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Probability

Topic 1 Classical Probability

Objective Questions I (Only one correct option)

1. A person throws two fair dice. He wins ₹ 15 for throwing a doublet (same numbers on the two dice), wins ₹ 12 when the throw results in the sum of 9, and loses ₹ 6 for any other outcome on the throw. Then, the expected gain/loss (in ₹) of the person is **(2019 Main, 12 April II)**

(a) $\frac{1}{2}$ gain (b) $\frac{1}{4}$ loss (c) $\frac{1}{2}$ loss (d) 2 gain

2. In a random experiment, a fair die is rolled until two fours are obtained in succession. The probability that the experiment will end in the fifth throw of the die is equal to **(2019 Main, 12 Jan I)**

(a) $\frac{175}{6^5}$ (b) $\frac{225}{6^5}$

(c) $\frac{200}{6^5}$ (d) $\frac{150}{6^5}$

3. If there of the six vertices of a regular hexagon are chosen at random, then the probability that the triangle formed with these chosen vertices is equilateral is **(2019 Main, 12 April I)**

(a) $\frac{1}{10}$ (b) $\frac{1}{5}$ (c) $\frac{3}{10}$ (d) $\frac{3}{20}$

4. Let $S = \{1, 2, \dots, 20\}$. A subset B of S is said to be "nice", if the sum of the elements of B is 203. Then, the probability that a randomly chosen subset of S is "nice", is **(2019 Main, 11 Jan II)**

(a) $\frac{6}{2^{20}}$ (b) $\frac{4}{2^{20}}$ (c) $\frac{7}{2^{20}}$ (d) $\frac{5}{2^{20}}$

5. If two different numbers are taken from the set {0, 1, 2, 3, …, 10}, then the probability that their sum as well as absolute difference are both multiple of 4, is **(2017 Main)**

(a) $\frac{6}{55}$ (b) $\frac{12}{55}$ (c) $\frac{14}{45}$ (d) $\frac{7}{55}$

6. Three randomly chosen non-negative integers x, y and z are found to satisfy the equation $x + y + z = 10$. Then the probability that z is even, is **(2017 Adv.)**

(a) $\frac{1}{2}$ (b) $\frac{36}{55}$ (c) $\frac{6}{11}$ (d) $\frac{5}{11}$

7. If 12 identical balls are to be placed in 3 different boxes, then the probability that one of the boxes contains excatly 3 balls, is **(2015 Main)**

(a) $\frac{55}{3}\left(\frac{2}{3}\right)^{11}$ (b) $55\left(\frac{2}{3}\right)^{10}$ (c) $220\left(\frac{1}{3}\right)^{12}$ (d) $22\left(\frac{1}{3}\right)^{11}$

8. Three boys and two girls stand in a queue. The probability that the number of boys ahead of every girl is atleast one more that the number of girls ahead of her, is **(2014 Adv)**

(a) 1/2 (b) 1/3 (c) 2/3 (d) 3/4

9. Four fair dice D_1, D_2, D_3 and D_4 each having six faces numbered 1, 2, 3, 4, 5 and 6 are rolled simultaneously. The probability that D_4 shows a number appearing on one of D_1, D_2 and D_3, is **(2012)**

(a) $\frac{91}{216}$ (b) $\frac{108}{216}$ (c) $\frac{125}{216}$ (d) $\frac{127}{216}$

10. Let ω be a complex cube root of unity with $\omega \neq 1$. A fair die is thrown three times. If r_1, r_2 and r_3 are the numbers obtained on the die, then the probability that $\omega^{r_1} + \omega^{r_2} + \omega^{r_3} = 0$, is **(2010)**

(a) 1/18 (b) 1/9 (c) 2/9 (d) 1/36

11. If three distinct numbers are chosen randomly from the first 100 natural numbers, then the probability that all three of them are divisible by both 2 and 3, is **(2004, 1M)**

(a) $\frac{4}{55}$ (b) $\frac{4}{35}$ (c) $\frac{4}{33}$ (d) $\frac{4}{1155}$

12. Two numbers are selected randomly from the set $S = \{1, 2, 3, 4, 5, 6\}$ without replacement one by one. The probability that minimum of the two numbers is less than 4, is **(2003, 1M)**

(a) 1/15 (b) 14/15 (c) 1/5 (d) 4/5

13. If the integers m and n are chosen at random between 1 and 100, then the probability that a number of the form $7^m + 7^n$ is divisible by 5, equals **(1999, 2M)**

(a) $\frac{1}{4}$ (b) $\frac{1}{7}$ (c) $\frac{1}{8}$ (d) $\frac{1}{49}$

14. Seven white balls and three black balls are randomly placed in a row. The probability that no two black balls are placed adjacently, equals **(1998, 2M)**

(a) $\frac{1}{2}$ (b) $\frac{7}{15}$ (c) $\frac{2}{15}$ (d) $\frac{1}{3}$

15. Three of the six vertices of a regular hexagon are chosen at rondom. The probability that the triangle with three vertices is equilateral, equals **(1995, 2M)**

(a) 1/2 (b) 1/5 (c) 1/10 (d) 1/20

16. Three identical dice are rolled. The probability that the same number will appear on each of them, is **(1984, 2M)**

(a) $\frac{1}{6}$ (b) $\frac{1}{36}$ (c) $\frac{1}{18}$ (d) $\frac{3}{28}$

17. Fifteen coupons are numbered 1, 2, ..., 15, respectively. Seven coupons are selected at random one at a time with replacement. The probability that the largest number appearing on a selected coupon is 9, is

(a) $\left(\frac{9}{16}\right)^6$ (b) $\left(\frac{8}{15}\right)^7$ (c) $\left(\frac{3}{5}\right)^7$ (d) None of these

Assertion and Reason

For the following questions, choose the correct answer from the codes (a), (b), (c) and (d) defined as follows.

(a) Statement I is true, Statement II is also true; Statement II is the correct explanation of Statement I

(b) Statement I is true, Statement II is also true; Statement II is not the correct explanation of Statement I

(c) Statement I is true; Statement II is false

(d) Statement I is false; Statement II is true

18. Consider the system of equations

$$ax + by = 0, cx + dy = 0,$$

where $a, b, c, d \in \{0, 1\}$.

Statement I The probability that the system of equations has a unique solution, is 3/8.

Statement II The probability that the system of equations has a solution, is 1. **(2008, 3M)**

Passage Based Problems

Passage

Box I contains three cards bearing numbers 1, 2, 3 ; box II contains five cards bearing numbers 1, 2, 3, 4, 5; and box III contains seven cards bearing numbers 1, 2, 3, 4, 5, 6, 7. A card is drawn from each of the boxes. Let x_i be the number on the card drawn from the ith box $i = 1, 2, 3$. **(2014 Adv.)**

19. The probability that $x_1 + x_2 + x_3$ is odd, is

(a) $\frac{29}{105}$ (b) $\frac{53}{105}$ (c) $\frac{57}{105}$ (d) $\frac{1}{2}$

20. The probability that x_1, x_2 and x_3 are in an arithmetic progression, is

(a) $\frac{9}{105}$ (b) $\frac{10}{105}$ (c) $\frac{11}{105}$ (d) $\frac{7}{105}$

Fill in the Blanks

21. Three faces of a fair die are yellow, two faces red and one face blue. The die is tossed three times. The probability that the colours, yellow, red and blue, appear in the first, second and the third tosses respectively, is...... . **(1992, 2M)**

22. If $\frac{1+3p}{3}, \frac{1-p}{4}$ and $\frac{1-2p}{2}$ are the probabilities of three mutually exclusive events, then the set of all values of p is... . **(1986, 2M)**

23. A determinant is chosen at random from the set of all determinants of order 2 with elements 0 or 1 only. The probability that the value of the determinant chosen is positive, is... . **(1982, 2M)**

True/False

24. If the letters of the word 'ASSASSIN' are written down at random in a row, the probability that no two S's occur together is 1/35.

Analytical and Descriptive Questions

25. An unbiased die, with faces numbered 1, 2, 3, 4, 5 and 6 is thrown n times and the list of n numbers showing up is noted. What is the probability that among the numbers 1, 2, 3, 4, 5 and 6 only three numbers appear in this list? **(2001, 5M)**

26. If p and q are chosen randomly from the set {1, 2, 3, 4, 5, 6, 7, 8, 9 and 10} with replacement, determine the probability that the roots of the equation $x^2 + px + q = 0$ are real. **(1997, 5M)**

27. In how many ways three girls and nine boys can be seated in two vans, each having numbered seats, 3 in the front and 4 at the back? How many seating arrangements are possible if 3 girls should sit together in a back row on adjacent seats? Now, if all the seating arrangements are equally likely, what is the probability of 3 girls sitting together in a back row on adjacent seats? **(1996, 5M)**

28. A box contains 2 fifty paise coins, 5 twenty five paise coins and a certain fixed number n (≥ 2) of ten and five paise coins. Five coins are taken out of the box at random. Find the probability that the total value of these 5 coins is less than one rupee and fifty paise. **(1988, 3M)**

29. Six boys and six girls sit in a row at random. Find the probability that

(i) the six girls sit together.

(ii) the boys and girls sit alternatively. **(1978, 3M)**

Topic 2 Addition and Subtraction Law of Probability

Objective Questions I (Only one correct option)

1. For three events A, B and C, if P (exactly one of A or B occurs) = P(exactly one of B or C occurs) = P (exactly one of C or A occurs) $= \frac{1}{4}$ and P (all the three events occur simultaneously) $= \frac{1}{16}$, then the probability that atleast one of the events occurs, is **(2017 Main)**

(a) $\frac{7}{32}$ (b) $\frac{7}{16}$ (c) $\frac{7}{64}$ (d) $\frac{3}{16}$

2. If $P(B) = \frac{3}{4}$, $P(A \cap B \cap \overline{C}) = \frac{1}{3}$ and $P(\overline{A} \cap B \cap \overline{C}) = \frac{1}{3}$, then $P(B \cap C)$ is equal to **(2002, 3M)**

(a) $\frac{1}{12}$ (b) $\frac{1}{6}$ (c) $\frac{1}{15}$ (d) $\frac{1}{9}$

3. If E and F are events with $P(E) \le P(F)$ and $P(E \cap F) > 0$, then which one is not correct? **(1998, 2M)**

(a) occurrence of $E \Rightarrow$ occurrence of F

(b) occurrence of $F \Rightarrow$ occurrence of E

(c) non-occurrence of $E \Rightarrow$ non-occurrence of F

(d) None of the above

4. For the three events A, B and C, P(exactly one of the events A or B occurs) = P(exactly one of the events B or C occurs) = P(exactly one of the events C or A occurs) $= p$ and P(all the three events occurs simultaneously) $= p^2$, where $0 < p < \frac{1}{2}$. Then, the probability of atleast one of the three events A, B and C occurring is **(1996, 2M)**

(a) $\frac{3p + 2p^2}{2}$ (b) $\frac{p + 3p^2}{4}$

(c) $\frac{p + 3p^2}{2}$ (d) $\frac{3p + 2p^2}{4}$

5. If $0 < P(A) < 1, 0 < P(B) < 1$ and $P(A \cup B) = P(A) + P(B) - P(A)\,P(B)$, then **(1995, 2M)**

(a) $P(B/A) = P(B) - P(A)$

(b) $P(A' - B') = P(A') - P(B')$

(c) $P(A \cup B)' = P(A)'P(B)'$

(d) $P(A/B) = P(A) - P(B)$

6. The probability that at least one of the events A and B occurs is 0.6. If A and B occur simultaneously with probability 0.2, then $P(\overline{A}) + P(\overline{B})$ is equal to **(1987, 2M)**

(a) 0.4 (b) 0.8 (c) 1.2 (d) 1.4

7. Two events A and B have probabilities 0.25 and 0.50, respectively. The probability that both A and B occur simultaneously is 0.14. Then, the probability that neither A nor B occurs, is **(1980, 1M)**

(a) 0.39 (b) 0.25

(c) 0.11 (d) None of these

Objective Questions II

(One or more than one correct option)

8. For two given events A and B, $P(A \cap B)$ is **(1988, 2M)**

(a) not less than $P(A) + P(B) - 1$

(b) not greater than $P(A) + P(B)$

(c) equal to $P(A) + P(B) - P(A \cup B)$

(d) equal to $P(A) + P(B) + P(A \cup B)$

9. If M and N are any two events, then the probability that exactly one of them occurs is **(1984, 3M)**

(a) $P(M) + P(N) - 2P(M \cap N)$

(b) $P(M) + P(N) - P(\overline{M \cup N})$

(c) $P(\overline{M}) + P(\overline{N}) - 2P(\overline{M} \cap \overline{N})$

(d) $P(M \cap \overline{N}) - P(\overline{M} \cap N)$

Fill in the Blanks

10. Three numbers are chosen at random without replacement from $\{1, 2, \ldots, 10\}$. The probability that the minimum of the chosen number is 3, or their maximum is 7, is … . **(1997C, 2M)**

11. $P(A \cup B) = P(A \cap B)$ if and only if the relation between $P(A)$ and $P(B)$ is… . **(1985, 2M)**

True/False

12. If the probability for A to fail in an examination is 0.2 and that of B is 0.3, then the probability that either A or B fails is 0.5. **(1989, 1M)**

Analytical and Descriptive Questions

13. In a certain city only two newspapers A and B are published, it is known that 25% of the city population reads A and 20% reads B, while 8% reads both A and B. It is also known that 30% of those who read A but not B look into advertisements and 40% of those who read B but not A look into advertisements while 50% of those who read both A and B look into advertisements. What is the percentage of the population reads an advertisement? **(1984, 4M)**

14. A, B, C are events such that

$$P_r(A) = 0.3,\ P_r(B) = 0.4,\ P_r(C) = 0.8,$$
$$P_r(AB) = 0.08,\ P_r(AC) = 0.28 \text{ and } P_r(ABC) = 0.09$$

If $P_r(A \cup B \cup C) \ge 0.75$, then show that $P_r(BC)$ lies in the interval [0.23, 0.48]. **(1983, 2M)**

15. A and B are two candidates seeking admission in IIT. The probability that A is selected is 0.5 and the probability that both A and B are selected is atmost 0.3. Is it possible that the probability of B getting selected is 0.9? **(1982, 2M)**

Pragraph Based Questions

There are five students S_1, S_2, S_3, S_4 and S_5 in a music class and for them there are five seats R_1, R_2, R_3, R_4 and R_5 arranged in a row, where initially the seat R_i is allotted to the student S_i, $i = 1, 2, 3, 4, 5$. But, on the examination day, the five students are randomly allotted the five seats.

(*There are two questions based on Paragraph, the question given below is one of them*) **(2018 Adv.)**

16. The probability that, on the examination day, the student S_1 gets the previously allotted seat R_1, and NONE of the remaining students gets the seat previously allotted to him/her is

(a) $\frac{3}{40}$ (b) $\frac{1}{8}$ (c) $\frac{7}{40}$ (d) $\frac{1}{5}$

17. For $i = 1, 2, 3, 4$, let T_i denote the event that the students S_i and S_{i+1} do NOT sit adjacent to each other on the day of the examination. Then, the probability of the event $T_1 \cap T_2 \cap T_3 \cap T_4$ is

(a) $\frac{1}{15}$ (b) $\frac{1}{10}$ (c) $\frac{7}{60}$ (d) $\frac{1}{5}$

Topic 3 Independent and Conditional Probability

Objective Questions I (Only one correct option)

1. Assume that each born child is equally likely to be a boy or a girl. If two families have two children each, then the conditional probability that all children are girls given that at least two are girls; is **(2019 Main, 10 April I)**

(a) $\frac{1}{17}$ (b) $\frac{1}{12}$ (c) $\frac{1}{10}$ (d) $\frac{1}{11}$

2. Four persons can hit a target correctly with probabilities $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ and $\frac{1}{8}$ respectively. If all hit at the target independently, then the probability that the target would be hit, is **(2019 Main, 9 April I)**

(a) $\frac{1}{192}$ (b) $\frac{25}{32}$ (c) $\frac{7}{32}$ (d) $\frac{25}{192}$

3. Let A and B be two non-null events such that $A \subset B$. Then, which of the following statements is always correct. **(2019 Main, 8 April I)**

(a) $P(A/B) = P(B) - P(A)$ (b) $P(A/B) \geq P(A)$
(c) $P(A/B) \leq P(A)$ (d) $P(A/B) = 1$

4. Two integers are selected at random from the set {1, 2, ... , 11}. Given that the sum of selected numbers is even, the conditional probability that both the numbers are even is **(2019 Main, 11 Jan I)**

(a) $\frac{2}{5}$ (b) $\frac{1}{2}$ (c) $\frac{7}{10}$ (d) $\frac{3}{5}$

5. An unbiased coin is tossed. If the outcome is a head, then a pair of unbiased dice is rolled and the sum of the numbers obtained on them is noted. If the toss of the coin results in tail, then a card from a well-shuffled pack of nine cards numbered 1, 2, 3, …, 9 is randomly picked and the number on the card is noted. The probability that the noted number is either 7 or 8 is **(2019 Main, 10 Jan I)**

(a) $\frac{15}{72}$ (b) $\frac{13}{36}$ (c) $\frac{19}{72}$ (d) $\frac{19}{36}$

6. Let two fair six-faced dice A and B be thrown simultaneously. If E_1 is the event that die A shows up four, E_2 is the event that die B shows up two and E_3 is the event that the sum of numbers on both dice is odd, then which of the following statements is not true? **(2016 Main)**

(a) E_1 and E_2 are independent
(b) E_2 and E_3 are independent
(c) E_1 and E_3 are independent
(d) E_1, E_2 and E_3 are independent

7. Let A and B be two events such that $P(\overline{A \cup B}) = \frac{1}{6}$, $P(A \cap B) = \frac{1}{4}$ and $P(\overline{A}) = \frac{1}{4}$, where $\overline{A}$ stands for the complement of the event A. Then , the events A and B are **(2014 Main)**

(a) independent but not equally likely
(b) independent and equally likely
(c) mutually exclusive and independent
(d) equally likely but not independent

8. Four persons independently solve a certain problem correctly with probabilities $\frac{1}{2}, \frac{3}{4}, \frac{1}{4}, \frac{1}{8}$. Then, the probability that the problem is solved correctly by atleast one of them, is **(2013 Adv)**

(a) $\frac{235}{256}$ (b) $\frac{21}{256}$ (c) $\frac{3}{256}$ (d) $\frac{253}{256}$

9. An experiment has 10 equally likely outcomes. Let A and B be two non-empty events of the experiment. If A consists of 4 outcomes, then the number of outcomes that B must have, so that A and B are independent, is

(a) 2, 4 or 8 (b) 3, 6 or 9 **(2008, 3M)**
(c) 4 or 8 (d) 5 or 10

10. Let E^c denotes the complement of an event E. If E, F, G are pairwise independent events with $P(G) > 0$ and $P(E \cap F \cap G) = 0$. Then, $P(E^c \cap F^c \mid G)$ equals **(2007, 3M)**

(a) $P(E^c) + P(F^c)$ (b) $P(E^c) - P(F^c)$
(c) $P(E^c) - P(F)$ (d) $P(E) - P(F^c)$

11. One Indian and four American men and their wives are to be seated randomly around a circular table. Then, the conditional probability that Indian man is seated adjacent to his wife given that each American man is seated adjacent to his wife, is **(2007, 3M)**

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{2}{5}$ (d) $\frac{1}{5}$

12. A fair die is rolled. The probability that the first time 1 occurs at the even throw, is **(2005, 1M)**

(a) 1/6 (b) 5/11 (c) 6/11 (d) 5/36

13. There are four machines and it is known that exactly two of them are faulty. They are tested, one by one, in a random order till both the faulty machines are identified. Then, the probability that only two tests are needed, is **(1998, 2M)**

(a) $\frac{1}{3}$ (b) $\frac{1}{6}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

14. A fair coin is tossed repeatedly. If tail appears on first four tosses, then the probability of head appearing on fifth toss equals **(1998, 2M)**

(a) $\frac{1}{2}$ (b) $\frac{1}{32}$ (c) $\frac{31}{32}$ (d) $\frac{1}{5}$

15. If from each of the three boxes containing 3 white and 1 black, 2 white and 2 black, 1 white and 3 black balls, one ball is drawn at random, then the probability that 2 white and 1 black balls will be drawn, is **(1998, 2M)**

(a) $\frac{13}{32}$ (b) $\frac{1}{4}$ (c) $\frac{1}{32}$ (d) $\frac{3}{16}$

16. The probability of India winning a test match against West Indies is 1/2. Assuming independence from match to match the probability that in a 5 match series India's second win occurs at third test, is **(1995, 2M)**

(a) 1/8 (b) 1/4 (c) 1/2 (d) 2/3

17. An unbiased die with faces marked 1, 2, 3, 4, 5 and 6 is rolled four times. Out of four face values obtained, the probability that the minimum face value is not less than 2 and the maximum face value is not greater than 5, is **(1993, 1M)**

(a) 16/81 (b) 1/81
(c) 80/81 (d) 65/81

18. A student appears for tests I, II and III. The student is successful if he passes either in tests I and II or tests I and III. The probabilities of the student passing in tests I, II and III are p, q and $\frac{1}{2}$, respectively. If the probability that the student is successful, is $\frac{1}{2}$, then **(1986, 2M)**

(a) $p = q = 1$ (b) $p = q = \frac{1}{2}$
(c) $p = 1, q = 0$ (d) $p = 1, q = \frac{1}{2}$

19. If A and B are two independent events such that $P(A) > 0$, and $P(B) \neq 1$, then $P(\overline{A}/\overline{B})$ is equal to **(1982, 2M)**

(a) $1 - P(A/B)$ (b) $1 - P(A/\overline{B})$
(c) $\frac{1 - P(A \cup B)}{P(B)}$ (d) $\frac{P(\overline{A})}{P(\overline{B})}$

20. The probability that an event A happens in one trial of an experiment, is 0.4. Three independent trials of the experiments are performed. The probability that the event A happens atleast once, is **(1980, 1M)**

(a) 0.936 (b) 0.784
(c) 0.904 (d) None of these

Objective Questions II

(One or more than one correct option)

21. Let X and Y be two events such that $P(X) = \frac{1}{3}$, $P(X/Y) = \frac{1}{2}$ and $P(Y/X) = \frac{2}{5}$. Then **(2017 Adv.)**

(a) $P(Y) = \frac{4}{15}$ (b) $P(X'/Y) = \frac{1}{2}$
(c) $P(X \cup Y) = \frac{2}{5}$ (d) $P(X \cap Y) = \frac{1}{5}$

22. If X and Y are two events such that $P(X/Y) = \frac{1}{2}$, $P(Y/X) = \frac{1}{3}$ and $P(X \cap Y)\frac{1}{6}$. Then, which of the following is/are correct? **(2012)**

(a) $P(X \cup Y) = 2/3$
(b) X and Y are independent
(c) X and Y are not independent
(d) $P(X^c \cap Y) = 1/3$

23. Let E and F be two independent events. The probability that exactly one of them occurs is $\frac{11}{25}$ and the probability of none of them occurring is $\frac{2}{25}$. If $P(T)$ denotes the probability of occurrence of the event T, then **(2011)**

(a) $P(E) = \frac{4}{5}, P(F) = \frac{3}{5}$ (b) $P(E) = \frac{1}{5}, P(F) = \frac{2}{5}$
(c) $P(E) = \frac{2}{5}, P(F) = \frac{1}{5}$ (d) $P(E) = \frac{3}{5}, P(F) = \frac{4}{5}$

24. The probabilities that a student passes in Mathematics, Physics and Chemistry are m, p and c, respectively. Of these subjects, the students has a 75% chance of passing in atleast one, a 50% chance of passing in atleast two and a 40% chance of passing in exactly two. Which of the following relations are true? **(1999, 3M) (2011)**

(a) $p + m + c = \frac{19}{20}$ (b) $p + m + c = \frac{27}{20}$
(c) $pmc = \frac{1}{10}$ (d) $pmc = \frac{1}{4}$

25. If $\overline{E}$ and $\overline{F}$ are the complementary events of E and F respectively and if $0 < P(F) < 1$, then **(1998, 2M)**

(a) $P(E/F) + P(\overline{E}/F) = 1$ (b) $P(E/F) + P(E/\overline{F}) = 1$
(c) $P(\overline{E}/F) + P(E/\overline{F}) = 1$ (d) $P(E/\overline{F}) + P(\overline{E}/\overline{F}) = 1$

26. Let E and F be two independent events. If the probability that both E and F happen is 1/12 and the probability that neither E nor F happen is 1/2. Then, **(1993, 2M)**

(a) $P(E) = 1/3, P(F) = 1/4$
(b) $P(E) = 1/2, P(F) = 1/6$
(c) $P(E) = 1/6, P(F) = 1/2$
(d) $P(E) = 1/4, P(F) = 1/3$

27. For any two events A and B in a sample space **(1991, 2M)**

(a) $P\left(\frac{A}{B}\right) \geq \frac{P(A) + P(B) - 1}{P(B)}$, $P(B) \neq 0$ is always true
(b) $P(A \cap \overline{B}) = P(A) - P(A \cap B)$ does not hold
(c) $P(A \cup B) = 1 - P(\overline{A})P(\overline{B})$, if A and B are independent
(d) $P(A \cup B) = 1 - P(\overline{A})P(\overline{B})$, if A and B are disjoint

28. If E and F are independent events such that $0 < P(E) < 1$ and $0 < P(F) < 1$, then **(1989, 2M)**

(a) E and F are mutually exclusive
(b) E and F^c(the complement of the event F) are independent
(c) E^c and F^c are independent
(d) $P(E/F) + P(E^c/F) = 1$

Fill in the Blanks

29. If two events A and B are such that $P(A^c) = 0.3, P(B) = 0.4$ and $P(A \cap B^c) = 0.5$, then $P[B/(A \cup B^c)] = \dots$. **(1994, 2M)**

30. Let A and B be two events such that $P(A) = 0.3$ and $P(A \cup B) = 0.8$. If A and B are independent events, then $P(B) = \dots$. **(1990, 2M)**

31. A pair of fair dice is rolled together till a sum of either 5 or 7 is obtained. Then, the probability that 5 comes before 7, is... . **(1989, 2M)**

32. Urn A contains 6 red and 4 black balls and urn B contains 4 red and 6 black balls. One ball is drawn at random from urn A and placed in urn B. Then, one ball is drawn at random from urn B and placed in urn A. If one ball is drawn at random from urn A, the probability that it is found to be red, is.... **(1988, 2M)**

33. A box contains 100 tickets numbered 1, 2, ...,100. Two tickets are chosen at random. It is given that the maximum number on the two chosen tickets is not more than 10. The maximum number on them is 5 with probability... . **(1985, 2M)**

Analytical and Descriptive Questions

34. If A and B are two independent events, prove that $P(A \cup B) \cdot P(A' \cap B') \le P(C)$, where C is an event defined that exactly one of A and B occurs. **(2004, 2M)**

35. A is targeting to B, B and C are targeting to A. Probability of hitting the target by A, B and C are $\frac{2}{3}, \frac{1}{2}$ and $\frac{1}{3}$, respectively. If A is hit, then find the probability that B hits the target and C does not. **(2003, 2M)**

36. For a student to qualify, he must pass atleast two out of three exams. The probability that he will pass the 1st exam is p. If he fails in one of the exams, then the probability of his passing in the next exam, is $\frac{p}{2}$ otherwise it remains the same. Find the probability that he will qualify. **(2003, 2M)**

37. A coin has probability p of showing head when tossed. It is tossed n times. Let p_n denotes the probability that no two (or more) consecutive heads occur. Prove that $p_1 = 1$, $p_2 = 1 - p^2$ and $p_n = (1-p).\ p_{n-1} + p(1-p)p_{n-2}, \forall\, n \ge 3$. **(2000, 5M)**

38. An unbiased coin is tossed. If the result in a head, a pair of unbiased dice is rolled and the number obtained by adding the numbers on the two faces is noted. If the result is a tail, a card from a well-shuffled pack of eleven cards numbered 2, 3, 4, ..., 12 is picked and the number on the card is noted. What is the probability that the noted number is either 7 or 8? **(1994, 5M)**

39. A lot contains 50 defective and 50 non-defective bulbs. Two bulbs are drawn at random, one at a time, with replacement. The events A, B, C are defined as :

A = (the first bulb is defective)

B = (the second bulb is non-defective)

C = (the two bulbs are both defective or both non-defective).

Determine whether
(i) A, B, C are pairwise independent.
(ii) A, B, C are independent. **(1992, 6M)**

40. In a multiple-choice question there are four alternative answers, of which one or more are correct. A candidate will get marks in the question only if he ticks the correct answers. The candidates decide to tick the answers at random, if he is allowed upto three chances to answer the questions, find the probability that he will get marks in the question. **(1985, 5M)**

41. A and B are two independent events. The probability that both A and B occur is $\frac{1}{6}$ and the probability that neither of them occurs is $\frac{1}{3}$. Find the probability of the occurrence of A. **(1984, 2M)**

42. Cards are drawn one by one at random from a well shuffled full pack of 52 playing cards until 2 aces are obtained for the first time. If N is the number of cards required to be drawn, then show that

$$P_r\{N = n\} = \frac{(n-1)(52-n)(51-n)}{50 \times 49 \times 17 \times 13}$$

where, $2 < n \le 50$. **(1983, 3M)**

43. An anti-aircraft gun can take a maximum of four shots at an enemy plane moving away from it. The probabilities of hitting the plane at the first, second, third and fourth shot are 0.4, 0.3, 0.2, and 0.1, respectively. What is the probability that the gun hits the plane? **(1981, 2M)**

44. A box contanis 2 black, 4 white and 3 red balls. One ball is drawn at random from the box and kept aside. From the remaining balls in the box, another ball is drawn at random and kept beside the first. This process is repeated till all the balls are drawn from the box. Find the probability that the balls drawn are in the sequence of 2 black, 4 white and 3 red. **(1979, 2M)**

Integer Answer Type Question

45. Of the three independent events E_1, E_2 and E_3, the probability that only E_1 occurs is α, only E_2 occurs is β and only E_3 occurs is γ. Let the probability p that none of events E_1, E_2 or E_3 occurs satisfy the equations

$(\alpha - 2\beta)$, $p = \alpha\beta$ and $(\beta - 3\gamma)$ $p = 2\beta\gamma$. All the given probabilities are assumed to lie in the interval (0, 1).

Then, $\dfrac{\text{probability of occurrence of } E_1}{\text{probability of occurrence of } E_3}$ is equal to

Passage Type Questions

Passage

Football teams T_1 and T_2 have to play two games against each other. It is assumed that the outcomes of the two games are independent. The probabilities of T_1 winning, drawing and losing a game against T_2 are $\frac{1}{2}, \frac{1}{6}$ and $\frac{1}{3}$, respectively. Each team gets 3 points for a win, 1 point for a draw and 0 point for a loss in a game. Let X and Y denote the total points scored by teams T_1 and T_2, respectively, after two games. **(2016 Adv.)**

46. $P(X > Y)$ is

(a) $\frac{1}{4}$ (b) $\frac{5}{12}$ (c) $\frac{1}{2}$ (d) $\frac{7}{12}$

47. $P(X = Y)$ is

(a) $\frac{11}{36}$ (b) $\frac{1}{3}$ (c) $\frac{13}{36}$ (d) $\frac{1}{2}$

Topic 4 Law of Total Probability and Baye's Theorem

Objective Question I (Only one correct option)

1. A pot contain 5 red and 2 green balls. At random a ball is drawn from this pot. If a drawn ball is green then put a red ball in the pot and if a drawn ball is red, then put a green ball in the pot, while drawn ball is not replace in the pot. Now we draw another ball randomnly, the probability of second ball to be red is **(2019 Main, 9 Jan II)**

(a) $\frac{27}{49}$ (b) $\frac{26}{49}$ (c) $\frac{21}{49}$ (d) $\frac{32}{49}$

2. A bag contains 4 red and 6 black balls. A ball is drawn at random from the bag, its colour is observed and this ball along with two additional balls of the same colour are returned to the bag. If now a ball is drawn at random from the bag, then the probability that this drawn ball is red, is **(2018 Main)**

(a) $\frac{3}{10}$ (b) $\frac{2}{5}$ (c) $\frac{1}{5}$ (d) $\frac{3}{4}$

3. A computer producing factory has only two plants T_1 and T_2. Plant T_1 produces 20% and plant T_2 produces 80% of the total computers produced. 7% of computers produced in the factory turn out to be defective. It is known that P(computer turns out to be defective, given that it is produced in plant T_1) $= 10P$ (computer turns out to be defective, given that it is produced in plant T_2), where $P(E)$ denotes the probability of an event E. A computer produced in the factory is randomly selected and it does not turn out to be defective. Then, the probability that it is produced in plant T_2, is **(2016 Adv.)**

(a) $\frac{36}{73}$ (b) $\frac{47}{79}$

(c) $\frac{78}{93}$ (d) $\frac{75}{83}$

4. A signal which can be green or red with probability $\frac{4}{5}$ and $\frac{1}{5}$ respectively, is received by station A and then transmitted to station B. The probability of each station receiving the signal correctly is $\frac{3}{4}$. If the signal received at station B is green, then the probability that the original signal green is **(2010)**

(a) $\frac{3}{5}$ (b) $\frac{6}{7}$ (c) $\frac{20}{23}$ (d) $\frac{9}{20}$

Objective Question II

(One or more than one correct option)

5. A ship is fitted with three engines E_1, E_2 and E_3. The engines function independently of each other with respective probabilities 1/2, 1/4 and 1/4. For the ship to be operational atleast two of its engines must function. Let X denotes the event that the ship is operational and let X_1, X_2 and X_3 denote, respectively the events that the engines E_1, E_2 and E_3 are functioning.

Which of the following is/are true? **(2012)**

(a) $P[X_1^c | X] = 3/16$

(b) P [exactly two engines of the ship are functioning] $= \frac{7}{8}$

(c) $P[X | X_2] = \frac{5}{16}$

(d) $P[X | X_1] = \frac{7}{16}$

Assertion and Reason

For the following questions, choose the correct answer from the codes (a), (b), (c) and (d) defined as follows.

(a) Statement I is true, Statement II is also true; Statement II is the correct explanation of Statement I

(b) Statement I is true, Statement II is also true; Statement II is not the correct explanation of Statement I

(c) Statement I is true; Statement II is false

(d) Statement I is false; Statement II is true

6. Let $H_1, H_2, \ldots, H_n$ be mutually exclusive events with $P(H_i) > 0$, $i = 1, 2, \ldots, n$. Let E be any other event with $0 < P(E) < 1$.

Statement I $P(H_i/E) > P(E/H_i) \cdot P(H_i)$ for $i = 1, 2, \ldots, n$

Statement II $\sum_{i=1}^{n} P(H_i) = 1$ **(2007, 3M)**

Passage Based Problems

Passage I

Let n_1 and n_2 be the number of red and black balls, respectively in box I. Let n_3 and n_4 be the number of red and black balls, respectively in box II. **(2015 Adv.)**

7. One of the two boxes, box I and box II was selected at random and a ball was drawn randomly out of this box. The ball was found to be red. If the probability that this red ball was drawn from box II, is $\frac{1}{3}$, then the correct option(s) with the possible values of n_1, n_2, n_3 and n_4 is/are

(a) $n_1 = 3, n_2 = 3, n_3 = 5, n_4 = 15$

(b) $n_1 = 3, n_2 = 6, n_3 = 10, n_4 = 50$

(c) $n_1 = 8, n_2 = 6, n_3 = 5, n_4 = 20$

(d) $n_1 = 6, n_2 = 12, n_3 = 5, n_4 = 20$

8. A ball is drawn at random from box I and transferred to box II. If the probability of drawing a red ball from box I, after this transfer, is $\frac{1}{3}$, then the correct option(s) with the possible values of n_1 and n_2 is/are

(a) $n_1 = 4$ and $n_2 = 6$ (b) $n_1 = 2$ and $n_2 = 3$

(c) $n_1 = 10$ and $n_2 = 20$ (d) $n_1 = 3$ and $n_2 = 6$

Passage II

Let U_1 and U_2 be two urns such that U_1 contains 3 white and 2 red balls and U_2 contains only 1 white ball. A fair coin is tossed. If head appears then 1 ball is drawn at random from U_1 and put into U_2. However, if tail appears then 2 balls are drawn at random from U_1 and put into U_2. Now, 1 ball is drawn at random from U_2. **(2011)**

9. The probability of the drawn ball from U_2 being white, is

(a) $\frac{13}{30}$ (b) $\frac{23}{30}$ (c) $\frac{19}{30}$ (d) $\frac{11}{30}$

10. Given that the drawn ball from U_2 is white, the probability that head appeared on the coin is

(a) $\frac{17}{23}$ (b) $\frac{11}{23}$ (c) $\frac{15}{23}$ (d) $\frac{12}{23}$

Passage III

A fair die is tossed repeatedly until a six is obtained. Let X denote the number of tosses required. **(2009)**

11. The probability that $X = 3$ equals

(a) $\frac{25}{216}$ (b) $\frac{25}{36}$ (c) $\frac{5}{36}$ (d) $\frac{125}{216}$

12. The probability that $X \geq 3$ equals

(a) $\frac{125}{216}$ (b) $\frac{25}{36}$ (c) $\frac{5}{36}$ (d) $\frac{25}{216}$

13. The conditional probability that $X \geq 6$ given $X > 3$ equals

(a) $\frac{125}{216}$ (b) $\frac{25}{216}$ (c) $\frac{5}{36}$ (d) $\frac{25}{36}$

Passage IV

There are n urns each containing $(n+1)$ balls such that the ith urn contains 'i' white balls and $(n+1-i)$ red balls. Let u_i be the event of selecting ith urn, $i = 1, 2, 3, \ldots, n$ and W denotes the event of getting a white balls. **(2006, 5M)**

14. If $P(u_i) \propto i$, where $i = 1, 2, 3, \ldots, n$, then $\lim_{n\to\infty} P(W)$ is equal to

(a) 1 (b) $\frac{2}{3}$

(c) $\frac{1}{4}$ (d) $\frac{3}{4}$

15. If $P(u_i) = c$, where c is a constant, then $P(u_n / W)$ is equal to

(a) $\frac{2}{n+1}$ (b) $\frac{1}{n+1}$ (c) $\frac{n}{n+1}$ (d) $\frac{1}{2}$

16. If n is even and E denotes the event of choosing even numbered urn $\left[P(u_i) = \frac{1}{n}\right]$, then the value of $P(W/E)$ is

(a) $\frac{n+2}{2n+1}$ (b) $\frac{n+2}{2(n+1)}$ (c) $\frac{n}{n+1}$ (d) $\frac{1}{n+1}$

Analytical and Descriptive Questions

17. A person goes to office either by car, scooter, bus or train probability of which being $\frac{1}{7}, \frac{3}{7}, \frac{2}{7}$ and $\frac{1}{7}$, respectively. Probability that he reaches offices late, if he takes car, scooter, bus or train is $\frac{2}{9}, \frac{1}{9}, \frac{4}{9}$ and $\frac{1}{9}$, respectively. Given that he reached office in time, then what is the probability that he travelled by a car? **(2005, 2M)**

18. A bag contains 12 red balls and 6 white balls. Six balls are drawn one by one without replacement of which at least 4 balls are white. Find the probability that in the next two drawn exactly one white ball is drawn. (Leave the answer in ${}^nC_r$). **(2004, 4M)**

19. A box contains N coins, m of which are fair and the rest are biased. The probability of getting a head when a fair coin is tossed, is 1/2, while it is 2/3 when a biased coin is tossed. A coin is drawn from the box at random and is tossed twice. The first time it shows head and the second time it shows tail. What is the probability that the coin drawn is fair? **(2002, 5M)**

20. An urn contains m white and n black balls. A ball is drawn at random and is put back into the urn along with k additional balls of the same colour as that of the ball drawn. A ball is again drawn at random. What is the probability that the ball drawn now is white? **(2001, 5M)**

21. Eight players $P_1, P_2, \ldots, P_8$ play a knock-out tournament. It is known that whenever the players P_i and P_j play, the player P_i will win if $i < j$. Assuming that the players are paired at random in each round, what is the probability that the player P_4 reaches the final? **(1999, 10M)**

22. Three players, A, B and C, toss a coin cyclically in that order (i.e. $A, B, C, A, B, C, A, B, \ldots$) till a head shows. Let p be the probability that the coin shows a head. Let α, β and γ be, respectively, the probabilities that A, B and C gets the first head. Prove that $\beta = (1-p)\alpha$. Determine α, β and γ (in terms of p). **(1998, 8M)**

23. Sixteen players $S_1, S_2, \ldots, S_{16}$ play in a tournament. They are divided into eight pairs at random from each pair a winner is decided on the basis of a game played between the two players of the pair. Assume that all the players are of equal strength.

(i) Find the probability that the player S_1 is among the eight winners.

(ii) Find the probability that exactly one of the two players S_1 and S_2 is among the eight winners. **(1997C, 5M)**

24. In a test an examinee either guesses or copies of knows the answer to a multiple choice question with four choices. The probability that he make a guess is $\frac{1}{3}$ and the probability that he copies the answer is $\frac{1}{6}$. The probability that his answer is correct given that he copied it, is $\frac{1}{8}$. Find the probability that he knew the answer to the question given that he correctly answered it. **(1991, 4M)**

25. An urn contains 2 white and 2 blacks balls. A ball is drawn at random. If it is white it is not replaced into the urn. Otherwise it is replaced along with another ball of the same colour. The process is repeated. Find the probability that the third ball drawn is black. **(1987, 4M)**

26. A lot contains 20 articles. The probability that the lot contains exactly 2 defective articles is 0.4 and the probability that the lot contains exactly 3 defective articles is 0.6. Articles are drawn from the lot at random one by one without replacement and are tested till all defective articles are found. What is the probability that the testing procedure ends at the twelfth testing? **(1986, 5M)**

Topic 5 Probability Distribution and Binomial Distribution

Objective Questions I (Only one correct option)

1. For an initial screening of an admission test, a candidate is given fifty problems to solve. If the probability that the candidate can solve any problem is $\frac{4}{5}$, then the probability that he is unable to solve less than two problem is **(2019 Main, 12 April II)**

(a) $\frac{201}{5}\left(\frac{1}{5}\right)^{49}$ (b) $\frac{316}{25}\left(\frac{4}{5}\right)^{48}$ (c) $\frac{54}{5}\left(\frac{4}{5}\right)^{49}$ (d) $\frac{164}{25}\left(\frac{1}{5}\right)^{48}$

2. Let a random variable X have a binomial distribution with mean 8 and variance 4. If $P(X \le 2) = \frac{k}{2^{16}}$, then k is equal to **(2019 Main, 12 April I)**

(a) 17 (b) 121 (c) 1 (d) 137

3. Minimum number of times a fair coin must be tossed so that the probability of getting atleast one head is more than 99% is **(2019 Main 10 April II)**

(a) 8 (b) 6 (c) 7 (d) 5

4. The minimum number of times one has to toss a fair coin so that the probability of observing atleast one head is atleast 90% is **(2019 Main, 8 April II)**

(a) 2 (b) 3 (c) 5 (d) 4

5. In a game, a man wins ₹ 100 if he gets 5 or 6 on a throw of a fair die and loses ₹ 50 for getting any other number on the die. If he decides to throw the die either till he gets a five or a six or to a maximum of three throws, then his expected gain/loss (in rupees) is **(2019 Main, 12 Jan II)**

(a) $\frac{400}{3}$ loss (b) $\frac{400}{9}$ loss (c) 0 (d) $\frac{400}{3}$ gain

6. If the probability of hitting a target by a shooter in any shot, is $\frac{1}{3}$, then the minimum number of independent shots at the target required by him so that the probability of hitting the target at least once is greater than $\frac{5}{6}$, is **(2019 Main, 10 Jan II)**

(a) 6 (b) 3 (c) 5 (d) 4

7. Two cards are drawn successively with replacement from a well shuffled deck of 52 cards. Let X denote the random variable of number of aces obtained in the two drawn cards. Then, $P(X = 1) + P(X = 2)$ equals **(2019 Main, 9 Jan I)**

(a) $\frac{25}{169}$ (b) $\frac{52}{169}$ (c) $\frac{49}{169}$ (d) $\frac{24}{169}$

8. A box contains 15 green and 10 yellow balls. If 10 balls are randomly drawn one-by-one with replacement, then the variance of the number of green balls drawn is **(2017 Main)**

(a) $\frac{12}{5}$ (b) 6 (c) 4 (d) $\frac{6}{25}$

9. A multiple choice examination has 5 questions. Each question has three alternative answers of which exactly one is correct. The probability that a student will get 4 or more correct answers just by guessing is **(2013 Main)**

(a) $\frac{17}{3^5}$ (b) $\frac{13}{3^5}$ (c) $\frac{11}{3^5}$ (d) $\frac{10}{3^5}$

10. India plays two matches each with West Indies and Australia. In any match the probabilities of India getting points 0, 1 and 2 are 0.45, 0.05 and 0.50, respectively. Assuming that the outcomes are independent. The probability of India getting at least 7 points, is **(1992, 2M)**

(a) 0.8750 (b) 0.0875 (c) 0.0625 (d) 0.0250

11. One hundred identical coins, each with probability p, of showing up heads are tossed once. If $0 < p < 1$ and the probability of heads showing on 50 coins is equal to that of heads showing on 51 coins, then the value of p is **(1988, 2M)**

(a) 1/2 (b) 49/101 (c) 50/101 (d) 51/101

Fill in the Blanks

12. If the mean and the variance of a binomial variate X are 2 and 1 respectively, then the probability that X takes a value greater than one is equal to… . **(1991, 2M)**

13. For a biased die the probabilities for the different faces to turn up are given below

Face	1	2	3	4	5	6
Probability	0.1	0.32	0.21	0.15	0.05	0.17

This die is tossed and you are told that either face 1 or face 2 has turned up. Then, the probability that it is face 1, is… . **(1981, 2M)**

Analytical & Descriptive Questions

14. Numbers are selected at random, one at a time, from the two-digit numbers 00, 01, 02, …, 99 with replacement. An event E occurs if and only if the product of the two digits of a selected number is 18. If four numbers are selected, find probability that the event E occurs at least 3 times. **(1993, 5M)**

15. A is a set containing n elements. A subset P of A is chosen at random. The set A is reconstructed by replacing the elements of P. A subset Q of A is again chosen at random. Find the probability that P and Q have no common elements. **(1991, 4M)**

16. Suppose the probability for A to win a game against B is 0.4. If A has an option of playing either a 'best of 3 games' or a 'best of 5 games" match against B, which option should choose so that the probability of his winning the match is higher? (no game ends in a draw). **(1989, 5M)**

17. A man takes a step forward with probability 0.4 and backwards with probability 0.6. Find the probability that at the end of eleven steps he is one step away from the starting point. **(1987, 3M)**

Integer Type Question

18. The minimum number of times a fair coin needs to be tossed, so that the probability of getting atleast two heads is atleast 0.96, is **(2015 Adv.)**

Answers

Topic 1

1. (c)	**2.** (a)	**3.** (a)	**4.** (d)
5. (a)	**6.** (c)	**7.** (a)	**8.** (a)
9. (a)	**10.** (c)	**11.** (d)	**12.** (d)
13. (a)	**14.** (b)	**15.** (c)	**16.** (b)
17. (d)	**18.** (b)	**19.** (b)	**20.** (c)
21. $\frac{1}{36}$	**22.** $\frac{1}{3} \le p \le \frac{1}{2}$	**23.** $\frac{3}{16}$	**24.** False
25. $\frac{(3^n - 3\cdot 2^n + 3) \times {^6C_3}}{6^n}$		**26.** 0.62	**27.** $\frac{1}{91}$
28. $1 - \frac{10\,(n+2)}{^{n+7}C_5}$		**29.** (i) $\frac{1}{132}$ (ii) $\frac{1}{462}$	

Topic 2

1. (b)	**2.** (a)	**3.** (c)	**4.** (a)
5. (c)	**6.** (c)	**7.** (a)	**8.** (a, b, c)
9. (a, c)	**10.** $\frac{11}{40}$	**11.** $P\,(A \cap B)$	**12.** False
13. 13.9%	**15.** No	**16.** (a)	**17.** (c)

Topic 3

1. (d)	**2.** (b)	**3.** (b)	**4.** (a)
5. (c)	**6.** (d)	**7.** (a)	**8.** (a)
9. (d)	**10.** (c)	**11.** (c)	**12.** (b)
13. (b)	**14.** (a)	**15.** (a)	**16.** (b)
17. (a)	**18.** (c)	**19.** (b)	**20.** (b)
21. (a, b)	**22.** (a,b)	**23.** (a, d)	**24.** (b, c)
25. (a, d)	**26.** (a, d)	**27.** (a, c)	**28.** (b, c, d)
29. $\frac{1}{4}$	**30.** $\frac{5}{7}$	**31.** $\frac{2}{5}$	**32.** $\frac{32}{55}$
33. $\frac{1}{9}$	**35.** $\frac{1}{2}$	**36.** $2p^2 - p^3$	**38.** $\frac{193}{792}$

39. (i) A, B and C are pairwise independent **40.** $\frac{1}{5}$

41. $\frac{1}{3}$ or $\frac{1}{2}$	**43.** 0.6976	**44.** $\frac{1}{1260}$	**45.** 6
46. (b)	**47.** (c)		

Topic 4

1. (d)	**2.** (b)	**3.** (c)	**4.** (c)
5. (b, d)	**6.** (d)	**7.** (b)	**8.** (d)
9. (b)	**10.** (d)	**11.** (a)	**12.** (b)`
13. (d)	**14.** (b)	**15.** (a)	**16.** (b)
17. $\frac{1}{7}$			

18. $\frac{^{12}C_2 \cdot {^6C_4}}{^{18}C_6} \cdot \frac{^{10}C_1 \cdot {^2C_1}}{^{12}C_2} + \frac{^{12}C_1 \cdot {^6C_5}}{^{18}C_6} \cdot \frac{^{11}C_1 \cdot {^1C_1}}{^{12}C_2}$ **19.** $\frac{9m}{8N + m}$

20. $\frac{m}{m+n}$ **21.** $\frac{4}{35}$

22. $\alpha = \frac{p}{1-(1-p)^3}, \beta = \frac{p\,(1-p)}{1-(1-p)^3}, \gamma = \frac{p - 2p^2 + p^3}{1(1-p)^3}$

23. (i) $\frac{1}{2}$ (ii) $\frac{8}{15}$	**24.** $\frac{24}{29}$	**25.** $\frac{23}{30}$	**26.** $\frac{99}{1900}$

Topic 5

1. (c)	**2.** (d)	**3.** (c)	**4.** (d)
5. (c)	**6.** (c)	**7.** (a)	**8.** (a)
9. (c)	**10.** (b)	**11.** (d)	**12.** $\frac{11}{16}$
13. $\frac{5}{21}$	**14.** $\frac{97}{25^4}$	**15.** $\left(\frac{3}{4}\right)^n$	
16. Best of 3 games		**17.** $^{11}C_6(0.24)^5$	**18.** (8)

Hints & Solutions

Topic 1 Classical Probability

1. It is given that a person wins ₹15 for throwing a doublet (1, 1) (2, 2), (3, 3), (4, 4), (5, 5), (6, 6) and win ₹12 when the throw results in sum of 9, i.e., when (3, 6), (4, 5), (5, 4), (6, 3) occurs.

Also, losses ₹6 for throwing any other outcome, i.e., when any of the rest $36 - 6 - 4 = 26$ outcomes occurs.

Now, the expected gain/loss

$= 15 \times P$ (getting a doublet) $+ 12 \times P$ (getting sum 9) $- 6 \times P$ (getting any of rest 26 outcome)

$$= \left(15 \times \frac{6}{36}\right) + \left(12 \times \frac{4}{36}\right) - \left(6 \times \frac{26}{36}\right)$$

$$= \frac{5}{2} + \frac{4}{3} - \frac{26}{6} = \frac{15 + 8 - 26}{6}$$

$$= \frac{23 - 26}{6} = -\frac{3}{6} = -\frac{1}{2}, \text{ means loss of ₹}\frac{1}{2}$$

2. Since, the experiment should be end in the fifth throw of the die, so total number of outcomes are 6^5.

Now, as the last two throws should be result in two fours

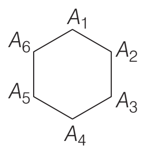
$\underset{\text{(i)}}{\_\_}\ \underset{\text{(ii)}}{\_\_}\ \underset{\text{(iii)}}{\_\_}\ \underset{\text{(iv)}}{4}\ \underset{\text{(v)}}{4}$

So, the third throw can be 1, 2, 3, 5 or 6 (not 4). Also, throw number (i) and (ii) can not take two fours in succession, therefore number of possibililites for throw (i) and (ii) $= 6^2 - 1 = 35$

[∵ when a pair of dice is thrown then (4, 4) occur only once].

Hence, the required probability $= \dfrac{5 \times 35}{6^5} = \dfrac{175}{6^5}$

3. Since, there is a regular hexagon, then the number of ways of choosing three vertices is ${}^6C_3$. And, there is only two ways i.e. choosing vertices of a regular hexagon alternate, here A_1, A_3, A_5 or A_2, A_4, A_6 will result in an equilateral triangle.



∴Required probability

$$= \frac{2}{{}^6C_3} = \frac{2}{\frac{6!}{3!3!}} = \frac{2 \times 3 \times 2 \times 3 \times 2}{6 \times 5 \times 4 \times 3 \times 2 \times 1} = \frac{1}{10}$$

4. Number of subset of $S = 2^{20}$

Sum of elements in S is $1 + 2 + \ldots.. + 20 = \dfrac{20(21)}{2} = 210$

$$\left[\because 1 + 2 + \ldots\ldots + n = \frac{n(n+1)}{2}\right]$$

Clearly, the sum of elements of a subset would be 203, if we consider it as follows

$S - \{7\}, S - \{1, 6\}\ S - \{2, 5\}, S - \{3, 4\}$

$S - \{1, 2, 4\}$

∴Number of favourables cases = 5

Hence, required probability $= \dfrac{5}{2^{20}}$

5. Total number of ways of selecting 2 different numbers from $\{0, 1, 2, \ldots, 10\} = {}^{11}C_2 = 55$

Let two numbers selected be x and y.

Then, $\quad x + y = 4m \quad \ldots$(i)

and $\quad x - y = 4n \quad \ldots$(ii)

$\Rightarrow \quad 2x = 4(m + n)$ and $2y = 4(m - n)$

$\Rightarrow \quad x = 2(m + n)$ and $y = 2(m - n)$

∴x and y both are even numbers.

x	y
0	4, 8
2	6, 10
4	0, 8
6	2, 10
8	0, 4
10	2, 6

∴Required probability $= \dfrac{6}{55}$

6. Sample space $\to {}^{12}C_2$

Number of possibilities for z is even.

$z = 0 \Rightarrow {}^{11}C_1$

$z = 2 \Rightarrow {}^{9}C_1$

$z = 4 \Rightarrow {}^{7}C_1$

$z = 6 \Rightarrow {}^{5}C_1$

$z = 8 \Rightarrow {}^{3}C_1$

$z = 10 \Rightarrow {}^{1}C_1$

Total = 36

∴ Probability $= \dfrac{36}{66} = \dfrac{6}{11}$

7. We have mentioned that boxes are different and one particular box has 3 balls.

Then, number of ways $= \dfrac{{}^{12}C_3 \times 2^9}{3^{12}} = \dfrac{55}{3}\left(\dfrac{2}{3}\right)^{11}$

8. Total number of ways to arrange 3 boys and 2 girls are 5!.

According to given condition, following cases may arise.

B	G	G	B	B
G	G	B	B	B
G	B	G	B	B
G	B	B	G	B
B	G	B	G	B

So, number of favourable ways $= 5 \times 3! \times 2! = 60$

∴ $\quad$ Required probability $= \dfrac{60}{120} = \dfrac{1}{2}$

9. **PLAN** As one of the dice shows a number appearing on one of P_1, P_2 and P_3.

Thus, three cases arise.

(i) All show same number.

(ii) Number appearing on D_4 appears on any one of D_1, D_2 and D_3.

(iii) Number appearing on D_4 appears on any two of D_1, D_2 and D_3.

Sample space $= 6 \times 6 \times 6 \times 6 = 6^4$ favourable events

$= $ Case I or Case II or Case III

***Case* I** First we should select one number for D_4 which appears on all i.e. ${}^6C_1 \times 1$.

***Case* II** For D_4 there are ${}^6C_1$ ways. Now, it appears on any one of D_1, D_2 and D_3 i.e. ${}^3C_1 \times 1$.

For other two there are 5×5 ways.

$\Rightarrow \quad {}^6C_1 \times {}^3C_1 \times 1 \times 5 \times 5$

***Case* III** For D_4 there are ${}^6C_1$ ways now it appears on any two of D_1, D_2 and D_3

$\Rightarrow \quad {}^3C_2 \times 1^2$

For other one there are 5 ways.

$\Rightarrow \quad {}^6C_1 \times {}^3C_2 \times 1^2 \times 5$

$$\text{Thus, probability} = \frac{{}^6C_1 + {}^6C_1 \times {}^3C_1 \times 5^2 + {}^6C_1 \times {}^3C_2 \times 5}{6^4} = \frac{6\,(1 + 75 + 15)}{6^4} = \frac{91}{216}$$

10. **Sample space** A dice is thrown thrice, $n(s) = 6 \times 6 \times 6$.

Favorable events $\omega^{r_1} + \omega^{r_2} + \omega^{r_3} = 0$

i.e. (r_1, r_2, r_3) are ordered 3 triples which can take values,

$$\left.\begin{matrix}(1,2,3), & (1,5,3), & (4,2,3), & (4,5,3)\\ (1,2,6), & (1,5,6), & (4,2,6), & (4,5,6)\end{matrix}\right\}\text{i.e. 8 ordered pairs}$$

and each can be arranged in $3!$ ways $= 6$

$$\therefore \quad n(E) = 8 \times 6 \Rightarrow P(E) = \frac{8 \times 6}{6 \times 6 \times 6} = \frac{2}{9}$$

11. Since, three distinct numbers are to be selected from first 100 natural numbers.

$\Rightarrow \quad n(S) = {}^{100}C_3$

$E_{(\text{favourable events})} =$ All three of them are divisible by both 2 and 3 .

$\Rightarrow$ Divisible by 6 i.e. {6, 12, 18, …, 96}

Thus, out of 16 we have to select 3.

$\therefore \quad n(E) = {}^{16}C_3$

$$\therefore \quad \text{Required probability} = \frac{{}^{16}C_3}{{}^{100}C_3} = \frac{4}{1155}$$

12. Here, two numbers are selected from {1, 2, 3, 4, 5, 6}

$\Rightarrow n(S) = 6 \times 5$ {as one by one without replacement}

Favourable events = the minimum of the two numbers is less than 4. $n(E) = 6 \times 4$ {as for the minimum of the two is less than 4 we can select one from (1, 2, 3, 4) and other from (1, 2, 3, 4, 5, 6)

$$\therefore \quad \text{Required probability} = \frac{n(E)}{n(S)} = \frac{24}{30} = \frac{4}{5}$$

13. $7^1 = 7,\ 7^2 = 49,\ 7^3 = 343,\ 7^4 = 2401, \ \ldots$

Therefore, for 7^r, $r \in N$ the number ends at unit place 7, 9, 3, 1, 7, …

$\therefore \quad 7^m + 7^n$ will be divisible by 5 if it end at 5 or 0.

But it cannot end at 5.

Also for end at 0.

For this m and n should be as follows

	m	n
1	$4r$	$4r-2$
2	$4r-1$	$4r-3$
3	$4r-2$	$4r$
4	$4r-3$	$4r-1$

For any given value of m, there will be 25 values of n.

Hence, the probability of the required event is

$$\frac{100 \times 25}{100 \times 100} = \frac{1}{4}$$

NOTE Power of prime numbers have cyclic numbers in their unit place.

14. The number of ways of placing 3 black balls without any restriction is ${}^{10}C_3$. Since, we have total 10 places of putting 10 balls in a row. Now, the number of ways in which no two black balls put together is equal to the number of ways of choosing 3 places marked '—' out of eight places.

—W—W—W—W —W—W—W—

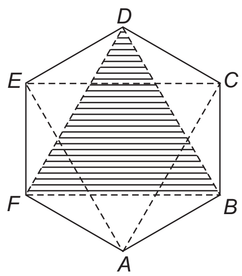
This can be done in ${}^8C_3$ ways.

$$\therefore \quad \text{Required probability} = \frac{{}^8C_3}{{}^{10}C_3} = \frac{8 \times 7 \times 6}{10 \times 9 \times 8} = \frac{7}{15}$$

15. Three vertices out of 6 can be chosen in ${}^6C_3$ ways.

So, total ways $= {}^6C_3 = 20$

Only two equilateral triangles can be formed ΔAEC and ΔBFD.



$\therefore$ Favourable ways $= 2$

So, required probability

$$= \frac{2}{20} = \frac{1}{10}$$

16. Since, three dice are rolled.

$\therefore$ Total number of cases $S = 6 \times 6 \times 6 = 216$

and the same number appear on each of them $= {}^6C_1 = 6$

$$\therefore \quad \text{Required probability} = \frac{6}{216} = \frac{1}{36}$$

17. Since, there are 15 possible cases for selecting a coupon and seven coupons are selected, the total number of cases of selecting seven coupons $= 15^7$

It is given that the maximum number on the selected coupon is 9, therefore the selection is to be made from the coupons numbered 1 to 9. This can be made in 9^7 ways. Out of these 9^7 cases, 8^7 does not contain the number 9.

Thus, the favourable number of cases $= 9^7 - 8^7$.

$\therefore$ Required probability $= \dfrac{9^7 - 8^7}{15^7}$

18. The number of all possible determinants of the form

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = 2^4 = 16$$

Out of which only 10 determinants given by

$$\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}, \begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix}, \begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix}, \begin{vmatrix} 0 & 0 \\ 1 & 1 \end{vmatrix}, \begin{vmatrix} 0 & 1 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 0 \\ 1 & 0 \end{vmatrix}, \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix}, \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix}, \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix}$$

Vanish and remaining six determinants have non-zero values. Hence, the required probability $= \dfrac{6}{16} = \dfrac{3}{8}$

Statement I is true.

Statement II is also true as the homogeneous equations have always a solution and Statement II is not the correct explanation of Statement I.

19. **PLAN** Probability $= \dfrac{\text{Number of favourable outcomes}}{\text{Number of total outcomes}}$

As, $x_1 + x_2 + x_3$ is odd.

So, all may be odd or one of them is odd and other two are even.

$\therefore$ Required probability

$$= \frac{{}^2C_1 \times {}^3C_1 \times {}^4C_1 + {}^1C_1 \times {}^2C_1 \times {}^4C_1 + {}^2C_1 \times {}^2C_1 \times {}^3C_1 + {}^1C_1 \times {}^3C_1 \times {}^3C_1}{{}^3C_1 \times {}^5C_1 \times {}^7C_1}$$

$$= \frac{24 + 8 + 12 + 9}{105}$$

$$= \frac{53}{105}$$

20. Since, x_1, x_2, x_3 are in AP.

$\therefore \quad x_1 + x_3 = 2x_2$

So, $x_1 + x_3$ should be even number.

Either both x_1 and x_3 are odd or both are even.

$\therefore$ Required probability $= \dfrac{{}^2C_1 \times {}^4C_1 + {}^1C_1 \times {}^3C_1}{{}^3C_1 \times {}^5C_1 \times {}^7C_1}$

$$= \frac{11}{105}$$

21. According to given condition,

P (yellow at the first toss) $= \dfrac{3}{6} = \dfrac{1}{2}$

P (red at the second toss) $= \dfrac{2}{6} = \dfrac{1}{3}$

and P (blue at the third toss) $= \dfrac{1}{6}$

Therefore, the probability of the required event

$$= \frac{1}{2} \times \frac{1}{3} \times \frac{1}{6} = \frac{1}{36}$$

22. Since, $\dfrac{1+3p}{3}, \dfrac{1-p}{4}$ and $\dfrac{1-2p}{2}$ are the probability of mutually exclusive events.

$$\therefore \quad \frac{1+3p}{3} + \frac{1-p}{4} + \frac{1-2p}{2} \le 1$$

$$\Rightarrow \quad 4 + 12p + 3 - 3p + 6 - 12p \le 12$$

$$\Rightarrow \quad 13 - 3p \le 12$$

$$\Rightarrow \quad p \ge \frac{1}{3} \quad \text{...(i)}$$

and $0 \le \dfrac{1+3p}{3} \le 1, 0 \le \dfrac{1-p}{4} \le 1, 0 \le \dfrac{1-2p}{2} \le 1$

$$\Rightarrow \quad 0 \le 1 + 3p \le 3, 0 \le 1 - p \le 4, 0 \le 1 - 2p \le 2$$

$$\Rightarrow \quad -\frac{1}{3} \le p \le \frac{2}{3}, 1 \ge p \ge -3, \frac{1}{2} \ge p \ge -\frac{1}{2} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), $1/3 \le p \le 1/2$

23. Since, determinant is of order 2×2 and each element is 0 or 1 only.

$$\therefore \quad n(S) = 2^4 = 16$$

and the determinant is positive are

$$\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}$$

$$\therefore \quad n(E) = 3$$

Thus, the required probability $= \dfrac{3}{16}$

24. Total number of ways to arrange 'ASSASSIN' is $\dfrac{8!}{4! \cdot 2!}$.

First we fix the position ⊗A ⊗ A ⊗ I⊗ N ⊗.

Number of ways in which no two S's occur together

$$= \frac{4!}{2!} \times {}^5C_4$$

$\therefore$ Required probability $= \dfrac{4! \times 5 \times 4! \times 2!}{2! \times 8!} = \dfrac{1}{14}$

Hence, it is a false statement.

25. Let us define a onto function F from $A : [r_1, r_2, \dots, r_n]$ to B: [1, 2, 3], where $r_1, r_2, \dots, r_n$ are the readings of n throws and 1, 2, 3 are the numbers that appear in the n throws.

Number of such functions, $M = N - [n(1) - n(2) + n(3)]$

where, N = total number of functions

and $n(t)$ = number of function having exactly t elements in the range.

Now, $\quad N = 3^n, n(1) = 3 \cdot 2^n, n(2) = 3, n(3) = 0$

$\Rightarrow \quad M = 3^n - 3 \cdot 2^n + 3$

Hence, the total number of favourable cases

$$= (3^n - 3 \cdot 2^n + 3) \cdot {}^6C_3$$

$\therefore$ Required probability $= \dfrac{(3^n - 3 \cdot 2^n + 3) \times {}^6C_3}{6^n}$

26. The required probability = 1 – (probability of the event that the roots of $x^2 + px + q = 0$ are non-real).

The roots of $x^2 + px + q = 0$ will be non-real if and only if $p^2 - 4q < 0$, i.e. if $p^2 < 4q$

The possible values of p and q can be possible according to the following table.

Value of q	Value of p	Number of pairs of p, q
1	1	1
2	1, 2	2
3	1, 2, 3	3
4	1, 2, 3	3
5	1, 2, 3, 4	4
6	1, 2, 3, 4	4
7	1, 2, 3, 4, 5	5
8	1, 2, 3, 4, 5	5
9	1, 2, 3, 4, 5	5
10	1, 2, 3, 4, 5, 6	6

Therefore, the number of possible pairs = 38

Also, the total number of possible pairs is $10 \times 10 = 100$

$\therefore$ The required probability $= 1 - \dfrac{38}{100} = 1 - 0.38 = 0.62$

27. We have 14 seats in two vans and there are 9 boys and 3 girls. The number of ways of arranging 12 people on 14 seats without restriction is

$$^{14}P_{12} = \frac{14!}{2!} = 7(13!)$$

Now, the number of ways of choosing back seats is 2.

and the number of ways of arranging 3 girls on adjacent seats is 2(3!) and the number of ways of arranging 9 boys on the remaining 11 seats is $^{11}P_9$ ways.

Therefore, the required number of ways

$$= 2 \cdot (2 \cdot 3!) \cdot {}^{11}P_9 = \frac{4 \cdot 3!\,11!}{2!} = 12!$$

Hence, the probability of the required event

$$= \frac{12!}{7 \cdot 13!} = \frac{1}{91}$$

28. There are $(n + 7)$ coins in the box out of which five coins can be taken out in $^{n+7}C_5$ ways.

The total value of 5 coins can be equal to or more than one rupee and fifty paise in the following ways.

(i) When one 50 paise coin and four 25 paise coins are chosen.

(ii) When two 50 paise coins and three 25 paise coins are chosen.

(iii) When two 50 paise coins, 2 twenty five paise coins and one from n coins of ten and five paise.

$\therefore$ The total number of ways of selecting five coins so that the total value of the coins is not less than one rupee and fifty paise is

$$({}^2C_1 \cdot {}^5C_5 \cdot {}^nC_0) + ({}^2C_2 \cdot {}^5C_3 \cdot {}^nC_0) + ({}^2C_2 \cdot {}^5C_2 \cdot {}^nC_1)$$
$$= 10 + 10 + 10n = 10(n + 2)$$

So, the number of ways of selecting five coins, so that the total value of the coins is less than one rupee and fifty paise is $^{n+7}C_5 - 10(n + 2)$

$$\therefore \text{ Required probability} = \frac{^{n+7}C_5 - 10(n+2)}{^{n+7}C_5} = 1 - \frac{10(n+2)}{^{n+7}C_5}$$

29. (i) The total number of arrangements of six boys and six girls = 12!

$$\therefore \text{ Required probability} = \frac{6! \times 7!}{(12)!} = \frac{1}{132}$$

[since, we consider six girls at one person]

(ii) Required probability $= \dfrac{2 \times 6! \times 6!}{(12)!} = \dfrac{1}{462}$

Topic 2 Addition and Subtraction Law of Probability

1. We have, P (exactly one of A or B occurs)

$$= P(A \cup B) - P(A \cap B)$$
$$= P(A) + P(B) - 2P(A \cap B)$$

According to the question,

$$P(A) + P(B) - 2P(A \cap B) = \frac{1}{4} \quad \ldots\text{(i)}$$

$$P(B) + P(C) - 2P(B \cap C) = \frac{1}{4} \quad \ldots\text{(ii)}$$

and
$$P(C) + P(A) - 2P(C \cap A) = \frac{1}{4} \quad \ldots\text{(iii)}$$

On adding Eqs. (i), (ii) and (iii), we get

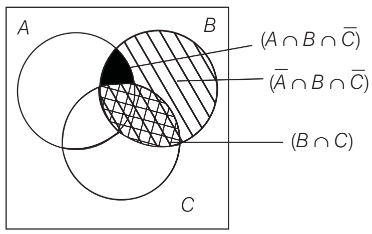
$$2[P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A)] = \frac{3}{4}$$

$$\Rightarrow P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) = \frac{3}{8}$$

$\therefore P$ (atleast one event occurs)

$$= P(A \cup B \cup C)$$
$$= P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$
$$= \frac{3}{8} + \frac{1}{16} = \frac{7}{16} \quad \left[\because P(A \cap B \cap C) = \frac{1}{16}\right]$$

2. Given, $P(B) = \dfrac{3}{4}$, $P(A \cap B \cap \overline{C}) = \dfrac{1}{3}$



and
$$P(\overline{A} \cap B \cap \overline{C}) = \frac{1}{3}$$

which can be shown in Venn diagram.

$$\therefore P(B \cap C) = P(B) - \{P(A \cap B \cap \overline{C} + P(\overline{A} \cap B \cap \overline{C}))\}$$

$$= \frac{3}{4} - \left(\frac{1}{3} + \frac{1}{3}\right) = \frac{3}{4} - \frac{2}{3} = \frac{1}{12}$$

3. It is given that, $P(E) \le P(F) \Rightarrow E \subseteq F$ ...(i)

and $P(E \cap F) > 0 \Rightarrow E \subset F$ ...(ii)

(a) occurrence of $E \Rightarrow$ occurrence of F [from Eq. (i)]
(b) occurrence of $F \Rightarrow$ occurrence of E [from Eq. (ii)]
(c) non-occurrence of $E \Rightarrow$ occurrence of F
Hence, option (c) is not correct. [from Eq. (i)]

4. We know that,

P (exactly one of A or B occurs)

$$= P(A) + P(B) - 2P(A \cap B)$$

$\therefore$ $P(A) + P(B) - 2P(A \cap B) = p$ ...(i)

Similarly, $P(B) + P(C) - 2P(B \cap C) = p$ ...(ii)

and $P(C) + P(A) - 2P(C \cap A) = p$ ...(iii)

On adding Eqs. (i), (ii) and (iii), we get

$$2\,[P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A)] = 3p$$

$\Rightarrow P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) = \frac{3p}{2}$ ...(v)

It also given that, $P(A \cap B \cap C) = p^2$ ...(v)

$\therefore$ P(at least one of the events A, B, and C occurs)

$$= P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$

$$= \frac{3p}{2} + p^2 \quad \text{[from Eqs. (iv) and (v)]}$$

$$= \frac{3p + 2p^2}{2}$$

5. Since, $P(A \cap B) = P(A) \cdot P(B)$

It means A and B are independent events, so A' and B' are also independent.

$\therefore$ $P(A \cup B)' = P(A' \cap B') = P(A)' \cdot P(B)'$

Alternate Solution

$P(A \cup B)' = 1 - P(A \cup B) = 1 - \{P(A) + P(B) - P(A) \cdot P(B)\}$

$= \{1 - P(A)\}\{1 - P(B)\} = P(A)' \, P(B)'$

6. Given, $P(A \cup B) = 0.6$, $P(A \cap B) = 0.2$

$\therefore$ $P(\overline{A}) + P(\overline{B}) = [1 - P(A)] + [1 - P(B)]$

$= 2 - [P(A) + P(B)]$

$= 2 - [P(A \cup B) + P(A \cap B)]$

$= 2 - [0.6 + 0.2] = 1.2$

7. Given, $P(A) = 0.25$, $P(B) = 0.50$, $P(A \cap B) = 0.14$

$\therefore$ $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.25 + 0.50 - 0.14 = 0.61$

Now, $P(\overline{A \cup B}) = 1 - P(A \cup B) = 1 - 0.61 = 0.39$

8. We know that,

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

Also, $P(A \cup B) \le 1$

$\therefore$ $P(A \cap B)_{\text{min.}}$, when $P(A \cup B)_{\text{max}} = 1$

$\Rightarrow$ $P(A \cap B) \ge P(A) + P(B) - 1$

$\therefore$ Option (a) is true.

Again, $P(A \cup B) \ge 0$

$\therefore$ $P(A \cap B)_{\text{max.}}$, when $P(A \cup B)_{\text{min.}} = 0$

$\Rightarrow$ $P(A \cap B) \le P(A) + P(B)$

$\therefore$ Option (b) is true.

Also, $P(A \cap B) = P(A) + P(B) - P(A \cup B)$, Thus, (c) is also correct.

Hence, (a), (b), (c) are correct options.

9. P(exactly one of M, N occurs)

$= P\{(M \cap \overline{N}) \cup (\overline{M} \cap N)\} = P(M \cap \overline{N}) + P(\overline{M} \cap N)$

$= P(M) - P(M \cap N) + P(N) - P(M \cap N)$

$= P(M) + P(N) - 2P(M \cap N)$

Also, P(exactly one of them occurs)

$= \{1 - P(\overline{M} \cap \overline{N})\}\{1 - P(\overline{M} \cup \overline{N})\}$

$= P(\overline{M} \cup \overline{N}) - P(\overline{M} \cap \overline{N}) = P(\overline{M}) + P(\overline{N}) - 2P(\overline{M} \cap \overline{N})$

Hence, (a) and (c) are correct answers.

10. Let E_1 be the event getting minimum number 3 and E_2 be the event getting maximum number 7.

Then, $P(E_1) = P$ (getting one number 3 and other two from numbers 4 to 10)

$$= \frac{^1C_1 \times {}^7C_2}{^{10}C_3} = \frac{7}{40}$$

$P(E_2) = P$(getting one number 7 and other two from numbers 1 to 6)

$$= \frac{^1C_1 \times {}^6C_2}{^{10}C_3} = \frac{1}{8}$$

and $P(E_1 \cap E_2) = P$(getting one number 3, second number 7 and third from 4 to 6)

$$= \frac{^1C_1 \times {}^1C_1 \times {}^3C_1}{^{10}C_3} = \frac{1}{40}$$

$\therefore$ $P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$$= \frac{7}{40} + \frac{1}{8} - \frac{1}{40} = \frac{11}{40}$$

11. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

If $P(A \cup B) = P(A \cap B)$,

then $P(A)$ and $P(B)$ are equals.

Since, $P(A \cup B) = P(A \cap B) \Rightarrow A$ and B are equals sets

Thus, $P(A)$ and $P(B)$ is equal to $P(A \cap B)$.

12. Given, P (A fails in examination) $= 0.2$

and P (B fails in examination) $= 0.3$

$P(A \cap B) = P(A) P(B) = (0.2)\,(0.3)$

$\therefore$ $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.2 + 0.3 - 0.06 = 0.44$

Hence, it is a false statement.

13. Let $P(A)$ and $P(B)$ denote respectively the percentage of city population that reads newspapers A and B.

Then,

$$P(A)=\frac{25}{100}=\frac{1}{4},\ P(B)=\frac{20}{100}=\frac{1}{5},$$

$$P(A\cap B)=\frac{8}{100}=\frac{2}{25},$$

$$P(A\cap \overline{B})=P(A)-P(A\cap B)=\frac{1}{4}-\frac{2}{25}=\frac{17}{100},$$

$$P(\overline{A}\cap B)=P(B)-P(A\cap B)=\frac{1}{5}-\frac{2}{25}=\frac{3}{25}$$

Let $P(C)$ be the probability that the population who reads advertisements.

$\therefore\ P(C)=30\%$ of $P(A\cap\overline{B})+40\%$ of $P(\overline{A}\cap B)$ $+\ 50\%$ of $P(A\cap B)$

[since, $A\cap\overline{B},\overline{A}\cap B$ and $A\cap B$ are all mutually exclusive]

$$\Rightarrow\ P(C)=\frac{3}{10}\times\frac{17}{100}+\frac{2}{5}\times\frac{3}{25}+\frac{1}{2}\times\frac{2}{25}=\frac{139}{1000}=13.9\%$$

14. We know that,

$P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)$ $-P(C\cap A)+P(A\cap B\cap C)=P(A\cup B\cup C)$

$\Rightarrow 0.3+0.4+0.8-\{0.08+0.28+P(BC)\}+0.09$ $=P(A\cup B\cup C)$

$\Rightarrow\ 1.23-P(BC)=P(A\cup B\cup C)$

where, $0.75\le P(A\cup B\cup C)\le 1$

$\Rightarrow\ 0.75\le 1.23-P(BC)\le 1$

$\Rightarrow\ -0.48\le -P(BC)\le -0.23$

$\Rightarrow\ 0.23\le P(BC)\le 0.48$

15. Given, $P(A)=0.5$ and $P(A\cap B)\le 0.3$

$\Rightarrow\ P(A)+P(B)-P(A\cup B)\le 0.3$

$\Rightarrow\ P(B)\le 0.3+P(A\cup B)-P(A)\le P(A\cup B)-0.2$

[since, $P(A\cup B)\le 1\ \Rightarrow\ P(A\cup B)-0.2\le 0.8$]

$\therefore\ P(B)\le 0.8$

$\Rightarrow\ P(B)$ cannot be 0.9.

16. Here, five students S_1,S_2,S_3,S_4 and S_5 and five seats R_1,R_2,R_3,R_4 and R_5

$\therefore$Total number of arrangement of sitting five students is $5!=120$

Here, S_1 gets previously alloted seat R_1

$\therefore S_2,S_3,S_4$ and S_5 not get previously seats.

Total number of way S_2,S_3,S_4 and S_5 not get previously seats is

$$4!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}\right)=24\left(1-1+\frac{1}{2}-\frac{1}{6}+\frac{1}{24}\right)$$

$$=24\left(\frac{12-4+1}{24}\right)=9$$

$$\therefore\ \text{Required probability}=\frac{9}{120}=\frac{3}{40}$$

17. Here, $n(T_1\cap T_2\cap T_3\cap T_4)$

Total $=-\,n(\overline{T_1}\cup\overline{T_2}\cup\overline{T_3}\cup\overline{T_4})$

$\Rightarrow n(T_1\cap T_2\cap T_3\cap T_4)$

$=5!-[{}^4C_1\,4!2!-({}^3C_1\,3!2!+{}^3C_1\,3!2!2!)$ $+({}^2C_1\,2!2!+{}^4C_1\,2\cdot 2!)-2]$

$\Rightarrow\ n(T_1\cap T_2\cap T_3\cap T_4)$

$=120-[192-(36+72)+(8+16)-2]$

$=120-[192-108+24-2]=14$

$$\therefore\ \text{Required probability}=\frac{14}{120}=\frac{7}{60}$$

Topic 3 Independent and Conditional Probability

1. Let event B is being boy while event G being girl.

According to the question, $P(B)=P(G)=\frac{1}{2}$

Now, required conditional probability that all children are girls given that at least two are girls, is

$$=\frac{\text{All 4 girls}}{(\text{All 4 girls})+(\text{exactly 3 girls }+1\text{ boy})+(\text{exactly 2 girls }+2\text{ boys})}$$

$$=\frac{\left(\frac{1}{2}\right)^4}{\left(\frac{1}{2}\right)^4+{}^4C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)+{}^4C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2}=\frac{1}{1+4+6}=\frac{1}{11}$$

2. **Key Idea** Use $P(\overline{A})=1-P(A)$ and condition of independent events i.e $P(A\cap B)=P(A)\cdot P(B)$

Given that probability of hitting a target independently by four persons are respectively

$$P_1=\frac{1}{2},\ P_2=\frac{1}{3},\ P_3=\frac{1}{4}\text{ and }P_4=\frac{1}{8}$$

Then, the probability of not hitting the target is

$$=\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{8}\right)$$

[$\because$ events are independent]

$$=\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\times\frac{7}{8}=\frac{7}{32}$$

Therefore, the required probability of hitting the target $=1-$ (Probability of not hitting the target)

$$=1-\frac{7}{32}=\frac{25}{32}$$

3. We know that, $P(A/B)=\frac{P(A\cap B)}{P(B)}$

[by the definition of conditional probability]

$\because\ A\subset B$

$\Rightarrow\ A\cap B=A$

$$\therefore\ P(A/B)=\frac{P(A)}{P(B)}\quad\ldots\text{(i)}$$

As we know that, $0\le P(B)\le 1$

$$\therefore\ 1\le\frac{1}{P(B)}<\infty\ \Rightarrow\ P(A)\le\frac{P(A)}{P(B)}<\infty$$

$$\Rightarrow\ \frac{P(A)}{P(B)}\ge P(A)\quad\ldots\text{(ii)}$$

Now, from Eqs (i) and (ii), we get

$$P(A/B)\ge P(A)$$

4. In {1, 2, 3, ..., 11} there are 5 even numbers and 6 odd numbers. The sum even is possible only when both are odd or both are even.

Let A be the event that denotes both numbers are even and B be the event that denotes sum of numbers is even. Then, $n(A) = {}^5C_2$ and $n(B) = {}^5C_2 + {}^6C_2$

Required probability

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{{}^5C_2/{}^{11}C_2}{\frac{({}^6C_2 + {}^5C_2)}{{}^{11}C_2}}$$

$$= \frac{{}^5C_2}{{}^6C_2 + {}^5C_2} = \frac{10}{15+10} = \frac{2}{5}$$

5. Clearly, $P(H) = \text{Probability of getting head} = \frac{1}{2}$

and $P(T) = \text{Probability of getting tail} = \frac{1}{2}$

Now, let E_1 be the event of getting a sum 7 or 8, when a pair of dice is rolled.

Then, $E_1 = \{(6, 1), (5, 2), (4, 3), (3, 4), (2, 5), (1, 6), (6, 2), (5, 3), (4, 4), (3, 5), (2, 6)\}$

$\Rightarrow P(E_1) =$ Probability of getting 7 or 8 when a pair of dice is thrown $= \frac{11}{36}$

Also, let $P(E_2) =$ Probability of getting 7 or 8 when a card is picked from cards numbered 1, 2, ..., 9 $= \frac{2}{9}$

$\therefore$ Probability that the noted number is 7 or 8

$$= P((H \cap E_1) \text{ or } (T \cap E_2))$$
$$= P(H \cap E_1) + P(T \cap E_2)$$

[$\because$ $(H \cap E_1)$ and $(T \cap E_2)$ are mutually exclusive]

$$= P(H) \cdot P(E_1) + P(T) \cdot P(E_2)$$

[$\because$ $\{H, E_1\}$ and $\{T, E_2\}$ both are sets of independent events]

$$= \frac{1}{2} \times \frac{11}{36} + \frac{1}{2} \times \frac{2}{9} = \frac{19}{72}$$

6. Clearly, $E_1 = \{(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)\}$

$E_2 = \{(1,2), (2,2), (3,2), (4,2), (5,2), (6,2)\}$

and $E_3 = \{(1, 2), (1, 4), (1, 6), (2, 1), (2, 3), (2, 5), (3, 2), (3, 4), (3, 6), (4, 1), (4, 3), (4, 5), (5, 2), (5, 4), (5, 6), (6, 1), (6, 3), (6, 5)\}$

$\Rightarrow$ $P(E_1) = \frac{6}{36} = \frac{1}{6}$, $P(E_2) = \frac{6}{36} = \frac{1}{6}$

and $P(E_3) = \frac{18}{36} = \frac{1}{2}$

Now, $P(E_1 \cap E_2) = P$ (getting 4 on die A and 2 on die B)

$$= \frac{1}{36} = P(E_1) \cdot P(E_2)$$

$P(E_2 \cap E_3) = P$ (getting 2 on die B and sum of numbers on both dice is odd)

$$= \frac{3}{36} = P(E_2) \cdot P(E_3)$$

$P(E_1 \cap E_3) = P$ (getting 4 on die A and sum of numbers on both dice is odd)

$$= \frac{3}{36} = P(E_1) \cdot P(E_3)$$

and $P(E_1 \cap E_2 \cap E_3) = P$ [getting 4 on die A, 2 on die B and sum of numbers is odd]

$$= P(\text{impossible event}) = 0$$

Hence, E_1, E_2 and E_3 are not independent.

7. Given, $P(\overline{A \cup B}) = \frac{1}{6}$, $P(A \cap B) = \frac{1}{4}$, $P(\overline{A}) = \frac{1}{4}$

$\therefore$ $P(A \cup B) = 1 - P(\overline{A \cup B}) = 1 - \frac{1}{6} = \frac{5}{6}$

and $P(A) = 1 - P(\overline{A}) = 1 - \frac{1}{4} = \frac{3}{4}$

$\therefore$ $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\frac{5}{6} = \frac{3}{4} + P(B) - \frac{1}{4}$$

$\Rightarrow$ $P(B) = \frac{1}{3} \Rightarrow A$ and B are not equally likely

$$P(A \cap B) = P(A) \cdot P(B) = \frac{1}{4}$$

So, events are independent.

8. **PLAN** It is simple application of independent event, to solve a certain problem or any type of compitition each event in independent of other.

Formula used

$P(A \cap B) = P(A) \cdot P(B)$, when A and B are independent events.

Probability that the problem is solved correctly by atleast one of them = 1 – (problem is not solved by all)

$\therefore$ P (problem is solved) $= 1 - P$ (problem is not solved)

$$= 1 - P(\overline{A}) \cdot P(\overline{B}) \cdot P(\overline{C}) \cdot P(\overline{D})$$

$$= 1 - \left(\frac{1}{2} \cdot \frac{1}{4} \cdot \frac{3}{4} \cdot \frac{7}{8}\right) = 1 - \frac{21}{256} = \frac{235}{256}$$

9. Since, $P(A) = \frac{2}{5}$

For independent events,

$$P(A \cap B) = P(A)P(B)$$

$\Rightarrow$ $P(A \cap B) \le \frac{2}{5}$

$\Rightarrow$ $P(A \cap B) = \frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{4}{10}$

[maximum 4 outcomes may be in $A \cap B$]

(i) Now, $P(A \cap B) = \frac{1}{10}$

$\Rightarrow$ $P(A) \cdot P(B) = \frac{1}{10}$

$\Rightarrow$ $P(B) = \frac{1}{10} \times \frac{5}{2} = \frac{1}{4}$, not possible

(ii) Now, $P(A \cap B) = \frac{2}{10}$ $\Rightarrow$ $\frac{2}{5} \times P(B) = \frac{2}{10}$

$\Rightarrow$ $P(B) = \frac{5}{10}$, outcomes of $B = 5$

(iii) Now, $P(A \cap B) = \frac{3}{10}$

$\Rightarrow \quad P(A)P(B) = \frac{3}{10} \Rightarrow \frac{2}{5} \times P(B) = \frac{3}{10}$

$P(B) = \frac{3}{4}$, not possible

(iv) Now, $\quad P(A \cap B) = \frac{4}{10} \Rightarrow P(A) \cdot p(B) = \frac{4}{10}$

$\Rightarrow \quad P(B) = 1$, outcomes of $B = 10$.

10. $$P\left(\frac{E^c \cap F^c}{G}\right) = \frac{P(E^c \cap F^c \cap G)}{P(G)}$$
$$= \frac{P(G) - P(E \cap G) - P(G \cap F)}{P(G)}$$
$$= \frac{P(G)[1 - P(E) - P(F)]}{P(G)} \quad [\because P(G) \neq 0]$$
$$= 1 - P(E) - P(F) = P(E^c) - P(F)$$

11. Let E = event when each American man is seated adjacent to his wife

and A = event when Indian man is seated adjacent to his wife

Now, $\quad n(A \cap E) = (4!) \times (2!)^5$

Even when each American man is seated adjacent to his wife.

Again, $\quad n(E) = (5!) \times (2!)^4$

$\therefore \quad P\left(\frac{A}{E}\right) = \frac{n(A \cap E)}{n(E)} = \frac{(4!) \times (2!)^5}{(5!) \times (2!)^4} = \frac{2}{5}$

Alternate Solution

Fixing four American couples and one Indian man in between any two couples; we have 5 different ways in which his wife can be seated, of which 2 cases are favourable.

$\therefore$ Required probability $= \frac{2}{5}$

12. Let E be the event of getting 1 on a die.

$\Rightarrow P(E) = \frac{1}{6}$ and $P(\overline{E}) = \frac{5}{6}$

$\therefore P$ (first time 1 occurs at the even throw)

$= t_2$ or t_4 or t_6 or t_8 ... and so on

$= \{P(\overline{E})P(E)\} + \{P(\overline{E})P(\overline{E})P(\overline{E})P(E)\} + \dots \infty$

$$= \left(\frac{5}{6}\right)\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^5\left(\frac{1}{6}\right) + \dots \infty = \frac{\frac{5}{36}}{1 - \frac{25}{36}} = \frac{5}{11}$$

13. Probability that only two tests are needed = Probability that the first machine tested is faulty × Probability that the second machine tested is faulty $= \frac{2}{4} \times \frac{1}{3} = \frac{1}{6}$

14. The event that the fifth toss results in a head is independent of the event that the first four tosses result in tails.

$\therefore$ Probability of the required event $= 1/2$

15. P (2 white and 1 black) $= P(W_1W_2B_3$ or $W_1B_2W_3$ or $B_1W_2W_3)$

$= P(W_1W_2B_3) + P(W_1B_2W_3) + P(B_1W_2W_3)$

$= P(W_1)P(W_2)P(B_3) + P(W_1)P(B_2)P(W_3) + P(B_1)P(W_2)P(W_3)$

$= \frac{3}{4} \cdot \frac{2}{4} \cdot \frac{3}{4} + \frac{3}{4} \cdot \frac{2}{4} \cdot \frac{1}{4} + \frac{1}{4} \cdot \frac{2}{4} \cdot \frac{1}{4} = \frac{1}{32}(9 + 3 + 1) = \frac{13}{32}$

16. Given, $\quad P$ (India wins) $= 1/2$

$\therefore \quad P$ (India losses) $= 1/2$

Out of 5 matches India's second win occurs at third test.

$\Rightarrow$ India wins third test and simultaneously it has won one match from first two and lost the other.

$\therefore$ Required probability $= P(LWW) + P(WLW)$

$= \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^3 = \frac{1}{4}$

17. Let A = getting not less than 2 and not greater than 5

$\Rightarrow \quad A = \{2, 3, 4, 5\} \quad \Rightarrow P(A) = \frac{4}{6}$

But die is rolled four times, therefore the probability in getting four throws

$= \left(\frac{4}{6}\right)\left(\frac{4}{6}\right)\left(\frac{4}{6}\right)\left(\frac{4}{6}\right) = \frac{16}{81}$

18. Let A, B and C denote the events of passing the tests I, II and III, respectively.

Evidently A, B and C are independent events.

According to given condition,

$\frac{1}{2} = P[(A \cap B) \cup (A \cap C)]$

$= P(A \cap B) + P(A \cap C) - P(A \cap B \cap C)$

$= P(A)P(B) + P(A) \cdot P(C) - P(A) \cdot P(B) \cdot P(C)$

$= pq + p \cdot \frac{1}{2} - pq \cdot \frac{1}{2}$

$\Rightarrow 1 = 2pq + p - pq \quad \Rightarrow \quad 1 = p(q + 1) \quad \dots$(i)

The values of option (c) satisfy Eq. (i).

[Infact, Eq. (i) is satisfied for infinite number of values of p and q. If we take any values of q such that $0 \leq q \leq 1$, then, p takes the value $\frac{1}{q+1}$. It is evident that, $0 < \frac{1}{q+1} \leq 1$ i.e. $0 < p \leq 1$. But we have to choose correct answer from given ones.]

19. Since, $P(A/\overline{B}) + P(\overline{A}/\overline{B}) = 1$

$\therefore \quad P(\overline{A}/\overline{B}) = 1 - P(A/\overline{B})$

20. Given that, $P(A) = 0.4$, $P(\overline{A}) = 0.6$

P(the event A happens at least once)

$= 1 - P$ (none of the event happens)

$= 1 - (0.6)(0.6)(0.6) = 1 - 0.216 = 0.784$

21. $P(X)=\frac{1}{3}$

$$P\left(\frac{X}{Y}\right)=\frac{P(X\cap Y)}{P(Y)}=\frac{1}{2}$$

$$P\left(\frac{Y}{X}\right)=\frac{P(X\cap Y)}{P(X)}=\frac{2}{5}$$

$$P(X\cap Y)=\frac{2}{15}$$

$$P(Y)=\frac{4}{15}$$

$$P\left(\frac{X'}{Y}\right)=\frac{P(Y)-P(X\cap Y)}{P(Y)}=\frac{\frac{4}{15}-\frac{2}{15}}{\frac{4}{15}}=\frac{1}{2}$$

$$P(X\cup Y)=\frac{1}{3}+\frac{4}{15}-\frac{2}{15}=\frac{7}{15}=\frac{7}{15}$$

22. PLAN

(i) Conditional probability, i.e. $P(A/B)=\frac{P(A\cap B)}{P(B)}$

(ii) $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

(iii) Independent event, then $P(A\cap B)=P(A)\cdot P(B)$

Here, $P(X/Y)=\frac{1}{2}$, $P\left(\frac{Y}{X}\right)=\frac{1}{3}$

and $P(X\cap Y)=6$

$\therefore \quad P\left(\frac{X}{Y}\right)=\frac{P(X\cap Y)}{P(Y)}$

$\Rightarrow \quad \frac{1}{2}=\frac{1/6}{P(Y)} \Rightarrow P(Y)=\frac{1}{3}$ ...(i)

$P\left(\frac{Y}{X}\right)=\frac{1}{3} \Rightarrow \frac{P(X\cap Y)}{P(X)}=\frac{1}{3}$

$\Rightarrow \quad \frac{1}{6}=\frac{1}{3}P(X)$

$\therefore \quad P(X)=\frac{1}{2}$ ...(ii)

$P(X\cup Y)=P(X)+P(Y)-P(X\cap Y)$

$=\frac{1}{2}+\frac{1}{3}-\frac{1}{6}=\frac{2}{3}$ ...(iii)

$P(X\cap Y)=\frac{1}{6}$ and $P(X)\cdot P(Y)=\frac{1}{2}\cdot\frac{1}{3}=\frac{1}{6}$

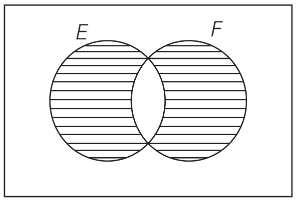
$\Rightarrow \quad P(X\cap Y)=P(X)\cdot P(Y)$

i.e. independent events

$\therefore \quad P(X^c\cap Y)=P(Y)-P(X\cap Y)$

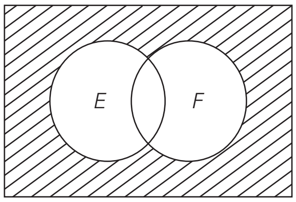
$=\frac{1}{3}-\frac{1}{6}=\frac{1}{6}$

23.



$P(E\cup F)-P(E\cap F)=\frac{11}{25}$ ...(i)

[i.e. only E or only F]



Neither of them occurs $=\frac{2}{25}$

$\Rightarrow \quad P(\overline{E}\cap\overline{F})=\frac{2}{25}$ ...(ii)

From Eq. (i), $P(E)+P(F)-2P(E\cap F)=\frac{11}{25}$ ...(iii)

From Eq. (ii), $(1-P(E))(1-P(F))=\frac{2}{25}$

$\Rightarrow \quad 1-P(E)-P(F)+P(E)\cdot P(F)=\frac{2}{25}$ ...(iv)

From Eqs. (iii) and (iv),

$$P(E)+P(F)=\frac{7}{5} \text{ and } P(E)\cdot P(F)=\frac{12}{25}$$

$\therefore \quad P(E)\cdot\left[\frac{7}{5}-P(E)\right]=\frac{12}{25}$

$\Rightarrow \quad (P(E))^2-\frac{7}{5}P(E)+\frac{12}{25}=0$

$\Rightarrow \quad \left[P(E)-\frac{3}{5}\right]\left[P(E)-\frac{4}{5}\right]=0$

$\therefore \quad P(E)=\frac{3}{5}$ or $\frac{4}{5} \Rightarrow P(F)=\frac{4}{5}$ or $\frac{3}{5}$

24. Let A, B and C respectively denote the events that the student passes in Maths, Physics and Chemistry.

It is given,

$P(A)=m$, $P(B)=p$ and $P(C)=c$ and

P (passing atleast in one subject)

$=P(A\cup B\cup C)=0.75$

$\Rightarrow \quad 1-P(A'\cap B'\cap C')=0.75$

$\because \quad [P(A)=1-P(\overline{A})$

and $\quad [P(\overline{A\cup B\cup C})=P(A'\cap B'\cap C')]$

$\Rightarrow 1-P(A')\cdot P(B')\cdot P(C')=0.75$

$\because$ A, B and C are independent events, therefore A', B' and C' are independent events.

$\Rightarrow \quad 0.75=1-(1-m)(1-p)(1-c)$

$\Rightarrow \quad 0.25=(1-m)(1-p)(1-c)$ ...(i)

Also, P (passing exactly in two subjects)= 0.4

$\Rightarrow P(A\cap B\cap\overline{C} \cup A\cap\overline{B}\cap C \cup \overline{A}\cap B\cap C)=0.4$

$\Rightarrow P(A\cap B\cap\overline{C})+P(A\cap\overline{B}\cap C)+P(\overline{A}\cap B\cap C)=0.4$

$\Rightarrow P(A)\,P(B)\,P(\overline{C})+P(A)P(\overline{B})\,P(C)$

$+P(\overline{A})\,P(B)\,P(C)=0.4$

$\Rightarrow \quad pm(1-c)+p(1-m)c+(1-p)mc=0.4$

$\Rightarrow \quad pm-pmc+pc-pmc+mc-pmc=0.4$ ...(ii)

Again, P (passing atleast in two subjects) $= 0.5$

$\Rightarrow P(A \cap B \cap \overline{C}) + P(A \cap \overline{B} \cap C)$

$+ P(\overline{A} \cap B \cap C) + P(A \cap B \cap C) = 0.5$

$\Rightarrow \quad pm(1-c) + pc(1-m) + cm(1-p) + pcm = 0.5$

$\Rightarrow pm - pcm + pc - pcm + cm - pcm + pcm = 0.5$

$\Rightarrow \quad (pm + pc + mc) - 2pcm = 0.5$ ...(iii)

From Eq. (ii),

$pm + pc + mc - 3pcm = 0.4$ ...(iv)

From Eq. (i),

$0.25 = 1 - (m + p + c) + (pm + pc + cm) - pcm$ ...(v)

On solving Eqs. (iii), (iv) and (v), we get

$p + m + c = 1.35 = 27/20$

Therefore, option (b) is correct.

Also, from Eqs. (ii) and (iii), we get $pmc = 1/10$

Hence, option (c) is correct.

25. (a) $P(E/F) + P(\overline{E}/F) = \dfrac{P(E \cap F)}{P(F)} + \dfrac{P(\overline{E} \cap F)}{P(F)}$

$= \dfrac{P(E \cap F) + P(\overline{E} \cap F)}{P(F)}$

$= \dfrac{P(F)}{P(F)} = 1$

Therefore, option (a) is correct.

(b) $P(E/F) + P(E/\overline{F}) = \dfrac{P(E \cap F)}{P(F)} + \dfrac{P(E \cap \overline{F})}{P(\overline{F})}$

$= \dfrac{P(E \cap F)}{P(F)} + \dfrac{P(E \cap \overline{F})}{1 - P(F)} \neq 1$

Therefore, option (b) is not correct.

(c) $P(\overline{E}/F) + P(E/\overline{F}) = \dfrac{P(\overline{E} \cap F)}{P(F)} + \dfrac{P(E \cap \overline{F})}{P(\overline{F})}$

$= \dfrac{P(\overline{E} \cap F)}{P(F)} + \dfrac{P(E \cap \overline{F})}{1 - P(F)} \neq 1$

Therefore, option (c) is not correct.

(d) $P(E/\overline{F}) + P(\overline{E}/\overline{F}) = \dfrac{P(E \cap \overline{F})}{P(\overline{F})} + \dfrac{P(\overline{E} \cap \overline{F})}{P(\overline{F})}$

$= \dfrac{P(E \cap \overline{F}) + P(\overline{E} \cap \overline{F})}{P(\overline{F})}$

$= \dfrac{P(\overline{F})}{P(\overline{F})} = 1$

Therefore, option (d) is correct.

26. Both E and F happen $\Rightarrow P(E \cap F) = \dfrac{1}{12}$

and neither E nor F happens $\Rightarrow P(\overline{E} \cap \overline{F}) = \dfrac{1}{2}$

But for independent events, we have

$P(E \cap F) = P(E)\,P(F) = \dfrac{1}{12}$ ...(i)

and $\quad P(\overline{E} \cap \overline{F}) = P(\overline{E})\,P(\overline{F})$

$= \{1 - P(E)\}\{(1 - P(F)\}$

$= 1 - P(E) - P(F) + P(E)P(F)$

$\Rightarrow \quad \dfrac{1}{2} = 1 - \{P(E) + P(F)\} + \dfrac{1}{12}$

$\Rightarrow \quad P(E) + P(F) = 1 - \dfrac{1}{2} + \dfrac{1}{12} = \dfrac{7}{12}$ ...(ii)

On solving Eqs. (i) and (ii), we get

either $\quad P(E) = \dfrac{1}{3}$ and $P(F) = \dfrac{1}{4}$

or $\quad P(E) = \dfrac{1}{4}$ and $P(F) = \dfrac{1}{3}$

27. We know that,

$P\left(\dfrac{A}{B}\right) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{P(A) + P(B) - P(A \cup B)}{P(B)}$

Since, $\quad P(A \cup B) < 1$

$\Rightarrow \quad -P(A \cup B) > -1$

$\Rightarrow \quad P(A) + P(B) - P(A \cup B) > P(A) + P(B) - 1$

$\Rightarrow \quad \dfrac{P(A) + P(B) - P(A \cup B)}{P(B)} > \dfrac{P(A) + P(B) - 1}{P(B)}$

$\Rightarrow \quad P\left(\dfrac{A}{B}\right) > \dfrac{P(A) + P(B) - 1}{P(B)}$

Hence, option (a) is correct.

The choice (b) holds only for disjoint i.e. $P(A \cap B) = 0$

Finally, $\quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= P(A) + P(B) - P(A) \cdot P(B)$,

if A, B are independent

$= 1 - \{1 - P(A)\}\{1 - P(B)\} = 1 - P(\overline{A}) \cdot P(\overline{B})$

Hence, option (c) is correct, but option (d) is not correct.

28. Since, E and F are independent events. Therefore, $P(E \cap F) = P(E) \cdot P(F) \neq 0$, so E and F are not mutually exclusive events.

Now, $P(E \cap \overline{F}) = P(E) - P(E \cap F) = P(E) - P(E) \cdot P(F)$

$= P(E)\,[1 - P(F)] = P(E) \cdot P(\overline{F})$

and $\quad P(\overline{E} \cap \overline{F}) = P(\overline{E \cup F}) = 1 - P(E \cup F)$

$= 1 - [1 - P(\overline{E}) \cdot P(\overline{F})]$

[$\because$ E and F are independent]

$= P(\overline{E}) \cdot P(\overline{F})$

So, E and $\overline{F}$ as well as $\overline{E}$ and $\overline{F}$ are independent events.

Now, $P(E/F) + P(\overline{E}/F) = \dfrac{P(E \cap F) + P(\overline{E} \cap F)}{P(F)}$

$= \dfrac{P(F)}{P(F)} = 1$

29. $P(A^c) = 0.3$ [given]

$\Rightarrow \quad P(A) = 0.7$

$P(B) = 0.4$ [given]

$\Rightarrow \quad P(B^c) = 0.6$ and $P(A \cap B^c) = 0.5$ [given]

Now, $\quad P(A \cup B^c) = P(A) + P(B^c) - P(A \cap B^c)$

$= 0.7 + 0.6 - 0.5 = 0.8$

$\therefore\ P\,[B/(A \cup B^c] = \dfrac{P\{B \cap (A \cup B^c)\}}{P\,(A \cup B^c)}$

$= \dfrac{P\{(B \cap A) \cup (B \cap B^c)\}}{0.8} = \dfrac{P\{(B \cap A) \cup \phi\}}{0.8} = \dfrac{P(B \cap A)}{0.8}$

$$= \frac{1}{0.8}[P(A) - P(A \cap B^c)]$$
$$= \frac{0.7 - 0.5}{0.8} = \frac{0.2}{0.8} = \frac{1}{4}$$

30. $P(A \cup B) = P(A) + P(B) - P(A)P(B)$, as A and B are independent events.

$\Rightarrow \quad 0.8 = (0.3) + P(B) - (0.3)P(B)$

$\Rightarrow \quad 0.5 = (0.7)P(B) \Rightarrow P(B) = \frac{5}{7}$

31. 5 can be thrown in 4 ways and 7 can be thrown in 6 ways, hence number of ways of throwing neither 5 nor 7 is $36 - (4 + 6) = 26$

$\therefore$ Probability of throwing a five in a single throw with a pair of dice $= \frac{4}{36} = \frac{1}{9}$ and probability of throwing neither 5 nor 7 $= \frac{26}{36} = \frac{13}{18}$

Hence, required probability

$$= \left(\frac{1}{9}\right) + \left(\frac{13}{18}\right)\left(\frac{1}{9}\right) + \left(\frac{13}{18}\right)^2\left(\frac{1}{9}\right) + \dots = \frac{\frac{1}{9}}{1 - \frac{13}{18}} = \frac{2}{5}$$

32. Let R be drawing a red ball and B for drawing a black ball, then required probability

$$= RRR + RBR + BRR + BBR$$
$$= \left(\frac{6}{10} \times \frac{5}{11} \times \frac{6}{10}\right) + \left(\frac{6}{10} \times \frac{6}{11} \times \frac{5}{10}\right) + \left(\frac{4}{10} \times \frac{4}{11} \times \frac{7}{10}\right) + \left(\frac{4}{10} \times \frac{7}{11} \times \frac{6}{10}\right)$$
$$= \frac{640}{1100} = \frac{32}{55}$$

33. Let A be the event that the maximum number on the two chosen tickets is not more than 10, and B be the event that the minimum number on them is 5

$$\therefore \quad P(A \cap B) = \frac{{}^5C_1}{{}^{100}C_2}$$

and $$P(A) = \frac{{}^{10}C_2}{{}^{100}C_2}$$

Then $$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{{}^5C_1}{{}^{10}C_2} = \frac{1}{9}$$

34. Here, $P(A \cup B) \cdot P(A' \cap B')$

$\Rightarrow \quad \{P(A) + P(B) - P(A \cap B)\}\{P(A') \cdot P(B')\}$

[since A, B are independent, so A', B' are independent]

$\therefore \; P(A \cup B) \cdot P(A' \cap B') \le \{P(A) + P(B)\} \cdot \{P(A') \cdot P(B')\}$

$= P(A) \cdot P(A') \cdot P(B') + P(B) \cdot P(A') \cdot P(B')$

$\le P(A) \cdot P(B') + P(B) \cdot P(A') \quad \dots$(i)

$[\because P(A') \le 1 \text{ and } P(B') \le 1]$

$\Rightarrow \quad P(A \cup B) \cdot P(A' \cap B') \le P(A) \cdot P(B') + P(B) \cdot P(A')$

$\Rightarrow \quad P(A \cup B) \cdot P(A' \cap B') \le P(C)$

$[\because P(C) = P(A) \cdot P(B') + P(B) \cdot P(A')]$

35. Given, $P(A)$ = probability that A will hit $B = \frac{2}{3}$

$P(B)$ = probability that B will hit $A = \frac{1}{2}$

$P(C)$ = probability that C will hit $A = \frac{1}{3}$

$P(E)$ = probability that A will be hit

$\Rightarrow \quad P(E) = 1 - P(\overline{B}) \cdot P(\overline{C}) = 1 - \frac{1}{2} \cdot \frac{2}{3} = \frac{2}{3}$

Probability if A is hit by B and not by C

$$= P(B \cap \overline{C} / E) = \frac{P(B) \cdot P(\overline{C})}{P(E)} = \frac{\frac{1}{2} \cdot \frac{2}{3}}{\frac{2}{3}} = \frac{1}{2}$$

36. Let E_i denotes the event that the students will pass the ith exam, where $i = 1, 2, 3$

and E denotes the student will qualify.

$$\therefore \quad P(E) = [P(E_1) \times P(E_2 / E_1)] + [P(E_1) \times P(E_2'/E_1) \times P(E_3 / E_2')] + [P(E_1') \times P(E_2 / E'_1) \times P(E_3 / E_2)]$$
$$= p^2 + p(1 - p) \cdot \frac{p}{2} + (1 - p) \cdot \frac{p}{2} \cdot p$$
$$\Rightarrow \quad P(E) = \frac{2p^2 + p^2 - p^3 + p^2 - p^3}{2} = 2p^2 - p^3$$

37. Since, p_n denotes the probability that no two (or more) consecutive heads occur.

$\Rightarrow$ p_n denotes the probability that 1 or no head occur.

For $n = 1$, $p_1 = 1$ because in both cases we get less than two heads (H, T).

For $n = 2$, $p_2 = 1 - p$ (two heads simultaneously occur).

$= 1 - p(HH) = 1 - pp = 1 - p^2$

For $n \ge 3$, $p_n = p_{n-1}(1 - p) + p_{n-2}(1 - p)p$

$\Rightarrow \quad p_n = (1 - p)p_{n-1} + p(1 - p)p_{n-2}$

Hence proved.

38. Let, E_1 = the event noted number is 7

E_2 = the event noted number is 8

H = getting head on coin

T = be getting tail on coin

$\therefore$ By law of total probability,

$P(E_1) = P(H) \cdot P(E_1 / H) + P(T) \cdot P(E_1 / T)$

and $P(E_2) = P(H) \cdot P(E_2 / H) + P(T) \cdot P(E_2 / T)$

where, $P(H) = 1/2 = P(T)$

$P(E_1/H)$ = probability of getting a sum of 7 on two dice

Here, favourable cases are

$\{(1, 6), (6, 1), (2, 5), (5, 2), (3, 4), (4, 3)\}$.

$$\therefore \quad P(E_1 / H) = \frac{6}{36} = \frac{1}{6}$$

Also, $P(E_1 / T)$ = probability of getting 7 numbered card out of 11 cards

$$= \frac{1}{11}$$

$P(E_2/H)$ = probability of getting a sum of 8 on two dice

Here, favourable cases are

$$\{(2,6),(6,2),(4,4),(5,3),(3,5)\}.$$

$$\therefore \quad P(E_2/H)=\frac{5}{36}$$

$P(E_2/T)$ = probability of getting '8' numbered card out of 11 cards

$= 1/11$

$$\therefore \quad P(E_1)=\left(\frac{1}{2}\times\frac{1}{6}\right)+\left(\frac{1}{2}\times\frac{1}{11}\right)=\frac{1}{12}+\frac{1}{22}=\frac{17}{132}$$

and
$$P(E_2)=\left(\frac{1}{2}\times\frac{5}{36}\right)+\left(\frac{1}{2}\times\frac{1}{11}\right)$$
$$=\frac{1}{2}\left(\frac{91}{396}\right)=\frac{91}{729}$$

Now, E_1 and E_2 are mutually exclusive events.

Therefore,

$$P(E_1 \text{ or } E_2)=P(E_1)+P(E_2)=\frac{17}{132}+\frac{91}{792}=\frac{193}{792}$$

39. Let D_1 denotes the occurrence of a defective bulb in Ist draw.

Therefore, $P(D_1)=\frac{50}{100}=\frac{1}{2}$

and let D_2 denotes the occurrence of a defective bulb in IInd draw.

Therefore, $P(D_2)=\frac{50}{100}=\frac{1}{2}$

and let N_1 denotes the occurrence of non-defective bulb in Ist draw.

Therefore, $P(N_1)=\frac{50}{100}=\frac{1}{2}$

Again, let N_2 denotes the occurrence of non-defective bulb in IInd draw.

Therefore, $P(N_2)=\frac{50}{100}=\frac{1}{2}$

Now, D_1 is independent with N_1 and D_2 is independent with N_2.

According to the given condition,

A = {the first bulb is defective} $=\{D_1D_2, D_1N_2\}$

B = {the second bulb is non-defective} $=\{D_1N_2, N_1N_2\}$

and C = {the two bulbs are both defective} $=\{D_1D_2, N_1N_2\}$

Again, we know that,

$$A\cap B=\{D_1N_2\},\ B\cap C=\{N_1N_2\}.$$
$$C\cap A=\{D_1D_2\} \text{ and } A\cap B\cap C=\phi$$

Also,
$$P(A)=P\{D_1D_2\}+P\{D_1N_2\}$$
$$=P(D_1)P(D_2)+P(D_1)P(N_2)$$
$$=\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)=\frac{1}{2}$$

Similarly, $P(B)=\frac{1}{2}$ and $P(C)=\frac{1}{2}$

Also, $P(A\cap B)=P(D_1N_2)=P(D_1)P(N_2)=\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)=\frac{1}{4}$

Similarly, $P(B\cap C)=\frac{1}{4}$, $P(C\cap A)=\frac{1}{4}$

and $P(A\cap B\cap C)=0$.

Since, $P(A\cap B)=P(A)P(B)$, $P(B\cap C)=P(B)P(C)$

and $P(C\cap A)=P(C)P(A)$.

Therefore, A, B and C are pairwise independent.

Also, $P(A\cap B\cap C)\neq P(A)P(B)P(C)$ therefore A, B and C cannot be independent.

40. The total number of ways to answer the question

$$={}^4C_1+{}^4C_2+{}^4C_3+{}^4C_4=2^4-1=15$$

P(getting marks) = P(correct answer in I chance) + P(correct answer in II chance) + P(correct answer in III chance)

$$=\frac{1}{15}+\left(\frac{14}{15}\cdot\frac{1}{14}\right)+\left(\frac{14}{15}\cdot\frac{13}{14}\cdot\frac{1}{13}\right)=\frac{3}{15}=\frac{1}{5}$$

41. Given, $P(A)\cdot P(B)=\frac{1}{6}$, $P(\overline{A})\cdot P(\overline{B})=\frac{1}{3}$

$$\therefore \quad [1-P(A)]\,[1-P(B)]=\frac{1}{3}$$

Let $P(A)=x$ and $P(B)=y$

$$\Rightarrow \quad (1-x)(1-y)=\frac{1}{3} \text{ and } xy=\frac{1}{6}$$
$$\Rightarrow \quad 1-x-y+xy=\frac{1}{3} \text{ and } xy=\frac{1}{6}$$
$$\Rightarrow \quad x+y=\frac{5}{6} \text{ and } xy=\frac{1}{6}$$
$$\Rightarrow \quad x\left(\frac{5}{6}-x\right)=\frac{1}{6}$$
$$\Rightarrow \quad 6x^2-5x+1=0$$
$$\Rightarrow \quad (3x-1)(2x-1)=0$$
$$\Rightarrow \quad x=\frac{1}{3} \text{ and } \frac{1}{2}$$
$$\therefore \quad P(A)=\frac{1}{3} \text{ or } \frac{1}{2}$$

42. $P(N$th draw gives 2nd ace)

= P{ 1 ace and $(n-2)$ other cards are drawn in $(N-1)$ draws} $\times P${Nth draw is 2nd ace}

$$=\frac{4\cdot(48)!\cdot(n-1)!\,(52-n)!}{(52)!\cdot(n-2)!\,(50-n)!}\cdot\frac{3}{(53-n)}$$
$$=\frac{4(n-1)(52-n)(51-n)\cdot 3}{52\cdot 51\cdot 50\cdot 49}$$
$$=\frac{(n-1)\,(52-n)\,(51-n)}{50\cdot 49\cdot 17\cdot 13}$$

43. Let $P(H_1)=0.4$, $P(H_2)=0.3$, $P(H_3)=0.2$, $P(H_4)=0.1$

P (gun hits the plane)

$= 1 - P$(gun does not hit the plane)

$$=1-P(\overline{H}_1)\cdot P(\overline{H}_2)\cdot P(\overline{H}_3)\cdot P(\overline{H}_4)$$
$$=1-(0.6)\,(0.7)\,(0.8)\,(0.9)=1-0.3024=0.6976$$

44. Since, the drawn balls are in the sequence black, black, white, white, white, white, red, red and red.

Let the corresponding probabilities be

$$p_1, p_2, \ldots, p_9$$

Then, $p_1 = \frac{2}{9}, \; p_2 = \frac{1}{8}, p_3 = \frac{4}{7}, p_4 = \frac{3}{6}, p_5 = \frac{2}{5}$

$$p_6 = \frac{1}{4}, p_7 = \frac{3}{3}, p_8 = \frac{2}{2}, p_9 = 1$$

$\therefore$ Required probabilitie

$$p_1 \cdot p_2 \cdot p_3 \cdot \ldots \cdot p_9$$
$$= \left(\frac{2}{9}\right)\left(\frac{1}{8}\right)\left(\frac{4}{7}\right)\left(\frac{3}{6}\right)\left(\frac{2}{5}\right)\left(\frac{1}{4}\right)\left(\frac{3}{3}\right)\left(\frac{2}{2}\right)(1) = \frac{1}{1260}$$

45. **PLAN**

Forthe events to be independent,

$P(E_1 \cap E_2 \cap E_3) = P(E_1) \cdot P(E_2) \cdot P(E_3)$

$P(E_1 \cap \bar{E}_2 \cap \bar{E}_3) = P(\text{only } E_1 \text{ occurs})$

$= P(E_1) \cdot (1 - P(E_2))\,(1 - P(E_3))$

Let x, y and z be probabilities of E_1, E_2 and E_3, respectively.

$\therefore \quad \alpha = x(1-y)(1-z) \quad \ldots$(i)

$\beta = (1-x) \cdot y(1-z) \quad \ldots$(ii)

$\gamma = (1-x)(1-y)z \quad \ldots$(iii)

$\Rightarrow \quad p = (1-x)(1-y)(1-z) \quad \ldots$(iv)

Given, $(\alpha - 2\beta)p = \alpha\beta$ and $(\beta - 3\gamma)p = 2\beta\gamma \quad \ldots$(v)

From above equations, $x = 2y$ and $y = 3z$

$\therefore \quad x = 6z$

$\Rightarrow \quad \frac{x}{z} = 6$

46. Here, $P(X > Y) = P(T_1 \text{win})\, P(T_1 \text{ win})$

$+ P(T_1 \text{ win})\, P(\text{draw}) + P(\text{draw})\, P(T_1 \text{ win})$

$= \left(\frac{1}{2} \times \frac{1}{2}\right) + \left(\frac{1}{2} \times \frac{1}{6}\right) + \left(\frac{1}{6} \times \frac{1}{2}\right) = \frac{5}{12}$

47. $P[X = Y] = P(\text{draw}) \cdot P(\text{draw})$

$+ P(T_1 \text{ win})\, P(T_2 \text{ win}) + P(T_2 \text{ win}) \cdot P(T_1 \text{ win})$

$= (1/6 \times 1/6) + (1/2 \times 1/3) + (1/3 \times 1/2) = 13/36$

Topic 4 Law of Total Probability and Baye's Theorem

1. Let A be the event that ball drawn is given and B be the event that ball drawn is red.

$\therefore \quad P(A) = \frac{2}{7}$ and $P(B) = \frac{5}{7}$

Again, let C be the event that second ball drawn is red.

$\therefore \quad P(C) = P(A) \;\; P(C/A) + P(B) P \;\; (C/B)$

$$= \frac{2}{7} \times \frac{6}{7} + \frac{5}{7} \times \frac{4}{7}$$
$$= \frac{12+40}{49} = \frac{32}{49}$$

2. **Key idea** Use the theorem of total probability

Let E_1 = Event that first ball drawn is red

E_2 = Event that first ball drawn is black

A = Event that second ball drawn is red

$P(E_1) = \frac{4}{10}, P\left(\frac{A}{E_1}\right) = \frac{6}{12}$

$\Rightarrow \quad P(E_2) = \frac{6}{10}, P\left(\frac{A}{E_2}\right) = \frac{4}{12}$

By law of total probability

$$P(A) = P(E_1) \times P\left(\frac{A}{E_1}\right) + P(E_2) \times P\left(\frac{A}{E_2}\right)$$
$$= \frac{4}{10} \times \frac{6}{12} + \frac{6}{10} \times \frac{4}{12} = \frac{24+24}{120} = \frac{48}{120} = \frac{2}{5}$$

3. Let $x = P$ (computer turns out to be defective, given that it is produced in plant T_2)

$\Rightarrow \quad x = P\left(\frac{D}{T_2}\right) \quad \ldots$(i)

where, $D =$ Defective computer

$\therefore P$ (computer turns out to be defective given that is produced in plant $T_1) = 10x$

i.e. $\quad P\left(\frac{D}{T_1}\right) = 10x \quad \ldots$(ii)

Also, $\quad P(T_1) = \frac{20}{100}$ and $P(T_2) = \frac{80}{100}$

Given, P (defective computer) $= \frac{7}{100}$

i.e. $\quad P(D) = \frac{7}{100}$

Using law of total probability,

$$P(D) = 9(T_1) \cdot P\left(\frac{D}{T_1}\right) + P(T_2) \cdot P\left(\frac{D}{T_2}\right)$$

$\therefore \quad \frac{7}{100} = \left(\frac{20}{100}\right) \cdot 10x + \left(\frac{80}{100}\right) \cdot x$

$\Rightarrow \quad 7 = (280)x \quad \Rightarrow \quad x = \frac{1}{40} \quad \ldots$(iii)

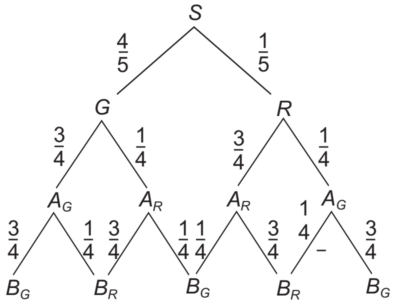
$\therefore \quad P\left(\frac{D}{T_2}\right) = \frac{1}{40}$ and $P\left(\frac{D}{T_1}\right) = \frac{10}{40}$

$\Rightarrow P\left(\frac{\bar{D}}{T_2}\right) = 1 - \frac{1}{40} = \frac{39}{40}$ and $P\left(\frac{\bar{D}}{T_1}\right) = 1 - \frac{10}{40} = \frac{30}{40} \quad \ldots$(iv)

Using Baye's theorem,

$$P\left(\frac{T_2}{\bar{D}}\right) = \frac{P(T_2 \cap \bar{D})}{P(T_1 \cap \bar{D}) + P(T_2 \cap \bar{D})}$$
$$= \frac{P(T_2) \cdot P\left(\frac{\bar{D}}{T_2}\right)}{P(T_1) \cdot P\left(\frac{\bar{D}}{T_1}\right) + P(T_2) \cdot P\left(\frac{\bar{D}}{T_2}\right)}$$
$$= \frac{\frac{80}{100} \cdot \frac{39}{40}}{\frac{20}{100} \cdot \frac{30}{40} + \frac{80}{100} \cdot \frac{39}{40}} = \frac{78}{93}$$

4. From the tree diagram, it follows that



$$P(B_G) = \frac{46}{80}$$

$$P(B_G|G) = \frac{10}{16} = \frac{5}{8}$$

$$P(B_G \cap G) = \frac{5}{8} \times \frac{4}{5} = \frac{1}{2}$$

$$\therefore \quad P(G|B_G) = \frac{\frac{1}{2}}{P(B_G)} = \frac{1}{2} \times \frac{80}{46} = \frac{20}{23}$$

5. **PLAN** It is based on law of total probability and Bay's Law.

Description of Situation It is given that ship would work if atleast two of engines must work. If X be event that the ship works. Then, $X \Rightarrow$ either any two of E_1, E_2, E_3 works or all three engines E_1, E_2, E_3 works.

Given, $P(E_1) = \frac{1}{2}, P(E_2) = \frac{1}{4}, P(E_3) = \frac{1}{4}$

$$\therefore \quad P(X) = \begin{Bmatrix} P(E_1 \cap E_2 \cap \overline{E}_3) + P(E_1 \cap \overline{E}_2 \cap E_3) \\ + P(\overline{E}_1 \cap E_2 \cap E_3) + P(E_1 \cap E_2 \cap E_3) \end{Bmatrix}$$

$$= \left(\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{3}{4} + \frac{1}{2}\cdot\frac{3}{4}\cdot\frac{1}{4} + \frac{1}{2}\cdot\frac{1}{4}\cdot\frac{1}{4}\right) + \left(\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{1}{4}\right)$$

$$= 1/4$$

Now, (a) $P(X_1^c / X)$

$$= P\left(\frac{X_1^c \cap X}{P(X)}\right) = \frac{P(\overline{E}_1 \cap E_2 \cap E_3)}{P(X)} = \frac{\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{1}{4}}{\frac{1}{4}} = \frac{1}{8}$$

(b) P (exactly two engines of the ship are functioning)

$$= \frac{P(E_1 \cap E_2 \cap \overline{E}_3) + P(E_1 \cap \overline{E}_2 \cap E_3) + P(\overline{E}_1 \cap E_2 \cap E_3)}{P(X)}$$

$$= \frac{\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{3}{4} + \frac{1}{2}\cdot\frac{3}{4}\cdot\frac{1}{4} + \frac{1}{2}\cdot\frac{1}{4}\cdot\frac{1}{4}}{\frac{1}{4}} = \frac{7}{8}$$

(c) $$P\left(\frac{X}{X_2}\right) = \frac{P(X \cap X_2)}{P(X_2)}$$

$$= \frac{P\text{ (ship is operating with } E_2 \text{ function)}}{P(X_2)}$$

$$= \frac{P(E_1 \cap E_2 \cap \overline{E}_3) + P(\overline{E}_1 \cap E_2 \cap E_3) + P(E_1 \cap E_2 \cap E_3)}{P(E_2)}$$

$$= \frac{\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{3}{4} + \frac{1}{2}\cdot\frac{1}{4}\cdot\frac{1}{4} + \frac{1}{2}\cdot\frac{1}{4}\cdot\frac{1}{4}}{\frac{1}{4}} = \frac{5}{8}$$

(d) $$P(X/X_1) = \frac{P(X \cap X_1)}{P(X_1)} = \frac{\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{1}{4} + \frac{1}{2}\cdot\frac{3}{4}\cdot\frac{1}{4} + \frac{1}{2}\cdot\frac{1}{4}\cdot\frac{3}{4}}{1/2}$$

$$= \frac{7}{16}$$

6. **Statement I** If $P(H_i \cap E) = 0$ for some i, then

$$P\left(\frac{H_i}{E}\right) = P\left(\frac{E}{H_i}\right) = 0$$

If $P(H_i \cap E) \neq 0, \forall \quad i = 1, 2, \ldots, n$, then

$$P\left(\frac{H_i}{E}\right) = \frac{P(H_i \cap E)}{P(H_i)} \times \frac{P(H_i)}{P(E)}$$

$$= \frac{P\left(\frac{E}{H_i}\right) \times P(H_i)}{P(E)} > P\left(\frac{E}{H_i}\right) \cdot P(H_i) \quad [\because 0 < P(E) < 1]$$

Hence, Statement I may not always be true.

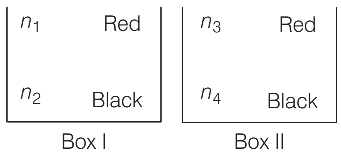
Statement II Clearly, $H_1 \cup H_2 \cup \ldots \cup H_n = S$ [sample space]

$$\Rightarrow \quad P(H_1) + P(H_2) + \ldots + P(H_n) = 1$$

Hence, Statement II is ture.

Passage I

7.



Let $\quad A =$ Drawing red ball

$$\therefore \quad P(A) = P(B_1) \cdot P(A/B_1) + P(B_2) \cdot P(A/B_2)$$

$$= \frac{1}{2}\left(\frac{n_1}{n_1 + n_2}\right) + \frac{1}{2}\left(\frac{n_3}{n_3 + n_4}\right)$$

Given, $$P(B_2/A) = \frac{1}{3}$$

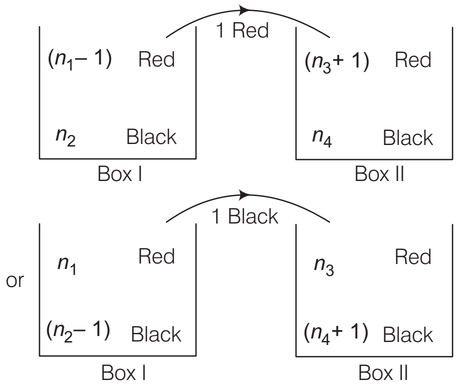
$$\Rightarrow \quad \frac{P(B_2) \cdot P(B_2 \cap A)}{P(A)} = \frac{1}{3}$$

$$\Rightarrow \quad \frac{\frac{1}{2}\left(\frac{n_3}{n_3 + n_4}\right)}{\frac{1}{2}\left(\frac{n_1}{n_1 + n_2}\right) + \frac{1}{2}\left(\frac{n_3}{n_3 + n_4}\right)} = \frac{1}{3}$$

$$\Rightarrow \quad \frac{n_3(n_1 + n_2)}{n_1(n_3 + n_4) + n_3(n_1 + n_2)} = \frac{1}{3}$$

Now, check options, then clearly options (a) and (b) satisfy.

8.



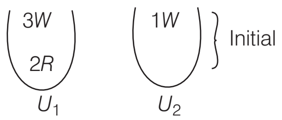
$\therefore P\,(\text{drawing red ball from } B_1) = \dfrac{1}{3}$

$$\Rightarrow \left(\frac{n_1-1}{n_1+n_2-1}\right)\left(\frac{n_1}{n_1+n_2}\right)+\left(\frac{n_2}{n_1+n_2}\right)\left(\frac{n_1}{n_1+n_2-1}\right)=\frac{1}{3}$$

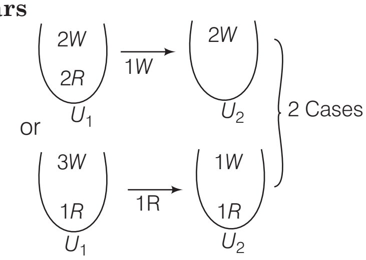
$$\Rightarrow \frac{n_1^2+n_1n_2-n_1}{(n_1+n_2)(n_1+n_2-1)}=\frac{1}{3}$$

Clearly, options (c) and (d) satisfy.

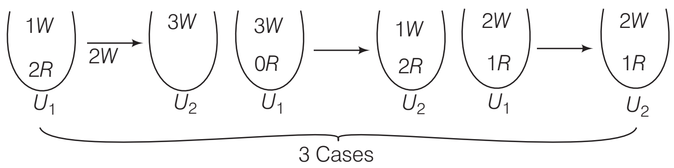
Passage II



Head appears



Tail appears



9. Now, probability of the drawn ball from U_2 being white is

$$P\,(\text{white}/U_2)=P(H)\cdot\left\{\frac{{}^3C_1}{{}^5C_1}\times\frac{{}^2C_1}{{}^2C_1}+\frac{{}^2C_1}{{}^5C_1}\times\frac{{}^1C_1}{{}^2C_1}\right\}$$

$$+\,P\,(T)\left\{\frac{{}^3C_2}{{}^5C_2}\times\frac{{}^3C_2}{{}^3C_2}+\frac{{}^2C_2}{{}^5C_2}\times\frac{{}^1C_1}{{}^3C_2}+\frac{{}^3C_1\cdot{}^2C_1}{{}^5C_2}\times\frac{{}^2C_1}{{}^3C_2}\right\}$$

$$=\frac{1}{2}\left\{\frac{3}{5}\times1+\frac{2}{5}\times\frac{1}{2}\right\}$$

$$+\frac{1}{2}\left\{\frac{3}{10}\times1+\frac{1}{10}\times\frac{1}{3}+\frac{6}{10}\times\frac{2}{3}\right\}=\frac{23}{30}$$

10. P (Head appeared/white from U_2)

$$=P(H)\,.\,\frac{\left\{\dfrac{{}^3C_1}{{}^5C_1}\times\dfrac{{}^2C_1}{{}^2C_1}+\dfrac{{}^2C_1}{{}^5C_1}\times\dfrac{{}^1C_1}{{}^2C_1}\right\}}{23/30}$$

$$=\frac{1}{2}\,\frac{\left\{\dfrac{3}{5}\times1+\dfrac{2}{5}\times\dfrac{1}{2}\right\}}{20/30}$$

$$=\frac{12}{23}$$

Passage III

11. $P\,(X=3)=\dfrac{5}{6}\cdot\dfrac{5}{6}\cdot\dfrac{1}{6}=\dfrac{25}{216}$

12. $P\,(X\geq3)=\dfrac{5}{6}\cdot\dfrac{5}{6}\cdot1=\dfrac{25}{36}$

13. $P\{(X\geq6)/(X>3)\}=\dfrac{P\{(X>3)/(X\geq6)\}\cdot P\,(X\geq6)}{P\,(X>3)}$

$$=\frac{1\cdot\left[\left(\frac{5}{6}\right)^5\cdot\left(\frac{1}{6}\right)+\left(\frac{5}{6}\right)^6\cdot\left(\frac{1}{6}\right)+\ldots\infty\right]}{\left[\left(\frac{5}{6}\right)^3\cdot\frac{1}{6}+\left(\frac{5}{6}\right)^4\cdot\frac{1}{6}+\ldots\infty\right]}=\frac{25}{36}$$

Passage IV

14. Here, $P(u_i)=ki, \Sigma\,P(u_i)=1$

$$\Rightarrow\qquad k=\frac{2}{n(n+1)}$$

$$\lim_{n\to\infty}P(W)=\lim_{n\to\infty}\sum_{i=1}^{n}\frac{2i^2}{n\,(n+1)^2}$$

$$=\lim_{n\to\infty}\frac{2n\,(n+1)(2n+1)}{6n\,(n+1)^2}=2/3$$

15. $P\left(\dfrac{u_n}{W}\right)=\dfrac{\dfrac{n}{n+1}}{\dfrac{\Sigma\,i}{n+1}}=\dfrac{2}{n+1}$

16. $P\left(\dfrac{W}{E}\right)=\dfrac{2+4+6+\ldots}{\dfrac{n\,(n+1)}{2}}=\dfrac{n+2}{2\,(n+1)}$

17. As, the statement shows problem is to be related to Baye's law.

Let C, S, B, T be the events when person is going by car, scooter, bus or train, respectively.

$\therefore\quad P(C)=\dfrac{1}{7}, P(S)=\dfrac{3}{7}, P(B)=\dfrac{2}{7}, P(T)=\dfrac{1}{7}$

Again, L be the event of the person reaching office late.

$\therefore \overline{L}$ be the event of the person reaching office in time.

Then, $P\left(\dfrac{\overline{L}}{C}\right)=\dfrac{7}{9}, P\left(\dfrac{\overline{L}}{S}\right)=\dfrac{8}{9}, P\left(\dfrac{\overline{L}}{B}\right)=\dfrac{5}{9}$

and $\quad P\left(\dfrac{\overline{L}}{T}\right)=\dfrac{8}{9}$

$$\therefore \quad P\left(\frac{C}{L}\right) = \frac{P\left(\frac{\bar{L}}{C}\right)\cdot P(C)}{P\left(\frac{\bar{L}}{C}\right)\cdot P(C) + P\left(\frac{\bar{L}}{S}\right)\cdot P(S) + P\left(\frac{\bar{L}}{B}\right)\cdot P(B) + P\left(\frac{\bar{L}}{T}\right)\cdot P(T)}$$

$$= \frac{\frac{7}{9}\times\frac{1}{7}}{\frac{7}{9}\times\frac{1}{7}+\frac{8}{9}\times\frac{3}{7}+\frac{5}{9}\times\frac{2}{7}+\frac{8}{9}\times\frac{1}{7}} = \frac{1}{7}$$

18. Let A_1 be the event exactly 4 white balls have been drawn. A_2 be the event exactly 5 white balls have been drawn.

A_3 be the event exactly 6 white balls have been drawn.

B be the event exactly 1 white ball is drawn from two draws. Then,

$$P(B) = P\left(\frac{B}{A_1}\right)P(A_1) + P\left(\frac{B}{A_2}\right)P(A_2) + P\left(\frac{B}{A_3}\right)P(A_3)$$

But $P\left(\frac{B}{A_3}\right) = 0$

[since, there are only 6 white balls in the bag]

$$\therefore \quad P(B) = P\left(\frac{B}{A_1}\right)P(A_1) + P\left(\frac{B}{A_2}\right)P(A_2)$$

$$= \frac{{}^{12}C_2\cdot{}^6C_4}{{}^{18}C_6}\cdot\frac{{}^{10}C_1\cdot{}^2C_1}{{}^{12}C_2} + \frac{{}^{12}C_1\cdot{}^6C_5}{{}^{18}C_6}\cdot\frac{{}^{11}C_1\cdot{}^1C_1}{{}^{12}C_2}$$

19. Let E be the event that coin tossed twice, shows head at first time and tail at second time and F be the event that coin drawn is fair.

$$P(F/E) = \frac{P(E/F)\cdot P(F)}{P(E/F)\cdot P(F) + P(E/F')\cdot P(F')}$$

$$= \frac{\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{m}{N}}{\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{m}{N} + \frac{2}{3}\cdot\frac{1}{3}\cdot\frac{N-m}{N}}$$

$$= \frac{\frac{m}{4}}{\frac{m}{4} + \frac{2(N-m)}{9}} = \frac{9m}{8N+m}$$

20. Let W_1 = ball drawn in the first draw is white.

B_1 = ball drawn in the first draw in black.

W_2 = ball drawn in the second draw is white.

Then , $P(W_2) = P(W_1)\, P(W_2/W_1) + P(B_1)P(W_2/B_1)$

$$= \left(\frac{m}{m+n}\right)\left(\frac{m+k}{m+n+k}\right) + \left(\frac{n}{m+n}\right)\left(\frac{m}{m+n+k}\right)$$

$$= \frac{m(m+k)+mn}{(m+n)(m+n+k)} = \frac{m(m+k+n)}{(m+n)(m+n+k)} = \frac{m}{m+n}$$

21. The number of ways in which $P_1, P_2, \ldots, P_8$ can be paired in four pairs

$$= \frac{1}{4!}[({}^8C_2)({}^6C_2)({}^4C_2)({}^2C_2)]$$

$$= \frac{1}{4!}\times\frac{8!}{2!6!}\times\frac{6!}{2!4!}\times\frac{4!}{2!2!}\times 1$$

$$= \frac{1}{4!}\times\frac{8\times7}{2!\times1}\times\frac{6\times5}{2!\times1}\times\frac{4\times3}{2!\times1} = \frac{8\times7\times6\times5}{2\cdot2\cdot2\cdot2} = 105$$

Now, atleast two players certainly reach the second round between P_1, P_2 and P_3 and P_4 can reach in final if exactly two players play against each other between P_1, P_2, P_3 and remaining player will play against one of the players from P_5, P_6, P_7, P_8 and P_4 plays against one of the remaining three from $P_5 \ldots P_8$.

This can be possible in

$${}^3C_2\times{}^4C_1\times{}^3C_1 = 3\cdot4\cdot3 = 36 \text{ ways}$$

$\therefore$ Probability that P_4 and exactly one of $P_5 \ldots P_8$ reach second round $= \frac{36}{105} = \frac{12}{35}$

If P_1, P_i, P_4 and P_j, where $i = 2$ or 3 and $j = 5$ or 6 or 7 reach the second round, then they can be paired in 2 pairs in $\frac{1}{2!}({}^4C_2)({}^2C_2) = 3$ ways. But P_4 will reach the final, if P_1 plays against P_i and P_4 plays against P_j.

Hence, the probability that P_4 will reach the final round from the second $= \frac{1}{3}$

$\therefore$ Probability that P_4 will reach the final is $\frac{12}{35}\times\frac{1}{3} = \frac{4}{35}$.

22. Let $q = 1 - p$ = probability of getting the tail. We have,

α = probability of A getting the head on tossing firstly

$$= P(H_1 \text{ or } T_1T_2T_3H_4 \text{ or } T_1T_2T_3T_4T_5T_6H_7 \text{ or } \ldots)$$

$$= P(H) + P(H)P(T)^3 + P(H)P(T)^6 + \ldots$$

$$= \frac{P(H)}{1-P(T)^3} = \frac{p}{1-q^3}$$

Also,

β = probability of B getting the head on tossing secondly

$$= P(T_1H_2 \text{ or } T_1T_2T_3T_4H_5 \text{ or } T_1T_2T_3T_4T_5T_6T_7H_8 \text{ or } \ldots)$$

$$= P(H)[P(T) + P(H)P(T)^4 + P(H)P(T)^7 + \ldots]$$

$$= P(T)[P(H) + P(H)P(T)^3 + P(H)P(T)^6 + \ldots]$$

$$= q\,\alpha = (1-p)\,\alpha = \frac{p(1-p)}{1-q^3}$$

Again, we have

$$\alpha + \beta + \gamma = 1$$

$$\Rightarrow \quad \gamma = 1 - (\alpha+\beta) = 1 - \frac{p + p(1-p)}{1-q^3}$$

$$= 1 - \frac{p + p(1-p)}{1-(1-p)^3}$$

$$= \frac{1-(1-p)^3 - p - p(1-p)}{1-(1-p)^3}$$

$$\gamma = \frac{1-(1-p)^3 - 2p + p^2}{1-(1-p)^3} = \frac{p - 2p^2 + p^3}{1-(1-p)^3}$$

Also, $\alpha = \dfrac{p}{1-(1-p)^3},\ \beta = \dfrac{p(1-p)}{1-(1-p)^3}$

23. (i) Probability of S_1 to be among the eight winners

$= $ (Probability of S_1 being a pair)

$\times$ (Probability of S_1 winning in the group)

$= 1 \times \dfrac{1}{2} = \dfrac{1}{2}$ [since, S_1 is definitely in a group]

(ii) If S_1 and S_2 are in the same pair, then exactly one wins.

If S_1 and S_2 are in two pairs separately, then exactly one of S_1 and S_2 will be among the eight winners. If S_1 wins and S_2 loses or S_1 loses and S_2 wins.

Now, the probability of S_1, S_2 being in the same pair and one wins

$= $ (Probability of S_1, S_2 being the same pair)

$\times$ (Probability of anyone winning in the pair).

and the probability of S_1, S_2 being the same pair

$$= \frac{n\,(E)}{n\,(S)}$$

where, $n\,(E) =$ the number of ways in which 16 persons can be divided in 8 pairs.

$$\therefore \quad n\,(E) = \frac{(14)!}{(2!)^7 \cdot 7!} \text{ and } n\,(S) = \frac{(16)!}{(2!)^8 \cdot 8!}$$

$\therefore$ Probability of S_1 and S_2 being in the same pair

$$= \frac{(14)! \cdot (2!)^8 \cdot 8!}{(2!)^7 \cdot 7! \cdot (16)!} = \frac{1}{15}$$

The probability of any one wining in the pairs of $S_1, S_2 = P$ (certain event) $= 1$

$\therefore$ The pairs of S_1, S_2 being in two pairs separately and S_1 wins, S_2 loses + The probability of S_1, S_2 being in two pairs separately and S_1 loses, S_2 wins.

$$= \left[1 - \frac{\frac{(14)!}{(2!)^7 \cdot 7!}}{\frac{(16)!}{(2!)^8 \cdot 8!}}\right] \times \frac{1}{2} \times \frac{1}{2} + \left[1 - \frac{\frac{(14)!}{(2!)^7 \cdot 7!}}{\frac{(16)!}{(2!)^8 \cdot 8!}}\right] \times \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{1}{2} \times \frac{14 \times (14)!}{15 \times (14)!} = \frac{7}{15}$$

$$\therefore \text{ Required probability} = \frac{1}{15} + \frac{7}{15} = \frac{8}{15}$$

24. Let E_1, E_2, E_3 and A be the events defined as

$E_1 =$ the examinee guesses the answer

$E_2 =$ the examinee copies the answer

$E_3 =$ the examinee knows the answer

and $A =$ the examinee answer correctly

We have, $P(E_1) = \dfrac{1}{3},\ P(E_2) = \dfrac{1}{6}$

Since, E_1, E_2, E_3 are mutually exclusive and exhaustive events.

$\therefore \quad P(E_1) + P(E_2) + P(E_3) = 1$

$$\Rightarrow \quad P(E_3) = 1 - \frac{1}{3} - \frac{1}{6} = \frac{1}{2}$$

If E_1 has already occured, then the examinee guesses. Since, there are four choices out of which only one is correct, therefore the probability that he answer correctly given that he has made a guess is 1/4.

i.e. $P(A/E_1) = \dfrac{1}{4}$

It is given that, $P(A/E_2) = \dfrac{1}{8}$

and $P(A/E_3) =$ probability that he answer correctly given that he know the answer $= 1$

By Baye's theorem, we have

$$P(E_3/A) = \frac{P(E_3) \cdot P(A/E_3)}{\left[\begin{array}{r} P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2) \\ + P(E_3) \cdot P(A/E_3) \end{array}\right]}$$

$$\therefore \quad P(E_3/A) = \frac{\frac{1}{2} \times 1}{\left(\frac{1}{3} \times \frac{1}{4}\right) + \left(\frac{1}{6} \times \frac{1}{8}\right) + \left(\frac{1}{2} \times 1\right)} = \frac{24}{29}$$

25. Let $B_i = i$th ball drawn is black.

$W_i = i$th ball drawn is white, where $i = 1, 2$

and $A =$ third ball drawn is black.

We observe that the black ball can be drawn in the third draw in one of the following mutually exclusive ways.

(i) Both first and second balls drawn are white and third ball drawn is black.

i.e. $(W_1 \cap W_2) \cap A$

(ii) Both first and second balls are black and third ball drawn is black.

i.e. $(B_1 \cap B_2) \cap A$

(iii) The first ball drawn is white, the second ball drawn is black and the third ball drawn is black.

i.e. $(W_1 \cap B_2) \cap A$

(iv) The first ball drawn is black, the second ball drawn is white and the third ball drawn is black.

i.e. $(B_1 \cap W_2) \cap A$

$$\therefore \quad P(A) = P[\{(W_1 \cap W_2) \cap A\} \cup \{(B_1 \cap B_2) \cap A\} \cup \{(W_1 \cap B_2) \cap A\} \cup \{(B_1 \cap W_2) \cap A\}]$$

$$= P\{(W_1 \cap W_2) \cap A\} + P\{(B_1 \cap B_2) \cap A\} + P\{(W_1 \cap B_2) \cap A\} + P\{(B_1 \cap W_2) \cap A\}$$

$$= P(W_1 \cap W_2) \cdot P(A/(W_1 \cap W_2)) + P(B_1 \cap B_2)$$

$$\therefore \quad P(A/(B_1 \cap B_2)) + P(W_1 \cap B_2) \cdot P(A/(W_1 \cap B_2)) + P(B_1 \cap W_2) \cdot P(A/(B_1 \cap W_2))$$

$$= \left(\frac{2}{4} \times \frac{1}{3}\right) \times 1 + \left(\frac{2}{4} \times \frac{3}{5}\right) \times \frac{4}{6} + \left(\frac{2}{4} \times \frac{2}{3}\right) \times \frac{3}{4} + \left(\frac{2}{4} \times \frac{2}{5}\right) \times \frac{3}{4}$$

$$= \frac{1}{6} + \frac{1}{5} + \frac{1}{4} + \frac{3}{20} = \frac{23}{30}$$

26. The testing procedure may terminate at the twelfth testing in two mutually exclusive ways.

I : When lot contains 2 defective articles.

II : When lot contains 3 defective articles.

Let A = testing procedure ends at twelth testing

A_1 = lot contains 2 defective articles

A_2 = lot contains 3 defective articles

$\therefore$ Required probability

$$= P(A_1)\cdot P(A/A_1) + P(A_2)\cdot P(A/A_2)$$

Here, $P(A/A_1)$ = probability that first 11 draws contain 10 non-defective and one-defective and twelfth draw contains a defective article.

$$= \frac{{}^{18}C_{10} \times {}^{2}C_1}{{}^{20}C_{11}} \times \frac{1}{9} \quad \ldots\text{(i)}$$

$P(A/A_2)$ = probability that first 11 draws contains 9 non-defective and 2-defective articles and twelfth draw contains defective $= \frac{{}^{17}C_9 \times {}^{3}C_2}{{}^{20}C_{11}} \times \frac{1}{9}$ ... (ii)

$\therefore$ Required probability

$$= (0.4)P(A/A_1) + 0.6\,P(A/A_2)$$

$$= \frac{0.4 \times {}^{18}C_{10} \times {}^{2}C_1}{{}^{20}C_{11}} \times \frac{1}{9} + \frac{0.6 \times {}^{17}C_9 \times {}^{3}C_2}{{}^{20}C_{11}} \times \frac{1}{9} = \frac{99}{1900}$$

Topic 5 Probability Distribution and Binomial Distribution

1. Given that, there are 50 problems to solve in an admission test and probability that the candidate can solve any problem is $\frac{4}{5} = q$ (say). So, probability that the candidate cannot solve a problem is $p = 1 - q = 1 - \frac{4}{5} = \frac{1}{5}$.

Now, let X be a random variable which denotes the number of problems that the candidate is unable to solve. Then, X follows binomial distribution with parameters $n = 50$ and $p = \frac{1}{5}$.

Now, according to binomial probability distribution concept

$$P(X = r) = {}^{50}C_r \left(\frac{1}{5}\right)^r \left(\frac{4}{5}\right)^{50-r}, \; r = 0, 1, \ldots, 50$$

$\therefore$ Required probability

$$= P(X < 2) = P(X = 0) + P(X = 1)$$

$$= {}^{50}C_0 \left(\frac{4}{5}\right)^{50} + {}^{50}C_1 \frac{4^{49}}{(5)^{50}} = \left(\frac{4}{5}\right)^{49}\left(\frac{4}{5} + \frac{50}{5}\right) = \frac{54}{5}\left(\frac{4}{5}\right)^{49}$$

2. Let for the given random variable 'X' the binomial probability distribution have n-number of independent trials and probability of success and failure are p and q respectively. According to the question,

Mean $= np = 8$ and variance $= npq = 4$

$$\therefore \quad q = \frac{1}{2} \Rightarrow p = 1 - q = \frac{1}{2}$$

Now, $n \times \frac{1}{2} = 8 \Rightarrow n = 16$

$$P(X = r) = {}^{16}C_r\left(\frac{1}{2}\right)^{16}$$

$$\therefore \quad P(X \le 2) = P(X = 0) + P(X = 1) + P(X = 2)$$

$$= {}^{16}C_0\left(\frac{1}{2}\right)^{16} + {}^{16}C_1\left(\frac{1}{2}\right)^{16} + {}^{16}C_2\left(\frac{1}{2}\right)^{16}$$

$$= \frac{1 + 16 + 120}{2^{16}} = \frac{137}{2^{16}} = \frac{k}{2^{16}} \quad \text{(given)}$$

$$\Rightarrow \quad k = 137$$

3. As we know probability of getting a head on a toss of a fair coin is $P(H) = \frac{1}{2} = p$ (let)

Now, let n be the minimum numbers of toss required to get at least one head, then required probability = 1 – (probability that on all 'n' toss we are getting tail)

$$= 1 - \left(\frac{1}{2}\right)^n \quad \left[\because P(\text{tail}) = P(\text{Head}) = \frac{1}{2}\right]$$

According to the question,

$$1 - \left(\frac{1}{2}\right)^n > \frac{99}{100} \Rightarrow \left(\frac{1}{2}\right)^n < 1 - \frac{99}{100}$$

$$\Rightarrow \quad \left(\frac{1}{2}\right)^n < \frac{1}{100} \Rightarrow 2^n > 100$$

$$\Rightarrow \quad n = 7 \quad \text{[for minimum]}$$

4. The required probability of observing atleast one head

$$= 1 - P\text{(no head)}$$

$$= 1 - \frac{1}{2^n} \quad \text{[let number of toss are } n\text{]}$$

$$\left[\because P(\text{Head}) = P(\text{Tail}) = \frac{1}{2}\right]$$

According to the question, $1 - \frac{1}{2^n} \ge \frac{90}{100}$

$$\Rightarrow \frac{1}{2^n} \le \frac{1}{10} \quad \Rightarrow \quad 2^n \ge 10 \quad \Rightarrow \quad n \ge 4$$

So, minimum number of times one has to toss a fair coin so that the probability of observing atleast one head is atleast 90% is 4.

5. Let p and q represents the probability of success and failure in a trial respectively. Then,

$p = P(5 \text{ or } 6) = \frac{2}{6} = \frac{1}{3}$ and $q = 1 - p = \frac{4}{6} = \frac{2}{3}$.

Now, as the man decides to throw the die either till he gets a five or a six or to a maximum of three throws, so he can get the success in first, second and third throw or not get the success in any of the three throws.

So, the expected gain/loss (in ₹)

$$= (p \times 100) + qp(-50 + 100) + q^2p(-50 - 50 + 100) + q^3\,(-50 - 50 - 50)$$

$$= \left(\frac{1}{3} \times 100\right) + \left(\frac{2}{3} \times \frac{1}{3}\right)(50) + \left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)(0) + \left(\frac{2}{3}\right)^3 (-150)$$

$= \frac{100}{3} + \frac{100}{9} + 0 - \frac{1200}{27}$

$= \frac{900 + 300 - 1200}{27} = \frac{1200 - 1200}{27} = 0$

6. The probability of hitting a target at least once
$= 1 -$ (probability of not hitting the target in any trial)
$= 1 - {}^nC_0\, p^0 q^n$
where n is the number of independent trials and p and q are the probability of success and failure respectively.
[by using binomial distribution]

Here, $p = \frac{1}{3}$ and $q = 1 - p = 1 - \frac{1}{3} = \frac{2}{3}$

According to the question, $1 - {}^nC_0 \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^n > \frac{5}{6}$

$\Rightarrow \left(\frac{2}{3}\right)^n < 1 - \frac{5}{6} \Rightarrow \left(\frac{2}{3}\right)^n < \frac{1}{6}$

Clearly, minimum value of n is 5.

7. Let p = probability of getting an ace in a draw = probability of success

and q = probability of not getting an ace in a draw = probability of failure

Then, $p = \frac{4}{52} = \frac{1}{13}$

and $q = 1 - p = 1 - \frac{1}{13} = \frac{12}{13}$

Here, number of trials, $n = 2$

Clearly, X follows binomial distribution with parameter $n = 2$ and $p = \frac{1}{13}$.

Now, $P(X = x) = {}^2C_x \left(\frac{1}{13}\right)^x \left(\frac{12}{13}\right)^{2-x}$, $x = 0, 1, 2$

$\therefore$ $P(X = 1) + P(X = 2)$

$= {}^2C_1\left(\frac{1}{13}\right)^1\left(\frac{12}{13}\right) + {}^2C_2\left(\frac{1}{13}\right)^2\left(\frac{12}{13}\right)^0$

$= 2\left(\frac{12}{169}\right) + \frac{1}{169}$

$= \frac{24}{169} + \frac{1}{169} = \frac{25}{169}$

8. Given box contains 15 green and 10 yellow balls.
$\therefore$ Total number of balls $= 15 + 10 = 25$

P(green balls) $= \frac{15}{25} = \frac{3}{5} = p$ = Probability of success

P(yellow balls) $= \frac{10}{25} = \frac{2}{5} = q$ = Probability of unsuccess

and $n = 10$ = Number of trials.

$\therefore$ Variance $= npq = 10 \times \frac{3}{5} \times \frac{2}{5} = \frac{12}{5}$

9. Probability of guessing a correct answer, $p = \frac{1}{3}$ and probability of guessing a wrong answer, $q = 2/3$

$\therefore$ The probability of guessing a 4 or more correct answers $= {}^5C_4\left(\frac{1}{3}\right)^4 \cdot \frac{2}{3} + {}^5C_5\left(\frac{1}{3}\right)^5 = 5 \cdot \frac{2}{3^5} + \frac{1}{3^5} = \frac{11}{3^5}$

10. India play 4 matches and getting at least 7 points. It can only be possible in *WWWD* or *WWWW* position, where W represents two points and D represents one point.

Therefore, the probability of the required event

$= {}^4C_3\ (0.05)\ (0.5)^3 + {}^4C_4 (0.5)^4$

$= [4(0.05) + 0.5]\,(0.5)^3 = 0.0875$

11. Let X be the number of coins showing heads. Let X be a binomial variate with parameters $n = 100$ and p.

Since, $P(X = 50) = P(X = 51)$

$\Rightarrow$ ${}^{100}C_{50} p^{50} (1 - p)^{50} = {}^{100}C_{51} (p)^{51} (1 - p)^{49}$

$\Rightarrow$ $\frac{(100)!}{(50!)\,(50!)} \cdot \frac{(51!) \times (49!)}{100!} = \frac{p}{1 - p} \Rightarrow \frac{p}{1 - p} = \frac{51}{50}$

$\Rightarrow$ $p = \frac{51}{101}$

12. For Binomial distribution, mean $= np$

and variance $= npq$

$\therefore$ $np = 2$ and $npq = 1$ [given]

$\Rightarrow$ $q = 1/2$ and $p + q = 1$

$\Rightarrow$ $p = 1/2$

$\therefore$ $n = 4$, $p = q = 1/2$

Now, $P\ (X > 1) = 1 - \{P\ (X = 0) + P\ (X = 1)\}$

$= 1 - {}^4C_0\left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^4 - {}^4C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^3$

$= 1 - \frac{1}{16} - \frac{4}{16} = \frac{11}{16}$

13. Probability (face 1) $= \frac{0.1}{0.1 + 0.32} = \frac{0.1}{0.42} = \frac{5}{21}$

14. Let E be the event that product of the two digits is 18, therefore required numbers are 29 , 36, 63 and 92.

Hence, $p = P(E) = \frac{4}{100}$

and probability of non-occurrence of E is

$q = 1 - P(E) = 1 - \frac{4}{100} = \frac{96}{100}$

Out of the four numbers selected, the probability that the event E occurs atleast 3 times, is given as

$P = {}^4C_3 p^3 q + {}^4C_4 p^4$

$= 4\left(\frac{4}{100}\right)^3\left(\frac{96}{100}\right) + \left(\frac{4}{100}\right)^4 = \frac{97}{25^4}$

15. Since, set A contains n elements. So, it has 2^n subsets.

$\therefore$ Set P can be chosen in 2^n ways, similarly set Q can be chosen in 2^n ways.

$\therefore$ P and Q can be chosen in $(2^n)(2^n) = 4^n$ ways.

Suppose, P contains r elements, where r varies from 0 to n. Then, P can be chosen in ${}^nC_r$ ways, for 0 to be disjoint from A, it should be chosen from the set of all subsets of set consisting of remaining $(n - r)$ elements. This can be done in 2^{n-r} ways.

$\therefore$ P and Q can be chosen in ${}^nC_r \cdot 2^{n-r}$ ways.

But, r can vary from 0 to n.

$\therefore$ Total number of disjoint sets P and Q

$$= \sum_{r=0}^{n} {}^nC_r 2^{n-r} = (1+2)^n = 3^n$$

Hence, required probability $= \dfrac{3^n}{4^n} = \left(\dfrac{3}{4}\right)^n$

16. ***Case* I** When A plays 3 games against B.

In this case, we have $n = 3$, $p = 0.4$ and $q = 0.6$

Let X denote the number of wins. Then,

$$P(X = r) = {}^3C_r (0.4)^r (0.6)^{3-r};\ r = 0, 1, 2, 3$$

$\therefore$ P_1 = probability of winning the best of 3 games

$= P(X \ge 2)$

$= P(X = 2) + P(X = 3)$

$= {}^3C_2 (0.4)^2 (0.6)^1 + {}^3C_3 (0.4)^3 (0.6)^0$

$= 0.288 + 0.064 = 0.352$

***Case* II** When A plays 5 games against B.

In this case, we have

$$n = 5,\ p = 0.4 \quad \text{and} \quad q = 0.6$$

Let X denotes the number of wins in 5 games.

Then,

$$P(X = r) = {}^5C_r (0.4)^r (0.6)^{5-r}, \text{ where } r = 0, 1, 2 \dots, 5$$

$\therefore$ P_2 = probability of winning the best of 5 games

$= P(X \ge 3)$

$= P(X = 3) + P(X = 4) + P(X = 5)$

$= {}^5C_3 (0.4)^3 (0.6)^2 + {}^5C_4 (0.4)^4 (0.6) + {}^5C_5 (0.4)^5 (0.6)^0$

$= 0.2304 + 0.0768 + 0.1024 = 0.31744$

Clearly, $P_1 > P_2$. Therefore, first option i.e. 'best of 3 games' has higher probability of winning the match.

17. The man will be one step away from the starting point, if

(i) either he is one step ahead or (ii) one step behind the starting point.

The man will be one step ahead at the end of eleven steps, if he moves six steps forward and five steps backward. The probability of this event is ${}^{11}C_6 (0.4)^6 (0.6)^5$.

The man will be one step behind at the end of eleven steps, if he moves six steps backward and five steps forward. The probability of this event is ${}^{11}C_6 (0.6)^6 (0.4)^5$.

$\therefore$ Required probability

$$= {}^{11}C_6 (0.4)^6 (0.6)^5 + {}^{11}C_6 (0.6)^6 (0.4)^5 = {}^{11}C_6 (0.24)^5$$

18. Using Binomial distribution,

$P(X \ge 2) = 1 - P(X = 0) - P(X = 1)$

$$= 1 - \left(\frac{1}{2}\right)^n - \left[{}^nC_1 \cdot \left(\frac{1}{2}\right) \cdot \left(\frac{1}{2}\right)^{n-1}\right]$$

$$= 1 - \frac{1}{2^n} - {}^nC_1 \cdot \frac{1}{2^n} = 1 - \left(\frac{1+n}{2^n}\right)$$

Given, $P(X \ge 2) \ge 0.96$

$\therefore \quad 1 - \dfrac{(n+1)}{2^n} \ge \dfrac{24}{25}$

$\Rightarrow \quad \dfrac{n+1}{2^n} \le \dfrac{1}{25}$

$\therefore \quad n = 8$