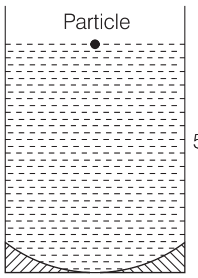
12

Optics

Topic 1 Reflection of Light

Objective Questions I (Only one correct option)

1. A concave mirror has radius of curvature of 40 cm. It is at the bottom of a glass that has water filled up to 5 cm (see figure). If a small particle is floating on the surface of water, its image as seen, from directly above the glass, is at a distance d from the surface of water. The value of d is close to



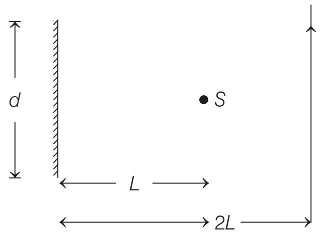
[Refractive index of water = 1.33] **(2019 Main, 12 April I)**

(a) 6.7 cm (b) 13.4 cm (c) 8.8 cm (d) 11.7 cm

2. A concave mirror for face viewing has focal length of 0.4 m. The distance at which you hold the mirror from your face in order to see your image upright with a magnification of 5 is **(2019 Main, 9 April I)**

(a) 0.16 m (b) 1.60 m (c) 0.32 m (d) 0.24 m

3. A point source of light, S is placed at a distance L in front of the centre of plane mirror of width d which is hanging vertically on a wall. A man walks in front of the mirror along a line parallel to the mirror, at a distance $2L$ as shown below. The distance over which the man can see the image of the light source in the mirror is **(2019 Main, 12 Jan I)**



(a) $\frac{d}{2}$ (b) d (c) $3d$ (d) $2d$

4. The plane mirrors (M_1 and M_2) are inclined to each other such that a ray of light incident on mirror M_1 and parallel to the mirror M_2 is reflected from mirror M_2 parallel to the mirror M_1. The angle between the two mirror is **(2019 Main, 9 Jan II)**

(a) 45° (b) 75° (c) 90° (d) 60°

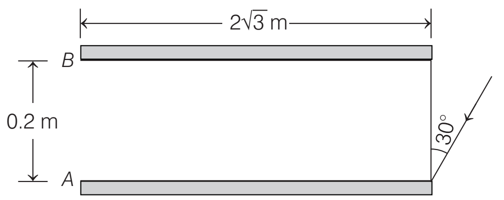
5. A ray of light travelling in the direction $\frac{1}{2}(\hat{\mathbf{i}} + \sqrt{3}\hat{\mathbf{j}})$ is incident on a plane mirror. After reflection, it travels along the direction $\frac{1}{2}(\hat{\mathbf{i}} - \sqrt{3}\hat{\mathbf{j}})$. The angle of incidence is

(a) 30° (b) 45° **(2013 Adv.)**
(c) 60° (d) 75°

6. In an experiment to determine the focal length (f) of a concave mirror by the u-v method, a student places the object pin A on the principal axis at a distance x from the pole P. The student looks at the pin and its inverted image from a distance keeping his/her eye in line with PA. When the student shifts his/her eye towards left, the image appears to the right of the object pin. Then **(2007, 3M)**

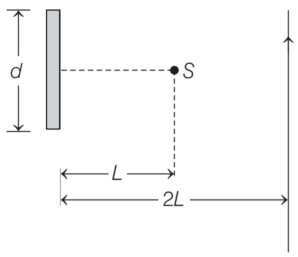
(a) $x < f$ (b) $f < x < 2f$ (c) $x = 2f$ (d) $x > 2f$

7. Two plane mirrors A and B are aligned parallel to each other, as shown in the figure. A light ray is incident at an angle 30° at a point just inside one end of A. The plane of incidence coincides with the plane of the figure. The maximum number of times the ray undergoes reflections (including the first one) before it emerges out is **(2002, 2M)**



(a) 28 (b) 30
(c) 32 (d) 34

8. A point source of light S, placed at a distance L in front of the centre of a plane mirror of width d, hangs vertically on a wall. A man walks in front of the mirror along a line parallel to the mirror at a distance $2L$ from it as shown. The greatest distance over which he can see the image of the light source in the mirror is **(2000, 2M)**



(a) $d/2$ (b) d (c) $2d$ (d) $3d$

9. A short linear object of length b lies along the axis of a concave mirror of focal length f at a distance u from the pole of the mirror. The size of the image is approximately equal to

(a) $b\left(\frac{u-f}{f}\right)^{1/2}$ (b) $b\left(\frac{f}{u-f}\right)^{1/2}$

(c) $b\left(\frac{u-f}{f}\right)$ (d) $b\left(\frac{f}{u-f}\right)^{2}$

Assertion and Reason

Mark your answer as

(a) If Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I.

(b) If Statement I is true, Statement II is true; Statement II is not a correct explanation for Statement I.

(c) If Statement I is true; Statement II is false.

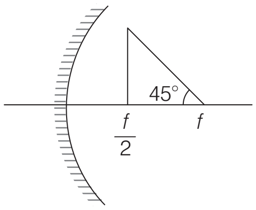
(d) If Statement I is false; Statement II is true.

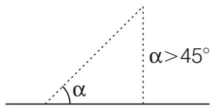
10. **Statement I** The formula connecting u, v and f for a spherical mirror is valid only for mirrors whose sizes are very small compared to their radii of curvature. **(2007, 3M)**

Statement II Laws of reflection are strictly valid for plane surfaces, but not for large spherical surfaces.

Objective Question II (One or more correct option)

11. A wire is bent in the shape of a right angled triangle and is placed in front of a concave mirror of focal length f as shown in the figure. Which of the figures shown in the four options qualitatively represent(s) the shape of the image of the bent wire? (These figures are not to scale.) **(2018 Adv.)**



(a)


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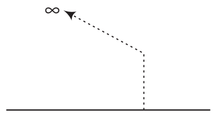

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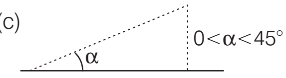
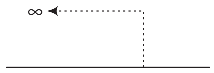


(d)


12. A student performed the experiment of determination of focal length of a concave mirror by u-v method using an optical bench of length 1.5 m. The focal length of the mirror used is 24 cm. The maximum error in the location of the image can be 0.2 cm. The 5 sets of (u, v) values recorded by the student (in cm) are : (42, 56), (48, 48), (60, 40), (66, 33), (78, 39). The data set(s) that cannot come from experiment and is (are) incorrectly recorded, is (are) **(2009)**

(a) (42, 56)
(b) (48, 48)
(c) (66, 33)
(d) (78, 39)

Fill in the Blank

13. A thin rod of length $f/3$ is placed along the optic axis of a concave mirror of focal length f such that its image which is real and elongated, just touches the rod. The magnification is …… **(1991, 1M)**

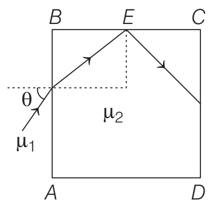
Integer Answer Type Question

14. Image of an object approaching a convex mirror of radius of curvature 20 m along its optical axis is observed to move from $\frac{25}{3}$ m to $\frac{50}{7}$ m in 30 s. What is the speed of the object in km h^{-1}? **(2010)**

Topic 2 Refraction of Light and TIR

Objective Questions I (Only one correct option)

1. A transparent cube of side d, made of a material of refractive index μ_2, is immersed in a liquid of refractive index $\mu_1 (\mu_1 < \mu_2)$. A ray is incident on the face AB at an angle θ (shown in the figure). Total internal reflection takes place at point E on the face BC.

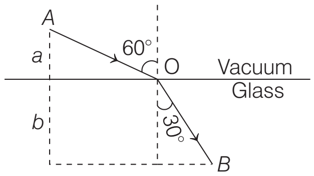


Then, θ must satisfy **(2019 Main, 12 April II)**

(a) $\theta < \sin^{-1}\frac{\mu_1}{\mu_2}$ (b) $\theta > \sin^{-1}\sqrt{\frac{\mu_2^2}{\mu_1^2}-1}$

(c) $\theta < \sin^{-1}\sqrt{\frac{\mu_2^2}{\mu_1^2}-1}$ (d) $\theta > \sin^{-1}\frac{\mu_1}{\mu_2}$

2. A ray of light AO in vacuum is incident on a glass slab at angle 60° and refracted at angle 30° along OB as shown in the figure. The optical path length of light ray from A to B is **(2019 Main, 11 April I)**



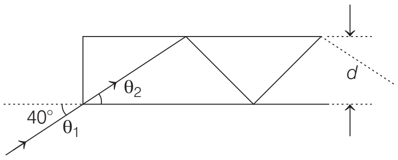
(a) $\frac{2\sqrt{3}}{a} + 2b$

(b) $2a + \frac{2b}{3}$

(c) $2a + 2b$

(d) $2a + \frac{2b}{\sqrt{3}}$

3. In figure, the optical fibre is $l = 2$ m long and has a diameter of $d = 20\,\mu$m. If a ray of light is incident on one end of the fibre at angle $\theta_1 = 40°$, the number of reflections it makes before emerging from the other end is close to (refractive index of fibre is 1.31 and $\sin 40° = 0.64$) **(Main 2019, 8 April I)**

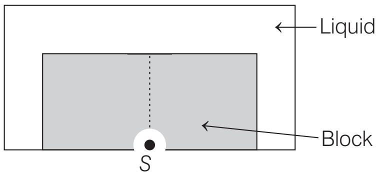


(a) 55000 (b) 66000 (c) 45000 (d) 57000

4. A green light is incident from the water to the air-water interface at the critical angle (θ). Select the correct statement. **(2014 Main)**

(a) The entire spectrum of visible light will come out of the water at an angle of 90° to the normal
(b) The spectrum of visible light whose frequency is less than that of green light will come out of the air medium
(c) The spectrum of visible light whose frequency is more than that of green light will come out to the air medium
(d) The entire spectrum of visible light will come out of the water at various angles to the normal

5. A point source S is placed at the bottom of a transparent block of height 10 mm and refractive index 2.72. It is immersed in a lower refractive index liquid as shown in the figure. It is found that the light emerging from the block to the liquid forms a circular bright spot of diameter 11.54 mm on the top of the block. The refractive index of the liquid is **(2014 Adv.)**



(a) 1.21 (b) 1.30 (c) 1.36 (d) 1.42

6. A ball is dropped from a height of 20 m above the surface of water in a lake. The refractive index of water is $\frac{4}{3}$. A fish inside the lake, in the line of fall of the ball, is looking at the ball. At an instant, when the ball is 12.8 m above the water surface, the fish sees the speed of ball as **(2009)**

(a) 9 ms^{-1} (b) 12 ms^{-1} (c) 16 ms^{-1} (d) 21.33 ms^{-1}

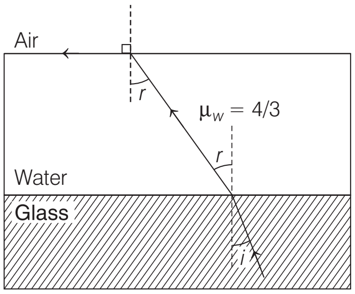
7. A ray of light travelling in water is incident on its surface open to air. The angle of incidence is θ, which is less than the critical angle. Then there will be **(2007, 3M)**

(a) only a reflected ray and no refracted ray
(b) only a refracted ray and no reflected ray
(c) a reflected ray and a refracted ray and the angle between them would be less than $180° - 2\theta$
(d) a reflected ray and a refracted ray and the angle between them would be greater than $180° - 2\theta$

8. A point object is placed at the centre of a glass sphere of radius 6 cm and refractive index 1.5. The distance of the virtual image from the surface of the sphere is **(2004, 2M)**

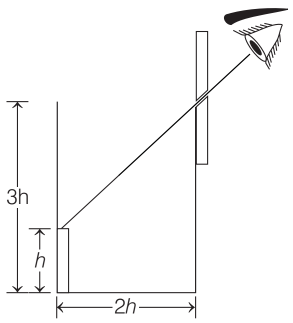
(a) 2 cm (b) 4 cm
(c) 6 cm (d) 12 cm

9. A ray of light is incident at the glass-water interface at an angle i, it emerges finally parallel to the surface of water, then the value of μ_g would be **(2003, 2M)**



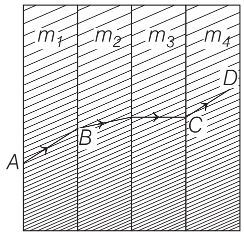
(a) $(4/3)\sin i$ (b) $1/(\sin i)$ (c) 4/3 (d) 1

10. An observer can see through a pin-hole the top end of a thin rod of height h, placed as shown in the figure. The beaker height is $3h$ and its radius h. When the beaker is filled with a liquid up to a height $2h$, he can see the lower end of the rod. Then the refractive index of the liquid is **(2002, 2M)**



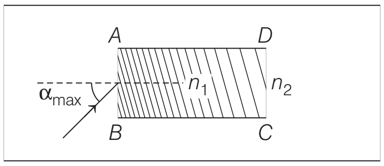
(a) $\frac{5}{2}$ (b) $\sqrt{\frac{5}{2}}$ (c) $\sqrt{\frac{3}{2}}$ (d) $\frac{3}{2}$

11. A ray of light passes through four transparent media with refractive indices μ_1, μ_2, μ_3 and μ_4 as shown in the figure. The surfaces of all media are parallel. If the emergent ray CD is parallel to the incident ray AB, we must have **(2001, 2M)**



(a) $\mu_1 = \mu_2$ (b) $\mu_2 = \mu_3$ (c) $\mu_3 = \mu_4$ (d) $\mu_4 = \mu_1$

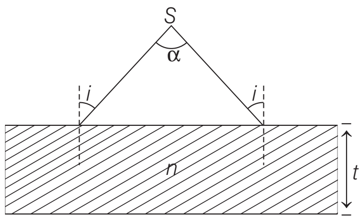
12. A rectangular glass slab $ABCD$ of refractive index n_1 is immersed in water of refractive index $n_2 (n_1 > n_2)$. A ray of light is incident at the surface AB of the slab as shown. The maximum value of the angle of incidence α_{max}, such that the ray comes out only from the other surface CD, is given by **(2000, 2M)**



(a) $\sin^{-1}\left[\frac{n_1}{n_2}\cos\left(\sin^{-1}\frac{n_2}{n_1}\right)\right]$ (b) $\sin^{-1}\left[n_1\cos\left(\sin^{-1}\frac{1}{n_2}\right)\right]$

(c) $\sin^{-1}\left(\frac{n_1}{n_2}\right)$ (d) $\sin^{-1}\left(\frac{n_2}{n_1}\right)$

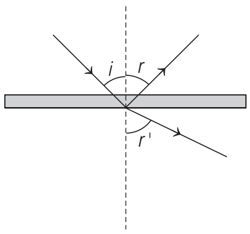
13. A diverging beam of light from a point source S having divergence angle α falls symmetrically on a glass slab as shown. The angles of incidence of the two extreme rays are equal. If the thickness of the glass slab is t and its refractive index is n, then the divergence angle of the emergent beam is



(a) zero (b) α **(2000,2M)**
(c) $\sin^{-1}(1/n)$ (d) $2\sin^{-1}(1/n)$

14. A spherical surface of radius of curvature R, separates air (refractive index 1.0) from glass (refractive index 1.5). The centre of curvature is in the glass. A point object P placed in air is found to have a real image Q in the glass. The line PQ cuts the surface at a point O and $PO = OQ$. The distance PO is equal to **(1998, 2M)**
(a) $5R$ (b) $3R$ (c) $2R$ (d) $1.5R$

15. A ray of light from a denser medium strikes a rarer medium at an angle of incidence i (see figure). The reflected and refracted rays make an angle of 90° with each other. The angles of reflection and refraction are r and r'. The critical angle is **(1983, 1M)**

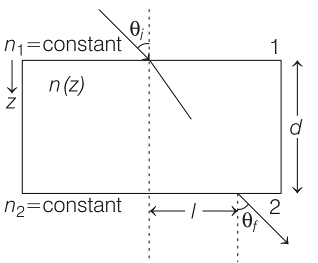


(a) $\sin^{-1}(\tan r)$ (b) $\sin^{-1}(\cot i)$
(c) $\sin^{-1}(\tan r')$ (d) $\tan^{-1}(\sin i)$

16. When a ray of light enters a glass slab from air **(1980, 1M)**
(a) its wavelength decreases
(b) its wavelength increases
(c) its frequency increases
(d) neither its wavelength nor its frequency changes

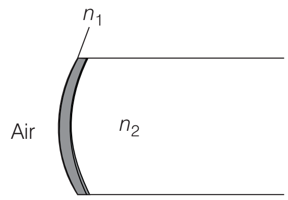
Objective Questions II (One or more correct option)

17. A transparent slab of thickness d has a refractive index $n(z)$ that increases with z. Here, z is the vertical distance inside the slab, measured from the top. The slab is placed between two media with uniform refractive indices n_1 and $n_2 (> n_1)$, as shown in the figure. A ray of light is incident with angle θ_i from medium 1 and emerges in medium 2 with refraction angle θ_f with a lateral displacement l. **(2016 Main)**



Which of the following statement(s) is (are) true?
(a) l is independent on $n(z)$
(b) $n_1 \sin\theta_i = (n_2 - n_1)\sin\theta_f$
(c) $n_1 \sin\theta_i = n_2 \sin\theta_f$
(d) l is independent of n_2

18. A transparent thin film of uniform thickness and refractive index $n_1 = 1.4$ is coated on the convex spherical surface of radius R at one end of a long solid glass cylinder of refractive index $n_2 = 1.5$, as shown in the figure. Rays of light parallel to the axis of the cylinder traversing through the film from air to glass get focused at distance f_1 from the film, while rays of light traversing from glass to air get focused at distance f_2 from the film. Then **(2014 Adv.)**

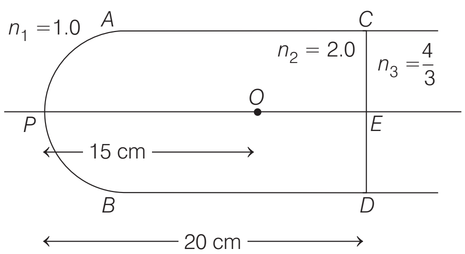


(a) $|f_1| = 3R$ (b) $|f_1| = 2.8R$
(c) $|f_2| = 2R$ (d) $|f_2| = 1.4R$

19. A ray of light travelling in a transparent medium falls on a surface separating the medium from air at an angle of incidence 45°. The ray undergoes total internal reflection. If n is the refractive index of the medium with respect to air, select the possible value (s) of n from the following **(1998, 2M)**
(a) 1.3 (b) 1.4
(c) 1.5 (d) 1.6

Fill in the Blanks

20. A slab of material of refractive index 2 shown in figure has a curved surface APB of radius of curvature 10 cm and a plane surface CD. On the left of APB is air and on the right of CD is water with refractive indices as given in the figure. An object O is placed at a distance of 15 cm from the pole P as shown. The distance of the final image of O from P, as viewed from the left is ...... **(1991, 2M)**



21. A monochromatic beam of light of wavelength 6000Å in vacuum enters a medium of refractive index 1.5. In the medium its wavelength is ......, and its frequency is ...... **(1985, 2M)**

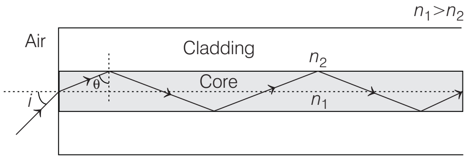
22. A light wave of frequency 5×10^{14} Hz enters a medium of refractive index 1.5. In the medium the velocity of the light wave is ...... and its wavelength is ...... **(1983, 2M)**

Passage Based Questions

Passage 1

Light guidance in an optical fibre can be understood by considering a structure comprising of thin solid glass cylinder of refractive index n_1 surrounded by a medium of lower refractive index n_2. The light guidance in the structure takes place due to successive total internal reflections at the interface of the media n_1 and n_2 as shown in the figure. All rays with the angle of incidence i less than a particular value i_m are confined in the medium of refractive index n_1. The numerical aperture (NA) of the structure is defined as $\sin i_m$.

23. For two structures namely S_1 with $n_1 = \frac{\sqrt{45}}{4}$ and $n_2 = \frac{3}{2}$, and S_2 with $n_1 = \frac{8}{5}$ and $n_2 = \frac{7}{5}$ and taking the refractive index of water to be $\frac{4}{3}$ and that to air to be 1, the correct options is/are **(2015 Adv.)**



(a) NA of S_1 immersed in water is the same as that of S_2 immersed in a liquid of refractive index $\frac{16}{3\sqrt{15}}$

(b) NA of S_1 immersed in liquid of refractive index $\frac{6}{\sqrt{15}}$ is the same as that of S_2 immersed in water

(c) NA of S_1 placed in air is the same as that S_2 immersed in liquid of refractive index $\frac{4}{\sqrt{15}}$

(d) NA of S_1 placed in air is the same as that of S_2 placed in water

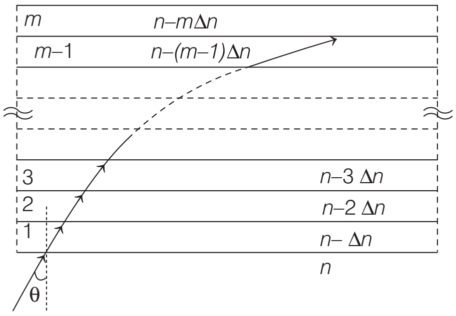
24. If two structures of same cross-sectional area, but different numerical apertures NA_1 and $NA_2 (NA_2 < NA_1)$ are joined longitudinally, the numerical aperture of the combined structure is **(2015 Adv.)**

(a) $\frac{NA_1 NA_2}{NA_1 + NA_2}$ (b) $NA_1 + NA_2$

(c) NA_1 (d) NA_2

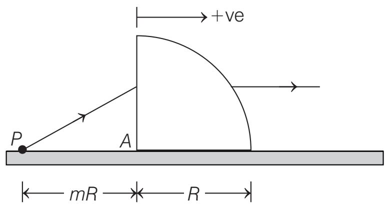
Integer Answer Type Question

25. A monochromatic light is travelling in a medium of refractive index $n = 1.6$. It enters a stack of glass layers from the bottom side at an angle $\theta = 30°$. The interfaces of the glass layers are parallel to each other. The refractive indices of different glass layers are monotonically decreasing as $n_m = n - m\Delta n$, where n_m is the refractive index of the mth slab and $\Delta n = 0.1$ (see the figure). The ray is refracted out parallel to the interface between the $(m-1)$th and mth slabs from the right side of the stack. What is the value of m? **(2017 Adv.)**



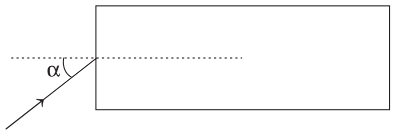
Analytical & Descriptive Questions

26. A quarter cylinder of radius R and refractive index 1.5 is placed on a table. A point object P is kept at a distance of mR from it. Find the value of m for which a ray from P will emerge parallel to the table as shown in figure. **(1999, 5M)**



27. The x-y plane is the boundary between two transparent media. Medium-1 with $z \geq 0$ has a refractive index $\sqrt{2}$ and medium-2 with $z \leq 0$ has a refractive index $\sqrt{3}$. A ray of light in medium-1 given by vector $\mathbf{A} = 6\sqrt{3}\hat{\mathbf{i}} + 8\sqrt{3}\hat{\mathbf{j}} - 10\hat{\mathbf{k}}$ is incident on the plane of separation. Find the unit vector in the direction of the refracted ray in medium-2. **(1999, 10 M)**

28. Light is incident at an angle α on one planar end of a transparent cylindrical rod of refractive index n. Determine the least value of n so that the light entering the rod does not emerge from the curved surface of the rod irrespective of the value of α. **(1992,8M)**



29. A parallel beam of light travelling in water (refractive index $= 4/3$) is refracted by a spherical air bubble of radius 2 mm situated in water. Assuming the light rays to be paraxial. **(1988, 6M)**

(a) Find the position of the image due to refraction at the first surface and the position of the final image.

(b) Draw a ray diagram showing the positions of both the images.

30. A right angled prism is to be made by selecting a proper material and the angles A and B $(B \leq A)$, as shown in figure. It is desired that a ray of light incident on the face AB emerges parallel to the incident direction after two internal reflections. **(1987, 7M)**

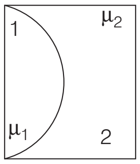
(a) What should be the minimum refractive index n for this to be possible ?

(b) For $n = 5/3$ is it possible to achieve this with the angle B equal to 30 degrees ?

Topic 3 Lens Theory

Objective Questions I (Only one correct option)

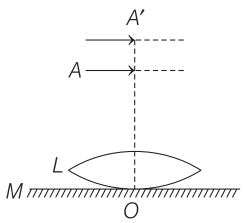
1. One plano-convex and one plano-concave lens of same radius of curvature R but of different materials are joined side by side as shown in the figure. If the refractive index of the material of 1 is μ_1 and that of 2 is μ_2, then the focal length of the combination is **(2019 Main, 10 April I)**



(a) $\dfrac{2R}{\mu_1 - \mu_2}$ (b) $\dfrac{R}{2-(\mu_1 - \mu_2)}$

(c) $\dfrac{R}{2(\mu_1 - \mu_2)}$ (d) $\dfrac{R}{\mu_1 - \mu_2}$

2. A thin convex lens L (refractive index = 1.5) is placed on a plane mirror M. When a pin is placed at A, such that $OA = 18$ cm, its real inverted image is formed at A itself, as shown in figure. When a liquid of refractive index μ_l is put between the lens and the mirror, the pin has to be moved to A', such that $OA' = 27$ cm, to get its inverted real image at A' itself. The value of μ_l will be **(2019 Main, 9 April II)**



(a) $\sqrt{3}$ (b) $\sqrt{2}$

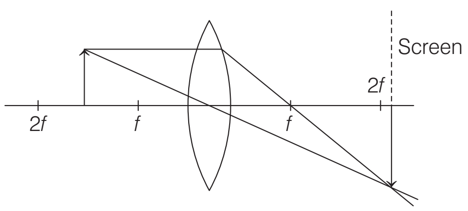
(c) $\dfrac{4}{3}$ (d) $\dfrac{3}{2}$

3. A convex lens of focal length 20 cm produces images of the same magnification 2 when an object is kept at two distances x_1 and x_2 $(x_1 > x_2)$ from the lens. The ratio of x_1 and x_2 is **(2019 Main, 9 April II)**

(a) 5 : 3 (b) 2 : 1

(c) 4 : 3 (d) 3 : 1

4. Formation of real image using a biconvex lens is shown below. If the whole set up is immersed in water without disturbing the object and the screen positions, what will one observe on the screen? **(2019 Main, 12 Jan II)**



(a) No Change (b) Magnified image

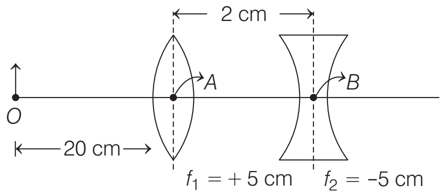
(c) Image disappears (d) Erect real image

5. A plano-convex lens (focal length f_2, refractive index μ_2, radius of curvature R) fits exactly into a plano-concave lens (focal length f_1, refractive index μ_1, radius of curvature R). Their plane surfaces are parallel to each other. Then, the focal length of the combination will be **(2019 Main, 12 Jan II)**

(a) $f_1 - f_2$ (b) $\dfrac{R}{\mu_2 - \mu_1}$

(c) $f_1 + f_2$ (d) $\dfrac{2f_1 f_2}{f_1 + f_2}$

6. What is the position and nature of image formed by lens combination shown in figure? (where, f_1 and f_2 are focal lengths) **(2019 Main, 12 Jan I)**



(a) $\dfrac{20}{3}$ cm from point B at right, real

(b) 70 cm from point B at right, real

(c) 40 cm from point B at right, real

(d) 70 cm from point B at left, virtual

7. An object is at a distance of 20 m from a convex lens of focal length 0.3 m. The lens forms an image of the object. If the object moves away from the lens at a speed of 5 m/s, the speed and direction of the image will be **(2019 Main, 11 Jan I)**

(a) 3.22×10^{-3} m/s towards the lens

(b) 0.92×10^{-3} m/s away from the lens

(c) 2.26×10^{-3} m/s away from the lens

(d) 1.16×10^{-3} m/s towards the lens

8. A plano-convex lens of refractive index μ_1 and focal length f_1 is kept in contact with another plano-concave lens of refractive index μ_2 and focal length f_2. If the radius of curvature of their spherical faces is R each and $f_1 = 2f_2$, then μ_1 and μ_2 are related as **(2019 Main, 10 Jan I)**

(a) $3\mu_2 - 2\mu_1 = 1$ (b) $2\mu_2 - \mu_1 = 1$

(c) $2\mu_1 - \mu_2 = 1$ (d) $\mu_1 + \mu_2 = 3$

9. A convex lens is put 10 cm from a light source and it makes a sharp image on a screen, kept 10 cm from the lens. Now, a glass block (refractive index is 1.5) of 1.5 cm thickness is placed in contact with the light source.

To get the sharp image again, the screen is shifted by a distance d. Then, d is **(2019 Main, 9 Jan I)**

(a) 0

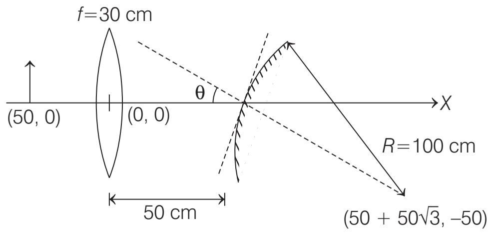
(b) 1.1 cm away from the lens

(c) 0.55 cm away from the lens

(d) 0.55 cm towards the lens

10. A diverging lens with magnitude of focal length 25 cm is placed at a distance of 15 cm from a converging lens of magnitude of focal length 20 cm. A beam of parallel light falls on the diverging lens. The final image formed is **(2017 Main)**
(a) virtual and at a distance of 40 cm from convergent lens
(b) real and at a distance of 40 cm from the divergent lens
(c) real and at a distance of 6 cm from the convergent lens
(d) real and at a distance of 40 cm from convergent lens

11. A small object is placed 50 cm to the left of a thin convex lens of focal length 30 cm. A convex spherical mirror of radius of curvature 100 cm is placed to the right of the lens at a distance of 50 cm. The mirror is tilted such that the axis of the mirror is at an angle $\theta = 30°$ to the axis of the lens, as shown in the figure. **(2016 Adv.)**

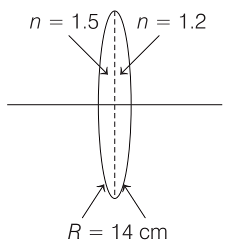


If the origin of the coordinate system is taken to be at the centre of the lens, the coordinates (in cm) of the point (x, y) at which the image is formed are
(a) $(125/3, 25/\sqrt{3})$ (b) $(50 - 25\sqrt{3}, 25)$
(c) $(0, 0)$ (d) $(25, 25\sqrt{3})$

12. A thin convex lens made from crown glass $(\mu = 3/2)$ has focal length f. When it is measured in two different liquids having refractive indices 4/3 and 5/3. It has the focal lengths f_1 and f_2, respectively. The correct relation between the focal length is **(2014 Main)**
(a) $f_1 = f_2 < f$
(b) $f_1 > f$ and f_2 becomes negative
(c) $f_2 > f$ and f_1 becomes negative
(d) f_1 and f_2 both become negative

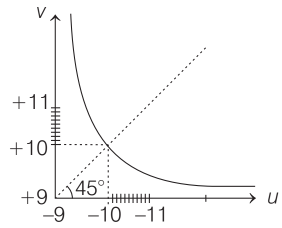
13. The image of an object, formed by a plano-convex lens at a distance of 8 m behind the lens, is real and is one-third the size of the object. The wavelength of light inside the lens is 2/3 times the wavelength in free space. The radius of the curved surface of the lens is **(2013 Adv.)**
(a) 1 m (b) 2 m (c) 3 m (d) 6 m

14. A bi-convex lens is formed with two thin plano-convex lenses as shown in the figure. Refractive index n of the first lens is 1.5 and that of the second lens is 1.2. Both the curved surfaces are of the same radius of curvature $R = 14$ cm.



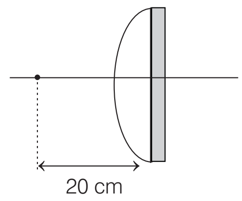
For this bi-convex lens, for an object distance of 40 cm, the image distance will be **(2012)**
(a) – 280.0 cm (b) 40.0 cm
(c) 21.5 cm (d) 13.3 cm

15. The graph between object distance u and image distance v for a lens is given below. The focal length of the lens is **(2006, 3M)**



(a) 5 ± 0.1 (b) 5 ± 0.05
(c) 0.5 ± 0.1 (d) 0.5 ± 0.05

16. A point object is placed at a distance of 20 cm from a thin plano-convex lens of focal length 15 cm. The plane surface of the lens is now silvered. The image created by the system is at **(2006, 3M)**

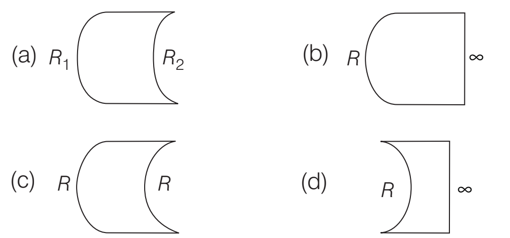


(a) 60 cm to the left of the system
(b) 60 cm to the right of the system
(c) 12 cm to the left of the system
(d) 12 cm to the right of the system

17. A convex lens is in contact with concave lens. The magnitude of the ratio of their focal length is $\frac{3}{2}$. Their equivalent focal length is 30 cm. What are their individual focal lengths ? **(2005, 2M)**
(a) –75, 50 (b) –10, 15 (c) 75, 50 (d) –15, 10

18. The size of the image of an object, which is at infinity, as formed by a convex lens of focal length 30 cm is 2 cm. If a concave lens of focal length 20 cm is placed between the convex lens and the image at a distance of 26 cm from the convex lens, calculate the new size of the image. **(2003, 2M)**
(a) 1.25 cm (b) 2.5 cm (c) 1.05 cm (d) 2 cm

19. Which one of the following spherical lenses does not exhibit dispersion? The radii of curvature of the surfaces of the lenses are as given in the diagrams **(2002, 2M)**



20. A hollow double concave lens is made of very thin transparent material. It can be filled with air or either of two liquids L_1 or L_2 having refracting indices n_1 and n_2 respectively $(n_2 > n_1 > 1)$. The lens will diverge a parallel beam of light if it is filled with **(2000, 2M)**
(a) air and placed in air
(b) air and immersed in L_1
(c) L_1 and immersed in L_2
(d) L_2 and immersed in L_1

21. A concave lens of glass, refractive index 1.5 has both surfaces of same radius of curvature R. On immersion in a medium of refractive index 1.75, it will behave as a **(1999, 2M)**
(a) convergent lens of focal length 3.5 R
(b) convergent lens of focal length 3.0 R
(c) divergent lens of focal length 3.5 R
(d) divergent lens of focal length 3.0 R

22. A real image of a distant object is formed by a plano-convex lens on its principal axis. Spherical aberration **(1998, 2M)**
(a) is absent
(b) is smaller if the curved surface of the lens faces the object
(c) is smaller if the plane surface of the lens faces the object
(d) is the same whichever side of the lens faces the object

23. An eye specialist prescribes spectacles having combination of convex lens of focal length 40 cm in contact with a concave lens of focal length 25 cm. The power of this lens combination in diopters is **(1997, 2M)**
(a) + 1.5 (b) – 1.5 (c) + 6.67 (d) – 6.67

24. Spherical aberration in a thin lens can be reduced by **(1994, 2M)**
(a) using a monochromatic light
(b) using a doublet combination
(c) using a circular annular mark over the lens
(d) increasing the size of the lens

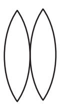
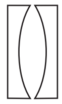
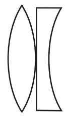
25. A convex lens of focal length 40 cm is in contact with a concave lens of focal length 25 cm. The power of the combination is **(1982, 3M)**
(a) – 1.5 D (b) – 6.5 D (c) + 6.5 D (d) + 6.67 D

Objective Question II (One or more correct option)

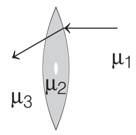
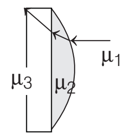
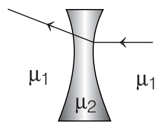
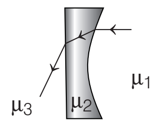
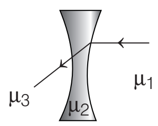
26. A plano-convex lens is made of material of refractive index n. When a small object is placed 30 cm away in front of the curved surface of the lens, an image of double the size of the object is produced. Due to reflection from the convex surface of the lens, another faint image is observed at a distance of 10 cm away from the lens. Which of the following statement(s) is (are) true? **(2016 Adv.)**
(a) The refractive index of the lens is 2.5
(b) The radius of curvature of the convex surface is 45 cm
(c) The faint image is erect and real
(d) The focal length of the lens is 20 cm

Match the Columns

27. Four combinations of two thin lenses are given in Column I. The radius of curvature of all curved surfaces is r and the refractive index of all the lenses is 1.5. Match lens combinations in Column I with their focal length in Column II and select the correct answer using the codes given below the lists. **(2014 Adv.)**

Column I		Column II	
A.		p.	$2r$
B.		q.	$r/2$
C.		r.	$-r$
D.		s.	r

28. Two transparent media of refractive indices μ_1 and μ_3 have a solid lens shaped transparent material of refractive index μ_2 between them as shown in figures in **Column II**. A ray traversing these media is also shown in the figures. In **Column I** different relationships between μ_1, μ_2 and μ_3 are given. Match them to the ray diagram shown in **Column II**. **(2010)**

	Column I		Column II
(A)	$\mu_1 < \mu_2$	(p)	
(B)	$\mu_1 > \mu_2$	(q)	
(C)	$\mu_2 = \mu_3$	(r)	
(D)	$\mu_2 > \mu_3$	(s)	
		(t)	

Fill in the Blanks

29. Two thin lenses, when in contact, produce a combination of power + 10 D. When they are 0.25 m apart, the power reduces to + 6 D. The focal length of the lenses are …… m and ……m. **(1997, 2M)**

30. A thin lens of refractive index 1.5 has a focal length of 15 cm in air. When the lens is placed in a medium of refractive index 4/3, its focal length will become …… cm. **(1987, 2M)**

31. A convex lens A of focal length 20 cm and a concave lens B of focal length 5 cm are kept along the same axis with a distance d between them. If a parallel beam of light falling on A leaves B as parallel beam, then d is equal to ...... cm. **(1985, 2M)**

True / False

32. A parallel beam of white light fall on a combination of a concave and a convex lens, both of the same material. Their focal lengths are 15 cm and 30 cm respectively for the mean wavelength in white light. On the same side of the lens system, one sees coloured patterns with violet colour nearer to the lens. **(1988, 2M)**

33. A convex lens of focal length 1 m and a concave lens of focal length 0.25 m are kept 0.75 m apart. A parallel beam of light first passes through the convex lens, then through the concave lens and comes to a focus 0.5 m away from the concave lens. **(1983, 2M)**

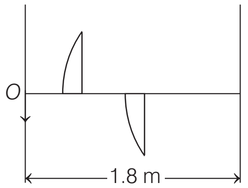
Integer Answer Type Question

34. The focal length of a thin biconvex lens is 20 cm. When an object is moved from a distance of 25 cm in front of it to 50 cm, the magnification of its image changes from m_{25} to m_{50}. The ratio $\frac{m_{25}}{m_{50}}$ is **(2010)**

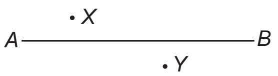
Analytical & Descriptive Questions

35. An object is approaching a thin convex lens of focal length 0.3 m with a speed of 0.01 m/s. Find the magnitudes of the rates of change of position and lateral magnification of image when the object is at a distance of 0.4 m from the lens. **(2004, 4M)**

36. A thin plano-convex lens of focal length f is split into two halves. One of the halves is shifted along the optical axis. The separation between object and image planes is 1.8 m. The magnification of the image formed by one of the half lens is 2. Find the focal length of the lens and separation between the halves. Draw the ray diagram for image formation. **(1996, 5M)**

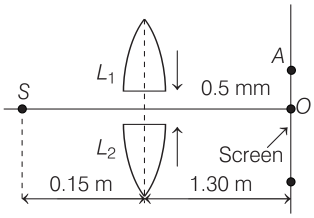


37. An image Y is formed of point object X by a lens whose optic axis is AB as shown in figure.



Draw a ray diagram to locate the lens and its focus. If the image Y of the object X is formed by a concave mirror (having the same optic axis as AB) instead of lens, draw another ray diagram to locate the mirror and its focus. Write down the steps of construction of the ray diagrams. **(1994, 6M)**

38. In given figure, S is a monochromatic point source emitting light of wavelength $\lambda = 500$ nm. A thin lens of circular shape and focal length 0.10 m is cut into two identical halves L_1 and L_2 by a plane passing through a diameter. The two halves are placed symmetrically about the central axis SO with a gap of 0.5 mm. The distance along the axis from S to L_1 and L_2 is 0.15 m while that from L_1 and L_2 to O is 1.30 m. The screen at O is normal to SO. **(1993, 5+1M)**



(a) If the third intensity maximum occurs at the point A on the screen, find the distance OA.

(b) If the gap between L_1 and L_2 is reduced from its original value of 0.5 mm, will the distance OA increase, decrease, or remain the same.

39. A plano-convex lens has a thickness of 4 cm. When placed on a horizontal table, with the curved surface in contact with it, the apparent depth of the bottom most point of the lens is found to be 3 cm. If the lens is inverted such that the plane face is in contact with the table, the apparent depth of the centre of the plane face is found to be 25/8 cm. Find the focal length of the lens. Assume thickness to be negligible while finding its focal length. **(1984, 6M)**

40. The radius of curvature of the convex face of a planoconvex lens is 12 cm and its $\mu = 1.5$. **(1979)**

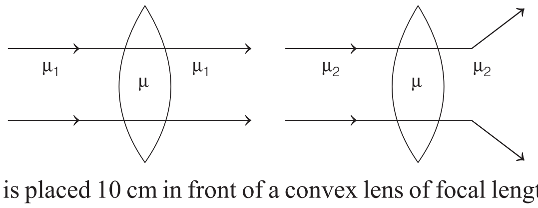
(a) Find the focal length of the lens. The plane face of the lens is now silvered.

(b) At what distance from the lens will parallel rays incident on the convex surface converge ?

(c) Sketch the ray diagram to locate the image, when a point object is placed on the axis 20 cm from the lens.

(d) Calculate the image distance when the object is placed as in (c)

41. What is the relation between the refractive indices μ_1 and μ_2? If the behaviour of light rays is as shown in the figure. **(1979)**



42. A pin is placed 10 cm in front of a convex lens of focal length 20 cm and made of a material of refractive index 1.5. The convex surface of the lens farther away from the pin is silvered and has a radius of curvature of 22 cm. Determine the position of the final image. Is the image real or virtual ? **(1978)**

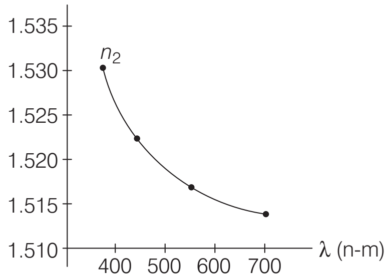
Topic 4 Prism

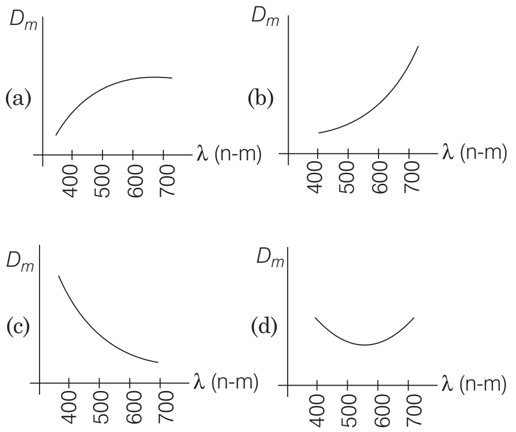
Objective Questions I (Only one correct option)

1. A monochromatic light is incident at a certain angle on an equilateral triangular prism and suffers minimum deviation. If the refractive index of the material of the prism is $\sqrt{3}$, then the angle of incidence is **(2019 Main, 11 Jan II)**

(a) 45º (b) 90º (c) 60º (d) 30º

2. The variation of refractive index of a crown glass thin prism with wavelength of the incident light is shown. Which of the following graphs is the correct one, if D_m is the angle of minimum deviation ? **(2019 Main, 11 Jan I)**

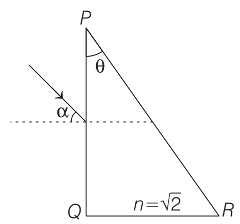




3. In an experiment for determination of refractive index of glass of a prism by i-δ plot, it was found that a ray incident at an angle 35° suffers a deviation of 40° and that it emerges at an angle 79°. In that case, which of the following is closest to the maximum possible value of the refractive index? **(2016 Main)**

(a) 1.5 (b) 1.6 (c) 1.7 (d) 1.8

4. A parallel beam of light is incident from air at an angle α on the side PQ of a right angled triangular prism of refractive index $n=\sqrt{2}$. Light undergoes total internal reflection in the prism at the face PR when α has a minimum value of 45°.



The angle θ of the prism is **(2016 Adv.)**

(a) 15° (b) 22.5° (c) 30° (d) 45°

5. Monochromatic light is incident on a glass prism of angle A. If the refractive index of the material of the prism is μ, a ray incident at an angle θ, on the face AB would get transmitted through the face AC of the prism provided **(2015 Main)**

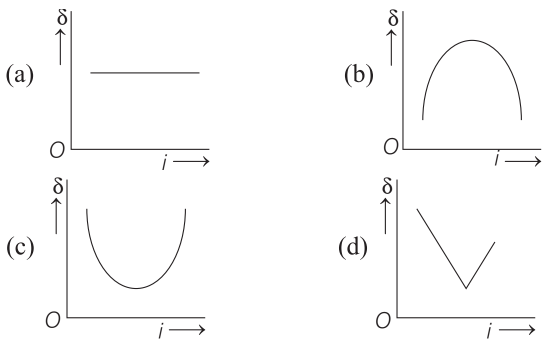
(a) $\theta < \cos^{-1}\left[\mu \sin\left\{A + \sin^{-1}\left(\frac{1}{\mu}\right)\right\}\right]$

(b) $\theta < \sin^{-1}\left[\mu \sin\left\{A - \sin^{-1}\left(\frac{1}{\mu}\right)\right\}\right]$

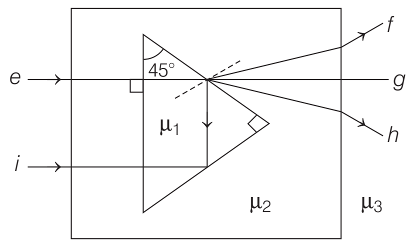
(c) $\theta > \cos^{-1}\left[\mu \sin\left\{A + \sin^{-1}\left(\frac{1}{\mu}\right)\right\}\right]$

(d) $\theta > \sin^{-1}\left[\mu \sin\left\{A - \sin^{-1}\left(\frac{1}{\mu}\right)\right\}\right]$

6. The graph between angle of deviation (δ) and angle of incidence (i) for a triangular prism is represented by **(2013 Main)**



7. A right angled prism of refractive index μ_1 is placed in a rectangular block of refractive index μ_2, which is surrounded by a medium of refractive index μ_3, as shown in the figure, A ray of light 'e' enters the rectangular block at normal incidence. Depending upon the relationships between μ_1, μ_2 and μ_3, it takes one of the four possible paths 'ef', 'eg', 'eh' or 'ei'. **(2013 Adv.)**



Match the paths in Column I with conditions of refractive indices in Column II and select the correct answer using the codes given below the lists.

Column I		Column II	
P.	$e \to f$	1.	$\mu_1 > \sqrt{2}\mu_2$
Q.	$e \to g$	2.	$\mu_2 > \mu_1$ and $\mu_2 > \mu_3$
R.	$e \to h$	3.	$\mu_1 = \mu_2$
S.	$e \to i$	4.	$\mu_2 < \mu_1 < \sqrt{2}\mu_2$ and $\mu_2 > \mu_3$

Codes

	P	Q	R	S
(a)	2	3	1	4
(b)	1	2	4	3
(c)	4	1	2	3
(d)	2	3	4	1

8. Two beams of red and violet colours are made to pass separately through a prism (angle of the prism is 60°). In the position of minimum deviation, the angle of refraction will be

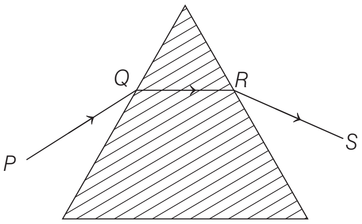
(a) 30° for both the colours **(2008, 3M)**

(b) greater for the violet colour

(c) greater for the red colour

(d) equal but not 30° for both the colours

9. A ray of light is incident on an equilateral glass prism placed on a horizontal table. For minimum deviation which of the following is true ? **(2004, 2M)**



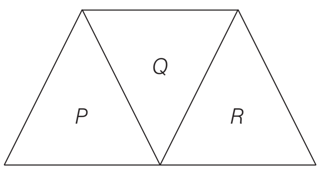
(a) PQ is horizontal

(b) QR is horizontal

(c) RS is horizontal

(d) Either PQ or RS is horizontal

10. A given ray of light suffers minimum deviation in an equilateral prism P. Additional prisms Q and R of identical shape and of the same material as P are now added as shown in the figure. The ray will suffer **(2001,2M)**



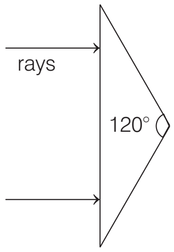
(a) greater deviation

(b) no deviation

(c) same deviation as before

(d) total internal reflection

11. An isosceles prism of angle 120° has a refractive index 1.44. Two parallel rays of monochromatic light enter the prism parallel to each other in air as shown. The rays emerging from the opposite face **(1995, 2M)**



(a) are parallel to each other

(b) are diverging

(c) make an angle $2[\sin^{-1}(0.72) - 30°]$ with each other

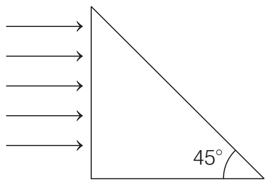
(d) make an angle $2\sin^{-1}(0.72)$ with each other

12. A thin prism P_1 with angle 4° and made from glass of refractive index 1.54 is combined with another thin prism P_2 made from glass of refractive index 1.72 to produce dispersion without deviation. The angle of the prism P_2 is **(1990, 2M)**

(a) 5.33° (b) 4°

(c) 3° (d) 2.6°

13. A beam of light consisting of red, green and blue colours is incident on a right-angled prism. The refractive indices of the material of the prism for the above red, green and blue wavelengths are 1.39, 1.44 and 1.47 respectively. The prism will **(1989, 2M)**



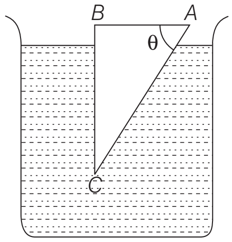
(a) separate the red colour from the green and blue colours

(b) separate the blue colour from the red and green colours

(c) separate all the three colours from one another

(d) not separate even partially any colour from the other two colours.

14. A glass prism of refractive index 1.5 is immersed in water (refractive index 4/3). A light beam incident normally on the face AB is totally reflected to reach the face BC if **(1981, 3M)**



(a) $\sin\theta > 8/9$ (b) $2/3 < \sin\theta < 8/9$

(c) $\sin\theta < 2/3$ (d) None of these

Objective Question II (One or more correct option)

15. For an isosceles prism of angle A and refractive index μ, it is found that the angle of minimum deviation $\delta_m = A$. Which of the following options is/are correct? **(2017 Adv.)**

(a) For the angle of incidence $i_1 = A$, the ray inside the prism is parallel to the base of the prism

(b) At minimum deviation, the incident angle i_1 and the refracting angle r_1 at the first refracting surface are related by $r_1 = \left(\frac{i_i}{2}\right)$

(c) For this prism, the emergent ray at the second surface will be tangential to the surface when the angle of incidence at the first surface is $i_1 = \sin^{-1}\left[\sin A\sqrt{4\cos^2\frac{A}{2} - 1} - \cos A\right]$

(d) For this prism, the refractive index μ and the angle prism A are related as $A = \frac{1}{2}\cos^{-1}\left(\frac{\mu}{2}\right)$

Fill in the Blanks

16. A ray of light is incident normally on one of the faces of a prism of apex angle 30° and refractive index $\sqrt{2}$. The angle of deviation of the ray is …… degrees. **(1997, 2M)**

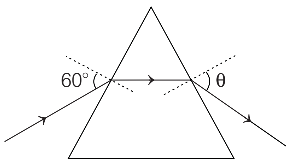
17. A ray of light undergoes deviation of 30° when incident on an equilateral prism of refractive index $\sqrt{2}$. The angle made by the ray inside the prism with the base of the prism is …… **(1992, 1M)**

True / False

18. A beam of white light passing through a hollow prism give no spectrum. **(1983, 2M)**

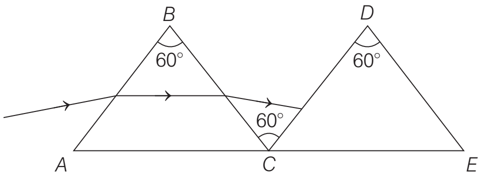
Integer Answer Type Question

19. A monochromatic beam of light is incident at 60° on one face of an equilateral prism of refractive index n and emerges from the opposite face making an angle $\theta(n)$ with the normal (see figure). For $n = \sqrt{3}$ the value of θ is 60° and $\frac{d\theta}{dn} = m$. The value of m is **(2015 Adv.)**



Analytical & Descriptive Questions

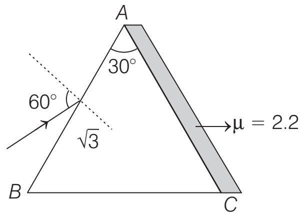
20. A ray of light is incident on a prism ABC of refractive index $\sqrt{3}$ as shown in figure. **(2005, 4M)**



(a) Find the angle of incidence for which the deviation of light ray by the prism ABC is minimum.

(b) By what angle the second identical prism must be rotated, so that the final ray suffers net minimum deviation.

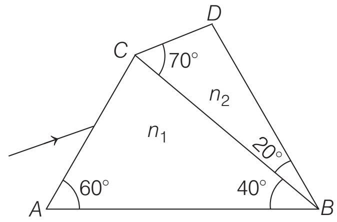
21. A prism of refracting angle 30° is coated with a thin film of transparent material of refractive index 2.2 on face AC of the prism. A light of wavelength 6600 Å is incident on face AB such that angle of incidence is 60°. Find **(2003, 4M)**



(a) the angle of emergence and

(b) the minimum value of thickness of the coated film on the face AC for which the light emerging from the face has maximum intensity. (Given refractive index of the material of the prism is $\sqrt{3}$)

22. A prism of refractive index n_1 and another prism of refractive index n_2 are stuck together with a gap as shown in the figure. The angles of the prism are as shown. n_1 and n_2 depend on λ, the wavelength of light according to :



$$n_1 = 1.20 + \frac{10.8 \times 10^4}{\lambda^2} \text{ and}$$

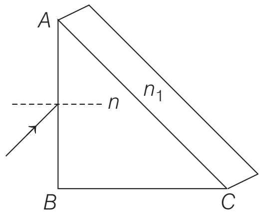
$$n_2 = 1.45 + \frac{1.80 \times 10^4}{\lambda^2}$$

where λ is in nm. **(1998, 8M)**

(a) Calculate the wavelength λ_0 for which rays incident at any angle on the interface BC pass through without bending at that interface.

(b) For light of wavelength λ_0, find the angle of incidence i on the face AC such that the deviation produced by the combination of prisms is minimum.

23. A right angled prism (45°-90°-45°) of refractive index n has a plane of refractive index n_1 $(n_1 < n)$ cemented to its diagonal face. The assembly is in air. The ray is incident on AB. **(1996, 3M)**



(a) Calculate the angle of incidence at AB for which the ray strikes the diagonal face at the critical angle.

(b) Assuming $n = 1.352$, calculate the angle of incidence at AB for which the refracted ray passes through the diagonal face undeviated.

24. A ray of light is incident at an angle of 60° on one face of a prism which has an angle of 30°. The ray emerging out of the prism makes an angle of 30° with the incident ray. Show that the emergent ray is perpendicular to the face through which it emerges and calculate the refractive index of the material of the lens. **(1978)**

Topic 5 Optical Instruments

Objective Questions I (Only one correct option)

1. The value of numerical aperture of the objective lens of a microscope is 1.25. If light of wavelength 5000Å is used, the minimum separation between two points, to be seen as distinct, will be **(2019 Main, 12 April I)**

(a) $0.24\,\mu m$ (b) $0.38\,\mu m$ (c) $0.12\,\mu m$ (d) $0.48\,\mu m$

2. Diameter of the objective lens of a telescope is 250 cm. For light of wavelength 600 nm coming from a distant object, the limit of resolution of the telescope is close to **(2019 Main, 9 April II)**

(a) 3.0×10^{-7} rad (b) 2.0×10^{-7} rad
(c) 1.5×10^{-7} rad (d) 4.5×10^{-7} rad

3. Calculate the limit of resolution of a telescope objective having a diameter of 200 cm, if it has to detect light of wavelength 500 nm coming from a star. **(2019 Main, 8 April II)**

(a) 610×10^{-9} rad (b) 305×10^{-9} rad
(c) 457.5×10^{-9} rad (d) 152.5×10^{-9} rad

4. An observer looks at a distant tree of height 10 m with a telescope of magnifying power of 20. To observer the tree appears **(2016 Main)**

(a) 10 times taller (b) 10 times nearer
(c) 20 times taller (d) 20 times nearer

5. The box of a pin hole camera of length L, has a hole of radius a. It is assumed that when the hole is illuminated by a parallel beam of light of wavelength λ the spread of the spot (obtained on the opposite wall of the camera) is the sum of its geometrical spread and the spread due to diffraction. The spot would then have its minimum size (say $b_{\min}$) when **(2016 Main)**

(a) $a = \dfrac{\lambda^2}{L}$ and $b_{\min} = \left(\dfrac{2\lambda^2}{L}\right)$

(b) $a = \sqrt{\lambda L}$ and $b_{\min} = \left(\dfrac{2\lambda^2}{L}\right)$

(c) $a = \sqrt{\lambda L}$ and $b_{\min} = \sqrt{4\lambda L}$

(d) $a = \dfrac{\lambda^2}{L}$ and $b_{\min} = \sqrt{4\lambda L}$

6. In a compound microscope, the intermediate image is **(2000, 2M)**

(a) virtual, erect and magnified
(b) real, erect and magnified
(c) real, inverted and magnified
(d) virtual, erect and reduced

7. The focal lengths of the objective and the eyepiece of a compound microscope are 2.0 cm and 3.0 cm respectively. The distance between the objective and the eyepiece is 15.0 cm. The final image formed by the eyepiece is at infinity. The two lenses are thin. The distance in cm of the object and the image produced by the objective, measured from the objective lens, are respectively **(1995,2M)**

(a) 2.4 and 12.0 (b) 2.4 and 15.0
(c) 2.0 and 12.0 (d) 2.0 and 3.0

8. An astronomical telescope has an angular magnification of magnitude 5 for far objects. The separation between the objective and the eyepiece is 36 cm and the final image is formed at infinity. The focal length f_o of the objective and the focal length f_e of the eyepiece are **(1989, 2M)**

(a) $f_o = 45$ cm and $f_e = -9$ cm
(b) $f_o = 50$ cm and $f_e = 10$ cm
(c) $f_o = 7.2$ cm and $f_e = 5$ cm
(d) $f_o = 30$ cm and $f_e = 6$ cm

Objective Question II (One or more correct option)

9. A planet is observed by an astronomical refracting telescope having an objective of focal length 16 m and an eyepiece of focal length 2 cm **(1992, 2M)**

(a) the distance between the objective and the eyepiece is 16.02 m
(b) the angular magnification of the planet is –800
(c) the image of the planet is inverted
(d) the objective is larger than the eyepiece

Fill in the Blank

10. The resolving power of electron microscope is higher than that of an optical microscope because the wavelength of electrons is …… than the wavelength of visible light. **(1992, 1M)**

Topic 6 Wave Optics

Objective Questions I (Only one correct option)

1. A system of three polarisers P_1, P_2, P_3 is set up such that the pass axis of P_3 is crossed with respect to that of P_1. The pass axis of P_2 is inclined at 60° to the pass axis of P_3. When a beam of unpolarised light of intensity I_0 is incident on P_1, the intensity of light transmitted by the three polarisers is I. The ratio (I_0 / I) equals (nearly) **(2019 Main, 12 April II)**

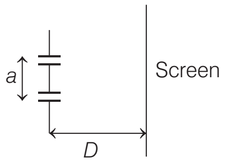
(a) 5.33 (b) 16.00 (c) 10.67 (d) 1.80

2. In a double slit experiment, when a thin film of thickness t having refractive index μ is introduced in front of one of the slits, the maximum at the centre of the fringe pattern shifts by one fringe width. The value of t is (λ is the wavelength of the light used) **(2019 Main, 12 April I)**

(a) $\dfrac{2\lambda}{(\mu - 1)}$ (b) $\dfrac{\lambda}{2(\mu - 1)}$ (c) $\dfrac{\lambda}{(\mu - 1)}$ (d) $\dfrac{\lambda}{(2\mu - 1)}$

3. In a Young's double slit experiment, the ratio of the slit's width is 4 : 1. The ratio of the intensity of maxima to minima, close to the central fringe on the screen, will be **(2019 Main, 10 April II)**
(a) 4 : 1 (b) 25 : 9
(c) 9 : 1 (d) $(\sqrt{3}+1)^4 : 16$

4. The figure shows a Young's double slit experimental setup. It is observed that when a thin transparent sheet of thickness t and refractive index μ is put in front of one of the slits, the central maximum gets shifted by a distance equal to n fringe widths. If the wavelength of light used is λ, t will be **(2019 Main, 9 April I)**



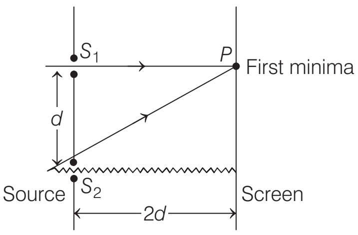
(a) $\frac{2nD\lambda}{a(\mu-1)}$ (b) $\frac{2D\lambda}{a(\mu-1)}$ (c) $\frac{D\lambda}{a(\mu-1)}$ (d) $\frac{nD\lambda}{a(\mu-1)}$

5. In an interference experiment, the ratio of amplitudes of coherent waves is $\frac{a_1}{a_2}=\frac{1}{3}$. The ratio of maximum and minimum intensities of fringes will be **(2019 Main, 8 April I)**
(a) 2 (b) 18 (c) 4 (d) 9

6. In a double-slit experiment, green light (5303 Å) falls on a double slit having a separation of 19.44 μ-m and a width of 4.05 μ-m. The number of bright fringes between the first and the second diffraction minima is **(2019 Main, 11 Jan II)**
(a) 5 (b) 10 (c) 9 (d) 4

7. In a Young's double slit experiment, the path difference at a certain point on the screen between two interfering waves is $\frac{1}{8}$th of wavelength. The ratio of the intensity at this point to that at the centre of a bright fringe is close to **(2019 Main, 11 Jan I)**
(a) 0.80 (b) 0.74 (c) 0.94 (d) 0.85

8. Consider a Young's double slit experiment as shown in figure



What should be the slit separation d in terms of wavelength λ such that the first minima occurs directly in front of the slit (S_1)? **(2019 Main, 10 Jan II)**
(a) $\frac{\lambda}{2(5-\sqrt{2})}$ (b) $\frac{\lambda}{(5-\sqrt{2})}$
(c) $\frac{\lambda}{2(\sqrt{5}-2)}$ (d) $\frac{\lambda}{(\sqrt{5}-2)}$

9 In a Young's double slit experiment with slit separation 0.1 mm, one observes a bright fringe at angle $\frac{1}{40}$ rad by using light of wavelength λ_1. When the light of the wavelength λ_2 is used a bright fringe is seen at the same angle in the same set up. Given that λ_1 and λ_2 are in visible range (380 n-m to 740 n-m), their values are **(2019 Main, 10 Jan I)**
(a) 380 n-m, 525 n-m (b) 400 n-m, 500 n-m
(c) 380 n-m, 500 n-m (d) 625 n-m, 500 n-m

10 In a Young's double slit experiment, the slits are placed 0.320 mm apart. Light of wavelength $\lambda = 500$ n-m is incident on the slits. The total number of bright fringes that are observed in the angular range $-30° \le \theta \le 30°$ is **(2019 Main, 9 Jan II)**
(a) 320 (b) 321
(c) 640 (d) 641

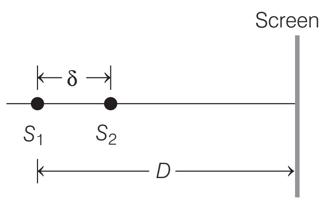
11 Two coherent sources produce waves of different intensities which interfere. After interference, the ratio of the maximum intensity to the minimum intensity is 16. The intensity of the waves are in the ratio **(2019 Main, 9 Jan I)**
(a) 16 : 9 (b) 5 : 3 (c) 25 : 9 (d) 4 : 1

12. Unpolarised light of intensity I passes through an ideal polariser A. Another identical polariser B is placed behind A. The intensity of light beyond B is found to be $\frac{I}{2}$. Now, another identical polariser C is placed between A and B. The intensity beyond B is now found to be $\frac{1}{8}$. The angle between polariser A and C is **(2018 Main)**
(a) 60° (b) 0° (c) 30° (d) 45°

13. The angular width of the central maximum in a single slit diffraction pattern is 60°. The width of the slit is 1 μm. The slit is illuminated by monochromatic plane waves. If another slit of same width is made near it, Young's fringes can be observed on a screen placed at a distance 50 cm from the slits. If the observed fringe width is 1 cm, what is slit separation distance?
(i.e. distance between the centres of each slit.) **(2018 Main)**
(a) 100 μm (b) 25 μm (c) 50 μm (d) 75 μm

14. In a Young's double slit experiment, slits are separated by 0.5 mm and the screen is placed 150 cm away. A beam of light consisting of two wavelengths, 650 nm and 520 nm, is used to obtain interference fringes on the screen. The least distance from the common central maximum to the point where the bright fringes due to both the wavelengths coincide, is **(2017 Main)**
(a) 7.8 mm (b) 9.75 mm
(c) 15.6 mm (d) 1.56 mm

15. Two coherent point sources S_1 and S_2 are separated by a small distance d as shown. The fringes obtained on the screen will be **(2013 Main)**



(a) points (b) straight lines
(c) semi-circle (d) concentric circles

16. In the Young's double slit experiment using a monochromatic light of wavelength λ the path difference (in terms of an integer n) corresponding to any point having half the peak intensity is **(2013 Adv.)**

(a) $(2n+1)\frac{\lambda}{2}$ (b) $(2n+1)\frac{\lambda}{4}$

(c) $(2n+1)\frac{\lambda}{8}$ (d) $(2n+1)\frac{\lambda}{16}$

17. A beam of unpolarised light of intensity I_0 is passed through a polaroid A and then through another polaroid B which is oriented so that its principal plane makes an angle of 45° relative to that of A. The intensity of the emergent light is

(a) I_0 (b) $I_0/2$ **(2013 Main)**

(c) $I_0/4$ (d) $I_0/8$

18. Young's double slit experiment is carried out by using green, red and blue light, one colour at a time. The fringe widths recorded are β_G, β_R and β_B respectively.

Then, **(2012)**

(a) $\beta_G > \beta_B > \beta_R$ (b) $\beta_B > \beta_G > \beta_R$

(c) $\beta_R > \beta_B > \beta_G$ (d) $\beta_R > \beta_G > \beta_B$

19. A biconvex lens of focal length f forms a circular image of radius r of sun in focal plane. Then which option is correct ?

(a) $\pi r^2 \propto f$ **(2006, 3M)**

(b) $\pi r^2 \propto f^2$

(c) If lower half part is convered by black sheet, then area of the image is equal to $\pi r^2/2$

(d) If f is doubled, intensity will increase

20. In Young's double slit experiment intensity at a point is (1/4) of the maximum intensity. Angular position of this point is

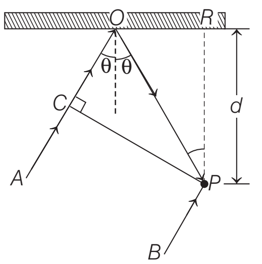
(a) $\sin^{-1}\left(\frac{\lambda}{d}\right)$ (b) $\sin^{-1}\left(\frac{\lambda}{2d}\right)$ **(2005, 2M)**

(c) $\sin^{-1}\left(\frac{\lambda}{3d}\right)$ (d) $\sin^{-1}\left(\frac{\lambda}{4d}\right)$

21. In a YDSE bi-chromatic light of wavelengths 400 nm and 560 nm are used. The distance between the slits is 0.1 mm and the distance between the plane of the slits and the screen is 1 m. The minimum distance between two successive regions of complete darkness is **(2004, 2M)**

(a) 4 mm (b) 5.6 mm (c) 14 mm (d) 28 mm

22. In the adjacent diagram, CP represents a wavefront and AO and BP, the corresponding two rays. Find the condition of θ for constructive interference at P between the ray BP and reflected ray OP **(2003, 2M)**



(a) $\cos\theta = \frac{3\lambda}{2d}$ (b) $\cos\theta = \frac{\lambda}{4d}$

(c) $\sec\theta - \cos\theta = \frac{\lambda}{d}$ (d) $\sec\theta - \cos\theta = \frac{4\lambda}{d}$

23. In the ideal double-slit experiment, when a glass-plate (refractive index 1.5) of thickness t is introduced in the path of one of the interfering beams (wavelength λ), the intensity at the position where the central maximum occurred previously remains unchanged. The minimum thickness of the glass-plate is **(2002, 2M)**

(a) 2λ (b) $\frac{2\lambda}{3}$ (c) $\frac{\lambda}{3}$ (d) λ

24. In a Young's double slit experiment, 12 fringes are observed to be formed in a certain segment of the screen when light of wavelength 600 nm is used. If the wavelength of light is changed to 400 nm, number of fringes observed in the same segment of the screen is given by **(2001, 2M)**

(a) 12 (b) 18 (c) 24 (d) 30

25. Two beams of light having intensities I and $4I$ interfere to produce a fringe pattern on a screen. The phase difference between the beams is $\pi/2$ at point A and π at point B. Then the difference between resultant intensities at A and B is **(2001, 2M)**

(a) $2I$ (b) $4I$ (c) $5I$ (d) $7I$

26. In a double slit experiment instead of taking slits of equal widths, one slit is made twice as wide as the other, then in the interference pattern **(2000, 2M)**

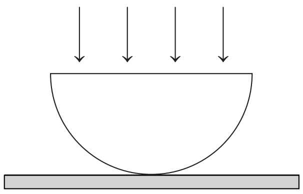
(a) the intensities of both the maxima and the minima increases

(b) the intensity of the maxima increases and the minima has zero intensity

(c) the intensity of maxima decreases and that of minima increases

(d) the intensity of maxima decreases and the minima has zero intensity

27. A thin slice is cut out of a glass cylinder along a plane parallel to its axis. The slice is placed on a flat plate as shown. The observed interference fringes from this combination shall be



(a) straight **(1999, 2M)**

(b) circular

(c) equally spaced

(d) having fringe spacing which increases as we go outwards

28. Yellow light is used in a single slit diffraction experiment with slit width of 0.6 mm. If yellow light is replaced by X-rays, then the observed pattern will reveal **(1999, 2M)**

(a) that the central maximum is narrower

(b) more number of fringes

(c) less number of fringes

(d) no diffraction pattern

29. A parallel monochromatic beam of light is incident normally on a narrow slit. A diffraction pattern is formed on a screen placed perpendicular to the direction of the incident beam. At the first minimum of the diffraction pattern, the phase difference between the rays coming from the two edges of the slit is **(1998, 2M)**

(a) zero (b) $\pi / 2$ (c) π (d) 2π

30. A narrow slit of width 1 mm is illuminated by monochromatic light of wavelength 600 nm. The distance between the first minima on either side of a screen at a distance of 2 m is **(1994, 1M)**

(a) 1.2 cm (b) 1.2 mm
(c) 2.4 cm (d) 2.4 mm

31. Two coherent monochromatic light beams of intensities I and $4I$ are superposed. The maximum and minimum possible intensities i n the resulting beam are **(1988, 1M)**

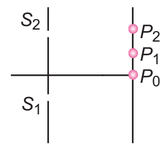
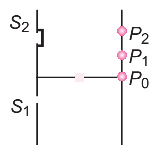
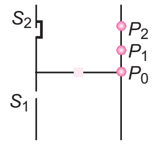
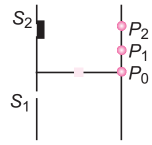
(a) $5I$ and I
(b) $5I$ and $3I$
(c) $9I$ and I
(d) $9I$ and $3I$

32. In Young's double slit experiment, the separation between the slits is halved and the distance between the slits and the screen is doubled. The fringe width is **(1981, 2M)**

(a) unchanged (b) halved
(c) doubled (d) quadrupled

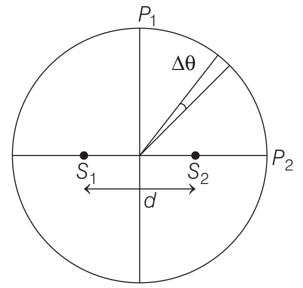
Match the Column

33. **Column I** shows four situations of standard Young's double slit arrangement with the screen placed far away from the slits S_1 and S_2. In each of these cases $S_1P_0 = S_2P_0, S_1P_1 - S_2P_1 = \frac{\lambda}{4}$ and $S_1P_2 - S_2P_2 = \lambda / 2$, where λ is the wavelength of the light used. In the cases B, C and D, a transparent sheet of refractive index μ and thickness t is pasted on slit S_2. The thickness of the sheets are different in different cases. The phase difference between the light waves reaching a point P on the screen from the two slits is denoted by $\delta(P)$ and the intensity by $I(P)$. Match each situation given in Column I with the statement(s) in Column II valid for that situation. **(2009)**

Column I		Column II
(A)		(p) $\delta(P_0) = 0$
(B) $(\mu - 1)t = \frac{\lambda}{4}$		(q) $\delta(P_1) = 0$
(C) $(\mu - 1)t = \frac{\lambda}{2}$		(r) $I(P_1) = 0$
(D) $(\mu - 1)t = \frac{3\lambda}{4}$		(s) $I(P_0) > I(P_1)$
		(t) $I(P_2) > I(P_1)$

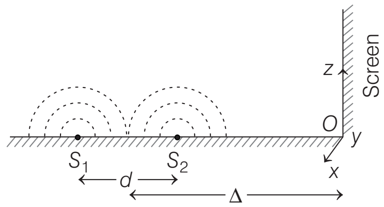
Objective Questions II (One or more correct option)

34. Two coherent monochromatic point sources S_1 and S_2 of wavelength $\lambda = 600$ nm are placed symmetrically on either side of the centre of the circle as shown. The sources are separated by a distance $d = 1.8$ mm. This arrangement produces interference fringes visible as alternate bright and dark spots on the circumference of the circle. The angular separation between two consecutive bright spots is $\Delta\theta$. Which of the following options is/are correct? **(2017 Adv.)**



(a) The angular separation between two consecutive bright spots decreases as we move from P_1 to P_2 along the first quadrant
(b) A dark spot will be formed at the point P_2
(c) The total number of fringes produced between P_1 and P_2 in the first quadrant is close to 3000
(d) At P_2 the order of the fringe will be maximum

35. While conducting the Young's double slit experiment, a student replaced the two slits with a large opaque plate in the x-y plane containing two small holes that act as two coherent point sources (S_1, S_2) emitting light of wavelength 600 mm. The student mistakenly placed the screen parallel to the x-z plane (for $z > 0$) at a distance $D = 3$m from the mid-point of S_1S_2, as shown schematically in the figure. The distance between the source $d = 0.6003$ mm . The origin O is at the intersection of the screen and the line joining S_1S_2. **(2016 Adv.)**



Which of the following is (are) true of the intensity pattern on the screen?

(a) Semi circular bright and dark bands centered at point O
(b) The region very close to the point O will be dark
(c) Straight bright and dark bands parallel to the X-axis
(d) Hyperbolic bright and dark bands with foci symmetrically placed about O in the x-direction

36. A light source, which emits two wavelengths $\lambda_1 = 400\,\text{nm}$ and $\lambda_2 = 600\,\text{nm}$, is used in a Young's double-slit experiment. If recorded fringe widths for λ_1 and λ_2 are β_1 and β_2 and the number of fringes for them within a distance y on one side of the central maximum are m_1 and m_2, respectively, then **(2014 Adv.)**

(a) $\beta_2 > \beta_1$
(b) $m_1 > m_2$
(c) from the central maximum, 3rd maximum of λ_2 overlaps with 5th minimum of λ_1
(d) the angular separation of fringes of λ_1 is greater than λ_2

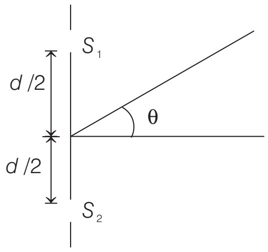
37. Using the expression $2d\sin\theta = \lambda$, one calculates the values of d by measuring the corresponding angles θ in the range 0 to 90°. The wavelength λ is exactly known and the error in θ is constant for all values of θ. As θ increases from 0° **(2013 Adv.)**

(a) the absolute error in d remains constant
(b) the absolute error in d increases
(c) the fractional error in d remains constant
(d) the fractional error in d decreases

38. In a Young's double slit experiment, the separation between the two slits is d and the wavelength of the light is λ. The intensity of light falling on slit 1 is four times the intensity of light falling on slit 2. Choose the correct choice (s). **(2008, 4M)**

(a) If $d = \lambda$, the screen will contain only one maximum
(b) If $\lambda < d < 2\lambda$, at least one more maximum (besides the central maximum) will be observed on the screen
(c) If the intensity of light falling on slit 1 is reduced so that it becomes equal to that of slit 2, the intensities of the observed dark and bright fringes will increase
(d) If the intensity of light falling on slit 2 is increased so that it becomes equal to that of slit 1, the intensities of the observed dark and bright fringes will increase

39. In an interference arrangement similar to Young's double-slit experiment, the slits S_1 and S_2 are illuminated with coherent microwave sources, each of frequency 10^6 Hz. The sources are synchronized to have zero phase difference. The slits are separated by a distance $d = 150.0\,\text{m}$. The intensity $I(\theta)$ is measured as a function of θ, where θ is defined as shown. If I_0 is the maximum intensity, then $I(\theta)$ for $0 \le \theta \le 90°$ is given by **(1995, 2M)**



(a) $I(\theta) = I_0/2$ for $\theta = 30°$
(b) $I(\theta) = I_0/4$ for $\theta = 90°$
(c) $I(\theta) = I_0$ for $\theta = 0°$
(d) $I(\theta)$ is constant for all values of θ

40. White light is used to illuminate the two slits in a Young's double slit experiment. The separation between the slits is b and the screen is at a distance $d\ (>> b)$ from the slits. At a point on the screen directly in front of one of the slits, certain wavelengths are missing. Some of these missing wavelengths are **(1984, 2M)**

(a) $\lambda = b^2/d$ (b) $\lambda = 2b^2/d$
(c) $\lambda = b^2/3d$ (d) $\lambda = 2b^2/3d$

41. In the Young's double slit experiment, the interference pattern is found to have an intensity ratio between the bright and dark fringes as 9. This implies that **(1982, 2M)**

(a) the intensities at the screen due to the two slits are 5 units and 4 units respectively
(b) the intensities at the screen due to the two slits are 4 units and 1 unit respectively
(c) the amplitude ratio is 3
(d) the amplitude ratio is 2

Fill in the Blanks

42. A slit of width d is placed in front of a lens of focal length 0.5 m and is illuminated normally with light of wavelength 5.89×10^{-7} m. The first diffraction minima on either side of the central diffraction maximum are separated by 2×10^{-3} m. The width d of the slit is …… m. **(1997, 1M)**

43. In Young's double-slit experiment, the two slits act as coherent sources of equal amplitude A and of wavelength λ. In another experiment with the same set-up the two slits are sources of equal amplitude A and wavelength λ, but are incoherent. The ratio of the intensity of light at the mid-point of the screen in the first case to that in the second case is …… **(1986, 2M)**

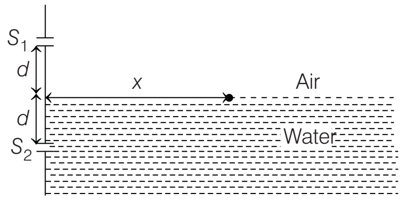
True / False

44. In a Young's double slit experiment performed with a source of white light, only black and white fringes are observed. **(1987, 2M)**

45. Two slits in a Young's double slit experiment are illuminated by two different sodium lamps emitting light of the same wavelength. No interference pattern will be observed on the screen. **(1984, 2M)**

Integer Answer Type Question

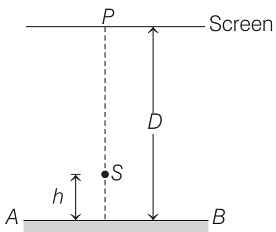
46. A Young's double slit interference arrangement with slits S_1 and S_2 is immersed in water (refractive index $= 4/3$) as shown in the figure. The positions of maxima on the surface of water are given by $x^2 = p^2m^2\lambda^2 - d^2$, where λ is the wavelength of light in air (refractive index $= 1$), $2d$ is the separation between the slits and m is an integer. The value of p is **(2015 Adv.)**



Analytical & Descriptive Questions

47. In a Young's double slit experiment, two wavelengths of 500 nm and 700 nm were used. What is the minimum distance from the central maximum where their maximas coincide again? Take $D/d = 10^3$. Symbols have their usual meanings. **(2004, 4M)**

48. A point source S emitting light of wavelength 600 nm is placed at a very small height h above a flat reflecting surface AB (see figure). The intensity of the reflected light is 36% of the incident intensity. Interference fringes are observed on a screen placed parallel to the reflecting surface at a very large distance D from it. **(2002,5M)**

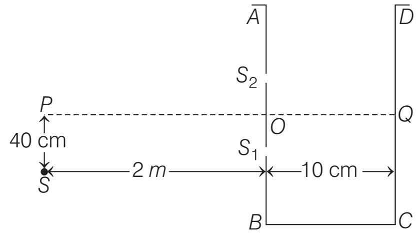


(a) What is the shape of the interference fringes on the screen?

(b) Calculate the ratio of the minimum to the maximum intensities in the interference fringes formed near the point P (shown in the figure).

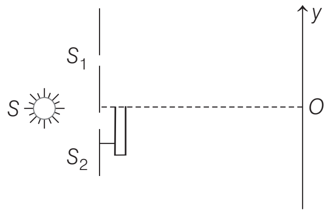
(c) If the intensity at point P corresponds to a maximum, calculate the minimum distance through which the reflecting surface AB should be shifted so that the intensity at P again becomes maximum.

49. A vessel $ABCD$ of 10 cm width has two small slits S_1 and S_2 sealed with identical glass plates of equal thickness. The distance between the slits is 0.8 mm. POQ is the line perpendicular to the plane AB and passing through O, the middle point of S_1 and S_2. A monochromatic light source is kept at S, 40 cm below P and 2 m from the vessel, to illuminate the slits as shown in the figure alongside. Calculate the position of the central bright fringe on the other wall CD with respect to the line OQ. Now, a liquid is poured into the vessel and filled upto OQ. The central bright fringe is found to be at Q. Calculate the refractive index of the liquid. **(2001, 5M)**



50. A glass plate of refractive index 1.5 is coated with a thin layer of thickness t and refractive index 1.8. Light of wavelength λ travelling in air is incident normally on the layer. It is partly reflected at the upper and the lower surfaces of the layer and the two reflected rays interfere. Write the condition for their constructive interference. If $\lambda = 648$ nm, obtain the least value of t for which the rays interfere constructively. **(2000, 4M)**

51. The Young's double slit experiment is done in a medium of refractive index 4/3. A light of 600 nm wavelength is falling on the slits having 0.45 mm separation. The lower slit S_2 is covered by a thin glass sheet of thickness 10.4 μm and refractive index 1.5. The interference pattern is observed on a screen placed 1.5 m from the slits as shown in the figure.



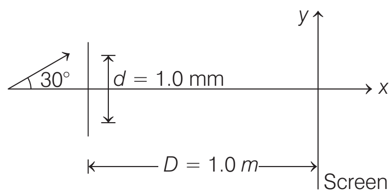
(a) Find the location of central maximum (bright fringe with zero path difference) on the y-axis.

(b) Find the light intensity of point O relative to the maximum fringe intensity.

(c) Now, if 600 nm light is replaced by white light of range 400 to 700 nm, find the wavelengths of the light that form maxima exactly at point O.

(All wavelengths in the problem are for the given medium of refractive index 4/3. Ignore dispersion) **(1999, 10M)**

52. A coherent parallel beam of microwaves of wavelength $\lambda = 0.5$ mm falls on a Young's double slit apparatus. The separation between the slits is 1.0 mm. The intensity of microwaves is measured on a screen placed parallel to the plane of the slits at a distance of 1.0 m from it as shown in the figure. **(1998, 8M)**



(a) If the incident beam falls normally on the double slit apparatus, find the y-coordinates of all the interference minima on the screen.

(b) If the incident beam makes an angle of 30° with the x-axis (as in the dotted arrow shown in figure), find the y-coordinates of the first minima on either side of the central maximum.

53. In a Young's experiment, the upper slit is covered by a thin glass plate of refractive index 1.4, while the lower slit is covered by another glass plate, having the same thickness as the first one but having refractive index 1.7. Interference pattern is observed using light of wavelength 5400 Å. It is found that the point P on the screen, where the central maximum ($n = 0$) fall before the glass plates were inserted, now has 3/4 the original intensity. It is further observed that what used to be the fifth maximum earlier lies below the point P while the sixth minima lies above P. Calculate the thickness of glass plate. (Absorption of light by glass plate may be neglected). **(1997, 5M)**

54. In Young's experiment, the source is red light of wavelength 7×10^{-7} m. When a thin glass plate of refractive index 1.5 at this wavelength is put in the path of one of the interfering beams, the central bright fringe shifts by 10^{-3} m to the position previously occupied by the 5th bright fringe. Find the thickness of the plate. When the source is now changed to green light of wavelength 5×10^{-7} m, the central fringe shifts to a position initially occupied by the 6th bright fringe due to red light. Find the refractive index of glass for green light. Also estimate the change in fringe width due to the change in wavelength. **(1997C, 5M)**

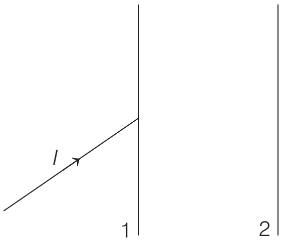
55. A double slit apparatus is immersed in a liquid of refractive index 1.33. It has slit separation of 1 mm and distance between the plane of slits and screen is 1.33 m. The slits are illuminated by a parallel beam of light whose wavelength in air is 6300 Å . **(1996, 3M)**

(a) Calculate the fringe width.

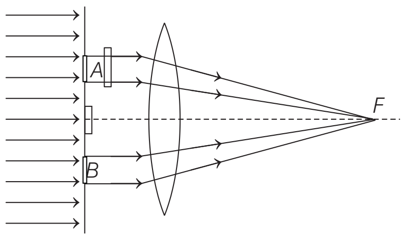
(b) One of the slits of the apparatus is covered by a thin glass sheet of refractive index 1.53. Find the smallest thickness of the sheet to bring the adjacent minimum as the axis.

56. Angular width of central maximum in the Fraunhofer diffraction pattern of a slit is measured. The slit is illuminated by light of wavelength 6000 Å. When the slit is illuminated by light of another wavelength, the angular width decreases by 30%. Calculate the wavelength of this light. The same decrease in the angular width of central maximum is obtained when the original apparatus is immersed in a liquid. Find refractive index of the liquid. **(1996, 2M)**

57. A narrow monochromatic beam of light of intensity I is incident on a glass plate as shown in figure. Another identical glass plate is kept close to the first one-and parallel to it. Each glass plate reflects 25 per cent of the light incident on it and transmits the remaining. Find the ratio of the minimum and maximum intensities in the interference pattern formed by the two beams obtained after one reflection at each plate. **(1990, 7M)**



58. In a modified Young's double slit experiment, a monochromatic uniform and parallel beam of light of wavelength 6000 Å and intensity $(10/\pi)$ Wm^{-2} is incident normally on two apertures A and B of radii 0.001 m and 0.002 m respectively. A perfectly transparent film of thickness 2000 Å and refractive index 1.5 for the wavelength of 6000Å is placed in front of aperture A (see figure). Calculate the power (in W) received at the focal spot F of the lens. The lens is symmetrically placed with respect to the apertures. Assume that 10% of the power received by each aperture goes in the original direction and is brought to the focal spot. **(1989, 8M)**



59. A beam of light consisting of two wavelengths, 6500Å and 5200 Å is used to obtain interference fringe in a Young's double slit experiment. **(1985, 6M)**

(a) Find the distance of the third bright fringe on the screen from the central maximum for wavelength 6500 Å.

(b) What is the least distance from the central maximum where the bright fringes due to both the wavelengths coincide?

The distance between the slits is 2 mm and the distance between the plane of the slits and the screen is 120 cm.

60. In Young's double slit experiment using monochromatic light the fringe pattern shifts by a certain distance on the screen when a mica sheet of refractive index 1.6 and thickness 1.964 microns is introduced in the path of one of the interfering waves. The mica sheet is then removed and the distance between the slits and the screen is doubled. It is found that the distance between successive maxima (or minima) now is the same as the observed fringe shift upon the introduction of the mica sheet. Calculate the wavelength of the monochromatic light used in the experiment. **(1983, 6M)**

Topic 7 Miscellaneous Problems

Objective Questions I (Only one correct option)

1. A convex lens (of focal length 20 cm) and a concave mirror, having their principal axes along the same lines, are kept 80 cm apart from each other. The concave mirror is to the right of the convex lens. When an object is kept at a distance of 30 cm to the left of the convex lens, its image remains at the same position even if the concave mirror is removed. The maximum distance of the object for which this concave mirror, by itself would produce a virtual image would be **(2019 Main, 8 April II)**

(a) 25 cm (b) 20 cm (c) 10 cm (d) 30 cm

2. An upright object is placed at a distance of 40 cm in front of a convergent lens of focal length 20 cm. A convergent mirror of focal length 10 cm is placed at a distance of 60 cm on the other side of the lens. The position and size of the final image will be **(2019 Main, 8 April I)**

(a) 20 cm from the convergent mirror, same size as the object
(b) 40 cm from the convergent mirror, same size as the object
(c) 40 cm from the convergent lens, twice the size of the object
(d) 20 cm from the convergent mirror,twice size of the object

3. A light wave is incident normally on a glass slab of refractive index 1.5. If 4% of light gets reflected and the amplitude of the electric field of the incident light is 30 V/m, then the amplitude of the electric field for the wave propogating in the glass medium will be **(2019 Main, 12 Jan I)**

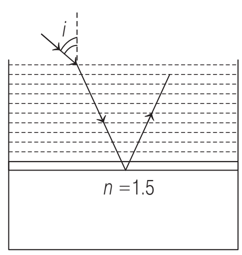
(a) 30 V/m (b) 6 V/m (c) 10 V/m (d) 24 V/m

4. The eye can be regarded as a single refracting surface. The radius of curvature of this surface is equal to that of cornea (7.8 mm). This surface separates two media of refractive indices 1 and 1.34. Calculate the distance from the refracting surface at which a parallel beam of light will come to focus. **(2019 Main, 10 Jan II)**

(a) 4.0 cm (b) 2 cm (c) 3.1 cm (d) 1 cm

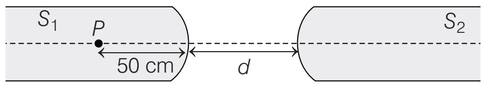
5. Consider a tank made of glass (refractive index is 1.5) with a thick bottom. It is filled with a liquid of refractive index μ. A student finds that, irrespective of what the incident angle i (see figure) is for a beam of light entering the liquid, the light reflected from the liquid glass interface is never completely polarised.

For this to happen, the minimum value of μ is **(2019 Main, 9 Jan I)**



(a) $\frac{3}{\sqrt{5}}$ (b) $\frac{5}{\sqrt{3}}$ (c) $\frac{4}{3}$ (d) $\sqrt{\frac{5}{3}}$

6. Two identical glass rods S_1 and S_2 (refractive index = 1.5) have one convex end of radius of curvature 10 cm. They are placed with the curved surfaces at a distance d as shown in the figure, with their axes (shown by the dashed line) aligned. When a point source of light P is placed inside rod S_1 on its axis at a distance of 50 cm from the curved face, the light rays emanating from it are found to be parallel to the axis inside S_2. The distance d is **(2015 Adv.)**



(a) 60 cm (b) 70 cm
(c) 80 cm (d) 90 cm

7. Assuming human pupil to have a radius of 0.25 cm and a comfortable viewing distance of 25 cm, the minimum separation between two objects that human eye can resolve at 500 nm wavelength is **(2015 Main)**

(a) 30 μm (b) 1 μm
(c) 100 μm (d) 300 μm

8. On a hot summer night, the refractive index of air is smallest near the ground and increases with height from the ground. When a light beam is directed horizontally, the Huygens principle leads us to conclude that as it travels, the light beam **(2015 Main)**

(a) becomes narrower
(b) goes horizontally without any deflection
(c) bends upwards
(d) bends downwards

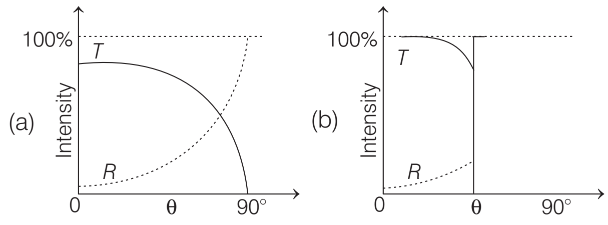
9. Two beams, A and B, of plane polarised light with mutually perpendicular planes of polarisation are seen through a polaroid. From the position when the beam A has maximum intensity (and beam B has zero intensity), a rotation of polaroid through 30° makes the two beams appear equally bright. If the initial intensities of the two beams are I_A and I_B respectively, then I_A / I_B equals **(2014 Main)**

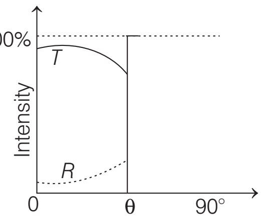
(a) 3 (b) 3/2 (c) 1 (d) 1/3

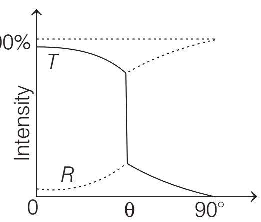
10. Diameter of a plano-convex lens is 6 cm and thickness at the centre is 3 mm. If speed of light in material of lens is 2×10^8 m/s, the focal length of the lens is **(2013 Main)**

(a) 15 cm (b) 20 cm (c) 30 cm (d) 10 cm

11. A light ray travelling in glass medium is incident on glass-air interface at an angle of incidence θ. The reflected (R) and transmitted (T) intensities, both as function of θ, are plotted. The correct sketch is **(2011)**



(c)


(d)


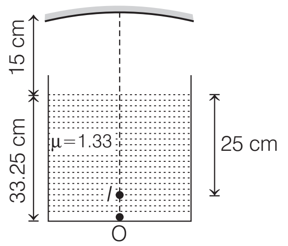
12. A biconvex lens of focal length 15 cm is in front of a plane mirror. The distance between the lens and the mirror is 10 cm. A small object is kept at a distance of 30 cm from the lens. The final image is **(2010)**

(a) virtual and at a distance of 16 cm from the mirror
(b) real and at a distance of 16 cm from the mirror
(c) virtual and at a distance of 20 cm from the mirror
(d) real and at a distance of 20 cm from the mirror

13. A light beam is travelling from Region I to Region IV (Refer Figure). The refractive index in Regions I, II, III and IV are $n_0, \frac{n_0}{2}, \frac{n_0}{6}$ and $\frac{n_0}{8}$, respectively. The angle of incidence θ for which the beam just misses entering Region IV is **(2008, 3M)**

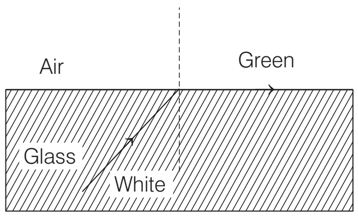
(a) $\sin^{-1}\left(\frac{3}{4}\right)$ (b) $\sin^{-1}\left(\frac{1}{8}\right)$ (c) $\sin^{-1}\left(\frac{1}{4}\right)$ (d) $\sin^{-1}\left(\frac{1}{3}\right)$

14. A container is filled with water ($\mu = 1.33$) up to a height of 33.25 cm. A concave mirror is placed 15 cm above the water level and the image of an object placed at the bottom is formed 25 cm below the water level. The focal length of the mirror is **(2005, 2M)**



(a) 10 cm (b) 15 cm
(c) 20 cm (d) 25 cm

15. White light is incident on the interface of glass and air as shown in the figure. If green light is just totally internally reflected then the emerging ray in air contains. **(2004, 2M)**



(a) yellow, orange, red (b) violet, indigo, blue
(c) all colours (d) all colours except green

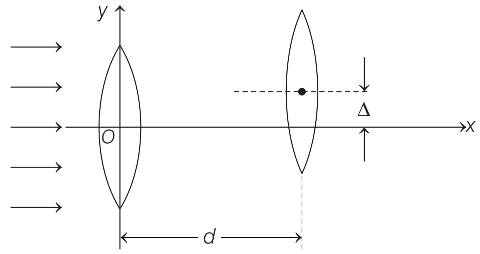
16. A concave mirror is placed on a horizontal table with its axis directed vertically upwards. Let O be the pole of the mirror and C its centre of curvature. A point object is placed at C. It has a real image, also located at C. If the mirror is now filled with water, the image will be

(a) real and will remain at C **(1998, 2M)**
(b) real and located at a point between C and ∞
(c) virtual and located at a point between C and O
(d) real and located at a point between C and O

17. A diminished image of an object is to be obtained on a screen 1.0 m from it. This can be achieved by placing

(a) a plane mirror **(1995, 2M)**
(b) a convex mirror of suitable focal length
(c) a convex lens of focal length less than 0.25 m
(d) a concave lens of suitable focal length

18. Two thin convex lenses of focal lengths f_1 and f_2 are separated by a horizontal distance d (where $d < f_1, d < f_2$) and their centres are displaced by a vertical separation Δ as shown in the figure. **(1993)**



Taking the origin of coordinates, O, at the centre of the first lens, the x and y-coordinates of the focal point of this lens system, for a parallel beam of rays coming from the left, are given by **(1993; 2M)**

(a) $x = \frac{f_1 f_2}{f_1 + f_2}$, $y = \Delta$
(b) $x = \frac{f_1(f_2 + d)}{f_1 + f_2 - d}$, $y = \frac{\Delta}{f_1 + f_2}$
(c) $x = \frac{f_1 f_2 + d(f_1 - d)}{f_1 + f_2 - d}$, $y = \frac{\Delta(f_1 - d)}{f_1 + f_2 - d}$
(d) $x = \frac{f_1 f_2 + d(f_1 - d)}{f_1 + f_2 - d}$, $y = 0$

Numerical Value

19. Sunlight of intensity $1.3\,\text{kWm}^{-2}$ is incident normally on a thin convex lens of focal length 20 cm. Ignore the energy loss of light due to the lens and assume that the lens aperture size is much smaller than its focal length. The average intensity of light, in kW m^{-2}, at a distance 22 cm from the lens on the other side is .... **(2018 Adv.)**

Passage Based Questions

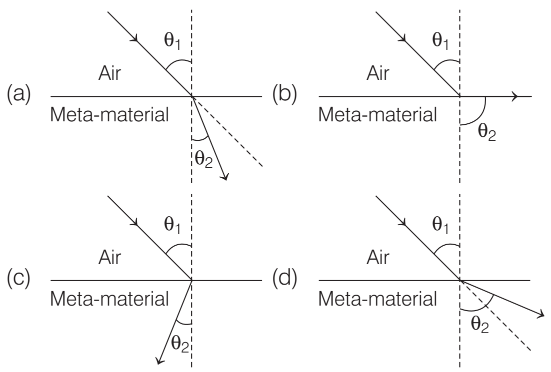
Passage 1

Most materials have the refractive index, $n > 1$. So, when a light ray from air enters a naturally occurring material, then by Snell's law, $\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$, it is understood that the refracted ray bends towards the normal. But it never emerges on the same side of the normal as the incident ray. According to electromagnetism, the refractive index of the medium is given by the relation, $n = \left(\frac{c}{v}\right) = \pm\sqrt{\varepsilon_r \mu_r}$, where c is the speed of electromagnetic waves in vacuum, v its speed in the medium, ε_r and μ_r are the relative permittivity and permeability of the medium respectively.

In normal materials, both ε_r and μ_r are positive, implying positive n for the medium. When both ε_r and μ_r are negative, one must choose the negative root of n. Such negative refractive index materials can now be artificially prepared and are called meta-materials. They exhibit significantly different optical behaviour, without violating any physical laws. Since n is negative, it results in a change in the direction of propagation of the refracted light. However, similar to normal materials, the frequency of light remains unchanged upon refraction even in meta-materials. **(2012)**

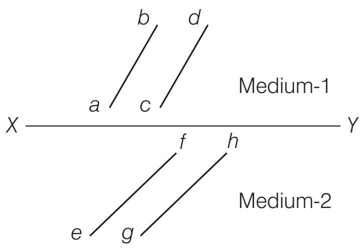
20. Choose the correct statement.
(a) The speed of light in the meta-material is $v = c\,|n|$
(b) The speed of light in the meta-material is $v = \dfrac{c}{|n|}$
(c) The speed of light in the meta-material is $v = c$
(d) The wavelength of the light in the meta-material (λ_m) is given by $\lambda_m = \lambda_{\text{air}}\,|n|$, where λ_{air} is the wavelength of the light in air.

21. For light incident from air on a meta-material, the appropriate ray diagram is



Passage 2

The figure shows a surface XY separating two transparent media, medium-1 and medium-2. The lines ab and cd represent wavefronts of a light wave travelling in medium-1 and incident on XY. The lines ef and gh represent wavefronts of the light wave in medium-2 after refraction.



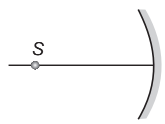
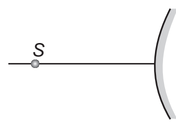
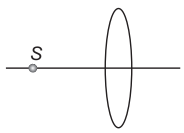
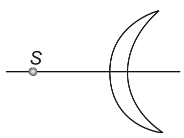
22. Light travels as a **(2007, 4M)**
(a) parallel beam in each medium
(b) convergent beam in each medium
(c) divergent beam in each medium
(d) divergent beam in one medium and convergent beam in the other medium

23. The phases of the light wave at c, d, e and f are ϕ_c, ϕ_d, ϕ_e and ϕ_f respectively. It is given that $\phi_c \neq \phi_f$ **(2007, 4M)**
(a) ϕ_c cannot be equal to ϕ_d
(b) ϕ_d can be equal to ϕ_e
(c) $(\phi_d - \phi_f)$ is equal to $(\phi_c - \phi_e)$
(d) $(\phi_d - \phi_c)$ is not equal to $(\phi_f - \phi_e)$

24. Speed of light is **(2007, 4M)**
(a) the same in medium-1 and medium-2
(b) larger in medium-1 than in medium-2
(c) larger in medium-2 than in medium-1
(d) different at b and d

Match the Columns

25. An optical component and an object S placed along its optic axis are given in Column I. The distance between the object and the component can be varied. The properties of images are given in Column II. Match all the properties of images from Column II with the appropriate components given in Column I. **(2008, 7M)**

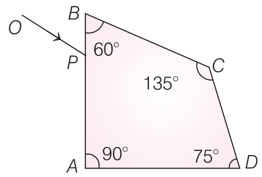
	Column I		Column II
(A)		(p)	Real image
(B)		(q)	Virtual image
(C)		(r)	Magnified image
(D)		(s)	Image at infinity

26. Some laws/processes are given in Column I. Match these with the physical phenomena given in Column II. **(2006, 4M)**

	Column I		Column II
(A)	Intensity of light received by lens	(p)	radius of aperture (R)
(B)	Angular magnification	(q)	dispersion of lens
(C)	Length of telescope	(r)	focal length f_o, f_e
(D)	Sharpness of image	(s)	spherical aberration

Objective Questions II (One or more correct option)

27. A ray OP of monochromatic light is incident on the face AB of prism $ABCD$ near vertex B at an incident angle of 60° (see figure). If the refractive index of the material of the prism is $\sqrt{3}$, which of the following is (are) correct? **(2010)**



(a) The ray gets totally internally reflected at face CD
(b) The ray comes out through face AD
(c) The angle between the incident ray and the emergent ray is 90°
(d) The angle between the incident ray and the emergent ray is 120°

28. Which of the following form(s) a virtual and erect image for all positions of the object ? **(1996, 2M)**
(a) Convex lens (b) Concave lens
(c) Convex mirror (d) Concave mirror

29. A converging lens is used to form an image on a screen. When the upper half of the lens is covered by an opaque screen **(1986, 2M)**
(a) half of the image will disappear
(b) complete image will be formed
(c) intensity of the image will increase
(d) intensity of the image will decrease

Fill in the Blanks

30. If ε_0 and μ_0 are, respectively, the electric permittivity and magnetic permeability of free space, ε and μ the corresponding quantities in a medium, the index of refraction of the medium in terms of the above parameters is ……… **(1992, 1M)**

31. A point source emits sound equally in all directions in a non-absorbing medium. Two points P and Q are at a distance 9 m and 25 m respectively from the source. The ratio of amplitudes of the waves at P and Q is …… **(1989, 2M)**

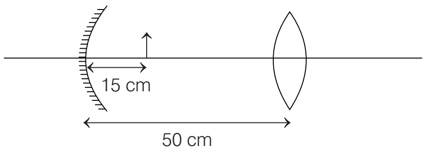
True / False

32. The intensity of light at a distance r from the axis of a long cylindrical source is inversely proportional to r. **(1981, 2M)**

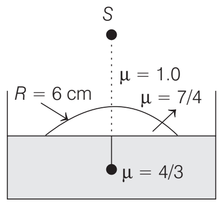
Integer Answer Type Questions

33. Consider a concave mirror and a convex lens (refractive index = 1.5) of focal length 10 cm each, separated by a distance of 50 cm in air (refractive index = 1) as shown in the figure. An object is placed at a distance of 15 cm from the mirror. Its erect image formed by this combination has magnification M_1. **(2015 Adv.)**

When the set-up is kept in a medium of refractive index $\frac{7}{6}$, the magnification becomes M_2. The magnitude $\left|\frac{M_2}{M_1}\right|$ is



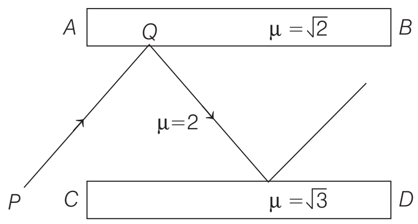
34. Water (with refractive index = $\frac{4}{3}$) in a tank is 18 cm deep. Oil of refractive index $\frac{7}{4}$ lies on water making a convex surface of radius of curvature $R = 6$ cm as shown. Consider oil to act as a thin lens. An object S is placed 24 cm above water surface. The location of its image is at x cm above the bottom of the tank. Then x is. **(2011)**



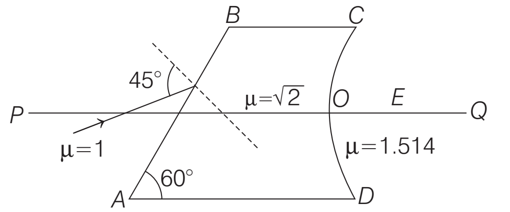
35. A large glass slab $\left(\mu = \frac{5}{3}\right)$ of thickness 8 cm is placed over a point source of light on a plane surface. It is seen that light emerges out of the top surface of the slab from a circular area of radius R cm. What is the value of R? **(2010)**

Analytical & Descriptive Questions

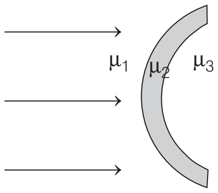
36. AB and CD are two slabs. The medium between the slabs has refractive index 2. Find the minimum angle of incidence of Q, so that the ray is totally reflected by both the slabs. **(2005, 2M)**



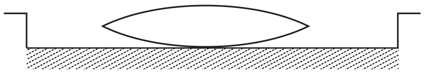
37. Figure shows an irregular block of material of refractive index $\sqrt{2}$. A ray of light strikes the face AB as shown in the figure. After refraction it is incident on a spherical surface CD of radius of curvature 0.4 m and enters a medium of refractive index 1.514 to meet PQ at E. Find the distance OE upto two places of decimal. **(2004, 2M)**



38. In the figure, light is incident on a thin lens as shown. The radius of curvature for both the surfaces is R. Determine the focal length of this system. **(2003, 2M)**



39. A thin biconvex lens of refractive index 3/2 is placed on a horizontal plane mirror as shown in the figure. The space between the lens and the mirror is then filled with water of refractive index 4/3. It is found that when a point object is placed 15 cm above the lens on its principal axis, the object coincides with its own image. On repeating with another liquid, the object and the image again coincide at a distance 25 cm from the lens. Calculate the refractive index of the liquid. **(2001, 5M)**

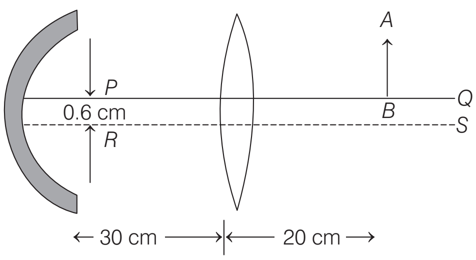


40. The refractive indices of the crown glass for blue and red light are 1.51 and 1.49 respectively and those of the flint glass are 1.77 and 1.73 respectively. An isosceles prism of angle 6° is made of crown glass. A beam of white light is incident at a small angle on this prism. The other flint glass isosceles prism is combined with the crown glass prism such that there is no deviation of the incident light.

(a) Determine the angle of the flint glass prism.

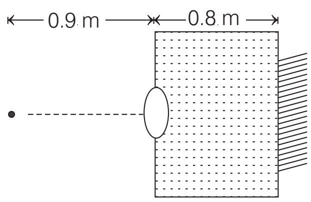
(b) Calculate the net dispersion of the combined system. **(2001, 5M)**

41. A convex lens of focal length 15 cm and a concave mirror of focal length 30 cm are kept with their optic axis PQ and RS parallel but separated in vertical direction by 0.6 cm as shown.



The distance between the lens and mirror is 30 cm. An upright object AB of height 1.2 cm is placed on the optic axis PQ of the lens at a distance of 20 cm from the lens. If $A'B'$ is the image after refraction from the lens and the reflection from the mirror, find the distance of $A'B'$ from the pole of the mirror and obtain its magnification. Also locate positions of A' and B' with respect to the optic axis RS. **(2000, 6M)**

42. A thin equiconvex lens of glass of refractive index $\mu = 3/2$ and of focal length 0.3 m in air is sealed into an opening at one end of a tank filled with water $\mu = 4/3$. On the opposite side of the lens, a mirror is placed inside the tank on the tank wall perpendicular to the lens axis, as shown in figure. The separation between the lens and the mirror is 0.8 m. A small object is placed outside the tank in front of lens. Find the position (relative to the lens) of the image of the object formed by the system **(1997C, 5M)**



43. A ray of light travelling in air is incident at grazing angle (Incident angle = 90°) on a long rectangular slab of a transparent medium of thickness $t = 1.0$ m. The point of incidence is the origin $A(0,0)$. The medium has a variable index of refraction $n(y)$ given by

$$n(y) = [ky^{3/2} + 1]^{1/2} \text{ where } k = 1.0\,(\text{m})^{-3/2}.$$

The refractive index of air is 1.0. **(1995, 10 M)**

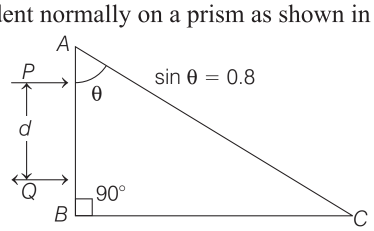
(a) Obtain a relation between the slope of the trajectory of the ray at a point $B(x, y)$ in the medium and the incident angle at that point.

(b) Obtain an equation for the trajectory $y(x)$ of the ray in the medium.

(c) Determine the coordinates (x_1, y_1) of the point P, where the ray intersects the upper surface of the slab-air boundary.

(d) Indicate the path of the ray subsequently.

44. Two parallel beams of light P and Q (separation d) containing radiations of wavelengths 4000 Å and 5000Å (which are mutually coherent in each wavelength separately) are incident normally on a prism as shown in figure.



The refractive index of the prism as a function of wavelength is given by the relation, $\mu(\lambda) = 1.20 + \dfrac{b}{\lambda^2}$ where λ is in Å and b is positive constant. The value of b is such that the condition for total reflection at the face AC is just satisfied for one wavelength and is not satisfied for the other. **(1991, 2+2+4M)**

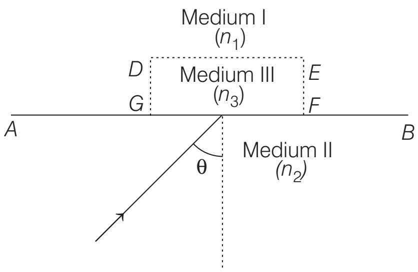
(a) Find the value of b.

(b) Find the deviation of the beams transmitted through the face AC.

(c) A convergent lens is used to bring these transmitted beams into focus. If the intensities of the upper and the lower beams immediately after transmission from the face AC, are $4I$ and I respectively, find the resultant intensity at the focus.

45. Monochromatic light is incident on a plane interface AB between two media of refractive indices n_1 and n_2 $(n_2 > n_1)$ at an angle of incidence θ as shown in the figure.

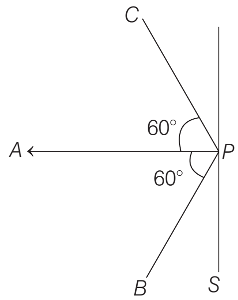
The angle θ is infinitesimally greater than the critical angle for the two media so that total internal reflection takes place. Now if a transparent slab $DEFG$ of uniform thickness and of refractive index n_3 is introduced on the interface (as shown in the figure), show that for any value of n_3 all light will ultimately be reflected back again into medium II.



Consider separately the cases : **(1986, 6M)**

(a) $n_3 < n_1$ and (b) $n_3 > n_1$

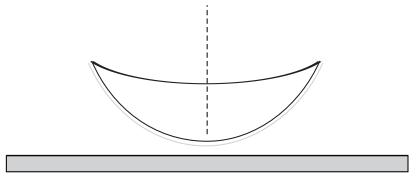
46. Screen S is illuminated by two point sources A and B. Another source C sends a parallel beam of light towards point P on the screen (see figure). Line AP is normal to the screen and the lines AP, BP and CP are in one plane. The distances AP, BP and CP are in one plane. The radiant powers of sources A and B are 90 W and 180 W respectively. The beam from C is of intensity 20 W/m^2. Calculate intensity at P on the screen. **(1982, 5M)**



47. The convex surface of a thin concavo-convex lens of glass of refractive index 1.5 has a radius of curvature 20 cm. The concave surface has a radius of curvature 60 cm. The convex side is silvered and placed on a horizontal surface. **(1981, 2M)**

(a) Where should a pin be placed on the optic axis such that its image is formed at the same place?

(b) If the concave part is filled with water of refractive index 4/3, find the distance through which the pin should be moved, so that the image of the pin again coincides with the pin.



Answers

Topic 1

1. (c) **2.** (c) **3.** (c) **4.** (d)
5. (a) **6.** (b) **7.** (b) **8.** (d)
9. (d) **10.** (c) **11.** (d) **12.** (c, d)
13. (–1.5) **14.** 3

Topic 2

1. (c) **2.** (c) **3.** (d)
4. (d) **5.** (c) **6.** (c) **7.** (c)
8. (c) **9.** (b) **10.** (b) **11.** (d)
12. (a) **13.** (b) **14.** (a)
15. (a) **16.** (a) **17.** (a, c, d) **18.** (a, c)
19. (c, d) **20.** 30 cm to the right of P. Image will be virtual
21. 4000Å, 5×10^{14} Hz
22. 2×10^8 m/s, 4×10^{-7}m
23. (a,c) **24.** (d) **25.** 8
26. $\frac{4}{3}$ **27.** $\frac{1}{5\sqrt{2}}(3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 5\hat{\mathbf{k}})$
28. $\sqrt{2}$ **29.** (a) – 6 mm, –5mm
30. (a) $\sqrt{2}$ (b) No

Topic 3

1. (d) **2.** (c) **3.** (d) **4.** (d) **5.** (c)
6. (b) **7.** (d) **8.** (c) **9.** (c)
10. (b) **11.** (d) **12.** (b) **13.** (c)
14. (b) **15.** (b) **16.** (c) **17.** (d)
18. (b) **19.** (c) **20.** (d) **21.** (a)
22. (b) **23.** (b) **24.** (c) **25.** (a)
26. (a, d) **27.** (A) → p; (B) → s; (C) → r; (D) → p
28. (A) → p, r; (B) → q, s, t; (C) → p, r, t; (D) → q, s
29. 0.125, 0.5 **30.** 60 **31.** 15
32. T **33.** F **34.** 6
35. 0.09 m/s, 0.3/s **36.** 0.4 m, 0.6 m **37.** See the hints

38. (a) 1 mm (b) Increase **39.** 75 cm

40. (a) + 24 cm (b) at 12 cm (d) $v = -30$ cm

41. $\mu_1 < \mu_2$ **42.** at a distance of 11 cm, virtual

Topic 4

1. (c) **2.** (c)

3. (a) **4.** (a) **5.** (d) **6.** (c)
7. (d) **8.** (a) **9.** (b) **10.** (c)
11. (c) **12.** (c) **13.** (a) **14.** (a)
15. (a, b, c) **16.** 15° **17.** 30° **18.** T
19. (2) **20.** (a) 60° (b) 60°

21. (a) zero (b) 1500Å

22. (a) 600 nm (b) $\sin^{-1}\left[\frac{3}{4}\right]$

23. (a) $i_1 = \sin^{-1}\left\{\frac{1}{\sqrt{2}}\left(\sqrt{n^2 - n_1^2} - n_1\right)\right\}$

(b) 73° **24.** $\mu = \sqrt{3}$

Topic 5

1. (a) **2.** (a) **3.** (b)

4. (d) **5.** (c) **6.** (c) **7.** (a)
8. (d) **9.** (a, b, c, d) **10.** smaller

Topic 6

1. (c) **2.** (c) **3.** (c) **4.** (*)
5. (c) **6.** (a) **7.** (d) **8.** (c)
9. (d) **10.** (d) **11.** (c) **12.** (d)
13. (b) **14.** (a) **15.** (d) **16.** (b)
17. (c) **18.** (d) **19.** (b) **20.** (c)
21. (d) **22.** (b) **23.** (a) **24.** (b)
25. (b) **26.** (a) **27.** (a) **28.** (d)
29. (d) **30.** (d) **31.** (c) **32.** (d)

33. (A) $\rightarrow$ p, s; (B) $\rightarrow$ q; (C) $\rightarrow$ t; (D) $\rightarrow$ r, s, t

34. (c, d) **35.** (a, b) **36.** (a, b, c) **37.** (d)
38. (a, b) **39.** (a, c) **40.** (a, c) **41.** (b, d)
42. 2.945×10^{-4} **43.** 2 **44.** F **45.** T

46. (3) **47.** 3.5 mm

48. (a) circular (b) $\frac{1}{16}$ (c) 300 nm

49. 2 cm above point Q on side CD, $\mu = 1.0016$

50. $3.6t = \left(n - \frac{1}{2}\right)\lambda$ with $n = 1, 2, 3 \ldots$, 90 nm

51. (a) 4.33 mm (b) $I = \frac{3I_{max}}{4}$ (c) 650 nm; 433.33nm

52. (a) $\pm$ 0.26 m, $\pm$ 1.13 m (b) 0.26 m, 1.13 m

53. 9.3 μm

54. 7×10^{-6}m, 1.6, -5.71×10^{-5}m

55. (a) 0.63 mm (b) 1.579 μm

56. 4200 Å, 1.43 **57.** $\frac{1}{49}$ **58.** 7×10^{-6} W

59. (a) 1.17 mm (b) 1.56 mm **60.** 5892Å

Topic 7

1. (c) **2.** (*) **3.** (d) **4.** (c)
5. (a) **6.** (b) **7.** (a) **8.** (c)
9. (d) **10.** (c) **11.** (c) **12.** (b)
13. (b) **14.** (c) **15.** (a) **16.** (d)
17. (c) **18.** (c) **19.** (130.0) **20.** (b)
21. (c) **22.** (a) **23.** (c) **24.** (b)

25. (A) $\rightarrow$ p, q, r, s; (B) $\rightarrow$ q; (C) $\rightarrow$ p, q, r, s; (D) $\rightarrow$ p, q, r, s

26. (A) $\rightarrow$ p; (B) $\rightarrow$ r; (C) $\rightarrow$ r; (D) $\rightarrow$ p, q, r

27. (a, b, c) **28.** (b, c) **29.** (b, d) **30.** $\sqrt{\frac{\varepsilon\mu}{\varepsilon_0\mu_0}}$

31. $\frac{25}{9}$ **32.** T **33.** (7) **34.** (2)

35. 6 **36.** 60° **37.** 6.06 m

38. $\frac{\mu_3 R}{\mu_3 - \mu_1}$ **39.** 1.6 **40.** (a) 4° (b) – 0.04°

41. 15 cm, $-3/2$

42. 0.9 m from the lens (rightwards) or 0.1 m behind the mirror

43. (a) Slope = cot i (b) $4y^{1/4} = x$ (c) (4m, 1m)
(d) the ray will emerge grazingly

44. (a) $b = 8 \times 10^5 (\text{Å})^2$ (b) $\delta_{4000\text{Å}} = 37°, \delta_{5000\text{Å}} = 27.13°$ (c) $9I$

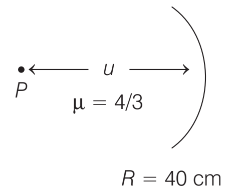
46. 13.97 W/m^2

47. (a) 15cm (b) 1.16 cm (downwards) reflection of light

Hints & Solutions

Topic 1 Reflection of Light

1. In the given case,



$u = -5\,\text{cm}$

Focal length,

$$f = \frac{-R}{2} = \frac{-40}{2} = -20\,\text{cm}$$

Now, using mirror formula,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

$$= -\frac{1}{20} + \frac{1}{5} = +\frac{3}{20}$$

$$\Rightarrow \quad v = +\frac{20}{3}\,\text{cm}$$

For the light getting refracted at water surface, this image will act as an object.

So, distance of object, $d = 5\,\text{cm} + \frac{20}{3}\,\text{cm} = \frac{35}{3}\,\text{cm}$

(below the surface). Let's assume final image at distance d after refraction.

$$\frac{d'}{d} = \frac{\mu_2}{\mu_1} \Rightarrow d' = d\left(\frac{\mu_2}{\mu_1}\right) = \left(\frac{35}{3}\,\text{cm}\right)\left(\frac{1}{\frac{4}{3}}\right)$$

$$= \frac{35}{3} \times \frac{3}{4}\,\text{cm} = \frac{35}{4}\,\text{cm} = 8.75\,\text{cm} \approx 8.8\,\text{cm}$$

2. Given, focal length of concave mirror,

$$f = -\,0.4\text{ m}$$

Magnification = 5

We know that, magnification produced by a mirror,

$$m = -\frac{\text{image distance}}{\text{object distance}}$$

$$\Rightarrow \quad \frac{v}{u} = -\,5 \text{ or } v = -\,5u$$

Using mirror formula,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

Substituting the given values in the above equation, we get

$$\frac{1}{-\,5u} + \frac{1}{u} = -\frac{1}{0.4}$$

$$\Rightarrow \quad \frac{4}{5u} = -\frac{1}{0.4}$$

$$\Rightarrow \quad u = -\frac{1.6}{5} = -\,0.32\text{ m}$$

Alternate Solution

Magnification produced by a mirror can also be given as

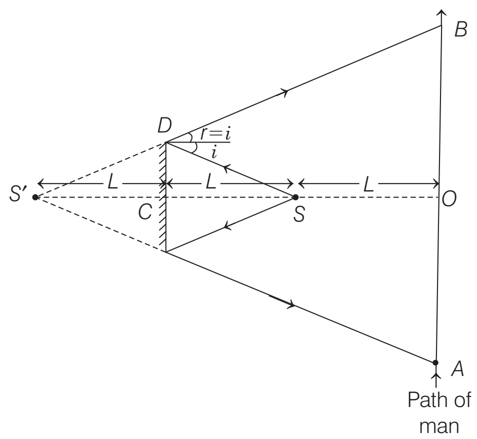
$$m = \frac{f}{f-u}$$

Substituting the given values, we get

$$5 = \frac{-0.4}{-0.4 - u}$$

or $\quad u = -\,0.32\text{ m}$

3. Light from mirror is reflected in a straight line and It is appear to come from its image formed at same distance (as that of source) behind the mirror as shown in the ray diagram below.



From ray diagram in similar triangles, $\Delta S'CD$ and $\Delta S'OB$, we have

$$\frac{S'C}{S'O} = \frac{CD}{OB}$$

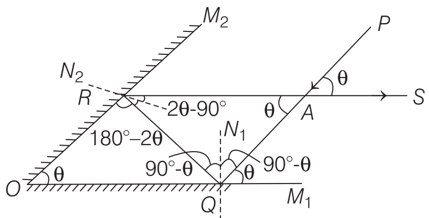
So, $\quad OB = \frac{CD \times S'O}{S'C} = \frac{\frac{d}{2} \times 3L}{L} = \frac{3d}{2}$

Also, $\quad OA = \frac{3d}{2}$

So, distance over which man can see the image S' is

$$\frac{3d}{2} + \frac{3d}{2} = 3d$$

4. The given condition is shown in the figure given below, where two plane mirror inclined to each other such that a ray of light incident on the first mirror (M_1) and parallel to the second mirror (M_2) is finally reflected from second mirror (M_2) parallel to the first mirror.



where, PQ = incident ray parallel to the mirror M_2,

QR = reflected ray from the mirror M_1,

RS = reflected ray from the mirror M_2 which is parallel to the M_1 and θ = angle between M_1 and M_2.

According to geometry,

$\angle PAS = \angle PQM_1 = \theta$ (angle on same line)

$\angle AQN_1$ = angle of incident = $90 - \theta$

$\angle N_1QR$ = angle of reflection = $(90 - \theta)$.

Therefore, for triangle ΔORQ, (according to geometry)

$$\angle\theta + \angle\theta + \angle ORQ = 180°$$

$$\angle ORQ = 180° - 2\theta \qquad \text{...(i)}$$

For normal N_2,

angle of incident = angle of reflection

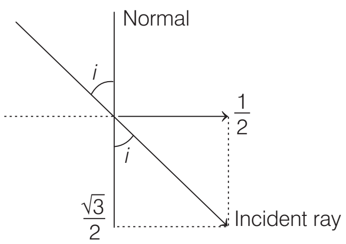
$$= 2\theta - 90° \qquad \text{...(ii)}$$

Therefore, for the triangle ΔRAQ

$$\Rightarrow \quad 4\theta - 180° + 180° - 2\theta + \theta = 180$$

$$3\theta = 180° \Rightarrow \theta = 60°$$

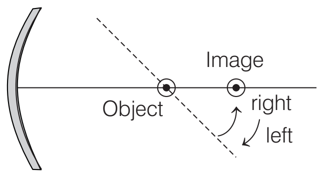
5.



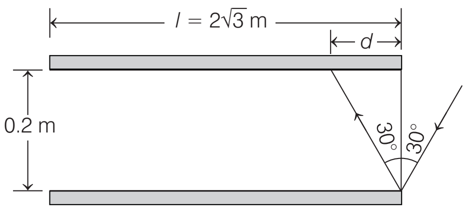
$$\tan i = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \qquad i = 30°$$

6. Since object and image move in opposite directions, the positioning should be as shown in the figure. Object lies between focus and centre of curvature $f < x < 2f$.



7. $d = 0.2 \tan 30° = \dfrac{0.2}{\sqrt{3}}$

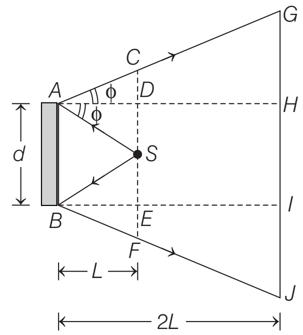


$$\frac{l}{d} = \frac{2\sqrt{3}}{0.2/\sqrt{3}} = 30$$

Therefore, maximum number of reflections are 30.

NOTE Answer of this question may be 30 or 31.

8. The ray diagram will be as follows :



$$HI = AB = d$$

$$DS = CD = \frac{d}{2}$$

Since, $AH = 2AD$

$$\therefore \qquad GH = 2CD = 2\frac{d}{2} = d$$

Similarly, $\quad IJ = d$

$$\therefore \qquad GJ = GH + HI + IJ$$

$$= d + d + d = 3d$$

9. From the mirror formula

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \qquad (f = \text{constant}) \text{ ...(i)}$$

$$-v^{-2}dv - u^{-2}du = 0$$

or $$|dv| = \left|\frac{v^2}{u^2}\right| |du| \qquad \text{...(ii)}$$

Here, $|dv|$ = size of image

$|du|$ = size of object (short) lying along the axis $= b$

Further, from Eq. (i), we can find

$$\frac{v^2}{u^2} = \left(\frac{f}{u - f}\right)^2$$

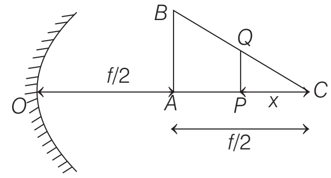
Substituting these values in Eq. (ii), we get

$$\text{Size of image } = b\left(\frac{f}{u - f}\right)^2$$

$\therefore$ Correct option is (d).

10. Laws of reflection can be applied to any type of surface.

11. Image of point A



$$\frac{PQ}{x} = \frac{AB}{f/2} \Rightarrow PQ = \frac{2(AB)x}{f}$$

For A :

$$\frac{1}{v} + \frac{1}{[-(f/2)]} = \frac{1}{-f} \Rightarrow v = f$$

$$\Rightarrow \quad \frac{I_{AB}}{AB} = -\frac{v}{u} = -\frac{f}{\left(-\frac{f}{2}\right)}$$

$$\Rightarrow \quad I_{AB} = 2\,AB$$

For height of PQ,

$$\frac{1}{v} + \frac{1}{-[(f-x)]} = \frac{1}{-f}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{(f-x)} - \frac{1}{f} \Rightarrow v = \frac{f(f-x)}{x}$$

$$\Rightarrow \quad \frac{I_{PQ}}{PQ} = -\frac{v}{u} = \frac{f(f-x)}{x[(f-x)]} = \left(\frac{f}{x}\right)$$

$$\Rightarrow \quad I_{PQ} = \frac{f}{x}PQ = \left(\frac{f}{x}\right)\left(\frac{2(AB)x}{f}\right)\left[\because PQ = \frac{2(AB)x}{f}\right]$$

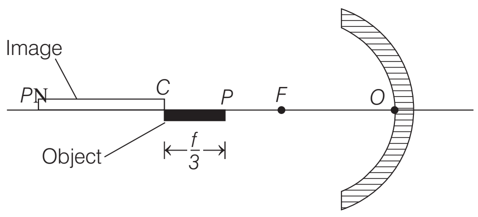
$$I_{PQ} = 2AB$$

(Size of image is independent of x. So, final image will be of same height terminating at infinity.)

12. Values of options (c) and (d) don't match with the mirror formula.

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

13. F = focus, C = centre of curvature



When the object lies between F and C, image is real, elongated and inverted. As one end of rod just touches its image, this end should lie at C. Because image of object at C is at C itself.

Let P' be the image of other end of rod P.

For P

$$u = -(2f - f/3) = -\frac{5f}{3}$$

Applying the mirror formula : $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$$\text{or} \quad \frac{1}{v} - \frac{3}{5f} = \frac{1}{-f} \quad \Rightarrow \quad \frac{1}{v} = \frac{3}{5f} - \frac{1}{f}$$

$$\text{or} \quad v = -\frac{5f}{2} \quad \text{or} \quad OP' = \frac{5f}{2}$$

$\therefore$ Length of image of rod

$$CP' = OP' - OC = \frac{5f}{2} - 2f = \frac{f}{2}$$

$$\therefore \quad \text{Magnification} = -\left(\frac{f/2}{f/3}\right) = -1.5$$

Here, negative sign implies that image is inverted.

14. Using mirror formula twice,

$$\frac{1}{+25/3} + \frac{1}{-u_1} = \frac{1}{+10}$$

$$\text{or} \quad \frac{1}{u_1} = \frac{3}{25} - \frac{1}{10}$$

$$\text{or} \quad u_1 = 50\text{ m and } \frac{1}{(+50/7)} + \frac{1}{-u_2} = \frac{1}{+10}$$

$$\therefore \quad \frac{1}{u_2} = \frac{7}{50} - \frac{1}{10} \text{ or } u_2 = 25\text{ m}$$

$$\text{Speed of object} = \frac{u_1 - u_2}{\text{time}} = \frac{25}{30}\text{ ms}^{-1} = 3\text{ kmh}^{-1}$$

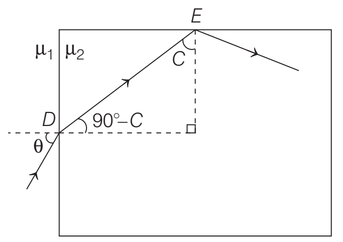
$\therefore$ Answer is 3.

Topic 2 Refraction of Light and TIR

1. **Key Idea** The critical angle is defined as the angle of incidence that provides an angle of refraction of 90°.

So, $\theta_c = \sin^{-1}\frac{\mu_2}{\mu_1}$

For total internal reflection, angle of incidence(i) at medium interface must be greater than critical angle (C).



where,

$$\sin C = \frac{\mu_1}{\mu_2} \quad \ldots\text{(i)}$$

Now, in given arrangement,

at point D,

$$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} \quad \text{(Snell's law)}$$

$$\Rightarrow \quad \frac{\sin\theta}{\sin(90° - C)} = \frac{\mu_2}{\mu_1} \Rightarrow \frac{\sin\theta}{\cos C} = \frac{\mu_2}{\mu_1}$$

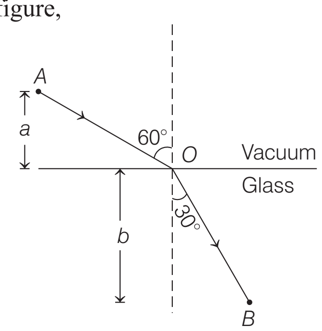
$$\Rightarrow \quad \sin\theta = \frac{\mu_2}{\mu_1}\cdot\cos C = \frac{\mu_2}{\mu_1}\sqrt{1-\sin^2 C} \quad \text{[from Eq. (i)]}$$

$$= \frac{\mu_2}{\mu_1}\sqrt{1-\frac{\mu_1^2}{\mu_2^2}} = \sqrt{\frac{\mu_2^2}{\mu_1^2} - 1} \Rightarrow \theta = \sin^{-1}\sqrt{\left(\frac{\mu_2^2}{\mu_1^2} - 1\right)}$$

For TIR at E, $i > C$

$$\Rightarrow \quad \theta < \sin^{-1}\sqrt{\left(\frac{\mu_2^2}{\mu_1^2} - 1\right)}$$

2. From the figure,



$$\cos 60° = \frac{a}{AO}$$

$$\Rightarrow \quad AO = \frac{a}{\cos 60°} = 2a \quad \text{...(i)}$$

and $$\cos 30° = \frac{b}{BO}$$

or $$BO = \frac{b}{\cos 30°} = \frac{2}{\sqrt{3}}\, b \quad \text{... (ii)}$$

Optical path length of light ray

$$= AO + \mu\,(BO) \quad \text{... (iii)}$$

Here, μ can be determined using Snell's law, i.e.

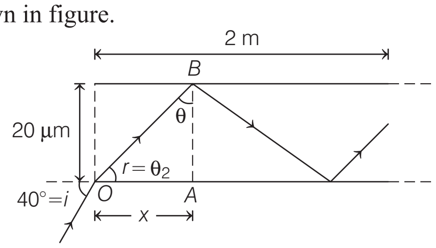
$$\mu = \frac{\sin 60°}{\sin 30°} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3} \quad \text{... (iv)}$$

Substituting the values from Eqs. (i), (ii) and (iv) in Eq. (iii), we get

$$\therefore \quad \text{Optical path} = 2a + (\sqrt{3} \times \frac{2}{\sqrt{3}} b)$$

$$= 2a + 2b$$

3. Total internal reflection occurs through given glass rod as shown in figure.



From Snell's law, $n_1 \sin i = n_2 \sin r$

where, $n_1 = 1$, $n_2 = 1.31$ and $i = 40°$

So, we get

$$1 \sin 40° = 1.31 \sin r \Rightarrow \sin r = \frac{0.64}{1.31} = 0.49 \approx 0.5$$

So, $r = 30°$

From ΔOAB, $\theta = 90 - r = 60°$

Now, $\tan\theta = \dfrac{x}{20\,\mu\text{m}}$

$$\Rightarrow \quad x = 20\sqrt{3}\ \mu\text{m} \qquad [\because \tan 60° = \sqrt{3}]$$

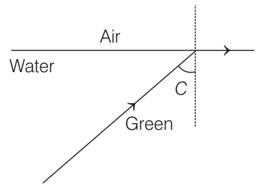
One reflection occurs in $20\sqrt{3}$ μm.

$\therefore$ Total number of reflections occurring in 2m

$$= n = \frac{2\text{m}}{20\sqrt{3}\ \mu\text{m}} = \frac{2}{20\sqrt{3} \times 10^{-6}}$$

$$= 57735 \text{ reflections} \approx 57000 \text{ reflections}$$

4. For total internal reflection of light take place, following conditions must be obeyed.



(*i*) The ray must travel from denser to rarer medium.

(*ii*) Angle of incidence (θ) must be greater than or equal to critical angle (C)

i.e. $$C = \sin^{-1}\left[\frac{\mu_{\text{rarer}}}{\mu_{\text{denser}}}\right]$$

Here, $\sin C = \dfrac{1}{n_{\text{water}}}$ and $n_{\text{water}} = a + \dfrac{b}{\lambda^2}$

If frequency is less $\Rightarrow \lambda$ is greater and hence, RI $n_{(\text{water})}$ is less and therefore, critical angle increases. So, they do not suffer reflection and come out at angle less than 90°.

5. At point Q angle of incidence is critical angle θ_C, where

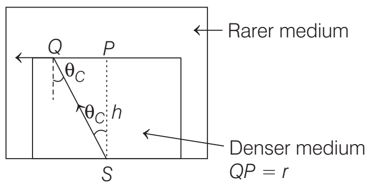
$$\sin\theta_C = \frac{\mu_l}{\mu_{\text{block}}}$$

In ΔPQS, $$\sin\theta_C = \frac{r}{\sqrt{r^2 + h^2}}$$

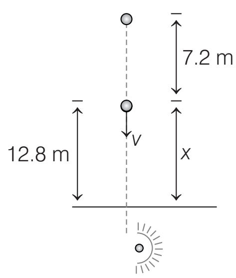
$$\therefore \quad \frac{\mu_l}{\mu_{\text{block}}} = \frac{r}{\sqrt{r^2 + h^2}}$$

$$\Rightarrow \quad \mu_l = \frac{r}{\sqrt{r^2 + h^2}} \times 2.72$$

$$= \frac{5.77}{11.54} \times 2.72 = 1.36$$



6. $v = \sqrt{2gh} = \sqrt{2 \times 10 \times 7} = 12\ \text{ms}^{-1}$



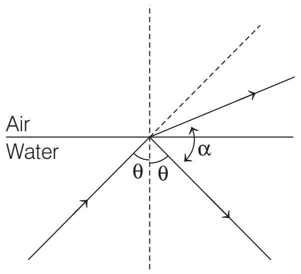
In this case when eye is inside water,

$$x_{\text{app.}} = \mu x$$

$$\therefore \qquad \frac{dx_{\text{app.}}}{dt} = \mu \cdot \frac{dx}{dt}$$

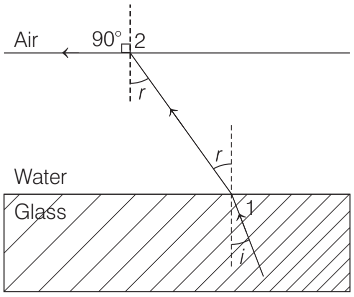
or $$v_{\text{app.}} = \mu v = \frac{4}{3} \times 12 = 16 \text{ ms}^{-1}$$

7. Since $\theta < \theta_c$, both reflection and refraction will take place. From the figure we can see that angle between reflected and refracted rays α is less than $180° - 2\theta$.



8. When the object is placed at the centre of the glass sphere, the rays from the object fall normally on the surface of the sphere and emerge undeviated.

9. Applying Snell's law ($\mu \sin i$ = constant) at 1 and 2, we have



$$\mu_1 \sin i_1 = \mu_2 \sin i_2$$

Here, $$\mu_1 = \mu_{\text{glass}},\ i_1 = i$$
$$\mu_2 = \mu_{\text{air}} = 1 \text{ and } i_2 = 90°$$

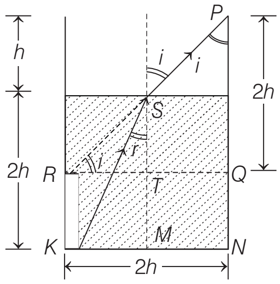
$$\therefore \qquad \mu_g \sin i = (1)(\sin 90°) \text{ or } \mu_g = \frac{1}{\sin i}$$

10. $$PQ = QR = 2h \Rightarrow \angle i = 45°$$

$$\therefore \qquad ST = RT = h = KM = MN$$

So, $$KS = \sqrt{h^2 + (2h)^2} = h\sqrt{5}$$

$$\therefore \qquad \sin r = \frac{h}{h\sqrt{5}} = \frac{1}{\sqrt{5}}$$



$$\therefore \qquad \mu = \frac{\sin i}{\sin r} = \frac{\sin 45°}{1/\sqrt{5}} = \sqrt{\frac{5}{2}}$$

11. Applying Snell's law at B and C,

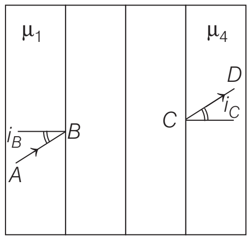
$\mu \sin i$ = constant or

$$\mu_1 \sin i_B = \mu_4 \sin i_C$$

But $AB \parallel CD$

$$\therefore \qquad i_B = i_C$$

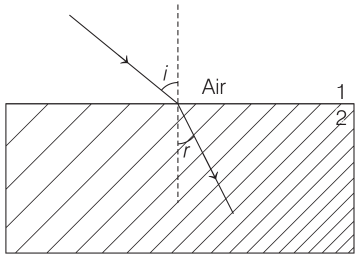
or $$\mu_1 = \mu_4$$



NOTE The Snell's law, which the students read in their plus two syllabus $\mu = \frac{\sin i}{\sin r}$ is derived from

$\mu \sin i$ = constant or $\mu_1 \sin i_1 = \mu_2 \sin i_2$

Here : $\mu_1 = 1, \mu_2 = \mu$, or $\sin i = \mu \sin r$ or $\mu = \dfrac{\sin i}{\sin r}$



12. Rays come out only from CD, means rays after refraction from AB get total internally reflected at AD.

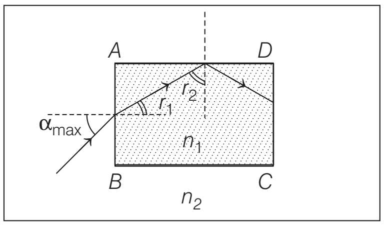
From the figure

$$r_1 + r_2 = 90°$$

$$\therefore \qquad r_1 = 90° - r_2$$

$$(r_1)_{\max} = 90° - (r_2)_{\min} \text{ and } (r_2)_{\min} = \theta_C$$

(for total internal reflection at AD)



where, $$\sin\theta_C = \frac{n_2}{n_1}$$

or $$\theta_C = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

$$\therefore \qquad (r_1)_{\max} = 90° - \theta_C$$

Now, applying Snell's law at face AB

$$\frac{n_1}{n_2} = \frac{\sin \alpha_{\max}}{\sin (r_1)_{\max}}$$

$$= \frac{\sin \alpha_{\max}}{\sin (90° - \theta_C)}$$

$$= \frac{\sin \alpha_{max}}{\cos\theta_C}$$

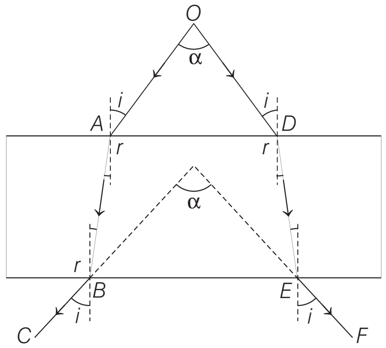
or $$\sin \alpha_{max} = \frac{n_1}{n_2}\cos\theta_C$$

$\therefore$ $$\alpha_{max} = \sin^{-1}\left[\frac{n_1}{n_2}\cos\theta_C\right]$$

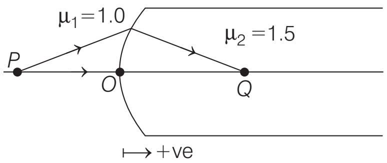
$$= \sin^{-1}\left[\frac{n_1}{n_2}\cos\left(\sin^{-1}\frac{n_2}{n_1}\right)\right]$$

13. Divergence angle will remain unchanged because in case of a glass slab every emergent ray is parallel to the incident ray. However, the rays are displaced slightly towards outer side.

(In the figure $OA \parallel BC$ and $OD \parallel EF$)



14. Let us say $PO = OQ = X$



Applying $$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

Substituting the values with sign

$$\frac{1.5}{+X} - \frac{1.0}{-X} = \frac{1.5 - 1.0}{+R}$$

(Distances are measured from O and are taken as positive in the direction of ray of light)

$\therefore$ $$\frac{2.5}{X} = \frac{0.5}{R}$$

$\therefore$ $$X = 5R$$

15. $r + r' + 90° = 180°$

$\therefore$ $$r' = 90° - r$$

Further, $$i = r$$

Applying Snell's law, $\mu_D \sin i = \mu_R \sin r'$

or $\mu_D \sin r = \mu_R \sin(90° - r) = \mu_R \cos r$

$\therefore$ $$\frac{\mu_R}{\mu_D} = \tan r,\ \theta_C = \sin^{-1}\left(\frac{\mu_R}{\mu_D}\right) = \sin^{-1}(\tan r)$$

16. $\lambda = \dfrac{v}{f}$

In moving from air to glass, f remains unchanged while v decreases. Hence, λ should decrease.

17. From Snell's law,

$$n \sin\theta = \text{constant}$$

$\therefore$ $n_1 \sin\theta_i = n_2 \sin\theta_f$

Further, l will depend on n_1 and $n(z)$. But it will be independent of n_2.

18. $\dfrac{1}{f_{film}} = (n_1 - 1)\left(\dfrac{1}{R} - \dfrac{1}{R}\right) \Rightarrow f_{film} = \infty$ (infinite)

$\therefore$ There is no effect of presence of film.

From Air to Glass

Using the equation $$\frac{n_2}{v} - \frac{1}{u} = \frac{n_2 - 1}{R}$$

$$\frac{1.5}{v} - \frac{1}{\infty} = \frac{1.5 - 1}{R} \Rightarrow v = 3R$$

$\therefore$ $$f_1 = 3R$$

From Glass to Air Again using the same equation

$$\frac{1}{v} - \frac{n_2}{u} = \frac{1 - n_2}{-R} \Rightarrow \frac{1}{v} - \frac{1.5}{\infty} = \frac{1 - 1.5}{-R} \Rightarrow v = 2R$$

$\therefore$ $$f_2 = 2R$$

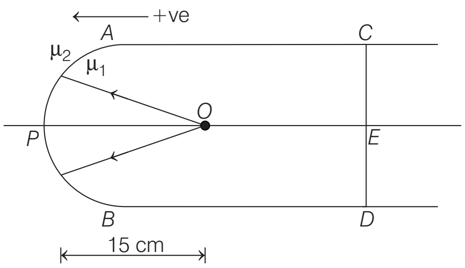
19. For total internal reflection to take place
Angle of incidence, $i >$ critical angle, θ_c

or $\sin i > \sin\theta_c$ or $\sin 45° > 1/n$

or $\dfrac{1}{\sqrt{2}} > \dfrac{1}{n}$ or $n > \sqrt{2}$ or $n > 1.414$

Therefore, possible values of n can be 1.5 or 1.6 in the given options.

20. Rays starting from O will suffer single refraction from spherical surface APB. Therefore, applying



$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{1.0}{v} - \frac{2.0}{-15} = \frac{1.0 - 2.0}{-10}$$

$$\frac{1}{v} = \frac{1}{10} - \frac{1}{7.5}$$

or $$v = -30\text{ cm}$$

Therefore, image of O will be formed at 30 cm to the right of P. Note that image will be virtual. There will be no effect of CED.

21. $\lambda_{\text{medium}} = \dfrac{\lambda_{\text{air}}}{\mu} = \dfrac{6000}{1.5} = 4000\ \text{Å}$

$$f = \frac{v_{\text{air}}}{\lambda_{\text{air}}} = \frac{v_{\text{medium}}}{\lambda_{\text{medium}}} = \frac{3.0\times 10^{8}}{6.0\times 10^{-7}} = 5.0\times 10^{14}\ \text{Hz}$$

Frequency remains unchanged.

22. $v = \dfrac{c}{\mu} = \dfrac{3\times 10^{8}}{1.5} = 2.0\times 10^{8}\ \text{m/s}$

$$\lambda = \frac{v}{f} = \frac{2.0\times 10^{8}}{5.0\times 10^{14}} = 4.0\times 10^{-7}\ \text{m}$$

23. $\dfrac{4}{3}\sin i = \dfrac{\sqrt{45}}{4}\sin(90-\theta_c) = \dfrac{\sqrt{45}}{4}\cos\theta_c$

$$\sin\theta_c = \frac{n_2}{n_1}$$

$$\therefore \qquad \cos\theta_c = \sqrt{1-\left(\frac{n_2}{n_1}\right)^2}$$

$$\Rightarrow \qquad \frac{4}{3}\sin i = \frac{\sqrt{45}}{4}\,\frac{3}{\sqrt{45}}$$

$$\sin i = \frac{9}{16}$$

In second case,

$$\sin\theta_c = \frac{n_2}{n_1} = \frac{7}{8} \Rightarrow \cos\theta_c = \frac{\sqrt{15}}{8}$$

$$\frac{16}{3\sqrt{15}}\sin i = \frac{8}{5}\sin(90-\theta_c)$$

Simplifying we get,

$$\sin i = \frac{9}{16}$$

Same approach can be adopted for other options. Correct answers are (a) and (c).

24. (1) $\sin i_m = n_1 \sin(90° - \theta_c)$

$\Rightarrow \sin i_m = n_1 \cos\theta_c$

$\Rightarrow \quad NA = n_1\sqrt{1-\sin^2\theta_c}$

$$= n_1\sqrt{1-\frac{n_2^2}{n_1^2}} = \sqrt{n_1^2 - n_2^2}$$

Substituting the values we get,

$$NA_1 = \frac{3}{4} \text{ and } NA_2 = \frac{\sqrt{15}}{5} = \sqrt{\frac{3}{4}}$$

$$NA_2 < NA_1$$

Therefore, the numerical aperture of combined structure is equal to the lesser of the two numerical aperture, which is NA_2.

25. But this value of refractive index is not possible.

$$1.6\sin\theta = (n - m\Delta n)\sin 90°$$

$$1.6\sin\theta = n - m\Delta n$$

$$1.6\times\frac{1}{2} = 1.6 - m(0.1)$$

$$0.8 = 1.6 - m\,(0.1)$$

$$m\times 0.1 = 0.8$$

$$m = 8$$

26. Applying $\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2-\mu_1}{R}$

First on plane surface

$$\frac{1.5}{AI_1} - \frac{1}{-mR} = \frac{1.5-1}{\infty} = 0 \qquad (R=\infty)$$

$$\therefore \qquad AI_1 = -(1.5\,mR)$$

Then, on curved surface

$$\frac{1}{\infty} - \frac{1.5}{-(1.5\,mR + R)} = \frac{1-1.5}{-R}$$

[$v = \infty$, because final image is at infinity]

$$\Rightarrow \qquad \frac{1.5}{(1.5\,m+1)R} = \frac{0.5}{R}$$

$$\Rightarrow \qquad 3 = 1.5m + 1$$

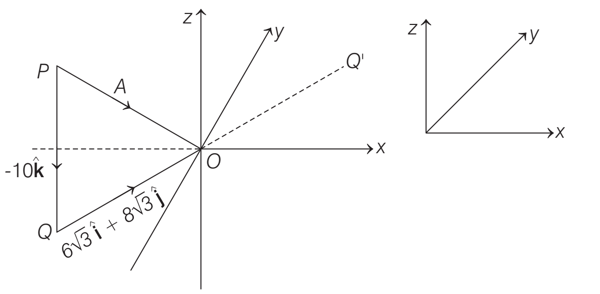
$$\Rightarrow \qquad \frac{3}{2}m = 2$$

$$\text{or} \qquad m = \frac{4}{3}$$

27. Incident ray $\mathbf{A} = 6\sqrt{3}\,\hat{\mathbf{i}} + 8\sqrt{3}\,\hat{\mathbf{j}} - 10\hat{\mathbf{k}}$

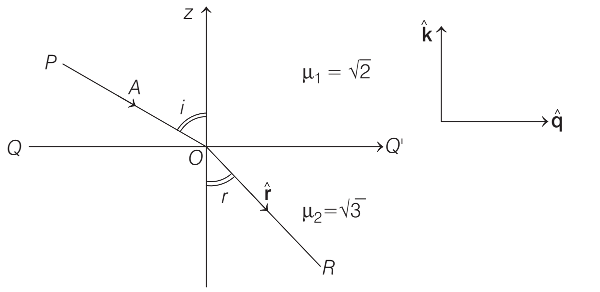
$= (6\sqrt{3}\,\hat{\mathbf{i}} + 8\sqrt{3}\,\hat{\mathbf{j}}) + (-10\hat{\mathbf{k}})$

$= \mathbf{QO} + \mathbf{PQ}$ (As shown in figure)



Note that **QO** is lying on x-y plane.

Now, QQ' and Z-axis are mutually perpendicular. Hence, we can show them in two-dimensional figure as below.



Vector **A** makes an angle i with z-axis, given by

$$i = \cos^{-1}\left\{\frac{10}{\sqrt{(10)^2 + (6\sqrt{3})^2 + (8\sqrt{3})^2}}\right\} = \cos^{-1}\left\{\frac{1}{2}\right\}$$

$$i = 60°$$

Unit vector in the direction of QOQ' will be

$$\hat{\mathbf{q}} = \frac{6\sqrt{3}\,\hat{\mathbf{i}} + 8\sqrt{3}\,\hat{\mathbf{j}}}{\sqrt{(6\sqrt{3})^2 + (8\sqrt{3})^2}}$$

$$= \frac{1}{5}(3\hat{\mathbf{i}} + 4\hat{\mathbf{j}})$$

Snell's law gives

$$\frac{\sqrt{3}}{\sqrt{2}} = \frac{\sin i}{\sin r} = \frac{\sin 60°}{\sin r}$$

$$\therefore \qquad \sin r = \frac{\sqrt{3}/2}{\sqrt{3}/\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\therefore \qquad r = 45°$$

Now, we have to find a unit vector in refracted ray's direction OR. Say it is $\hat{\mathbf{r}}$ whose magnitude is 1. Thus,

$$\hat{\mathbf{r}} = (1\sin r)\hat{\mathbf{q}} - (1\cos r)\hat{\mathbf{k}}$$

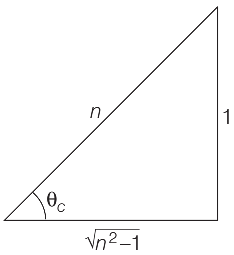
$$= \frac{1}{\sqrt{2}}[\hat{\mathbf{q}} - \hat{\mathbf{k}}] = \frac{1}{\sqrt{2}}\left[\frac{1}{5}(3\hat{\mathbf{i}} + 4\hat{\mathbf{j}}) - \hat{\mathbf{k}}\right]$$

$$\hat{\mathbf{r}} = \frac{1}{5\sqrt{2}}(3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 5\hat{\mathbf{k}}).$$

28. $\sin\theta_c = \dfrac{1}{n}$ (θ_c = critical angle)

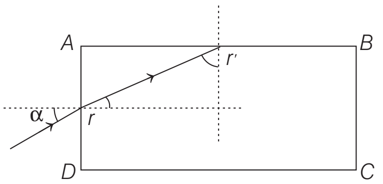
$$r' = 90° - r \quad \Rightarrow \quad (r')_{\min} = 90° - (r)_{\max}$$

and $$n = \frac{\sin (i)_{\max}}{\sin (r)_{\max}} = \frac{\sin 90°}{\sin (r)_{\max}} \qquad (\because\ i_{\max} = 90°)$$



Then, $$\sin (r)_{\max} = \frac{1}{n} = \sin\theta_c$$

$$(r)_{\max} = \theta_c \quad \text{or} \quad (r')_{\min} = (90° - \theta_c)$$



Now, if minimum value of r' i.e. $90° - \theta_c$ is greater than θ_c, then obviously all values of r' will be greater than θ_c *i.e.*, total internal reflection will take place at face AB in all conditions. Therefore, the necessary condition is

$$(r')_{\min} \geq \theta_c$$

or $$(90° - \theta_c) \geq \theta_c$$

or $$\sin (90° - \theta_c) \geq \sin\theta_c$$

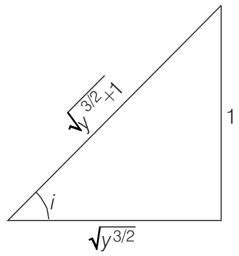
or $$\cos\theta_c \geq \sin\theta_c$$

or $$\cot\theta_c \geq 1$$

or $$\sqrt{n^2 - 1} \geq 1$$

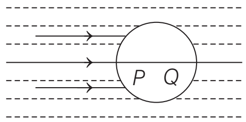
or $$n^2 \geq 2$$

or $$n \geq \sqrt{2}$$



Therefore, minimum value of n is $\sqrt{2}$.

29. (a)



Applying $\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{R}$, one by one on two spherical surfaces.

First on left surface

$$\frac{1}{v_1} - \frac{4/3}{\infty} = \frac{1 - 4/3}{+2}$$

or $$\frac{1}{v_1} = -\frac{1}{6}$$

or $$v_1 = -6 \text{ mm}$$

i.e. first image will be formed at 6 mm towards left of P

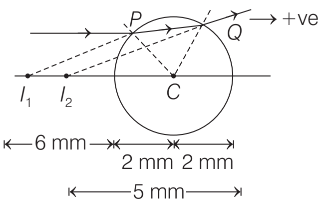
Second on right surface Now, distance of first image I_1 from Q will be 10 mm (towards left).

$$\frac{4/3}{v_2} - \frac{1}{-10} = \frac{4/3 - 1}{-2}$$

or $$\frac{4}{3v_2} = -\frac{1}{6} - \frac{1}{10} = -\frac{4}{15}$$

or $$v_2 = -5 \text{mm}$$

(b) The ray diagram is shown in figure.

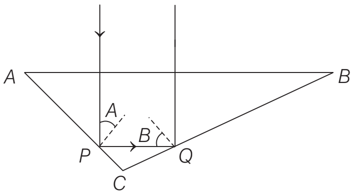


NOTE

- At P and Q both normal will pass through C.
- At P ray of light is travelling from a denser medium (water) to rarer medium (air) therefore, ray of light will bend away from the normal and on extending meet at I_1. Similarly at Q, ray of light bends towards the normal.
- Both the images I_1 and I_2 are virtual.

30. (a) At P, angle of incidence $i_A = A$

and at Q, angle of incidence $i_B = B$



If TIR satisfies for the smaller angle of incidence than for larger angle of incidence is automatically satisfied.

$$B \le A \quad \therefore \quad i_B \le i_A$$

Maximum value of B can be 45°. Therefore, if condition of TIR is satisfied, then condition of TIR will be satisfied for all value of i_A and i_B

Thus, $45° \ge \theta_c$

or $\sin 45° \ge \sin\theta_c$

or $\frac{1}{\sqrt{2}} \ge \frac{1}{\mu}$ or $\mu \ge \sqrt{2}$

$\therefore$ Minimum value of μ or n is $\sqrt{2}$.

(b) For $n = \frac{5}{3}, \sin\theta_c = \frac{1}{n} = \sin^{-1}\left(\frac{3}{5}\right) \approx 37°$

If $B = 30°$, then $i_B = 30°$

then $A = 60°$ or $i_A = 60°$

$$i_A > \theta_c \text{ but } i_B < \theta_c$$

i.e. TIR will take place at A but not at B.

or we write : $\sin i_B < \sin\theta_c < \sin i_A$

or $\sin 30° < \frac{3}{5} < \sin 60°$

or $0.5 < 0.6 < 0.86$

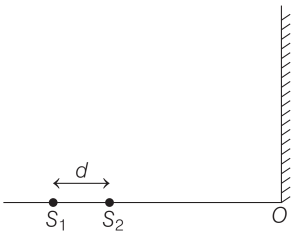
Topic 3 Lens Theory

1. Focal length of a lens is given as

$$\frac{1}{f} = (\mu_1 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$\therefore$ Focal length of plano-convex lens, i.e. lens 1,

$R_1 = \infty$ and $R = -R$



$\Rightarrow \quad \frac{1}{f_1} = \frac{\mu_1 - 1}{R}$

or $\quad f_1 = \frac{R}{(\mu_1 - 1)}$ ... (i)

Similarly, focal length of plano-concave lens, i.e. lens 2,

$$R_1 = -R \text{ and } R_2 = \infty$$

$\Rightarrow \quad \frac{1}{f_2} = -\frac{(\mu_2 - 1)}{R}$

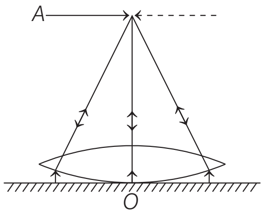
or $\quad f_2 = \frac{-R}{(\mu_2 - 1)}$ ... (ii)

From Eqs. (i) and (ii), net focal length is

$$\frac{1}{f} = \frac{\mu_1 - 1}{R} - \frac{\mu_2 - 1}{R} = \frac{\mu_1 - \mu_2}{R}$$

$\Rightarrow \quad f = \frac{R}{\mu_1 - \mu_2}$

2. Light from plane mirror is reflected back on it's path, so that image of A coincides with A itself.



This would happen when rays refracted by the convex lens falls normally on the plane mirror, i.e. the refracted rays form a beam parallel to principal axis of the lens. Hence, the object would then be considered at the focus of convex lens.

$\therefore$ Focal length of curvature of convex lens is, $f_1 = 18$ cm

With liquid between lens and mirror, image is again coincides with object, so the second measurement is focal length of combination of liquid lens and convex lens.

$$\therefore \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f_{eq}} \Rightarrow \frac{1}{18} + \frac{1}{f_2} = \frac{1}{27} \Rightarrow f_2 = -54 \text{ cm}$$

For convex lens by lens maker's formula, we have

$$\frac{1}{f} = (\mu - 1)\left(\frac{2}{R}\right) \Rightarrow \frac{1}{18} = 0.5 \times \frac{2}{R} \Rightarrow R = 18 \text{ cm}$$

and for plano-convex liquid lens, we have

$$\frac{1}{f} = (\mu_l - 1)\left(\frac{-1}{R}\right) \Rightarrow -\frac{1}{54} = (\mu_l - 1)\left(\frac{-1}{18}\right)$$

$$\Rightarrow \mu_l = 1 + \frac{1}{3} = \frac{4}{3}$$

3. Since, image formed by a convex lens can be real or virtual in nature.

Thus, $\quad m = +2$ or -2.

First let us take image to be real in nature, then

$m = -2 = \frac{v}{u}$, where v is image distance and u is object distance.

$\Rightarrow \quad v = -2x_1$ [Taking $u = x_1$]

Now, by using lens equation,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{-2x_1} - \frac{1}{x_1} = \frac{1}{20} \Rightarrow \frac{-3}{2x_1} = \frac{1}{20}$$

$\Rightarrow \quad x_1 = -30$ cm

Now, let us take image to be virtual in nature, then

$$m = 2 = \frac{v}{u} \Rightarrow v = 2x_2 \qquad \text{[Taking } u = x_2\text{]}$$

Again, by using lens equation,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{2x_2} - \frac{1}{x_2} = \frac{1}{20}$$

$$\Rightarrow \qquad \frac{-1}{2x_2} = \frac{1}{20} \quad \Rightarrow x_2 = -10 \text{ cm}$$

So, the ratio of x_1 and x_2 is

$$\frac{x_1}{x_2} = \frac{-30}{-10} = 3:1$$

Alternate Solution

Magnification for a lens can also be written as

$$m = \left(\frac{f}{f+u}\right)$$

When $m = -2$ (for real image)

$$-2 = \frac{f}{f + x_1}$$

$$\Rightarrow \qquad -2f + (-2)x_1 = f \text{ or } x_1 = -\frac{3f}{2}$$

Similarly, when $m = +2$ (for virtual image)

$$+2 = \frac{f}{f + x_2} \Rightarrow 2f + 2x_2 = f \text{ or } x_2 = \frac{-f}{2}$$

Now, the ratio of x_1 and x_2 is

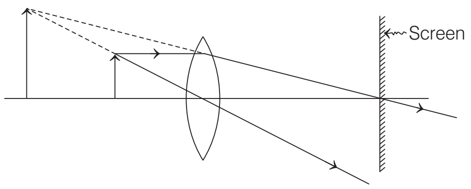
$$\frac{x_1}{x_2} = \frac{\frac{-3f}{2}}{-f/2} = \frac{3}{1}$$

4. When given set up is immersed in water, focal length of lens increases,

Note
$$\left\{\begin{array}{l} \dfrac{f_{\text{liquid}}}{f_{\text{air}}} = \dfrac{n_{ga} - 1}{n_{gl} - 1} = \dfrac{n_{ga} - 1}{\left(\dfrac{n_{ga}}{n_{la}} - 1\right)} \\ \text{Now, } n_{ga} = \dfrac{3}{2} \text{ and } n_{la} = \dfrac{4}{3} \\ \therefore f_{\text{liquid}} = f_{\text{air}} \left(\dfrac{\frac{1}{2}}{\frac{1}{8}}\right) = 4 f_{\text{air}} \end{array}\right.$$

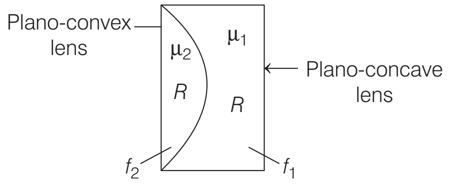
As focal length increases there is no focussing of image on screen

$\therefore$ image will disappear



Actually a virtual image is formed on same side of object.

5. Given combination is as shown below



As lenses are in contact, equivalent focal length of combination is

$$\frac{1}{f_{\text{eq}}} = \frac{1}{f_1} + \frac{1}{f_2}$$

Using lens Maker's formula,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Here, $$\frac{1}{f_1} = (\mu_1 - 1)\left(\frac{1}{-R} - \frac{1}{\infty}\right) = \frac{(1 - \mu_1)}{R}$$

and $$\frac{1}{f_2} = (\mu_2 - 1)\left(\frac{1}{\infty} - \frac{1}{-R}\right) = \frac{(\mu_2 - 1)}{R}$$

$\therefore$ $$\frac{1}{f_{\text{eq}}} = \left(\frac{\mu_2 - 1}{R}\right) + \left(\frac{1 - \mu_1}{R}\right)$$

$$= \frac{\mu_2 - 1 + 1 - \mu_1}{R} = \frac{\mu_2 - \mu_1}{R}$$

So, $$f_{\text{eq}} = \frac{R}{\mu_2 - \mu_1}.$$

6. For lens A, object distance, $u = -20$ cm

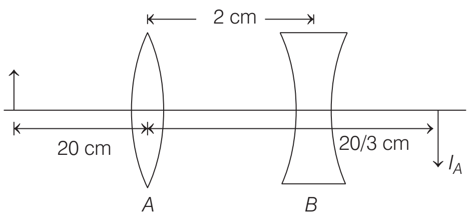
Focal length, $f = +5$ cm

From lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

We have, $$\frac{1}{v} = \frac{1}{5} - \frac{1}{20}$$

$$\frac{1}{v} = \frac{20 - 5}{20 \times 5} = \frac{15}{100}$$

$$v = \frac{20}{3} \text{ cm}$$



For lens B, image of A is object for B.

$\therefore$ $$u = \frac{20}{3} - 2 = +\frac{14}{3} \text{ cm}$$

$$f = -5 \text{ cm}$$

Now, from lens formula, we have

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{v}=\frac{1}{-5}+\frac{3}{14}$$

$$\frac{1}{v}=\frac{15-14}{5\times 14}$$

$$v = 70\,\text{cm}$$

Hence, image is on right of lens B and is real in nature.

7. Lens formula is given as

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u} \qquad \ldots\text{(i)}$$

$$\Rightarrow \quad \frac{1}{v}=\frac{1}{f}+\frac{1}{u}$$

$$\Rightarrow \quad \frac{uf}{u+f}=v$$

$$\Rightarrow \quad \frac{v}{u}=\frac{f}{u+f} \qquad \ldots\text{(ii)}$$

Now, by differentiating Eq. (i), we get

$$0=-\frac{1}{v^2}\cdot\frac{dv}{dt}+\frac{1}{u^2}\cdot\frac{du}{dt}$$

[$\because$ f (focal length of a lens is constant)]

$$\text{or} \quad \frac{dv}{dt}=\frac{v^2}{u^2}\,du/dt$$

$$\Rightarrow \quad \frac{dv}{dt}=\left(\frac{f}{u+f}\right)^2\cdot\frac{du}{dt} \qquad \text{[using Eq. (ii)]}$$

Given, $f = 0.3$ m, $u = -20$ m, $du/dt = 5$ m / s

$$\therefore \quad \frac{dv}{dt}=\left(\frac{0.3}{0.3-20}\right)^2\times 5=\left(\frac{3}{197}\right)^2\times 5$$

$$= 1.16\times 10^{-3}\ \text{m / s}$$

Thus, the image is moved with a speed of 1.16×10^{-3} m / s towards the lens.

8. Given, $\quad f_1 = 2f_2$

$$\Rightarrow \quad \frac{1}{|f_1|}=\frac{1}{|2f_2|} \qquad \ldots\text{(i)}$$

Using lens Maker's formula, we get

$$\frac{1}{f_1}=(\mu_1-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\frac{1}{f_2}=(\mu_2-1)\left(\frac{1}{R_1'}-\frac{1}{R_2'}\right)$$

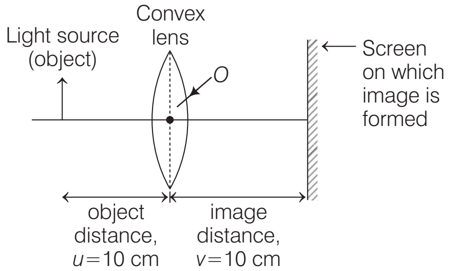
$$\Rightarrow \left|(\mu_1-1)\left(\frac{1}{\infty}-\frac{1}{-R}\right)\right|=\left|\frac{1}{2}(\mu_2-1)\left(\frac{1}{-R}-\frac{1}{\infty}\right)\right|$$

[$\because$ usingEq. (i)]

$$\Rightarrow \quad \frac{\mu_1-1}{R}=\frac{\mu_2-1}{2R}$$

$$\Rightarrow \quad 2\mu_1-\mu_2=1$$

9. Initially, when a light source (i.e. an object) is placed at 10 cm from the convex mirror and an image is form on the screen as shown in the figure below,



Since, $u=v$ which can only be possible in the situation when the object is place at '$2f$' of the lens.

So, that the image can form at '$2f$' only on the other side of the lens.

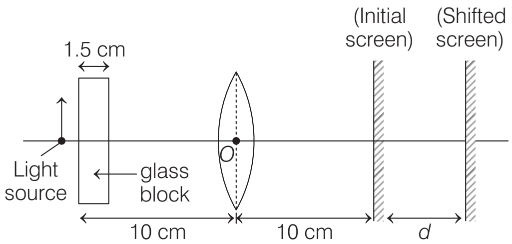
Thus, the distance from the optical centre (O) of the lens and $2f$ is

$$2\times \text{focal length } (f) = \text{object distance}$$

$$\Rightarrow \quad 2\times f = 10$$

$$\text{or} \quad f = 5\,\text{cm}$$

Now, when a glass block is placed in contact with the light source i.e., object, then the situation is shown in the figure given below



Then due to the block, the position of the object in front of the lens would now be shifted due to refraction of the light source rays through the block.

The shift in the position of the object is given as

$$x=\left(1-\frac{1}{\mu}\right)t$$

where, μ is the refractive index of the block and t is its thickness.

$$\Rightarrow \quad x=\left(1-\frac{1}{1.5}\right)1.5=\left(1-\frac{2}{3}\right)\frac{3}{2}$$

$$=\frac{1}{3}\times\frac{3}{2}=\frac{1}{2}=0.5\,\text{cm}$$

$\therefore$ The new object distance of the light source in front of the lens will be

$$u' = 10-0.5 = 9.5\,\text{cm}$$

Since, the focal length of the lens is 5 cm.

Therefore, the image distance of the light source now can be given as,

$$\frac{1}{v'} = \frac{1}{f} + \frac{1}{u'} \qquad \text{(using lens formula)}$$

Substituting the values, we get

$$\frac{1}{v'} = \frac{1}{5} + \left(\frac{1}{-9.5}\right) = \frac{+9.5 - 5}{47.5} = \frac{4.5}{47.5}$$

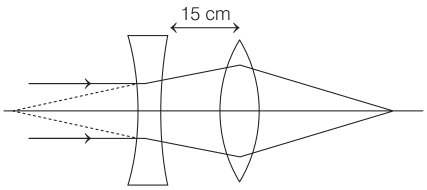
or $v' = 10.55$ cm

$\therefore$ The value of $d = v' - v = 10.55 - 10$

$= 0.55$ cm, away from the lens

Focal length in the above question can be calculated by using lens formula i.e. $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

10.



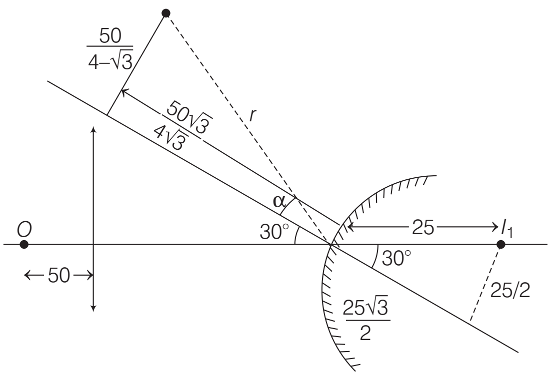
Here, $f_1 = -25$ cm, $f_2 = 20$ cm

For diverging lens, $v = -25$ cm

For converging lens, $u = -(15 + 25) = -40$ cm

$$\therefore \quad \frac{1}{v} - \frac{1}{-40} = \frac{1}{+20} \Rightarrow \frac{1}{v} = \frac{1}{20} - \frac{1}{40} = \frac{1}{40} \Rightarrow v = 40 \text{ cm}$$

11.



For Lens

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow v = \frac{uf}{u+f}$$

$$\Rightarrow \quad v = \frac{(-50)(30)}{-50+30} = 75$$

For Mirror

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow v = \frac{uf}{u-f}$$

$$\Rightarrow \quad v = \frac{\left(\frac{25\sqrt{3}}{2}\right)(50)}{\frac{25\sqrt{3}}{2} - 50} = \frac{-50\sqrt{3}}{4-\sqrt{3}}$$

$$\Rightarrow \quad m = -\frac{v}{u} = \frac{h_2}{h_1} \Rightarrow h_2 = -\left(\frac{\frac{-50\sqrt{3}}{4-\sqrt{3}}}{\frac{25\sqrt{3}}{2}}\right) \cdot \frac{25}{2}$$

$$\Rightarrow \quad h_2 = \frac{+50}{4-\sqrt{3}}$$

The x-coordinate of the images

$= 50 - v\cos 30 + h_2 \cos 60 \approx 25$

The y-coordinate of the images

$= v\sin 30 + h_2 \sin 60 \approx 25\sqrt{3}$

12. It is based on lens maker's formula and its magnification.

i.e. $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

According to lens maker's formula, when the lens in the air.

$$\frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f} = \frac{1}{2x} \Rightarrow f = 2x$$

Here, $\left(\frac{1}{x} = \frac{1}{R_1} - \frac{1}{R_2}\right)$

In case of liquid, where refractive index is $\frac{4}{3}$ and $\frac{5}{3}$, we get

Focal length in first liquid

$$\frac{1}{f_1} = \left(\frac{\mu_s}{\mu_{l_1}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \Rightarrow \frac{1}{f_1} = \left(\frac{\frac{3}{2}}{\frac{4}{3}} - 1\right)\frac{1}{x}$$

$\Rightarrow f_1$ is positive.

$$\frac{1}{f_1} = \frac{1}{8x} = \frac{1}{4(2x)} = \frac{1}{4f}$$

$$\Rightarrow \quad f_1 = 4f$$

Focal length in second liquid

$$\frac{1}{f_2} = \left(\frac{\mu_s}{\mu_{l_2}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \quad \frac{1}{f_2} = \left(\frac{\frac{3}{2}}{\frac{5}{3}} - 1\right)\left(\frac{1}{x}\right)$$

$\Rightarrow f_2$ is negative.

13. $\mu = \frac{\lambda_{\text{air}}}{\lambda_{\text{medium}}} = \frac{1}{(2/3)} = \frac{3}{2}$

Further, $|m| = \frac{1}{3} = \left|\frac{v}{u}\right|$

$\therefore \quad |v| = \frac{|u|}{3}$

$\Rightarrow \quad u = -24$ m (Real object)

$\therefore \quad v = +8$ m (Real image)

Now, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} = (\mu - 1)\left(\frac{1}{+R} - \frac{1}{\infty}\right)$

$\therefore$ $\frac{1}{8} + \frac{1}{24} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{R}\right)$

$\therefore$ $R = 3\text{ m}$

14. Using the lens formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} \text{ or } \frac{1}{v} = \frac{1}{u} + \frac{1}{f_1} + \frac{1}{f_2}$$

$$= \frac{1}{u} + (n_1 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) + (n_2 - 1)\left(\frac{1}{R_1'} - \frac{1}{R_2'}\right)$$

Substituting the values, we get

$$\frac{1}{v} = \frac{1}{-40} + (1.5 - 1)\left(\frac{1}{14} - \frac{1}{\infty}\right) + (1.2 - 1)\left(\frac{1}{\infty} - \frac{1}{-14}\right)$$

Solving this equations, we get

$$v = +40\text{ cm}$$

15. From the lens formula,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \text{ we have,}$$

$$\frac{1}{f} = \frac{1}{10} - \frac{1}{-10}$$

or $f = +5$

Further, $\Delta u = 0.1$
and $\Delta v = 0.1$ (from the graph)

Now, differentiating the lens formula, we have

$$\frac{\Delta f}{f^2} = \frac{\Delta v}{v^2} + \frac{\Delta u}{u^2}$$

or $$\Delta f = \left(\frac{\Delta v}{v^2} + \frac{\Delta u}{u^2}\right) f^2$$

Substituting the values, we have

$$\Delta f = \left(\frac{0.1}{10^2} + \frac{0.1}{10^2}\right)(5)^2 = 0.05$$

$\therefore$ $f \pm \Delta f = 5 \pm 0.05$

16. Refraction from lens : $\frac{1}{v_1} - \frac{1}{-20} = \frac{1}{15}$

$\therefore$ $v = 60\text{ cm}$ $\xrightarrow{}$ + ve direction

i.e. first image is formed at 60 cm to the right of lens system.

Reflection from mirror

After reflection from the mirror, the second image will be formed at a distance of 60 cm to the left of lens system.

Refraction from lens

$$\frac{1}{v_3} - \frac{1}{60} = \frac{1}{15} \qquad \longleftarrow + \text{ ve direction}$$

or $v_3 = 12\text{ cm}$

Therefore, the final image is formed at 12 cm to the left of the lens system.

17. Let focal length of convex lens is $+f$, then focal length of concave lens would be $-\frac{3}{2}f$.

From the given condition,

$$\frac{1}{30} = \frac{1}{f} - \frac{2}{3f} = \frac{1}{3f}$$

$\therefore$ $f = 10\text{ cm}$

Therefore, focal length of convex lens = + 10 cm
and that of concave lens = − 15 cm

18. Image formed by convex lens at I_1 will act as a virtual object for concave lens. For concave lens

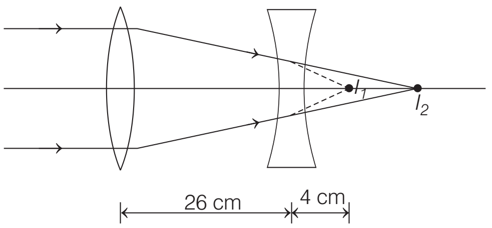
$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

or $$\frac{1}{v} - \frac{1}{4} = \frac{1}{-20}$$

or $v = 5\text{ cm}$

Magnification for concave lens

$$m = \frac{v}{u} = \frac{5}{4} = 1.25$$



As size of the image at I_1 is 2 cm. Therefore, size of image at I_2 will be $2 \times 1.25 = 2.5$ cm.

19. $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

For no dispersion, $d\left\{\frac{1}{f}\right\} = 0$

or $R_1 = R_2$

20. The lens makers' formula is

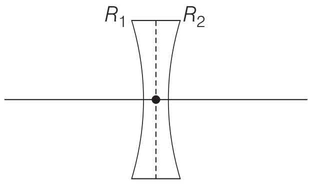
$$\frac{1}{f} = \left(\frac{n_L}{n_m} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

where, n_L = Refractive index of lens and
n_m = Refractive index of medium.

In case of double concave lens, R_1 is negative and R_2 is positive. Therefore, $\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ will be negative.

For the lens to be diverging in nature, focal length f should be negative or $\left(\frac{n_L}{n_m} - 1\right)$ should be positive or $n_L > n_m$ but since $n_2 > n_1$ (given), therefore the lens should be filled with L_2 and immersed in L_1.

21. $R_1 = -R,\ R_2 = +R,\ \mu_g = 1.5$ and $\mu_m = 1.75$



$$\therefore \quad \frac{1}{f} = \left(\frac{\mu_g}{\mu_m} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

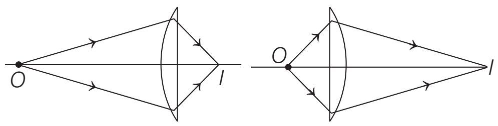
Substituting the values, we have

$$\frac{1}{f} = \left(\frac{1.5}{1.75} - 1\right)\left(\frac{1}{-R} - \frac{1}{R}\right) = \frac{1}{3.5R}$$

$$\therefore \quad f = +3.5R$$

Therefore, in the medium it will behave like a convergent lens of focal length $3.5R$. It can be understood as, $\mu_m > \mu_g$, the lens will change its behaviour.

22. In general spherical aberration is minimum when the total deviation produced by the system is equally divided on all refracting surfaces. A plano-convex lens is used for this purpose. In order that the total deviation be equally divided on two surfaces, it is essential that more parallel beam (or the incident and refracted) be incident on the convex side. Thus, when the object is far away from the lens, incident rays will be more parallel than the refracted rays, therefore, the object should face the convex side,but if the object is near the lens, the object should face the plane side. This has been shown in figure.



23. The focal length of combination is given by

$$\frac{1}{F} = \frac{1}{40} - \frac{1}{25} \qquad \left(\frac{1}{F} = \frac{1}{F_1} + \frac{1}{F_2}\right)$$

or $$F = -\frac{200}{3}\ \text{cm}$$

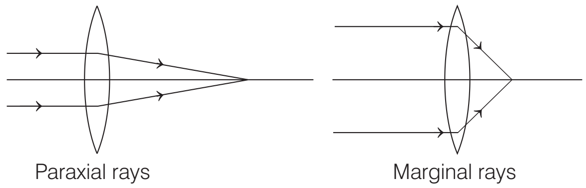
$$= -\frac{2}{3}\ \text{m}$$

$\therefore$ Power of the combination in dioptres,

$$P = -\frac{3}{2} \qquad \left[P = \frac{1}{F(\text{m})}\right]$$

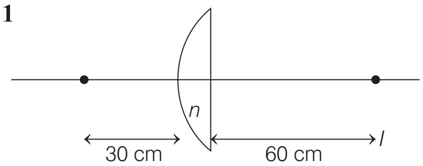
$$= -1.5$$

24. Spherical aberration is caused due to spherical nature of lens. Paraxial and marginal rays are focused at different places on the axis of the lens. Therefore, image so formed is blurred. This aberration can be reduced by either stopping paraxial rays or marginal rays, which can be done by using a circular annular mark over the lens.



25. $P = P_1 + P_2 = \dfrac{1}{f_1(\text{m})} + \dfrac{1}{f_2(\text{m})} = \dfrac{1}{0.4} - \dfrac{1}{0.25} = -1.5\ \text{D}$

26. **Case 1**



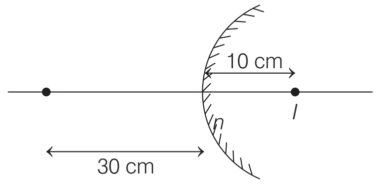
Using lens formula,

$$\frac{1}{60} + \frac{1}{30} = \frac{1}{f_1} \Rightarrow \frac{1}{f_1} = \frac{1}{60} + \frac{2}{60}$$

$$\Rightarrow \quad f_1 = 20\ \text{cm}$$

Further, $\dfrac{1}{f_1} = (n-1)\left(\dfrac{1}{R} - \dfrac{1}{\infty}\right) \Rightarrow f_1 = \dfrac{R}{n-1} = +20\ \text{cm}$

Case 2



Using mirror formula

$$\frac{1}{10} - \frac{1}{30} = \frac{1}{f_2}$$

$$\frac{3}{30} - \frac{1}{30} = \frac{1}{f_2} = \frac{2}{30}$$

$$f_2 = 15 = \frac{R}{2} \Rightarrow R = 30$$

$$R = 30\ \text{cm}$$

$$\frac{R}{n-1} + 20\ \text{cm} = \frac{30}{n-1}$$

$$\Rightarrow \quad = 2n - 2 = 3 \Rightarrow f_1 = +20\ \text{cm}$$

Refractive index of lens is 2.5.

Radius of curvature of convex surface is 30 cm.

Faint image is erect and virtual focal length of lens is 20 cm.

27. (P)  $\dfrac{1}{f} = \left(\dfrac{3}{2} - 1\right)\left(\dfrac{1}{r} + \dfrac{1}{r}\right) = \dfrac{1}{r} \Rightarrow f = r$

 $\Rightarrow \dfrac{1}{f_{\text{eq}}} = \dfrac{1}{f} + \dfrac{1}{f} = \dfrac{2}{r} \Rightarrow f_{\text{eq}} = \dfrac{r}{2}$

(Q)  $\dfrac{1}{f} = \left(\dfrac{3}{2} - 1\right)\left(\dfrac{1}{r}\right) \Rightarrow f = 2r$

 $\Rightarrow \dfrac{1}{\phi} + \dfrac{1}{\phi} = \dfrac{2}{\phi} = \dfrac{1}{\rho} \Rightarrow f_{\text{eq}} = r$

(R) $\frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(-\frac{1}{r}\right) = -\frac{1}{2r} \Rightarrow f = -2r$

$\Rightarrow \frac{1}{f_{eq}} = \frac{1}{f} + \frac{1}{f} = -\frac{2}{2r} \Rightarrow f_{eq} = -r$

(S) $\Rightarrow \frac{1}{f_{eq}} = \frac{1}{r} + \frac{1}{-2r} = \frac{1}{2r} \Rightarrow f_{eq} = 2r$

28. (A)$\rightarrow$ since $\mu_1 < \mu_2$, the ray of light will bend towards normal after first refraction.

(B)$\rightarrow$ $\mu_1 > \mu_2$, the ray of light will bend away from the normal after first refraction.

(C)$\rightarrow$ since $\mu_2 = \mu_3$ means in second refraction there will be no change in the path of ray of light.

(D)$\rightarrow$ Since $\mu_2 > \mu_3$, ray of light will bend away from the normal after second refraction.

Therefore the correct options are as under.

(A) $\rightarrow$ p, r (B) $\rightarrow$ q, s, t (C) $\rightarrow$ p, r, t (D) $\rightarrow$ q, s

29. When the lenses are in contact, the power of the system is

$$P = P_1 + P_2 \text{ or } P_1 + P_2 = 10 \quad \text{...(i)}$$

When lenses are separated by a distance $d = 0.25 \text{ m} = \frac{1}{4} \text{m}$

The power is $P = P_1 + P_2 - d\, P_1 P_2$

or $$P_1 + P_2 - \frac{P_1 P_2}{4} = 6 \quad \text{...(ii)}$$

Solving Eqs. (i) and (ii), we can find that $P_1 = 8$ D and $P_2 = 2$ D

$\therefore$ $$f_1 = \frac{1}{8} \text{m} = 0.125 \text{m}$$

$\Rightarrow$ $$f_2 = \frac{1}{2} \text{m} = 0.5 \text{m}$$

30. $$\frac{1}{f_{air}} = (1.5 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{...(i)}$$

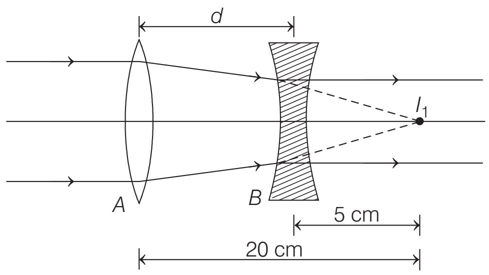
$$\frac{1}{f_{medium}} = \left(\frac{1.5}{4/3} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{...(ii)}$$

Dividing Eq. (i) by Eq. (ii), we get

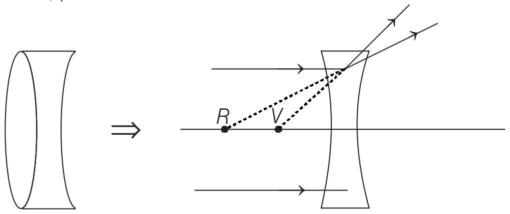
$$\frac{f_{medium}}{f_{air}} = 4$$

$\therefore$ $$f_{medium} = 4 f_{air} = 4 \times 15 = 60 \text{cm}$$

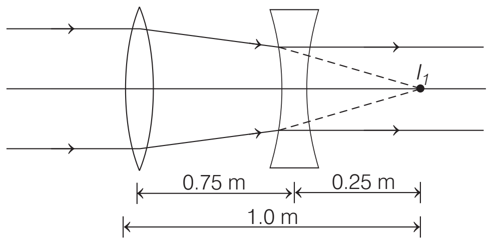
31. At I_1, second focus of convex lens should coincide the first focus of concave lens.



32. Focal length of concave is less, i.e. power of concave lens will be more. Hence, the combination will behave like a concave lens. Further, μ_V is greater than all other colours. Hence, f_V will be least.



33. At I_1, second focus of convex lens coincides with first focus of concave lens. Hence, rays will become parallel to the optic axis after refraction from both the lenses.



34. $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ or $\frac{u}{v} - 1 = \frac{u}{f}$ or $\frac{u}{v} = \left(\frac{u+f}{f}\right)$

$\therefore$ $$m = \frac{v}{u} = \left(\frac{f}{u+f}\right) \Rightarrow \frac{m_{25}}{m_{50}} = \frac{\left(\frac{20}{-25+20}\right)}{\left(\frac{20}{-50+20}\right)} = 6$$

$\therefore$ Answer is 6.

35. Differentiating the lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ with respect to time, we get

$$-\frac{1}{v^2}.\frac{dv}{dt} + \frac{1}{u^2}.\frac{du}{dt} = 0 \quad (\text{as } f = \text{constant})$$

$\therefore$ $$\left(\frac{dv}{dt}\right) = \left(\frac{v^2}{u^2}\right).\frac{du}{dt} \quad \text{...(i)}$$

Further, substituting proper values in lens formula, we have

$$\frac{1}{v} + \frac{1}{0.4} = \frac{1}{0.3} \quad (u = -0.4 \text{ m}, f = 0.3 \text{ m})$$

or $$v = 1.2 \text{m}$$

Putting the values in Eq. (i), we get

Magnitude of rate of change of position of image $= 0.09$ m/s

Lateral magnification, $m = \frac{v}{u}$

$\therefore$ $$\frac{dm}{dt} = \frac{u.\frac{dv}{dt} - v.\frac{du}{dt}}{u^2} = \frac{(-0.4)(0.09) - (1.2)(0.01)}{(0.4)^2}$$

$$= -0.3/\text{s}$$

$\therefore$ Magnitude of rate of change of lateral magnification

$$= 0.3/\text{s}$$

36. For both the halves, position of object and image is same. Only difference is of magnification. Magnification for one of the halves is given as 2(> 1). This can be for the first one, because for this, $|v| > |u|$. Therefore, magnification, $|m| = |v/u| > 1$. So, for the first half

$$|v/u| = 2 \quad \text{or} \quad |v| = 2|u|$$

Let $u = -x$ then $v = +2x$ and $|u| + |v| = 1.8\,\text{m}$

i.e. $3x = 1.8$ m or $x = 0.6$ m

Hence, $u = -0.6$ m and $v = +1.2$ m.

Using, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{1.2} - \frac{1}{-0.6} = \frac{1}{0.4}$

$\therefore \quad f = 0.4$ m

For the second half

$$\frac{1}{f} = \frac{1}{1.2 - d} - \frac{1}{-(0.6 + d)}$$

or $$\frac{1}{0.4} = \frac{1}{1.2 - d} + \frac{1}{0.6 + d}$$

Solving this, we get $d = 0.6$ m.

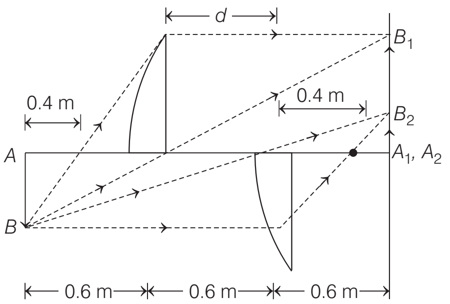
Magnification for the second half will be

$$m_2 = \frac{v}{u} = \frac{0.6}{-(1.2)} = -\frac{1}{2}$$

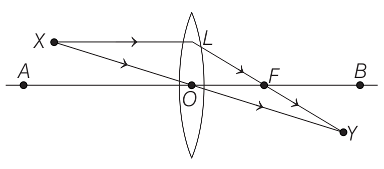
and magnification for the first half is

$$m_1 = \frac{v}{u} = \frac{1.2}{-(0.6)} = -2$$

The ray diagram is as follows :



37. **Steps I** In case of a lens

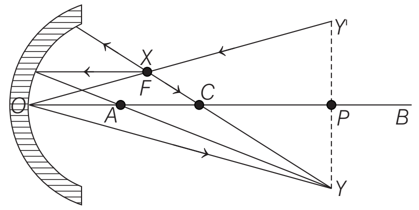


(a) Join X and Y. The point O, where the line XY cuts the optic axis AB, is the optical centre of the lens.

(b) Draw a line parallel to AB from point X. Let it cuts the lens at L. Join L and Y. The point F where the line LY cuts the optic axis AB is the focus of the lens F.

NOTE As the image is inverted, lens should be a convex because a concave lens always forms a virtual and erect image.

Step II In case of a concave mirror



(a) Draw a line YY' perpendicular to AB from point Y. Let it cuts the line AB at point P. Locate a point Y' such that $PY = PY'$.

(b) Extend the line XY'. Let it cuts the line AB at point O. Then O is the pole of the mirror.

(c) Join X and Y. The point C, where the line XY cuts the optic axis AB, is the centre of curvature of the mirror.

(d) The centre point F of OC is the focus of the mirror.

38. (a) For the lens, $u = -0.15$ m ; $f = +0.10$ m

Therefore, using $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ we have

$$\frac{1}{v} = \frac{1}{u} + \frac{1}{f} = \frac{1}{(-0.15)} + \frac{1}{(0.10)} \quad \text{or} \quad v = 0.3\,\text{m}$$

Linear magnification, $m = \frac{v}{u} = \frac{0.3}{-0.15} = -2$

Hence, two images S_1 and S_2 of S will be formed at 0.3 m from the lens as shown in figure. Image S_1 due to part 1 will be formed at 0.5 mm above its optic axis ($m = -2$). Similarly, S_2 due to part 2 is formed 0.5 mm below the optic axis of this part as shown.

Hence, d = distance between S_1 and S_2 = 1.5 mm

$$D = 1.30 - 0.30 = 1.0\,\text{m} = 10^3\,\text{mm}$$

$$\lambda = 500\,\text{nm} = 5 \times 10^{-4}\,\text{mm}$$

Therefore, fringe width,

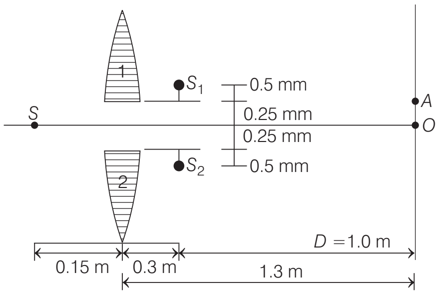
$$\omega = \frac{\lambda D}{d} = \frac{(5 \times 10^{-4})(10^3)}{(1.5)} = \frac{1}{3}\,\text{mm}$$

Now, as the point A is at the third maxima

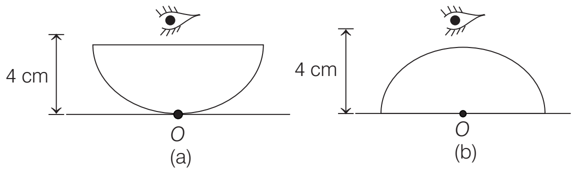
$OA = 3\omega = 3(1/3)$ mm or $OA = 1$ mm

NOTE The language of the question is slightly confusing. The third intensity maximum may be understood as second order maximum (zero order, first order and the second order). In that case $OA = 2\omega = 2(1/3)$ mm = 0.67 mm.

(b) If the gap between L_1 and L_2 is reduced, d will decrease. Hence, the fringe width ω will increase or the distance OA will increase.



39. **Refer figure (a)**



In this case refraction of the rays starting from O takes place from a plane surface. So, we can use

$$d_{\text{app}} = \frac{d_{\text{actual}}}{\mu} \quad \text{or} \quad 3 = \frac{4}{\mu} \quad \text{or} \quad \mu = \frac{4}{3}$$

Refer figure (b) In this case refraction takes place from a spherical surface. Hence, applying

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

we have, $$\frac{1}{(-25/8)} - \frac{4/3}{-4} = \frac{1-4/3}{-R}$$

or $$\frac{1}{3R} = \frac{1}{3} - \frac{8}{25} = \frac{1}{75}$$

$\therefore$ $$R = 25\,\text{cm}$$

Now, to find the focal length we will use the lens Maker's formula

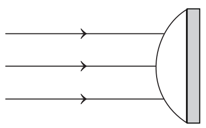
$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \left(\frac{4}{3} - 1\right)\left(\frac{1}{\infty} - \frac{1}{-25}\right) = \frac{1}{75}$$

$\therefore \; f = 75\,\text{cm}$

40. (a) $$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.5 - 1)\left(\frac{1}{12} - \frac{1}{\infty}\right) = \frac{1}{24}$$

$\therefore \quad f = +24\,\text{cm}$

(b) The system will behave like a mirror of power given by



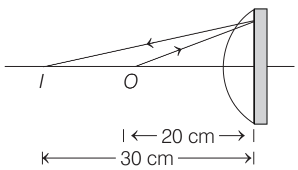
$$P = 2P_L + P_M$$

$\therefore$ $$-\frac{1}{F\,(\text{m})} = 2\left(\frac{1}{0.24}\right) + 0$$

$\therefore$ $$F = -0.12\,\text{m} = -12\,\text{cm}$$

Hence, the system will behave like a concave mirror of focal length 12 cm. Therefore parallel rays will converge at a distance of 12 cm (to the left) of the system.

(c)



(d) Using mirror formula

$$\frac{1}{v} - \frac{1}{20} = \frac{-1}{12}$$

Solving, we get $v = -30\,\text{cm}$.

Therefore the image will be formed at a distance of 30 cm to the left of system.

41. From first figure it is clear that

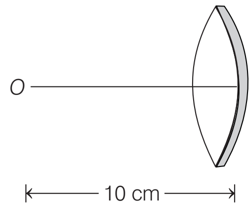
$$\mu = \mu_1$$

From second figure it is clear that

$$\mu < \mu_2$$

$\therefore$ $$\mu_1 < \mu_2$$

42. The given system behaves like a mirror of power given by



$$P = 2P_L + P_M$$

or $$-\frac{1}{F} = 2\left(\frac{1}{0.2}\right) + \left(\frac{-2}{0.22}\right)$$

As $$P_L = \frac{1}{f(\text{m})}$$

and $$P_M = \frac{-1}{f(\text{m})} = \frac{-2}{R(\text{m})}$$

Solving this equation, we get

$$F = -1.1\,\text{m} = -110\,\text{cm}$$

i.e. the system behaves as a concave mirror of focal length 18.33 cm.

Using the mirror formula

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \text{ we have}$$

$$\frac{1}{v} - \frac{1}{10} = -\frac{1}{110}$$

or $$v = 11\,\text{cm}$$

i.e. virtual image will be formed at a distance of 11 cm.

Topic 4 Prism

1. Given, refractive index of material of prism $n = \sqrt{3}$, prism angle $A = 60°$

Method 1

Using prism formula,

$$n = \frac{\sin\left(\frac{A+\delta}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$\Rightarrow \quad \sqrt{3} = \frac{\sin\left(\frac{60+\delta}{2}\right)}{\sin 30°}$$

$$\Rightarrow \quad \sin\left(\frac{60+\delta}{2}\right) = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad \sin\left(\frac{60+\delta}{2}\right) = \sin 60°$$

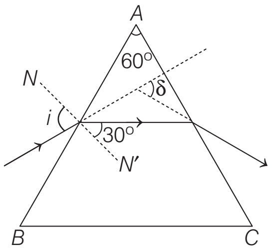
or $$\frac{60+\delta}{2} = 60$$

or angle of minimum deviation $\delta = 60°$

Incident angle, $i = \frac{60 + \delta}{2} = 60°$

Method 2 For minimum deviation, ray should pass symmetrically (i.e. parallel to the base of the equilateral prism)

$\Rightarrow$ From geometry of given figure, we have, $r = 30°$



Using Snell's law,

$$n = \frac{\sin i}{\sin r}$$

$$\sin i = n \sin r = \sqrt{3} \sin 30°$$

$$\Rightarrow \quad \sin i = \frac{\sqrt{3}}{2} \text{ or } i = 60°$$

2. For a crown glass thin prism i.e., prism with small angle, the angle of minimum deviation is given as,

$$D_m = (n-1)A$$

where, A is prism angle and n is refractive index.

$\Rightarrow D_m \propto n$ … (i)

Since from the given graph, the value of 'n' decrease with the increase in 'λ'. Thus, from relation (i), we can say that, D_m will also decrease with the increase in 'λ'.

$\therefore$ Hence, option (c) is correct.

3. $$\delta = (i_1 + i_2) - A$$

$$\Rightarrow \quad 40° = (35° + 79°) - A$$

$$\Rightarrow \quad A = 74°$$

Now, we know that

$$\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

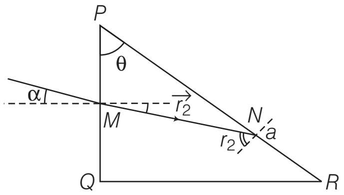
It we take the given deviation as the minimum deviation then,

$$\mu = \frac{\sin\left(\frac{74° + 40°}{2}\right)}{\sin\left(\frac{74°}{2}\right)} = 1.51$$

The given deviation may or may not be the minimum deviation. Rather it will be less than this value. Therefore, μ will be less than 1.51.

Hence, maximum possible value of refractive index is 1.51.

4.



Applying Snell's law at M,

$$n = \frac{\sin\alpha}{\sin r_1} \Rightarrow \sqrt{2} = \frac{\sin 45°}{\sin r_1}$$

$$\Rightarrow \quad \sin r_1 = \frac{\sin 45°}{\sqrt{2}} = \frac{1/\sqrt{2}}{\sqrt{2}} = \frac{1}{2}$$

$$r_1 = 30°$$

$$\sin\theta_c = \frac{1}{n} = \frac{1}{\sqrt{2}} \Rightarrow \theta_c = 45°$$

Let us take $r_2 = \theta_c = 45°$ for just satisfying the condition of TIR.

In ΔPNM,

$$\theta + 90 + r_1 + 90 - r_2 = 180°$$

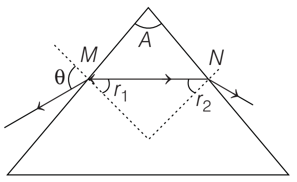
$$\text{or} \quad \theta = r_2 - r_1 = 45° - 30° = 15°$$

NOTE If $\alpha > 45°$ (the given value). Then, $r_1 > 30°$ (the obtained value)

$\therefore$ $r_2 - r_1 = \theta$ or $r_2 = \theta + r_1$

or TIR will take place. So, for taking TIR under all conditions α should be greater than 45° or this is the minimum value of α.

5.



Applying Snell's law at M,

$$\mu = \frac{\sin\theta}{\sin r_1}$$

$$\therefore \quad r_1 = \sin^{-1}\left(\frac{\sin\theta}{\mu}\right) \text{ or } \sin r_1 = \frac{\sin\theta}{\mu}$$

$$\text{Now,} \quad r_2 = A - r_1 = A - \sin^{-1}\left(\frac{\sin\theta}{\mu}\right)$$

Ray of light would get transmitted form face AC if

$$r_2 < \theta_c \quad \text{or} \quad A - \sin^{-1}\left(\frac{\sin\theta}{\mu}\right) < \theta_c$$

$$\text{where,} \quad \theta_c = \sin^{-1}\left(\frac{1}{\mu}\right)$$

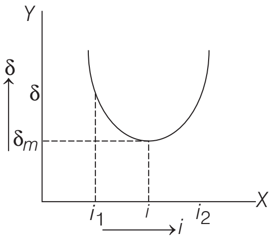
$$\therefore \quad \sin^{-1}\left(\frac{\sin\theta}{\mu}\right) > A - \theta_c$$

or $$\frac{\sin\theta}{\mu} > \sin(A-\theta_c)$$

$\therefore$ $$\theta > \sin^{-1}[\mu\sin(A-\theta_c)]$$

or $$\theta > \sin^{-1}\left[\mu\sin\left\{A-\sin^{-1}\left(\frac{1}{\mu}\right)\right\}\right]$$

6. We know that the angle of deviation depends upon the angle of incidence. If we determine experimentally, the angles of deviation corresponding to different angles of incidence and then plot i (on-X-axis) and δ (on-Y-axis), we get a curve as shown in figure.



Clearly if angle of incidence is gradually increased, from a small value, the angle of deviation first decreases, becomes minimum for a particular angle of incidence and then begins to increase.

7. For $e \to i$

$\Rightarrow$ $$45° > \theta_c$$

$\Rightarrow$ $$\sin 45° > \sin\theta_c$$

$\Rightarrow$ $$\frac{1}{\sqrt{2}} > \frac{\mu_2}{\mu_1}$$

$\Rightarrow$ $$\mu_1 > \sqrt{2}\mu_2$$

For $e \to f$

angle of refraction is lesser than angle of incidence, so $\mu_2 > \mu_1$ and then $\mu_2 > \mu_3$

For $e \to g$, $\mu_1 = \mu_2$

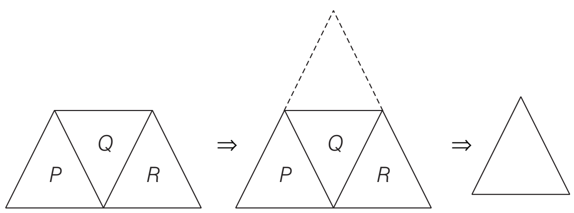
for $e \to h, \mu_2 < \mu_1 < \sqrt{2}\,\mu_2$ and $\mu_2 > \mu_3$

8. At minimum deviation ($\delta = \delta_m$):

$$r_1 = r_2 = \frac{A}{2} = \frac{60°}{2} = 30° \quad \text{(For both colours)}$$

9. During minimum deviation the ray inside the prism is parallel to the base of the prism in case of an equilateral prism.

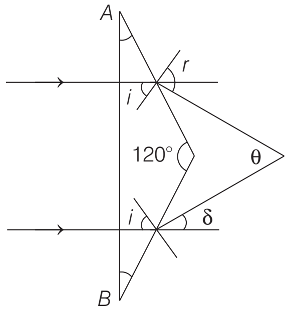
10. Figure (a) is part of an equilateral prism of figure (b) as shown in figure which is a magnified image of figure (c). Therefore, the ray will suffer the same deviation in figure (a) and figure (c).



(a) (b) (c)

NOTE Questions are often asked based on part of a prism. For example, section shown in figure (a) is part of a prism shown in figure (c).

11. The diagramatic representation of the given problem is shown in figure.



From figure it follows that $\angle i = \angle A = 30°$

From Snell's law, $n_1 \sin i = n_2 \sin r$

or $$\sin r = \frac{1.44\sin 30°}{1} = 0.72$$

Now, $$\angle\delta = \angle r - \angle i = \sin^{-1}(0.72) - 30°$$

$\therefore$ $$\theta = 2(\angle\delta) = 2\{\sin^{-1}(0.72) - 30°\}$$

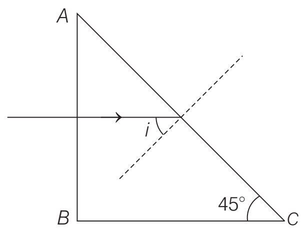
12. Deviation $\delta = (\mu - 1)A$

Given, $\delta_{\text{net}} = 0$

$\therefore$ $$(\mu_1 - 1)A_1 = (\mu_2 - 1)A_2$$

or $$A_2 = \frac{(\mu_1 - 1)}{(\mu_2 - 1)}A_1 = \frac{(1.54-1)}{(1.72-1)}(4°) = 3°$$

13. The colours for which $i > \theta_c$, will get total internal reflection : $i > \theta_c$ or $\sin i > \sin\theta_c$



$i = 45°$

or $\sin 45° > \frac{1}{\mu}$ or $\frac{1}{\sqrt{2}} > \frac{1}{\mu}$

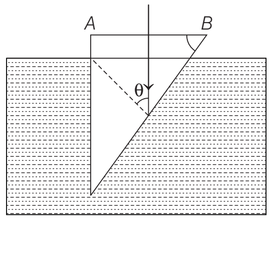
or for which $\mu > \sqrt{2}$ or $\mu > 1.414$

Hence, the rays for which $\mu > 1.414$ will get TIR.

For green and blue $\mu > 1.414$, so they will suffer TIR on face AC. Only red comes out from this face.

14. Let θ_c be the critical angle at face BC, then

$$\sin\theta_c = \frac{\mu_w}{\mu_g} = \frac{4/3}{3/2} = \frac{8}{9}$$



Angle of incidence at face BC is $i = \theta$

Total internal reflection (TIR) will take place on this surface if,

$$i > \theta_c \text{ or } \theta > \theta_c \text{ or } \sin\theta > \sin\theta_c \text{ or } \sin\theta > \frac{8}{9}$$

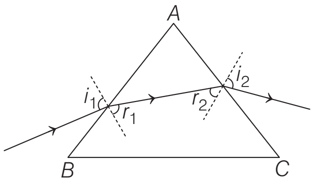
15 The minimum deviation produced by a prism

$\delta_m = 2i - A = A$

$\therefore\ i_1 = i_2 = A$ and

$r_1 = r_2 = A/2$

$\therefore\ r_1 = i_1/2$



Now, using Snell's law

$\sin A = \mu \sin A/2$

$\Rightarrow\ \mu = 2\cos(A/2)$

For this prism when the emergent ray at the second surface is tangential to the surface

$$i_2 = \pi/2 \Rightarrow r_2 = \theta_c \Rightarrow r_1 = A - \theta_c$$

so, $\sin i_1 = \mu \sin(A - \theta_c)$

so, $$i_1 = \sin^{-1}\left[\sin A\sqrt{4\cos^2\frac{A}{2} - 1} - \cos A\right]$$

For minimum deviation through isosceles prism, the ray inside the prism is parallel to the base of the prism if $\angle B = \angle C$.

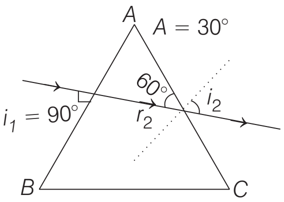
But it is not necessarily parallel to the base if,

$$\angle A = \angle B \text{ or } \angle A = \angle C$$

16. Ray falls normally on the face AB. Therefore, it will pass undeviated through AB.

$\therefore\ r_2 = 90° - 60° = 30°$

$\mu = \sqrt{2} = \dfrac{\sin i_2}{\sin r_2}$



$i_2 = 45°$

Deviation $= i_2 - r_2 = 45° - 30° = 15°$

(Deviation at face AC only)

17. Let δ_m be the angle of minimum deviation. Then

$$\mu = \frac{\sin\left(\dfrac{A+\delta_m}{2}\right)}{\sin(A/2)} \quad (A = 60° \text{ for an equilateral prism})$$

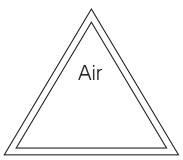
$$\therefore\ \sqrt{2} = \frac{\sin\left(\dfrac{60° + \delta_m}{2}\right)}{\sin\left(\dfrac{60°}{2}\right)}$$

Solving this we get $\delta_m = 30°$

The given deviation is also 30° (i.e. δ_m)

Under minimum deviation, the ray inside the prism is parallel to base for an equilateral prism.

18. Through a thin glass slab ray of light almost passes undeviated. A hollow prism can be assumed to be made up of three thin glass slabs as shown in figure.



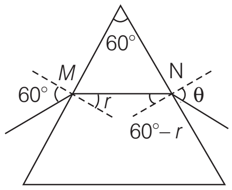
19. Applying Snell's law at M and N,

$$\sin 60° = n \sin r \qquad \text{...(i)}$$

$$\sin\theta = n\sin(60 - r) \qquad \text{...(ii)}$$

Differentiating we get

$$\cos\theta\frac{d\theta}{dn} = -n\cos(60-r)\frac{dr}{dn} + \sin(60-r)$$



Differentiating Eq. (i),

$$n\cos r\frac{dr}{dn} + \sin r = 0$$

or $$\frac{dr}{dn} = -\frac{\sin r}{n\cos r} = \frac{-\tan r}{n}$$

$$\Rightarrow\ \cos\theta\frac{d\theta}{dn} = -n\cos(60-r)\left(\frac{-\tan r}{n}\right) + \sin(60-r)$$

$$\frac{d\theta}{dn} = \frac{1}{\cos\theta}[\cos(60-r)\tan r + \sin(60-r)]$$

Form Eq. (i), $r = 30°$ for $n = \sqrt{3}$

$$\Rightarrow\ \frac{d\theta}{dn} = \frac{1}{\cos 60}(\cos 30 \times \tan 30 + \sin 30) = 2\left(\frac{1}{2} + \frac{1}{2}\right) = 2$$

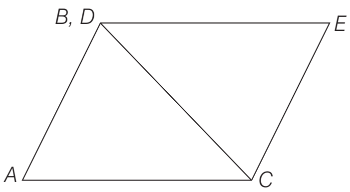
20. (a) At minimum deviation, $r_1 = r_2 = 30°$

$\therefore$ From Snell's law

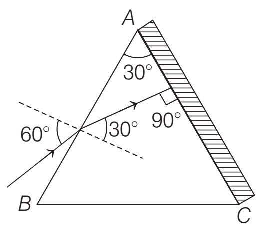
$$\mu = \frac{\sin i_1}{\sin r_1} \text{ or } \sqrt{3} = \frac{\sin i_1}{\sin 30°}$$

$$\therefore\ \sin i_1 = \frac{\sqrt{3}}{2} \text{ or } i_1 = 60°$$

(b) In the position shown net deviation suffered by the ray of light should be minimum. Therefore, the second prism should be rotated by 60° (anti-clockwise).



21. (a) $\sin i_1 = \mu \sin r_1$



or $\quad \sin 60° = \sqrt{3}\sin r_1$

$\therefore \quad \sin r_1 = \dfrac{1}{2}$ or $r_1 = 30°$

Now, $\quad r_1 + r_2 = A$

$\therefore \quad r_2 = A - r_1 = 30° - 30° = 0°$

Therefore, ray of light falls normally on the face AC and angle of emergence $i_2 = 0°$.

(b) Multiple reflections occur between surface of film. Intensity will be maximum if constructive interference takes place in the transmitted wave.

For maximum thickness

$$\Delta x = 2\mu t = \lambda \qquad (t = \text{thickness})$$

$$\therefore \quad t = \frac{\lambda}{2\mu} = \frac{6600}{2\times 2.2} = 1500\,\text{Å}$$

22. $n_1 = 1.20 + \dfrac{10.8\times 10^4}{\lambda^2}$ and $n_2 = 1.45 + \dfrac{1.80\times 10^4}{\lambda^2}$

Here, λ is in nm.

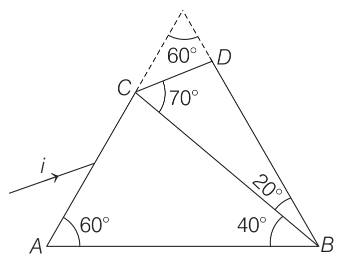
(a) The incident ray will not deviate at BC if $n_1 = n_2$

$$\Rightarrow 1.20 + \frac{10.8\times 10^4}{\lambda_0^2} = 1.45 + \frac{1.80\times 10^4}{\lambda_0^2}$$

$$\Rightarrow \frac{9\times 10^4}{\lambda_0^2} = 0.25 \quad \text{or} \quad \lambda_0 = \frac{3\times 10^2}{0.5}$$

or $\quad \lambda_0 = 600\,\text{nm}$

(b) The given system is a part of an equilateral prism of prism angle 60° as shown in figure.



At minimum deviation,

$$r_1 = r_2 = \frac{60°}{2} = 30° = r\,(\text{say})$$

$$\therefore \quad n_1 = \frac{\sin i}{\sin r}$$

$$\therefore \quad \sin i = n_1.\sin 30°$$

$$\sin i = \left\{1.20 + \frac{10.8\times 10^4}{(600)^2}\right\}\left(\frac{1}{2}\right) = \frac{1.5}{2} = \frac{3}{4}$$

or $\quad i = \sin^{-1}(3/4)$

23. (a) Critical angle θ_c at face AC will be given by

$$\theta_c = \sin^{-1}\left(\frac{n_1}{n}\right)$$

or $\quad \sin\theta_c = \dfrac{n_1}{n}$

Now, it is given that $r_2 = \theta_c$

$\therefore \quad r_1 = A - r_2 = (45° - \theta_c)$

Applying Snell's law at face AB, we have

$$n = \frac{\sin i_1}{\sin r_1} \quad \text{or} \quad \sin i_1 = n\sin r_1$$

$\therefore \quad i_1 = \sin^{-1}(n\sin r_1)$

Substituting value of r_1, we get

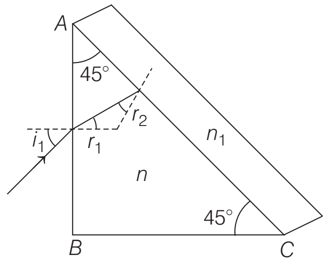
$$i_1 = \sin^{-1}\{n\sin(45° - \theta_c)\}$$

$$= \sin^{-1}\{n(\sin 45°\cos\theta_c - \cos 45°\sin\theta_c)\}$$

$$= \sin^{-1}\left\{\frac{n}{\sqrt{2}}(\sqrt{1-\sin^2\theta_c} - \sin\theta_c)\right\}$$

$$= \sin^{-1}\left\{\frac{n}{\sqrt{2}}\left(\sqrt{1-\frac{n_1^2}{n^2}} - \frac{n_1}{n}\right)\right\}$$

$$i_1 = \sin^{-1}\left\{\frac{1}{\sqrt{2}}(\sqrt{n^2-n_1^2} - n_1)\right\}$$



Therefore, required angle of incidence (i_1) at face AB for which the ray strikes at AC at critical angle is

$$i_1 = \sin^{-1}\left\{\frac{1}{\sqrt{2}}(\sqrt{n^2-n_1^2} - n_1)\right\}$$

(b) The ray will pass undeviated through face AC when either $n_1 = n$ or $r_2 = 0°$ i.e. ray falls normally on face AC.

Here $n_1 \neq n$ (because $n_1 < n$ is given)

$\therefore \quad r_2 = 0°$

or $\quad r_1 = A - r_2 = 45° - 0° = 45°$

Now applying Snell's law at face AB, we have $n = \dfrac{\sin i_1}{\sin r_1}$

or $\quad 1.352 = \dfrac{\sin i_1}{\sin 45°}$

$\therefore \quad \sin i_1 = (1.352)\left(\dfrac{1}{\sqrt{2}}\right),$

$\quad \sin i_1 = 0.956$

$\therefore \quad i_1 = \sin^{-1}(0.956) \approx 73°$

Therefore, required angle of incidence is

$$i = 73°$$

24. Given $i_1 = 60°$, $A = 30°$, $\delta = 30°$

From the relation

$$\delta = (i_1 + i_2) - A$$

we have, $\quad i_2 = 0°$

i.e. the ray is perpendicular to the face from which it emerges.

Further, $i_2 = 0°$

$\therefore$ $r_2 = 0°$

$$r_1 + r_2 = A$$

$$r_1 = A = 30°$$

$$\mu = \frac{\sin i_1}{\sin r_1} = \frac{\sin 60°}{\sin 30°} = \sqrt{3}$$

Topic 5 Optical Instruments

1. **Key Idea** Numerical Aperture (NA) of the microscope is given by
$\text{NA} = \frac{0.61\lambda}{d}$
Here, d = minimum separation between two points to be seen as distinct and λ = wavelength of light.

Given, $\lambda = 5000\text{Å} = 5000 \times 10^{-10}$ m and $\text{NA} = 1.25$

Now, $d = \frac{0.61\lambda}{\text{NA}} = \frac{0.61 \times 5000 \times 10^{-10}}{1.25}$

or $d = \frac{3.05}{1.25} \times 10^{-7}$ m

$= 2.4 \times 10^{-7}$ m

or $d = 0.24\,\mu\text{m}$

2 Limit of resolution for a telescope from Rayleigh's criteria is

$$\theta_R = \frac{1.22\,\lambda}{D}$$

Here, $D = 250\text{ cm} = 250 \times 10^{-2}$ m

and $\lambda = 600\text{ nm} = 600 \times 10^{-9}$ m

So, limit of resolution is

$$\theta_R = \frac{1.22 \times 600 \times 10^{-9}}{250 \times 10^{-2}}$$

$$= 2.93 \times 10^{-7}\text{ rad}$$

$$\approx 3 \times 10^{-7}\text{ rad}$$

3 For a telescope, limit of resolution is

given by $\Delta\theta = \frac{1.22\lambda}{D}$

Here, $\lambda = 500\text{ nm} = 500 \times 10^{-9}$ m,

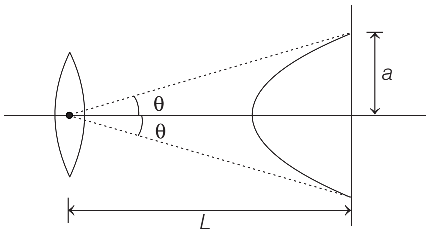
$D = 200\text{ cm} = 200 \times 10^{-2}$ m

So, limit of resolution is $\Delta\theta = \frac{1.22 \times 500 \times 10^{-9}}{200 \times 10^{-2}}$

$$= 305 \times 10^{-9}\text{ rad}$$

4. Telescope resolves and brings objects closer. Hence, telescope with magnifying power of 20, the tree appears 20 times nearer.

5.

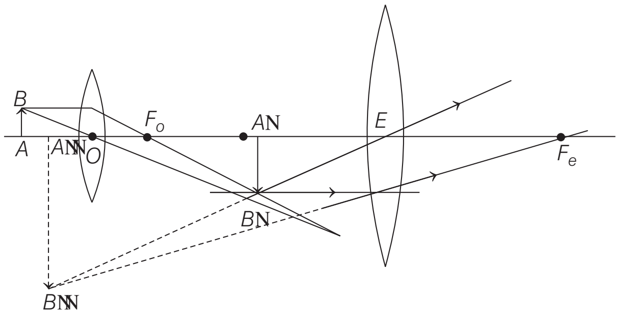


$$a \sin\theta \approx \lambda$$

$$a\left(\frac{a}{L}\right) \approx \lambda \Rightarrow a = \sqrt{\lambda L}$$

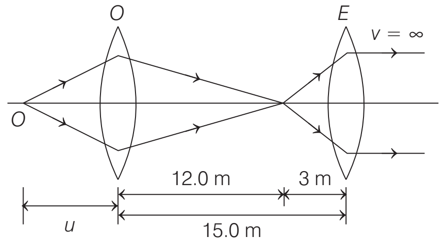
Spread $= 2a = \sqrt{4\lambda L}$

6. The ray diagram is as follows



From the figure it is clear that image formed by objective (or the intermediate image) is real, inverted and magnified.

7. Since, the final image is formed at infinity, the image formed by the objective will be at the focal point of the eyepiece, which is 3.0 cm. The image formed by the objective will be at a distance of 12.0 cm (= 15.0 cm – 3.0 cm) from the objective.



If u is the distance of the object from the objective, we have

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{u} + \frac{1}{12.0} = \frac{1}{2.0}$$

$$\Rightarrow \quad u = \frac{(12.0)\,(2.0)}{12.0 - 2.0}$$

$$= \frac{24.0}{10.0} = 2.4\text{cm}$$

8. Image formed by objective (I_1) is at second focus of it because objective is focussed for distant objects. Therefore,

$$P_1 I_1 = f_o$$

Further I_1 should lie at first focus of eyepiece because final image is formed at infinity.

$\therefore \quad P_2 I_1 = f_e$

Given $\quad P_1 P_2 = 36\,\text{cm}$

$\therefore \quad f_o + f_e = 36 \quad \ldots(\text{i})$

Further angular magnification is given as 5. Therefore,

$$\frac{f_o}{f_e} = 5 \quad \ldots(\text{ii})$$

Solving Eqs. (i) and (ii), we get

$$f_o = 30\,\text{cm} \text{ and } f_e = 6\,\text{cm}$$

9. Distance between objective and eyepiece

$$L = f_o + f_e = (16 + 0.02)\,\text{m} = 16.02\,\text{m}$$

Angular magnification

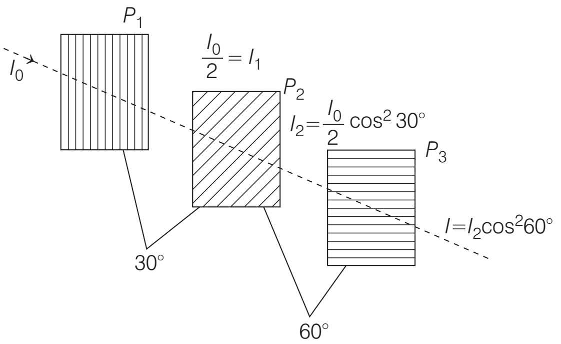
$$M = -f_o/f_e = -16/0.02 = -800$$

Image is inverted and objective is larger than the eyepiece.

10. The resolving power of a microscope is inversely proportional to the wavelength of the wave used. de-Broglie matter wave is used in case of an electron microscope whose wavelength is less than the wavelength of visible light used in optical microscope.

Topic 6 Wave Optics

1.



When unpolarised light pass through polaroid P_1, intensity obtained is

$$I_1 = \frac{I_0}{2}$$

where, I_0 = intensity of incident light.

Now, this transmitted light is polarised and it pass through polariser P_2. So, intensity I_2 transmitted is obtained by Malus law.

$$\Rightarrow \quad I_2 = I_1 \cos^2\theta$$

As angle of pass axis of P_1 and P_3 is 90° and angle of pass axis of P_2 and P_3 is 60°, so angle between pass axis of P_1 and P_2 is $(90° - 60°) = 30°$. So,

$$I_2 = \frac{I_0}{2}\cos^2 30° = \frac{I_0}{2} \times \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{8} I_0$$

When this light pass through third polariser P_3, intensity I transmitted is again obtained by Malus law.

So, $\quad I = I_2 \cos^2 60° = \left(\frac{3}{8} I_0\right) \cos^2 60°$

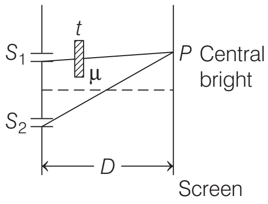
$$= \frac{3}{8} I_0 \times \left(\frac{1}{2}\right)^2 = \frac{3}{32} I_0$$

So, ratio $\frac{I_0}{I} = \frac{32}{3} = 10.67$

2 As we know,

Path difference introduced by thin film,

$$\Delta = (\mu - 1)t \quad \ldots(\text{i})$$



and if fringe pattern shifts by one frings width, then path difference,

$$\Delta = 1 \times \lambda = \lambda \quad \ldots(\text{ii})$$

So, from Eqs. (i) and (ii), we get

$$(\mu - 1)t = \lambda \Rightarrow t = \frac{\lambda}{\mu - 1}$$

Alternate Solution

Path difference introduced by the thin film of thickness t and refractive index μ is given by

$$\Delta = (\mu - 1)t$$

$\therefore$ Position of the fringe is

$$x = \frac{\Delta D}{d} = \frac{(\mu - 1)tD}{d} \quad \ldots(\text{i})$$

Fringe width of one fringe is given by

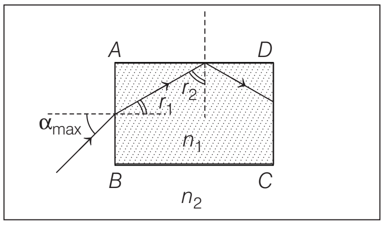
$$\beta = \frac{\lambda D}{d} \quad \ldots(\text{ii})$$

Given that $x = \beta$, so from Eqs. (i) and (ii), we get

$$\Rightarrow \quad \frac{(\mu - 1)\,tD}{d} = \frac{\lambda D}{d}$$

$$\Rightarrow \quad (\mu - 1)t = \lambda \text{ or } t = \frac{\lambda}{(\mu - 1)}$$

3.



Key Idea Amplitude of light is directly proportional to area through which light is passing.

For same length of slits,

$$\text{amplitude} \propto (\text{width})^{1/2}$$

Also, $$\text{intensity} \propto (\text{amplitude})^2$$

In YDSE, ratio of intensities of maxima and minima is given by

$$\frac{I_{\max}}{I_{\min}} = \frac{(\sqrt{I_1} + \sqrt{I_2})^2}{(\sqrt{I_1} - \sqrt{I_2})^2}$$

where, I_1 and I_2 are the intensities obtained from two slits, respectively.

$$\Rightarrow \quad \frac{I_{\max}}{I_{\min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2}$$

where, a_1 and a_2 are light amplitudes from slits 1 and 2, respectively.

$$\Rightarrow \quad \frac{I_{\max}}{I_{\min}} = \frac{(\sqrt{W_1} + \sqrt{W_2})^2}{(\sqrt{W_1} - \sqrt{W_2})^2}$$

where, W_1 and W_2 are the widths of slits, respectively.

Here, $\left(\frac{W_1}{W_2}\right) = \left(\frac{a_1}{a_2}\right)^2 = \frac{4}{1} \Rightarrow \sqrt{\frac{W_1}{W_2}} = \frac{2}{1}$

So, we have

$$\frac{I_{\max}}{I_{\min}} = \left(\frac{\sqrt{W_1} + \sqrt{W_2}}{\sqrt{W_1} - \sqrt{W_2}}\right)^2 = \left(\frac{\sqrt{\frac{W_1}{W_2}} + 1}{\sqrt{\frac{W_1}{W_2}} - 1}\right)^2$$

$$= \left(\frac{2+1}{2-1}\right)^2 = 9:1$$

4. Path difference introduced by a slab of thickness t and refractive index μ is given by

$$\Delta = (\mu - 1)\,t$$

Position of the fringe is $x = \frac{\Delta D}{d} = \frac{(\mu - 1)tD}{d}$

Also, fringe width is given by

$$\beta = \frac{\lambda D}{d}$$

According to the question,

$$n\beta = x$$

$$\Rightarrow \quad \frac{n\lambda D}{d} = (\mu - 1)t\frac{D}{d}$$

$$\Rightarrow n\lambda = (\mu - 1)t \quad \text{or } t = \frac{n\lambda}{(\mu - 1)}$$

Alternate Solution

Path difference, $\Delta = \frac{xd}{D} = (\mu - 1)t$

or $$t = \frac{xd}{(\mu - 1)D}$$

Also, $$\beta = \frac{\lambda D}{d} \Rightarrow \frac{d}{D} = \frac{\lambda}{\beta}$$

$$\therefore \quad t = \frac{x\lambda}{(\mu - 1)\beta} = \frac{n\lambda D \times \lambda}{d(\mu - 1)\beta} \qquad [\because x = n\beta]$$

$$\Rightarrow \quad t = \frac{n\lambda}{(\mu - 1)}$$

So, no option is correct.

5. In Young's double slit experiment, ratio of maxima and minima intensity is given by

$$\frac{I_{\max}}{I_{\min}} = \left(\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2 = \left(\frac{\sqrt{I_1/I_2} + 1}{\sqrt{I_1/I_2} - 1}\right)^2$$

As, intensity $(I) \propto [\text{amplitude }(a)]^2$

$$\therefore \quad \frac{I_1}{I_2} = \left(\frac{a_1}{a_2}\right)^2 = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

So, $$\frac{I_{\max}}{I_{\min}} = \left(\frac{\frac{1}{3} + 1}{\frac{1}{3} - 1}\right)^2 = 4:1$$

6. Here, wavelength of light used $(\lambda) = 5303\,\text{Å}$.

Distance between two slit $(d) = 19.44\ \mu\text{-m}$

Width of single slit (a)= $4.05\,\mu\text{m}$

Here, angular width between first and second diffraction minima

$$\theta \simeq \frac{\lambda}{a}$$

and angular width of a fringe due to double slit is

$$\theta' = \frac{\lambda}{d}$$

$\therefore$ Number of fringes between first and second diffraction minima, $n = \frac{\theta}{\theta'} = \frac{\frac{\lambda}{a}}{\frac{\lambda}{d}} = \frac{d}{a} = \frac{19.44}{4.05} = 4.81$ or $n \simeq 5$

$\therefore$ 5 interfering bright fringes lie between first and second diffraction minima.

7. Let intensity of each wave is I_0.

Then, intensity at the centre of bright fringe will be $4I_0$.

Given, path difference, $\Delta x = \lambda/8$

$\therefore$ Phase difference, $\phi = \Delta x \times \frac{2\pi}{\lambda}$

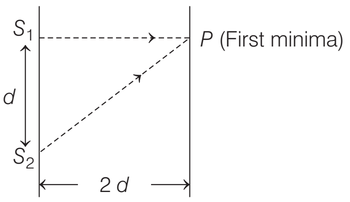
$$\Rightarrow \quad \phi = \frac{\lambda}{8} \times \frac{2\pi}{\lambda} \text{ or } \phi = \pi/4$$

Intensity of light at this point,

$$I' = I_0 + I_0 + 2I_0 \cos(\pi/4) = 2I_0 + \sqrt{2}\,I_0 = 3.41\,I_0$$

Now, $$\frac{I'}{4I_0} = \frac{3.41}{4} = 0.85$$

8. In the given case, figure for first minima will be as shown below



We know that condition for minima in Young's double slit experiment is path difference,

$$\Delta x = (2n-1)\lambda/2$$

For first minima, $n = 1$

$$\Rightarrow \quad \Delta x = \lambda/2 \quad \dots \text{(i)}$$

Path difference between the rays coming from virtual sources S_1 and S_2 at point 'P' will be

$$\Delta x = S_2P - S_1P \quad \dots \text{(ii)}$$

From triangle S_1S_2P,

$$S_1P = 2d \quad \dots \text{(iii)}$$

and $\quad (S_2P^2) = (S_1S_2)^2 + (S_1P)^2 = d^2 + (2d)^2$

$$\Rightarrow \quad (S_2P^2) = 5d^2 \text{ or } S_2P = \sqrt{5}\,d \quad \dots \text{(iv)}$$

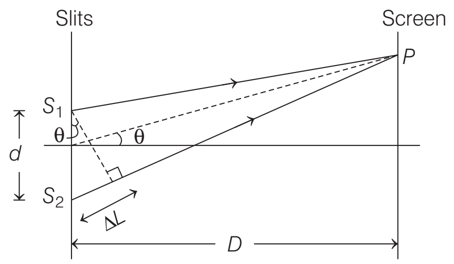
Substituting the values from Eqs. (iii) and (iv) in Eq. (ii), we get

$$\Delta x = \sqrt{5}\,d - 2d \quad \dots \text{(v)}$$

From Eqs. (i) and (v), we get

$$\sqrt{5}\,d - 2d = \lambda/2 \Rightarrow d = \frac{\lambda}{2(\sqrt{5}-2)}$$

9. **Key Idea** In a YDSE, path difference between 2 rays, reaching at some common point P located at angular position 'θ' as shown in the figure below is



$$\Delta L = d \sin\theta$$

For small value of θ; $\sin\theta \approx \theta$

So, path difference $= \Delta L = d\theta$

For a bright fringe at same angular position 'θ', both of the rays from slits S_1 and S_2 are in phase.

Hence, path difference is an integral multiple of wavelength of light used.

i.e., $\quad \Delta L = n\lambda$

or $\quad d\theta = n\lambda \Rightarrow \lambda = \dfrac{d\theta}{n}$

Here, $\theta = \dfrac{1}{40}$ rad, $d = 0.1$ mm

Hence, $\lambda = \dfrac{0.1}{40n}\text{ mm} = \dfrac{0.1\times 10^{-3}\text{ m}}{40n}$

$$= \frac{0.1\times 10^{-3}\times 10^{9}}{40n}\text{ n-m}$$

$$\Rightarrow \quad \lambda = \frac{2500}{n}\text{ n-m}$$

So, with light of wavelength λ_1 we have

$$\lambda_1 = \frac{2500}{n_1}\text{ (n-m)}$$

and with light of wavelength λ_2, we have

$$\lambda_2 = \frac{2500}{n_2}\text{ (n-m)}$$

Now, choosing different integral values for n_1 and n_2, (i.e., $n_1, n_2 = 1, 2, 3 \dots$ etc) we find that for

$$n_1 = 4,\ \lambda_1 = \frac{2500}{4} = 625\text{ n-m}$$

and for $n_2 = 5$,

$$\lambda_2 = \frac{2500}{5} = 500\text{ n-m}$$

These values lie in given interval 500 n-m to 625 n-m.

10. In Young's double slit experiment, the condition of bright fringe and dark fringe are,

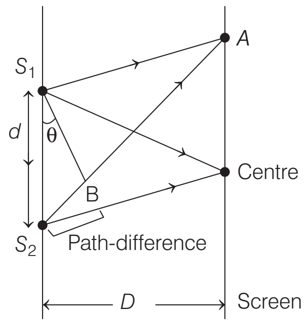
for bright fringes (maxima), path difference $= n\lambda$

$$d\sin\theta = n\lambda$$

for dark fringes (minima),

path-difference $= (2n-1)\dfrac{\lambda}{2}$

$$d\sin\theta = (2n-1)\frac{\lambda}{2}$$



For the given question, distance between slits

$$(d) = 0.320\text{ mm}$$

Wavelength of light used $(\lambda) = 500$ n-m

Angular range for bright fringe $(\theta) = -30^\circ$ to 30°

Hence, for bright-fringe,

$$n\lambda = d\sin\theta$$

$$n = \frac{d\sin\theta}{\lambda} = \frac{0.320\times 10^{-3}\times \sin 30^\circ}{500\times 10^{-9}}$$

$$n_{max} = 320$$

$\therefore$ Total number of maxima between the two lines are

$$n = (n_{max} \times 2) + 1$$

Here, $n = (320 \times 2) + 1$

$$n = 641$$

11. Let the intensity of two coherent sources be I_1 and I_2, respectively. It is given that,

$$\frac{\text{maximum intensity}}{\text{minimum intensity}} = \frac{I_{max}}{I_{min}} = \frac{16}{1}$$

Since, we know

$$I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$$

and $I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2$

$\therefore$ We can write,

$$\frac{I_{max}}{I_{min}} = \frac{(\sqrt{I_1} + \sqrt{I_2})^2}{(\sqrt{I_1} - \sqrt{I_2})^2} = 16$$

$$\Rightarrow \quad \frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}} = \frac{\sqrt{16}}{1} = \frac{4}{1}$$

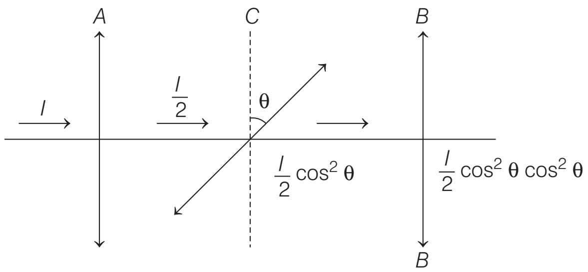
$$\sqrt{I_1} + \sqrt{I_2} = 4\sqrt{I_1} - 4\sqrt{I_2}$$

$$\Rightarrow \quad 5\sqrt{I_2} = 3\sqrt{I_1} \Rightarrow \frac{\sqrt{I_1}}{\sqrt{I_2}} = \frac{5}{3}$$

Squaring on both the sides, we get

$$\frac{I_1}{I_2} = \frac{25}{9}$$

12.



Using the relation, $I = I_0 \cos^2\theta$

We have,

$$\frac{I}{2}(\cos^2\theta)^2 = \frac{I}{8} \quad \Rightarrow \quad \cos^2\theta = \frac{1}{2}$$

or $\cos\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45°$

13. In single slit diffraction pattern, $\lambda = b\sin\theta$

At $\theta = 30°$,

$$\lambda = \frac{b}{2} = \frac{1\times10^{-6}}{2} = 5\times10^{-7}\,\text{m}$$

In YDSE,

fringe width, $\omega = \frac{\lambda D}{d} \Rightarrow d = \frac{\lambda D}{\omega} = \frac{5\times10^{-7}\times0.5}{1\times10^{-2}}$

$$d = 25\times10^{-6}\text{ m} = 25\mu\text{m}$$

14. Let nth bright fringes coincides, then

$$y_{n_1} = y_{n_2}$$

$$\Rightarrow \quad \frac{n_1\lambda_1 D}{d} = \frac{n_2\lambda_2 D}{d} \Rightarrow \frac{n_1}{n_2} = \frac{\lambda_1}{\lambda_2} = \frac{520}{650} = \frac{4}{5}$$

Hence, distance from the central maxima is

$$y = \frac{n_1\lambda_1 D}{d} = \frac{4\times650\times10^{-9}\times1.5}{0.5\times10^{-3}} = 7.8\,\text{mm}$$

15. A fringe is a locus of points having constant path difference from the two coherent sources S_1 and S_2. It will be concentric circle.

16. $$I = I_{max}\cos^2\frac{\phi}{2} \qquad \ldots\text{(i)}$$

Given, $$I = \frac{I_{max}}{2} \qquad \ldots\text{(ii)}$$

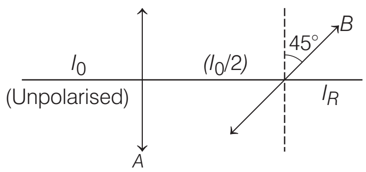
$\therefore$ From Eqs. (i) and (ii), we have

$$\phi = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$$

Or path difference, $\Delta x = \left(\frac{\lambda}{2\pi}\right)\cdot\phi$

$$\therefore \quad \Delta x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4} .., \left(\frac{2n+1}{4}\right)\lambda$$

17. Relation between intensities is



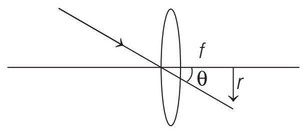
$$I_R = \left(\frac{I_0}{2}\right)\cos^2(45°) = \frac{I_0}{2}\times\frac{1}{2} = \frac{I_0}{4}$$

18. Fringe width $\beta = \frac{\lambda D}{d}$ or $\beta \propto \lambda$

Now, $\lambda_R > \lambda_G > \lambda_B$

$\therefore$ $\beta_R > \beta_G > \beta_B$

19.



$$r = f\tan\theta$$

or $r \propto f$

$\therefore$ $\pi r^2 \propto f^2$

20. $$I = I_{max}\cos^2\left(\frac{\phi}{2}\right)$$

$$\therefore \quad \frac{I_{max}}{4} = I_{max}\cos^2\frac{\phi}{2}$$

$$\cos\frac{\phi}{2} = \frac{1}{2}$$

or $\quad \dfrac{\phi}{2} = \dfrac{\pi}{3}$

$\therefore \quad \phi = \dfrac{2\pi}{3} = \left(\dfrac{2\pi}{\lambda}\right)\Delta x \quad \ldots$(i)

where $\quad \Delta x = d\sin\theta$

Substituting in Eq. (i), we get

$$\sin\theta = \frac{\lambda}{3d} \text{ or } \theta = \sin^{-1}\left(\frac{\lambda}{3d}\right)$$

21. Let nth minima of 400 nm coincides with mth minima of 560 nm, then

$$(2n-1)\left(\frac{400}{2}\right) = (2m-1)\left(\frac{560}{2}\right)$$

or $\quad \dfrac{2n-1}{2m-1} = \dfrac{7}{5} = \dfrac{14}{10} = \ldots$

i.e. 4th minima of 400 nm coincides with 3rd minima of 560 nm.

Location of this minima is,

$$Y_1 = \frac{(2\times 4-1)(1000)(400\times 10^{-6})}{2\times 0.1}$$

$$= 14 \text{ mm}$$

Next 11th minima of 400 nm will coincide with 8th minima of 560 nm.

Location of this minima is,

$$Y_2 = \frac{(2\times 11-1)(1000)(400\times 10^{-6})}{2\times 0.1} = 42 \text{ mm}$$

$\therefore \quad$ Required distance $= Y_2 - Y_1 = 28$ mm

22. $PR = d$

$\therefore \quad PO = d\sec\theta$

and $\quad CO = PO\cos 2\theta = d\sec\theta\cos 2\theta$

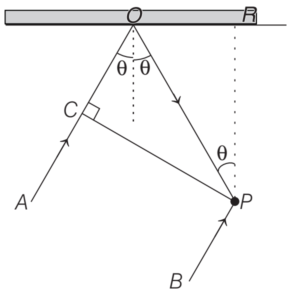
path difference between the two rays is,

$$\Delta x = PO + OC = (d\sec\theta + d\sec\theta\cos 2\theta)$$

phase difference between the two rays is

$\Delta\phi = \pi$ (one is reflected, while another is direct)

Therefore, condition for constructive interference should be



$$\Delta x = \frac{\lambda}{2}, \frac{3\lambda}{2} \ldots$$

or $\quad d\sec\theta\,(1+\cos 2\theta) = \dfrac{\lambda}{2}$

or $\quad \left(\dfrac{d}{\cos\theta}\right)(2\cos^2\theta) = \dfrac{\lambda}{2}$ or $\cos\theta = \dfrac{\lambda}{4d}$

23. Path difference due to slab should be integral multiple of λ or

$$\Delta x = n\lambda \quad \text{or} \quad (\mu-1)t = n\lambda \quad n = 1, 2, 3, \ldots$$

or $\quad t = \dfrac{n\lambda}{\mu-1}$

For minimum value of t, $n = 1$

$\therefore \quad t = \dfrac{\lambda}{\mu-1} = \dfrac{\lambda}{1.5-1} = 2\lambda$

24. Fringe width, $\omega = \dfrac{\lambda D}{d} \propto \lambda$

When the wavelength is decreased from 600 nm to 400 nm, fringe width will also decrease by a factor of $\dfrac{4}{6}$ or $\dfrac{2}{3}$ or the number of fringes in the same segment will increase by a factor of 3/2.

Therefore, number of fringes observed in the same segment

$$= 12 \times \frac{3}{2} = 18$$

NOTE Since $\omega \propto \lambda$, therefore, if YDSE apparatus is immersed in a liquid of refractive index μ, the wavelength λ and thus the fringe width will decrease μ times.

25. $\quad I(\phi) = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi \quad \ldots$(i)

Here, $\quad I_1 = I$ and $I_2 = 4I$

At point A, $\phi = \dfrac{\pi}{2}$

$\therefore \quad I_A = I + 4I = 5I$

At point B, $\phi = \pi$

$\therefore \quad I_B = I + 4I - 4I = I$

$\therefore \quad I_A - I_B = 4I$

NOTE Eq. (i) for resultant intensity can be applied only when the sources are coherent. In the question it is given that the rays interfere. Interference takes place only when the sources are coherent. That is why we applied equation number (i). When the sources are incoherent, the resultant intensity is given by $I = I_1 + I_2$

26. In interference we know that

$$I_{\max} = (\sqrt{I_1} + \sqrt{I_2})^2$$

and $\quad I_{\min} = (\sqrt{I_1} \sim \sqrt{I_2})^2$

Under normal conditions (when the widths of both the slits are equal)

$$I_1 \approx I_2 = I \text{ (say)}$$

$\therefore \quad I_{\max} = 4I \quad$ and $\quad I_{\min} = 0$

When the width of one of the slits is increased. Intensity due to that slit would increase, while that of the other will remain same. So, let :

$$I_1 = I$$

and $\quad I_2 = \eta I \quad (\eta > 1)$

Then, $\quad I_{\max} = I\,(1+\sqrt{\eta})^2 > 4I$

and $\quad I_{\min} = I\,(\sqrt{\eta}-1)^2 > 0$

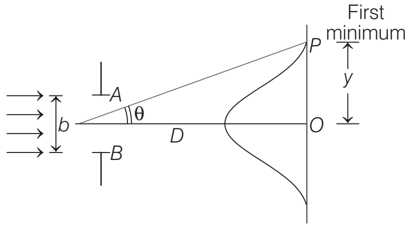
$\therefore$ Intensity of both maxima and minima is increased.

27. Locus of equal path difference are the lines running parallel to the axis of the cylinder. Hence, straight fringes are obtained.

NOTE Circular rings (also called Newton's rings) are observed in interference pattern when a plano-convex lens of large focal length is placed with its convex surface in contact with a plane glass plate because locus of equal path difference in this case is a circle.

28. Diffraction is obtained when the slit width is of the order of wavelength of light (or any electromagnetic wave) used. Here, wavelength of X-rays (1-100 Å) << slit width (0.6 mm). Therefore, no diffraction pattern will be observed.

29. At first minima, $b\sin\theta = \lambda$



or $\quad b\,\theta = \lambda$ or $b\left(\dfrac{y}{D}\right) = \lambda$

or $\quad y = \dfrac{\lambda D}{b}$

or $\quad \dfrac{yb}{D} = \lambda \qquad$ ... (i)

Now, at P (First minima) path difference between the rays reaching from two edges (A and B) will be

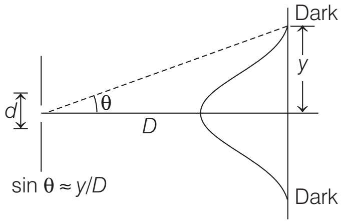
$$\Delta x = \frac{yb}{D} \qquad \left(\text{Compare with } \Delta x = \frac{yd}{D} \text{ in YDSE}\right)$$

or $\quad \Delta x = \lambda \qquad$ [From Eq. (i)]

Corresponding phase difference (ϕ) will be

$$\phi = \left(\frac{2\pi}{\lambda}\right).\Delta x,\ \phi = \frac{2\pi}{\lambda}.\lambda = 2\pi$$

30. For first dark fringe on either side $d\sin\theta = \lambda$



or $\quad \dfrac{dy}{D} = \lambda$

$\therefore \quad y = \dfrac{\lambda D}{d}$

Therefore, distance between two dark fringes on either side

$$= 2y = \frac{2\lambda D}{d}$$

Substituting the values, we have

$$\text{Distance} = \frac{2(600\times 10^{-6}\text{ mm})(2\times 10^{3}\text{ mm})}{(1.0\text{ mm})} = 2.4\text{ mm}$$

31. $I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2 = (\sqrt{4I} + \sqrt{I})^2 = 9I$

$I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2 = (\sqrt{4I} - \sqrt{I})^2 = I$

32. $\omega = \dfrac{\lambda D}{d}$

d is halved and D is doubled

$\therefore$ Fringe width ω will become four times.

$\therefore$ Correct option is (d).

33. (A) $\rightarrow$ (p, s) $\rightarrow$ Intensity at P_0 is maximum. It will continuously decrease from P_0 towards P_2

(B) $\rightarrow$ (q) $\rightarrow$ Path difference due to slap will be compensated by geometrical path difference. Hence, $\delta(P_1) = 0$.

(C) $\rightarrow$ (t) $\rightarrow$ $\delta(P_0) = \dfrac{\lambda}{2}, \delta(P_1) = \dfrac{\lambda}{2} - \dfrac{\lambda}{4} = \dfrac{\lambda}{4}$ and

$\delta(P_2) = \dfrac{\lambda}{2} - \dfrac{\lambda}{3} = \dfrac{\lambda}{6}$. When path difference increases from 0 to $\dfrac{\lambda}{2}$, intensity will decrease from maximum to zero. Hence, in this case,

$$I(P_2) > I(P_1) > I(P_0)$$

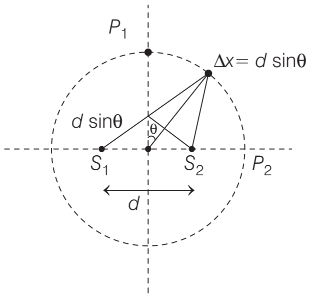
(D) $\rightarrow$ (r, s, t,)

$$\delta(P_0) = \frac{3\lambda}{4}, \delta(P_1) = \frac{3\lambda}{4} - \frac{\lambda}{4} = \frac{\lambda}{2}$$

and $\quad \delta(P_2) = \dfrac{3\lambda}{4} - \dfrac{\lambda}{3} = \dfrac{5\lambda}{12}$.

In this case $I(P_1) = 0$..

34.



$$\lambda = 600\text{ nm}$$

at $P_1 \qquad \Delta x = 0$

at $P_2 \qquad \Delta x = 1.8\text{ mm} = n\lambda$

Number of maximas will be $n = \dfrac{\Delta x}{\lambda} = \dfrac{1.8\text{ mm}}{600\text{ nm}} = 3000$

at $P_2 \qquad \Delta x = 3000\lambda$

Hence, bright fringe will be formed.

At P_2, 3000th maxima is formed.

For (a) option

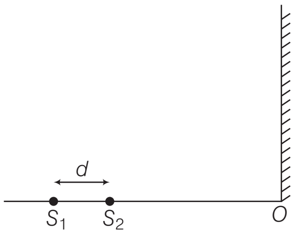
$$\Delta x = d\sin\theta \Rightarrow d\Delta x = d\cos d\theta$$

$$R\lambda = d\cos\theta R d\theta \Rightarrow R d\theta = \frac{R\lambda}{d\cos\theta}$$

As we move from P_1 to P_2

$\theta\uparrow \cos\theta\downarrow R d\theta\uparrow$

35.



Path difference at point $O = d = 0.6003$ mm

$= 600300$ nm.

This path difference is equal to $\left(1000\lambda + \frac{\lambda}{2}\right)$.

$\Rightarrow$ Minima is formed at point O.

Line S_1S_2 and screen are $\perp$ to each other so fringe pattern is circular (semi-circular because only half of screen is available)

36. Fringe width $\beta = \frac{\lambda D}{d}$

or $\qquad \beta \propto \lambda$

$\therefore \qquad \lambda_2 > \lambda_1$

So $\qquad \beta_2 > \beta_1$

Number of fringes in a given width

$$m = \frac{y}{\beta} \quad \text{or} \quad m \propto \frac{1}{\beta}$$

$\Rightarrow \qquad m_2 < m_1$ as $\beta_2 > \beta_1$

Distance of 3rd maximum of λ_2 from central maximum

$$= \frac{3\lambda_2 D}{d} = \frac{1800D}{d}$$

Distance of 5th minimum of λ_1 from central maximum

$$= \frac{9\lambda_1 D}{2d} = \frac{1800D}{d}$$

So, 3rd maximum of λ_2 will overlap with 5th minimum of λ_1.

Angular separation (or angular fringe width) $= \frac{\lambda}{d} \propto \lambda$

$\Rightarrow$ Angular separation for λ_1 will be lesser.

37. $d = \frac{\lambda}{2\sin\theta}$ ln $d = \ln\lambda - \ln 2 - \ln\sin\theta$

$$\frac{\Delta(d)}{d} = 0 - 0 - \frac{1}{\sin\theta} \times \cos\theta\,(\Delta\theta)$$

Fractional error $= \left|\frac{\Delta d}{d}\right| = (\cot\theta)\,\Delta\theta$

Absolute error $\Delta d = (d\cot\theta)\,\Delta\theta$

$$= \left(\frac{\lambda}{2\sin\theta}\right)\left(\frac{\cos\theta}{\sin\theta}\right)\Delta\theta$$

Now, given that $\Delta\theta =$ constant

As θ increases, $\sin\theta$ increases, $\cos\theta$ and $\cot\theta$ decrease.

$\therefore$ Both fractional and absolute errors decrease.

38. For $d = \lambda$, there will be only one, central maxima.

For $\lambda < d < 2\lambda$, there will be three maximas on the screen corresponding to path difference, $\Delta x = 0$ and $\Delta x = \pm\lambda$.

39. The intensity of light is $I(\theta) = I_0\cos^2\left(\frac{\delta}{2}\right)$

where, $\qquad \delta = \frac{2\pi}{\lambda}(\Delta x) = \left(\frac{2\pi}{\lambda}\right)(d\sin\theta)$

(a) For $\theta = 30°$

$$\lambda = \frac{c}{\nu} = \frac{3\times10^8}{10^6} = 300 \text{ m and } d = 150 \text{ m}$$

$$\delta = \left(\frac{2\pi}{300}\right)(150)\left(\frac{1}{2}\right) = \frac{\pi}{2}$$

$$\therefore \qquad \frac{\delta}{2} = \frac{\pi}{4}$$

$$\therefore \qquad I(\theta) = I_0\cos^2\left(\frac{\pi}{4}\right) = \frac{I_0}{2}$$ [option (a)]

(b) For $\qquad \theta = 90°$

$$\delta = \left(\frac{2\pi}{300}\right)(150)(1) = \pi$$

or $\qquad \frac{\delta}{2} = \frac{\pi}{2}$ and $I(\theta) = 0$

(c) For $\theta = 0°$, $\delta = 0$ or $\frac{\delta}{2} = 0$

$$\therefore \qquad I(\theta) = I_0$$ [option (c)]

40. At P (directly infront of S_1) $y = b/2$

$\therefore$ Path difference,

$$\Delta X = S_2P - S_1P = \frac{y\cdot(b)}{d} = \frac{\left(\frac{b}{2}\right)(b)}{d} = \frac{b^2}{2d}$$

Those wavelengths will be missing for which

$$\Delta X = \frac{\lambda_1}{2}, \frac{3\lambda_2}{2}, \frac{5\lambda_3}{2} \ldots$$

$$\therefore \qquad \lambda_1 = 2\Delta x = \frac{b^2}{d}$$

$$\lambda_2 = \frac{2\Delta x}{3} = \frac{b^2}{3d}$$

$$\lambda_3 = \frac{2\Delta x}{5} = \frac{b^2}{5d}$$

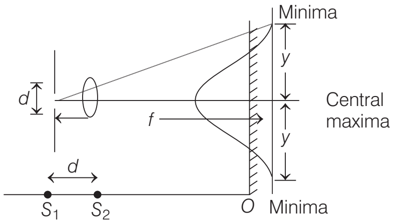
41. $\frac{I_{max}}{I_{min}} = \frac{(\sqrt{I_1}+\sqrt{I_2})^2}{(\sqrt{I_1}-\sqrt{I_2})^2} = \left(\frac{\sqrt{I_1/I_2}+1}{\sqrt{I_1/I_2}-1}\right)^2 = 9$ (Given)

Solving this, we have $\frac{I_1}{I_2} = 4$ but $I \propto A^2$

$$\therefore \qquad \frac{A_1}{A_2} = 2$$

$\therefore$ Correct options are (b) and (d).

42. Given



$$2y = 2 \times 10^{-3} \text{ m}$$
$$y = 1 \times 10^{-3} \text{ m}$$

First minima is obtained at

$$d \sin\theta = \lambda \text{ but } \sin\theta \approx \tan\theta = \frac{y}{f}$$

$$\therefore \quad d\left(\frac{y}{f}\right) = \lambda$$

$$d = \frac{\lambda f}{y} = \frac{5.89 \times 10^{-7} \times 0.5}{1 \times 10^{-3}}$$
$$= 2.945 \times 10^{-4} \text{ m}$$

43. In case of YDSE, at mid-point intensity will be $I_{max} = 4I_0$

In the second case when sources are incoherent, the intensity will be $I = I_0 + I_0 = 2I_0$

Therefore, the desired ratio is

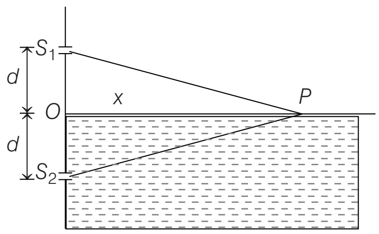
$$\frac{4I_0}{2I_0} = 2$$

Here, I_0 is the intensity due to one slit.

44. With white light we get coloured fringes (not only black and white) with centre as white.

45. To obtain interference, sources must be coherent. Two different light sources can never be coherent.

46.



$$\mu(S_2P) - S_1P = m\lambda$$

$$\Rightarrow \mu\sqrt{d^2 + x^2} - \sqrt{d^2 + x^2} = m\lambda$$

$$\Rightarrow (\mu - 1)\sqrt{d^2 + x^2} = m\lambda$$

$$\Rightarrow \left(\frac{4}{3} - 1\right)\sqrt{d^2 + x^2} = m\lambda$$

or $$\sqrt{d^2 + x^2} = 3m\lambda$$

Squaring this equation we get,

$$x^2 = 9m^2\lambda^2 - d^2$$

$$\Rightarrow p^2 = 9 \quad \text{or} \quad p = 3$$

47. Let n_1 bright fringe corresponding to wavelength $\lambda_1 = 500$ nm coincides with n_2 bright fringe corresponding to wavelength $\lambda_2 = 700$ nm.

$$\therefore \quad n_1 \frac{\lambda_1 D}{d} = n_2 \frac{\lambda_2 D}{d}$$

or $$\frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1} = \frac{7}{5}$$

This implies that 7th maxima of λ_1 coincides with 5th maxima of λ_2. Similarly 14th maxima of λ_1 will coincide with 10th maxima of λ_2 and so on.

$$\therefore \text{ Minimum distance} = \frac{n_1\lambda_1 D}{d} = 7 \times 5 \times 10^{-7} \times 10^3$$
$$= 3.5 \times 10^{-3} \text{ m} = 3.5 \text{ mm}$$

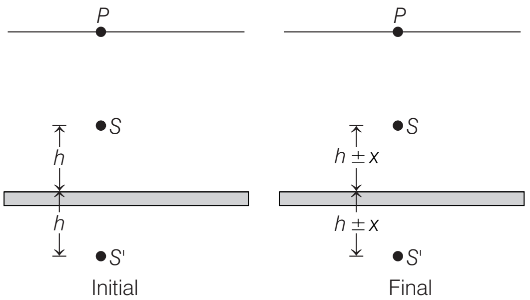
48. (a) Shape of the interference fringes will be circular.

(b) Intensity of light reaching on the screen directly from the source $I_1 = I_0$ (say) and intensity of light reaching on the screen after reflecting from the mirror is $I_2 = 36\%$ of $I_0 = 0.36I_0$

$$\therefore \quad \frac{I_1}{I_2} = \frac{I_0}{0.36I_0} = \frac{1}{0.36} \text{ or } \sqrt{\frac{I_1}{I_2}} = \frac{1}{0.6}$$

$$\therefore \quad \frac{I_{min}}{I_{max}} = \frac{\left(\sqrt{\frac{I_1}{I_2}} - 1\right)^2}{\left(\sqrt{\frac{I_1}{I_2}} + 1\right)^2} = \frac{\left(\frac{1}{0.6} - 1\right)^2}{\left(\frac{1}{0.6} + 1\right)^2} = \frac{1}{16}$$

(c) Initially path difference at P between two waves reaching from S and S' is $2h$.



Therefore, for maximum intensity at P :

$$2h = \left(n - \frac{1}{2}\right)\lambda \qquad \dots \text{(i)}$$

Now, let the source S is path difference will be $2h + 2x$ or $2h - 2x$. So, for displaced by x (away or towards mirror) then nemaximum intensity at P

$$2h + 2x = \left[n + 1 - \frac{1}{2}\right]\lambda \qquad \dots \text{(ii)}$$

or $$2h - 2x = \left[n - 1 - \frac{1}{2}\right]\lambda \qquad \dots \text{(iii)}$$

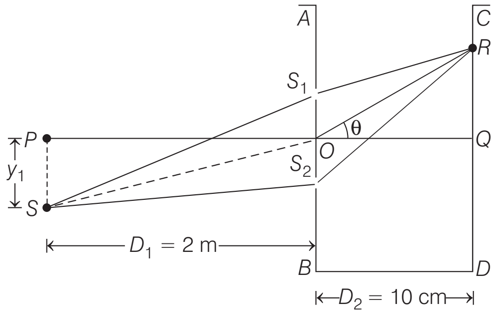
Solving Eqs. (i) and (ii) or Eqs. (i) and (iii), we get

$$x = \frac{\lambda}{2} = \frac{600}{2} = 300 \text{ nm}$$

NOTE Here, we have taken the condition of maximum intensity at P as : Path difference $\Delta x = \left(n - \frac{1}{2}\right)\lambda$

Because the reflected beam suffers a phase difference of π.

49. Given $y_1 = 40\,\text{cm}$, $D_1 = 2\text{ m} = 200\,\text{cm}$, $D_2 = 10\,\text{cm}$



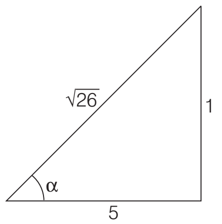
$$\tan\alpha = \frac{y_1}{D_1} = \frac{40}{200} = \frac{1}{5} \Rightarrow \therefore\ \alpha = \tan^{-1}(1/5)$$

$$\sin\alpha = \frac{1}{\sqrt{26}} \approx \frac{1}{5} = \tan\alpha$$

Path difference between SS_1 and SS_2 is

$$\Delta X_1 = SS_1 - SS_2 \text{ or } \Delta X_1 = d\sin\alpha = (0.8\,\text{mm})\left(\frac{1}{5}\right)$$

or $\quad \Delta X_1 = 0.16\,\text{mm} \quad \ldots\text{(i)}$



Now, let at point R on the screen, central bright fringe is observed (*i.e.,* net path difference = 0).

Path difference between S_2R and S_1R would be

$$\Delta X_2 = S_2R - S_1R$$

or $\quad \Delta X_2 = d\sin\theta \quad \ldots\text{(ii)}$

Central bright fringe will be observed when net path difference is zero.

or $\quad \Delta X_2 - \Delta X_1 = 0$

$$\Delta X_2 = \Delta X_1$$

or $\quad d\sin\theta = 0.16$

or $\quad (0.8)\sin\theta = 0.16$

or $\quad \sin\theta = \frac{0.16}{0.8} = \frac{1}{5}$

$$\tan\theta = \frac{1}{\sqrt{24}} \approx \sin\theta = \frac{1}{5}$$

Hence, $\quad \tan\theta = \frac{y_2}{D_2} = \frac{1}{5}$

$\therefore \quad y_2 = \frac{D_2}{5} = \frac{10}{5} = 2\,\text{cm}$

Therefore, central bright fringe is observed at 2 cm above point Q on side CD.

Alternate solution

ΔX at R will be zero if $\Delta X_1 = \Delta X_2$

or $\quad d\sin\alpha = d\sin\theta$

or $\quad \alpha = \theta$

or $\quad \tan\alpha = \tan\theta$

$\Rightarrow \quad \frac{y_1}{D_1} = \frac{y_2}{D_2}$

or $\quad y_2 = \frac{D_2}{D_1}\cdot y_1 = \left(\frac{10}{200}\right)(40)\,\text{cm}$

or $\quad y_2 = 2\,\text{cm}$

(b) The central bright fringe will be observed at point Q. If the path difference created by the liquid slab of thickness $t = 10\,\text{cm}$ or 100 mm is equal to ΔX_1, so that the net path difference at Q becomes zero.

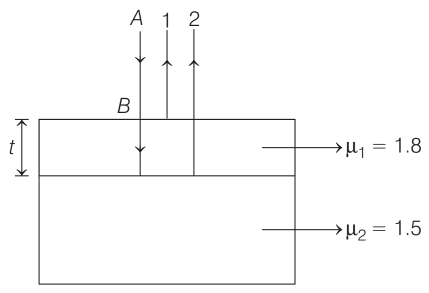
So, $\quad (\mu - 1)\,t = \Delta X_1$

or $\quad (\mu - 1)(100) = 0.16$

or $\quad \mu - 1 = 0.0016$

or $\quad \mu = 1.0016$

50. Incident ray AB is partly reflected as ray 1 from the upper surface and partly reflected as ray 2 from the lower surface of the layer of thickness t and refractive index $\mu_1 = 1.8$ as shown in figure. Path difference between the two rays would by



$$\Delta x = 2\mu_1\, t = 2(1.8)\,t = 3.6\,t$$

Ray 1 is reflected from a denser medium, therefore, it undergoes a phase change of π, whereas the ray 2 gets reflected from a rarer medium, therefore, there is no change in phase of ray 2.

Hence, phase difference between rays 1 and 2 would be $\Delta\phi = \pi$. Therefore, condition of constructive interference will be

$$\Delta x = \left(n - \frac{1}{2}\right)\lambda \text{ where } n = 1, 2, 3\ldots \text{ or } 3.6\,t = \left(n - \frac{1}{2}\right)\lambda$$

Least value of t is corresponding to $n = 1$ or

$$t_{\text{min}} = \frac{\lambda}{2 \times 3.6}$$

or $\quad t_{\text{min}} = \frac{648}{7.2}\,\text{nm}$

or $\quad t_{\text{min}} = 90\,\text{nm}$

NOTE

- For a wave (whether it is sound or electromagnetic), a medium is denser or rarer is decided from the speed of wave in that medium. In denser medium speed of wave is less. For example, water is rarer for sound, while denser for light compared to air because speed of sound in water is more than in air, while speed of light is less.
- In transmission/refraction, no phase change takes place. In reflection, there is a change of phase of π when it is reflected by a denser medium and phase change is zero if it is reflected by a rarer medium.
- If two waves in phase interfere having a path difference of Δx; then condition of maximum intensity would be $\Delta x = n\lambda$ where $n = 0, 1, 2, \ldots$
- But if two waves, which are already out of phase (a phase difference of π) interfere with path difference Δx, then condition of maximum intensity will be $\Delta x = \left(n - \frac{1}{2}\right)\lambda$ where $n = 1, 2, \ldots$

51. Given, $\lambda = 600 \text{ nm} = 6 \times 10^{-7}$ m,

$d = 0.45 \text{ mm} = 0.45 \times 10^{-3}$ m,

$D = 1.5$ m.

Thickness of glass sheet, $t = 10.4\ \mu\text{m} = 10.4 \times 10^{-6}$ m

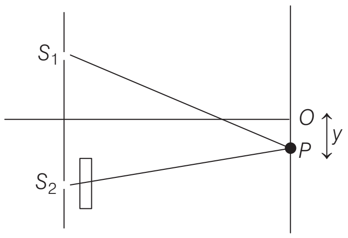
Refractive index of medium, $\mu_m = 4/3$

and refractive index of glass sheet, $\mu_g = 1.5$

(a) Let central maximum is obtained at a distance y below point O. Then $\Delta x_1 = S_1P - S_2P = \frac{yd}{D}$

Path difference due to glass sheet

$$\Delta x_2 = \left(\frac{\mu_g}{\mu_m} - 1\right) t$$



Net path difference will be zero when

$$\Delta x_1 = \Delta x_2$$

or $$\frac{yd}{D} = \left(\frac{\mu_g}{\mu_m} - 1\right) t$$

$\therefore$ $$y = \left(\frac{\mu_g}{\mu_m} - 1\right) t\frac{D}{d}$$

Substituting the values, we have

$$y = \left(\frac{1.5}{4/3} - 1\right)\frac{10.4 \times 10^{-6}(1.5)}{0.45 \times 10^{-3}}$$

$$y = 4.33 \times 10^{-3} \text{ m}$$

or we can say $y = 4.33$ mm.

(b) At O, $\Delta x_1 = 0$ and $\Delta x_2 = \left(\frac{\mu_g}{\mu_m} - 1\right) t$

$\therefore$ Net path difference, $\Delta x = \Delta x_2$

Corresponding phase difference, $\Delta\phi$

or simple $\phi = \frac{2\pi}{\lambda}.\Delta x$

Substituting the values, we have

$$\phi = \frac{2\pi}{6 \times 10^{-7}}\left(\frac{1.5}{4/3} - 1\right)(10.4 \times 10^{-6})$$

$$\phi = \left(\frac{13}{3}\right)\pi$$

Now, $$I(\phi) = I_{\max}\cos^2\left(\frac{\phi}{2}\right)$$

$\therefore$ $$I = I_{\max}\cos^2\left(\frac{13\pi}{6}\right)$$

$$I = \frac{3}{4} I_{\max}$$

(c) At O, path difference is $\Delta x = \Delta x_2 = \left(\frac{\mu_g}{\mu_m} - 1\right) t$

For maximum intensity at O

$\Delta x = n\lambda$ (Here $n = 1, 2, 3, \ldots$)

$\therefore$ $\lambda = \frac{\Delta x}{1}, \frac{\Delta x}{2}, \frac{\Delta x}{3} \ldots$ and so on

$$\Delta x = \left(\frac{1.5}{4/3} - 1\right)(10.4 \times 10^{-6} \text{ m})$$

$$= \left(\frac{1.5}{4/3} - 1\right)(10.4 \times 10^{3} \text{ nm})$$

$$\Delta x = 1300 \text{ nm}$$

$\therefore$ Maximum intensity will be corresponding to

$$\lambda = 1300 \text{ nm}, \frac{1300}{2} \text{ nm}, \frac{1300}{3} \text{ nm}, \frac{1300}{4} \text{ nm} \ldots$$

$= 1300$ nm, 650 nm, 433.33 nm, 325 nm ...

The wavelengths in the range 400 to 700 nm are 650 nm and 433.33 nm.

52. Given, $\lambda = 0.5$ mm, $d = 1.0$ mm, $D = 1$ m

(a) When the incident beam falls normally :

Path difference between the two rays S_2P and S_1P is

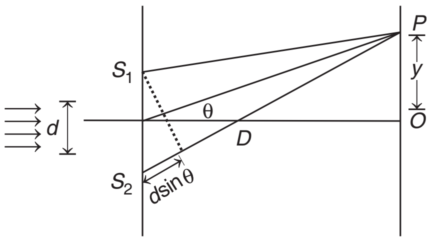
$$\Delta x = S_2P - S_1P \approx d\sin\theta$$

For minimum intensity,

$$d\sin\theta = (2n-1)\frac{\lambda}{2}, \text{ where } n = 1, 2, 3, \ldots$$

or $$\sin\theta = \frac{(2n-1)\lambda}{2d}$$

$$= \frac{(2n-1)0.5}{2 \times 1.0} = \frac{2n-1}{4}$$



As $\sin\theta \le 1$ therefore $\frac{(2n-1)}{4} \le 1$ or $n \le 2.5$

So, n can be either 1 or 2.

When $n = 1, \sin\theta_1 = \frac{1}{4}$ or $\tan\theta_1 = \frac{1}{\sqrt{15}}$

$$n = 2, \sin\theta_2 = \frac{3}{4}$$

or $\tan\theta_2 = \frac{3}{\sqrt{7}}$

$\therefore \quad y = D\tan\theta = \tan\theta \quad (D = 1\text{ m})$

So, the position of minima will be

$$y_1 = \tan\theta_1 = \frac{1}{\sqrt{15}}\text{ m} = 0.26\text{ m}$$

$$y_2 = \tan\theta_2 = \frac{3}{\sqrt{7}}\text{ m} = 1.13\text{ m}$$

And as minima can be on either side of centre O.

Therefore there will be four minimas at positions $\pm$ 0.26 m and $\pm$1.13 m on the screen.

(b) When $\alpha = 30°$, path difference between the rays before reaching S_1 and S_2 is

$$\Delta x_1 = d\sin\alpha = (1.0)\sin 30° = 0.5\text{ mm} = \lambda$$

So, there is already a path difference of λ between the rays.

Position of central maximum Central maximum is defined as a point where net path difference is zero. So,

$$\Delta x_1 = \Delta x_2$$

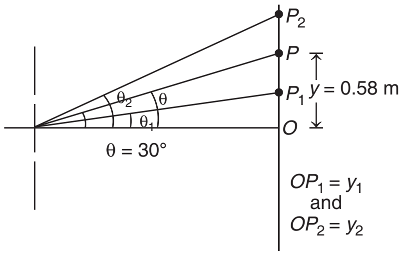
or $d\sin\alpha = d\sin\theta$

or $\theta = \alpha = 30°$

or $\tan\theta = \frac{1}{\sqrt{3}} = \frac{y_0}{D}$

$\therefore \quad y_0 = \frac{1}{\sqrt{3}}\text{ m} \quad (D = 1\text{ m})$

$$y_0 = 0.58\text{ m}$$



At point P, $\quad \Delta x_1 = \Delta x_2$

Above point P $\quad \Delta x_2 > \Delta x_1$ and

Below point P $\quad \Delta x_1 > \Delta x_2$

Now, let P_1 and P_2 be the minimas on either side of central maxima. Then, for P_2

$$\Delta x_2 - \Delta x_1 = \frac{\lambda}{2}$$

or $\Delta x_2 = \Delta x_1 + \frac{\lambda}{2} = \lambda + \frac{\lambda}{2} = \frac{3\lambda}{2}$

or $d\sin\theta_2 = \frac{3\lambda}{2}$

or $\sin\theta_2 = \frac{3\lambda}{2d} = \frac{(3)(0.5)}{(2)(1.0)} = \frac{3}{4}$

$\therefore \quad \tan\theta_2 = \frac{3}{\sqrt{7}} = \frac{y_2}{D}$

or $y_2 = \frac{3}{\sqrt{7}} = 1.13\text{ m}$

Similarly by for P_1

$$\Delta x_1 - \Delta x_2 = \frac{\lambda}{2} \text{ or } \Delta x_2 = \Delta x_1 - \frac{\lambda}{2} = \lambda - \frac{\lambda}{2} = \frac{\lambda}{2}$$

or $d\sin\theta_1 = \frac{\lambda}{2}$

or $\sin\theta_1 = \frac{\lambda}{2d} = \frac{(0.5)}{(2)(1.0)} = \frac{1}{4}$

$\therefore \quad \tan\theta_1 = \frac{1}{\sqrt{15}} = \frac{y_1}{D}$

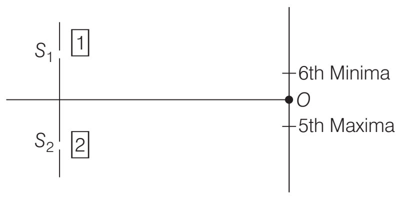
or $y_1 = \frac{1}{\sqrt{15}}\text{ m} = 0.26\text{ m}$

Therefore, y-coordinates of the first minima on either side of the central maximum are $y_1 = 0.26$ m and $y_2 = 1.13$ m.

NOTE In this problem $\sin\theta \approx \tan\theta \approx \theta$ is not valid as θ is large.

53. $\mu_1 = 1.4$ and $\mu_2 = 1.7$ and let t be the thickness of each glass plates.

Path difference at O, due to insertion of glass plates will be



$$\Delta x = (\mu_2 - \mu_1)t = (1.7 - 1.4)\,t = 0.3\,t \quad \text{... (i)}$$

Now, since 5th maxima (earlier) lies below O and 6th minima lies above O.

This path difference should lie between 5λ and $5\lambda + \frac{\lambda}{2}$

So, let $\quad \Delta x = 5\lambda + \Delta \quad$ ... (ii)

where $\quad \Delta < \frac{\lambda}{2}$

Due to the path difference Δx, the phase difference at O will be

$$\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{\lambda}(5\lambda + \Delta)$$

$$= (10\pi + \frac{2\pi}{\lambda}.\Delta) \quad \text{...(iii)}$$

Intensity at O is given $\frac{3}{4}I_{max}$ and since

$$I(\phi) = I_{max}\cos^2\left(\frac{\phi}{2}\right)$$

$$\therefore \quad \frac{3}{4} I_{\max} = I_{\max} \cos^2\left(\frac{\phi}{2}\right)$$

or $$\frac{3}{4} = \cos^2\left(\frac{\phi}{2}\right) \quad \ldots \text{(iv)}$$

From Eqs. (iii) and (iv), we find that

$$\Delta = \frac{\lambda}{6}$$

ie, $$\Delta x = 5\lambda + \frac{\lambda}{6} = \frac{31}{6}\lambda = 0.3\, t$$

$$\therefore \quad t = \frac{31\lambda}{6(0.3)} = \frac{(31)(5400 \times 10^{-10})}{1.8}$$

or $$t = 9.3 \times 10^{-6} \text{ m} = 9.3\, \mu\text{m}$$

54. (a) Path difference due to the glass slab,

$$\Delta x = (\mu - 1)t = (1.5 - 1)\, t = 0.5t$$

Due to this slab, 5 red fringes have been shifted upwards. Therefore,

$$\Delta x = 5\lambda_{\text{red}}$$

or $$0.5t = (5)(7 \times 10^{-7} \text{m})$$

$\therefore$ t = thickness of glass slab $= 7 \times 10^{-6}$ m

(b) Let μ' be the refractive index for green light then

$$\Delta x' = (\mu' - 1)\, t$$

Now the shifting is of 6 fringes of red light. Therefore,

$$\Delta x' = 6\lambda_{\text{red}}$$

$$\therefore \quad (\mu' - 1)\, t = 6\lambda_{\text{red}}$$

$$\therefore \quad (\mu' - 1) = \frac{(6)(7 \times 10^{-7})}{7 \times 10^{-6}} = 0.6$$

$$\therefore \quad \mu' = 1.6$$

(c) In part (a), shifting of 5 bright fringes was equal to 10^{-3} m. Which implies that

$$5\omega_{\text{red}} = 10^{-3} \text{m} \quad \text{(Here, } \omega = \text{Fringe width)}$$

$$\therefore \quad \omega_{\text{red}} = \frac{10^{-3}}{5} \text{m} = 0.2 \times 10^{-3} \text{m}$$

Now since $\omega = \dfrac{\lambda D}{d}$

or $$\omega \propto \lambda$$

$$\therefore \quad \frac{\omega_{\text{green}}}{\omega_{\text{red}}} = \frac{\lambda_{\text{green}}}{\lambda_{\text{red}}}$$

$$\therefore \quad \omega_{\text{green}} = \omega_{\text{red}} \frac{\lambda_{\text{green}}}{\lambda_{\text{red}}} = (0.2 \times 10^{-3})\left(\frac{5 \times 10^{-7}}{7 \times 10^{-7}}\right)$$

$$\omega_{\text{green}} = 0.143 \times 10^{-3} \text{m}$$

$$\therefore \quad \Delta\omega = \omega_{\text{green}} - \omega_{\text{red}} = (0.143 - 0.2) \times 10^{-3} \text{m}$$

$$\Delta\omega = -5.71 \times 10^{-5} \text{m}$$

55. Given, $\mu = 1.33$, $d = 1$ mm, $D = 1.33$ m,

$$\lambda = 6300 \text{ Å}$$

(a) Wavelength of light in the given liquid :

$$\lambda' = \frac{\lambda}{\mu} = \frac{6300}{1.33} \text{Å} \approx 4737 \text{ Å}$$

$$= 4737 \times 10^{-10} \text{ m}$$

$\therefore$ Fringe width, $\omega = \dfrac{\lambda' D}{d}$

$$\omega = \frac{(4737 \times 10^{-10} \text{ m})(1.33 \text{ m})}{(1 \times 10^{-3} \text{ m})} = 6.3 \times 10^{-4} \text{ m}$$

$$\omega = 0.63 \text{ mm}$$

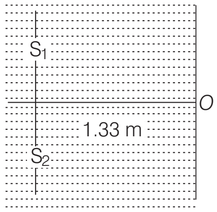
(b) Let t be the thickness of the glass slab.

Path difference due to glass slab at centre O.

$$\Delta x = \left(\frac{\mu_{\text{glass}}}{\mu_{\text{liquid}}} - 1\right) t = \left(\frac{1.53}{1.33} - 1\right) t$$

or $$\Delta x = 0.15\, t$$

Now, for the intensity to be minimum at sO, this path difference should be equal to $\dfrac{\lambda'}{2}$

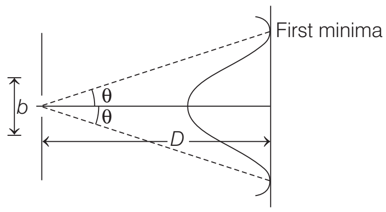


$$\therefore \quad \Delta x = \frac{\lambda'}{2} \quad \text{or } 0.15\, t = \frac{4737}{2} \text{Å}$$

$$\therefore \quad t = 15790 \text{ Å} \quad \text{or} \quad t = 1.579\, \mu\text{m}$$

56. (a) Given, $\lambda = 6000$ Å

Let b be the width of slit and D the distance between screen and slit.



First minima is obtained at $b \sin\theta = \lambda$

or $b\theta = \lambda$ as $\sin\theta \approx \theta$ or $\theta = \dfrac{\lambda}{b}$

Angular width of first maxima $= 2\theta = \dfrac{2\lambda}{b} \propto \lambda$

Angular width will decrease by 30% when λ is also decreased by 30%.

Therefore, new wavelength

$$\lambda' = \left\{(6000) - \left(\frac{30}{100}\right) 6000\right\} \text{ Å}$$

$$\lambda' = 4200 \text{ Å}$$

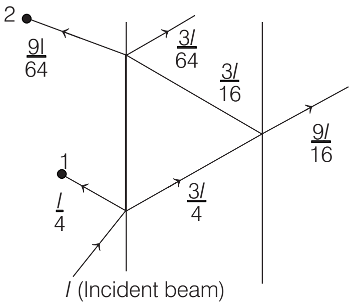
(b) When the apparatus is immersed in a liquid of refractive index μ, the wavelength is decreased μ times. Therefore,

$$4200\ \text{Å} = \frac{6000\ \text{Å}}{\mu}$$

$$\therefore \quad \mu = \frac{6000}{4200}$$

or $$\mu = 1.429 \simeq 1.43$$

57. Each plate reflects 25% and transmits 75%.



Incident beam has an intensity I. This beam undergoes multiple reflections and refractions. The corresponding intensity after each reflection and refraction (transmission) are shown in figure.

Interference pattern is to take place between rays 1 and 2.

$$I_1 = \frac{I}{4} \quad \text{and} \quad I_2 = 9I/64$$

$$\therefore \quad \frac{I_{\min}}{I_{\max}} = \left(\frac{\sqrt{I_1} - \sqrt{I_2}}{\sqrt{I_1} + \sqrt{I_2}}\right)^2 = \frac{1}{49}$$

58. Power received by aperture A,

$$P_A = I\,(\pi r_A^2) = \frac{10}{\pi}(\pi)(0.001)^2 = 10^{-5}\ \text{W}$$

Power received by aperture B,

$$P_B = I\,(\pi r_B^2) = \frac{10}{\pi}(\pi)(0.002)^2$$
$$= 4 \times 10^{-5}\ \text{W}$$

Only 10% of P_A and P_B goes to the original direction. Hence, 10% of $P_A = 10^{-6} = P_1$ (say)

and 10% of $P_B = 4 \times 10^{-6} = P_2$ (say)

Path difference created by slab

$$\Delta x = (\mu - 1)\,t$$
$$= (1.5 - 1)(2000) = 1000\text{Å}$$

Corresponding phase difference,

$$\phi = \frac{2\pi}{\lambda} \cdot \Delta x = \frac{2\pi}{6000} \times 1000 = \frac{\pi}{3}$$

Now, resultant power at the focal point

$$P = P_1 + P_2 + 2\sqrt{P_1 P_2}\cos\phi$$
$$= 10^{-6} + 4 \times 10^{-6} + 2\sqrt{(10^{-6})(4 \times 10^{-6})}\cos\frac{\pi}{3}$$
$$= 7 \times 10^{-6}\ \text{W}$$

59. (a) The desired distance will be

$$y_1 = 3\omega_1 = 3\left(\frac{\lambda_1 D}{d}\right) = \frac{(3)(6500 \times 10^{-10})(1.2)}{(2 \times 10^{-3})}$$
$$= 11.7 \times 10^{-4}\text{m} = 1.17\ \text{mm}$$

(b) Let n_1 bright fringe of λ_1 coincides with n_2 bright fringe of λ_2. Then,

$$\frac{n_1 \lambda_1 D}{d} = \frac{n_2 \lambda_2 D}{d} \quad \text{or} \quad \frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1} = \frac{5200}{6500} = \frac{4}{5}$$

Therefore, 4th bright of λ_1 coincides with 5th bright of λ_2. Similarly, 8th bright of λ_1 will coincide with 10th bright of λ_2 and so on. The least distance from the central maximum will therefore corresponding to 4th bright of λ_1 (or 5th bright of λ_2.) Hence,

$$Y_{\min} = \frac{4\lambda_1 D}{d} = \frac{4(6500 \times 10^{-10})(1.2)}{(2 \times 10^{-3})}$$
$$= 15.6 \times 10^{-4}\ \text{m} = 1.56\ \text{mm}$$

60. Shifting of fringes due to introduction of slab in the path of one of the slits, comes out to be,

$$\Delta y = \frac{(\mu - 1)tD}{d} \qquad \ldots\text{(i)}$$

Now, the distance between the screen and slits is doubled. Hence, the new fringe width will become.

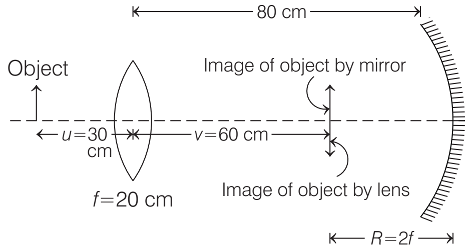
$$\omega' = \frac{\lambda(2D)}{d} \qquad \ldots\text{(ii)}$$

Given, $\Delta y = \omega'$ or $\dfrac{(\mu - 1)\,tD}{d} = \dfrac{2\lambda D}{d}$

$$\therefore \quad \lambda = \frac{(\mu - 1)t}{2} = \frac{(1.6 - 1)(1.964 \times 10^{-6})}{2}$$
$$= 0.5892 \times 10^{-6}\ \text{m} = 5892\ \text{Å}$$

Topic 7 Miscellaneous Problems

1. The given situation can be drawn as shown below



For lens formula, $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$

Substituting given values, we get

$$\frac{1}{v} - \frac{1}{-30} = \frac{1}{20}$$

$$\Rightarrow \quad v = 60\ \text{cm}$$

So, this image is at a distance of $80 - 60 = 20\ \text{cm}$ from the mirror.

As, the image formed by the mirror coincides with image formed by the lens. This condition is only possible, if any object that has been placed in front of concave mirror is at centre of curvature, i.e. at $2f$.

So, radius of curvature of mirror is $R = 20\,\text{cm}$

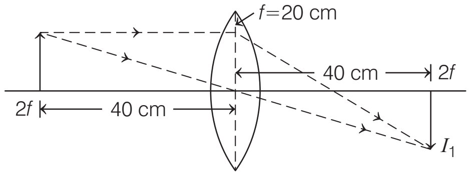
$\therefore$ Focal length of mirror, $f = \dfrac{R}{2} = 10\,\text{cm}$

As, for virtual image, the object is to kept between pole and focus of the mirror.

$\therefore$ The maximum distance of the object for which this concave mirror by itself produce a virtual image would be 10 cm.

2. In given system of lens and mirror, position of object O in front of lens is at a distance $2f$.

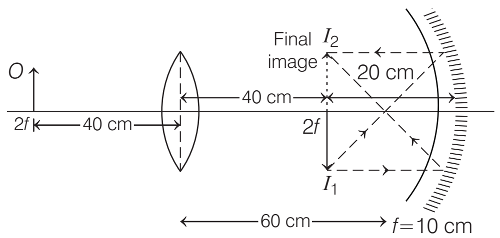
i.e. $u = 2f = 40\,\text{cm}$



So, image (I_1) formed is real, inverted and at a distance, $v = 2f = 2 \times 20 = 40$ cm, (behind lens) magnification, $m_1 = \dfrac{v}{u} = \dfrac{40}{40} = 1$

Thus, size of image is same as that of object.

This image (I_1) acts like a real object for mirror.



As object distance for mirror is $u = C = 2f = -20\,\text{cm}$

where, C = centre of curvature.

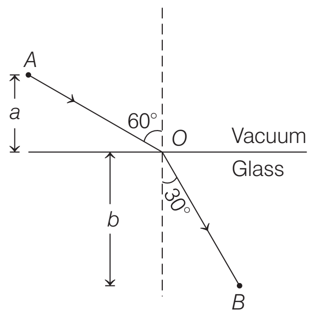
So, image (I_2) formed by mirror is at $2f$.

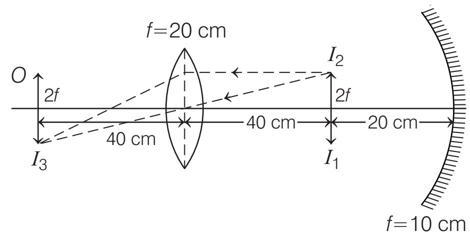
$\therefore$ For mirror $v = 2f = 2(-10) = -20\,\text{cm}$

Magnification, $m_2 = -\dfrac{v}{u} = -\dfrac{(-20)}{(-20)} = -1$

Thus, image size is same as that of object.

The image I_2 formed by the mirror will act like an object for lens.





As the object is at $2f$ distance from lens, so image (I_3) will be formed at a distance $2f$ or 40 cm. Thus, magnification,

$$m_3 = \frac{v}{u} = \frac{40}{40} = 1$$

So, final magnification, $m = m_1 \times m_2 \times m_3 = -1$

Hence, final image (I_3) is real, inverted of same size as that of object and coinciding with object.

3. Energy of a light wave $\propto$ Intensity of the light wave

Since, intensity $= \varepsilon v A^2$

where, ε is the permittivity of the medium in which light is travelling with velocity v and A is its amplitude.

Since, only 4% of the energy of the light gets reflected.

$\therefore$ 96% of the energy of the light is transmitted.

$E_{\text{transmitted}}(E_t) = 96\%$ of $E_{\text{incident}}(E_i)$

$$\varepsilon_0 \varepsilon_r v A_t^2 = \frac{96}{100} \times \varepsilon_0 \times c \times A_1^0$$

$$A_t^2 = \frac{96}{100} \cdot \frac{\varepsilon_0}{\varepsilon_r} \cdot \frac{c}{v} A_i^2$$

$$A_t^2 = \frac{96}{100} \cdot \frac{v^2}{c^2} \cdot \frac{c}{v} A_i^2 \qquad \left[\because \sqrt{\frac{\varepsilon_0}{\varepsilon_r}} = \frac{v}{c}\right]$$

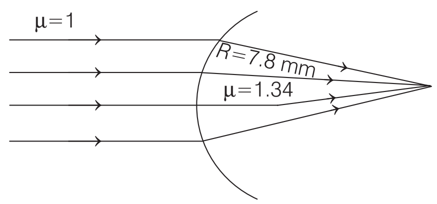
$$A_t^2 = \frac{96}{100} \cdot \frac{v}{c} \cdot A_i^2$$

$$= \frac{96}{100} \times \frac{1}{3/2} \times 30^2 \qquad \left[\because \mu = \frac{c}{v} = 1.5\right]$$

$$A_t = \sqrt{\frac{64}{100} \times 30^2}$$

$$A_t = 24 \text{ V/m}$$

4. The given condition is shown in the figure below



where, a parallel beam of light is coming from air ($\mu = 1$) to a spherical surface (eye) of refractive index 1.34.

Radius of curvature of this surface is 7.8 mm.

From the image formation formula for spherical surface, i.e. relation between object, image and radius of curvature.

$$\frac{\mu_r}{v} - \frac{\mu_i}{u} = \frac{\mu_r - \mu_i}{R} \quad \dots \text{(i)}$$

Given, $\mu_r = 1.34$, $\mu_i = 1$, $u = \infty$ (– ve) and

$$R = 7.8$$

Substituting the given values, we get

$$\frac{1.34}{v} + \frac{1}{\infty} = \frac{1.34 - 1}{7.8}$$

or $$\frac{1.34}{v} = \frac{0.34}{7.8}$$

$$\Rightarrow \quad v = \frac{1.34 \times 7.8}{0.34} \text{ mm}$$

$$\Rightarrow \quad v = \frac{4}{3} \times 3 \times 7.8 \text{ mm}$$

($\because$ approximately $1.34 = 4/3$ and $0.34 = 1/3$)

$$\Rightarrow \quad v = 31.2 \text{ mm or } 3.12 \text{ cm}$$

5. **Key Idea** When a beam of unpolarised light is reflected from a transparent medium of refractive index μ, then the reflected light is completely plane polarised at a certain angle of incidence i_B, which is known as Brewster's angle.

In the given condition, the light reflected irrespective of an angle of incidence is never completely polarised. So,

$$i_C > i_B$$

where, i_C is the critical angle.

$$\Rightarrow \quad \sin i_C < \sin i_B \quad \dots\text{(i)}$$

From Brewster's law, we know that

$$\tan i_B = {}^w\mu_g = \frac{\mu_{\text{glass}}}{\mu_{\text{water}}} = \frac{1.5}{\mu} \quad \dots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{1}{\mu} < \frac{1.5}{\sqrt{(1.5)^2 + (\mu)^2}}$$

$$\Rightarrow \quad \sqrt{(1.5)^2 + \mu^2} < 1.5\,\mu$$

$$\mu^2 + (1.5)^2 < (1.5\,\mu)^2 \quad \text{or} \quad \mu < \frac{3}{\sqrt{5}}$$

$\therefore$ The minimum value of μ should be $\frac{3}{\sqrt{5}}$.

6. $R = 10$ cm

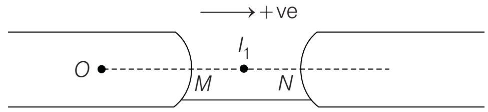
Applying $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ two times

$$\frac{1}{v} - \frac{1.5}{-50} = \frac{1 - 1.5}{-10}$$

$$\frac{1}{v} + \frac{1.5}{50} = \frac{0.5}{10}$$

$$\frac{1}{v} = \frac{0.5}{10} - \frac{1.5}{50} = \frac{2.5 - 1.5}{50} \Rightarrow v = 50$$

$$MN = d,\ MI_1 = 50 \text{ cm},$$
$$NI_1 = (d - 50) \text{ cm}$$

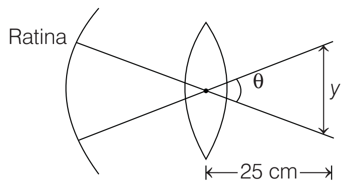


Again, $$\frac{1.5}{\infty} - \frac{1}{-(d - 50)} = \frac{1.5 - 1}{10}$$

$$\frac{1}{d - 50} = \frac{1}{20}$$

$$d = 70$$

7.



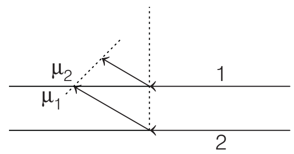
Resolving angle of naked eye is given by

$$\theta = \frac{1.22\lambda}{D}$$

$$\therefore \quad \frac{y}{25 \times 10^{-2}} = \frac{1.22 \times 500 \times 10^{-9}}{0.25 \times 2 \times 10^{-2}}$$

$$\therefore \quad y = 30 \times 10^{-6} \text{ m}$$
$$= 30\,\mu\text{m}$$

8.



$$\mu_2 > \mu_1$$

Dotted line is the normal.

According to Huygen's principle, each point on wavefront behaves as a point source of light.

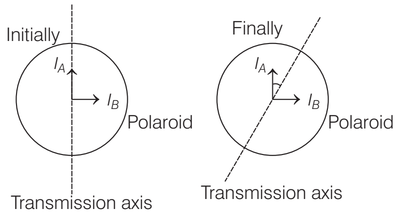
Ray 2 will travel faster than 1 as $\mu_2 > \mu_1$. So, beam will bend upwards.

9. By law of Malus i.e. $I = I_0 \cos^2\theta$

Now, $$I_{A'} = I_A \cos^2 30°$$
$$I_{B'} = I_B \cos^2 60°$$

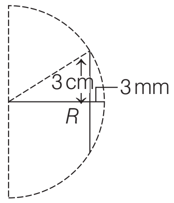
As, $$I'_A = I'_B$$

$$I_A \cos^2 30° = I_B \cos^2 60°$$



$$\Rightarrow \quad I_A \frac{3}{4} = I_B \frac{1}{4} \quad \Rightarrow \quad \frac{I_A}{I_B} = \frac{1}{3}$$

10. By Pythagoras theorem



$$R^2 = (3)^2 + (R - 0.3)^2 \Rightarrow R \approx 15\,\text{cm}$$

Refractive index of material of lens $\mu = \dfrac{c}{v}$

Here c = speed of light in vacuum $= 3 \times 10^8$ m/s

v = speed of light in material of lens $= 2 \times 10^8$ m/s

$$= \frac{3 \times 10^8}{2 \times 10^8} = \frac{3}{2}$$

From lens maker's formula

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

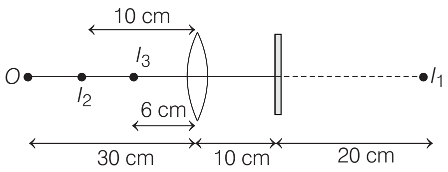
Here, $R_1 = R$ and $R_2 = \infty$ (For plane surface)

$$\frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{15}\right)$$

$\Rightarrow \qquad f = 30\,\text{cm}$

11. After critical angle reflection will be 100% and transmission is 0%. Options (b) and (c) satisfy this condition. But option (c) is the correct option. Because in option (b) transmission is given 100% at $\theta = 0°$, which is not true.

12. Object is placed at distance $2f$ from the lens. So first image I_1 will be formed at distance $2f$ on other side. This image I_1 will behave like a virtual object for mirror. The second image I_2 will be formed at distance 20 cm in front of the mirror, or at distance 10 cm to the left hand side of the lens.



Now applying lens formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\therefore \qquad \frac{1}{v} - \frac{1}{+10} = \frac{1}{+15}$$

or $\qquad v = 6$ cm

Therefore, the final image is at distance 16 cm from the mirror. But, this image will be real.

This is because ray of light is travelling from right to left.

13. Critical angle from region III to region IV

$$\sin\theta_C = \frac{n_0/8}{n_0/6} = \frac{3}{4}$$

Now, applying Snell's law in region I and region III

$$n_0 \sin\theta = \frac{n_0}{6}\sin\theta_C$$

or
$$\sin\theta = \frac{1}{6}\sin\theta_C$$
$$= \frac{1}{6}\left(\frac{3}{4}\right) = \frac{1}{8}$$

$$\therefore \qquad \theta = \sin^{-1}\left(\frac{1}{8}\right)$$

14. Distance of object from mirror

$$= 15 + \frac{33.25}{1.33} = 40\,\text{cm}$$

Distance of image from mirror $= 15 + \dfrac{25}{1.33}$

$$= 33.8\,\text{cm}$$

For the mirror, $\qquad \dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f}$

$$\therefore \qquad \frac{1}{-33.8} + \frac{1}{-40} = \frac{1}{f}$$

$$\therefore \qquad f = -18.3\,\text{cm}$$

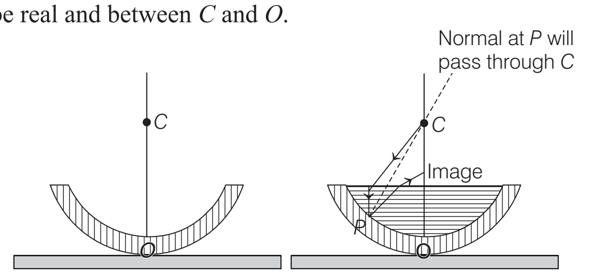
$\therefore$ Most suitable answer is (c).

15. Critical angle $\theta_C = \sin^{-1}\left(\dfrac{1}{\mu}\right)$

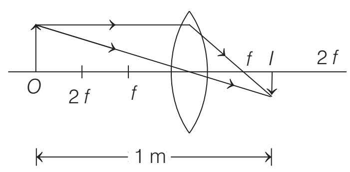
Wavelength increases in the sequence of VIBGYOR. According to Cauchy's formula refractive index (μ) decreases as the wavelength increases. Hence the refractive index will increase in the sequence of ROYGBIV. The critical angle θ_C will thus increase in the same order VIBGYOR.

For green light the incidence angle is just equal to the critical angle. For yellow, orange and red the critical angle will be greater than the incidence angle. So, these colours will emerge from the glass air interface.

16. The ray diagram is shown in figure. Therefore, the image will be real and between C and O.



17. Image can be formed on the screen if it is real. Real image of reduced size can be formed by a concave mirror or a convex lens.

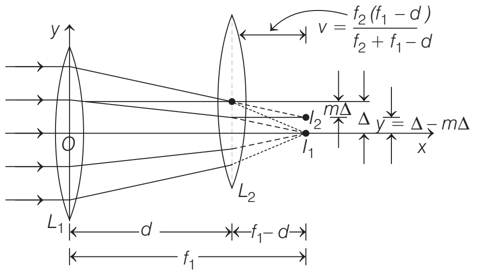


A diminished real image is formed by a convex lens when the object is placed beyond $2f$ and the image of such object is formed between f and $2f$ on other side.

Thus, $\quad d > (2f + 2f)$

or $\quad 4f < 0.1\,\text{m}$ or $\quad f < 0.25\text{ m}$

18. From the first lens parallel beam of light is focused at its focus i.e. at a distance f_1 from it. This image I_1 acts as virtual object for second lens L_2. Therefore, for L_2



$$u = +(f_1 - d),\ f = +f_2$$

$$\therefore \quad \frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{f_2} + \frac{1}{f_1 - d}$$

Hence, $\quad v = \dfrac{f_2(f_1 - d)}{f_2 + f_1 - d}$

Therefore, x-coordinate of its focal point will be

$$x = d + v = d + \frac{f_2(f_1 - d)}{f_2 + f_1 - d}$$

$$= \frac{f_1 f_2 + d(f_1 - d)}{f_1 + f_2 - d}$$

Linear magnification for L_2

$$m = \frac{v}{u} = \frac{f_2(f_1 - d)}{f_2 + f_1 - d} \cdot \frac{1}{f_1 - d}$$

$$= \frac{f_2}{f_2 + f_1 - d}$$

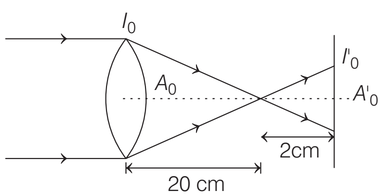
Therefore, second image will be formed at a distance of $m\Delta$ or $\left(\dfrac{f_2}{f_2 + f_1 - d}\right).\Delta$ below its optic axis.

Therefore, y-coordinate of the focus of system will be

$$y = \Delta - \left(\frac{f_2\,\Delta}{f_2 + f_1 - d}\right)$$

or $\quad y = \dfrac{(f_1 - d).\Delta}{f_2 + f_1 - d}$

19.



$$\frac{A_0'}{A_0} = \left(\frac{2}{20}\right)^2 = \frac{1}{100}$$

$$\Rightarrow \quad A_0' = \frac{A_0}{100}$$

$$P = I_0 A_0 = I_0' A_0'$$

$$\Rightarrow \quad I_0' = \frac{I_0 A_0}{\dfrac{A_0}{100}}$$

$$\Rightarrow \quad = 100 I_0 = 130\,\text{kW/m}^2$$

20. Since value of n in meta-material is negative.

$$\therefore \quad v = \frac{c}{|n|}$$

21. According to the paragraph, refracted ray in meta-material should be on same side of normal.

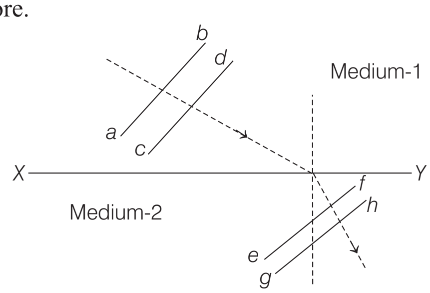
22. Wavefronts are parallel in both media. Therefore, light which is perpendicular to wavefront travels as a parallel beam in each medium.

23. All points on a wavefront are at the same phase.

$$\therefore \quad \phi_d = \phi_c \text{ and } \phi_f = \phi_e$$

$$\therefore \quad \phi_d - \phi_f = \phi_c - \phi_e$$

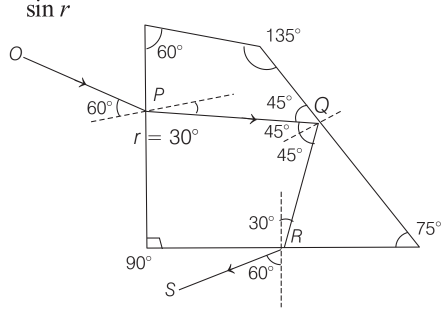
24. In medium-2 wavefront bends away from the normal after refraction. Therefore, ray of light which is perpendicular to wavefront bends towards the normal in medium-2 during refraction. So, medium-2 is denser or its speed in medium-1 is more.



25. **(A)**, **(C)** and **(D)** In case of concave mirror or convex lens image can be real, virtual, diminished magnified or of same size.

(B) In case of convex mirror image is always virtual (for real object).

27. $\sqrt{3} = \dfrac{\sin 60°}{\sin r}$



$\therefore \qquad r = 30°$

$$\theta_C = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right) \text{ or } \sin\theta_C = \frac{1}{\sqrt{3}}$$

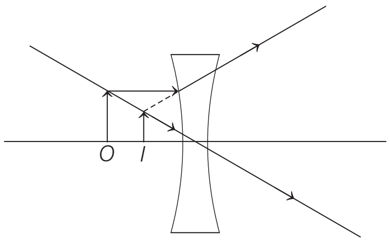
$$= 0.577$$

At point Q, angle of incidence inside the prism is $i = 45°$.

Since $\sin i = \frac{1}{\sqrt{2}}$ is greater than $\sin\theta_C = \frac{1}{\sqrt{2}}$, ray gets totally internally reflected at face CD. Path of ray of light after point Q is shown in figure.

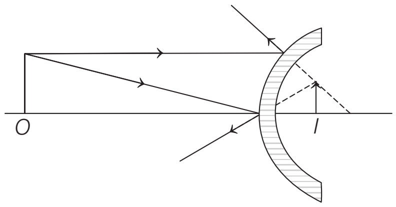
From the figure, we can see that angle between incident ray OP and emergent ray RS is 90°.

28. For a lens $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, i.e. $\frac{1}{v} = \frac{1}{f} + \frac{1}{u}$



For a concave lens, f and u are negative, i.e. v will always be negative and image will always be virtual.

For a mirror :



$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \text{ i.e. } \frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

Here, f is positive and u is negative for a convex mirror. Therefore, v is always positive and image is always virtual.

29. When upper half of the lens is covered, image is formed by the rays coming from lower half of the lens. Or image will be formed by less number of rays. Therefore, intensity of image will decrease. But complete image will be formed.

30. Speed of light in vacuum, $c = \frac{1}{\sqrt{\varepsilon_0\mu_0}}$

and speed of light in medium, $v = \frac{1}{\sqrt{\varepsilon\mu}}$

Therefore, refractive index of the medium is

$$\mu = \frac{c}{v} = \frac{1/\sqrt{\varepsilon_0\mu_0}}{1/\sqrt{\varepsilon\mu}} = \sqrt{\frac{\varepsilon\mu}{\varepsilon_0\mu_0}}$$

31. $I \propto \frac{1}{r^2}$ (in case of point source)

and $I \propto A^2$

$\Rightarrow \qquad A \propto \frac{1}{r}$

or $\qquad \frac{A_1}{A_2} = \frac{r_2}{r_1} = \frac{25}{9}$

32. At a distance r from a line source of power P and length l, the intensity will be,

$$I = \frac{P}{S} = \frac{P}{2\pi rl}$$

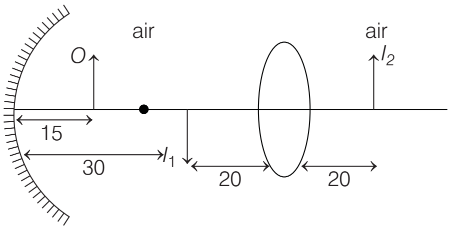
or $\qquad I \propto \frac{1}{r}$

33. Case I

Reflection from mirror

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} \quad \Rightarrow \quad \frac{1}{-10} = \frac{1}{v} + \frac{1}{-15}$$

$\Rightarrow \qquad v = -30$



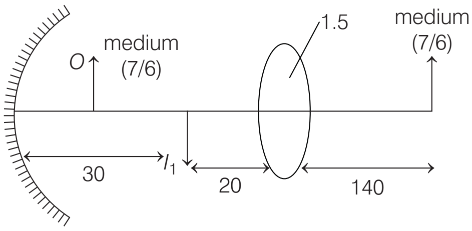
For lens $\qquad \frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

$$\frac{1}{10} = \frac{1}{v} - \frac{1}{-20}$$

$$v = 20$$

$$|M_1| = \left|\frac{v_1}{u_1}\right|\left|\frac{v_2}{u_2}\right| = \left(\frac{30}{15}\right)\left(\frac{20}{20}\right) = 2 \times 1 = 2 \qquad \text{(in air)}$$

Case II For mirror, there is no change.



For lens, $\qquad \frac{1}{f_{air}} = \left(\frac{3/2}{1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$$\frac{1}{f_{medium}} = \left(\frac{3/2}{7/6} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

with $\qquad f_{air} = 10\,\text{cm}$

We get $\qquad \frac{1}{f_{medium}} = \frac{4}{70}\,\text{cm}^{-1}$

$$\frac{1}{v} - \frac{1}{-20} = \frac{4}{70}$$

$$\frac{1}{v} + \frac{1}{20} = \left(\frac{2}{7}\right)\left(\frac{2}{10}\right) = \frac{4}{70}$$

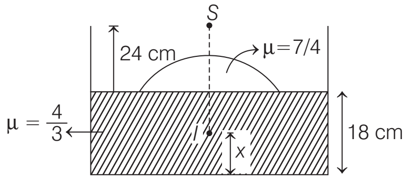
$$\frac{1}{v} = \frac{4}{70} - \frac{1}{20} \quad \Rightarrow \quad v = 140,$$

$$|M_2| = \left|\frac{v_1}{u_1}\right|\left|\frac{v_2}{u_2}\right| = \left(\frac{30}{15}\right)\left(\frac{140}{20}\right),$$

$$= (2)\left(\frac{140}{20}\right) = 14$$

$$\Rightarrow \quad \left|\frac{M_2}{M_1}\right| = \frac{14}{2} = 7$$

34.



Two refractions will take place, first from spherical surface and the other from the plane surface.

So, applying

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

two times with proper sign convention.

Ray of light is travelling downwards. Therefore, downward direction is taken as positive direction.

$$\frac{7/4}{v} - \frac{1.0}{-24} = \frac{7/4 - 1.0}{+6} \quad \ldots\text{(i)}$$

$$\frac{4/3}{(18-x)} - \frac{7/4}{v} = \frac{4/3 - 7/4}{\infty} \quad \ldots\text{(ii)}$$

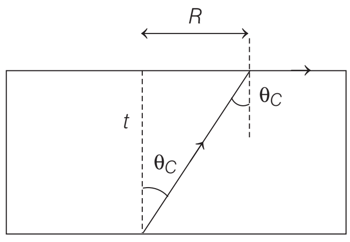
Solving these equations, we get, $x = 2\,\text{cm}$

$\therefore$ Answer is 2.

35. $\frac{R}{t} = \tan\theta_C$ or $R = t\,(\tan\theta_C)$

But, $\sin\theta_C = \frac{1}{\mu} = \frac{3}{5}$

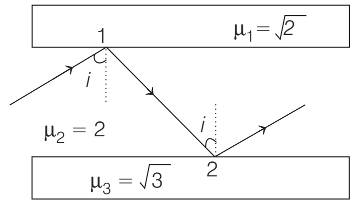
$\therefore \quad \tan\theta_C = \frac{3}{4}$



$\therefore \quad R = \frac{3}{4}t = \frac{3}{4}(8\,\text{cm}) = 6$ cm

Hence the answer is 6.

36. Critical angles at 1 and 2



$$\theta_{C_1} = \sin^{-1}\left(\frac{\mu_1}{\mu_2}\right) = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45°$$

$$\theta_{C_2} = \sin^{-1}\left(\frac{\mu_3}{\mu_2}\right) = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 60°$$

Therefore, minimum angle of incidence for total internal reflection to take place on both slabs should be 60°.

$$i_{\min} = 60°$$

37. Applying Snell's law on face AB,

$$(1)\sin 45° = (\sqrt{2})\sin r$$

$\therefore \quad \sin r = \frac{1}{2}$

or $\quad r = 30°$

i.e. ray becomes parallel to AD inside the block.

Now applying,

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R} \text{ on face } CD,$$

$$\frac{1.514}{OE} - \frac{\sqrt{2}}{\infty} = \frac{1.514 - \sqrt{2}}{0.4}$$

Solving this equation, we get $OE = 6.06\,\text{m}$

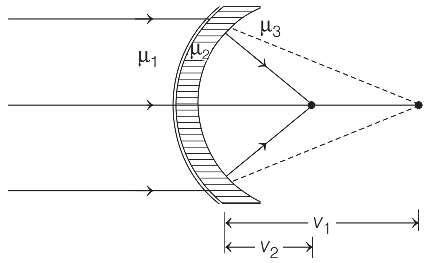
38. For refraction at first surface,

$$\frac{\mu_2}{v_1} - \frac{\mu_1}{-\infty} = \frac{\mu_2 - \mu_1}{+R} \quad \ldots\text{(i)}$$

For refraction at second surface,

$$\frac{\mu_3}{v_2} - \frac{\mu_2}{v_1} = \frac{\mu_3 - \mu_2}{+R} \quad \ldots\text{(ii)}$$

Adding Eqs. (i) and (ii), we get



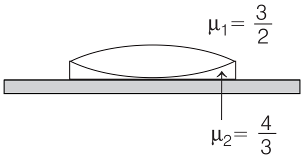
$$\frac{\mu_3}{v_2} = \frac{\mu_3 - \mu_1}{R}$$

or $\quad v_2 = \frac{\mu_3 R}{\mu_3 - \mu_1}$

Therefore, focal length of the given lens system is

$$\frac{\mu_3 R}{\mu_3 - \mu_1}.$$

39. Let R be the radius of curvature of both the surfaces of the equi-convex lens. In the first case :



Let f_1 be the focal length of equi-convex lens of refractive index μ_1 and f_2 the focal length of plano-concave lens of refractive index μ_2. The focal length of the combined lens system will be given by

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$= (\mu_1 - 1)\left(\frac{1}{R} - \frac{1}{-R}\right) + (\mu_2 - 1)\left(\frac{1}{-R} - \frac{1}{\infty}\right)$$

$$= \left(\frac{3}{2} - 1\right)\left(\frac{2}{R}\right) + \left(\frac{4}{3} - 1\right)\left(-\frac{1}{R}\right)$$

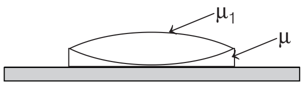
$$= \frac{1}{R} - \frac{1}{3R} = \frac{2}{3R} \quad \text{or} \quad F = \frac{3R}{2}$$

Now, image coincides with the object when ray of light retraces its path or it falls normally on the plane mirror. This is possible only when object is at focus of the lens system.

Hence, $F = 15\,\text{cm}$ (Distance of object = 15 cm)

or $\frac{3R}{2} = 15\,\text{cm}$ or $R = 10\,\text{cm}$

In the second case, let μ be the refractive index of the liquid filled between lens and mirror and let F' be the focal length of new lens system. Then,



$$\frac{1}{F'} = (\mu_1 - 1)\left(\frac{1}{R} - \frac{1}{-R}\right) + (\mu - 1)\left(\frac{1}{-R} - \frac{1}{\infty}\right)$$

or $$\frac{1}{F'} = \left(\frac{3}{2} - 1\right)\left(\frac{2}{R}\right) - \frac{(\mu - 1)}{R}$$

or $$= \frac{1}{R} - \frac{\mu - 1}{R} = \frac{(2 - \mu)}{R}$$

$\therefore$ $$F' = \frac{R}{2 - \mu} = \frac{10}{2 - \mu} \qquad (\because R = 10\,\text{cm})$$

Now, the image coincides with object when it is placed at 25 cm distance.

Hence, $F' = 25$

or $\frac{10}{2 - \mu} = 25$

or $50 - 25\mu = 10$

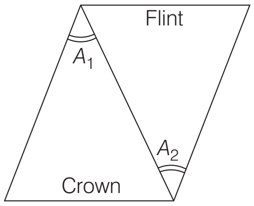
or $25\mu = 40$

$\therefore$ $\mu = \frac{40}{25} = 1.6$

or $\mu = 1.6$

40. (a) When angle of prism is small and angle of incidence is also small, the deviation is given by $\delta = (\mu - 1)A$.

Net deviation by the two prisms is zero. So,



$$\delta_1 + \delta_2 = 0$$

or $$(\mu_1 - 1)A_1 + (\mu_2 - 1)A_2 = 0 \qquad \dots \text{(i)}$$

Here, μ_1 and μ_2 are the refractive indices for crown and flint glasses respectively.

Hence, $$\mu_1 = \frac{1.51 + 1.49}{2} = 1.5$$

and $$\mu_2 = \frac{1.77 + 1.73}{2} = 1.75$$

A_1 = Angle of prism for crown glass = 6°

Substituting the values in Eq. (i), we get

$$(1.5 - 1)(6^\circ) + (1.75 - 1)A_2 = 0$$

This gives $A_2 = -4^\circ$

Hence, angle of flint glass prism is 4° (Negative sign shows that flint glass prism is inverted with respect to the crown glass prism.)

(b) Net dispersion due to the two prisms is

$$= (\mu_{b_1} - \mu_{r_1})A_1 + (\mu_{b_2} - \mu_{r_2})A_2$$
$$= (1.51 - 1.49)(6^\circ) + (1.77 - 1.73)(-4^\circ) = -0.04^\circ$$

$\therefore$ Net dispersion is – 0.04°

41. (a) Rays coming from object AB first refract from the lens and then reflect from the mirror.

Refraction from the lens

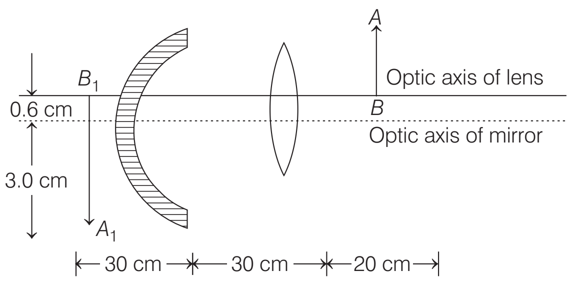
$$u = -20\,\text{cm}, \; f = +15\,\text{cm}$$

Using lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} - \frac{1}{-20} = \frac{1}{15}$

$\therefore$ $v = +60\,\text{cm}$

and linear magnification, $m_1 = \frac{v}{u} = \frac{+60}{-20} = -3$

i.e. first image formed by the lens will be at 60 cm from it (or 30 cm from mirror) towards left and 3 times magnified but inverted. Length of first image A_1B_1 would be $1.2 \times 3 = 3.6\,\text{cm}$ (inverted).



Reflection from mirror Image formed by lens (A_1B_1) will behave like a virtual object for mirror at a distance of 30 cm from it as shown. Therefore $u = +30$ cm, $f = -30$ cm.

Using mirror formula, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ or $\frac{1}{v}+\frac{1}{30}=-\frac{1}{30}$

$\therefore \quad v = -15$ cm

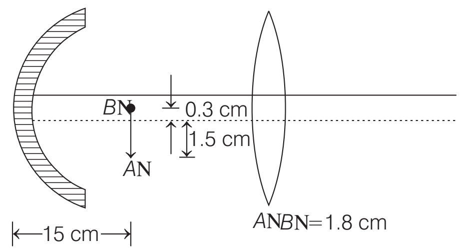
and linear magnification,

$$m_2 = -\frac{v}{u} = -\frac{-15}{+30} = +\frac{1}{2}$$

i.e. final image $A'B'$ will be located at a distance of 15 cm from the mirror (towards right) and since magnification is $+\frac{1}{2}$, length of final image would be $3.6 \times \frac{1}{2} = 1.8$ cm.

$\therefore \quad A'B' = 1.8$ cm

Point B_1 is 0.6 cm above the optic axis of mirror, therefore, its image B' would be $(0.6)\left(\frac{1}{2}\right) = 0.3$ cm above optic axis. Similarly, point A_1 is 3 cm below the optic axis, therefore, its image A' will be $3 \times \frac{1}{2} = 1.5$ cm below the optic axis as shown below



Total magnification of the image,

$$m = m_1 \times m_2 = (-3)\left(+\frac{1}{2}\right) = -\frac{3}{2}$$

$$\therefore \quad A'B' = (m)(AB) = \left(-\frac{3}{2}\right)(1.2) = -1.8 \text{ cm}$$

Note that, there is no need of drawing the ray diagram if not asked in the question.

NOTE With reference to the pole of an optical instrument (whether it is a lens or a mirror) the coordinates of the object (X_o, Y_o) are generally known to us. The corresponding coordinates of image (X_i, Y_i) are found as follows

X_i is obtained using $\frac{1}{v} \pm \frac{1}{u} = \frac{1}{f}$

Here, v is actually X_i and u is X_o ie, the above formula can be written as $\frac{1}{X_i} \pm \frac{1}{X_o} = \frac{1}{f}$

Similarly, Y_i is obtained from $m = \frac{I}{O}$

Here, I is Y_i and O is Y_o *i.e.,* the above formula can be written as $m = Y_i / Y_o$ or $Y_i = mY_o$.

42. From lens maker's formula,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

we have $\quad \frac{1}{0.3} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{R} - \frac{1}{-R}\right)$

(Here, $R_1 = R$ and $R_2 = -R$)

$R = 0.3$

Now applying $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ at air glass surface, we get

$$\frac{3/2}{v_1} - \frac{1}{-(0.9)} = \frac{3/2-1}{0.3}$$

$\therefore \quad v_1 = 2.7$ m

i.e. first image I_1 will be formed at 2.7 m from the lens. This will act as the virtual object for glass water surface. Therefore, applying $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ at glass water surface, we have

$$\frac{4/3}{v_2} - \frac{3/2}{2.7} = \frac{4/3 - 3/2}{-0.3}$$

$\therefore \quad v_2 = 1.2$ m

i.e. second image I_2 is formed at 1.2 m from the lens or 0.4 m from the plane mirror. This will act as a virtual object for mirror. Therefore, third real image I_3 will be formed at a distance of 0.4 m in front of the mirror after reflection from it. Now this image will work as a real object for water-glass interface. Hence, applying

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

we get $\quad \frac{3/2}{v_4} - \frac{4/3}{-(0.8-0.4)} = \frac{3/2 - 4/3}{0.3}$

$\therefore \quad v_4 = -0.54$ m

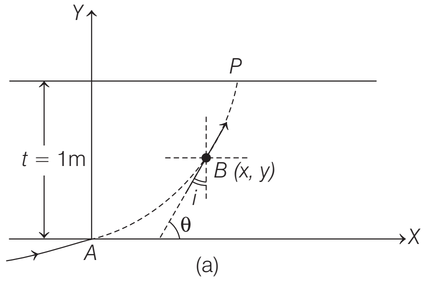
i.e. fourth image is formed to the right of the lens at a distance of 0.54 m from it. Now finally applying the same formula for glass-air surface,

$$\frac{1}{v_5} - \frac{3/2}{-0.54} = \frac{1 - 3/2}{-0.3}$$

$\therefore \quad v_5 = -0.9$ m

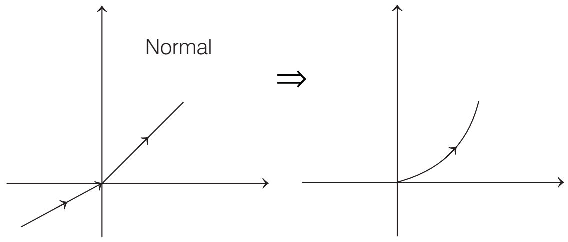
i.e. position of final image is 0.9 m relative to the lens (rightwards) or the image is formed 0.1 m behind the mirror.

43. (a) Refractive index is a function of y. It varies along Y-axis i.e. the boundary separating two media is parallel to X-axis or normal at any point will be parallel to Y-axis.



(a)

Secondly, refractive index increases as y is increased. Therefore, ray of light is travelling from rarer to denser medium i.e. it will bend towards the normal and shape of its trajectory will be as shown below.



(b)

Now, refer to figure (a)

Let i be the angle of incidence at any point B on its path

$$\theta = 90° - i \quad \text{or} \quad \tan\theta = \tan(90° - i) = \cot i$$

or slope $= \cot i$

(b) but $\tan\theta = \dfrac{dy}{dx} \Rightarrow \therefore \dfrac{dy}{dx} = \cot i$ ... (i)

Applying Snell's law at A and B

$n_A \sin i_A = n_B \sin i_B \Rightarrow n_A = 1$ because $y = 0$

$\sin i_A = 1$ because $i_A = 90°$ (Grazing incidence)

$$n_B = \sqrt{ky^{3/2} + 1} = \sqrt{y^{3/2} + 1}$$

because $k = 1.0(\text{m})^{-3/2}$

$$\therefore \quad (1)(1) = \sqrt{(y^{3/2} + 1)} \sin i \Rightarrow \sin i = \frac{1}{\sqrt{y^{3/2} + 1}}$$

$$\therefore \quad \cot i = \sqrt{y^{3/2}} \quad \text{or} \quad y^{3/4} \qquad \text{... (ii)}$$

Equating Eqs. (i) and (ii), we get

$$\frac{dy}{dx} = y^{3/4} \quad \text{or} \quad y^{-3/4} dy = dx$$

$$\text{or} \int_0^y y^{-3/4} dy = \int_0^x dx \quad \text{or} \quad 4y^{1/4} = x \qquad \text{... (iii)}$$

The required equation of trajectory is $4y^{1/4} = x$.

(c) At point P, where the ray emerges from the slab

$$y = 1.0 \text{ m}$$

$$\therefore \quad x = 4.0 \text{ m} \qquad \text{[From Eq. (iii)]}$$

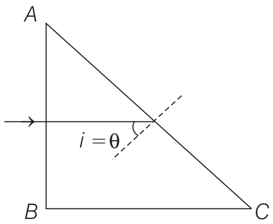
Therefore, coordinates of point P are

$$P = (4.0 \text{ m}, 1.0 \text{ m})$$

(d) As $n_A \sin i_A = n_P \sin i_p$ and as $n_A = n_P = 1$

Therefore, $i_P = i_A = 90°$ i.e. the ray will emerge parallel to the boundary at P i.e. at grazing emergence.

44. (a) Total internal reflection (TIR) will take place first for that wavelength for which critical angle is small or μ is large.



From the given expression of μ, it is more for the wavelength for which value of λ is less.

Thus, condition of TIR is just satisfied for 4000 Å.

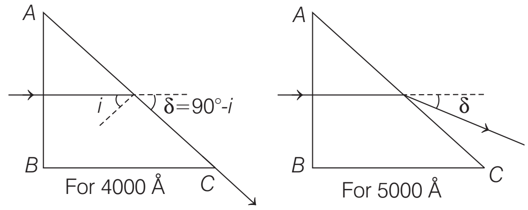
or $\quad i = \theta_c$ for 4000Å or $\theta = \theta_c$ or $\sin\theta = \sin\theta_c$

or $\quad 0.8 = \dfrac{1}{\mu}$ (for 4000Å)

$$\text{or} \quad 0.8 = \frac{1}{1.20 + \dfrac{b}{(4000)^2}}$$

Solving this equation, we get $b = 8.0 \times 10^5 (\text{Å})^2$

(b)



For, 4000Å condition of TIR is just satisfied. Hence, it will emerge from AC, just grazingly.

or $\delta_{4000\text{Å}} = 90° - i = 90° - \sin^{-1}(0.8) \approx 37°$

For 5000Å $\mu = 1.2 + \dfrac{b}{\lambda^2} = 1.2 + \dfrac{8.0 \times 10^5}{(5000)^2} = 1.232$

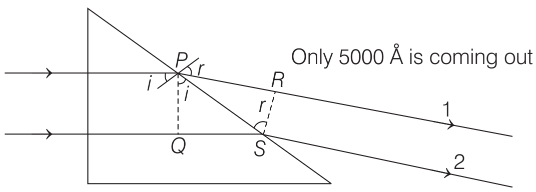
Applying $\quad \mu = \dfrac{\sin i_{\text{air}}}{\sin i_{\text{medium}}} \quad$ or $\quad 1.232 = \dfrac{\sin i_{\text{air}}}{\sin\theta}$

$$= \frac{\sin i_{\text{air}}}{0.8} \Rightarrow i_{\text{air}} = 80.26°$$

$$\therefore \quad \delta_{5000\text{Å}} = i_{\text{air}} - i_{\text{medium}}$$

$$= 80.26° - \sin^{-1}(0.8) = 27.13°$$

(c)



Path difference between rays 1 and 2

$$\Delta x = \mu(QS) - PR \qquad \text{…(i)}$$

Further, $\quad \dfrac{QS}{PS} = \sin i \Rightarrow \dfrac{PR}{PS} = \sin r$

$$\therefore \quad \frac{PR/PS}{QS/PS} = \frac{\sin r}{\sin i} = \mu \Rightarrow \mu(QS) = PR$$

Substituting in Eq. (i), we get $\Delta x = 0$.

$\therefore$ Phase difference between rays 1 and 2 will be zero.

Or these two rays will interfere constructively. So, maximum intensity will be obtained from their interference.

or $\quad I_{\max} = (\sqrt{I_1} + \sqrt{I_2})^2 = (\sqrt{4I} + \sqrt{I})^2 = 9I$

NOTE In this question we have written,

$$\mu = \frac{\sin r}{\sin i} \text{ not } \frac{\sin i}{\sin r}$$

because in medium angle with normal is i and in air angle with normal is r.

or $\quad \mu = \dfrac{\sin i_{\text{air}}}{\sin i_{\text{medium}}}$

45. Given θ is slightly greater than $\sin^{-1}\left(\frac{n_1}{n_2}\right)$

(a) **When $n_3 < n_1$**

i.e. $n_3 < n_1 < n_2$

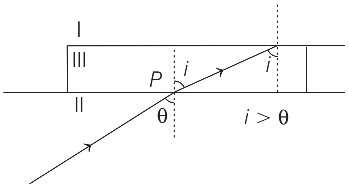
or $\frac{n_3}{n_2} < \frac{n_1}{n_2}$ or $\sin^{-1}\left(\frac{n_3}{n_2}\right) < \sin^{-1}\left(\frac{n_1}{n_2}\right)$

Hence, critical angle for III and II will be less than the critical angle for II and I. So, if TIR is taking place between I and II, then TIR will definitely take place between I and III.

(b) **When $n_3 > n_1$** Now two cases may arise :

Case 1 $n_1 < n_3 < n_2$

In this case there will be no TIR between II and III but TIR will take place between III and I. This is because



Ray of light first enters from II to III *ie,* from denser to rarer.

$\therefore$ $i > \theta$

Applying Snell's law at P

$$n_2 \sin\theta = n_3 \sin i \text{ or } \sin i = \left(\frac{n_2}{n_3}\right)\sin\theta$$

Since, $\sin\theta$ is slightly greater than $\frac{n_1}{n_2}$

$\therefore$ $\sin i$ is slightly greater than $\frac{n_2}{n_3} \times \frac{n_1}{n_2}$ or $\frac{n_1}{n_3}$

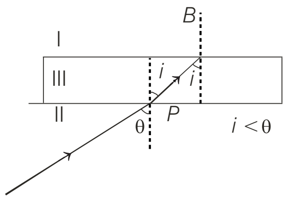
but $\frac{n_1}{n_3}$ is nothing but $\sin(\theta_c)_{\text{I, III}}$

$\therefore$ $\sin(i)$ is slightly greater than $\sin(\theta_c)_{\text{I, III}}$

Or TIR will now take place on I and III and the ray will be reflected back.

Case 2 $n_1 < n_2 < n_3$

This time while moving from II to III, ray of light will bend towards normal. Again applying Snell's law at P



$$n_2 \sin\theta = n_3 \sin i$$

$\Rightarrow$ $\sin i = \frac{n_2}{n_3}\sin\theta$

Since, $\sin\theta$ slightly greater than $\frac{n_1}{n_2}$

$\sin i$ will be slightly greater than $\frac{n_2}{n_3} \times \frac{n_1}{n_2}$ or $\frac{n_1}{n_3}$

but $\frac{n_1}{n_3}$ is $\sin(\theta_c)_{\text{I, III}}$

i.e. $\sin i > \sin(\theta_c)_{\text{I, III}}$ or $i > (\theta_c)_{\text{I, III}}$

Therefore, TIR will again take place between I and III and the ray will be reflected back.

NOTE Case I and case 2 of $n_3 > n_1$ can be explained by one equation only. But two cases are deliberately formed for better understanding of refraction, Snell's law and total internal reflection (TIR).

46. Resultant intensity at P

$$I_P = I_A + I_B + I_C$$
$$= \frac{P_A}{4\pi(PA)^2} + \frac{P_B}{4\pi(PB)^2}\cos 60^\circ + I_C \cos 60^\circ$$
$$= \frac{90}{4\pi(3)^2} + \frac{180}{4\pi\,(1.5)^2}\cos 60^\circ + 20\cos 60^\circ$$
$$= 0.79 + 3.18 + 10 = 13.97\ \text{W/m}^2$$

47. (a) Image of object will coincide with it if ray of light after refraction from the concave surface fall normally on concave mirror so formed by silvering the convex surface. Or image after refraction from concave surface should form at centre of curvature of concave mirror or at a distance of 20 cm on same side of the combination. Let x be the distance of pin from the given optical system.

Applying, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

With proper signs, $\frac{1.5}{-20} - \frac{1}{-x} = \frac{1.5 - 1}{-60}$

or $\frac{1}{x} = \frac{3}{40} - \frac{1}{120} = \frac{8}{120} \Rightarrow x = \frac{120}{8} = 15\ \text{cm}$

(b) Now, before striking with the concave surface, the ray is first refracted from a plane surface. So, let x be the distance of pin, then the plane surface will form its image at a distance $\frac{4}{3}x$ ($h_{\text{app.}} = \mu h$) from it.

Now, using $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ with proper signs,

we have $\frac{1.5}{-20} - \frac{4/3}{-\frac{4x}{3}} = \frac{1.5 - 4/3}{-60}$

or $\frac{1}{x} = \frac{3}{40} - \frac{1}{360}$ or $x = 13.84\ \text{cm}$

$\therefore$ $\Delta x = x_1 - x_2 = 15\ \text{cm} - 13.84\ \text{cm}$

$= 1.16\ \text{cm}$ (downwards)