4

Permutations and Combinations

Topic 1 General Arrangement

Objective Questions I (Only one correct option)

1. The number of four-digit numbers strictly greater than 4321 that can be formed using the digits 0, 1, 2, 3, 4, 5 (repetition of digits is allowed) is **(2019 Main, 8 April II)**

(a) 306 (b) 310
(c) 360 (d) 288

2. How many 3×3 matrices M with entries from $\{0, 1, 2\}$ are there, for which the sum of the diagonal entries of $M^T M$ is 5 ? **(2017 Adv.)**

(a) 198 (b) 162 (c) 126 (d) 135

3. The number of integers greater than 6000 that can be formed using the digits 3, 5, 6, 7 and 8 without repetition is **(2015 Main)**

(a) 216 (b) 192
(c) 120 (d) 72

4. The number of seven-digit integers, with sum of the digits equal to 10 and formed by using the digits 1, 2 and 3 only, is **(2009)**

(a) 55 (b) 66 (c) 77 (d) 88

5. How many different nine-digit numbers can be formed from the number 22 33 55 888 by rearranging its digits so that the odd digits occupy even positions? **(2000, 2M)**

(a) 16 (b) 36
(c) 60 (d) 180

6. An n-digit number is a positive number with exactly n digits. Nine hundred distinct n-digit numbers are to be formed using only the three digits 2,5 and 7. The smallest value of n for which this is possible, is **(1998, 2M)**

(a) 6 (b) 7 (c) 8 (d) 9

7. In a collage of 300 students, every student reads 5 newspapers and every newspaper is read by 60 students. The number of newspapers is **(1998, 2M)**

(a) atleast 30
(b) atmost 20
(c) exactly 25
(d) None of the above

8. A five digits number divisible by 3 is to be formed using the numbers 0, 1 , 2, 3 , 4 and 5, without repetition. The total number of ways this can be done, is **(1989, 2M)**

(a) 216 (b) 240
(c) 600 (d) 3125

9. Eight chairs are numbered 1 to 8. Two women and three men wish to occupy one chair each.

First the women choose the chairs from amongst the chairs marked 1 to 4 and then the men select the chairs from amongst the remaining. The number of possible arrangements is

(a) ${}^6C_3 \times {}^4C_2$ (b) ${}^4P_2 \times {}^4P_3$ **(1982, 2M)**
(c) ${}^4C_2 + {}^4P_3$ (d) None of these

10. The different letters of an alphabet are given. Words with five letters are formed from these given letters. Then, the number of words which have at least one letter repeated, is **(1980, 2M)**

(a) 69760 (b) 30240
(c) 99748 (d) None

Analytical & Descriptive Question

11. Eighteen guests have to be seated half on each side of a long table. Four particular guests desire to sit on one particular side and three other on the other side. Determine the number of ways in which the sitting arrangements can be made. **(1991, 4M)**

Match the Column

Match the conditions/expressions in Column I with statement in Column II.

12. Consider all possible permutations of the letters of the word ENDEANOEL. **(2008, 6M)**

	Column I		Column II
A.	The number of permutations containing the word ENDEA, is	p.	5!
B.	The number of permutations in which the letter E occurs in the first and the last positions, is	q.	$2\times 5!$
C.	The number of permutations in which none of the letters D, L, N occurs in the last five positions, is	r.	$7\times 5!$
D.	The number of permutations in which the letters A, E, O occur only in odd positions, is	s.	$21\times 5!$

Topic 2 Properties of Combinational and General Selections

Objective Questions I (Only one correct option)

1. The number of ways of choosing 10 objects out of 31 objects of which 10 are identical and the remaining 21 are distinct, is **(2019 Main, 12 April I)**

(a) $2^{20}-1$ (b) 2^{21} (c) 2^{20} (d) $2^{20}+1$

2. Suppose that 20 pillars of the same height have been erected along the boundary of a circular stadium. If the top of each pillar has been connected by beams with the top of all its non-adjacent pillars, then the total number of beams is **(2019 Main, 10 April II)**

(a) 180 (b) 210 (c) 170 (d) 190

3. Some identical balls are arranged in rows to form an equilateral triangle. The first row consists of one ball, the second row consists of two balls and so on. If 99 more identical balls are added to the total number of balls used in forming the equilateral triangle, then all these balls can be arranged in a square whose each side contains exactly 2 balls less than the number of balls each side of the triangle contains. Then, the number of balls used to form the equilateral triangle is **(2019 Main, 9 April II)**

(a) 262 (b) 190 (c) 225 (d) 157

4. There are m men and two women participating in a chess tournament. Each participant plays two games with every other participant. If the number of games played by the men between themselves exceeds the number of games played between the men and the women by 84, then the value of m is **(2019 Main, 12 Jan II)**

(a) 12 (b) 11 (c) 9 (d) 7

5. If ${}^nC_4$, ${}^nC_5$ and ${}^nC_6$ are in AP, then n can be **(2019 Main, 12 Jan II)**

(a) 9 (b) 11 (c) 14 (d) 12

6. If $\sum_{r=0}^{25}\{{}^{50}C_r\cdot {}^{50-r}C_{25-r}\}=K({}^{50}C_{25})$, then, K is equal to **(2019 Main, 10 Jan II)**

(a) 2^{24} (b) $2^{25}-1$ (c) 2^{25} (d) $(25)^2$

7. If $\sum_{i=1}^{20}\left(\frac{{}^{20}C_{i-1}}{{}^{20}C_i+{}^{20}C_{i-1}}\right)^3=\frac{k}{21}$, then k equals **(2019 Main, 10 Jan I)**

(a) 100 (b) 400 (c) 200 (d) 50

8. A man X has 7 friends, 4 of them are ladies and 3 are men. His wife Y also has 7 friends, 3 of them are ladies and 4 are men. Assume X and Y have no common friends. Then, the total number of ways in which X and Y together can throw a party inviting 3 ladies and 3 men, so that 3 friends of each of X and Y are in this party, is **(2017 Main)**

(a) 485 (b) 468 (c) 469 (d) 484

9. Let $S=\{1,2,3,\ldots\ldots,9\}$. For $k=1,2,\ldots\ldots 5$, let N_k be the number of subsets of S, each containing five elements out of which exactly k are odd. Then $N_1+N_2+N_3+N_4+N_5=$ **(2017 Adv.)**

(a) 210 (b) 252 (c) 126 (d) 125

10. A debate club consists of 6 girls and 4 boys. A team of 4 members is to be selected from this club including the selection of a captain (from among these 4 members) for the team. If the team has to include atmost one boy, the number of ways of selecting the team is **(2016 Adv.)**

(a) 380 (b) 320 (c) 260 (d) 95

11. Let T_n be the number of all possible triangles formed by joining vertices of an n-sided regular polygon. If $T_{n+1}-T_n=10$, then the value of n is **(2013 Main)**

(a) 7 (b) 5 (c) 10 (d) 8

12. If r, s, t are prime numbers and p, q are the positive integers such that LCM of p, q is $r^2s^4t^2$, then the number of ordered pairs (p, q) is **(2006, 3M)**

(a) 252 (b) 254 (c) 225 (d) 224

13. The value of the expression ${}^{47}C_4+\sum_{j=1}^{5}{}^{52-j}C_3$ is **(1980, 2M)**

(a) ${}^{47}C_5$ (b) ${}^{52}C_5$
(c) ${}^{52}C_4$ (d) None of these

Match Type Question

14. In a high school, a committee has to be formed from a group of 6 boys $M_1, M_2, M_3, M_4, M_5, M_6$ and 5 girls G_1, G_2, G_3, G_4, G_5.

(i) Let α_1 be the total number of ways in which the committee can be formed such that the committee has 5 members, having exactly 3 boys and 2 girls.

(ii) Let α_2 be the total number of ways in which the committee can be formed such that the committee has at least 2 members, and having an equal number of boys and girls.

(iii) Let α_3 be the total number of ways in which the committee can be formed such that the committee has 5 members, at least 2 of them being girls.

(iv) Let α_4 be the total number of ways in which the committee can be formed such that the committee has 4 members, having at least 2 girls such that both M_1 and G_1 are NOT in the committee together. **(2018 Adv.)**

	List-I		List-II
P.	The value of α_1 is	1.	136
Q.	The value of α_2 is	2.	189
R.	The value of α_3 is	3.	192
S.	The value of α_4 is	4.	200
		5.	381
		6.	461

The correct option is

(a) P $\to$ 4; Q $\to$ 6; R $\to$ 2; S $\to$ 1
(b) P $\to$ 1; Q $\to$ 4; R $\to$ 2; S $\to$ 3
(c) P $\to$ 4; Q $\to$ 6; R $\to$ 5; S $\to$ 2
(d) P $\to$ 4; Q $\to$ 2; R $\to$ 3; S $\to$ 1

Integer Answer Type Question

15. Let $n \geq 2$ be an integer. Take n distinct points on a circle and join each pair of points by a line segment. Colour the line segment joining every pair of adjacent points by blue and the rest by red. If the number of red and blue line segments are equal, then the value of n is **(2014 Adv)**

Fill in the Blanks

16. Let A be a set of n distinct elements. Then, the total number of distinct functions from A to A is…and out of these… are onto functions. **(1985, 2M)**

17. In a certain test, a_i students gave wrong answers to at least i questions, where $i = 1, 2, \dots, k$. No student gave more that k wrong answers. The total number of wrong answers given is … . **(1982, 2M)**

True/False

18. The product of any r consecutive natural numbers is always divisible by $r\,!$. **(1985, 1M)**

Analytical & Descriptive Questions

19. A committee of 12 is to be formed from 9 women and 8 men. In how many ways this can be done if at least five women have to be included in a committee ? In how many of these committees

(i) the women are in majority?
(ii) the men are in majority? **(1994, 4M)**

20. A student is allowed to select atmost n books from n collection of $(2n + 1)$ books. If the total number of ways in which he can select at least one books is 63, find the value of n. **(1987, 3M)**

21. A box contains two white balls, three black balls and four red balls. In how many ways can three balls be drawn from the box, if at least one black ball is to be included in the draw ? **(1986, $2\frac{1}{2}$ M)**

22. 7 relatives of a man comprises 4 ladies and 3 gentlemen, his wife has also 7 relatives ; 3 of them are ladies and 4 gentlemen. In how many ways can they invite a dinner party of 3 ladies and 3 gentlemen so that there are 3 of man's relative and 3 of the wife's relatives? **(1985, 5M)**

23. m men and n women are to be seated in a row so that no two women sit together. If $m > n$, then show that the number of ways in which they can be seated, is

$$\frac{m\,!\,(m+1)\,!}{(m-n+1)\,!}.$$

(1983, 2M)

24. mn squares of equal size are arranged to form a rectangle of dimension m by n where m and n are natural numbers. Two squares will be called 'neighbours' if they have exactly one common side. A natural number is written in each square such that the number in written any square is the arithmetic mean of the numbers written in its neighbouring squares. Show that this is possible only if all the numbers used are equal. **(1982, 5M)**

25. If ${}^nC_{r-1} = 36$, ${}^nC_r = 84$ and ${}^nC_{r+1} = 126$, then find the values of n and r. **(1979, 3M)**

Topic 3 Multinomial, Repeated Arrangement and Selection

Objective Question I (Only one correct option)

1. The number of 6 digits numbers that can be formed using the digits 0, 1, 2,5, 7 and 9 which are divisible by 11 and no digit is repeated, is **(2019 Main, 10 April I)**

(a) 60 (b) 72
(c) 48 (d) 36

2. A committee of 11 members is to be formed from 8 males and 5 females. If m is the number of ways the committee is formed with at least 6 males and n is the number of ways the committee is formed with atleast 3 females, then **(2019 Main, 9 April I)**

(a) $m = n = 68$ (b) $m + n = 68$
(c) $m = n = 78$ (d) $n = m - 8$

3. Consider three boxes, each containing 10 balls labelled 1, 2, …, 10. Suppose one ball is randomly drawn from each of the boxes. Denote by n_i, the label of the ball drawn from the ith box, $(i = 1, 2, 3)$. Then, the number of ways in which the balls can be chosen such that $n_1 < n_2 < n_3$ is **(2019 Main, 12 Jan I)**

(a) 82 (b) 120 (c) 240 (d) 164

4. The number of natural numbers less than 7,000 which can be formed by using the digits 0, 1, 3, 7, 9 (repitition of digits allowed) is equal to **(2019 Main, 9 Jan II)**

(a) 374 (b) 375 (c) 372 (d) 250

5. Consider a class of 5 girls and 7 boys. The number of different teams consisting of 2 girls and 3 boys that can be formed from this class, if there are two specific boys A and B, who refuse to be the members of the same team, is **(2019 Main, 9 Jan I)**

(a) 350 (b) 500 (c) 200 (d) 300

6. If all the words (with or without meaning) having five letters, formed using the letters of the word SMALL and arranged as in a dictionary, then the position of the word SMALL is **(2016 Main)**

(a) 46th (b) 59th
(c) 52nd (d) 58th

7. The letters of the word COCHIN are permuted and all the permutations are arranged in an alphabetical order as in an English dictionary. The number of words that appear before the word COCHIN, is **(2007, 3M)**

(a) 360 (b) 192 (c) 96 (d) 48

Numerical Value

8. The number of 5 digit numbers which are divisible by 4, with digits from the set $\{1, 2, 3, 4, 5\}$ and the repetition of digits is allowed, is .... **(2018 Adv.)**

Integer Answer Type Questions

9. Words of length 10 are formed using the letters A, B, C, D, E, F, G, H, I, J. Let x be the number of such words where no letter is repeated; and let y be the number of such words where exactly one letter is repeated twice and no other letter is repeated. Then, $\frac{y}{9x} =$ **(2017 Adv.)**

10. Let n be the number of ways in which 5 boys and 5 girls can stand in a queue in such a way that all the girls stand consecutively in the queue. Let m be the number of ways in which 5 boys and 5 girls can stand in a queue in such a way that exactly four girls stand consecutively in the queue. Then, the value of $\frac{m}{n}$ is **(2015 Adv.)**

11. Let $n_1 < n_2 < n_3 < n_4 < n_5$ be positive integers such that $n_1 + n_2 + n_3 + n_4 + n_5 = 20$. The number of such distinct arrangements $(n_1, n_2, n_3, n_4, n_5)$ is **(2014 Adv.)**

Fill in the Blanks

12. Let n and k be positive integers such that $n \geq \frac{k(k+1)}{2}$. The number of solutions $(x_1, x_2, \dots, x_k)$, $x_1 \geq 1$, $x_2 \geq 2, \dots, x_k \geq k$ for all integers satisfying $x_1 + x_2 + \dots + x_k = n$ is … **(1996, 2M)**

13. Total number of ways in which six '+' and four '–' signs can be arranged in a line such that no two '–'signs occur together is… . **(1988, 2M)**

Analytical & Descriptive Question

14. Five balls of different colours are to be placed in three boxes of different sizes. Each box can hold all five. In how many different ways can we place the balls so that no box remains empty? **(1981, 4M)**

Topic 4 Distribution of Object into Group

Objective Questions I (Only one correct option)

1. A group of students comprises of 5 boys and n girls. If the number of ways, in which a team of 3 students can randomly be selected from this group such that there is at least one boy and at least one girl in each team, is 1750, then n is equal to **(2019 Main, 12 April II)**

(a) 28 (b) 27
(c) 25 (d) 24

2. Let S be the set of all triangles in the xy-plane, each having one vertex at the origin and the other two vertices lie on coordinate axes with integral coordinates. If each triangle in S has area 50 sq. units, then the number of elements in the set S is **(2019 Main, 9 Jan II)**

(a) 36 (b) 32
(c) 18 (d) 9

3. From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf, so that the dictionary is always in the middle. The number of such arrangements is **(2018 Main)**

(a) atleast 1000
(b) less than 500
(c) atleast 500 but less than 750
(d) atleast 750 but less than 1000

4. The total number of ways in which 5 balls of different colours can be distributed among 3 persons so that each person gets at least one ball, is **(2012)**

(a) 75 (b) 150 (c) 210 (d) 243

5. The number of arrangements of the letters of the word BANANA in which the two N's do not appear adjacently, is **(2002, 1M)**

(a) 40 (b) 60 (c) 80 (d) 100

Analytical & Descriptive Questions

6. Using permutation or otherwise, prove that $\dfrac{n^2!}{(n!)^n}$ is an integer, where n is a positive integer. **(2004, 2M)**

7. In how many ways can a pack of 52 cards be

(i) divided equally among four players in order

(ii) divided into four groups of 13 cards each

(iii) divided in 4 sets, three of them having 17 cards each and the fourth just one card? **(1979, 3M)**

Topic 5 Dearrangement and Number of Divisors

Objective Question I (Only one correct option)

1. Number of divisors of the form $(4n+2)$, $n \geq 0$ of the integer 240 is **(1998, 2M)**

(a) 4 (b) 8
(c) 10 (d) 3

Fill in the Blank

2. There are four balls of different colours and four boxes of colours, same as those of the balls. The number of ways in which the balls, one each in a box, could be placed such that a ball does not go to a box of its own colour is.... . **(1992, 2M)**

Answers

Topic 1

1. (b) **2.** (a) **3.** (b) **4.** (c)
5. (c) **6.** (b) **7.** (c) **8.** (a)
9. (d) **10.** (a) **11.** ${}^9P_4 \times {}^9P_3\,(11)!$
12. (A → p; B → s; C → q ; D → q)

Topic 2

1. (c) **2.** (c) **3.** (b) **4.** (a)
5. (c) **6.** (c) **7.** (a) **8.** (a)
9. (c) **10.** (a) **11.** (b) **12.** (c)
13. (c) **14.** (c) **15.** (5)
16. $n^n, \sum_{r=1}^{n} (-1)^{n-r}\, {}^nC_r\, (r)^n$ **17.** $2^n - 1$ **18.** (True)
19. 6062, (i) 2702 (ii) 1008 **20.** $n = 3$ **21.** (64)
22. (485) **25.** ($n = 9$ and $r = 3$)

Topic 3

1. (a) **2.** (c) **3.** (b) **4.** (a)
5. (d) **6.** (d) **7.** (c) **8.** (625)
9. (5) **10.** (5) **11.** (7)
12. $\frac{1}{2}(2n - k^2 + k - 2)$ **13.** (35 ways) **14.** (300)

Topic 4

1. (c) **2.** (a) **3.** (a) **4.** (b)
5. (a) **7.** (i) $\dfrac{(52)!}{(13!)^4}$ (ii) $\dfrac{(52)!}{4!\,(13!)^4}$ (iii) $\dfrac{(52)!}{3!\,(17)^3}$

Topic 5

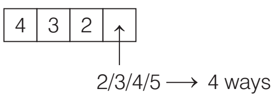
1. (a) **2.** (9)

Hints & Solutions

Topic 1 General Arrangement

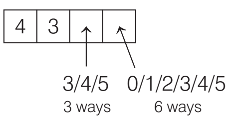
1. Following are the cases in which the 4-digit numbers strictly greater than 4321 can be formed using digits 0, 1, 2, 3, 4, 5 (repetition of digits is allowed)

Case-I



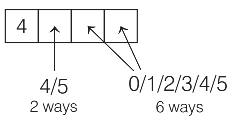
4 numbers

Case-II



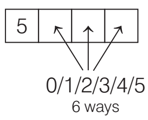
3×6=18 numbers

Case-III



2×6×6=72 numbers

Case-IV



6×6×6=216 numbers

So, required total numbers = 4 + 18 + 72 + 216 = 310

2. Sum of diagonal entries of M^TM is $\sum a_i^2$.

$$\sum_{i=1}^{9} a_i^2 = 5$$

Possibilities

I. 2, 1, 0, 0, 0, 0, 0, 0, 0, which gives $\frac{9!}{7!}$ matrices

II. 1, 1, 1, 1, 1, 0, 0, 0, 0, which gives $\frac{9!}{4! \times 5!}$ matrices

Total matrices $= 9 \times 8 + 9 \times 7 \times 2 = 198$

3. The integer greater than 6000 may be of 4 digits or 5 digits. So, here two cases arise.

***Case* I** When number is of 4 digits.

Four-digit number can start from 6, 7 or 8.

Thus, total number of 4-digit numbers, which are greater than $6000 = 3 \times 4 \times 3 \times 2 = 72$

***Case* II** When number is of 5 digits.

Total number of five-digit numbers which are greater than $6000 = 5! = 120$

$\therefore$ Total number of integers $= 72 + 120 = 192$

4. There are two possible cases

***Case* I** Five 1's, one 2's, one 3's

Number of numbers $= \frac{7!}{5!} = 42$

***Case* II** Four 1's, three 2's

Number of numbers $= \frac{7!}{4!3!} = 35$

$\therefore$ Total number of numbers $= 42 + 35 = 77$

5. X—X—X—X—X. The four digits 3, 3, 5, 5 can be arranged at (—) places in $\frac{4!}{2!2!} = 6$ ways.

The five digits 2, 2, 8, 8, 8 can be arranged at (X) places in $\frac{5!}{2!3!}$ ways $= 10$ ways.

Total number of arrangements $= 6 \times 10 = 60$

[since, events A and B are independent, therefore $A \cap B = A \times B$]

6. Distinct n-digit numbers which can be formed using digits 2, 5 and 7 are 3^n. We have to find n, so that

$$3^n \geq 900$$

$$\Rightarrow \quad 3^{n-2} \geq 100$$

$$\Rightarrow \quad n - 2 \geq 5$$

$\Rightarrow$ $n \geq 7$, so the least value of n is 7.

7. Let n be the number of newspapers which are read by the students.

Then, $\quad 60n = (300) \times 5$

$\Rightarrow \quad n = 25$

8. Since, a five-digit number is formed using the digits {0, 1, 2, 3, 4 and 5} divisible by 3 i.e. only possible when sum of the digits is multiple of three.

***Case* I** Using digits 0, 1, 2, 4, 5

Number of ways $= 4 \times 4 \times 3 \times 2 \times 1 = 96$

***Case* II** Using digits 1, 2, 3, 4, 5

Number of ways $= 5 \times 4 \times 3 \times 2 \times 1 = 120$

$\therefore$ Total numbers formed $= 120 + 96 = 216$

9. Since, the first 2 women select the chairs amongst 1 to 4 in ${}^4P_2$ ways. Now, from the remaining 6 chairs, three men could be arranged in ${}^6P_3$.

$\therefore$ Total number of arrangements $= {}^4P_2 \times {}^6P_3$.

10. Total number of five letters words formed from ten different letters $= 10 \times 10 \times 10 \times 10 \times 10 = 10^5$

Number of five letters words having no repetition

$$= 10 \times 9 \times 8 \times 7 \times 6 = 30240$$

$\therefore$ Number of words which have at least one letter repeated $= 10^5 - 30240 = 69760$

11. Let the two sides be A and B. Assume that four particular guests wish to sit on side A. Four guests who wish to sit on side A can be accommodated on nine chairs in ${}^9P_4$ ways and three guests who wish to sit on side B can be accommodated in ${}^9P_3$ ways. Now, the remaining guests are left who can sit on 11 chairs on both the sides of the table in (11!) ways. Hence, the total number of ways in which 18 persons can be seated $= {}^9P_4 \times {}^9P_3 \times (11)!$.

12. A. If ENDEA is fixed word, then assume this as a single letter. Total number of letters = 5

Total number of arrangements $= 5!$.

B. If E is at first and last places, then total number of permutations $= 7!/2! = 21 \times 5!$

C. If D, L, N are not in last five positions

$\leftarrow$ D, L, N, N $\rightarrow$ $\leftarrow$ E, E, E, A, O $\rightarrow$

Total number of permutations $= \frac{4!}{2!} \times \frac{5!}{3!} = 2 \times 5!$

D. Total number of odd positions = 5

Permutations of AEEEO are $\frac{5!}{3!}$.

Total number of even positions = 4

$\therefore$ Number of permutations of N, N, D, L $= \frac{4!}{2!}$

$\Rightarrow$ Total number of permutations $= \frac{5!}{3!} \times \frac{4!}{2!} = 2 \times 5!$

Topic 2 Properties of Combinational and General Selections

1. Given that, out of 31 objects 10 are identical and remaining 21 are distinct, so in following ways, we can choose 10 objects.

0 identical + 10 distincts, number of ways $= 1 \times {}^{21}C_{10}$

1 identical + 9 distincts, number of ways $= 1 \times {}^{21}C_9$

2 identicals + 8 distincts, number of ways $= 1 \times {}^{21}C_8$

. . . . . .

. . . . . .

So, total number of ways in which we can choose 10 objects is

$$^{21}C_{10} + {}^{21}C_9 + {}^{21}C_8 + \ldots + {}^{21}C_0 = x \text{(let)} \quad \ldots \text{(i)}$$

$$\Rightarrow {}^{21}C_{11} + {}^{21}C_{12} + {}^{21}C_{13} + \ldots + {}^{21}C_{21} = x \quad \ldots \text{(ii)}$$

$[\because {}^nC_r = {}^nC_{n-r}]$

On adding both Eqs. (i) and (ii), we get

$$2x = {}^{21}C_0 + {}^{21}C_1 + {}^{21}C_2 + \ldots + {}^{21}C_{10} + {}^{21}C_{11} + {}^{21}C_{12} + \ldots + {}^{21}C_{21}$$

$$\Rightarrow \quad 2x = 2^{21} \Rightarrow \quad x = 2^{20}$$

2. It is given that, there are 20 pillars of the same height have been erected along the boundary of a circular stadium.

Now, the top of each pillar has been connected by beams with the top of all its non-adjacent pillars, then total number of beams = number of diagonals of 20-sided polygon.

$\because$ $^{20}C_2$ is selection of any two vertices of 20-sided polygon which included the sides as well.

So, required number of total beams = $^{20}C_2 - 20$

[$\because$ the number of diagonals in a n-sided closed polygon = $^nC_2 - n$]

$$= \frac{20 \times 19}{2} - 20$$
$$= 190 - 20 = 170$$

3. Let there are n balls used to form the sides of equilateral triangle.

According to the question, we have

$$\frac{n(n+1)}{2} + 99 = (n-2)^2$$

$$\Rightarrow \quad n^2 + n + 198 = 2[n^2 - 4n + 4]$$
$$\Rightarrow \quad n^2 - 9n - 190 = 0$$
$$\Rightarrow \quad n^2 - 19n + 10n - 190 = 0$$
$$\Rightarrow \quad (n-19)(n+10) = 0$$
$$\Rightarrow \quad n = 19, -10$$
$$\Rightarrow \quad n = 19 \quad [\because \text{number of balls } n > 0]$$

Now, number of balls used to form an equilateral triangle is $\frac{n(n+1)}{2}$

$$= \frac{19 \times 20}{2} = 190.$$

4. Since, there are m-men and 2-women and each participant plays two games with every other participant.

$\therefore$ Number of games played by the men between themselves $= 2 \times {}^mC_2$

and the number of games played between the men and the women $= 2 \times {}^mC_1 \times {}^2C_1$

Now, according to the question,

$$2\,{}^mC_2 = 2\,{}^mC_1\,{}^2C_1 + 84$$

$$\Rightarrow \quad \frac{m!}{2!(m-2)!} = m \times 2 + 42$$
$$\Rightarrow \quad m(m-1) = 4m + 84$$
$$\Rightarrow \quad m^2 - m = 4m + 84$$
$$\Rightarrow \quad m^2 - 5m - 84 = 0$$
$$\Rightarrow \quad m^2 - 12m + 7m - 84 = 0$$
$$\Rightarrow \quad m(m-12) + 7(m-12) = 0$$
$$\Rightarrow \quad m = 12 \quad [\because m > 0]$$

5. If $^nC_4$, $^nC_5$ and $^nC_6$ are in AP, then

$$2 \cdot {}^nC_5 = {}^nC_4 + {}^nC_6$$

[If a, b, c are in AP, then $2b = a + c$]

$$\Rightarrow 2\frac{n!}{5!(n-5)!} = \frac{n!}{4!(n-4)!} + \frac{n!}{6!(n-6)!} \quad \left[\because {}^nC_r = \frac{n!}{r!(n-r)!}\right]$$

$$\Rightarrow \frac{2}{5 \cdot 4!(n-5)\,(n-6)!} = \frac{1}{4!(n-4)\,(n-5)\,(n-6)!} + \frac{1}{6 \cdot 5 \cdot 4!\,(n-6)!}$$

$$\Rightarrow \frac{2}{5(n-5)} = \frac{1}{(n-4)\,(n-5)} + \frac{1}{30}$$

$$\Rightarrow \frac{2}{5(n-5)} = \frac{30 + (n-4)\,(n-5)}{30\,(n-4)\,(n-5)}$$

$$\Rightarrow 12\,(n-4) = 30 + n^2 - 9n + 20$$
$$\Rightarrow n^2 - 21n + 98 = 0$$
$$\Rightarrow n^2 - 14n - 7n + 98 = 0$$
$$\Rightarrow n(n-14) - 7(n-14) = 0$$
$$\Rightarrow (n-7)\,(n-14) = 0$$
$$\Rightarrow n = 7 \text{ or } 14$$

6. Given, $\sum_{r=0}^{25} \{{}^{50}C_r \cdot {}^{50-r}C_{25-r}\} = K\,{}^{50}C_{25}$

$$\Rightarrow \sum_{r=0}^{25} \left(\frac{50!}{r!(50-r)!} \times \frac{(50-r)!}{(25-r)!25!}\right) = K\,{}^{50}C_{25}$$

$$\Rightarrow \sum_{r=0}^{25} \left(\frac{50!}{25!25!} \times \frac{25!}{r!(25-r)!}\right) = K\,{}^{50}C_{25}$$

[on multiplying 25! in numerator and denominator.]

$$\Rightarrow {}^{50}C_{25} \sum_{r=0}^{25} {}^{25}C_r = K\,{}^{50}C_{25} \quad \left[\because {}^{50}C_{25} = \frac{50!}{25!25!}\right]$$

$$\Rightarrow K = \sum_{r=0}^{25} {}^{25}C_r = 2^{25}$$

$[\because {}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n = 2^n]$

$$\Rightarrow K = 2^{25}$$

7. Given,

$$\sum_{i=1}^{20} \left(\frac{{}^{20}C_{i-1}}{{}^{20}C_i + {}^{20}C_{i-1}}\right)^3 = \frac{k}{21}$$

$$\Rightarrow \sum_{i=1}^{20} \left(\frac{{}^{20}C_{i-1}}{{}^{21}C_i}\right)^3 = \frac{k}{21} \quad (\because {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r)$$

$$\Rightarrow \sum_{i=1}^{20} \left(\frac{{}^{20}C_{i-1}}{\frac{21}{i}\,{}^{20}C_{i-1}}\right)^3 = \frac{k}{21} \quad \left(\because {}^nC_r = \frac{n}{r}\,{}^{n-1}C_{r-1}\right)$$

$$\Rightarrow \sum_{i=1}^{20} \left(\frac{i}{21}\right)^3 = \frac{k}{21}$$

$$\Rightarrow \frac{1}{(21)^3} \sum_{i=1}^{20} i^3 = \frac{k}{21}$$

$$\Rightarrow \quad \frac{1}{(21)^3}\left[\frac{n(n+1)}{2}\right]^2_{n=20} = \frac{k}{21}$$

$$\left[\because 1^3 + 2^3 + \ldots + n^3 = \left[\frac{n(n+1)}{2}\right]^2\right]$$

$$\Rightarrow \quad k = \frac{21}{(21)^3}\left(\frac{20 \times 21}{2}\right)^2 = 100$$

$$\therefore \quad k = 100$$

8. Given, X has 7 friends, 4 of them are ladies and 3 are men while Y has 7 friends, 3 of them are ladies and 4 are men.

$\therefore$ Total number of required ways

$= {}^3C_3 \times {}^4C_0 \times {}^4C_0 \times {}^3C_3 + {}^3C_2 \times {}^4C_1 \times {}^4C_1 \times {}^3C_2$
$\quad + {}^3C_1 \times {}^4C_2 \times {}^4C_2 \times {}^3C_1 + {}^3C_0 \times {}^4C_3 \times {}^4C_3 \times {}^3C_0$

$= 1 + 144 + 324 + 16 = 485$

9. $N_i = {}^5C_k \times {}^4C_{5-k}$

$N_1 = 5 \times 1$
$N_2 = 10 \times 4$
$N_3 = 10 \times 6$
$N_4 = 5 \times 4$
$N_5 = 1$
$N_1 + N_2 + N_3 + N_4 + N_5 = 126$

10. We have, 6 girls and 4 boys. To select 4 members (atmost one boy)

i.e. (1 boy and 3 girls) or (4 girls) $= {}^6C_3 \cdot {}^4C_1 + {}^6C_4$ ...(i)

Now, selection of captain from 4 members $= {}^4C_1$ ...(ii)

$\therefore$ Number of ways to select 4 members (including the selection of a captain, from these 4 members) $= ({}^6C_3 \cdot {}^4C_1 + {}^6C_4)\,{}^4C_1$

$= (20 \times 4 + 15) \times 4 = 380$

11. Given, $T_n = {}^nC_3 \Rightarrow T_{n+1} = {}^{n+1}C_3$

$\therefore \quad T_{n+1} - T_n = {}^{n+1}C_3 - {}^nC_3 = 10$ [given]

$\Rightarrow {}^nC_2 + {}^nC_3 - {}^nC_3 = 10 \quad [\because {}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}]$

$\Rightarrow \quad {}^nC_2 = 10$

$\Rightarrow \quad n = 5$

12. Since, r, s, t are prime numbers.

$\therefore$ Selection of p and q are as under

p	*q*	**Number of ways**
r^0	r^2	1 way
r^1	r^2	1 way
r^2	r^0, r^1, r^2	3 ways

$\therefore$ Total number of ways to select, $r = 5$

Selection of s as under

s^0	s^4	1 way
s^1	s^4	1 way
s^2	s^4	1 way
s^3	s^4	1 way
s^4		5 ways

$\therefore$ Total number of ways to select $s = 9$

Similarly, the number of ways to select $t = 5$

$\therefore$ Total number of ways $= 5 \times 9 \times 5 = 225$

13. Here, ${}^{47}C_4 + \sum_{j=1}^{5} {}^{52-j}C_3$

$= {}^{47}C_4 + {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3$
$= ({}^{47}C_4 + {}^{47}C_3) + {}^{48}C_3 + {}^{49}C_3 + {}^{50}C_3 + {}^{51}C_3$
[using ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$]
$= ({}^{48}C_4 + {}^{48}C_3) + {}^{49}C_3 + {}^{50}C_3 + {}^{51}C_3$
$= ({}^{49}C_4 + {}^{49}C_3) + {}^{50}C_3 + {}^{51}C_3$
$= ({}^{50}C_4 + {}^{50}C_3) + {}^{51}C_3$
$= {}^{51}C_4 + {}^{51}C_3 = {}^{52}C_4$

14. Given 6 boys $M_1, M_2, M_3, M_4, M_5, M_6$ and 5 girls G_1, G_2, G_3, G_4, G_5

(i) $\alpha_1 \to$ Total number of ways of selecting 3 boys and 2 girls from 6 boys and 5 girls.
i..e, ${}^6C_3 \times {}^5C_2 = 20 \times 10 = 200$
$\therefore \quad \alpha_1 = 200$

(ii) $\alpha_2 \to$ Total number of ways selecting at least 2 member and having equal number of boys and girls
i.e., ${}^6C_1{}^5C_1 + {}^6C_2{}^5C_2 + {}^6C_3{}^5C_3 + {}^6C_4{}^5C_4 + {}^6C_5{}^5C_5$
$= 30 + 150 + 200 + 75 + 6 = 461$
$\Rightarrow \quad \alpha_2 = 461$

(iii) $\alpha_3 \to$ Total number of ways of selecting 5 members in which at least 2 of them girls
i.e., ${}^5C_2{}^6C_3 + {}^5C_3{}^6C_2 + {}^5C_4{}^6C_1 + {}^5C_5{}^6C_0$
$= 200 + 150 + 30 + 1 = 381$
$\alpha_3 = 381$

(iv) $\alpha_4 \to$ Total number of ways of selecting 4 members in which at least two girls such that M_1 and G_1 are not included together.
G_1 is included $\to {}^4C_1 \cdot {}^5C_2 + {}^4C_2 \cdot {}^5C_1 + {}^4C_3$
$= 40 + 30 + 4 = 74$
M_1 is included $\to {}^4C_2 \cdot {}^5C_1 + {}^4C_3 = 30 + 4 = 34$
G_1 and M_1 both are not included
${}^4C_4 + {}^4C_3 \cdot {}^5C_1 + {}^4C_2 \cdot {}^5C_2$
$1 + 20 + 60 = 81$
$\therefore$ Total number $= 74 + 34 + 81 = 189$
$\alpha_4 = 189$
Now, $P \to 4; Q \to 6; R \to 5; S \to 2$
Hence, option (c) is correct.

15. **PLAN** Number of line segment joining pair of adjacent point $= n$
Number of line segment obtained joining n points on a circle $= {}^nC_2$
Number of red line segments $= {}^nC_2 - n$

Number of blue line segments $= n$

$\therefore \quad {}^nC_2 - n = n$

$$\Rightarrow \quad \frac{n(n-1)}{2} = 2n$$

$\Rightarrow \quad n = 5$

16. Let $A = \{x_1, x_2, \dots, x_n\}$

$\therefore$ Number of functions from A to A is n^n and out of these $\sum_{r=1}^{n} (-1)^{n-r}\ {}^nC_r (r)^n$ are onto functions.

17. The number of students answering exactly $k\ (1 \le k \le n-1)$ questions wrongly is $2^{n-k} - 2^{n-k-1}$. The number of students answering all questions wrongly is 2^0.

Thus, total number of wrong answers

$$= 1\,(2^{n-1} - 2^{n-2}) + 2\,(2^{n-2} - 2^{n-3}) + \dots + (n-1)\,(2^1 - 2^0) + 2^0 \cdot n$$

$$= 2^{n-1} + 2^{n-2} + 2^{n-3} + \dots + 2^1 + 2^0 = 2^n - 1$$

18. Let r consecutive integers be $x+1, x+2, \dots, x+r$.

$$\therefore\quad (x+1)(x+2)\dots(x+r) = \frac{(x+r)(x+r-1)\dots(x+1)\,x!}{x!}$$

$$= \frac{(x+r)!}{(x)!}\cdot\frac{r!}{r!} = {}^{x+r}C_r \cdot (r)!$$

Thus, $(x+1)(x+2)\dots(x+r) = {}^{x+r}C_r \cdot (r)!$, which is clearly divisible by $(r)!$. Hence, it is a true statement.

19. Given that, there are 9 women and 8 men, a committee of 12 is to be formed including at least 5 women.

This can be done in

= (5 women and 7 men) + (6 women and 6 men) + (7 women and 5 men) + (8 women and 4 men) + (9 women and 3 men) ways

Total number of ways of forming committee

$$= ({}^9C_5 \cdot {}^8C_7) + ({}^9C_6 \cdot {}^8C_6) + ({}^9C_7 \cdot {}^8C_5) + ({}^9C_8 \cdot {}^8C_4) + ({}^9C_9 \cdot {}^8C_3)$$

$$= 1008 + 2352 + 2016 + 630 + 56 = 6062$$

(i) The women are in majority $= 2016 + 630 + 56 = 2702$

(ii) The man are in majority $= 1008$ ways

20. Since, student is allowed to select at most n books out of $(2n+1)$ books.

$$\therefore\quad {}^{2n+1}C_1 + {}^{2n+1}C_2 + \dots + {}^{2n+1}C_n = 63 \quad \dots \text{(i)}$$

We know ${}^{2n+1}C_0 + {}^{2n+1}C_1 + \dots + {}^{2n+1}C_{2n+1} = 2^{2n+1}$

$\Rightarrow 2\,({}^{2n+1}C_0 + {}^{2n+1}C_1 + {}^{2n+1}C_2 + \dots + {}^{2n+1}C_n) = 2^{2n+1}$

$$\Rightarrow\quad {}^{2n+1}C_1 + {}^{2n+1}C_2 + \dots + {}^{2n+1}C_n = (2^{2n} - 1) \quad \dots \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$2^{2n} - 1 = 63$$

$\Rightarrow\quad 2^{2n} = 64$

$\Rightarrow\quad 2n = 6$

$\Rightarrow\quad n = 3$

21. ***Case* I** When one black and two others balls are drawn.

$\Rightarrow$ Number of ways $= {}^3C_1 \cdot {}^6C_2 = 45$

***Case* II** When two black and one other balls are drawn

$\Rightarrow$ Number of ways $= {}^3C_2 \cdot {}^6C_1 = 18$

***Case* III** When all three black balls are drawn

$\Rightarrow$ Number of ways $= {}^3C_3 = 1$

$\therefore$ Total number of ways $= 45 + 18 + 1 = 64$

22. The possible cases are

***Case* I** A man invites 3 ladies and women invites 3 gentlemen.

Number of ways $= {}^4C_3 \cdot {}^4C_3 = 16$

***Case* II** A man invites (2 ladies, 1 gentleman) and women invites (2 gentlemen, 1 lady).

Number of ways $= ({}^4C_2 \cdot {}^3C_1) \cdot ({}^3C_1 \cdot {}^4C_2) = 324$

***Case* III** A man invites (1 lady, 2 gentlemen) and women invites (2 ladies, 1 gentleman).

Number of ways $= ({}^4C_1 \cdot {}^3C_2) \cdot ({}^3C_2 \cdot {}^4C_1) = 144$

***Case* IV** A man invites (3 gentlemen) and women invites (3 ladies).

Number of ways $= {}^3C_3 \cdot {}^3C_3 = 1$

$\therefore$ Total number of ways,

$$= 16 + 324 + 144 + 1 = 485$$

23. Since, m men and n women are to be seated in a row so that no two women sit together. This could be shown as

$$\times M_1 \times M_2 \times M_3 \times \dots \times M_m \times$$

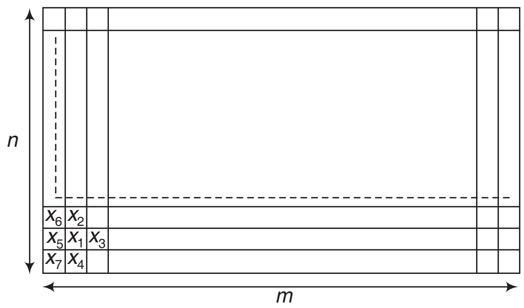
which shows there are $(m+1)$ places for n women.

$\therefore$ Number of ways in which they can be arranged

$$= (m)!\ {}^{m+1}P_n$$

$$= \frac{(m)! \cdot (m+1)!}{(m+1-n)!}$$

24. Let mn squares of equal size are arrange to form a rectangle of dimension m by n. Shown as, from figure.



neighbours of x_1 are $\{x_2, x_3, x_4, x_5\}$ x_5 are $\{x_1, x_6, x_7\}$ and x_7 are $\{x_5, x_4\}$.

$$\Rightarrow\quad x_1 = \frac{x_2 + x_3 + x_4 + x_5}{4}, \quad x_5 = \frac{x_1 + x_6 + x_7}{3}$$

and $\quad x_7 = \dfrac{x_4 + x_5}{2}$

$$\therefore\quad 4x_1 = x_2 + x_3 + x_4 + \frac{x_1 + x_6 + x_7}{3}$$

$$\Rightarrow\quad 12x_1 = 3x_2 + 3x_3 + 3x_4 + x_1 + x_6 + \frac{x_4 + x_5}{2}$$

$$\Rightarrow\quad 24x_1 = 6x_2 + 6x_3 + 6x_4 + 2x_1 + 2x_6 + x_4 + x_5$$

$$\Rightarrow\quad 22x_1 = 6x_2 + 6x_3 + 7x_4 + x_5 + 2x_6$$

where, $x_1, x_2, x_3, x_4, x_5, x_6$ are all the natural numbers and x_1 is linearly expressed as the sum of x_2, x_3, x_4, x_5, x_6 where sum of coefficients are equal only if, all observations are same.

$\Rightarrow \quad x_2 = x_3 = x_4 = x_5 = x_6$

$\Rightarrow$ All the numbers used are equal.

25. We know that, $\dfrac{{}^nC_r}{{}^nC_{r-1}} = \dfrac{n-r+1}{r}$

$\Rightarrow \quad \dfrac{84}{36} = \dfrac{7}{3} = \dfrac{n-r+1}{r}$ [given]

$\Rightarrow \quad 3n - 10r + 3 = 0$ ...(i)

Also given, $\dfrac{{}^nC_r}{{}^nC_{r+1}} = \dfrac{84}{126}$

$\Rightarrow \quad \dfrac{r+1}{n-r} = \dfrac{2}{3}$

$\Rightarrow \quad 2n - 5r - 3 = 0$ ...(ii)

On solving Eqs. (i) and (ii), we get

$$r = 3 \quad \text{and} \quad n = 9$$

Topic 3 Multinomial, Repeated Arrangement and Selection

1. **Key Idea** Use divisibility test of 11 and consider different situation according to given condition.

Since, the sum of given digits

$$0 + 1 + 2 + 5 + 7 + 9 = 24$$

Let the six-digit number be *abcdef* and to be divisible by 11, so the difference of sum of odd placed digits and sum of even placed digits should be either 0 or a multiple of 11 means $|(a + c + e) - (b + d + f)|$ should be either 0 or a multiple of 11.

Hence, possible case is $a + c + e = 12 = b + d + f$ (only)

Now, ***Case* I**

set $\{a, c, e\} = \{0, 5, 7\}$ and set $\{b, d, f\} = \{1, 2, 9\}$

So, number of 6-digits numbers $= (2 \times 2!) \times (3!) = 24$

[$\because a$ can be selected in ways only either 5 or 7].

***Case* II**

Set $\{a, c, e\} = \{1, 2, 9\}$ and set $\{b, d, f\} = \{0, 5, 7\}$

So, number of 6-digits numbers $= 3! \times 3! = 36$

So, total number of 6-digits numbers $= 24 + 36 = 60$

2. Since there are 8 males and 5 females. Out of these 13 members committee of 11 members is to be formed.

According to the question, m = number of ways when there is at least 6 males

$= ({}^8C_6 \times {}^5C_5) + ({}^8C_7 \times {}^5C_4) + ({}^8C_8 \times {}^5C_3)$

$= (28 \times 1) + (8 \times 5) + (1 \times 10)$

$= 28 + 40 + 10 = 78$

and n = number of ways when there is at least 3 females

$= ({}^5C_3 \times {}^8C_8) + ({}^5C_4 \times {}^8C_7) + ({}^5C_5 \times {}^8C_6)$

$= 10 \times 1 + 5 \times 8 + 1 \times 28 = 78$

So, $m = n = 78$

3. Given there are three boxes, each containing 10 balls labelled 1, 2, 3, ... , 10.

Now, one ball is randomly drawn from each boxes, and n_i denote the label of the ball drawn from the ith box, $(i = 1, 2, 3)$.

Then, the number of ways in which the balls can be chosen such that $n_1 < n_2 < n_3$ is same as selection of 3 different numbers from numbers $\{1, 2, 3, \ldots, 10\} = {}^{10}C_3 = 120$.

4. Using the digits 0, 1, 3, 7, 9

number of one digit natural numbers that can be formed $= 4$,

number of two digit natural numbers that can be formed $= 20$,

4×5

($\because$ 0 can not come in Ist box)

number of three digit natural numbers that can be formed $= 100$

$4 \times 5 \times 5$

and number of four digit natural numbers less than 7000, that can be formed $= 250$

$2 \times 5 \times 5 \times 5$

($\because$ only 1 or 3 can come in Ist box)

$\therefore$ Total number of natural numbers formed

$$= 4 + 20 + 100 + 250 = 374$$

5. Number of girls in the class = 5 and number of boys in the class = 7

Now, total ways of forming a team of 3 boys and 2 girls

$$= {}^7C_3 \cdot {}^5C_2 = 350$$

But, if two specific boys are in team, then number of ways $= {}^5C_1 \cdot {}^5C_2 = 50$

Required ways, i.e. the ways in which two specific boys are not in the same team $= 350 - 50 = 300$.

Alternate Method

Number of ways when A is selected and B is not

$$= {}^5C_2 \cdot {}^5C_2 = 100$$

Number of ways when B is selected and A is not

$$= {}^5C_2 \cdot {}^5C_2 = 100$$

Number of ways when both A and B are not selected

$$= {}^5C_3 \cdot {}^5C_2 = 100$$

$\therefore$ Required ways $= 100 + 100 + 100 = 300$.

6. Clearly, number of words start with A $= \frac{4!}{2!} = 12$

Number of words start with $L = 4! = 24$

Number of words start with $M = \frac{4!}{2!} = 12$

Number of words start with $SA = \frac{3!}{2!} = 3$

Number of words start with $SL = 3! = 6$

Note that, next word will be "SMALL".

Hence, the position of word "SMALL" is 58th.

7. Arrange the letters of the word COCHIN as in the order of dictionary CCHINO.

Consider the words starting from C.

There are 5! such words. Number of words with the two C's occupying first and second place $= 4!$.

Number of words starting with CH, CI, CN is 4! each.

Similarly, number of words before the first word starting with CO = 4! + 4! + 4! + 4! = 96.

The word starting with CO found first in the dictionary is COCHIN. There are 96 words before COCHIN.

8. A number is divisible by 4 if last 2 digit number is divisible by 4.

$\therefore$ Last two digit number divisible by 4 from (1, 2, 3, 4, 5) are 12, 24, 32, 44, 52

$\therefore$ The number of 5 digit number which are divisible by 4, from the digit (1, 2, 3, 4, 5) and digit is repeated is

$$5 \times 5 \times 5 \times (5 \times 1) = 625$$

9. $x = 10!$

$y = {}^{10}C_1 \times {}^9C_8 \times \frac{10!}{2!} = 10 \times 9 \times \frac{10!}{2} \Rightarrow \frac{y}{9x} = \frac{10}{2} = 5$

10. Here, __ B_1 __ B_2 __ B_3 __ B_4 __ B_5 __

Out of 5 girls, 4 girls are together and 1 girl is separate. Now, to select 2 positions out of 6 positions between boys $= {}^6C_2$ ...(i)

4 girls are to be selected out of $5 = {}^5C_4$ ...(ii)

Now, 2 groups of girls can be arranged in $2!$ ways. ...(iii)

Also, the group of 4 girls and 5 boys is arranged in $4! \times 5!$ ways. ...(iv)

Now, total number of ways $= {}^6C_2 \times {}^5C_4 \times 2! \times 4! \times 5!$

[from Eqs. (i), (ii), (iii) and (iv)]

$\therefore \quad m = {}^6C_2 \times {}^5C_4 \times 2! \times 4! \times 5!$

and $\quad n = 5! \times 6!$

$\Rightarrow \quad \frac{m}{n} = \frac{{}^6C_2 \times {}^5C_4 \times 2! \times 4! \times 5!}{6! \times 5!} = \frac{15 \times 5 \times 2 \times 4!}{6 \times 5 \times 4!} = 5$

11. **PLAN** Reducing the equation to a newer equation, where sum of variables is less. Thus, finding the number of arrangements becomes easier.

As, $\quad n_1 \geq 1,\ n_2 \geq 2,\ n_3 \geq 3,\ n_4 \geq 4,\ n_5 \geq 5$

Let $\quad n_1 - 1 = x_1 \geq 0,\ n_2 - 2 = x_2 \geq 0, \ldots, n_5 - 5 = x_5 \geq 0$

$\Rightarrow$ New equation will be

$x_1 + 1 + x_2 + 2 + \ldots + x_5 + 5 = 20$

$\Rightarrow \quad x_1 + x_2 + x_3 + x_4 + x_5 = 20 - 15 = 5$

Now, $\quad x_1 \leq x_2 \leq x_3 \leq x_4 \leq x_5$

x_1	x_2	x_3	x_4	x_5
0	0	0	0	5
0	0	0	1	4
0	0	0	2	3
0	0	1	1	3
0	0	1	2	2
0	1	1	1	2
1	1	1	1	1

So, 7 possible cases will be there.

12. The number of solutions of $x_1 + x_2 + \ldots + x_k = n$

$=$ Coefficient of t^n in $(t + t^2 + t^3 + \ldots)(t^2 + t^3 + \ldots)\ldots(t^k + t^{k+1} + \ldots)$

$=$ Coefficient of t^n in $t^{1+2+\ldots+k}(1 + t + t^2 + \ldots)^k$

Now, $\quad 1 + 2 + \ldots + k = \frac{k(k+1)}{2} = p \quad$ [say]

and $\quad 1 + t + t^2 + \ldots = \frac{1}{1-t}$

Thus, the number of required solutions

$=$ Coefficient of t^{n-p} in $(1-t)^{-k}$

$=$ Coefficient of t^{n-p} in $[1 + {}^kC_1 t + {}^{k+1}C_2 t^2 + {}^{k+2}C_3 t^3 + \ldots]$

$= {}^{k+n-p-1}C_{n-p} = {}^rC_{n-p}$

where, $r = k + n - p - 1 = k + n - 1 - \frac{1}{2}k(k+1)$

$= \frac{1}{2}(2k + 2n - 2 + k^2 - k) = \frac{1}{2}(2n - k^2 + k - 2)$

13. Since, six '+' signs are + + + + + +

$\therefore$ 4 negative sign has seven places to be arranged in

$\Rightarrow \quad {}^7C_4$ ways $= 35$ ways

14. Since, each box can hold five balls.

$\therefore$ Number of ways in which balls could be distributed so that none is empty, are (2, 2, 1) or (3, 1, 1).

i.e. $\quad ({}^5C_2\ {}^3C_2\ {}^1C_1 + {}^5C_3\ {}^2C_1\ {}^1C_1) \times 3!$

$= (30 + 20) \times 6 = 300$

Topic 4 Distribution of Object into Group

1. It is given that a group of students comprises of 5 boys and n girls. The number of ways, in which a team of 3 students can be selected from this group such that each team consists of at least one boy and at least one girls, is = (number of ways selecting one boy and 2 girls) + (number of ways selecting two boys and 1 girl)

$= ({}^5C_1 \times {}^nC_2)\ ({}^5C_2 \times {}^nC_1) = 1750$ [given]

$\Rightarrow \left(5 \times \frac{n(n-1)}{2}\right) + \left(\frac{5 \times 4}{2} \times n\right) = 1750$

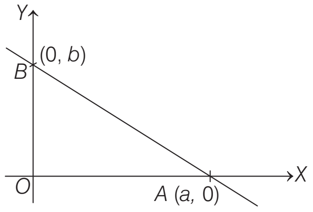
$\Rightarrow n(n-1) + 4n = \frac{2}{5} \times 1750 \Rightarrow n^2 + 3n = 2 \times 350$

$\Rightarrow n^2 + 3n - 700 = 0 \Rightarrow n^2 + 28n - 25n - 700 = 0$

$\Rightarrow n(n + 28) - 25(n + 28) = 0 \Rightarrow (n + 28)(n - 25) = 0$

$\Rightarrow n = 25 \quad [\because n \in N]$

2. According to given information, we have the following figure.



(Note that as a and b are integers so they can be negative also). Here $O(0,0)$, $A(a,0)$ and $B(0,b)$ are the three vertices of the triangle.

Clearly, $OA = |a|$ and $OB = |b|$.

$\therefore$ Area of $\Delta OAB = \frac{1}{2}|a||b|$.

But area of such triangles is given as 50 sq units.

$\therefore \quad \frac{1}{2}|a||b| = 50$

$\Rightarrow \quad |a||b| = 100 = 2^2 \cdot 5^2$

Number of ways of distributing two 2's in $|a|$ and $|b| = 3$

$\|a\|$	$\|b\|$
0	2
1	1
2	0

$\Rightarrow$ 3 ways

Similarly, number of ways of distributing two 5's in $|a|$ and $|b| = 3$ ways.

$\therefore$ Total number of ways of distributing 2's and 5's $= 3 \times 3 = 9$ ways

Note that for one value of $|a|$, there are 2 possible values of a and for one value of $|b|$, there are 2 possible values of b.

$\therefore$ Number of such triangles possible $= 2 \times 2 \times 9 = 36$.

So, number of elements in S is 36.

3. Given 6 different novels and 3 different dictionaries.

Number of ways of selecting 4 novels from 6 novels is

$$^6C_4 = \frac{6!}{2!4!} = 15$$

Number of ways of selecting 1 dictionary is from 3 dictionaries is $^3C_1 = \frac{3!}{1!2!} = 3$

$\therefore$ Total number of arrangement of 4 novels and 1 dictionary where dictionary is always in the middle, is

$$15 \times 3 \times 4! = 45 \times 24 = 1080$$

4.

Objects	Groups	Objects	Groups
Distinct	Distinct	Identical	Identical
Distinct	Identical	Identical	Distinct

Description of Situation Here, 5 distinct balls are distributed amongst 3 persons so that each gets at least one ball. i.e. Distinct $\rightarrow$ Distinct

So, we should make cases

***Case* I** $\left.\begin{matrix} A & B & C \\ 1 & 1 & 2 \end{matrix}\right\}$ ***Case* II** $\left.\begin{matrix} A & B & C \\ 1 & 2 & 2 \end{matrix}\right\}$

Number of ways to distribute 5 balls

$$= \left({}^5C_1 \cdot {}^4C_1 \cdot {}^3C_3 \times \frac{3!}{2!}\right) + \left({}^5C_1 \cdot {}^4C_2 \cdot {}^2C_2 \times \frac{3!}{2!}\right)$$

$$= 60 + 90 = 150$$

5. Total number of arrangements of word BANANA

$$= \frac{6!}{3!2!} = 60$$

The number of arrangements of words BANANA in which two N's appear adjacently $= \frac{5!}{3!} = 20$

Required number of arrangements $= 60 - 20 = 40$

6. Here, n^2 objects are distributed in n groups, each group containing n identical objects.

$\therefore$ Number of arrangements

$$= {}^{n^2}C_n \cdot {}^{n^2-n}C_n \cdot {}^{n^2-2n}C_n \cdot {}^{n^2-3n}C_n \cdot {}^{n^2-2n}C_n \dots {}^nC_n$$

$$= \frac{(n^2)!}{n!(n^2-n)!} \cdot \frac{(n^2-n)!}{n!(n^2-2n)!} \cdots \frac{n!}{n! \cdot 1} = \frac{(n^2)!}{(n!)^n}$$

$\Rightarrow$ Integer (as number of arrangements has to be integer).

7. (i) The number of ways in which 52 cards be divided equally among four players in order

$$= {}^{52}C_{13} \times {}^{39}C_{13} \times {}^{26}C_{13} \times {}^{13}C_{13} = \frac{(52)!}{(13!)^4}$$

(ii) The number of ways in which a pack of 52 cards can be divided equally into four groups of 13 cards each $= \frac{{}^{52}C_{13} \times {}^{39}C_{13} \times {}^{26}C_{13} \times {}^{13}C_{13}}{4!} = \frac{(52)!}{4!(13!)^4}$

(iii) The number of ways in which a pack of 52 cards be divided into 4 sets, three of them having 17 cards each and the fourth just one card

$$= \frac{{}^{52}C_{17} \times {}^{35}C_{18} \times {}^{18}C_{17} \times {}^1C_1}{3!} = \frac{(52)!}{3!(17)^3}$$

Topic 5 Dearrangement and Number of Divisors

1. Since, $240 = 2^4 \cdot 3 \cdot 5$

$\therefore$ Total number of divisors $= (4+1)(2)(2) = 20$

Out of these 2, 6, 10, and 30 are of the form $4n + 2$.

2. The number of ways in which the ball does not go its own colour box $= 4!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right)$

$$= 4!\left(\frac{1}{2} - \frac{1}{6} + \frac{1}{24}\right) = 24\left(\frac{12-4+1}{24}\right) = 9$$