight of load, $m = 4$ kg, $g = 3.1\,\pi\,\text{ms}^{-2}$

$$\therefore \quad \text{Tensile stress} = \frac{\text{Force}(F)}{\text{Area}(A)} = \frac{mg}{\pi r^2}$$

$$= \frac{4\times 3.1\times\pi}{\pi\times(2\times 10^{-3})^2} = 3.1\times 10^6 \text{Nm}^{-2}$$

5. When rubber cord is stretched, then it stores potential energy and when released, this potential energy is given to the stone as kinetic energy.



So, potential energy of stretched cord

= kinetic energy of stone

$$\Rightarrow \quad \frac{1}{2}Y\left(\frac{\Delta L}{L}\right)^2 A\cdot L = \frac{1}{2}mv^2$$

Here, $\Delta L = 20$ cm $= 0.2$ m, $L = 42$ cm $= 0.42$ m, $v = 20\,\text{ms}^{-1}$, $m = 0.02$ kg, $d = 6$ mm $= 6\times 10^{-3}$ m

$$\therefore \quad A = \pi r^2 = \pi\left(\frac{d}{2}\right)^2 = \pi\left(\frac{6\times 10^{-3}}{2}\right)^2$$

$$= \pi(3\times 10^{-3})^2 = 9\pi\times 10^{-6} \text{ m}^2$$

On substituting values, we get

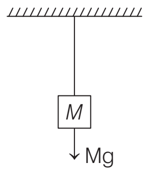
$$Y = \frac{mv^2 L}{A(\Delta L)^2} = \frac{0.02\times(20)^2\times 0.42}{9\pi\times 10^{-6}\times(0.2)^2} \approx 3.0\times 10^6 \text{Nm}^{-2}$$

So, the closest value of Young's modulus is 10^6 Nm^{-2}.

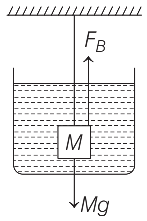
6. When load M is attached to wire, extension in length of wire is

$$\Delta l_1 = \frac{Mgl}{A.Y} \quad \ldots\text{(i)}$$

where, Y is the Young's modulus of the wire.



when load is immersed in liquid of relative density 2, increase in length of wire as shown in the figure is



$$\Delta l_2 = \frac{(Mg - F_B)l}{A.Y}$$

where, F_B = Buoyant force

$$\therefore \quad \Delta l_2 = \frac{\left(Mg - Mg\cdot\frac{\rho_l}{\rho_b}\right)l}{A.Y} \qquad \left[\because F_B = V\rho_l g = \frac{Mg}{\rho_b}\rho_l g\right]$$

Here given that, $\dfrac{\rho_l}{\rho_b} = \dfrac{2}{8} = \dfrac{1}{4}$

$$\text{So,} \quad \Delta l_2 = \frac{\left(\frac{3}{4}Mg\right)l}{A.Y} \quad \ldots\text{(ii)}$$

Dividing Eqs. (ii) by (i), we get

$$\frac{\Delta l_2}{\Delta l_1} = \frac{3}{4}$$

$$\Rightarrow \quad \Delta l_2 = \frac{3}{4}\times\Delta l_1$$

$$= \frac{3}{4}\times 4 \text{ mm} = 3 \text{ mm}$$

7. If a rod of length L and coefficient of linear expansion $\alpha/°C$, then with the rise in temperature by ΔT K, its change in length is given as,

$$\Delta L = L\alpha\ \Delta T \Rightarrow \frac{\Delta L}{L} = \alpha\ \Delta T \quad \text{...(i)}$$

Also, when a rod is subjected to some compressive force (F), then its' Young's modulus is given as

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{\frac{F}{A}}{\frac{\Delta L}{L}}$$

$$\frac{\Delta L}{L} = \frac{F}{YA} \quad \text{...(ii)}$$

Since, it is given that the length of the rod does not change. So, from Eqs. (i) and (ii), we get

$$\alpha\ \Delta T = \frac{F}{YA} \Rightarrow Y = \frac{F}{A\alpha\ \Delta T}$$

8. $$\Delta P = \frac{F}{A} = \frac{mg}{a}$$

Bulk modulus, $K = \dfrac{-\Delta P}{\Delta V/V}$

Here, $V = \dfrac{4}{3}\pi r^3$

and ΔV or $dV = (4\pi r^3)dr$

$$\Rightarrow \quad K = -\frac{\frac{mg}{A}}{\frac{4\pi r^2 dr}{\frac{4}{3}\pi r^3}} \quad \text{or} \quad \frac{dr}{r} = -\frac{mg}{3Ka}$$

9. $$T = 2\pi\sqrt{\frac{L}{g}} \quad \text{...(i)}$$

$$T_M = 2\pi\sqrt{\frac{L+\Delta L}{g}}$$

Here, $\Delta L = \dfrac{FL}{AY} = \dfrac{MgL}{AY}$

$$\therefore \quad T_M = 2\pi\sqrt{\frac{L + \frac{MgL}{AY}}{g}} \quad \text{...(ii)}$$

Solving Eqs. (i) and (ii), we get

$$\frac{1}{Y} = \frac{A}{Mg}\left[\left(\frac{T_M}{T}\right)^2 - 1\right]$$

10. $$\Delta l = \frac{FL}{AY} = \frac{FL}{(\pi r^2)Y} \Rightarrow \Delta l \propto \frac{L}{r^2}$$

$$\therefore \quad \frac{\Delta l_1}{\Delta l_2} = \frac{L/R^2}{2L/(2R)^2} = 2$$

11. From the definition of bulk modulus, $B = \dfrac{-dp}{(dV/V)}$

Substituting the values, we have

$$B = \frac{(1.165 - 1.01)\times 10^5}{(10/100)} = 1.55\times 10^5\ \text{Pa}$$

12. $\Delta l = \left(\dfrac{l}{YA}\right).\ w$

i.e. graph is a straight line passing through origin (as shown in question also), the slope of which is $\dfrac{l}{YA}$.

$$\therefore \quad \text{Slope} = \left(\frac{l}{YA}\right)$$

$$\therefore \quad Y = \left(\frac{l}{A}\right)\left(\frac{1}{\text{slope}}\right) = \left(\frac{1.0}{10^{-6}}\right)\frac{(80-20)}{(4-1)\times 10^{-4}}$$

$$= 2.0\times 10^{11}\ \text{N/m}^2$$

13. In isothermal process

$$pV = \text{constant}$$

$$\therefore \quad pdV + Vdp = 0 \quad \text{or} \quad \left(\frac{dp}{dV}\right) = -\left(\frac{p}{V}\right)$$

$$\therefore \quad \text{Bulk modulus, } B = -\left(\frac{dp}{dV/V}\right) = -\left(\frac{dp}{dV}\right)V$$

$$\therefore \quad B = -\left[\left(-\frac{p}{V}\right)V\right] = p$$

$$\therefore \quad B = p$$

NOTE Adiabatic bulk modulus is given by $B = \gamma P$.

14. Thermal stress $\sigma = Y\ \alpha\ \Delta\theta$

Given, $\sigma_1 = \sigma_2$

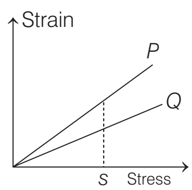
$$\therefore \quad Y_1\alpha_1\Delta\theta = Y_2\alpha_2\Delta\theta \quad \text{or} \quad \frac{Y_1}{Y_2} = \frac{\alpha_2}{\alpha_1} = \frac{3}{2}$$

---

15. $$\Delta l = \frac{Fl}{AY} = \frac{Fl}{\left(\frac{\pi d^2}{4}\right)Y} \quad \text{or} \quad (\Delta l) \propto \frac{l}{d^2}$$

Now, $\dfrac{l}{d^2}$ is maximum in option (a).

16. $Y = \dfrac{\text{stress}}{\text{strain}}$



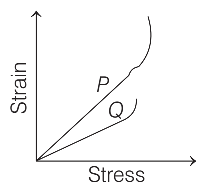
or $Y \propto \dfrac{1}{\text{strain}}$

(for same stress say σ)

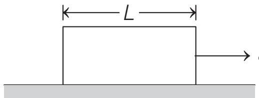
$(\text{strain})_Q < (\text{strain})_P$

$\Rightarrow Y_Q > Y_P$

So, P is more ductile than Q. Further, from the given figure we can also see that breaking stress of P is more than Q. So, it has more tensile strength.

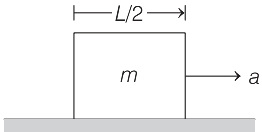


17. Let A be the area of cross-section of the rod. FBD of rod at mid-point



Mass m = volume × density

$$= \left(\frac{L}{2} . A\right)\rho$$



$\therefore \quad T = m\alpha = \left(\frac{L}{2}A\rho\alpha\right)$

$\therefore$ Stress $= \frac{T}{A} = \frac{1}{2}\rho\alpha\, L$

18. $\Delta p = \frac{Mg}{A} \quad \Rightarrow \quad \left|\frac{\Delta V}{V}\right| = \frac{\Delta p}{k} = \frac{Mg}{Ak}$

Now as, $\quad V = \frac{4}{3}\pi R^3$ or $V \propto R^3$

$\therefore \quad \frac{\Delta V}{V} = 3\left(\frac{\Delta R}{R}\right)$

or $\quad \frac{\Delta R}{R} = \frac{1}{3}\left(\frac{\Delta V}{V}\right) \quad$ ...(i)

$\therefore$ From Eq. (i)

$$\frac{\Delta R}{R} = \frac{Mg}{3Ak}$$

19. $\quad W = \frac{1}{2}Kx^2$

Here, $\quad K = \frac{YA}{L}$

$\therefore \quad W = \frac{1}{2}\left(\frac{YA}{L}\right)x^2$

20. Young's modulus of elasticity is given by

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{F/A}{l/L} = \frac{FL}{lA} = \frac{FL}{l\left(\frac{\pi d^2}{4}\right)}$$

Substituting the values, we get

$$Y = \frac{50 \times 1.1 \times 4}{(1.25 \times 10^{-3}) \times \pi \times (5.0 \times 10^{-4})^2}$$

$$= 2.24 \times 10^{11} \text{ N/m}^2$$

Now, $\quad \frac{\Delta Y}{Y} = \frac{\Delta L}{L} + \frac{\Delta l}{l} + 2\frac{\Delta d}{d}$

$$= \left(\frac{0.1}{110}\right) + \left(\frac{0.001}{0.125}\right) + 2\left(\frac{0.001}{0.05}\right) = 0.0489$$

$$\Delta Y = (0.0489)Y = (0.0489) \times (2.24 \times 10^{11}) \text{ N/m}^2$$

$$= 1.09 \times 10^{10} \text{ N/m}^2$$

21. (a) The change in length due to decrease in temperature,

$$\Delta l_1 = L\alpha\, \Delta\theta = (0.5)(10^{-5})(0 - 100)$$

$$\Delta l_1 = -0.5 \times 10^{-3} \text{ m} \quad \text{...(i)}$$

Negative sign implies that length is decreasing. Now, let M be the mass attached to the lower end. Then, change in length due to suspension of load is

$$\Delta l_2 = \frac{(Mg)L}{AY} = \frac{(M)(10)(0.5)}{(4 \times 10^{-6})(10^{11})}$$

$$\Delta l_2 = (1.25 \times 10^{-5})M \quad \text{...(ii)}$$

$$\Delta l_1 + \Delta l_2 = 0$$

or $\quad (1.25 \times 10^{-5})M = (0.5 \times 10^{-3})$

$\therefore \quad M = \left(\frac{0.5 \times 10^{-3}}{1.25 \times 10^{-5}}\right)\text{kg}$

or $\quad M = 40 \text{ kg}$

(b) Energy stored,

At 0°C the natural length of the wire is less than its actual length; but since a mass is attached at its lower end, an elastic potential energy is stored in it. This is given by

$$U = \frac{1}{2}\left(\frac{AY}{L}\right)(\Delta l)^2 \quad \text{...(iii)}$$

Here, $\Delta l = |\Delta l_1| = \Delta l_2 = 0.5 \times 10^{-3}$ m

Substituting the values,

$$U = \frac{1}{2}\left(\frac{4 \times 10^{-6} \times 10^{11}}{0.5}\right)(0.5 \times 10^{-3})^2 = 0.1 \text{ J}$$

NOTE Comparing the equation

$$Y = \frac{F/A}{\Delta l/L} \quad \text{or} \quad F = \left(\frac{AY}{L}\right)\Delta l$$

with the spring equation $F = K \cdot \Delta x$, we find that equivalent spring constant of a wire is $k = \left(\frac{AY}{L}\right)$

Therefore, potential energy stored in it should be

$$U = \frac{1}{2}k(\Delta l)^2 = \frac{1}{2}\left(\frac{AY}{L}\right)(\Delta l)^2$$

Topic 2 Ideal Fluids at Rest

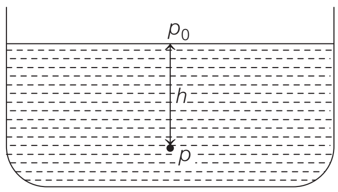
1. Pressure inside a fluid volume open to atmosphere is

$$p = p_0 + h\rho g$$

where, p = pressure at depth h, h = depth,

ρ = density of fluid

and $\quad g$ = acceleration due to gravity.



In problem given,

when $h = d_1$, pressure $p_1 = 5.05 \times 10^6$ Pa and when $h = d_2$, pressure $p_2 = 8.08 \times 10^6$ Pa

So, we have

$$p_1 = p_0 + d_1\rho g = 5.05 \times 10^6$$

and $$p_2 = p_0 + d_2\rho g = 8.08 \times 10^6$$

$\Rightarrow$ $$p_2 - p_1 = (d_2 - d_1)\rho g = 3.03 \times 10^6$$

$\Rightarrow$ $$d_2 - d_1 = \frac{3.03 \times 10^6}{\rho g}$$

Given, $\rho = 10^3\ \text{kgm}^{-3}$

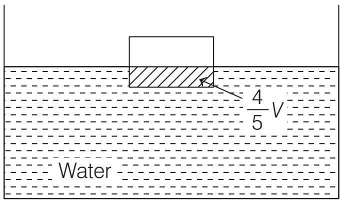
and $g = 10\ \text{ms}^{-2}$

$$\therefore\ d_2 - d_1 = \frac{3.03 \times 10^6}{10^3 \times 10} = 303\ \text{m} \approx 300\ \text{m}$$

2. *(a)* For a floating body,

upthrust = weight of the part of object, i.e. submerged in the fluid.

In first situation,



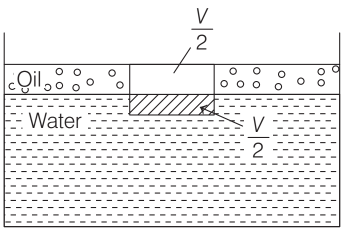
So, weight of block of volume V = weight of water of volume $\frac{4}{5}V \Rightarrow V\rho_b g = \frac{4}{5}V\rho_w g$

where, ρ_b = density of block

and ρ_w = density of water.

$$\Rightarrow \frac{\rho_b}{\rho_w} = \frac{4}{5} \quad \ldots \text{(i)}$$

In second situation,



So, weight of block of volume V = weight of oil of volume $\frac{V}{2}$ + weight of water of volume $\frac{V}{2}$.

$$\Rightarrow V\rho_b g = \frac{V}{2}\rho_o g + \frac{V}{2}\rho_w g$$

where, ρ_o = density of oil.

$$\Rightarrow 2\rho_b = \rho_o + \rho_w$$

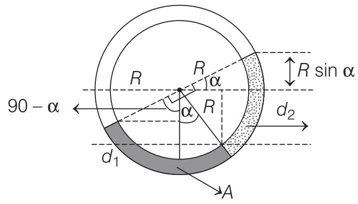
$$\Rightarrow \frac{2\rho_b}{\rho_w} = \frac{\rho_o}{\rho_w} + 1$$

$$\Rightarrow 2 \times \frac{4}{5} = \frac{\rho_o}{\rho_w} + 1 \quad \text{[using Eq. (i)]}$$

$$\Rightarrow \rho_0/\rho_w = 8/5 - 1 = 3/5 = 0.6$$

3. Equating pressure at A, we get

$$R\sin\alpha\ d_2 + R\cos\alpha\ d_2 + R(1-\cos\alpha)\ d_1 = R(1-\sin\alpha)\ d_1$$

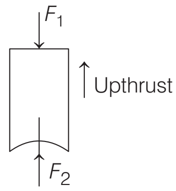


$$(\sin\alpha + \cos\alpha)\ d_2 = d_1(\cos\alpha - \sin\alpha)$$

$$\Rightarrow \frac{d_1}{d_2} = \frac{1+\tan\alpha}{1-\tan\alpha}$$

4. l will decrease, because the block moves up and h will decrease, because the coin will displace the volume of water (V_1) equal to its own volume when it is in the water whereas when it is on the block it will displace the volume of water (V_2) whose weight is equal to weight of coin and since density of coin is greater than the density of water, $V_1 < V_2$.

5. $F_2 - F_1$ = upthrust



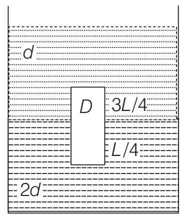
$\therefore$ $F_2 = F_1$ + upthrust

$$F_2 = (p_0 + \rho g h)\,\pi R^2 + V\rho g$$

$$= p_0\pi R^2 + \rho g\ (\pi R^2 h + V)$$

$\therefore$ Most appropriate option is (d).

6.



Considering vertical equilibrium of cylinder

Weight of cylinder = Upthrust due to upper liquid + upthrust due to lower liquid

$$\therefore\ (A/5)(L)Dg = (A/5)(3L/4)(d)g + (A/5)(L/4)(2d)(g)$$

$$\therefore\ D = \left(\frac{3}{4}\right)d + \left(\frac{1}{4}\right)(2d)$$

$$D = \frac{5}{4}d$$

7. Let density of material of sphere (in g/cm^3) be ρ.

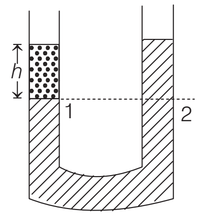
Applying the condition of floatation,

Weight = Upthrust

or $$V\rho g = \frac{V}{2}\rho_{\text{oil}} g + \frac{V}{2}\rho_{\text{Hg}} g$$

or $$\rho = \frac{\rho_{\text{oil}}}{2} + \frac{\rho_{\text{Hg}}}{2} = \frac{0.8}{2} + \frac{13.6}{2} = 7.2\ \text{g/cm}^3$$

8.



$$p_1 = p_2$$
$$p_0 + \rho_I gh = p_0 + \rho_{II} gh$$
$$\therefore \quad \rho_I = \rho_{II} \Rightarrow \rho_{II} = 1.1$$

9. In a freely falling system $g_{eff} = 0$ and since,

$$\text{Upthrust } = V_i \rho_L g_{eff}$$

(V_i = immersed volume, ρ_L = density of liquid)

$\therefore$ Upthrust = 0

10. $$w_{app} = w_{actual} - \text{Upthrust}$$

Upthrust $F = V_S \rho_L g$

Here, V_S = volume of solid, ρ_L = density of liquid

At higher temperature $F' = V_S' \rho_L' g$

$$\therefore \quad \frac{F'}{F} = \frac{V_S'}{V_S} \cdot \frac{\rho_L'}{\rho_L} = \frac{(1 + \gamma_S \Delta\theta)}{(1 + \gamma_L \Delta\theta)}$$

Since, $\gamma_S < \gamma_L$ (given)

$$\therefore \quad F' < F \text{ or } w_{app}' > w_{app}$$

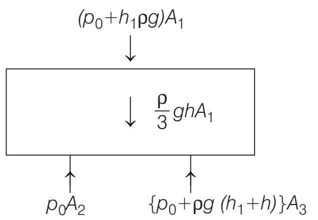
11. Let A_1 = Area of cross-section of cylinder = $4\pi r^2$

A_2 = Area of base of cylinder in air = πr^2

and A_3 = Area of base of cylinder in water

$= A_1 - A_2 = 3\pi r^2$

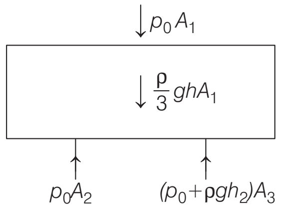
Drawing free body diagram of cylinder



Equating the net downward forces and net upward forces, we get, $h_1 = \frac{5}{3} h$.

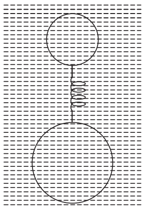
12. Again equating the forces, we get

$$h_2 = \frac{4h}{9}$$



13. For $h_2 < 4h/9$, buoyant force will further decrease. Hence, the cylinder remains at its original position.

14.



On small sphere

$$\frac{4}{3}\pi R^3(\rho)g + kx = \frac{4}{3}\pi R^3(2\rho)g \quad \dots\text{(i)}$$

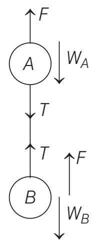
On second sphere (large)

$$\frac{4}{3}\pi R^3(3\rho)g = \frac{4}{3}\pi R^3(2\rho)g + kx \quad \dots\text{(ii)}$$

By Eqs. (i) and (ii), we get

$$x = \frac{4\pi R^3 \rho g}{3k}$$

15. F = Upthrust $= V d_F g$



Equilibrium of A,

$$V d_F g = T + W_A = T + V d_A g \quad \dots\text{(i)}$$

Equilibrium of B,

$$T + V d_F g = V d_B g \quad \dots\text{(ii)}$$

Adding Eqs. (i) and (ii), we get

$$2d_f = d_A + d_B$$

$\therefore$ Option (d) is correct

From Eq. (i) we can see that $d_F > d_A$ (as $T > 0$)

$\therefore$ Option (a) is correct.

From Eq. (ii) we can see that, $d_B > d_F$

$\therefore$ Option (b) is correct.

$\therefore$ Correct options are (a), (b) and (d).

16. Liquid will apply an upthrust on m. An equal force will be exerted (from Newton's third law) on the liquid. Hence, A will read less than 2 kg and B more than 5 kg. Therefore, the correct options are (b) and (c).

17. When ice melts level of water does not change. In case of lead, it was initially floating i.e. it would had displaced the water equal to the weight of lead. So, volume of water displaced would be,

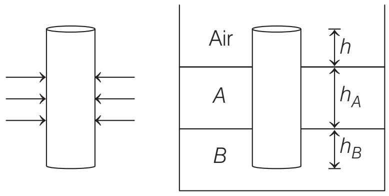
$$V_1 = \frac{m}{\rho_w} \ (m = \text{ mass of lead})$$

Now, when ice melts lead will sink and it would displace the water equal to the volume of lead itself. So, volume of water displaced in this case would be,

$V_2 = \frac{m}{\rho_l}$. Now, as $\rho_l > \rho_w, V_2 < V_1$ or level will fall.

18. On increasing the temperature of mercury its density will decrease. Hence, level of mercury in barometer tube will increase.

19. (a) Liquid A is applying the hydrostatic force on cylinder from all the sides. So, net force is zero.



(b) In equilibrium

Weight of cylinder = Net upthrust on the cylinder

Let s be the area of cross-section of the cylinder, then

weight $= (s)(h + h_A + h_B)\rho_{\text{cylinder}} g$

and upthrust on the cylinder

= upthrust due to liquid A + upthrust due to liquid B

$= sh_A\rho_A g + sh_B\rho_B g$

Equating these two,

$$s(h + h_A + h_B)\rho_{\text{cylinder}} g = sg(h_A\rho_A + h_B\rho_B)$$

or $\quad (h + h_A + h_B)\rho_{\text{cylinder}} = h_A\rho_A + h_B\rho_B$

Substituting,

$h_A = 1.2\,\text{cm}, h_B = 0.8\,\text{cm}$ and $\rho_A = 0.7\,\text{g/cm}^3$

$\rho_B = 1.2\ \text{g/cm}^3$ and $\rho_{\text{cylinder}} = 0.8\,\text{g/cm}^3$

In the above equation, we get $h = 0.25\text{cm}$

(c) Net upward force = extra upthrust $= sh\rho_B g$

$\therefore$ Net acceleration $a = \dfrac{\text{Force}}{\text{Mass of cylinder}}$

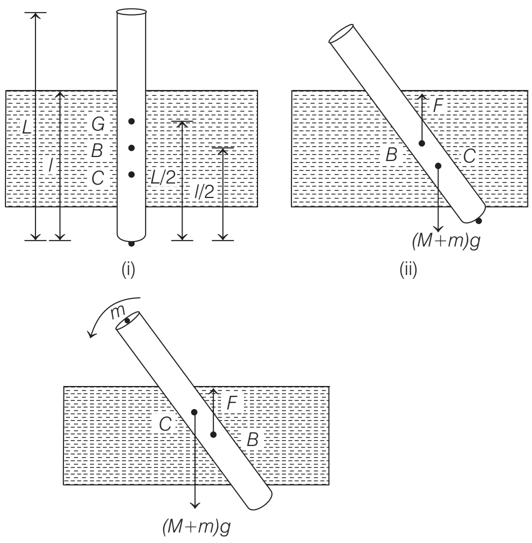
or $\quad a = \dfrac{sh\rho_B g}{s(h + h_A + h_B)\rho_{\text{cylinder}}}$

or $\quad a = \dfrac{h\rho_B g}{(h + h_A + h_B)\rho_{\text{cylinder}}}$

Substituting the values of h, h_A h_B, ρ_B and ρ_{cylinder},

we get, $a = \dfrac{g}{6}$ (upwards)

20. Let M = Mass of stick $= \pi R^2 \rho L$



l = Immersed length of the rod

G = CM of rod

B = Centre of buoyant force (F)

C = CM of rod + mass (m)

Y_{CM} = Distance of C from bottom of the rod

Mass m should be attached to the lower end because otherwise B will be below G and C will be above G and the torque of the couple of two equal and opposite forces F and $(M + m)g$ will be counter clockwise on displacing the rod leftwards. Therefore, the rod cannot be in rotational equilibrium. See the figure (iii).

Now, refer figures (i) and (ii).

For vertical equilibrium $Mg + mg = F$ (upthrust)

or $(\pi R^2 L)\rho g + mg = (\pi R^2\, l)\sigma\, g$

$$\therefore \qquad l = \left\{\frac{\pi R^2 L\rho + m}{\pi R^2\sigma}\right\}$$

Position of CM (of rod + m) from bottom

$$Y_{\text{CM}} = \frac{M.\frac{L}{2}}{M + m} = \frac{(\pi R^2 L\rho)\frac{L}{2}}{(\pi R^2 L\rho) + m}$$

Centre of buoyancy (B) is at a height of $\dfrac{l}{2}$ from the bottom.

We can see from figure (ii) that for rotational equilibrium of the rod, B should either lie above C or at the same level of B.

Therefore, $\qquad \dfrac{l}{2} \geq Y_{\text{CM}}$

or $\qquad \dfrac{\pi R^2 L\rho + m}{2\pi R^2\sigma} \geq \dfrac{(\pi R^2 L\rho)\frac{L}{2}}{(\pi R^2 L\rho) + m}$

or $\qquad m + \pi R^2 L\rho \geq \pi R^2 L\sqrt{\rho\sigma}$

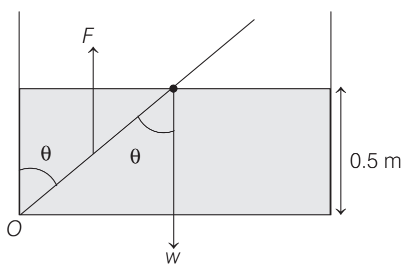
or $\qquad m \geq \pi R^2 L(\sqrt{\rho\sigma} - \rho)$

$\therefore$ Minimum value of m is $\pi R^2 L(\sqrt{\rho\sigma} - \rho)$.

21. Submerged length $= 0.5\sec\theta$, F = Upthrust, w = Weight

Three forces will act on the plank.

(a) Weight which will act at centre of plank.



(b) Upthrust which will act at centre of submerged portion.

(c) Force from the hinge at O.

Taking moments of all three forces about point O. Moment of hinge force will be zero.

$\therefore$ Moment of w (clockwise)

$$= \text{Moment of } F \text{ (anti-clockwise)}$$

$$\therefore \quad (Alg\rho)\frac{l}{2}\sin\theta = A\,(0.5\sec\theta)(\rho_w)(g)\left(\frac{0.5\sec\theta}{2}\right)\sin\theta$$

$$\therefore \quad \cos^2\theta = \frac{(0.5)^2\,(1)}{(l^2)(\rho)}$$

$$= \frac{(0.5)^2}{(0.5)} = \frac{1}{2} \quad (\text{as } l = 1\,\text{m})$$

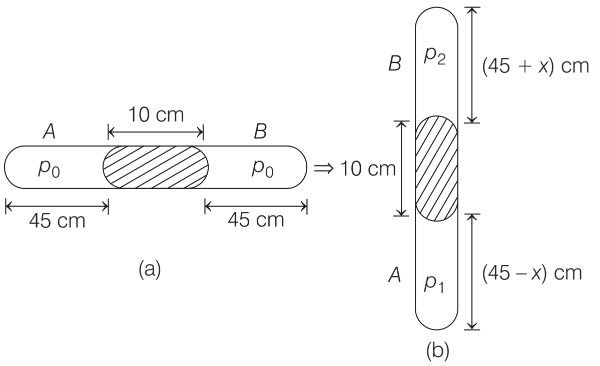
$$\therefore \quad \cos\theta = \frac{1}{\sqrt{2}}$$

or $\quad \theta = 45°$

22. When stones were floating with boat, they will be displacing water of volume (say V_1) whose weight should be equal to weight of stones. When the stones sink, they will displace water of volume (say V_2) whose volume is equal to the volume of stones. But since density of water is less than the density of stones.

$\therefore \quad V_1 > V_2 \quad$ or $\quad$ level will fall.

23. Let area of cross-section of the tube be A.



Applying $p_1V_1 = p_2V_2$ in A and B we have,

$$p_0(45)(A) = p_1\,(45 - x)\,A$$

or $\quad 76 \times 45 = p_1(45 - x) \quad \ldots$(i)

$$p_0(45)(A) = p_2\,(45 + x)\,A$$

$\therefore \quad 76 \times 45 = p_2(45 + x) \quad \ldots$(ii)

From Eqs. (i) and (ii), we get

$$(p_1 - p_2) = 76 \times 45\left(\frac{1}{45 - x} - \frac{1}{45 + x}\right)$$

From figure (b),

$$(p_1 - p_2)\,A = \text{Weight of 10 cm of Hg column}$$

or $\quad p_1 - p_2 =$ Pressure equivalent to 10 cm of Hg column

$$76 \times 45\left(\frac{1}{45 - x} - \frac{1}{45 + x}\right) = 10$$

Solving this equation, we get

$$x = 2.95\,\text{cm}$$

Topic 3 Ideal Fluids in Motion

1. **Key Idea** Terminal speed of a sphere falling in a viscous fluid is

$$v_T = \frac{2}{9}\frac{r^2}{\eta}(\rho_0 - \rho_f)g$$

where, η = coefficient of viscosity of fluid,
ρ_0 = density of falling sphere
and ρ_f = density of fluid.

As we know, if other parameters remains constant, terminal velocity is proportional to square of radius of falling sphere.

i.e. $\quad v_T \propto r^2 \quad \ldots$(i)

Now, when sphere of radius R is broken into 27 identical solid sphere of radius r, then Volume of sphere of radius $R = 27 \times$ Volume of sphere of radius r

$$\Rightarrow \quad \frac{4}{3}\pi R^3 = 27 \times \frac{4}{3}\pi r^3$$

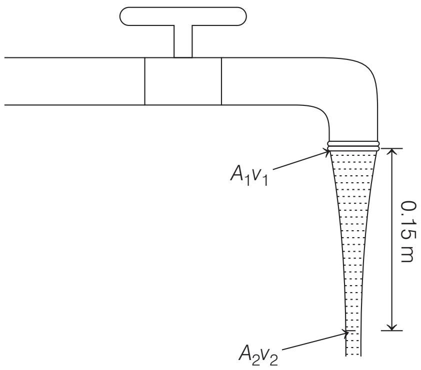
$$\Rightarrow \quad R = 3r$$

$$\Rightarrow \quad r = \frac{R}{3}$$

So, from Eq. (i), we have

$$\frac{v_1}{v_2} = \frac{R^2}{\left(\frac{R}{3}\right)^2} = 9$$

2 Given situation is as shown in the figure below



From equation of continuity,

$$A \propto \frac{1}{v}$$

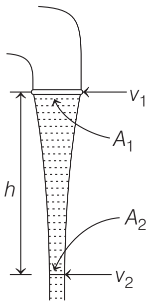
where, A = area of flow
and $\quad v$ = velocity of flow.

$\therefore$ Increase in speed of flow causes a decrease in area of flow.

Here given that height of fall, $h = 0.15$ m

Area, $\quad A = 10^{-4}\,\text{m}^2$

Initial speed, $\quad v = 1\,\text{ms}^{-1}$



Velocity of water stream below h height is

$$v_2 = \sqrt{v_1^2 + 2gh} \qquad [\because v^2 - u^2 = 2gh]$$

Substituting the given values, we get

$$= \sqrt{1^2 + 2 \times 10 \times 0.15} = \sqrt{4} = 2 \text{ ms}^{-1}$$

Now, from equation of continuity, we have

$$A_1 v_1 = A_2 v_2$$

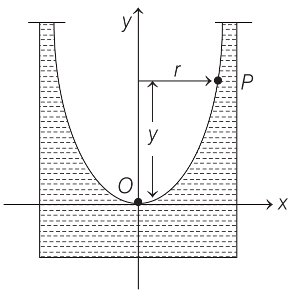
or $\quad A_2 = \dfrac{A_1 v_1}{v_2}$

$\therefore \quad A_2 = \dfrac{10^{-4} \times 1}{2}$

$$= 0.5 \times 10^{-4}$$

$$= 5 \times 10^{-5} \text{ m}^2$$

3 When liquid filled vessel is rotated the liquid profile becomes a paraboloid due to centripetal force, as shown in the figure below



Pressure at any point P due to rotation is

$$p_R = \frac{1}{2}\rho r^2 \omega^2$$

Gauge pressure at depth y is $p_G = -\rho g y$

If p_0 is atmospheric pressure, then total pressure at point P is

$$p = p_0 + \frac{1}{2}\rho r^2 \omega^2 - \rho g y$$

For any point on surface of rotating fluid,

$$p = p_0$$

Hence, for any surface point;

$$p_0 = p_0 + \frac{1}{2}\rho r^2 \omega^2 - \rho g y$$

or $\quad \dfrac{1}{2}\rho r^2 \omega^2 = \rho g y$

or $\quad y = \dfrac{r^2 \omega^2}{2g} \qquad \ldots \text{(i)}$

In the given case,

Angular speed, $\omega = 2\text{rps} = 2 \times 2\pi = 4\pi \text{ rad s}^{-1}$

Radius of vessel, $r = 5 \text{ cm} = 0.05 \text{ m}$

and $g = 10 \text{ ms}^{-2}$

Hence, substituting these values in Eq. (i), we get

$$y = \frac{\omega^2 r^2}{2g} = \frac{(4\pi)^2 (0.05)^2}{2 \times 10} = 0.02 \text{ m} = 2 \text{ cm}$$

4 Mass per unit time of a liquid flow is given by

$$\frac{dm}{dt} = \rho A v$$

where, ρ is density of liquid, A is area through which it is flowing and v is velocity.

$\therefore$ Rate of change in momentum of the 25% of liquid which loses all momentum is

$$\frac{dp_1}{dt} = \frac{1}{4}\left(\frac{dm}{dt}\right) v = \frac{1}{4}\rho A v^2 \qquad \ldots \text{(i)}$$

and the rate of change in momentum of the 25% of the liquid which comes back with same speed.

$$\frac{dp_2}{dt} = \frac{1}{4}\left(\frac{dm}{dt}\right) \times 2v = \frac{1}{2}\rho A v^2 \qquad \ldots \text{(ii)}$$

[$\because$ Net change in velocity is = $2v$]

$\therefore$ Net pressure on the mesh is

$$p = \frac{F_{\text{net}}}{A} = \frac{(dp_1/dt + dp_2/dt)}{A} \qquad \left[\because F = \frac{dp}{dt}\right]$$

$\therefore$ From Eqs. (i) and (ii), we get

$$p = \frac{3}{4}\rho v^2 A / A = \frac{3}{4}\rho v^2$$

5 As, level of water in tank remains constant with time, so

(water inflow rate) = (outflow rate of water)

$\Rightarrow 10^{-4} \text{ m}^3\text{s}^{-1}$ = Area of orifice × Velocity of outflow

$\Rightarrow 10^{-4} \text{ m}^3\text{s}^{-1} = 10^{-4} \times \sqrt{2gh}$

where, h = Height of water above the orifice or hole.

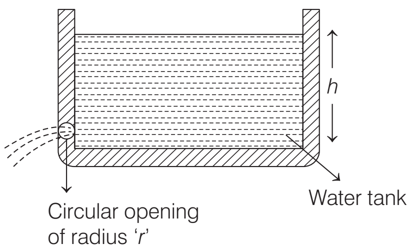
$\Rightarrow \sqrt{2gh} = 1$

or $\; 2 \times 9.8 \times h = 1$

$\Rightarrow \quad h = \dfrac{1}{19.6} \text{ m} = \dfrac{100}{19.6} \text{cm}$

or $\quad h = 5.1 \text{ cm}$

6 For the given condition, a water tank is open to air and its water level maintained.



Suppose the depth of the centre of the opening from level of water in tank is h and the radius of opening is r.

According to question, the water per minute through a circular opening flow rate $(Q) = 0.74\ \text{m}^3/\text{min}$

$$= \frac{0.74}{60}\ \text{m}^3/\text{s}$$

r = radius of circular opening = 2 cm

Here, the area of circular opening = $\pi(r^2)$

$$a = \pi \times (2 \times 10^{-2})^2 = 4\pi \times 10^{-4}\ \text{m}^2$$

Now flow rate through an area is given by

$$Q = \text{Area of circular opening} \times \text{Velocity of water}$$

$$Q = a \times v = \pi(r^2) \times v$$

$$\Rightarrow \quad \frac{0.74}{60} = (4\pi \times 10^{-4}) \times v \qquad \dots\text{(i)}$$

According to Torricelli's law (velocity of efflux)

$$v = \sqrt{2gh} \qquad \dots\text{(ii)}$$

Equation value of 'v' from (i) and (ii) we get,

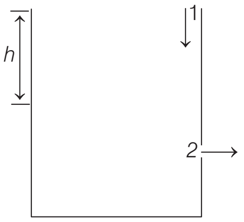
$$\sqrt{2gh} = \frac{0.74 \times 10^4}{60 \times 4\pi}$$

$$\Rightarrow \quad h = \left(\frac{0.74 \times 10^4}{60 \times 4\pi}\right)^2 \times \frac{1}{2g}$$

$$h \approx 4.8\ \text{m}$$

7. Applying continuity equation at 1 and 2, we have

$$A_1 v_1 = A_2 v_2 \qquad \dots\text{(i)}$$



Further applying Bernoulli's equation at these two points, we have

$$p_0 + \rho g h + \frac{1}{2}\rho v_1^2 = p_0 + 0 + \frac{1}{2}\rho v_2^2 \qquad \dots\text{(ii)}$$

Solving Eqs. (i) and (ii), we have $v_2^2 = \dfrac{2gh}{1 - \dfrac{A_2^2}{A_1^2}}$

Substituting the values, we have

$$v_2^2 = \frac{2 \times 10 \times 2.475}{1 - (0.1)^2} = 50\ \text{m}^2/\text{s}^2$$

8. Velocity of efflux at a depth h is given by $v = \sqrt{2gh}$. Volume of water flowing out per second from both the holes are equal.

$$\therefore \quad a_1 v_1 = a_2 v_2$$

or $\quad (L^2)\sqrt{2g(y)} = \pi R^2 \sqrt{2g\,(4y)}$ or $R = \dfrac{L}{\sqrt{2\pi}}$

9. From conservation of energy

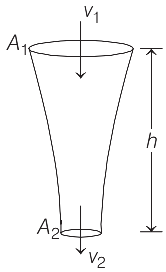
$$v_2^2 = v_1^2 + 2gh \qquad \dots\text{(i)}$$

[can also be found by applying Bernoulli's theorem]

From continuity equation

$$A_1 v_1 = A_2 v_2$$

$$v_2 = \left(\frac{A_1}{A_2}\right) v_1 \qquad \dots\text{(ii)}$$



Substituting value of v_2 from Eq. (ii) in Eq. (i),

$$\frac{A_1^2}{A_2^2}.\, v_1^2 = v_1^2 + 2gh \text{ or } A_2^2 = \frac{A_1^2\ v_1^2}{v_1^2 + 2gh}$$

$$\therefore \quad A_2 = \frac{A_1\ v_1}{\sqrt{v_1^2 + 2gh}}$$

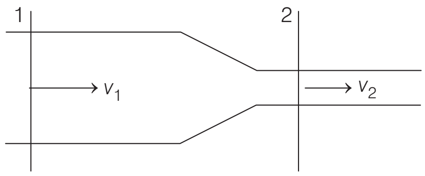
Substituting the given values

$$A_2 = \frac{(10^{-4})(1.0)}{\sqrt{(1.0)^2 + 2\,(10)\,(0.15)}}$$

$$A_2 = 5.0 \times 10^{-5}\ \text{m}^2$$

10. From continuity equation

$$A_1 v_1 = A_2 v_2$$



$$\therefore \quad v_2 = \left(\frac{A_1}{A_2}\right) v_1 = \left(\frac{10}{5}\right)(1)$$

$$= 2\ \text{m/s}$$

Applying Bernoulli's theorem at 1 and 2

$$p_2 + \frac{1}{2}\rho v_2^2 = p_1 + \frac{1}{2}\rho v_1^2$$

$$\therefore \quad p_2 = p_1 + \frac{1}{2}\rho\,(v_1^2 - v_2^2)$$

$$= \left(2000 + \frac{1}{2} \times 10^3\ (1 - 4)\right)$$

$$\therefore \quad p_2 = 500\ \text{Pa}$$

11. From equation of continuity (Av = constant)

$$\frac{\pi}{4}(8)^2\ (0.25) = \frac{\pi}{4}(2)^2\ (v) \qquad \dots\text{(i)}$$

Here, v is the velocity of water with which water comes out of the syringe (Horizontally).

Solving Eq. (i), we get $\quad v = 4$ m/s

The path of water after leaving the syringe will be a parabola. Substituting proper values in equation of trajectory.

$$y = x \tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

According to question, we have,

$$-1.25 = R\ \tan 0° - \frac{(10)\,(R^2)}{(2)\,(4)^2\cos^2 0°}$$

(R = horizontal range)

Solving this equation, we get $R = 2$ m

12. Given, $A_1 = 4 \times 10^{-3}\,\text{m}^2$, $A_2 = 8 \times 10^{-3}\,\text{m}^2$,

$h_1 = 2\,\text{m}$, $h_2 = 5\,\text{m}$,

$v_1 = 1\,\text{m/s}$ and $\rho = 10^3\,\text{kg/m}^3$

From continuity equation, we have

$$A_1 v_1 = A_2 v_2 \quad \text{or} \quad v_2 = \left(\frac{A_1}{A_2}\right) v_1$$

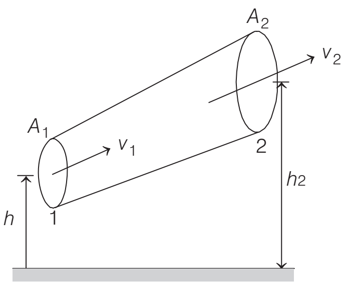
or $$v_2 = \left(\frac{4 \times 10^{-3}}{8 \times 10^{-3}}\right)(1\text{ m/s})$$

$\Rightarrow$ $$v_2 = \frac{1}{2}\text{ m/s}$$

Applying Bernoulli's equation at sections 1 and 2

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho\, v_2^2 + \rho g h_2$$

or $$p_1 - p_2 = \rho g\,(h_2 - h_1) + \frac{1}{2}\rho\,(v_2^2 - v_1^2) \qquad \text{...(i)}$$



(a) Work done per unit volume by the pressure as the fluid flows from P to Q.

$$W_1 = p_1 - p_2$$
$$= \rho g\,(h_2 - h_1) + \frac{1}{2}\rho\,(v_2^2 - v_1^2) \qquad \text{[From Eq. (i)]}$$
$$= \left\{(10^3)(9.8)(5-2) + \frac{1}{2}(10^3)\left(\frac{1}{4} - 1\right)\right\}$$
$$= [29400 - 375] = 29025\,\text{J/m}^3$$

(b) Work done per unit volume by the gravity as the fluid flows from P to Q.

$$W_2 = \rho g\,(h_1 - h_2)$$
$$= \{(10^3)(9.8)(2-5)\}$$

or $$W_2 = -29400\,\text{J/m}^3$$

13. (a) Mass of water = (Volume) (density)

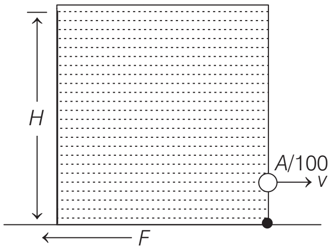
$\therefore$ $$m_0 = (AH)\rho$$

$\therefore$ $$H = \frac{m_0}{A\rho} \qquad \text{...(i)}$$

Velocity of efflux,

$$v = \sqrt{2gH} = \sqrt{2g\frac{m_0}{A\rho}}$$
$$= \sqrt{\frac{2m_0 g}{A\rho}}$$

Thrust force on the container due to draining out of liquid from the bottom is given by,



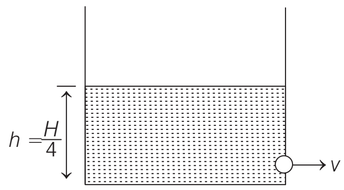
F = (density of liquid) (area of hole)(velocity of efflux)^2

$$F = \rho a v^2$$
$$F = \rho(A/100)v^2$$
$$= \rho(A/100)\left(\frac{2m_0 g}{A\rho}\right)$$
$$F = \frac{m_0 g}{50}$$

$\therefore$ Acceleration of the container, $a = F/m_0 = g/50$

(b) Velocity of efflux when 75% liquid has been drained out

i.e. height of liquid, $h = \dfrac{H}{4} = \dfrac{m_0}{4A\rho}$



$$v = \sqrt{2gh} = \sqrt{2g\left(\frac{m_0}{4A\rho}\right)}$$
$$v = \sqrt{\frac{m_0 g}{2A\rho}}$$

14. (a) (i) Considering vertical equilibrium of cylinder

Weight of cylinder = upthrust due to upper liquid + upthrust due to lower liquid

$$\therefore \left(\frac{A}{5}\right)(L)D.g = \left(\frac{A}{5}\right)\left(\frac{3L}{4}\right)(d)g + \left(\frac{A}{5}\right)\left(\frac{L}{4}\right)(2d)(g)$$

$$\therefore \quad D = \left(\frac{3}{4}\right)d + \left(\frac{1}{4}\right)(2d)$$

$$\Rightarrow \quad D = \frac{5}{4}d$$

(ii) Considering vertical equilibrium of two liquids and the cylinder.

$(p - p_0)A$ = weight of two liquids + weight of cylinder

$$\therefore \frac{\text{weight of two liquids + weight of cylinder}}{A} \qquad \text{...(i)}$$

Now, weight of cylinder

$$= \left(\frac{A}{5}\right)(L)(D)(g) = \left(\frac{A}{5}Lg\right)\left(\frac{5}{4}d\right) = \frac{ALdg}{4}$$

Weight of upper liquid $= \left(\dfrac{H}{2}Adg\right)$ and

Weight of lower liquid $= \frac{H}{2} A (2d) g$

$= HAgd$

$\therefore$ Total weight of two liquids $= \frac{3}{2} HAdg$

$\therefore$ From Eq. (i) pressure at the bottom of the container will be

$$p = p_0 + \frac{\left(\frac{3}{2}\right) HAdg + \frac{ALdg}{4}}{A}$$

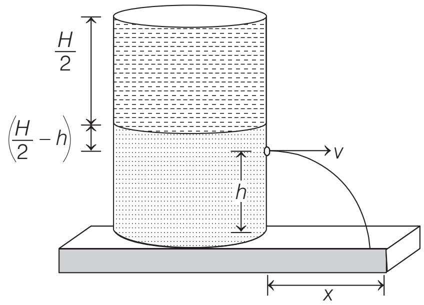
or $$p = p_0 + \frac{dg\,(6H + L)}{4}$$

(b) (i) Applying Bernoulli's theorem,

$$p_0 + dg\left(\frac{H}{2}\right) + 2dg\left(\frac{H}{2} - h\right) = p_0 + \frac{1}{2}(2d)\,v^2$$

Here, v is velocity of efflux at 2.
Solving this, we get

$$v = \sqrt{(3H - 4h)\frac{g}{2}}$$



(ii) Time taken to reach the liquid to the bottom will be

$$t = \sqrt{2h/g}$$

$\therefore$ Horizontal distance x travelled by the liquid is

$$x = vt = \sqrt{\left(3H - 4h\right)\frac{g}{2}}\left(\sqrt{\frac{2h}{g}}\right)$$

$$x = \sqrt{h\,(3H - 4h)}$$

(iii) For x to be maximum $\frac{dx}{dh} = 0$

or $$\frac{1}{2\sqrt{h(3H - 4h)}}(3H - 8h) = 0$$

or $$h = \frac{3H}{8}$$

Therefore, x will be maximum at $h = \frac{3H}{8}$

The maximum value of x will be

$$x_m = \sqrt{\left(\frac{3H}{8}\right)\left[3H - 4\left(\frac{3H}{8}\right)\right]}$$

$$x_m = \frac{3}{4}H$$

Topic 4 Surface Tension and Viscosity

1. Given,

$$\frac{T_{Hg}}{T_W} = 7.5, \quad \frac{\rho_{Hg}}{\rho_W} = 13.6$$

and $$\frac{\cos\theta_{Hg}}{\cos\theta_W} = \frac{\cos 135^\circ}{\cos 0^\circ} = -\frac{1}{\sqrt{2}}$$

Height of the fluid inside capillary tube is given by

$$h = \frac{2T\cos\theta}{\rho g r}$$

According to given situation, $h_w = h_{Hg}$

$$\therefore \quad \frac{2T_w \cos\theta_w}{\rho_w\, g\, r_w} = \frac{2T_{Hg}\cos\theta_{Hg}}{\rho_{Hg}\, g\, r_{Hg}}$$

$$\therefore \quad \frac{r_{Hg}}{r_w} = \left(\frac{T_{Hg}}{T_w}\right)\left(\frac{\cos\theta_{Hg}}{\cos\theta_w}\right)\left(\frac{\rho_w}{\rho_{Hg}}\right)$$

Given, $r_{Hg} = r_1$ and $r_w = r_2$, then

Substituting the given values, we get

$$\frac{r_{Hg}}{r_w} = \frac{r_1}{r_2} = 7.5 \times \frac{1}{\sqrt{2}} \times \frac{1}{13.6}$$

$$= 0.4 = 2/5$$

2. Height of liquid rise in capillary tube,

$$h = \frac{2T\cos\theta_c}{\rho r g}$$

$$\Rightarrow \quad h \propto \frac{1}{r}$$

So, when radius is doubled, height becomes half.

$\therefore$ $h' = h/2$

Now, density $(\rho) = \frac{\text{mass }(M)}{\text{volume }(V)}$

$$\Rightarrow \quad M = \rho \times \pi r^2 h$$

$$\therefore \quad M' = \rho\pi r'^2 h'$$

$$\therefore \quad \frac{M'}{M} = \frac{r'^2 h'}{r^2 h} = \frac{(2r)^2 (h/2)}{r^2 h} = 2$$

$$\Rightarrow \quad M' = 2M$$

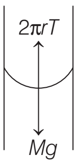
Alternate Solution

According to the given figure, force inside the capillary tube is

$2\pi rT = Mg \Rightarrow M \propto r$

When $r' = 2r$, then $M' = 2M$

3. By ascent formula, we have surface tension,



$$T = \frac{rhg}{2} \times 10^3 \,\frac{\text{N}}{\text{m}}$$

$$= \frac{dhg}{4} \times 10^3 \,\frac{\text{N}}{\text{m}}$$

$$\Rightarrow \quad \frac{\Delta T}{T} = \frac{\Delta d}{d} + \frac{\Delta h}{h}$$ [given, g is constant]

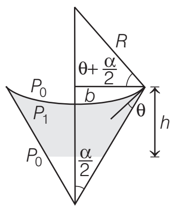
So, percentage $= \frac{\Delta T}{T} \times 100 = \left(\frac{\Delta d}{d} + \frac{\Delta h}{h}\right) \times 100$

$$= \left(\frac{0.01 \times 10^{-2}}{1.25 \times 10^{-2}} + \frac{0.01 \times 10^{-2}}{1.45 \times 10^{-2}} \right) \times 100$$

$$= 1.5\%$$

$$\therefore \quad \frac{\Delta T}{T} \times 100 = 1.5\%$$

4. Let R be the radius of the meniscus formed with a contact angle θ. By geometry, this radius makes an angle $\theta + \frac{\alpha}{2}$ with the horizontal and,



$$\cos\left(\theta + \frac{\alpha}{2}\right) = b/R \quad \text{...(i)}$$

Let P_0 be the atmospheric pressure and P_1 be the pressure just below the meniscus. Excess pressure on the concave side of meniscus of radius R is,

$$P_0 - P_1 = 2S/R \quad \text{...(ii)}$$

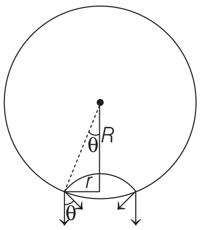
The hydrostatic pressure gives,

$$P_0 - P_1 = h\rho g \quad \text{...(iii)}$$

Eliminate $(P_0 - P_1)$ from second and third equations and substitute R from first equation to get,

$$h = \frac{2S}{\rho g R} = \frac{2S}{b\rho g} \cos\left(\theta + \frac{\alpha}{2}\right)$$

5. The bubble will detach if,



$$\int \sin\theta T \times dl = T(2\pi r)\sin\theta$$

Buoyant force $\geq$ Surface tension force

$$\frac{4}{3}\pi R^3 \rho_w g \geq \int T \times dl \sin\theta$$

$$(\rho_w)\left(\frac{4}{3}\pi R^3\right) g \geq (T)(2\pi r)\sin\theta$$

$$\Rightarrow \quad \sin\theta = \frac{r}{R}$$

Solving, $r = \sqrt{\frac{2\rho_w R^4 g}{3T}} = R^2 \sqrt{\frac{2\rho_w g}{3T}}$

No option matches with the correct answer.

6. Decrease in surface energy = heat required in vaporisation.

$$S = 4\pi r^2$$

$$\therefore \quad dS = 2(4\pi r)\,dr$$

$$\therefore \quad T(dS) = L(dm)$$

$$\therefore \quad T(2)(4\pi r)dr = L(4\pi r^2 dr)\rho$$

$$\therefore \quad r = \frac{2T}{\rho L}$$

7. $\Delta p_1 = \frac{4T}{r_1}$ and $\Delta p_2 = \frac{4T}{r_2}$

$$r_1 < r_2 \Rightarrow \Delta p_1 > \Delta p_2$$

$\therefore$ Air will flow from 1 to 2 and volume of bubble at end 1 will decrease.

Therefore, correct option is (b).

8. Force from right hand side liquid on left hand side liquid.

(i) Due to surface tension force $= 2RT$ (towards right)

(ii) Due to liquid pressure force

$$= \int_{x=0}^{x=h} (p_0 + \rho g h)(2R \cdot x)\,dx$$

$$= (2p_0 Rh + R\rho g h^2) \text{ (towards left)}$$

$\therefore$ Net force is $|2p_0 Rh + R\rho g h^2 - 2RT|$

9. $h = \frac{2\sigma\cos\theta}{r\rho g}$

(a) $\rightarrow h \propto \frac{1}{r}$

(b) h depends upon σ.

(c) If lift is going up with constant acceleration.

$$g_{\text{eff}} = (g + a) \quad \Rightarrow h = \frac{2\sigma\cos\theta}{r\rho(g + a)}$$

It means h decreases.

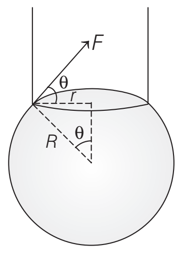
(d) h is proportional to $\cos\theta$.

10. Vertical force due to surface tension,

$$F_v = F\sin\theta = (T2\pi r)(r/R) = \frac{2\pi r^2 T}{R}$$

$\therefore$ Correct option is (c).

11. $\frac{2\pi r^2 T}{R} = mg = \frac{4}{3}\pi R^3 \cdot \rho \cdot g$



$$\therefore \quad R^4 = \frac{3r^2 T}{2\rho g} = \frac{3 \times (5 \times 10^{-4})^2 (0.11)}{2 \times 10^3 \times 10}$$

$$= 4.125 \times 10^{-12} \text{ m}^4$$

$$\therefore \quad R = 1.425 \times 10^{-3} \text{m} \approx 1.4 \times 10^{-3} \text{ m}$$

$\therefore$ Correct option is (a).

12. Surface energy, $E = (4\pi R^2)T$

$$= (4\pi)(1.4 \times 10^{-3})^2 (0.11)$$

$$= 2.7 \times 10^{-6} \text{ J}$$

$\therefore$ Correct option is (b).

13. From mass conservation,

$$\rho \cdot \frac{4}{3}\pi R^3 = \rho \cdot K \cdot \frac{4}{3}\pi r^3 \Rightarrow R = K^{1/3} r$$

$$\therefore \quad \Delta U = T\Delta A = T(K \cdot 4\pi r^2 - 4\pi R^2)$$

$$= T(K \cdot 4\pi R^2 K^{-2/3} - 4\pi R^2)$$

$$\Delta U = 4\pi R^2 T[K^{1/3} - 1]$$

Putting the values, we get

$$10^{-3} = \frac{10^{-1}}{4\pi} \times 4\pi \times 10^{-4}[K^{1/3} - 1]$$

$$100 = K^{1/3} - 1$$

$\Rightarrow \quad K^{1/3} \cong 100 = 10^2$

Given that $\quad K = 10^{\alpha}$

$\therefore \quad 10^{\alpha/3} = 10^2$

$\Rightarrow \quad \dfrac{\alpha}{3} = 2 \Rightarrow \alpha = 6$

14. Terminal velocity is given by

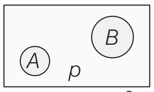
$$v_T = \frac{2}{9}\frac{r^2}{\eta}(d-\rho)g$$

$$\frac{v_P}{v_Q} = \frac{r_P^2}{r_Q^2} \times \frac{\eta_Q}{\eta_P} \times \frac{(d-\rho_P)}{(d-\rho_Q)}$$

$$= \left(\frac{1}{0.5}\right) \times \left(\frac{2}{3}\right) \times \frac{(8-0.8)}{(8-1.6)}$$

$$= 4 \times \frac{2}{3} \times \frac{7.2}{6.4} = 3$$

15. Although not given in the question, but we will have to assume that temperatures of A and B are same.



$p = 8\ \text{Nm}^{-2}$

$$\frac{n_B}{n_A} = \frac{p_B V_B / RT}{p_A V_A / RT} = \frac{p_B V_B}{p_A V_A}$$

$$= \frac{p + 4S / r_A \times 4/3\pi\,(r_A)^3}{(p + 4S/r_B) \times 4/3\pi\,(r_B)^3}$$

(S = surface tension)

Substituting the values, we get

$$\frac{n_B}{n_A} = 6$$

16. Terminal velocity $v_T = \dfrac{2r^2 g}{9\eta}(\rho_S - \rho_L)$

and viscous force $F = 6\pi\eta r v_T$

Rate of production of heat (power) : as viscous force is the only dissipative force.

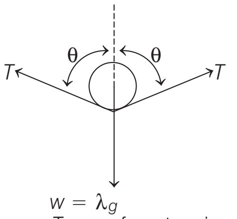
Hence,

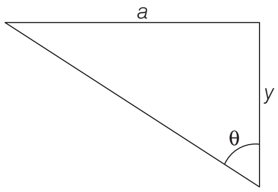
$$\frac{dQ}{dt} = F v_T = (6\pi\eta r v_T)(v_T) = 6\pi\eta r v_T^2$$

$$= 6\pi\eta r \left\{\frac{2}{9}\frac{r^2 g}{\eta}(\rho_S - \rho_L)\right\}^2$$

$$= \frac{8\pi g^2}{27\eta}(\rho_S - \rho_L)^2 r^5 \quad \text{or} \quad \frac{dQ}{dt} \propto r^5$$

17. Free body diagram of the wire is as shown in figure.





Considering the equilibrium of wire in vertical direction, we have

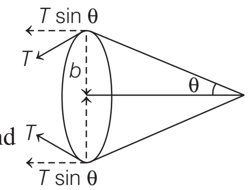
$$2Tl\cos\theta = \lambda l g \qquad \text{...(i)}$$

For $\quad y << a, \cos\theta \approx \dfrac{y}{a}$

Substituting the values in Eq. (i), we get

$$T = \frac{\lambda a}{2y}$$

18. Surface Tension force
$= 2\pi b \times 2T\sin\theta$



Mass of the air per second entering the bubble $= \rho A v$

Momentum of air per second = Force due to air $= \rho A v^2$

The bubble will separate from the tube when force due to moving air becomes equal to the surface tension force inside the bubble.

$$2\pi b \times 2T\sin\theta = \rho A v^2$$

putting $\sin\theta = \dfrac{b}{r}$, $A = \pi b^2$ and solving, we get

$$r = \frac{4T}{\rho v^2}$$

19. When the tube is not there,

$$p + p_0 + \frac{1}{2}\rho v_1^2 + \rho g H = \frac{1}{2}\rho v_2^2 + p_0$$

$$\therefore \quad p + \rho g H = \frac{1}{2}\rho(v_2^2 - v_1^2)$$

$$A_1 v_1 = A_2 v_2$$

or $\quad v_1 = \dfrac{A_2 v_2}{A_1}$

$$\therefore \quad p + \rho g H = \frac{1}{2} \times \rho \left[v_2^2 - \left(\frac{A_2}{A_1}v_2\right)^2\right]$$

$$= \frac{1}{2} \times \rho \times v_2^2\left[1 - \left(\frac{\pi\,(0.3)^2}{\pi\,(0.9)^2}\right)^2\right]$$

$$= \frac{1}{2} \times \rho \times (10)^2\left[1 - \frac{1}{81}\right]$$

$$= \frac{4 \times 10^3\,\rho}{81}$$

$$= \frac{4 \times 10^3 \times 900}{81}$$

$$= \frac{4}{9} \times 10^5\ \text{N/m}^2$$

This is also the excess pressure Δp.

By Poiseuille's equation, the rate of flow of liquid in the capillary tube

$$Q = \frac{\pi(\Delta p)\,a^4}{8\eta l}$$

$$\therefore \quad 8\times10^{-6} = \frac{(\pi a^2)(\Delta p)}{8\eta}\left(\frac{a^2}{l}\right)$$

$$\therefore \quad \eta = \frac{(\pi a^2)(\Delta p)\left(\frac{a^2}{l}\right)}{8\times 8\times10^{-6}}$$

Substituting the values, we have

$$\eta = \frac{(10^{-6})\left(\frac{4}{9}\times10^5\right)(2\times10^{-6})}{8\times8\times10^{-6}}$$

$$= \frac{1}{720}\text{ N-s/m}^2$$

Topic 5 Miscellaneous Problems

1. Reynolds' number for flow of a liquid is given by

$$R_e = \frac{\rho v D}{\eta}$$

where, velocity of flow,

$$v = \frac{\text{volume flow rate}}{\text{area of flow}} = \frac{V/t}{A}$$

So, $R_e = \frac{\rho VD}{\eta At} = \frac{\rho V 2r}{\eta\times\pi r^2\times t} = \frac{2\rho V}{\eta\pi rt}$

Here, ρ = density of water = $1000\,\text{kgm}^{-3}$

$$\frac{V}{t} = \frac{100\times10^{-3}}{60}\text{ m}^3\text{s}^{-1}$$

where, η = viscosity of water = 1×10^{-3} Pa-s

and r = radius of pipe = 5×10^{-2} m

$$R_e = \frac{2\times1000\times100\times10^{-3}}{1\times10^{-3}\times60\times3.14\times5\times10^{-2}}$$

$$= 212.3\times10^2 \approx 2.0\times10^4$$

So, order of Reynolds' number is of 10^4.

2 When soap bubble is being inflated and its temperature remains constant, then it follows Boyle's law, so

pV = constant (k)

$$\Rightarrow p = \frac{k}{V}$$

Differentiating above equation with time, we get

$$\frac{dp}{dt} = k.\frac{d}{dt}\left(\frac{1}{V}\right) \Rightarrow \frac{dp}{dt} = k\left(\frac{-1}{V^2}\right).\frac{dV}{dt}$$

It is given that, $\frac{dV}{dt} = c$ (a constant)

So, $$\frac{dp}{dt} = \frac{-kc}{V^2} \quad \ldots \text{(i)}$$

Now, from $\frac{dV}{dt} = c$; we get

$$dV = cdt$$

or $$\int dV = \int cdt \text{ or } V = ct \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{dp}{dt} = \frac{-kc}{c^2t^2} \text{ or } \frac{dp}{dt} = -\left(\frac{k}{c}\right)t^{-2} \Rightarrow dp = -\frac{k}{c}.t^{-2}dt$$

Integrating both sides, we get

$$\int dp = -\frac{k}{c}\int t^{-2}dt$$

$$p = -\frac{k}{c}.\left(\frac{t^{-2+1}}{-2+1}\right)$$

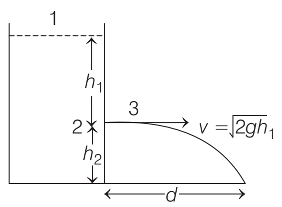
$$= -\frac{k}{c}.\frac{-1}{t} = \frac{k}{ct} \text{ or } p \propto \frac{1}{t}$$

Hence, p *versus* $\frac{1}{t}$ graph is a straight line, which is correctly represented in option (b).

3. Stress $= \frac{\text{Weight}}{\text{Area}} = \frac{9^3\times W_0}{9^2\times A_0} = 9\left(\frac{W_0}{A_0}\right)$

Hence, the stress increases by a factor of 9.

4. $d = 2\sqrt{h_1h_2} = \sqrt{4h_1h_2}$



This is independent of the value of g.

(A) $g_{\text{eff}} > g$ $\quad d = \sqrt{4h_1h_2} = 1.2$ m

(B) $g_{\text{eff}} < g$ $\quad d = \sqrt{4h_1h_2} = 1.2$ m

(C) $g_{\text{eff}} = g$ $\quad d = \sqrt{4h_1h_2} = 1.2$ m

(D) $g_{\text{eff}} = 0$

No water leaks out of jar. As there will be no pressure difference between top of the container and any other point.

$$p_1 = p_2 = p_3 = p_0$$

5. If the deformation is small, then the stress in a body is directly proportional to the corresponding strain.

According to Hooke's law i.e.

Young's modulus $(Y) = \frac{\text{Tensile stress}}{\text{Tensile strain}}$

So, $$Y = \frac{F/A}{\Delta L/L} = \frac{FL}{A\Delta L}$$

If the rod is compressed, then compressive stress and strain appear. Their ratio Y is same as that for tensile case.

Given, length of a steel wire (L) = 10 cm

Temperature (θ) = 100° C

As length is constant.

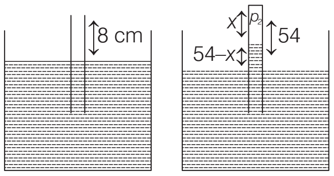
$\therefore$ Strain $= \frac{\Delta L}{L} = \alpha\,\Delta\theta$

Now, pressure = stress = $Y\times$ strain

[Given, $Y = 2\times10^{11}\text{ N/m}^2$ and $\alpha = 1.1\times10^{-5}\text{ K}^{-1}$]

$$= 2\times10^{11}\times1.1\times10^{-5}\times100 = 2.2\times10^8\text{ Pa}$$

6. **Key Idea** In this question, the system is accelerating horizontally i.e. no component of acceleration in vertical direction. Hence, the pressure in the vertical direction will remain unaffected.



i.e. $p_1 = p_0 + \rho g h$

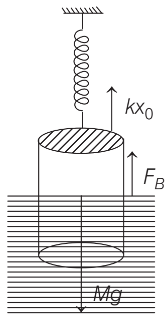
Again, we have to use the concept that the pressure in the same level will be same.

For air trapped in tube, $p_1 V_1 = p_2 V_2$

$$p_1 = p_{atm} = \rho g 76$$
$$V_1 = A \cdot 8 \quad [A = \text{area of cross-section}]$$
$$p_2 = p_{atm} - \rho g(54 - x) = \rho g(22 + x)$$
$$V_2 = A \cdot x \quad \rho g 76 \times 8A = \rho g(22 + x) A x$$
$$x^2 + 22x - 78 \times 8 = 0 \quad \Rightarrow x = 16\,\text{cm}$$

7. In equilibrium, Upward force = Downward force

$$kx_0 + F_B = mg$$



Here, kx_0 is restoring force of spring and F_B is buoyancy force.

$$kx_0 + \sigma \frac{L}{2} Ag = Mg$$

$$x_0 = \frac{Mg - \frac{\sigma LAg}{2}}{k} = \frac{Mg}{k}\left(1 - \frac{\sigma LA}{2M}\right)$$

8. Let V_1 = total material volume of shell

V_2 = total inside volume of shell and

x = fraction of V_2 volume filled with water.

In floating condition,

Total weight = Upthrust

$$\therefore \quad V_1 \rho_c g + (xV_2)(1)g = \left(\frac{V_1 + V_2}{2}\right)(1)g$$

or $$x = 0.5 + (0.5 - \rho_c)\frac{V_1}{V_2}$$

From here we can see that, $x > 0.5$ if $\rho_c < 0.5$

9. $k_1 = \left(\frac{\rho_{Fe}}{\rho_{Hg}}\right)_{0°C}$ and $k_2 = \left(\frac{\rho_{Fe}}{\rho_{Hg}}\right)_{60°C}$.

Here, ρ = Density

$$\therefore \quad \frac{k_1}{k_2} = \frac{(\rho_{Fe})_{0°C}}{(\rho_{Hg})_{0°C}} \times \left(\frac{\rho_{Hg}}{\rho_{Fe}}\right)_{60°C} = \frac{(1 + 60\gamma_{Fe})}{(1 + 60\gamma_{Hg})}$$

NOTE In this problem two concepts are used

(i) When a solid floats in a liquid, then

Fraction of volume submerged $(k) = \frac{\rho_{solid}}{\rho_{liquid}}$

This result comes from the fact that

Weight = Upthrust

$$V\rho_{solid}\, g = V_{submerged}\, \rho_{liquid}\, g$$

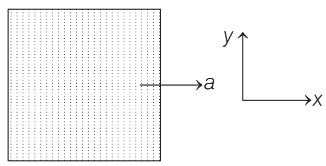
$$\therefore \quad \frac{V_{submerged}}{V} = \frac{\rho_{solid}}{\rho_{liquid}}$$

(ii) $\frac{\rho_{\theta°C}}{\rho_{0°C}} = \frac{1}{1 + \gamma \cdot \theta}$

This is because $\rho \propto \frac{1}{Volume}$ (mass remaining constant)

$$\therefore \quad \frac{\rho_{\theta°C}}{\rho_{0°C}} = \frac{V_{0°C}}{V_{\theta°C}} = \frac{V_{0°C}}{V_{0°C} + \Delta V} = \frac{V_{0°C}}{V_{0°C} + V_{0°C}\gamma\,\theta} = \frac{1}{1 + \gamma\,\theta}$$

10. If a fluid (gas or liquid) is accelerated in positive x-direction, then pressure decreases in positive x-direction. Change in pressure has following differential equation.

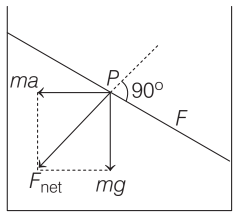


$$\frac{dp}{dx} = -\rho a$$

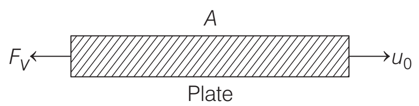
where, ρ is the density of the fluid. Therefore, pressure is lower in front side.

11. Net force on the free surface of the liquid in equilibrium (from accelerated frame) should be perpendicular to it.

Forces on a water particle P on the free surfaces have been shown in the figure. In the figure ma is the pseudo force.



12.



$$F_v = -\eta A\left(\frac{dv}{dy}\right)$$

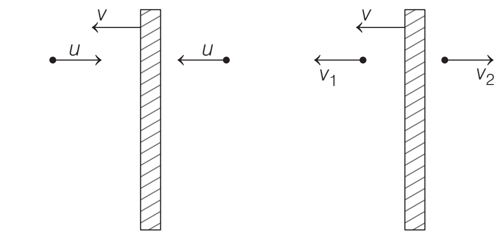
Since, height h of the liquid in tank is very small.

$$\Rightarrow \quad \frac{dv}{dy} = \frac{\Delta v}{\Delta y} = \left(\frac{u_0}{h}\right) \Rightarrow F_v = -(\eta)A\left(\frac{u_0}{h}\right)$$

$$F_v \propto \left(\frac{1}{h}\right), F_v \propto u_0, F \propto A, F_v \propto \eta$$

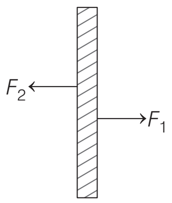
Statement a, c and are correct.

13.



Just before the collision   Just after the collision

$$v_1 = u + 2v$$
$$\Delta v_1 = (2u + 2v)$$
$$F_1 = \frac{dp_1}{dt}$$
$$= \rho A(u + v)(2u + 2v)$$
$$= 2\rho A(u + v)^2$$



$$v_2 = (u - 2v)$$
$$\Delta v_2 = (2u - 2v)$$
$$F_2 = \frac{dp_2}{dt}$$
$$= \rho A(u - v)(2u - 2v) = 2\rho A(u - v)^2$$

[ΔF is the net force due to the air molecules on the plate]

$$\Delta F = 2\rho A(4uv) = 8\rho Auv$$
$$P = \frac{\Delta F}{A} = 8\rho(uv)$$
$$F_{net} = (F - \Delta F) = ma$$

[m is mass of the plate]

$$F - (8\rho Au)v = ma$$

14. For floating, net weight of system = net upthrust

$\Rightarrow (\rho_1 + \rho_2)Vg = (\sigma_1 + \sigma_2)Vg$

Since string is taut, $\rho_1 < \sigma_1$ and $\rho_2 > \sigma_2$

$$\mathbf{v}_P = \frac{2r^2 g}{2\eta_2}(\sigma_2 - \rho_1) \quad \text{(upward terminal velocity)}$$

$$\mathbf{v}_Q = \frac{2r^2 g}{9\eta_1}(\rho_2 - \sigma_1) \quad \text{(downward terminal velocity)}$$

$$\left|\frac{\mathbf{v}_P}{\mathbf{v}_Q}\right| = \frac{\eta_1}{\eta_2}$$

Further, $\mathbf{v}_P \cdot \mathbf{v}_Q$ will be negative as they are opposite to each other.

15. From continuity equation, Av = constant

or $$A \propto \frac{1}{v}$$

16. From continuity equation,

$$A_1 v_1 = A_2 v_2$$

Here, $A_1 = 400 A_2$

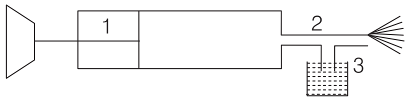
because $r_1 = 20 r_2$ and $A = \pi r^2$

$$\therefore \quad v_2 = \frac{A_1}{A_2}(v_1) = 400 v_1$$
$$= 400(5)\text{mm/s} = 2000 \text{ mm/s} = 2\text{m/s}$$

17. $$p_1 - p_2 = \frac{1}{2}\rho_a v_a^2 \Rightarrow p_3 - p_2 = \frac{1}{2}\rho_l v_l^2$$

$$p_3 = p_1$$

$$\therefore \quad \frac{1}{2}\rho_l v_l^2 = \frac{1}{2}\rho_a v_a^2 \Rightarrow v_l = \sqrt{\frac{\rho_a}{\rho_l}}\, v_a$$



$\therefore$ Volume flow rate $\propto \sqrt{\dfrac{\rho_a}{\rho_l}}$

18. As the bubble moves upwards, besides the buoyancy force (cause of which is pressure difference) only force of gravity and force of viscosity will act.

$\therefore$ Correct option is (d).

19. As there is no exhange of heat. Therefore, process is adiabatic. Applying,

$$Tp^{\frac{1-\gamma}{\gamma}} = \text{constant} \Rightarrow T_2 p_2^{\frac{1-\gamma}{\gamma}} = T_1 p_1^{\frac{1-\gamma}{\gamma}}$$

or $$T_2 = T_1\left(\frac{p_1}{p_2}\right)^{\frac{1-\gamma}{\gamma}} = T_1\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

Substituting the values, we have

$$T_2 = T_0\left[\frac{p_0 + \rho_l g(H - y)}{p_0 + \rho_l gH}\right]^{\frac{5/3-1}{5/3}}$$

$$= T_0\left[\frac{p_0 + \rho_l g(H - y)}{p_0 + \rho_l gH}\right]^{2/5}$$

$\therefore$ Correct option is (b).

20. Buoyancy force

$$F = \text{(volume of bubble)}(\rho_l)g = \left(\frac{nRT_2}{p_2}\right)\rho_l g$$

Here, $$T_2 = T_0\left[\frac{p_0 + \rho_e g\,(H - y)}{p_0 + \rho_l gh}\right]^{2/5}$$

and $$p_2 = p_0 + \rho_l g(H - y)$$

Substituting the values, we get

$$F = \frac{\rho_l nRgT_0}{(p_0 + \rho_l gH)^{2/5}[p_0 + \rho_l g(H - y)]^{3/5}}$$

$\therefore$ Correct option is (b)

21. The condition of floating is,

Weight = Upthrust

$$V\rho_1 g = V_i \rho_2 g$$

(ρ_1 = density of metal, ρ_2 = density of mercury)

$$\therefore \qquad \frac{V_i}{V} = \frac{\rho_1}{\rho_2}$$

= fraction of volume of metal submerged in mercury
= x (say)

Now, when the temperature is increased by ΔT.

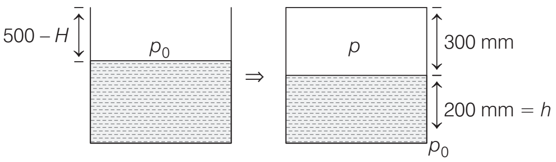
$$\rho_1' = \frac{\rho_1}{1+\gamma_1 \Delta T} \text{ and } \rho_2' = \frac{\rho_2}{1+\gamma_2 \Delta T}$$

$$\therefore \qquad x' = \left(\frac{\rho_1}{1+\gamma_1 \Delta T}\right)\left(\frac{1+\gamma_2 \Delta T}{\rho_2}\right) = \frac{\rho_1}{\rho_2}\left(\frac{1+\gamma_2 \Delta T}{1+\gamma_1 \Delta T}\right)$$

$$\therefore \qquad x' = x\left(\frac{1+\gamma_2 \Delta T}{1+\gamma_1 \Delta T}\right) \Rightarrow \frac{x'}{x} = \frac{1+\gamma_2 \Delta T}{1+\gamma_1 \Delta T}$$

22. When tube is heated the density of water at A will decrease, hence the water will rise up or it will circulate in clockwise direction.

23. In this question, we will have to assume that temperature of enclosed air about water is constant (or pV = constant)



$$p = p_0 - \rho g h \qquad \text{...(i)}$$

$$p_0[A(500-H)] = p[A(300)] \qquad \text{...(ii)}$$

Solving these two equations, we get $H = 206$ mm

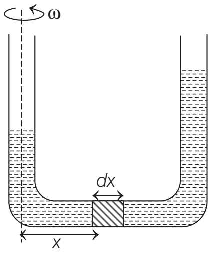
$\therefore$ Level fall = (206 − 200) mm = 6 mm

24. For circular motion of small element dx, we have

$$dF = (dm)\, x\, \omega^2$$

$$\therefore (dp)\, A = (\rho A dx)\cdot x\, \omega^2 \text{ or } dp = \rho\, \omega^2\, x \cdot dx$$

$$\therefore \int_{p_1}^{p_2} dp = \rho\, \omega^2 \int_0^L x\, dx \Rightarrow p_2 - p_1 = \frac{\rho\, \omega^2 L^2}{2}$$



$$\therefore \qquad \rho g H = \frac{\rho \omega^2 L^2}{2} \Rightarrow H = \frac{\omega^2 L^2}{2g}$$

25. In elastic collision with the surface, direction of velocity is reversed but its magnitude remains the same. Therefore,

$$\text{time of fall = time of rise or time of fall} = \frac{t_1}{2}$$

Hence, velocity of the ball just before it collides with liquid is

$$v = g\frac{t_1}{2} \qquad \text{...(i)}$$

Retardation inside the liquid,

$$a = \frac{\text{upthrust} - \text{weight}}{\text{mass}} = \frac{Vd_L g - Vdg}{Vd} = \left(\frac{d_L - d}{d}\right) g \quad \text{... (ii)}$$

Time taken to come to rest under this retardation will be

$$t = \frac{v}{a} = \frac{gt_1}{2a} = \frac{gt_1}{2\left(\frac{d_L - d}{d}\right)g} = \frac{dt_1}{2(d_L - d)}$$

Same will be the time to come back on the liquid surface. Therefore,

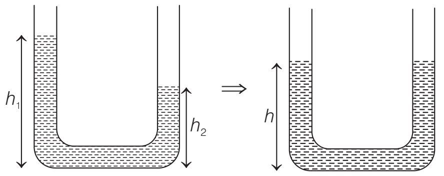
(a) t_2 = time the ball takes to came back to the position from where it was released

$$= t_1 + 2t = t_1 + \frac{dt_1}{d_L - d} = t_1\left[1 + \frac{d}{d_L - d}\right] \text{ or } t_2 = \frac{t_1 d_L}{d_L - d}$$

(b) The motion of the ball is periodic but not simple harmonic because the acceleration of the ball is g in air and $\left(\frac{d_L - d}{d}\right) g$ inside the liquid which is not proportional to the displacement, which is necessary and sufficient condition for SHM.

(c) When $d_L = d$, retardation or acceleration inside the liquid becomes zero (upthrust = weight). Therefore, the ball will continue to move with constant velocity $v = gt_1/2$ inside the liquid.

26. Let h be the level in equilibrium. Equating the volumes, we have



$$Ah_1 + Ah_2 = 2Ah$$

$$\therefore \qquad h = \left(\frac{h_1 + h_2}{2}\right)$$

Work done by gravity = $U_i - U_f$

$$W = \left(m_1 g \frac{h_1}{2} + m_2 g \frac{h_2}{2}\right) - (m_1 + m_2) g \frac{h}{2}$$

$$= \frac{Ah_1 \rho g h_1}{2} + \frac{Ah_2 \rho g h_2}{2} - [Ah_1 \rho + Ah_2 \rho] g \left(\frac{h_1 + h_2}{4}\right)$$

Simplifying this, we get

$$W = \frac{\rho A g}{4}(h_1 - h_2)^2$$