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Modern Physics

Topic 1 Bohr's Atomic Model

Objective Questions I (Only one correct option)

1. The electron in a hydrogen atom first jumps from the third excited state to the second excited state and subsequently to the first excited state. The ratio of the respective wavelengths λ_1 / λ_2 of the photons emitted in this process is **(Main 2019, 12 April II)**

(a) 20/7 (b) 27/5
(c) 7/5 (d) 9/7

2. An excited He^+ ion emits two photons in succession, with wavelengths 108.5 nm and 30.4 nm, in making a transition to ground state. The quantum number n corresponding to its initial excited state is [for photon of wavelength λ, energy $E = \frac{1240\,\text{eV}}{\lambda \text{ (in nm)}}$] **(Main 2019, 12 April I)**

(a) $n = 4$ (b) $n = 5$
(c) $n = 7$ (d) $n = 6$

3. In Li^{++}, electron in first Bohr orbit is excited to a level by a radiation of wavelength λ. When the ion gets de-excited to the ground state in all possible ways (including intermediate emissions), a total of six spectral lines are observed. What is the value of λ? **(Main 2019, 10 April II)**

[Take, $h = 6.63 \times 10^{-34}$ Js; $c = 3 \times 10^8$ ms^{-1}]

(a) 9.4 nm (b) 12.3 nm (c) 10.8 nm (d) 11.4 nm

4. A He^+ ion is in its first excited state. Its ionisation energy is **(Main 2019, 9 April II)**

(a) 54.40 eV (b) 13.60 eV (c) 48.36 eV (d) 6.04 eV

5. Taking the wavelength of first Balmer line in hydrogen spectrum ($n = 3$ to $n = 2$) as 660 nm, the wavelength of the 2$^{\text{nd}}$ Balmer line ($n = 4$ to $n = 2$) will be **(Main 2019, 9 April I)**

(a) 889.2 nm (b) 388.9 nm (c) 642.7 nm (d) 488.9 nm

6. Radiation coming from transitions $n = 2$ to $n = 1$ of hydrogen atoms fall on He^+ ions in $n = 1$ and $n = 2$ states. The possible transition of helium ions as they absorb energy from the radiation is **(Main 2019, 8 April I)**

(a) $n = 2 \rightarrow n = 3$ (b) $n = 1 \rightarrow n = 4$
(c) $n = 2 \rightarrow n = 5$ (d) $n = 2 \rightarrow n = 4$

7. A particle of mass m moves in a circular orbit in a central potential field $U(r) = \frac{1}{2}kr^2$. If Bohr's quantization conditions are applied, radii of possible orbitals and energy levels vary with quantum number n as **(Main 2019, 12 Jan I)**

(a) $r_n \propto n, E_n \propto n$ (b) $r_n \propto n^2, E_n \propto \frac{1}{n^2}$
(c) $r_n \propto \sqrt{n}, E_n \propto n$ (d) $r_n \propto \sqrt{n}, E_n \propto \frac{1}{n}$

8. In a hydrogen like atom, when an electron jumps from the M-shell to the L-shell, the wavelength of emitted radiation is λ. If an electron jumps from N-shell to the L-shell, the wavelength of emitted radiation will be **(Main 2019, 11 Jan II)**

(a) $\frac{27}{20}\lambda$ (b) $\frac{25}{16}\lambda$
(c) $\frac{20}{27}\lambda$ (d) $\frac{16}{25}\lambda$

9. A hydrogen atom, initially in the ground state is excited by absorbing a photon of wavelength 980 Å. The radius of the atom in the excited state in terms of Bohr radius a_0 will be (Take $hc = 12500$ eV-Å) **(Main 2019, 11 Jan I)**

(a) $4a_0$ (b) $9a_0$
(c) $16a_0$ (d) $25a_0$

10. If the series limit frequency of the Lyman series is ν_L, then the series limit frequency of the Pfund series is **(2018 Main)**

(a) $\nu_L / 25$ (b) $16\,\nu_L$
(c) $\frac{\nu_L}{16}$ (d) $\frac{\nu_L}{20}$

11. As an electron makes a transition from an excited state to the ground state of a hydrogen like atom/ion **(2015 Main)**

(a) kinetic energy, potential energy and total energy decrease
(b) kinetic energy decreases, potential energy increases but total energy remains same
(c) kinetic energy and total energy decrease but potential energy increases
(d) its kinetic energy increases but potential energy and total energy decrease

12. Hydrogen $({}_1H^1)$, deuterium $({}_1H^2)$, singly ionised helium $({}_2He^4)^+$ and doubly ionised lithium $({}_3Li^8)^{++}$ all have one electron around the nucleus. Consider an electron transition from $n = 2$ to $n = 1$. If the wavelengths of emitted radiation are $\lambda_1, \lambda_2, \lambda_3$ and λ_4, respectively for four elements, then approximately which one of the following is correct? **(2014 Main)**

(a) $4\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$
(b) $\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$
(c) $\lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$
(d) $\lambda_1 = 2\lambda_2 = 3\lambda_3 = 4\lambda_4$

13. In a hydrogen like atom electron makes transition from an energy level with quantum number n to another with quantum number $(n - 1)$. If $n >> 1$, the frequency of radiation emitted is proportional to **(2013 Main)**

(a) $\frac{1}{n}$ (b) $\frac{1}{n^2}$
(c) $\frac{1}{n^4}$ (d) $\frac{1}{n^3}$

14. The wavelength of the first spectral line in the Balmer series of hydrogen atom is 6561 Å. The wavelength of the second spectral line in the Balmer series of singly ionized helium atom is **(2011)**

(a) 1215 Å (b) 1640 Å
(c) 2430 Å (d) 4687 Å

15. The largest wavelength in the ultraviolet region of the hydrogen spectrum is 122 nm. The smallest wavelength in the infrared region of the hydrogen spectrum (to the nearest integer) is **(2007, 3M)**

(a) 802 nm (b) 823 nm
(c) 1882 nm (d) 1648 nm

16. A photon collides with a stationary hydrogen atom in ground state inelastically. Energy of the colliding photon is 10.2 eV. After a time interval of the order of micro second another photon collides with same hydrogen atom inelastically with an energy of 15 eV. What will be observed by the detector? **(2005, 2M)**

(a) 2 photons of energy 10.2 eV
(b) 2 photons of energy 1.4 eV
(c) One photon of energy 10.2 eV and an electron of energy 1.4 eV
(d) One photon of energy 10.2 eV and another photon of energy 1.4 eV

17. If the atom ${}_{100}Fm^{257}$ follows the Bohr's model and the radius of last orbit of ${}_{100}Fm^{257}$ is n times the Bohr radius, then find n **(2003, 2M)**

(a) 100 (b) 200 (c) 4 (d) 1/4

18. The electric potential between a proton and an electron is given by $V = V_0 \ln \frac{r}{r_0}$, where r_0 is a constant. Assuming Bohr's model to be applicable, write variation of r_n with n, n being the principal quantum number. **(2003, 2M)**

(a) $r_n \propto n$ (b) $r_n \propto \frac{1}{n}$ (c) $r_n \propto n^2$ (d) $r_n \propto \frac{1}{n^2}$

19. A hydrogen atom and a Li^{2+} ion are both in the second excited state. If l_H and l_{Li} are their respective electronic angular momenta, and E_H and E_{Li} their respective energies, then **(2002, 2M)**

(a) $l_H > l_{Li}$ and $|E_H| > |E_{Li}|$
(b) $l_H = l_{Li}$ and $|E_H| < |E_{Li}|$
(c) $l_H = l_{Li}$ and $|E_H| > |E_{Li}|$
(d) $l_H < l_{Li}$ and $|E_H| < |E_{Li}|$

20. The transition from the state $n = 4$ to $n = 3$ in a hydrogen like atom results in ultraviolet radiation. Infrared radiation will be obtained in the transition **(2001, 2M)**

(a) $2 \to 1$ (b) $3 \to 2$
(c) $4 \to 2$ (d) $5 \to 4$

21. The electron in a hydrogen atom makes a transition from an excited state to the ground state. Which of the following statement is true? **(2000, 2M)**

(a) Its kinetic energy increases and its potential and total energy decreases
(b) Its kinetic energy decreases, potential energy increases and its total energy remains the same
(c) Its kinetic and total energy decreases and its potential energy increases
(d) Its kinetic, potential and total energy decreases

22. Imagine an atom made up of proton and a hypothetical particle of double the mass of the electron but having the same charge as the electron. Apply the Bohr atom model and consider all possible transitions of this hypothetical particle to the first excited level. The longest wavelength photon that will be emitted has wavelength λ (given in terms of the Rydberg constant R for the hydrogen atom) equal to **(2000, 2M)**

(a) $9/5R$ (b) $36/5R$
(c) $18/5R$ (d) $4/R$

23. In hydrogen spectrum, the wavelength of H_α line is 656 nm; whereas in the spectrum of a distant galaxy H_α line wavelength is 706 nm. Estimated speed of galaxy with respect to earth is **(1999, 2M)**

(a) 2×10^8 m/s
(b) 2×10^7 m/s
(c) 2×10^6 m/s
(d) 2×10^5 m/s

24. As per Bohr model, the minimum energy (in eV) required to remove an electron from the ground state of doubly ionized Li atom $(Z = 3)$ is **(1997, 1M)**

(a) 1.51 (b) 13.6
(c) 40.8 (d) 122.4

25. Consider the spectral line resulting from the transition $n = 2 \to n = 1$ in the atoms and ions given below. The shortest wavelength is produced by **(1983, 1M)**

(a) hydrogen atom
(b) deuterium atom
(c) singly ionised helium
(d) doubly ionised lithium

Passage Based Questions

Passage 1

The key feature of Bohr's theory of spectrum of hydrogen atom is the quantisation of angular momentum when an electron is revolving around a proton. We will extend this to a general rotational motion to find quantised rotational energy of a diatomic molecule assuming it to be rigid. The rule to be applied is Bohr's quantisation condition.

26. A diatomic molecule has moment of inertia I. By Bohr's quantization condition its rotational energy in the nth level ($n = 0$ is not allowed) is **(2010)**

(a) $\frac{1}{n^2}\left(\frac{h^2}{8\pi^2 I}\right)$ (b) $\frac{1}{n}\left(\frac{h^2}{8\pi^2 I}\right)$

(c) $n\left(\frac{h^2}{8\pi^2 I}\right)$ (d) $n^2\left(\frac{h^2}{8\pi^2 I}\right)$

27. It is found that the excitation frequency from ground to the first excited state of rotation for the CO molecule is close to $\frac{4}{\pi} \times 10^{11}$ Hz. Then the moment of inertia of CO molecule about its centre of mass is close to (Take $h = 2\pi \times 10^{-34}$ J-s) **(2010)**

(a) 2.76×10^{-46} kg-m^2 (b) 1.87×10^{-46} kg-m^2

(c) 4.67×10^{-47} kg-m^2 (d) 1.17×10^{-47} kg-m^2

28. In a CO molecule, the distance between C (mass = 12 amu) and O (mass = 16 amu), where 1 amu $= \frac{5}{3} \times 10^{-27}$ kg, is close to **(2010)**

(a) 2.4×10^{-10} m (b) 1.9×10^{-10} m

(c) 1.3×10^{-10} m (d) 4.4×10^{-11} m

Passage 2

When a particle is restricted to move along x-axis between $x = 0$ and $x = a$, where a is of nanometer dimension, its energy can take only certain specific values. The allowed energies of the particle moving in such a restricted region, correspond to the formation of standing waves with nodes at its ends $x = 0$ and $x = a$. The wavelength of this standing wave is related to the linear momentum p of the particle according to the de-Broglie relation. The energy of the particle of mass m is related to its linear momentum as $E = \frac{p^2}{2m}$. Thus, the energy of the particle can be denoted by a quantum number n taking values 1, 2, 3, ...($n = 1$, called the ground state) corresponding to the number of loops in the standing wave.

Use the model described above to answer the following three questions for a particle moving in the line $x = 0$ to $x = a$. [Take $h = 6.6 \times 10^{-34}$ Js and $e = 1.6 \times 10^{-19}$ C]

29. The allowed energy for the particle for a particular value of n is proportional to **(2009)**

(a) a^{-2} (b) $a^{-3/2}$ (c) a^{-1} (d) a^2

30. If the mass of the particle is $m = 1.0 \times 10^{-30}$ kg and $a = 6.6$ nm, the energy of the particle in its ground state is closest to **(2009)**

(a) 0.8 meV (b) 8 meV

(c) 80 meV (d) 800 meV

31. The speed of the particle that can take discrete values is proportional to **(2009)**

(a) $n^{-3/2}$ (b) n^{-1}

(c) $n^{1/2}$ (d) n

Passage 3

In a mixture of H - He$^+$ gas (He$^+$ is singly ionized He atom), H atoms and He$^+$ ions are excited to their respective first excited states. Subsequently, H atoms transfer their total excitation energy to He$^+$ ions (by collisions). Assume that the Bohr model of atom is exactly valid.

32. The quantum number n of the state finally populated in He$^+$ ions is **(2008, 4M)**

(a) 2 (b) 3 (c) 4 (d) 5

33. The wavelength of light emitted in the visible region by He$^+$ ions after collisions with H atoms is **(2008, 4M)**

(a) 6.5×10^{-7} m (b) 5.6×10^{-7} m

(c) 4.8×10^{-7} m (d) 4.0×10^{-7} m

34. The ratio of the kinetic energy of the $n = 2$ electron for the H atom to that of He$^+$ ion is **(2008, 4M)**

(a) 1/4 (b) 1/2

(c) 1 (d) 2

Objective Questions II (One or more correct option)

35. Highly excited states for hydrogen-like atoms (also called Rydberg states) with nuclear charge Ze are defined by their principle quantum number n, where $n >> 1$. Which of the following statement(s) is (are) true? **(2016 Adv.)**

(a) Relative change in the radii of two consecutive orbitals does not depend on Z

(b) Relative change in the radii of two consecutive orbitals varies as $1/n$

(c) Relative change in the energy of two consecutive orbitals varies as $1/n^3$

(d) Relative change in the angular momenta of two consecutive orbitals varies as $1/n$

36. The radius of the orbit of an electron in a Hydrogen-like atom is $4.5\,a_0$ where a_0 is the Bohr radius. Its orbital angular momentum is $\frac{3h}{2\pi}$. It is given that h is Planck constant and R is Rydberg constant. The possible wavelength(s), when the atom de-excites, is (are) **(2013 Adv.)**

(a) $\frac{9}{32R}$ (b) $\frac{9}{16R}$ (c) $\frac{9}{5R}$ (d) $\frac{4}{3R}$

37. The electron in a hydrogen atom makes a transition $n_1 \to n_2$, where n_1 and n_2 are the principal quantum numbers of two states. Assume the Bohr's model to be valid. The time period of the electron in the initial state is eight times that in the final state. The possible values of n_1 and n_2 are **(1998, 2M)**

(a) $n_1 = 4, n_2 = 2$ (b) $n_1 = 8, n_2 = 2$
(c) $n_1 = 8, n_2 = 1$ (d) $n_1 = 6, n_2 = 3$

38. In the Bohr model of the hydrogen atoms **(1984, 2M)**

(a) the radius of the nth orbit is proportional n^2.
(b) the total energy of the electron in the nth orbit is inversely proportional to n.
(c) the angular momentum of the electron in an orbit is an integral multiple of h/π.
(d) the magnitude of the potential energy of the electron in any orbit is greater than its kinetic energy.

Numerical Value

39. Consider a hydrogen-like ionised atom with atomic number Z with a single electron. In the emission spectrum of this atom, the photon emitted in the $n = 2$ to $n = 1$ transition has energy 74.8 eV higher than the photon emitted in the $n = 3$ to $n = 2$ transition. The ionisation energy of the hydrogen atom is 13.6 eV. The value of Z is .... **(2018 Adv.)**

Fill in the Blanks

40. The recoil speed of a hydrogen atom after it emits a photon is going from $n = 5$ state to $n = 1$ state is .... m/s. **(1997, 1M)**

41. In the Bohr model of the hydrogen atom, the ratio of the kinetic energy to the total energy of the electron in a quantum state n is .... **(1992, 1M)**

Integer Answer Type Questions

42. An electron in a hydrogen atom undergoes a transition from an orbit with quantum number n_i to another with quantum number n_f. v_i and v_f are respectively the initial and final potential energies of the electron. If $\frac{v_i}{v_f} = 6.25$, then the smallest possible n_f is **(2017 Adv.)**

43. A hydrogen atom in its ground state is irradiated by light of wavelength 970Å. Taking $hc/e = 1.237 \times 10^{-6}$ eVm and the ground state energy of hydrogen atom as -13.6 eV, the number of lines present in the emission spectrum is **(2016 Adv.)**

44. Consider a hydrogen atom with its electro in the n^{th} orbital. An electromagnetic radiation of wavelength 90 nm is used to ionize the atom. If the kinetic energy of the ejected electron is 10.4 eV, then the value of n is ($hc = 1242$ eV nm) **(2015 Adv.)**

Analytical & Descriptive Questions

45. Wavelengths belonging to Balmer series lying in the range of 450 nm to 750 nm were used to eject photoelectrons from a metal surface whose work function is 2.0 eV. Find (in eV) the maximum kinetic energy of the emitted photoelectrons. (Take $hc = 1242$ eV nm.) **(2004, 4M)**

46. A hydrogen-like atom (described by the Bohrs model) is observed to emit six wavelengths, originating from all possible transitions between a group of levels. These levels have energies between – 0.85 eV and –0.544 eV (including both these values). **(2002, 5M)**

(a) Find the atomic number of the atom.
(b) Calculate the smallest wavelength emitted in these transitions.

(Take $hc = 1240$ eV-nm, ground state energy of hydrogen atom = -13.6 eV)

47. A hydrogen like atom of atomic number Z is in an excited state of quantum number $2n$. It can emit a maximum energy photon of 204 eV. If it makes a transition to quantum state n, a photon of energy 40.8 eV is emitted. Find n, Z and the ground state energy (in eV) of this atom. Also, calculate the minimum energy (in eV) that can be emitted by this atom during de-excitation. Ground state energy of hydrogen atom is –13.6 eV. **(2000, 6M)**

48. An electron in a hydrogen like atom is in an excited state. It has a total energy of −3.4 eV. Calculate **(1996, 3M)**

(a) the kinetic energy,
(b) the de-Broglie wavelength of the electron.

49. A hydrogen like atom (atomic number Z) is in a higher excited state of quantum number n. The excited atom can make a transition to the first excited state by successively emitting two photons of energy 10.2 eV and 17.0 eV respectively. Alternately, the atom from the same excited state can make a transition to the second excited state by successively emitting two photons of energies 4.25 eV and 5.95 eV respectively. **(1994, 6M)**
Determine the values of n and Z.
(Ionization energy of H-atom = 13.6 eV)

50. A particle of charge equal to that of an electron $-e$, and mass 208 times of the mass of the electron (called a mu-meson) moves in a circular orbit around a nucleus of charge $+3e$. (Take the mass of the nucleus to be infinite). Assuming that the Bohr model of the atom is applicable to this system, **(1988, 6M)**

(a) derive an expression for the radius of the n^{th} Bohr orbit.
(b) find the value of n for which the radius of the orbit is approximately the same as that of the first Bohr orbit for the hydrogen atom.
(c) find the wavelength of the radiation emitted when the mu-meson jumps from the third orbit to the first orbit.

(Rydberg's constant = 1.097×10^7 m^{-1})

51. A doubly ionised lithium atom is hydrogen-like with atomic number 3. **(1985, 6M)**

(a) Find the wavelength of the radiation required to excite the electron in Li^{2+} from the first to the third Bohr orbit. (Ionisation energy of the hydrogen atom equals 13.6 eV.)
(b) How many spectral lines are observed in the emission spectrum of the above excited system?

52. The ionization energy of a hydrogen like Bohr atom is 4 rydberg. **(1984, 4M)**

(a) What is the wavelength of the radiation emitted when the electron jumps from the first excited state to the ground state?

(b) What is the radius of the first orbit for this atom ?

53. Ultraviolet light of wavelengths 800 Å and 700 Å when allowed to fall on hydrogen atoms in their ground state is found to liberate electrons with kinetic energy 1.8 eV and 4.0 eV respectively. Find the value of Planck's constant. **(1983, 4M)**

54. Hydrogen atom in its ground state is excited by means of monochromatic radiation of wavelength 975° Å. How many different lines are possible in the resulting spectrum? Calculate the longest wavelength amongst them. You may assume the ionization energy for hydrogen atom as 13.6 eV. **(1982, 5M)**

55. A single electron orbits around a stationary nucleus of charge $+Ze$. Where Z is a constant and e is the magnitude of the electronic charge. It requires 47.2 eV to excite the electron from the second Bohr orbit to the third Bohr orbit. **(1981, 10M)**
Find

(a) the value of Z.

(b) the energy required to excite the electron from the third to the fourth Bohr orbit.

(c) the wavelength of the electromagnetic radiation required to remove the electron from the first Bohr orbit to infinity.

(d) the kinetic energy, potential energy and the angular momentum of the electron in the first Bohr orbit.

(e) the radius of the first Bohr orbit.

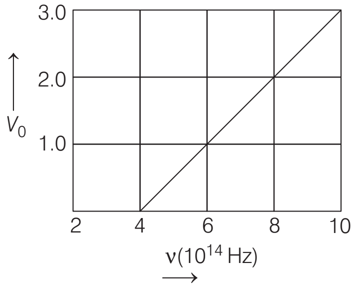
(The ionization energy of hydrogen atom $= 13.6$ eV, Bohr radius $= 5.3 \times 10^{-11}$ m, velocity of light $= 3 \times 10^8$ m/s. Planck's constant $= 6.6 \times 10^{-34}$ J-s).

Topic 2 Photo Electric Effect

Objective Questions I (Only one correct option)

1. The stopping potential V_0 (in volt) as a function of frequency (ν) for a sodium emitter, is shown in the figure. The work function of sodium, from the data plotted in the figure, will be

(Take, Planck's constant $(h) = 6.63 \times 10^{-34}$ J-s, electron charge, $e = 1.6 \times 10^{-19}$ C] **(Main 2019, 12 April I)**



(a) 1.82 eV (b) 1.66 eV
(c) 1.95 eV (d) 2.12 eV

2. A 2 mW laser operates at a wavelength of 500 nm. The number of photons that will be emitted per second is

[Given, Planck's constant $h = 6.6 \times 10^{-34}$ Js, speed of light $c = 3.0 \times 10^8$ m / s] **(Main 2019, 10 April II)**

(a) 1×10^{16} (b) 5×10^{15}
(c) 1.5×10^{16} (d) 2×10^{16}

3. In a photoelectric effect experiment, the threshold wavelength of light is 380 nm. If the wavelength of incident light is
260 nm, the maximum kinetic energy of emitted electrons will be

Given, $E \text{ (in eV)} = \dfrac{1237}{\lambda \text{ (in nm)}}$ **(Main 2019, 10 April I)**

(a) 15.1 eV (b) 3.0 eV
(c) 1.5 eV (d) 4.5 eV

4. The electric field of light wave is given as $\mathbf{E} = 10^{-3} \cos\left(\dfrac{2\pi x}{5 \times 10^{-7}} - 2\pi \times 6 \times 10^{14} t\right) \hat{\mathbf{x}}$ NC^{-1}. This light falls on a metal plate of work function 2eV. The stopping potential of the photoelectrons is

Given, $E \text{ (in eV)} = \dfrac{12375}{\lambda (\text{in Å})}$ **(Main 2019, 9 April I)**

(a) 0.48 V (b) 0.72 V
(c) 2.0 V (d) 2.48 V

5. When a certain photosensitive surface is illuminated with monochromatic light of frequency ν, the stopping potential for the photocurrent is $-V_0/2$. When the surface is illuminated by monochromatic light of frequency $\nu/2$, the stopping potential is $-V_0$. The threshold frequency for photoelectric emission is **(Main 2019, 12 Jan II)**

(a) $\dfrac{4}{3}\nu$ (b) 2ν
(c) $\dfrac{3\nu}{2}$ (d) $\dfrac{5\nu}{3}$

6. In a photoelectric experiment, the wavelength of the light incident on a metal is changed from 300 nm to 400 nm. The decrease in the stopping potential is close to $\left(\dfrac{hc}{e} = 1240 \text{ nmV}\right)$ **(Main 2019, 11 Jan II)**

(a) 0.5 V (b) 2.0 V
(c) 1.5 V (d) 1.0 V

7. A metal plate of area 1×10^{-4} m^2 is illuminated by a radiation of intensity 16 m W/m^2. The work function of the metal is 5 eV. The energy of the incident photons is 10 eV and only 10% of it produces photoelectrons. The number of emitted photoelectrons per second and their maximum energy, respectively will be (Take, 1 eV = 1.6×10^{-19} J)

(a) 10^{11} and 5 eV (b) 10^{12} and 5 eV
(c) 10^{10} and 5 eV (d) 10^{14} and 10 eV

8. The magnetic field associated with a light wave is given at the origin, by

$B = B_0 [\sin(3.14 \times 10^7) ct + \sin(6.28 \times 10^7) ct]$.

If this light falls on a silver plate having a work function of 4.7 eV, what will be the maximum kinetic energy of the photoelectrons? **(Main 2019, 10 Jan II)**

(Take, $c = 3 \times 10^8$ ms^{-1} and $h = 6.6 \times 10^{-34}$ J-s)

(a) 7.72 eV (b) 6.82 eV (c) 8.52 eV (d) 12.5 eV

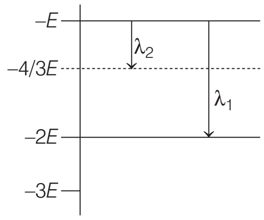
9. Surface of certain metal is first illuminated with light of wavelength $\lambda_1 = 350$ nm and then by light of wavelength $\lambda_2 = 540$ n-m. It is found that the maximum speed of the photoelectrons in the two cases differ by a factor of 2. The work function of the metal (in eV) is close to

(energy of photon $= \dfrac{1240}{\lambda \text{ (in n-m)}}$ eV) **(Main 2019, 9 Jan I)**

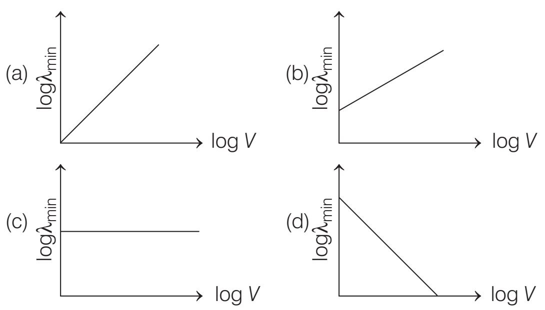
(a) 5.6 (b) 2.5 (c) 1.8 (d) 1.4

10. Some energy levels of a molecule are shown in the figure. The ratio of the wavelengths $r = \lambda_1 / \lambda_2$ is given by **(2017 Main)**

(a) $r = \dfrac{2}{3}$ (b) $r = \dfrac{3}{4}$ (c) $r = \dfrac{1}{3}$ (d) $r = \dfrac{4}{3}$



11. An electron beam is accelerated by a potential difference V to hit a metallic target to produce X-rays. It produces continuous as well as characteristic X-rays. If λ_{min} is the smallest possible wavelength of X-rays in the spectrum, the variation of $\log \lambda_{min}$ with $\log V$ is correctly represented in **(2017 Main)**



12. Radiation of wavelength λ, is incident on a photocell. The fastest emitted electron has speed v. If the wavelength is changed to $\dfrac{3\lambda}{4}$, the speed of the fastest emitted electron will be **(2016 Main)**

(a) $> v\left(\dfrac{4}{3}\right)^{1/2}$ (b) $< v\left(\dfrac{4}{3}\right)^{1/2}$
(c) $= v\left(\dfrac{4}{3}\right)^{1/2}$ (d) $= v\left(\dfrac{3}{4}\right)^{1/2}$

13. In a historical experiment to determine Planck's constant, a metal surface was irradiated with light of different wavelengths. The emitted photoelectron energies were measured by applying a stopping potential. The relevant data for the wavelength (λ) of incident light and the corresponding stopping potential (V_0) are given below: **(2016 Adv.)**

λ (μm)	V_0 (Volt)
0.3	2.0
0.4	1.0
0.5	0.4

Given that $c = 3 \times 10^8$ ms^{-1} and $e = 1.6 \times 10^{-19}$ C, Planck's constant (in units of J-s) found from such an experiment is)

(a) 6.0×10^{-34} (b) 6.4×10^{-34}
(c) 6.6×10^{-34} (d) 6.8×10^{-34}

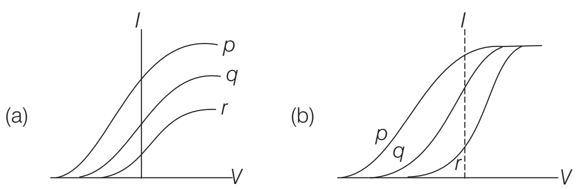
14. A metal surface is illuminated by light of two different wavelengths 248 nm and 310 nm. The maximum speeds of the photoelectrons corresponding to these wavelengths are u_1 and u_2, respectively. If the ratio $u_1 : u_2 = 2 : 1$ and $hc = 1240$ e V nm, the work function of the metal is nearly **(2014 Adv.)**

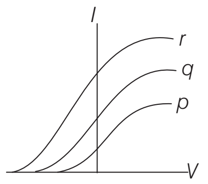
(a) 3.7 eV (b) 3.2 eV (c) 2.8 eV (d) 2.5 eV

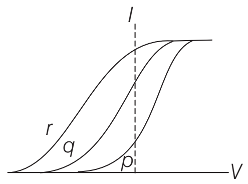
15. The radiation corresponding to $3 \rightarrow 2$ transition of hydrogen atom falls on a metal surface to produce photoelectrons. These electrons are made to enter a magnetic field of 3×10^{-4} T. If the radius of the largest circular path followed by these electrons is 10.0 mm, the work function of the metal is close to **(2014 Main)**

(a) 1.8 eV (b) 1.1 eV (c) 0.8 eV (d) 1.6 eV

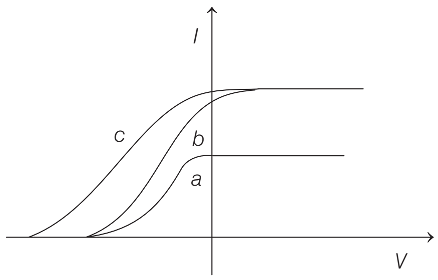
16. Photoelectric effect experiments are performed using three different metal plates p, q and r having work functions $\phi_p = 2.0$ eV, $\phi_q = 2.5$ eV and $\phi_r = 3.0$ eV, respectively. A light beam containing wavelengths of 550 nm, 450 nm and 350 nm with equal intensities illuminates each of the plates. The correct I-V graph for the experiment is **(2009)**



(c) 

(d) 

17. The figure shows the variation of photocurrent with anode potential for a photosensitive surface for three different radiations. Let I_a, I_b and I_c be the intensities and f_a, f_b and f_c be the frequencies for the curves *a, b* and *c* respectively **(2004, 2M)**



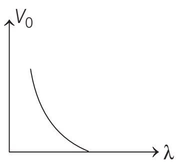
(a) $f_a = f_b$ and $I_a \neq I_b$ (b) $f_a = f_c$ and $I_a = I_c$
(c) $f_a = f_b$ and $I_a = I_b$ (d) $f_b = f_c$ and $I_b = I_c$

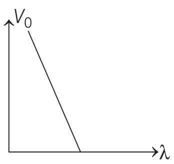
18. The work function of a substance is 4.0 eV. The longest wavelength of light that can cause photoelectron emission from this substance is approximately **(1998, 2M)**
(a) 540 nm (b) 400 nm (c) 310 nm (d) 220 nm

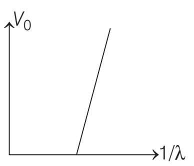
19. The maximum kinetic energy of photoelectrons emitted from a surface when photons of energy 6 eV fall on it is 4 eV. The stopping potential in volt is **(1997, 1M)**
(a) 2 (b) 4 (c) 6 (d) 10

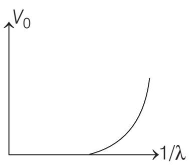
Objective Questions II (One or more correct option)

20. For photo-electric effect with incident photon wavelength λ, the stopping potential is V_0. Identify the correct variation(s) of V_0 with λ and $\frac{1}{\lambda}$. **(2015 Adv.)**

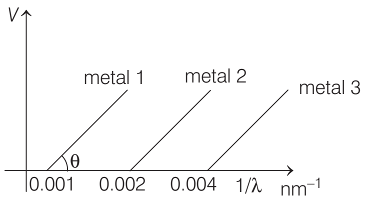
(a) 

(b) 

(c) 

(d) 

21. The graph between $1/\lambda$ and stopping potential (V) of three metals having work functions ϕ_1, ϕ_2 and ϕ_3 in an experiment of photoelectric effect is plotted as shown in the figure. Which of the following statement(s) is/are correct ? (Here, λ is the wavelength of the incident ray). **(2006, 5M)**



(a) Ratio of work functions $\phi_1 : \phi_2 : \phi_3 = 1 : 2 : 4$
(b) Ratio of work functions $\phi_1 : \phi_2 : \phi_3 = 4 : 2 : 1$
(c) $\tan\theta$ is directly proportional to hc/e, where h is Planck's constant and c is the speed of light
(d) The violet colour light can eject photoelectrons from metals 2 and 3

22. When photons of energy 4.25 eV strike the surface of a metal A, the ejected photoelectrons have maximum kinetic energy T_A expressed in eV and de-Broglie wavelength λ_A. The maximum kinetic energy of photoelectrons liberated from another metal B by photons of energy 4.70 eV is $T_B = (T_A - 1.50\,\text{eV})$. If the de-Broglie wavelength of these photoelectrons is $\lambda_B = 2\lambda_A$, then **(1994, 2M)**
(a) the work function of A is 2.25 eV
(b) the work function of B is 4.20 eV
(c) $T_A = 2.00$ eV
(d) $T_B = 2.75$ eV

23. When a monochromatic point source of light is at a distance of 0.2 m from a photoelectric cell, the cut-off voltage and the saturation current are respectively 0.6 V and 18.0 mA. If the same source is placed 0.6 m away from the photoelectric cell, then **(1992, 2M)**
(a) the stopping potential will be 0.2 V
(b) the stopping potential will be 0.6 V
(c) the saturation current will be 6.0 mA
(d) the saturation current will be 2.0 mA

24. Photoelectric effect supports quantum nature of light because **(1987, 2M)**
(a) there is a minimum frequency of light below which no photoelectrons are emitted
(b) the maximum kinetic energy of photoelectrons depends only on the frequency of light and not on its intensity
(c) even when the metal surface is faintly illuminated, the photoelectrons leave the surface immediately
(d) electric charge of the photoelectrons is quantized

25. The threshold wavelength for photoelectric emission from a material is 5200 Å. Photoelectrons will be emitted when this material is illuminated with monochromatic radiation from a **(1982, 3M)**
(a) 50 W infrared lamp (b) 1 W infrared lamp
(c) 50 W ultraviolet lamp (d) 1 W ultraviolet lamp

Integer Answer Type Questions

26. The work functions of silver and sodium are 4.6 and 2.3 eV, respectively. The ratio of the slope of the stopping potential *versus* frequency plot for silver to that of sodium is **(2013 Adv.)**

27. A silver sphere of radius 1 cm and work function 4.7 eV is suspended from an insulating thread in free-space. It is under continuous illumination of 200 nm wavelength light. As photoelectrons are emitted, the sphere gets charged and acquires a potential. The maximum number of photoelectrons emitted from the sphere is $A \times 10^Z$ (where $1 < A < 10$). The value of Z is **(2011)**

Fill in the Blank

28. The maximum kinetic energy of electrons emitted in the photoelectric effect is linearly dependent on the .... of the incident radiation. **(1984, 2M)**

True/False

29. In a photoelectric emission process, the maximum energy of the photoelectrons increases with increasing intensity of the incident light. **(1986, 3M)**

30. The kinetic energy of photoelectrons emitted by a photosensitive surface depends on the intensity of the incident radiation. **(1981, 2M)**

Analytical & Descriptive Questions

31. In a photoelectric experiment set-up, photons of energy 5 eV falls on the cathode having work function 3 eV. (a) If the saturation current is $i_A = 4\mu A$ for intensity $10^{-5}\ W/m^2$, then plot a graph between anode potential and current. (b) Also, draw a graph for intensity of incident radiation $2 \times 10^{-5}\ W/m^2$. **(2003, 2M)**

32. Two metallic plates A and B each of area $5 \times 10^{-4}\ m^2$, are placed parallel to each other at separation of 1 cm. Plate B carries a positive charge of 33.7×10^{-12} C. A monochromatic beam of light, with photons of energy 5 eV each, starts falling on plate A at $t = 0$ so that 10^{16} photons fall on it per square metre per second. Assume that one photoelectron is emitted for every 10^6 incident photons. Also assume that all the emitted photoelectrons are collected by plate B and the work function of plate A remains constant at the value 2 eV.
Determine **(2002, 5M)**

(a) the number of photoelectrons emitted up to $t = 10$ s,

(b) the magnitude of the electric field between the plates A and B at $t = 10$s and

(c) the kinetic energy of the most energetic photoelectrons emitted at $t = 10$ s when it reaches plate B.

Neglect the time taken by the photoelectron to reach plate B.
(Take $\varepsilon_0 = 8.85 \times 10^{-12}\ C^2 / N\text{-}m^2$).

33. Photoelectrons are emitted when 400 nm radiation is incident on a surface of work function 1.9 eV. These photoelectrons pass through a region containing α-particles. A maximum energy electron combines with an α-particle to form a He^+ ion, emitting a single photon in this process. He^+ ions thus formed are in their fourth excited state. Find the energies in eV of the photons lying in the 2 to 4 eV range, that are likely to be emitted during and after the combination. **(1999, 5M)**

[Take $h = 4.14 \times 10^{-15}$ eV-s]

34. In a photoelectric effect set-up a point of light of power 3.2×10^{-3} W emits monoenergetic photons of energy 5.0 eV. The source is located at a distance of 0.8 m from the centre of a stationary metallic sphere of work function 3.0 eV and of radius 8.0×10^{-3}m. The efficiency of photoelectrons emission is one for every 10^6 incident photons. Assume that the sphere is isolated and initially neutral and that photoelectrons are instantly swept away after emission. **(1995, 10M)**

(a) Calculate the number of photoelectrons emitted per second.

(b) Find the ratio of the wavelength of incident light to the de-Broglie wavelength of the fastest photoelectrons emitted.

(c) It is observed that the photoelectrons emission stops at a certain time t after the light source is switched on why ?

(d) Evaluate the time t.

35. Light from a discharge tube containing hydrogen atoms falls on the surface of a piece of sodium. The kinetic energy of the fastest photoelectrons emitted from sodium is 0.73 eV. The work function for sodium is 1.82 eV. **(1992, 10M)**

Find

(a) the energy of the photons causing the photoelectrons emission.

(b) the quantum numbers of the two levels involved in the emission of these photons.

(c) the change in the angular momentum of the electron in the hydrogen atom, in the above transition, and

(d) the recoil speed of the emitting atom assuming it to be at rest before the transition. (Ionization potential of hydrogen is 13.6 eV.)

36. A beam of light has three wavelengths 4144 Å, 4972 Å and 6216 Å with a total intensity of $3.6 \times 10^{-3}\ Wm^{-2}$ equally distributed amongst the three wavelengths. The beam falls normally on an area $1.0\ cm^2$ of a clean metallic surface of work function 2.3 eV. Assume that there is no loss of light by reflection and that each energetically capable photon ejects one electron. Calculate the number of photoelectrons liberated in two seconds. **(1989, 8M)**

Topic 3 Radioactivity

Objective Questions I (Only one correct option)

1. In a radioactive decay chain, the initial nucleus is $^{232}_{90}\text{Th}$. At the end, there are 6 α-particles and 4 β-particles which are emitted. If the end nucleus is $^{A}_{Z}X$, A and Z are given by **(Main 2019, 12 Jan II)**

(a) $A = 202; Z = 80$ (b) $A = 208; Z = 82$
(c) $A = 200; Z = 81$ (d) $A = 208; Z = 80$

2. Using a nuclear counter, the count rate of emitted particles from a radioactive source is measured. At $t = 0$, it was 1600 counts per second and $t = 8$ s, it was 100 counts per second. The count rate observed as counts per second at $t = 6$ s is close to **(Main 2019, 10 Jan I)**

(a) 400 (b) 200 (c) 150 (d) 360

3. In given time $t = 0$, Activity of two radioactive substances A and B are equal. After time t, the ratio of their activities $\frac{R_B}{R_A}$ decreases according to e^{-3t}. If the half life of A is In 2, the half-life of B will be **(Main 2019, 9 Jan II)**

(a) $4 \ln 2$ (b) $\frac{\ln 2}{4}$ (c) $\frac{\ln 2}{2}$ (d) $2 \ln 2$

4. A sample of radioactive material A, that has an activity of 10 mCi ($1 \text{ Ci} = 3.7 \times 10^{10}$ decays/s) has twice the number of nuclei as another sample of a different radioactive material B which has an activity of 20 mCi. The correct choices for half-lives of A and B would, then be respectively **(Main 2019, 9 Jan I)**

(a) 20 days and 10 days (b) 5 days and 10 days
(c) 10 days and 40 days (d) 20 days and 5 days

5. A radioactive nucleus A with a half-life T, decays into a nucleus B. At $t = 0$, there is no nucleus B. After sometime t, the ratio of the number of B to that of A is 0.3. Then, t is given by **(2017 Main)**

(a) $t = T \frac{\log 1.3}{\log 2}$ (b) $t = T \log 1.3$
(c) $t = \frac{T}{\log 1.3}$ (d) $t = \frac{T \log 2}{2 \log 1.3}$

6. Half-lives of two radioactive elements A and B are 20 min and 40 min, respectively. Initially, the samples have equal number of nuclei. After 80 min, the ratio of decayed numbers of A and B nuclei will be **(2016 Main)**

(a) 1 : 16 (b) 4 : 1 (c) 1 : 4 (d) 5 : 4

7. An accident in a nuclear laboratory resulted in deposition of a certain amount of radioactive material of half-life 18 days inside the laboratory. Tests revealed that the radiation was 64 times more than the permissible level required for safe operation of the laboratory. What is the minimum number of days after which the laboratory can be considered safe for use? **(2016 Adv.)**

(a) 64 (b) 90 (c) 108 (d) 120

8. A radioactive sample S_1 having an activity of 5 μCi has twice the number of nuclei as another sample S_2 which has an activity of 10 μCi. The half lives of S_1 and S_2 can be **(2008, 3M)**

(a) 20 yr and 5 yr, respectively
(b) 20 yr and 10 yr, respectively
(c) 10 yr each
(d) 5 yr each

9. Half-life of a radioactive substance A is 4 days. The probability that a nucleus will decay in two half-lives is **(2006, 3M)**

(a) $\frac{1}{4}$ (b) $\frac{3}{4}$ (c) $\frac{1}{2}$ (d) 1

10. After 280 days, the activity of a radioactive sample is 6000 dps. The activity reduces to 3000 dps after another 140 days. The initial activity of the sample in dps is **(2004, 2M)**

(a) 6000 (b) 9000
(c) 3000 (d) 24000

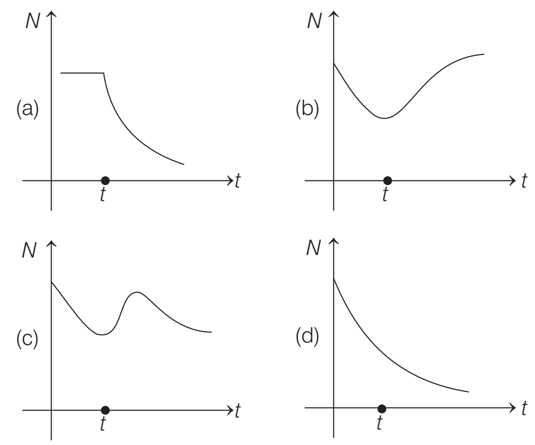
11. Which of the following processes represent a γ-decay ? **(2002, 2M)**

(a) $^{A}X_Z + \gamma \rightarrow {}^{A}X_{Z-1} + a + b$
(b) $^{A}X_Z + {}^{1}n_0 \rightarrow {}^{A-3}X_{Z-2} + c$
(c) $^{A}X_Z \rightarrow {}^{A}X_Z + f$
(d) $^{A}X_Z + e_{-1} \rightarrow {}^{A}X_{A-1} + g$

12. The half-life of ^{215}At is 100 μs. The time taken for the activity of a sample of ^{215}At to decay to $\frac{1}{16}$th of its initial value is **(2002, 2M)**

(a) 400 μs (b) 63 μs
(c) 40 μs (d) 300 μs

13. A radioactive sample consists of two distinct species having equal number of atoms initially. The mean life of one species is τ and that of the other is 5τ. The decay products in both cases are stable. A plot is made of the total number of radioactive nuclei as a function of time. Which of the following figure best represents the form of this plot ? **(2001, 2M)**



14. The electron emitted in beta radiation originates from
(a) inner orbits of atom **(2001, 2M)**
(b) free electrons existing in nuclei
(c) decay of a neutron in a nucleus
(d) photon escaping from the nucleus

15. Two radioactive materials X_1 and X_2 have decay constants $10\,\lambda$ and λ respectively. If initially they have the same number of nuclei, then the ratio of the number of nuclei of X_1 to that of X_2 will be $1/e$ after a time **(2000, 2M)**
(a) $1/10\lambda$ (b) $1/11\lambda$ (c) $11/10\lambda$ (d) $1/9\lambda$

16. The half-life period of a radioactive element x is same as the mean life time of another radioactive element y. Initially both of them have the same number of atoms. Then **(1999, 3M)**
(a) x and y have the same decay rate initially
(b) x and y decay at the same rate always
(c) y will decay at a faster rate than x
(d) x will decay at a faster rate than y

17. Which of the following is a correct statement ? **(1999, 2M)**
(a) Beta rays are same as cathode rays
(b) Gamma rays are high energy neutrons
(c) Alpha particles are singly ionized helium atoms
(d) Protons and neutrons have exactly the same mass

18. The half-life of ^{131} I is 8 days. Given a sample of ^{131}I at time $t = 0$, we can assert that **(1998, 2M)**
(a) no nucleus will decay before $t = 4$ days
(b) no nucleus will decay before $t = 8$ days
(c) all nuclei will decay before $t = 16$ days
(d) a given nucleus may decay at any time after $t = 0$

19. Masses of two isobars $_{29}\mathrm{Cu}^{64}$ and $_{30}\mathrm{Zn}^{64}$ are 63.9298 u and 63.9292 u respectively. It can be concluded from these data that **(1997C, 1M)**
(a) both the isobars are stable
(b) Zn^{64} is radioactive, decaying to Cu^{64} through β-decay
(c) Cu^{64} is radioactive, decaying to Zn^{64} through γ-decay
(d) Cu^{64} is radioactive, decaying to Zn^{64} through β-decay

20. Consider α-particles, β-particles and γ-rays each having an energy of 0.5 MeV. In increasing order of penetrating powers, the radiations are **(1994, 1M)**
(a) α, β, γ (b) α, γ, β (c) β, γ, α (d) γ, β, α

21. The decay constant of a radioactive sample is λ. The half-life and mean-life of the sample are respectively given by **(1989, 2M)**
(a) $1/\lambda$ and $(\ln 2)/\lambda$ (b) $(\ln 2)/\lambda$ and $1/\lambda$
(c) $\lambda\,(\ln 2)$ and $1/\lambda$ (d) $\lambda/(\ln 2)$ and $1/\lambda$

22. A freshly prepared radioactive source of half-life 2 h emits radiation of intensity which is 64 times the permissible safe level. The minimum time after which it would be possible to work safely with this source is **(1988, 2M)**
(a) 6 h (b) 12 h
(c) 24 h (d) 128 h

23. During a negative beta decay, **(1987, 2M)**
(a) an atomic electron is ejected
(b) an electron which is already present within the nucleus is ejected
(c) a neutron in the nucleus decays emitting an electron
(d) a part of the binding energy of the nucleus is converted into an electron

24. Beta rays emitted by a radioactive material are **(1983, 1M)**
(a) electromagnetic radiations
(b) the electrons orbiting around the nucleus
(c) charged particles emitted by the nucleus
(d) neutral particles

25. The half-life of the radioactive radon is 3.8 days. The time, at the end of which 1/20th of the radon sample will remain undecayed, is (given $\log_{10} e = 0.4343$) **(1981, 2M)**
(a) 3.8 days (b) 16.5 days (c) 33 days (d) 76 days

Objective Questions II (One or more than one)

26. In a radioactive decay chain, $^{232}_{90}\mathrm{Th}$ nucleus decays to $^{212}_{82}\mathrm{Pb}$ nucleus. Let N_α and N_β be the number of α and β- particles respectively, emitted in this decay process. Which of the following statements is (are) true? **(2018 Adv.)**
(a) $N_\alpha = 5$ (b) $N_\alpha = 6$ (c) $N_\beta = 2$ (d) $N_\beta = 4$

Integer Answer Type Questions

27. ^{131}I is an isotope of Iodine that β decays to an isotope of Xenon with a half-life of 8 days. A small amount of a serum labelled with ^{131}I is injected into the blood of a person. The activity of the amount of ^{131}I injected was 2.4×10^5 Becquerel (Bq). It is known that the injected serum will get distributed uniformly in the blood stream in less than half an hour. After 11.5 h, 2.5 ml of blood is drawn from the person's body, and gives an activity of 115 Bq. The total volume of blood in the person's body, in litres is approximately (you may use $e^2 \approx 1 + x$ for $|x| << 1$ and $\ln 2 \approx 0.7$). **(2017 Adv.)**

28. For a radioactive material, its activity A and rate of change of its activity R are defined as $A = -\dfrac{dN}{dt}$ and $R = -\dfrac{dA}{dt}$, where $N(t)$ is the number of nuclei at time t. Two radioactive source P(mean life τ) and Q (mean life 2τ) have the same activity at $t = 0$. Their rate of change of activities at $t = 2\tau$ are R_P and R_Q, respectively. If $\dfrac{R_P}{R_Q} = \dfrac{n}{e}$, then the value of n is **(2015 Adv.)**

29. A freshly prepared sample of a radioisotope of half-life 1386 s has activity 10^3 disintegrations per second. Given that $\ln 2 = 0.693$, the fraction of the initial number of nuclei (expressed in nearest integer percentage) that will decay in the first 80 s after preparation of the sample is **(2013 Adv.)**

30. The activity of a freshly prepared radioactive sample is 10^{10} disintegrations per second, whose mean life is 10^{-9} s. The mass of an atom of this radioisotope is 10^{-25} kg. The mass (in mg) of the radioactive sample is **(2011)**

Fill in the Blanks

31. In the nuclear process, ${}_6C^{11} \to {}_5B^{11} + \beta^+ + X$, X stands for .... **(1992, 1M)**

32. When boron nucleus $\left({}^{10}_{5}B\right)$ is bombarded by neutrons, α-particles are emitted. The resulting nucleus is of the element .... and has the mass number…… . **(1986, 2M)**

33. In the uranium radioactive series, the initial nucleus is ${}^{238}_{92}U$ and the final nucleus is ${}^{206}_{82}Pb$. When the uranium nucleus decays to lead, the number of α-particles emitted is ... and the number of β-particles emitted is ..... . **(1985, 2M)**

34. The radioactive decay rate of a radioactive element is found to be 10^3 disintegration/second at a certain time. If the half-life of the element is one second, the decay rate after one second is .... and after three seconds is........ . **(1983, 2M)**

Analytical & Descriptive Questions

35. A rock is 1.5×10^9 yr old. The rock contains ${}^{238}U$ which disintegrates to form ${}^{206}Pb$. Assume that there was no ${}^{206}Pb$ in the rock initially and it is the only stable product formed by the decay. Calculate the ratio of number of nuclei of ${}^{238}U$ to that of ${}^{206}Pb$ in the rock. Half-life of ${}^{238}U$ is 4.5×10^9 yr. $(2^{1/3} = 1.259)$ **(2004, 2M)**

36. A radioactive element decays by β-emission. A detector records n beta particles in 2 s and in next 2 s it records $0.75\, n$ beta particles. Find mean life correct to nearest whole number. Given $\ln |2| = 0.6931, \ln |3| = 1.0986$. **(2003, 2M)**

37. A radioactive nucleus X decays to a nucleus Y with a decay constant $\lambda_x = 0.1\, s^{-1}$, Y further decays to a stable nucleus Z with a decay constant $\lambda_y = 1/30\, s^{-1}$. Initially, there are only X nuclei and their number is $N_0 = 10^{20}$. Set-up the rate equations for the populations of X, Y and Z. The population of Y nucleus as a function of time is given by $N_y(t) = \{N_0 \lambda_x / (\lambda_x - \lambda_y)\} [\exp(-\lambda_y t) - \exp(-\lambda_x t)]$. Find the time at which N_Y is maximum and determine the populations X and Z at that instant. **(2001, 5M)**

38. Nuclei of a radioactive element A are being produced at a constant rate α. The element has a decay constant λ. At time $t = 0$, there are N_0 nuclei of the element. **(1998, 8M)**

(a) Calculate the number N of nuclei of A at time t.

(b) If $\alpha = 2N_0\lambda$, calculate the number of nuclei of A after one half-life of A and also the limiting value of N as $t \to \infty$.

39. At a given instant there are 25% undecayed radioactive nuclei in a sample. After 10 s the number of undecayed nuclei reduces to 12.5%. Calculate **(1996, 3M)**

(a) mean life of the nuclei,

(b) the time in which the number of undecayed nuclei will further reduce to 6.25% of the reduced number.

40. A small quantity of solution containing Na^{24} radio nuclide (half-life = 15 h) of activity 1.0 microcurie is injected into the blood of a person. A sample of the blood of volume 1 cm^3 taken after 5h shows an activity of 296 disintegrations per minute. Determine the total volume of the blood in the body of the person. Assume that the radioactive solution mixes uniformly in the blood of the person. **(1994, 6M)**

(1 curie $= 3.7 \times 10^{10}$ disintegrations per second)

41. There is a stream of neutrons with a kinetic energy of 0.0327 eV. If the half-life of neutrons is 700 s, what fraction of neutrons will decay before they travel a distance of 10 m? **(1986, 6M)**

42. A uranium nucleus (atomic number 92, mass number 238) emits an alpha particle and the resulting nucleus emits β-particle. What are the atomic number and mass number of the final nucleus? **(1982, 2M)**

Topic 4 X-Rays and de-Broglie Wavelength

Objective Questions I (Only one correct option)

1. Consider an electron in a hydrogen atom, revolving in its second excited state (having radius 4.65 Å). The de-Broglie wavelength of this electron is **(Main 2019, 12 April II)**

(a) 3.5 Å (b) 6.6 Å
(c) 12.9 Å (d) 9.7 Å

2. A particle P is formed due to a completely inelastic collision of particles x and y having de-Broglie wavelengths λ_x and λ_y, respectively. If x and y were moving in opposite directions, then the de-Broglie wavelength of P is **(Main 2019, 9 April II)**

(a) $\lambda_x - \lambda_y$ (b) $\dfrac{\lambda_x \lambda_y}{\lambda_x - \lambda_y}$ (c) $\dfrac{\lambda_x \lambda_y}{\lambda_x + \lambda_y}$ (d) $\lambda_x + \lambda_y$

3. Two particles move at right angle to each other. Their de-Broglie wavelengths are λ_1 and λ_2, respectively. The particles suffer perfectly inelastic collision. The de-Broglie wavelength λ of the final particle, is given by **(Main 2019, 8 April I)**

(a) $\dfrac{1}{\lambda^2} = \dfrac{1}{\lambda_1^2} + \dfrac{1}{\lambda_2^2}$

(b) $\lambda = \sqrt{\lambda_1 \lambda_2}$

(c) $\lambda = \dfrac{\lambda_1 + \lambda_2}{2}$

(d) $\dfrac{2}{\lambda} = \dfrac{1}{\lambda_1} + \dfrac{1}{\lambda_2}$

4. A particle A of mass 'm' and charge 'q' is accelerated by a potential difference of
50 V. Another particle B of mass '$4m$' and charge 'q' is accelerated by a potential difference of 2500 V. The ratio of de-Broglie wavelengths $\frac{\lambda_A}{\lambda_B}$ is close to **(Main 2019, 12 Jan I)**

(a) 4.47 (b) 10.00
(c) 0.07 (d) 14.14

5. If the de-Broglie wavelength of an electron is equal to 10^{-3} times, the wavelength of a photon of frequency 6×10^{14} Hz, then the speed of electron is equal to
(Take, speed of light $= 3 \times 10^8$ m/s,
Planck's constant $= 6.63 \times 10^{-34}$ J-s and
mass of electron $= 9.1 \times 10^{-31}$ kg) **(Main 2019, 11 Jan I)**

(a) 1.45×10^6 m/s (b) 1.8×10^6 m/s
(c) 1.1×10^6 m/s (d) 1.7×10^6 m/s

6. An electron from various excited states of hydrogen atom emit radiation to come to the ground state. Let λ_n, λ_g be the de-Broglie wavelength of the electron in the nth state and the ground state, respectively. Let Λ_n be the wavelength of the emitted photon in the transition from the nth state to the ground state. For large n, (A, B are constants) **(2018 main)**

(a) $\Lambda_n^2 \approx \lambda$ (b) $\Lambda_n \approx A + \frac{B}{\lambda_n^2}$
(c) $\Lambda_n \approx A + B\lambda_n^2$ (d) $\Lambda_n^2 \approx A + B\lambda_n^2$

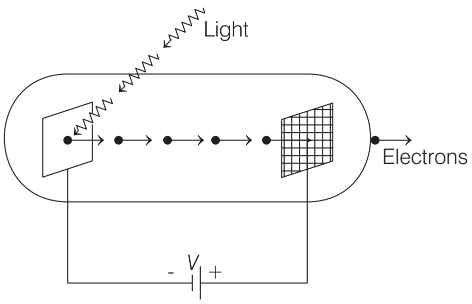
7. A particle A of mass m and initial velocity v collides with a particle B of mass $\frac{m}{2}$ which is at rest. The collision is held on, and elastic. The ratio of the de-Broglie wavelengths λ_A to λ_B after the collision is **(2017 Main)**

(a) $\frac{\lambda_A}{\lambda_B} = 2$ (b) $\frac{\lambda_A}{\lambda_B} = \frac{2}{3}$ (c) $\frac{\lambda_A}{\lambda_B} = \frac{1}{2}$ (d) $\frac{\lambda_A}{\lambda_B} = \frac{1}{3}$

8. A photoelectric material having work-function ϕ_0 is illuminated with light of wavelength $\lambda \left(\lambda < \frac{hc}{\phi_0}\right)$. The fastest photoelectron has a de-Broglie wavelength λ_d. A change in wavelength of the incident light by $\Delta\lambda$ results in a change $\Delta\lambda_d$ in λ_d. Then, the ratio $\frac{\Delta\lambda_d}{\Delta\lambda}$ is proportional to **(2017 Adv.)**

(a) $\frac{\lambda_d^2}{\lambda^2}$ (b) $\frac{\lambda_d}{\lambda}$ (c) $\frac{\lambda_d^3}{\lambda}$ (d) $\frac{\lambda_d^3}{\lambda^2}$

9. Light of wavelength λ_{ph} falls on a cathode plate inside a vacuum tube as shown in the figure. The work function of the cathode surface is ϕ and the anode is a wire mesh of conducting material kept at a distance d from the cathode. A potential difference V is maintained between the electrodes. If the minimum de Broglie wavelength of the electrons passing through the anode is λ_e, which of the following statements(s) is (are) true? **(2016 Adv.)**



(a) λ_e increases at the same rate as λ_{ph} for $\lambda_{ph} < hc/\phi$
(b) λ_e is approximately halved, if d is doubled
(c) λ_e decreases with increase in ϕ and λ_{ph}
(d) For large potential difference ($V >> \phi/e$), λ_e is approximately halved if V is made four times.

10. If λ_{Cu} is the wavelength of K_α, X-ray line of copper (atomic number 29) and λ_{MO} is the wavelength of the K_α, X-ray line of molybdenum (atomic number 42), then the ratio $\lambda_{Cu}/\lambda_{Mo}$ is close to **(2014 Adv.)**

(a) 1.99 (b) 2.14
(c) 0.50 (d) 0.48

11. Which one of the following statements is wrong in the context of X-rays generated from an X-ray tube ? **(2008, 3M)**

(a) Wavelength of characteristic X-rays decreases when the atomic number of the target increases
(b) Cut-off wavelength of the continuous X-rays depends on the atomic number of the target
(c) Intensity of the characteristic X-rays depends on the electrical power given to the X-ray tube
(d) Cut-off wavelength of the continuous X-rays depends on the energy of the electrons in the X-ray tube

12. Electrons with de-Broglie wavelength λ fall on the target in an X-ray tube. The cut-off wavelength of the emitted X-rays is **(2007, 3M)**

(a) $\lambda_0 = \frac{2mc\lambda^2}{h}$ (b) $\lambda_0 = \frac{2h}{mc}$
(c) $\lambda_0 = \frac{2m^2c^2\lambda^3}{h^2}$ (d) $\lambda_0 = \lambda$

13. K_α wavelength emitted by an atom of atomic number $Z = 11$ is λ. Find the atomic number for an atom that emits K_α radiation with wavelength 4λ **(2005, 2M)**

(a) $Z = 6$ (b) $Z = 4$
(c) $Z = 11$ (d) $Z = 44$

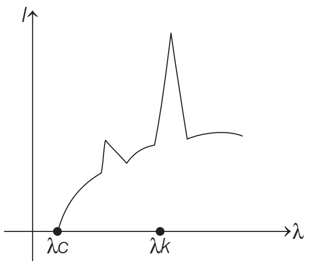
14. The energy of a photon is equal to the kinetic energy of a proton. The energy of the photon is E. Let λ_1 be the de-Broglie wavelength of the proton and λ_2 be the wavelength of the photon. The ratio $\frac{\lambda_1}{\lambda_2}$ is proportional to **(2004, 2M)**

(a) E^0 (b) $E^{1/2}$ (c) E^{-1} (d) E^{-2}

15. The potential difference applied to an X-ray tube is 5 kV and the current through it is 3.2 mA. Then the number of electrons striking the target per second is **(2002, 2M)**

(a) 2×10^{16} (b) 5×10^{6} (c) 1×10^{17} (d) 4×10^{15}

16. The intensity of X-rays from a coolidge tube is plotted against wavelength λ as shown in the figure. The minimum wavelength found is λ_c and the wavelength of the K_α line is λ_k. As the accelerating voltage is increased **(2001, 2M)**



(a) $\lambda_k - \lambda_c$ increases (b) $\lambda_k - \lambda_c$ decreases

(c) λ_k increases (d) λ_k decreases

17. Electrons with energy 80 keV are incident on the tungsten target of an X-ray tube. K-shell electrons of tungsten have 72.5 keV energy. X-rays emitted by the tube contain only **(2000, 2M)**

(a) a continuous X-ray spectrum (Bremsstrahlung) with a minimum wavelength of ≈ 0.155 Å

(b) a continuous X-ray spectrum (Bremsstrahlung) with all wavelengths

(c) the characteristic X-ray spectrum of tungsten

(d) a continuous X-ray spectrum (Bremsstrahlung) with a minimum wavelength of ≈ 0.155Å and the characteristic X-ray spectrum of tungsten

18. A particle of mass M at rest decays into two particles of masses m_1 and m_2 having non-zero velocities. The ratio of the de-Broglie wavelengths of the particles λ_1/λ_2 is **(1999, 2M)**

(a) m_1/m_2 (b) m_2/m_1 (c) 1 (d) $\sqrt{m_2}/\sqrt{m_1}$

19. X-rays are produced in an X-ray tube operating at a given accelerating voltage. The wavelength of the continuous X-rays has values from **(1998, 2M)**

(a) 0 to ∞ (b) λ_{min} to ∞ where $\lambda_{\text{min}} > 0$

(c) 0 to λ_{max} where $\lambda_{\text{max}} < \infty$

(d) λ_{min} to λ_{max} where $0 < \lambda_{\text{min}} < \lambda_{\text{max}} < \infty$

20. The K_α X-ray emission line of tungsten occurs at $\lambda = 0.021$ nm. The energy difference between K and L levels in this atoms is about **(1997C, 1M)**

(a) 0.51 MeV (b) 1.2 MeV (c) 59 keV (d) 13.6 eV

21. The X-ray beam coming from an X-ray tube will be **(1985, 2M)**

(a) monochromatic

(b) having all wavelengths smaller than a certain maximum wavelength

(c) having all wavelengths larger than a certain minimum wavelength

(d) having all wavelengths lying between a minimum and a maximum wavelength

22. The shortest wavelength of X-rays emitted from an X-ray tube depends on **(1982, 3M)**

(a) the current in the tube

(b) the voltage applied to the tube

(c) the nature of the gas in tube

(d) the atomic number of the target material

Assertion and Reason

Mark your answer as

(a) If Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I.

(b) If Statement I is true, Statement II is true; Statement II is not a correct explanation for Statement I.

(c) If Statement I is true; Statement II is false.

(d) If Statement I is false; Statement II is true.

23. **Statement I** If the accelerating potential in an X-ray tube is increased, the wavelengths of the characteristic X-rays do not change **(2007, 3M)**

Statement II When an electron beam strikes the target in an X-ray tube, part of the kinetic energy is converted into X-ray energy.

Objective Question II (One or more correct option)

24. The potential difference applied to an X-ray tube is increased. As a result, in the emitted radiation **(1988, 2M)**

(a) the intensity increases

(b) the minimum wavelength increases

(c) the intensity remains unchanged

(d) the minimum wavelength decreases

Integer Answer Type Questions

25. An electron in an excited state of Li^{2+} ion has angular momentum $\frac{3h}{2\pi}$. The de Broglie wavelength of the electron in this state is $p\pi a_0$ (where a_0 is the Bohr radius). The value of p is **(2015 Adv.)**

26. A proton is fired from very far away towards a nucleus with charge $Q = 120\,e$, where e is the electronic charge. It makes a closest approach of 10 fm to the nucleus. The de-Broglie wavelength (in units of fm) of the proton at its start is [Take the proton mass, $m_p = (5/3) \times 10^{-27}$ kg; $h/e = 4.2 \times 10^{-15}$ J-s/C;

$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ m/F; 1 fm $= 10^{-15}$ m] **(2013 Adv.)**

27. An α-particle and a proton are accelerated from rest by a potential difference of 100 V. After this, their de-Broglie wavelengths are λ_α and λ_p respectively. The ratio $\frac{\lambda_p}{\lambda_\alpha}$, to the nearest integer, is **(2010)**

Fill in the Blanks

28. The wavelength of K_α, X-rays produced by an X-ray tube is 0.76 Å. The atomic number of the anode material of the tube is .... **(1996, 2M)**

29. A potential difference of 20 kV is applied across an X-ray tube. The minimum wavelength of X-rays generated is .... Å. **(1996, 2M)**

30. In an X-ray tube, electrons accelerated through a potential difference of 15, 000 V strike a copper target. The speed of the emitted X-ray inside the tube is .... m/s. **(1992, 1M)**

31. The wavelength of the characteristic X-ray K_α line emitted by a hydrogen like element is 0.32D. The wavelength of the K_β line emitted by the same element will be .... **(1990, 2M)**

32. When the number of electrons striking the anode of an X-ray tube is increased the .... of the emitted X-rays increases, while when the speeds of the electrons striking the anode are increased the cut-off wavelength of the emitted X-rays .... **(1986, 2M)**

33. To produce characteristic X-rays using a tungsten target in an X-ray generator, the accelerating voltage should be greater than ...... V and the energy of the characteristic radiation is ...... eV. (The binding energy of the innermost electron in tungsten is 40 keV). **(1983, 2M)**

Analytical & Descriptive Questions

34. If the wavelength of the n^{th} line of Lyman series is equal to the de-Broglie wavelength of electron in initial orbit of a hydrogen like element ($Z = 11$). Find the value of n. **(2005)**

35. X-rays are incident on a target metal atom having 30 neutrons. The ratio of atomic radius of the target atom and ${}^4_2$He is $(14)^{1/3}$. **(2005, 4M)**
(a) Find the mass number of target atom.
(b) Find the frequency of K_α line emitted by this metal.
($R = 1.1 \times 10^7 \text{ m}^{-1}$, $c = 3 \times 10^8$ m/s)

36. The potential energy of a particle varies as

$$U(x) = E_0 \quad \text{for } 0 \le x \le 1$$
$$= 0 \quad \text{for } x > 1$$

For $0 \le x \le 1$, de-Broglie wavelength is λ_1 and for $x > 1$ the de-Broglie wavelength is λ_2. Total energy of the particle is $2E_0$. Find $\dfrac{\lambda_1}{\lambda_2}$. **(2005, 2M)**

37. Characteristic X-rays of frequency 4.2×10^{18} Hz are produced when transitions from L-shell to K-shell take place in a certain target material. Use Mosley's law to determine the atomic number of the target material. Given Rydberg's constant $R = 1.1 \times 10^7 \text{ m}^{-1}$. **(2003, 2M)**

38. Assume that the de-Broglie wave associated with an electron can form a standing wave between the atoms arranged in a one dimensional array with nodes at each of the atomic sites. It is found that one such standing wave is formed if the distance d between the atoms of the array is 2Å. A similar standing wave is again formed if d is increased to 2.5 Å but not for any intermediate value of d. Find the energy of the electron in eV and the least value of d for which the standing wave of the type described above can form. **(1997, 5M)**

Topic 5 Nuclear Physics

Objective Questions I (Only one correct option)

1. Half lives of two radioactive nuclei A and B are 10 minutes and 20 minutes, respectively. If initially a sample has equal number of nuclei, then after
60 minutes, the ratio of decayed numbers of nuclei A and B will be **(Main 2019, 12 April II)**
(a) 3 : 8 (b) 1 : 8 (c) 8 : 1 (d) 9 : 8

2. Two radioactive substances A and B have decay constants 5λ and λ, respectively. At $t = 0$, a sample has the same number of the two nuclei. The time taken for the ratio of the number of nuclei to become $\left(\dfrac{1}{e}\right)^2$ will be **(Main 2019, 10 April II)**
(a) $2/\lambda$ (b) $1/2\lambda$ (c) $1/4\lambda$ (d) $1/\lambda$

3. Two radioactive materials A and B have decay constants 10λ and λ, respectively. If initially they have the same number of nuclei, then the ratio of the number of nuclei of A to that of B will be $1/e$ after a time **(Main 2019, 10 April I)**
(a) $\dfrac{1}{11\lambda}$ (b) $\dfrac{11}{10\lambda}$ (c) $\dfrac{1}{9\lambda}$ (d) $\dfrac{1}{10\lambda}$

4. The ratio of mass densities of nuclei of ^{40}Ca and ^{16}O is close to **(Main 2019, 8 April II)**
(a) 5 (b) 2 (c) 0.1 (d) 1

5. Consider the nuclear fission

$$Ne^{20} \longrightarrow 2He^4 + C^{12}$$

Given that the binding energy/nucleon of Ne^{20}, He^4 and C^{12} are respectively, 8.03 MeV, 7.07 MeV and 7.86 MeV, identify the correct statement. **(Main 2019, 10 Jan II)**
(a) Energy of 3.6 MeV will be released.
(b) Energy of 12.4 MeV will be supplied.
(c) 8.3 MeV energy will be released.
(d) Energy of 11.9 MeV has to be supplied.

6. The electrostatic energy of Z protons uniformly distributed throughout a spherical nucleus of radius R is given by

$$E = \frac{3}{5}\frac{Z(Z-1)e^2}{4\pi\varepsilon_0 R}$$

The measured masses of the neutron, ${}^1_1$H, ${}^{15}_7$N and ${}^{15}_8$O are 1.008665 u, 1.007825 u, 15.000109 u and 15.003065 u, respectively. Given that the radii of both the $^{15}_{7}N$ and $^{15}_{8}O$ nuclei are same, $1\ u = 931.5\ MeV/c^2$ (c is the speed of light) and $e^2/(4\pi\varepsilon_0) = 1.44$ MeV fm. Assuming that the difference between the binding energies of $^{15}_{7}N$ and $^{15}_{8}O$ is purely due to the electrostatic energy, the radius of either of the nuclei is ($1\ \text{fm} = 10^{-15}$ m) **(2016 Adv.)**

(a) 2.85 fm (b) 3.03 fm (c) 3.42 fm (d) 3.80 fm

7. A fission reaction is given by $^{236}_{92}U \rightarrow {}^{140}_{54}Xe + {}^{94}_{38}Sr + x + y$, where x and y are two particles. Considering $^{236}_{92}U$ to be at rest, the kinetic energies of the products are denoted by K_{Xe}, K_{Sr}, K_x(2 MeV) and K_y(2 MeV), respectively. Let the binding energies per nucleon of $^{236}_{92}U$, $^{140}_{54}Xe$ and $^{94}_{38}Sr$ be 7.5 MeV, 8.5 MeV and 8.5 MeV, respectively. Considering different conservation laws, the correct options is/are **(2015 Adv.)**

(a) $x = n,\ y = n,\ K_{Sr} = 129$ MeV, $K_{Xe} = 86$ MeV
(b) $x = p,\ y = e^-,\ K_{Sr} = 129$ MeV, $K_{Xe} = 86$ MeV
(c) $x = p,\ y = n,\ K_{Sr} = 129$ MeV, $K_{Xe} = 86$ MeV
(d) $x = n,\ y = n,\ K_{Sr} = 86$ MeV, $K_{Xe} = 129$ MeV

8. In the options given below, let E denote the rest mass energy of a nucleus and n a neutron. The correct option is **(2007, 3M)**

(a) $E({}^{236}_{92}U) > E({}^{137}_{53}I) + E({}^{97}_{39}Y) + 2E(n)$
(b) $E({}^{236}_{92}U) < E({}^{137}_{53}I) + E({}^{97}_{39}Y) + 2E(n)$
(c) $E({}^{236}_{92}U) < E({}^{140}_{56}Ba) + E({}^{94}_{36}Kr) + 2E(n)$
(d) $E({}^{235}_{92}U) < E({}^{140}_{56}Ba) + E({}^{94}_{36}Kr) + E(n)$

9. If a star can convert all the He nuclei completely into oxygen nuclei. The energy released per oxygen nuclei is :
(Mass of the helium nucleus is 4.0026 amu and mass of oxygen nucleus is 15.9994 amu) **(2005, 2M)**

(a) 7.6 MeV
(b) 56.12 MeV
(c) 10.24 MeV
(d) 23.4 MeV

10. A nucleus with mass number 220 initially at rest emits an α-particle. If the Q value of the reaction is 5.5 MeV, calculate the kinetic energy of the α-particle **(2003, 2M)**

(a) 4.4 MeV (b) 5.4 MeV
(c) 5.6 MeV (d) 6.5 MeV

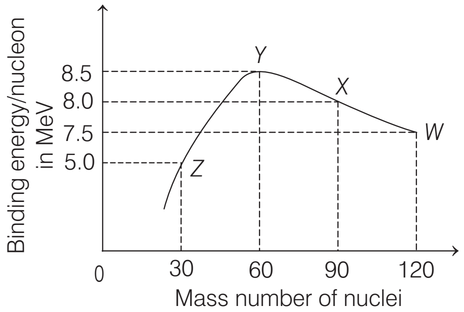
11. For uranium nucleus how does its mass vary with volume ? **(2003, 2M)**

(a) $m \propto V$ (b) $m \propto 1/V$
(c) $m \propto \sqrt{V}$ (d) $m \propto V^2$

12. Order of magnitude of density of uranium nucleus is ($m_p = 1.67 \times 10^{-27}$ kg) **(1999, 2M)**

(a) $10^{20}\ kg/m^3$
(b) $10^{17}\ kg/m^3$
(c) $10^{14}\ kg/m^3$
(d) $10^{11}\ kg/m^3$

13. Binding energy per nucleon *versus* mass number curve for nuclei is shown in figure. W, X, Y and Z are four nuclei indicated on the curve. The process that would release energy is **(1999, 2M)**



(a) $Y \rightarrow 2Z$ (b) $W \rightarrow X + Z$
(c) $W \rightarrow 2Y$ (d) $X \rightarrow Y + Z$

14. Fast neutrons can easily be slowed down by **(1994, 1M)**

(a) the use of lead shielding
(b) passing them through heavy water
(c) elastic collisions with heavy nuclei
(d) applying a strong electric field

15. A star initially has 10^{40} deuterons. It produces energy *via* the processes ${}_1H^2 + {}_1H^2 \rightarrow {}_1H^3 + p$ and ${}_1H^2 + {}_1H^3 \rightarrow {}_2He^4 + n$. If the average power radiated by the star is 10^{16} W, the deuteron supply of the star is exhausted in a time of the order of **(1993, 2M)**

(a) 10^6 s (b) 10^8 s
(c) 10^{12} s (d) 10^{16} s

The mass of the nuclei are as follows

$M(H^2) = 2.014$ amu; $M(n) = 1.008$ amu;
$M(p) = 1.007$ amu; $M(He^4) = 4.001$ amu.

16. During a nuclear fusion reaction **(1987, 2M)**

(a) a heavy nucleus breaks into two fragments by itself
(b) a light nucleus bombarded by thermal neutrons breaks up
(c) a heavy nucleus bombarded by thermal neutrons breaks up
(d) two light nuclei combine to give a heavier nucleus and possibly other products

17. The equation;

$4\,{}^1_1H \longrightarrow {}^4_2He^{2+} + 2e^- + 26$ MeV represents **(1983, 1M)**

(a) β-decay (b) γ-decay (c) fusion (d) fission

Passage Based Questions

Passage 1

The mass of a nucleus $^A_Z X$ is less that the sum of the masses of $(A - Z)$ number of neutrons and Z number of protons in the nucleus. The energy equivalent to the corresponding mass difference is known as the binding energy of the nucleus. A heavy nucleus of mass M can break into two light nuclei of masses m_1 and m_2 only if $(m_1 + m_2) < M$. Also two light nuclei of masses m_3 and m_4 can undergo complete fusion and form a heavy nucleus of mass M' only if $(m_3 + m_4) > M'$. The masses of some neutral atoms are given in the table below

(2013 Adv.)

$^1_1$H	1.007825u	$^2_1$H	2.014102u
$^6_3$Li	6.01513u	$^7_3$Li	7.016004u
$^{152}_{64}$Gd	151.919803u	$^{206}_{82}$Pb	205.974455u
$^3_1$H	3.016050u	$^4_2$He	4.002603u
$^{70}_{30}$Zn	69.925325u	$^{82}_{34}$Se	81.916709u

18. The correct statement is
(a) The nucleus $^6_3$Li can emit an alpha particle.
(b) The nucleus $^{210}_{84}$Po can emit a proton.
(c) Deuteron and alpha particle can undergo complete fusion.
(d) The nuclei $^{70}_{30}$Zn and $^{82}_{34}$Se can undergo complete fusion.

19. The kinetic energy (in keV) of the alpha particle, when the nucleus $^{210}_{84}$Po at rest undergoes alpha decay, is
(a) 5319 (b) 5422 (c) 5707 (d) 5818

Passage 2

Scientists are working hard to develop nuclear fusion reactor. Nuclei of heavy hydrogen, $^2_1$H known as deuteron and denoted by D can be thought of as a candidate for fusion reactor. The D-D reaction is $^2_1\text{H} + ^2_1\text{H} \rightarrow ^3_2\text{He} + n +$ energy. In the core of fusion reactor, a gas of heavy hydrogen is fully ionized into deuteron nuclei and electrons. This collection of $^4_1$H nuclei and electrons is known as plasma. The nuclei move randomly in the reactor core and occasionally come close enough for nuclear fusion to take place.

Usually, the temperatures in the reactor core are too high and no material wall can be used to confine the plasma. Special techniques are used which confine the plasma for a time t_0 before the particles fly away from the core. If n is the density (number/volume) of deuterons, the product nt_0 is called Lawson number. In one of the criteria, a reactor is termed successful if Lawson number is greater than 5×10^{14} s cm^{-3}.

It may be helpful to use the following : Boltzmann constant $k = 8.6 \times 10^{-5}$ eV/K; $\dfrac{e^2}{4\pi\varepsilon_0} = 1.44 \times 10^{-9}$ eVm.

20. In the core of nuclear fusion reactor, the gas becomes plasma because of **(2009)**
(a) strong nuclear force acting between the deuterons.
(b) Coulomb force acting between the deuterons.
(c) Coulomb force acting between deuteron-electron pairs.
(d) the high temperature maintained inside the reactor core.

21. Assume that two deuteron nuclei in the core of fusion reactor at temperature T are moving towards each other, each with kinetic energy 1.5 kT, when the separation between them is large enough to neglect Coulomb potential energy. Also neglect any interaction from other particles in the core. The minimum temperature T required for them to reach a separation of 4×10^{-15} m is in the range **(2009)**
(a) 1.0×10^9 K $< T < 2.0 \times 10^9$ K
(b) 2.0×10^9 K $< T < 3.0 \times 10^9$ K
(c) 3.0×10^9 K $< T < 4.0 \times 10^9$ K
(d) 4.0×10^9 K $< T < 5.0 \times 10^9$ K

22. Results of calculations for four different designs of a fusion reactor using D-D reaction are given below. Which of these is most promising based on Lawson criterion ? **(2009)**
(a) Deuteron density $= 2.0 \times 10^{12}$ cm^{-3}, confinement time $= 5.0 \times 10^{-3}$ s
(b) Deuteron density $= 8.0 \times 10^{14}$ cm^{-3}, confinement time $= 9.0 \times 10^{-1}$ s
(c) Deuteron density $= 4.0 \times 10^{23}$ cm^{-3}, confinement time $= 1.0 \times 10^{-11}$ s
(d) Deuteron density $= 1.0 \times 10^{24}$ cm^{-3}, confinement time $= 4.0 \times 10^{-12}$ s

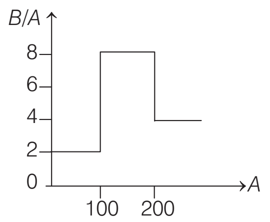
Match the Column

23. Some laws/processes are given in **Column I.** Match these with the physical phenomena given in **Column II.** **(2006,4M)**

	Column I		Column II
(A)	Nuclear fusion	(p)	Converts some matter into energy
(B)	Nuclear fission	(q)	Generally possible for nuclei with low atomic number
(C)	β-decay	(r)	Generally possible for nuclei with higher atomic number
(D)	Exothermic nuclear reaction	(s)	Generally possible for weak nuclear forces

Objective Questions II (One or more correct option)

24. Assume that the nuclear binding energy per nucleon (B/A) *versus* mass number (A) is as shown in the figure.
Use this plot to choose the correct choice(s) given below. **(2008, 4M)**



(a) Fusion of two nuclei with mass numbers lying in the range of $1 < A < 50$ will release energy.

(b) Fusion of two nuclei with mass numbers lying in the range of $51 < A < 100$ will release energy.

(c) Fission of a nucleus lying in the mass range of $100 < A < 200$ will release energy when broken into two equal fragments.

(d) Fission of a nucleus lying in the mass range of $200 < A < 260$ will release energy when broken into two equal fragments.

25. Let m_p be the mass of proton, m_n the mass of neutron. M_1 the mass of $^{20}_{10}$Ne nucleus and M_2 the mass of $^{40}_{20}$Ca nucleus. Then **(1998, 2M)**

(a) $M_2 = 2M_1$
(b) $M_2 > 2M_1$
(c) $M_2 < 2M_1$
(d) $M_1 < 10(m_n + m_p)$

26. Which of the following statements(s) is (are) correct ? **(1994, 2M)**

(a) The rest mass of a stable nucleus is less than the sum of the rest masses of its separated nucleons
(b) The rest mass of a stable nucleus is greater than the sum of the rest masses of its separated nucleons
(c) In nuclear fission, energy is released by fusing two nuclei of medium mass (approximately 100 amu)
(d) In nuclear fission, energy is released by fragmentation of a very heavy nucleus

27. The mass number of a nucleus is **(1986, 2M)**

(a) always less than its atomic number.
(b) always more than its atomic number.
(c) sometimes equal to its atomic number.
(d) sometimes more than and sometimes equal to its atomic number.

28. From the following equations pick out the possible nuclear fusion reactions **(1984, 3M)**

(a) $_6C^{13} + {}_1H^1 \longrightarrow {}_6C^{14} + 4.3$ MeV
(b) $_6C^{12} + {}_1H^1 \longrightarrow {}_7N^{13} + 2$ MeV
(c) $_7N^{14} + {}_1H^1 \longrightarrow {}_8O^{15} + 7.3$ MeV
(d) $_{92}U^{235} + {}_0n^1 \longrightarrow {}_{54}Xe^{140} + 36Sr^{94} + {}_0n^1 + {}_0n^1 + \gamma + 200$MeV

Fill in the Blanks

29. Consider the reaction : $^2_1H + ^2_1H = ^4_2He + Q$. Mass of the deuterium atom = 2.0141u. Mass of helium atom = 4.0024 u. This is a nuclear .... reaction in which the energy Q released is ...... MeV. **(1996, 2M)**

30. The binding energies per nucleon for deuteron ($_1H^2$) and helium ($_2He^4$) are 1.1 MeV and 7.0 MeV respectively. The energy released when two deuterons fuse to form a helium nucleus ($_2He^4$) is .... **(1988, 2M)**

True/False

31. The order of magnitude of the density of nuclear matter is 10^4 kg/m^3. **(1989, 2M)**

Integer Answers Type Question

32. The isotopes $^{12}_5$B having a mass 12.014 u undergoes β-decay to $^{12}_6$C. $^{12}_6$C has an excited state of the nucleus ($^{12}_6C^*$) at 4.041 MeV above its ground state. If $^{12}_5$B decays to $^{12}_5C^*$, the maximum kinetic energy of the β-particle in units of MeV is (1u = 931.5 MeV/c^2, where c is the speed of light in vacuum) **(2016 Adv.)**

33. A nuclear power plant supplying electrical power to a village uses a radioactive material of half life T years as the fuel. The amount of fuel at the beginning is such that the total power requirement of the village is 12.5% of the electrical power available from the plant at that time. If the plant is able to meet the total power needs of the village for a maximum period of nT years, then the value of n is **(2015 Adv.)**

Analytical & Descriptive Questions

34. In a nuclear reactor ^{235}U undergoes fission liberating 200 MeV of energy. The reactor has a 10% efficiency and produces 1000 MW power. If the reactor is to function for 10 yr, find the total mass of uranium required. **(2001, 5M)**

35. The element curium $^{248}_{96}$Cm has a mean life of 10^{13}s. Its primary decay modes are spontaneous fission and α-decay, the former with a probability of 8% and the latter with a probability of 92%, each fission releases 200 MeV of energy. The masses involved in decay are as follows : **(1997, 5M)**

$^{248}_{96}$Cm = 248.072220 u,
$^{244}_{94}$Pu = 244.064100 u and $^4_2$He = 4.002603 u. Calculate the power output from a sample of 10^{20} Cm atoms.
(1 u = 931 MeV/c^2)

36. A nucleus X, initially at rest, undergoes alpha-decay according to the equation. **(1991, 2+4+2M)**

$$^A_{92}X \to ^{228}_{Z}Y + \alpha$$

(a) Find the values of A and Z in the above process.
(b) The alpha particle produced in the above process is found to move in a circular track of radius 0.11m in a uniform magnetic field of 3 T. Find the energy (in MeV) released during the process and the binding energy of the parent nucleus X. Given that m (Y) = 228.03u ; $m(^1_0n)$ = 1.009u

$m(^4_2He)$ = 4.003 u ; $m(^1_1H)$ = 1.008 u.

37. It is proposed to use the nuclear fusion reaction;

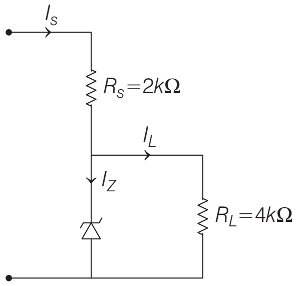
$$^2_1H + ^2_1H \to ^4_2He$$

in a nuclear reactor 200 MW rating. If the energy from the above reaction is used with a 25 per cent efficiency in the reactor, how many grams of deuterium fuel will be needed per day? (The masses of $^2_1$H and $^4_2$He are 2.0141 atomic mass units and 4.0026 atomic mass units respectively. **(1990, 8M)**

Topic 6 Semiconductor Devices, Diodes and Triodes

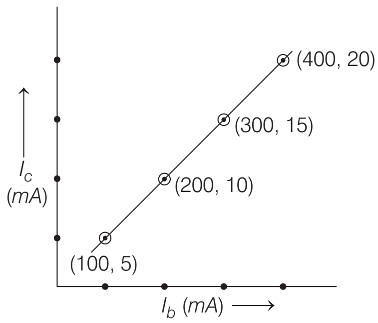
Objective Questions I (Only one correct option)

1. Figure shows a DC voltage regulator circuit, with a Zener diode of breakdown voltage = 6 V. If the unregulated input voltage varies between 10 V to 16 V, then what is the maximum Zener current? **(Main 2019, 12 April II)**



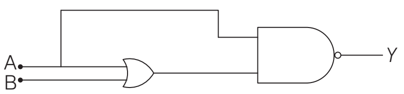
(a) 2.5 mA (b) 1.5 mA (c) 7.5 mA (d) 3.5 mA

2. The transfer characteristic curve of a transistor, having input and output resistance 100Ω and 100 kΩ respectively, is shown in the figure. The voltage and power gain, are respectively, **(Main 2019, 12 April I)**



(a) $2.5 \times 10^4, 2.5 \times 10^6$ (b) $5 \times 10^4, 5 \times 10^6$
(c) $5 \times 10^4, 5 \times 10^5$ (d) $5 \times 10^4, 2.5 \times 10^6$

3. The truth table for the circuit given in the figure is **(Main 2019, 12 April I)**



(a)

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	1

(b)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

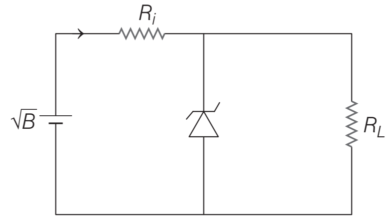
(c)

A	B	Y
0	0	1
0	1	1
1	0	0
1	1	0

(d)

A	B	Y
0	0	0
0	1	0
1	0	1
1	1	1

4. The figure represents a voltage regulator circuit using a Zener diode. The breakdown voltage of the Zener diode is 6 V and the load resistance is $R_L = 4$ kΩ. The series resistance of the circuit is $R_i = 1$ kΩ. If the battery voltage V_B varies from 8V to 16V, what are the minimum and maximum values of the current through Zener diode? **(Main 2019, 10 April II)**

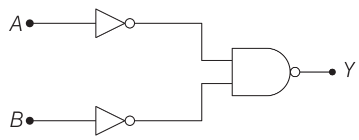


(a) 1.5 mA, 8.5 mA (b) 1 mA, 8.5 mA
(c) 0.5 mA, 8.5 mA (d) 0.5 mA, 6 mA

5. An *n-p-n* transistor operates as a common emitter amplifier, with a power gain of 60 dB. The input circuit resistance is 100 Ω and the output load resistance is 10 kΩ. The common emitter current gain β is **(Main 2019, 10 April I)**

(a) 10^2 (b) 6×10^2 (c) 10^4 (d) 60

6. The logic gate equivalent to the given logic circuit is **(Main 2019, 9 April II)**

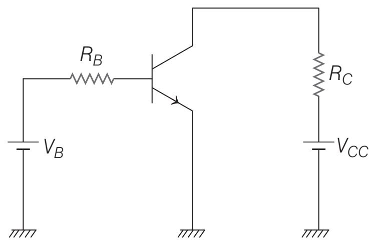


(a) NOR (b) NAND (c) OR (d) AND

7. An *n-p-n* transistor is used in common emitter configuration as an amplifier with 1 kΩ load resistance. Signal voltage of 10 mV is applied across the base-emitter. This produces a 3 mA change in the collector current and 15 μA change in the base current of the amplifier. The input resistance and voltage gain are **(Main 2019, 9 April I)**

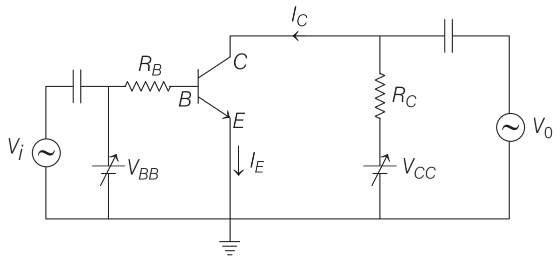
(a) 0.67 kΩ, 200 (b) 0.33 kΩ, 1.5
(c) 0.67 kΩ, 300 (d) 0.33 kΩ, 300

8. A common emitter amplifier circuit, built using an *n-p-n* transistor, is shown in the figure. Its DC current gain is 250, $R_C = 1$kΩ and $V_{CC} = 10$V. What is the minimum base current for V_{CE} to reach saturation? **(Main 2019, 8 April II)**



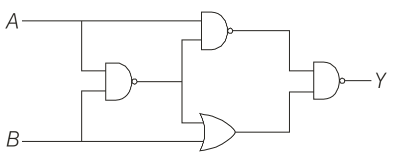
(a) 40 μA (b) 10 μA (c) 100 μA (d) 7 μA

9. In the figure, given that V_{BB} supply can vary from 0 to 5.0 V, $V_{CC} = 5$ V, $\beta_{DC} = 200$, $R_B = 100$ k Ω, $R_C = 1$ k Ω and $V_{BE} = 1.0$ V. The minimum base current and the input voltage at which the transistor will go to saturation, will be, respectively **(Main 2019, 12 Jan II)**



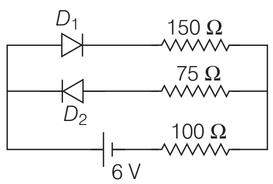
(a) 25 μA and 2.8 V (b) 25 μA and 3.5 V
(c) 20 μA and 3.5 V (d) 20 μA and 2.8 V

10. The output of the given logic circuit is **(Main 2019, 12 Jan I)**



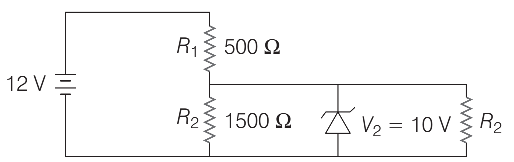
(a) $A\overline{B}$ (b) $\overline{A}B$ (c) $AB + \overline{AB}$ (d) $A\overline{B} + \overline{A}B$

11. The circuit shown below contains two ideal diodes, each with a forward resistance of 50 Ω. If the battery voltage is 6 V, the current through the 100 Ω resistance (in ampere) is **(Main 2019, 11 Jan II)**



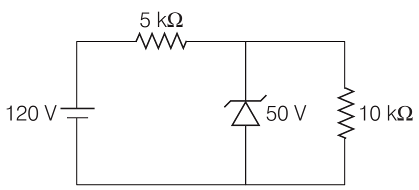
(a) 0.027 (b) 0.020 (c) 0.030 (d) 0.036

12. In the given circuit, the current through zener diode is closed to **(Main 2019, 11 Jan I)**



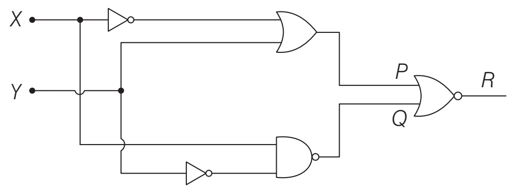
(a) 6.0 mA (b) 6.7 mA (c) 0 (d) 4.0 mA

13. For the circuit shown below, the current through the Zener diode is **(Main 2019, 10 Jan II)**



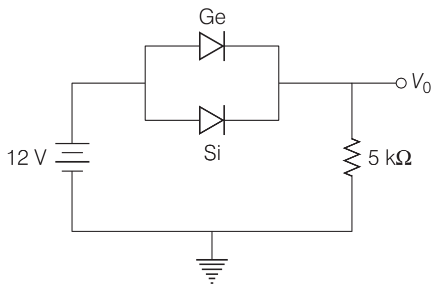
(a) 14 mA (b) zero (c) 5 mA (d) 9 mA

14. To get output '1' at R, for the given logic gate circuit, the input values must be **(Main 2019, 10 Jan I)**



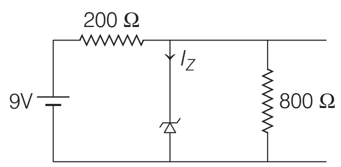
(a) $X = 0, Y = 0$ (b) $X = 1, Y = 0$
(c) $X = 1, Y = 1$ (d) $X = 0, Y = 1$

15. At 0.3V and 0.7 V, the diodes Ge and Si become conductor respectively. In given figure, if ends of diode Ge overturned, the change in potential V_0 will be **(Main 2019, 9 Jan II)**



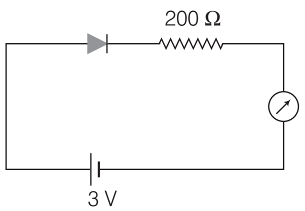
(a) 0.2 V (b) 0.6V (c) 0.4 V (d) 0.8V

16 The reverse breakdown voltage of a Zener diode is 5.6 V in the given circuit.



The current I_z through the Zener is **(2019 Main 8 April I)**
(a) 10 mA (b) 17 mA (c) 15 mA (d) 7 mA

17. The reading of the ammeter for a silicon diode in the given circuit is **(2018 Main)**

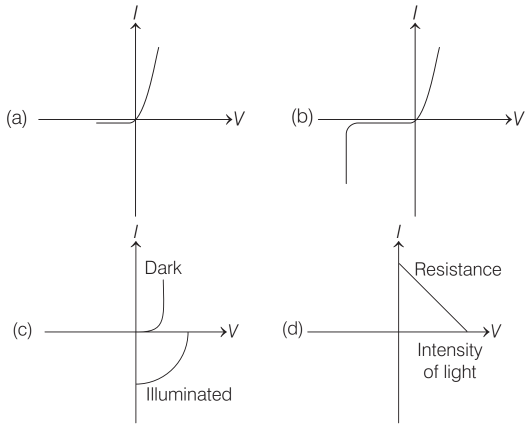


(a) 13.5 mA (b) 0 (c) 15 mA (d) 11.5 mA

18. In a common emitter amplifier circuit using an *n-p-n* transistor, the phase difference between the input and the output voltages will be **(2017 Main)**
(a) 90° (b) 135° (c) 180° (d) 45°

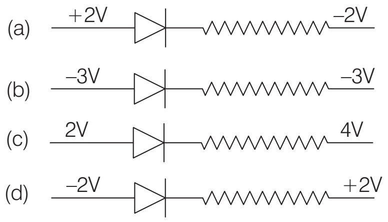
19. The temperature dependence of resistances of Cu and undoped Si in the temperature range 300-400 K, is best described by **(2016 Main)**
(a) linear increase for Cu, linear increase for Si
(b) linear increase for Cu, exponential increase for Si
(c) linear increase for Cu, exponential decrease for Si
(d) linear decrease for Cu, linear decrease for Si

20. Identify the semiconductor devices whose characteristics are as given below, in the order (a),(b),(c),(d). **(2016 Main)**

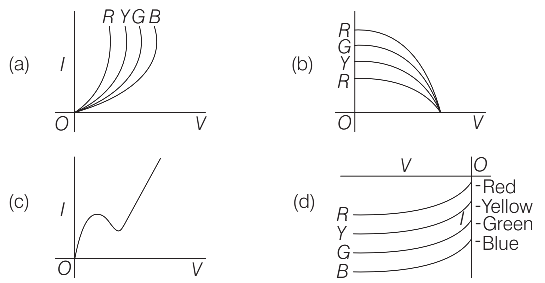


(a) Simple diode, Zener diode, Solar cell, Light dependent resistance
(b) Zener diode, Simple diode, Light dependent resistance, Solar cell
(c) Solar cell, Light dependent resistance, Zener diode, Simple diode
(d) Zener diode, Solar cell, Simple diode, Light dependent resistance

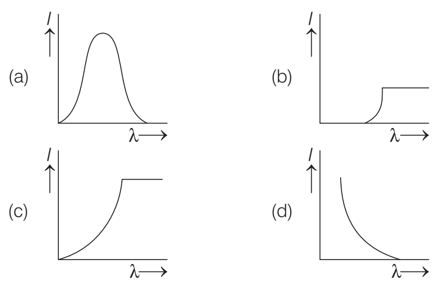
21. The forward biased diode connection is **(2014 Main)**



22. The I-V characteristic of an LED is **(2013 Main)**



23. The anode voltage of a photocell is kept fixed. The wavelength λ of the light falling on the cathode is gradually changed. The plate current I of photocell varies as follows **(2013 Main)**



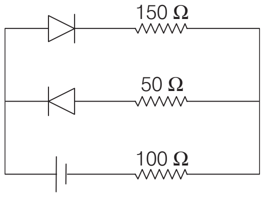
24. A diode detector is used to detect an amplitude modulated wave of 60% modulation by using a condenser of capacity 250 pF in parallel with a load resistance 100 kΩ. Find the maximum modulated frequency which could be detected by it. **(2013 Main)**

(a) 10.62 MHz (b) 10.62 kHz
(c) 5.31 MHz (d) 5.31 kHz

25. In a p-n junction diode not connected to any circuit
(a) the potential is the same everywhere. **(1998, 2M)**
(b) the p-type side is at a higher potential than the n-type side.
(c) there is an electric field at the junction directed from the n-side to the p-type side.
(d) there is an electric field at the junction directed from the p-type side to the n-type side.

26. Which of the following statements is not true ? **(1997, 1M)**
(a) The resistance of intrinsic semiconductors decreases with increase of temperature.
(b) Doping pure Si with trivalent impurities give p-type semiconductors.
(c) The majority carriers in n-type semiconductors are holes.
(d) A p-n junction can act as a semiconductor diode.

27. The circuit shown in the figure contains two diodes each with a forward resistance of 50 Ω and with infinite backward resistance. If the battery voltage is 6 V, the current through the 100 Ω resistance (in ampere) is



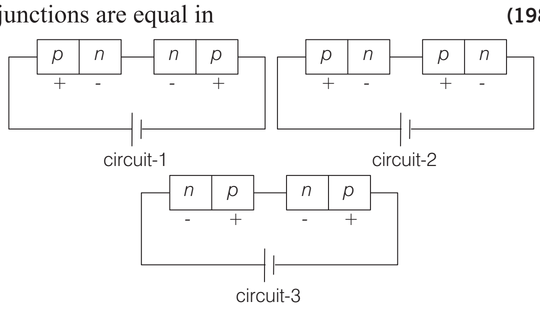
(1997, 1M)

(a) zero (b) 0.02 (c) 0.03 (d) 0.036

28. The dominant mechanisms for motion of charge carriers in forward and reverse biased silicon p-n junctions are **(1997C, 1M)**
(a) drift in forward bias, diffusion in reverse bias.
(b) diffusion in forward bias, drift in reverse bias.
(c) diffusion in both forward and reverse bias.
(d) drift in both forward and reverse bias.

29. The electrical conductivity of a semiconductor increases when electro magnetic radiation of wavelength shorter than 2480 nm is incident on it. The band gap (in eV) for the semiconductor is **(1997C, 1M)**
(a) 0.9 (b) 0.7 (c) 0.5 (d) 1.1

30. Two identical p-n junctions may be connected in series with a battery in three ways . The potential drops across the two p-n junctions are equal in **(1989, 2M)**



(a) circuit-1 and circuit-2
(b) circuit-2 and circuit-3
(c) circuit-3 and circuit-1
(d) circuit-1 only

31. For a given plate voltage, the plate current in a triode valve is maximum when the potential of **(1985, 2M)**
(a) the grid is positive and plate is negative
(b) the grid is zero and plate is positive
(c) the grid is negative and plate is positive
(d) the grid is positive and plate is positive

32. Select the correct statement from the following **(1984, 2M)**
(a) a diode can be used as a rectifier
(b) a triode cannot be used as a rectifier
(c) the current in a diode is always proportional to the applied voltage
(d) the linear portion of the I-V characteristic of a triode is used for amplification without distortion

Objective Questions II (One or more correct option)

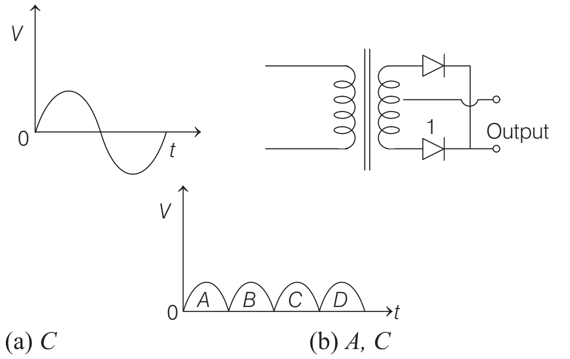
33. For a common-emitter configuration, if α and β have their usual meanings, the **incorrect** relationship between α and β is **(2016 Main)**

(a) $\frac{1}{\alpha} = \frac{1}{\beta} + 1$ (b) $\alpha = \frac{\beta}{1-\beta}$ (c) $\alpha = \frac{\beta}{1+\beta}$ (d) $\alpha = \frac{\beta^2}{1+\beta^2}$

34. A transistor is used in common emitter mode as an amplifier, then **(1998, 2M)**
(a) the base emitter junction is forward biased.
(b) the base emitter junction is reverse biased.
(c) the input signal is connected in series with the voltage applied to bias the base emitter junction.
(d) the input signal is connected in series with the voltage applied to bias the base collector junction.

35. Holes are charge carriers in **(1997)**
(a) intrinsic semiconductors
(b) ionic solids
(c) p-type semiconductors
(d) metals

36. A full wave rectifier circuit along with the output is shown in figure. The contribution (s) from the diode 1 is (are) **(1996, 2M)**



(a) C (b) A, C
(c) B, D (d) A, B, C, D

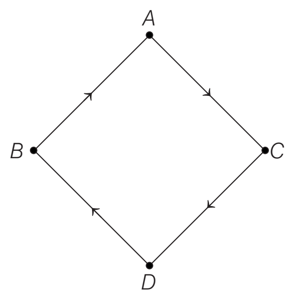
37. In an n-p-n transistor circuit, the collector current is 10 mA. If 90% of the electrons emitted reach the collector **(1992, 2M)**
(a) the emitter current will be 9 mA
(b) the emitter current will be 11mA
(c) the base current will be 1mA
(d) the base current will be −1mA

38. The impurity atoms with which pure silicon should be doped to make a p-type semiconductor are those of **(1988, 2M)**
(a) phosphorus (b) boron
(c) antimony (d) aluminium

Fill in the Blanks

39. In a .... biased p-n junction, the net flow of holes is from the n region to the p region. **(1993, 1M)**

40. For the given circuit shown in figure to act as full wave rectifier, the AC input should be connected across .... and the DC out put would appear across .... **(1991, 1M)**



41. ....biasing of p-n junction offers high resistance to current flow across the junction. The biasing is obtained by connecting the p-side to the .... terminal of the battery. **(1990, 2M)**

42. In the forward bias arrangement of a p-n junction rectifier, the p end is connected to the .... terminal of the battery and the direction of the current is from .... to ...... in the rectifier. **(1988, 2M)**

True/False

43. For a diode the variation of its anode current I_a with the anode voltage V_a at two different cathode temperatures T_1 and T_2 is shown in the figure. The temperature T_2 is greater than T_1. **(1986, 3M)**

Analytical & Descriptive Questions

44. A triode has plate characteristics in the form of parallel lines in the region of our interest. At a grid voltage of −1 V the anode current I (in mA) is given in terms of plate voltage V by the algebraic relation :

$$I = 0.125\ V - 7.5$$ **(1987, 7M)**

For grid voltage of −3 V, the current at anode voltage of 300 V is 5 mA. Determine the plate resistance (r_p) transconductance (g_m) and the amplification factor (μ) for the triode.

Topic 7 Miscellaneous Problems

Objective Questions I (Only one correct option)

1. A plane electromagnetic wave having a frequency $\nu = 23.9$ GHz propagates along the positive z-direction in free space. The peak value of the electric field is 60 V/m. Which among the following is the acceptable magnetic field component in the electromagnetic wave? **(Main 2019, 12 April II)**

(a) $\mathbf{B} = 2 \times 10^7 \sin(0.5 \times 10^3 z + 1.5 \times 10^{11} t)\,\hat{\mathbf{i}}$

(b) $\mathbf{B} = 2 \times 10^{-7} \sin(0.5 \times 10^3 z - 1.5 \times 10^{11} t)\,\hat{\mathbf{i}}$

(c) $\mathbf{B} = 60 \sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t)\,\hat{\mathbf{k}}$

(d) $\mathbf{B} = 2 \times 10^{-7} \sin(1.5 \times 10^2 x + 0.5 \times 10^{11} t)\,\hat{\mathbf{j}}$

2. In an amplitude modulator circuit, the carrier wave is given by $C(t) = 4\sin(20000\,\pi t)$ while modulating signal is given by, $m(t) = 2\sin(2000\,\pi t)$. The values of modulation index and lower side band frequency are **(Main 2019, 12 April II)**

(a) 0.5 and 10 kHz (b) 0.4 and 10 kHz
(c) 0.3 and 9 kHz (d) 0.5 and 9 kHz

3. An electromagnetic wave is represented by the electric field $\mathbf{E} = E_0\,\hat{\mathbf{n}} \sin[\omega t + (6y - 8z)]$. Taking unit vectors in x, y and z- directions to be $\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}}$, the direction of propagation $\hat{\mathbf{s}}$, is **(Main 2019, 12 April I)**

(a) $\hat{\mathbf{s}} = \frac{3\hat{\mathbf{i}} - 4\hat{\mathbf{j}}}{5}$ (b) $\hat{\mathbf{s}} = \frac{-4\hat{\mathbf{k}} + 3\hat{\mathbf{j}}}{5}$

(c) $\hat{\mathbf{s}} = \left(\frac{-3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}}{5}\right)$ (d) $\hat{\mathbf{s}} = \frac{4\hat{\mathbf{j}} - 3\hat{\mathbf{k}}}{5}$

4. Light is incident normally on a completely absorbing surface with an energy flux of 25 W cm^{-2}. If the surface has an area of 25 cm^2, the momentum transferred to the surface in 40 min time duration will be **(Main 2019, 10 April II)**

(a) 3.5×10^{-6} N-s (b) 6.3×10^{-4} N-s
(c) 1.4×10^{-6} N-s (d) 5.0×10^{-3} N-s

5. Given below in the left column are different modes of communication using the kinds of waves given in the right column.

A.	Optical fibre communication	P.	Ultrasound
B.	Radar	Q.	Infrared light
C.	Sonar	R.	Microwaves
D.	Mobile phones	S.	Radio waves

From the options given below, find the most appropriate match between entries in the left and the right column. **(Main 2019, 10 April I)**

(a) A-Q, B-S, C-R, D-P (b) A-S, B-Q, C-R, D-P
(c) A-Q, B-S, C-P, D-R (d) A-R, B-P, C-S, D-Q

6. The electric field of a plane electromagnetic wave is given by

$$\mathbf{E} = E_0\hat{\mathbf{i}} \cos(kz) \cos(\omega t)$$

The corresponding magnetic field $\mathbf{B}$ is then given by **(Main 2019, 10 April I)**

(a) $\mathbf{B} = \frac{E_0}{c}\hat{\mathbf{j}} \sin(kz) \sin(\omega t)$

(b) $\mathbf{B} = \frac{E_0}{c}\hat{\mathbf{j}} \sin(kz) \cos(\omega t)$

(c) $\mathbf{B} = \frac{E_0}{c}\hat{\mathbf{k}} \sin(kz) \cos(\omega t)$

(d) $\mathbf{B} = \frac{E_0}{c}\hat{\mathbf{j}} \cos(kz) \sin(\omega t)$

7. A message signal of frequency 100 MHz and peak voltage 100 V is used to execute amplitude modulation on a carrier wave of frequency 300 GHz and peak voltage 400 V. The modulation index and difference between the two side band frequencies are **(Main 2019, 10 April I)**

(a) 0.25 ; 1×10^8 Hz (b) 4 ; 1×10^8 Hz
(c) 0.25 ; 2×10^8 Hz (d) 4; 2×10^8 Hz

8. The physical sizes of the transmitter and receiver antenna in a communication system are **(Main 2019, 9 April II)**

(a) proportional to carrier frequency
(b) inversely proportional to modulation frequency
(c) independent of both carrier and modulation frequency
(d) inversely proportional to carrier frequency

9. 50 Q/m^2 energy density of sunlight is normally incident on the surface of a solar panel. Some part of incident energy (25%) is reflected from the surface and the rest is absorbed. The force exerted on 1 m^2 surface area will be close to ($c = 3 \times 10^8$ m/s) **(Main 2019, 9 April II)**

(a) 20×10^{-8} N (b) 35×10^{-8} N
(c) 15×10^{-8} N (d) 10×10^{-8} N

10. A signal $A\cos\omega t$ is transmitted using $v_0 \sin\omega_0 t$ as carrier wave. The correct amplitude modulated (AM) signal is **(Main 2019, 9 April I)**

(a) $(v_0 \sin\omega_0 t + A\cos\omega t$
(b) $(v_0 + A)\cos\omega t \sin\omega_0 t$
(c) $v_0 \sin[\omega_0(1 + 0.01A\sin\omega t)t]$
(d) $v_0 \sin\omega_0 t + \frac{A}{2}\sin(\omega_0 - \omega)t + \frac{A}{2}\sin(\omega_0 + \omega)t$

11. In a line of sight radio communication, a distance of about 50 km is kept between the transmitting and receiving antennas. If the height of the receiving antenna is 70 m, then the minimum height of the transmitting antenna should be

(Radius of the earth = 6.4×10^6 m) **(Main 2019, 8 April II)**

(a) 20 m (b) 32 m
(c) 40 m (d) 51 m

12. The magnetic field of an electromagnetic wave is given by

$\mathbf{B} = 1.6 \times 10^{-6} \cos(2 \times 10^7 z + 6 \times 10^{15} t)(2\hat{\mathbf{i}} + \hat{\mathbf{j}})\,\text{Wbm}^{-2}$

The associated electric field will be **(Main 2019, 8 April II)**

(a) $\mathbf{E} = 4.8 \times 10^2 \cos(2 \times 10^7 z - 6 \times 10^{15} t)(-2\hat{\mathbf{j}} + \hat{\mathbf{i}})\,\mathrm{Vm}^{-1}$

(b) $\mathbf{E} = 4.8 \times 10^2 \cos(2 \times 10^7 z - 6 \times 10^{15} t)(2\hat{\mathbf{j}} + \hat{\mathbf{i}})\,\mathrm{Vm}^{-1}$

(c) $\mathbf{E} = 4.8 \times 10^2 \cos(2 \times 10^7 z + 6 \times 10^{15} t)(\hat{\mathbf{i}} - 2\hat{\mathbf{j}})\,\mathrm{Vm}^{-1}$

(d) $\mathbf{E} = 4.8 \times 10^2 \cos(2 \times 10^7 z + 6 \times 10^{15} t)(-\hat{\mathbf{i}} + 2\hat{\mathbf{j}})\mathrm{Vm}^{-1}$

13. A nucleus A, with a finite de-Broglie wavelength λ_A, undergoes spontaneous fission into two nuclei B and C of equal mass. B flies in the same directions as that of A, while C flies in the opposite direction with a velocity equal to half of that of B. The de-Broglie wavelengths λ_B and λ_C of B and C respectively **(Main 2019, 8 April II)**

(a) $2\lambda_A, \lambda_A$ (b) $\frac{\lambda_A}{2}, \lambda_A$

(c) $\lambda_A, 2\lambda_A$ (d) $\lambda_A, \frac{\lambda_A}{2}$

14. The wavelength of the carrier waves in a modern optical fibre communication network is close to **(Main 2019, 8 April I)**

(a) 2400 nm (b) 1500 nm (c) 600 nm (d) 900 nm

15. A plane electromagnetic wave travels in free space along the x-direction. The electric field component of the wave at a particular point of space and time is $E = 6\mathrm{Vm}^{-1}$ along y-direction. Its corresponding magnetic field component, B would be **(Main 2019, 8 April I)**

(a) 2×10^{-8} T along z - direction
(b) 6×10^{-8} T along x - direction
(c) 6×10^{-8} T along z - direction
(d) 2×10^{-8} T along y - direction

16. To double the covering range of a TV transmission tower, its height should be multiplied by **(Main 2019, 12 Jan II)**

(a) $\sqrt{2}$ (b) 4 (c) 2 (d) $\frac{1}{\sqrt{2}}$

17. In a Frank-Hertz experiment, an electron of energy 5.6 eV passes through mercury vapour and emerges with an energy 0.7 eV. The minimum wavelength of photons emitted by mercury atoms is close to **(Main 2019, 12 Jan II)**

(a) 250 nm (b) 2020 nm
(c) 1700 nm (d) 220 nm

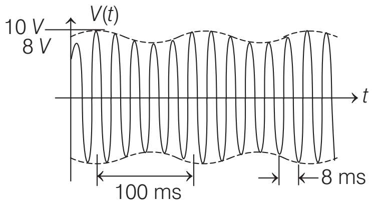
18. A 100 V carrier wave is made to vary between 160 V and 40 V by a modulating signal. What is the modulation index? **(Main 2019, 12 Jan I)**

(a) 0.4 (b) 0.5
(c) 0.6 (d) 0.3

19. A 27 mW laser beam has a cross-sectional area of 10 mm^2. The magnitude of the maximum electric field in this electromagnetic wave is given by
[Take, permittivity of space, $\varepsilon_0 = 9 \times 10^{-12}$ SI units and speed of light, $c = 3 \times 10^8$ m/s] **(Main 2019, 11 Jan II)**

(a) 1 kV/m (b) 0.7 kV/m
(c) 2 kV/m (d) 1.4 kV/m

20. An amplitude modulated signal is plotted below **(Main 2019, 11 Jan II)**



Which one of the following best describes the above signal?

(a) $[1 + 9\sin(2\pi \times 10^4 t)]\sin(2.5\pi \times 10^5 t)$V
(b) $[9 + \sin(2\pi \times 10^4 t)]\sin(2.5\pi \times 10^5 t)$V
(c) $[9 + \sin(4\pi \times 10^4 t)]\sin(5\pi \times 10^5 t)$V
(d) $[9 + \sin(2.5\pi \times 10^5 t)]\sin(2\pi \times 10^4 t)$V

21. An amplitude modulates signal is given by
$v(t) = 10[1 + 0.3\cos(2.2 \times 10^4 t)]\ \sin(5.5 \times 10^5 t)$.
Here, t is in seconds. The sideband frequencies (in kHz) are $\left(\text{Take, } \pi = \frac{22}{7}\right)$ **(Main 2019, 11 Jan I)**

(a) 892.5 and 857.5 (b) 89.25 and 85.75
(c) 178.5 and 171.5 (d) 1785 and 1715

22. An electromagnetic wave of intensity 50 Wm^{-2} enters in a medium of refractive index 'n' without any loss. The ratio of the magnitudes of electric fields and the ratio of the magnitudes of magnetic fields of the wave before and after entering into the medium are respectively, given by **(Main 2019, 11 Jan I)**

(a) $\left(\frac{1}{\sqrt{n}}, \sqrt{n}\right)$ (b) $(\sqrt{n}, \sqrt{n})$

(c) $\left(\frac{1}{\sqrt{n}}, \frac{1}{\sqrt{n}}\right)$ (d) $\left(\sqrt{n}, \frac{1}{\sqrt{n}}\right)$

23. The electric field of a plane polarised electromagnetic wave in free space at time $t = 0$ is given by an expression.

$$\mathbf{E}(x, y) = 10\hat{\mathbf{j}}\cos[(6x + 8z)]$$

The magnetic field $\mathbf{B}(x, z, t)$ is given by (where, c is the velocity of light) **(Main 2019, 10 Jan II)**

(a) $\frac{1}{c}(6\hat{\mathbf{k}} - 8\hat{\mathbf{i}})\cos[(6x + 8z + 10ct)]$

(b) $\frac{1}{c}(6\hat{\mathbf{k}} - 8\hat{\mathbf{i}})\cos[(6x + 8z - 10ct)]$

(c) $\frac{1}{c}(6\hat{\mathbf{k}} + 8\hat{\mathbf{i}})\cos[(6x - 8z + 10ct)]$

(d) $\frac{1}{c}(6\hat{\mathbf{k}} + 8\hat{\mathbf{i}})\cos[(6x + 8z - 10ct)]$

24. The modulation frequency of an AM radio station is 250 kHz, which is 10% of the carrier wave. If another AM station approaches you for license, what broadcast frequency will you allot ? **(Main 2019, 10 Jan II)**

(a) 2000 kHz (b) 2250 kHz
(c) 2900 kHz (d) 2750 kHz

25. If the magnetic field of a plane electromagnetic wave is given by

$$B = 100 \times 10^{-6} \sin\left[2\pi \times 2 \times 10^{15}\left(t - \frac{x}{c}\right)\right],$$

then the maximum electric field associated with it is (Take, the speed of light $= 3 \times 10^8$ m/s) **(Main 2019, 10 Jan I)**

(a) 6×10^4 N/C (b) 4×10^4 N/C
(c) 3×10^4 N/C (d) 4.5×10^4 N/C

26. A TV transmission tower has a height of 140 m and the height of the receiving antenna is 40 m. What is the maximum distance upto which signals can be broadcasted from this tower in LOS (Line of Sight) mode? (Take, radius of earth $= 6.4 \times 10^6$ m). **(Main 2019, 10 Jan I)**

(a) 65 km (b) 80 km (c) 40 km (d) 48 km

27. In an electron microscope, the resolution that can be achieved is of the order of wavelength of electrons used. To resolve a width of 7.5×10^{-12} m, the minimum electron energy required is close to **(Main 2019, 10 Jan I)**

(a) 500 keV (b) 1 keV
(c) 100 keV (d) 25 keV

28. In free space, the energy of electromagnetic wave in electric field is U_E and in magnetic field is U_B. Then **(Main 2019, 9 Jan II)**

(a) $U_E = U_B$ (b) $U_E > U_B$
(c) $U_E < U_B$ (d) $U_E = \frac{U_B}{2}$

29. In communication system, only one percent frequency of signal of wavelength 800 nm can be used as bandwidth. How many channal of 6MHz bandwidth can be broadcast this?
$(c = 3 \times 10^8 \text{ m/s}, h = 6.6 \times 10^{-34} \text{ J-s})$ **(Main 2019, 9 Jan II)**

(a) 3.75×10^6 (b) 3.86×10^6 (c) 6.25×10^5 (d) 4.87×10^5

30. An EM wave from air enters a medium. The electric fields are

$E_1 = E_{01}\hat{x}\cos\left[2\pi\nu\left(\frac{z}{c} - t\right)\right]$ in air and

$E_2 = E_{02}\hat{x}\cos[k(2z - ct)]$ in medium, where the wave number k and frequency ν refer to their values in air. The medium is non-magnetic.

If ε_{r_1} and ε_{r_2} refer to relative permittivities of air and medium respectively, which of the following options is correct?

(a) $\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = \frac{1}{2}$ (b) $\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = 4$
(c) $\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = 2$ (d) $\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = \frac{1}{4}$

31. A telephonic communication service is working at carrier frequency of 10 GHz. Only 10% of it is utilised for transmission. How many telephonic channels can be transmitted simultaneously, if each channel requires a bandwidth of 5 kHz? **(2018 Main)**

(a) 2×10^6 (b) 2×10^3 (c) 2×10^4 (d) 2×10^5

32. A signal of 5 kHz frequency is amplitude modulated on a carrier wave of frequency 2MHz. The frequencies of the resultant signal is/are **(2015 Main)**

(a) 2 MHz only
(b) 2005 kHz 2000 kHz and 1995 kHz
(c) 2005 kHz and 1995 kHz
(d) 2000 kHz and 1995 kHz

33. A red LED emits light at 0.1 W uniformly around it. The amplitude of the electric field of the light at a distance of 1 m from the diode is **(2015 Main)**

(a) 2.45 V/m (b) 1.73 V/m
(c) 5.48 V/m (d) 7.75 V/m

34. During the propagation of electromagnetic waves in a medium, **(2014 Main)**

(a) electric energy density is double of the magnetic energy density.
(b) electric energy density is half of the magnetic energy density.
(c) electric energy density is equal to the magnetic energy density.
(d) Both electric and magnetic energy densities are zero.

35. The magnetic field in a travelling electromagnetic wave has a peak value of 20 nT. The peak value of electric field strength is **(2013 Main)**

(a) 3V/m (b) 6 V/m
(c) 9 V/m (d) 12 V/m

36. A beam of electron is used in an YDSE experiment. The slit width is d. When the velocity of electron is increased, then **(2005, 2M)**

(a) no interference is observed
(b) fringe width increases
(c) fringe width decreases
(d) fringe width remains same

37. ^{22}Ne nucleus, after absorbing energy, decays into two α-particles and an unknown nucleus. The unknown nucleus is **(1999, 2M)**

(a) nitrogen (b) carbon (c) boron (d) oxygen

38. Four physical quantities are listed in **Column I**. Their values are listed in **Column II** in a random order **(1987, 2M)**

	Column I	Column II
A	Thermal energy of air molecules at room temperature.	(i) 0.02 eV
B	Binding energy of heavy nuclei per nucleon.	(ii) 2 eV
C	X-ray photon energy.	(iii) 10 keV
D	Photon energy of visible light.	(iv) 7 MeV

The correct matching of **Columns I** and **II** is given by :

	A	B	C	D
(a)	i	iv	iii	ii
(b)	i	iii	ii	iv
(c)	ii	i	iii	iv
(d)	ii	iv	i	iii

39. If elements with principal quantum number $n > 4$ were not allowed in nature, the number of possible elements would be **(1983, 1M)**

(a) 60 (b) 32 (c) 4 (d) 64

40. The plate resistance of a triode is $3 \times 10^3 \Omega$ and its mutual conductance is 1.5×10^{-3} A/V. The amplification factor of the triode is **(1981, 2M)**

(a) 5×10^{-5} (b) 4.5 (c) 45 (d) 2×10^5

Numerical Value

41. In a photoelectric experiment, a parallel beam of monochromatic light with power of 200 W is incident on a perfectly absorbing cathode of work function 6.25 eV. The frequency of light is just above the threshold frequency, so that the photoelectrons are emitted with negligible kinetic energy. Assume that the photoelectron emission efficiency is 100%. A potential difference of 500 V is applied between the cathode and the anode. All the emitted electrons are incident normally on the anode and are absorbed. The anode experiences a force $F = n \times 10^{-4}$N due to the impact of the electrons. The value of n is .... (Take mass of the electron, $m_e = 9 \times 10^{-31}$kg and eV $= 1.6 \times 10^{-19}$ J) **(2018 Adv.)**

Passage Based Questions

Passage

The β-decay process, discovered around 1900, is basically the decay of a neutron (n). In the laboratory, a proton (p) and an electron (e^-) are observed as the decay products of the neutron. Therefore, considering the decay of a neutron as a two-body decay process, it was predicted theoretically that the kinetic energy of the electron should be a constant. But experimentally, it was observed that the electron kinetic energy has a continuous spectrum. Considering a three-body decay process, *i.e.*, $n \to p + e^- + \bar{\nu}_e$, around 1930, Pauli explained the observed electron energy spectrum.

Assuming the anti-neutrino ($\bar{\nu}_e$) to be massless and possessing negligible energy, and the neutron to be at rest, momentum and energy conservation principles are applied. From this calculation, the maximum kinetic energy of the electron is 0.8×10^6 eV. The kinetic energy carried by the proton is only the recoil energy. **(2012)**

42. If the anti-neutrino had a mass of 3 eV/c^2 (where c is the speed of light) instead of zero mass, what should be the range of the kinetic energy K, of the electron?

(a) $0 \le K \le 0.8 \times 10^6$ eV (b) 3.0 eV $\le K \le 0.8 \times 10^6$ eV

(c) 3.0 eV $\le K < 0.8 \times 10^6$ eV (d) $0 \le K < 0.8 \times 10^6$ eV

43. What is the maximum energy of the anti-neutrino?

(a) Zero
(b) Much less than 0.8×10^6 eV
(c) Nearly 0.8×10^6 eV
(d) Much larger than 0.8×10^6 eV

Match the Columns

44. Match the nuclear processes given in Column I with the appropriate option(s) in Column II. **(2015 Adv.)**

Column I		Column II	
A.	Nuclear fusion	P.	absorption of thermal neutrons by $^{235}_{92}$U
B.	Fission in a nuclear reactor	Q.	$^{60}_{27}$Co nucleus
C.	β-decay	R.	Energy production in stars via hydrogen conversion to helium
D.	γ-ray emission	S.	Heavy water
		T.	Neutrino emission

45. Match Column I (fundamental experiment) with Column II (its conclusion) and select the correct option from the choices given below the list. **(2015 Main)**

Column I		Column II	
A	Franck-Hertz experiment	1	Particle nature of light
B	Photo-electric experiment	2	Discrete energy levels of atom
C	Davisson-Germer experiment	3	Wave nature of electron
		4	Structure of atom

	A	B	C
(a)	1	4	3
(b)	2	1	3
(c)	2	4	3
(d)	4	3	2

46. Match List I (Electromagnetic wave type) with List II (Its association/application) and select the correct option from the choices given below the lists. **(2014 Main)**

List I		List II	
A.	Infrared waves	1.	To treat muscular strain
B.	Radio waves	2.	For broadcasting
C.	X-rays	3.	To detect fracture of bones
D.	Ultraviolet	4.	Absorbed by the ozone layer of the atmosphere

Codes

	A	B	C	D
(a)	4	3	2	1
(b)	1	2	4	3
(c)	3	2	1	4
(d)	1	2	3	4

47. Match List-I of the nuclear process with List-II containing parent nucleus and one of the end products of each process and then select the correct answer using the codes given below the lists. **(2013 Adv.)**

	List I		List II
P.	Alpha decay	1.	${}^{15}_{8}O \longrightarrow {}^{17}_{7}N + \ldots$
Q.	β^+ decay	2.	${}^{238}_{92}U \rightarrow {}^{234}_{90}Th + \ldots$
R.	Fission	3.	${}^{185}_{83}Bi \rightarrow {}^{184}_{82}Pb + \ldots$
S.	Proton emission	4.	${}^{239}_{94}Pu \rightarrow {}^{140}_{57}La + \ldots$

Codes

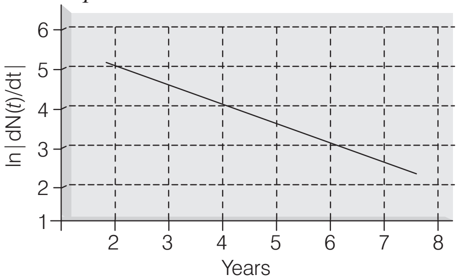
	P	Q	R	S
(a)	4	2	1	3
(b)	1	3	2	4
(c)	2	1	4	3
(d)	4	3	2	1

48. Some laws/processes are given in Column-I. Match these with the physical phenomena given in Column-II. **(2007, 6M)**

	Column I		Column II
(A)	Transition between two atomic energy levels	(p)	Characteristic X-rays
(B)	Electron emission from a material	(q)	Photoelectric effect
(C)	Moseley's law	(r)	Hydrogen spectrum

Integer Answer Type Question

49. To determine the half-life of a radioactive element, a student plots a graph of $\ln\left|\frac{dN(t)}{dt}\right|$ *versus* t. Here $\frac{dN(t)}{dt}$ is the rate of radioactive decay at time t. If the number of radioactive nuclei of this element decreases by a factor of p after 4.16 yr, the value of p is **(2010)**



Fill in the Blank

50. Atoms having the same ...... but different ...... are called isotopes. **(1986, 2M)**

Analytical & Descriptive Questions

51. A nucleus at rest undergoes a decay emitting an α-particle of de-Broglie wavelength, $\lambda = 5.76 \times 10^{-15}$ m. If the mass of the daughter nucleus is 223.610 amu and that of the α-particle is 4.002 amu. Determine the total kinetic energy in the final state. Hence obtain the mass of the parent nucleus in amu. (1 amu = 931.470 MeV/c^2) **(2001, 5M)**

52. When a beam of 10.6 eV photons of intensity 2.0 W/m^2 falls on a platinum surface of area 1.0×10^{-4} m^2 and work function 5.6 eV. 0.53% of the incident photons eject photoelectrons. Find the number of photoelectrons emitted per second and their minimum and maximum energies (in eV). Take 1 eV $= 1.6 \times 10^{-19}$ J. **(2000, 4M)**

53. In the following, **Column** I lists some physical quantities and the **Column** II gives approximate energy values associated with some of them. Choose the appropriate value of energy from **Column** II for each of the physical quantities in **Column** I and write the corresponding letters A, B, C etc., against the number (i), (ii) and (iii) etc., of the physical quantity. In the answer books in your answer the sequence of column I should be maintained : **(1997, 4M)**

	Column I		Column II
(i)	Energy of thermal neutrons	(A)	0.025 eV
(ii)	Energy of X-ray	(B)	0.5eV
(iii)	Binding energy per nucleon	(C)	3eV
(iv)	Photoelectric threshold of a metal	(D)	20 eV
		(E)	8 MeV
		(F)	10 keV

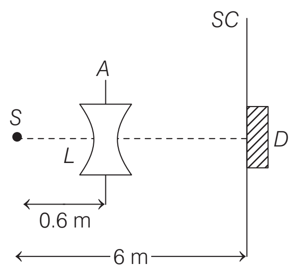
54. A neutron of kinetic energy 65 eV collides inelastically with a singly ionized helium atom at rest. It is scattered at an angle of 90° with respect to its original direction. **(1993, 9+1M)**

(a) Find the allowed values of the energy of the neutron and that of the atom after the collision.

(b) If the atom gets de-excited subsequently by emitting radiation, find the frequencies of the emitted radiation.

[Given : Mass of He atom = 4 × (mass of neutrons)
Ionization energy of H atom = 13.6 eV]

55. A monochromatic point source S radiating wavelength 6000 Å, with power 2 W, an aperture A of diameter 0.1 m and a large screen SC are placed as shown in figure. A photoemissive detector D of surface area 0.5 cm^2 is placed at the centre of the



screen. The efficiency of the detector for the photoelectron generation per incident photon is 0.9. **(1991, 2+4+2M)**

(a) Calculate the photon flux at the centre of the screen and the photocurrent in the detector.

(b) If the concave lens L of focal length 0.6 m is inserted in the aperture as shown, find the new values of photon flux and photocurrent. Assume a uniform average transmission of 80% from the lens.

(c) If the work function of the photoemissive surface is 1 eV, calculate the values of the stopping potential in the two cases (without and with the lens in the aperture).

56. Electrons in hydrogen-like atom ($Z = 3$) make transitions from the fifth to the fourth orbit and from the fourth to the third orbit. The resulting radiations are incident normally on a metal plate and eject photoelectrons. The stopping potential for the photoelectrons ejected by the shorter wavelength is 3.95 V. Calculate the work function of the metal, and the stopping potential for the photoelectrons ejected by the longer wavelength (Rydberg's constant = $1.094 \times 10^7 \text{ m}^{-1}$) **(1990, 7M)**

57. A gas of identical hydrogen-like atoms has some atoms in the lowest (ground) energy level A and some atoms in a particular upper (excited) energy level B and there are no atoms in any other energy level. The atoms of the gas make the transition to a higher energy level by absorbing monochromatic light of photon energy 2.7eV. Subsequently, the atoms emit radiation of only six different energy photons. Some of the emitted photons have an energy of 2.7 eV, some have more energy and some less than 2.7 eV. **(1989, 8M)**

(a) Find the principal quantum number of the initially excited level B.

(b) Find the ionization energy for the gas atoms.

(c) Find the maximum and the minimum energies of the emitted photons.

58. How many electrons, protons and neutrons are there in a nucleus of atomic number 11 and mass number 24?
Find, **(1982, 2M)**

(a) Number of electrons.
(b) Number of protons.
(c) Number of neutrons.

Answers

Topic 1

1. (a)	**2.** (b)	**3.** (c)	**4.** (b)
5. (d)	**6.** (d)	**7.** (c)	**8.** (c)
9. (c)	**10.** (a)		
11. (d)	**12.** (c)	**13.** (d)	**14.** (a)
15. (b)	**16.** (c)	**17.** (d)	**18.** (a)
19. (b)	**20.** (d)	**21.** (a)	**22.** (c)
23. (b)	**24.** (d)	**25.** (d)	**26.** (d)
27. (b)	**28.** (c)	**29.** (a)	**30.** (b)
31. (d)	**32.** (c)	**33.** (c)	**34.** (a)
35. (a, b, d)	**36.** (a, c)	**37.** (a, d)	**38.** (a, d)
39. 3			
40. 4.17	**41.** –1	**42.** (5)	**43.** 6
44. (2)	**45.** 0.55 eV	**46.** (a) $z = 3$ (b) 4052.3 nm	

47. $n = 2, Z = 4, -217.6$ eV, 10.58 eV

48. (a) 3.4 eV (b) 6.63 Å **49.** 6, 3

50. (a) $r_n = \dfrac{n^2 h^2 \varepsilon_0}{624\ \pi m_e e^2}$ (b) $n \approx 25$ (c) 0.546 Å

51. (a) 113.74 Å (b) 3 **52.** (a) 300 Å (b) 0.2645 Å

53. 6.6×10^{-34} J-s

54. Six, 1.875 μm

55. (a) 5 (b) 16.53 eV(c) 36.4 Å (d) 340 eV, – 680 eV, – 340 eV, $1.05 \times 10^{-34} \dfrac{\text{kg-m}^2}{\text{s}}$ (e) 1.06×10^{-11}m

Topic 2

1. (b)	**2.** (b)	**3.** (c)	**4.** (a)
5. (c)	**6.** (d)	**7.** (a)	**8.** (a)
9. (c)	**10.** (c)	**11.** (b)	**12.** (a)
13. (b)	**14.** (a)	**15.** (b)	**16.** (a)
17. (a)	**18.** (c)	**19.** (b)	**20.** (a, c)
21. (a, c)	**22.** (a, b, c)	**23.** (b, d)	**24.** (a, b, c)
25. (c, d)	**26.** 1	**27.** 7	**28.** Frequency
29. Question is incomplete		**30.** F	

32. (a) 5×10^7 (b) 2×10^3 N/C (c) 23 eV

33. During combination 3.4 eV. After combination 3.84 eV, 2.64 eV

34. (a) 10^5/s (b) 285.1 (d) 111 s

35. (a) 2.55 eV (b) $4 \to 2$ (c) $-\dfrac{h}{\pi}$ (d) 0.814 m/s

36. 1.1×10^{12}

Topic 3

1. (b)	**2.** (b)	**3.** (b)	**4.** (d)
5. (a)	**6.** (d)	**7.** (c)	**8.** (a)
9. (b)	**10.** (d)	**11.** (c)	**12.** (a)
13. (d)	**14.** (c)	**15.** (d)	**16.** (c)
17. (a)	**18.** (d)	**19.** (d)	**20.** (a)
21. (b)	**22.** (b)	**23.** (c)	**24.** (c)
25. (b)	**26.** (a,c)		
27. 5	**28.** 2	**29.** 4	**30.** 1
31. Neutrino	**32.** Lithium, 7	**33.** 8, 6	
34. 500 dPs, 125 dPs		**35.** 3.861	
36. 6.947 s			

37. (a) $\dfrac{dN_X}{dt} = -\lambda_X N_X, \dfrac{dN_Y}{dt} = \lambda_X N_X - \lambda_Y N_Y, \dfrac{dN_Z}{dt} = \lambda_Y N_Y$

(b) 16.48 s (c) $N_X = 1.92 \times 10^{19}$, $N_Z = 2.32 \times 10^{19}$

38. (a) $\frac{1}{\lambda}[\alpha - (\alpha - \lambda N_0)\, e^{-\lambda t}]$ (b) (i) $\frac{3}{2} N_0$ (ii) $2N_0$

39. (a) 14.43 s (b) 40 s **40.** 5.95 L

41. 3.96×10^{-6} **42.** (a) 91 (b) 234

Topic 4

1. (d) **2.** (b) **3.** (a) **4.** (d)
5. (a) **6.** (b)
7. (a) **8.** (d) **9.** (d) **10.** (b)
11. (b) **12.** (a) **13.** (a) **14.** (b)
15. (a) **16.** (a) **17.** (d) **18.** (c)
19. (b) **20.** (c) **21.** (c) **22.** (b)
23. (b) **24.** (a, d) **25.** 2 **26.** 7
27. 3 **28.** 39 **29.** 0.62 **30.** 3×10^8
31. 0.27 Å **32.** Intensity, decrease
33. 30×10^3, 30×10^3 **34.** $n = 24$
35. (a) 56 (b) 1.55×10^{18} Hz **36.** $\sqrt{2}$ **37.** 42
38. (a) 150.8 eV (b) 0.5 Å

Topic 5

1. (d) **2.** (b) **3.** (c) **4.** (d)
5. (*)
6. (c) **7.** (a) **8.** (a) **9.** (c)
10. (b) **11.** (a) **12.** (b) **13.** (c)
14. (b) **15.** (c) **16.** (d) **17.** (c)
18. (c) **19.** (a) **20.** (d) **21.** (a)
22. (b)
23. (A) $\rightarrow$ p, q (B) $\rightarrow$ p, r (C) $\rightarrow$ p, s (D) $\rightarrow$ p, q, r
24. (b, d) **25.** (c, d) **26.** (a, d) **27.** (c, d)
28. (b, c) **29.** Fusion, 24 **30.** 23.6 MeV **31.** F
32. 9 **33.** 3 **34.** 3.847×10^4kg
35. 3.32×10^{-5}W
36. (a) 232, 90 (b) 5.3 MeV, 1823.2 MeV
37. 120.26 g

Topic 6

1. (d) **2.** (d) **3.** (c) **4.** (c)
5. (a) **6.** (c) **7.** (c) **8.** (a)
9. (b) **10.** (a) **11.** (b) **12.** (c)
13. (d) **14.** (b) **15.** (c) **16.** (a)
17. (c)
18. (c) **19.** (c) **20.** (a) **21.** (a)
22. (a) **23.** (d) **24.** (b) **25.** (c)
26. (c) **27.** (b) **28.** (b) **29.** (c)
30. (b) **31.** (d) **32.** (a) **33.** (b, d)
34. (a, d) **35.** (a, c) **36.** (b, c) **37.** (b, c)
38. (b, d) **39.** Reverse **40.** B and D, A and C
41. Reverse, negative **42.** Positive, p-side, n-side
43. T **44.** 8kΩ, 12.5×10^{-3} A/V, 100

Topic 7

1. (b) **2.** (d) **3.** (c) **4.** (d)
5. (c) **6.** (a) **7.** (c) **8.** (d)
9. (a) **10.** (d) **11.** (b) **12.** (c)
13. (b) **14.** (b) **15.** (a) **16.** (b)
17. (a) **18.** (c) **19.** (d) **20.** (b)
21. (b) **22.** (d) **23.** (b) **24.** (a)
25. (c) **26.** (a) **27.** (d) **28.** (a)
29. (c) **30.** (d) **31.** (d)
32. (b) **33.** (a) **34.** (c) **35.** (b)
36. (c) **37.** (b) **38.** (a) **39.** (a)
40. (b) **41.** (24) **42.** (d) **43.** (c)
44. A$\rightarrow$ (R or RT), B $\rightarrow$ (PS) C$\rightarrow$ (Q, T), D $\rightarrow$ (R)
45. (b) **46.** (d) **47.** (c)
48. (A) $\rightarrow$ p, r (B) $\rightarrow$ p, q,s (C) $\rightarrow$ p (D) $\rightarrow$ q
49. 8 **50.** Atomic number, mass number
51. (a) 6.25 MeV, (b) 227.62 amu
52. 6.25×10^{11}, zero, 5.0 eV
53. (i) A(ii) F (iii) E (iv) C
54. (a) 6.36 eV, 0.312 eV (of neutron), 17.84 eV, 16.328 eV (of atom) (b) 1.82×10^{15}Hz, 11.67×10^{15} Hz, 9.84×10^{15}Hz
55. (a) 2.87×10^{13} s^{-1}m^{-2}, 2.07×10^{-10}A (b) 2.06×10^{13} s^{-1}m^{-2}, 1.483×10^{-10}A (c) 1.06 V in both cases
56. 2eV, 0.74 V
57. (a) 2 (b) 14.4 eV (c) 13.5 eV, 0.7 eV
58. Zero, 11, 13

Hints & Solutions

Topic 1 Bohr's Atomic Model

1. Wavelength λ of emitted photon as an electron transits from an initial energy level n_i to some final energy level n_f is given by Balmer's formula,

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

where, R = Rydberg constant.

In transition from $n = 4$ to $n = 3$, we have

$$\frac{1}{\lambda_1} = R\left(\frac{1}{3^2} - \frac{1}{4^2}\right)$$

$$= R\left(\frac{7}{9 \times 16}\right) \quad \ldots\text{(i)}$$

In transition from $n = 3$ to $n = 2$, we have

$$\frac{1}{\lambda_2} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

$$= R\left(\frac{5}{9 \times 4}\right) \quad \ldots\text{(ii)}$$

So, from Eqs. (i) and (ii), the ratio of $\frac{\lambda_1}{\lambda_2}$ is

$$\frac{\lambda_1}{\lambda_2} = \frac{\left(\frac{9 \times 16}{7R}\right)}{\left(\frac{9 \times 4}{5R}\right)} = \frac{20}{7}$$

2. Change in energy in transition from n to m stage is given by $(n > m)$,

$$E_n = -\frac{E_0 Z^2}{n^2}$$

Here, $Z = 2$

$$\Delta E_n = +13.6 \times 4\left[\frac{1}{m^2} - \frac{1}{n^2}\right] = \frac{hc}{\lambda} \quad \ldots\text{(i)}$$

Let it start from n to m and then m to ground.

So, in first case,

$$13.6 \times 4 \times \left(\frac{1}{m^2} - \frac{1}{n^2}\right) = \frac{hc}{108.5 \text{ nm}} \quad \ldots\text{(ii)}$$

and in second case,

$$13.6 \times 4 \times \left(\frac{1}{1^2} - \frac{1}{m^2}\right) = \frac{hc}{30.4 \text{ nm}}$$

$$\Rightarrow \quad \left(1 - \frac{1}{m^2}\right) = \frac{1240 \text{ eV}}{30.4 \times 13.6 \times 4} \quad \left(\text{Given, } E = \frac{1240 \text{ eV}}{\lambda \text{ (in nm)}}\right)$$

$$\Rightarrow \quad \left(1 - \frac{1}{m^2}\right) = 0.74980 \approx 0.75$$

or $$\frac{1}{m^2} = 1 - 0.75 = 0.25$$

$$\Rightarrow \quad m^2 = \frac{1}{0.25} = 4$$

Hence, $m = 2$

So, by putting the value of m in Eq. (ii), we get

$$13.6 \times 4 \times \left(\frac{1}{2^2} - \frac{1}{n^2}\right) = \frac{1240}{108.5} \text{ eV}$$

$$\Rightarrow \quad \left(\frac{1}{4} - \frac{1}{n^2}\right) = \frac{1240}{108.5 \times 13.6 \times 4}$$

$$\Rightarrow \quad \frac{1}{4} - \frac{1}{n^2} = 0.21$$

or $$\frac{1}{n^2} = 0.25 - 0.21 = 0.04$$

$$n^2 = \frac{1}{0.04} = 25$$

$$\Rightarrow \quad n^2 = 25$$

$$\Rightarrow \quad n = 5$$

3. Number of spectral lines produced as an excited electron falls to ground state $(n = 1)$ is,

$$N = \frac{n(n-1)}{2}$$

In given case, $N = 6$

$$\therefore \quad 6 = \frac{n(n-1)}{2}$$

$$\Rightarrow \quad n = 4$$

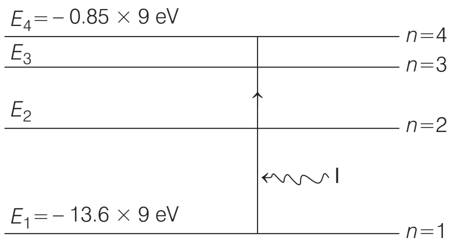
So, L^{++} electron is in it's 3rd excited state.

Now, using the expression of energy of an electron in nth energy level,

$$E_n = -\frac{13.6 Z^2}{n^2} \text{ eV}$$

where, Z is the atomic number.

$\therefore$ Energy levels of L^{++} electron are as shown



So, energy absorbed by electron from incident photon of wavelength λ is

$$\Delta E = \frac{hc}{\lambda} \Rightarrow (13.6 \times 9 - 0.85 \times 9) = \frac{hc}{\lambda}$$

$$\Rightarrow \quad \lambda = \frac{hc}{9(13.6 - 0.85)}$$

$$\Rightarrow \quad \lambda = \frac{1240 \text{ eV-nm}}{9 \times 12.75 \text{ eV}} = 10.8 \text{ nm}$$

4. Energy of a hydrogen atom like ion by Bohr's model is

$$E_n = -13.6\frac{Z^2}{n^2}$$

where, Z = atomic number
and n = principal quantum number.
For a He^+ ion in first excited state,

$n = 2$, $Z = 2$

$$\therefore \quad E_2 = -13.6 \times \frac{4}{4} = -13.6 \text{ eV}$$

So, it's ionisation energy $= -(E_2) = 13.6$ eV.

5. Expression for the energy of the hydrogenic electron states for atoms of atomic number Z is given by

$$E = h\nu = \frac{Z^2 me^4}{8h^2 E_0^2}\left[\frac{1}{m^2} - \frac{1}{n^2}\right] \qquad \text{Here, } m < n)$$

or $$\frac{hc}{\lambda} = \frac{Z^2 me^4}{8h^2 E_0^2}\left[\frac{1}{m^2} - \frac{1}{n^2}\right] \Rightarrow \frac{1}{\lambda} \propto \left(\frac{1}{m^2} - \frac{1}{n^2}\right) Z^2$$

For first case,

$\lambda = 660$ nm, $m = 2$ and $n = 3$

$$\therefore \quad \frac{1}{660 \text{ nm}} \propto \left[\frac{1}{(2)^2} - \frac{1}{(3)^2}\right] Z^2$$

$$\Rightarrow \quad \frac{1}{660 \text{ nm}} \propto \left(\frac{1}{4} - \frac{1}{9}\right) Z^2 \text{ or } \frac{5}{36} Z^2 \qquad \ldots\text{(i)}$$

For second case, transition is from $n = 4$ to $n = 2$, i.e. $m = 2$ and $n = 4$

$$\therefore \quad \frac{1}{\lambda} \propto \left(\frac{1}{(2)^2} - \frac{1}{(4)^2}\right) Z^2 \Rightarrow \frac{1}{\lambda} \propto \left(\frac{1}{4} - \frac{1}{16}\right) Z^2$$

or $$\frac{1}{\lambda} \propto \frac{3}{16} Z^2 \qquad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

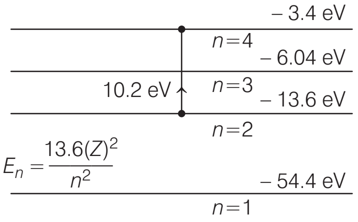
$$\frac{\lambda}{660 \text{ nm}} = \frac{5}{36} \times \frac{16}{3}$$

$$\Rightarrow \quad \lambda = \frac{80}{108} \times 660 \text{ nm} = 488.9 \text{ nm}$$

6. De-excitation energy of hydrogen electron in transition $n = 2$ to $n = 1$ is

$$E = 13.6 \times \left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \text{eV} = 13.6\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = 10.2 \text{ eV}$$

Now, energy levels of helium ion's (He^+) electron are
(For helium, $Z = 2$)



So, a photon of energy 10.2 eV can cause a transition $n = 2$ to $n = 4$ in a He^+ ion.

Alternate Solution

For He^+ ion, when in $n = 1$ state,

$$10.2 = 13.6 \times 2^2\left(\frac{1}{1^2} - \frac{1}{n^2}\right) \Rightarrow n = 1$$

Thus, no transition takes place.
Similarly, when in $n = 2$ state,

$$10.2 = 13.6 \times 2^2\left(\frac{1}{2^2} - \frac{1}{n^2}\right) \Rightarrow n = 4$$

7. As, for conservative fields $\mathbf{F} = -\left(\frac{d\mathbf{U}}{d\mathbf{r}}\right)$

$\therefore$ Magnitude of force on particle is

$$\Rightarrow \quad F = \frac{dU}{dr} = \frac{d}{dr}\left(\frac{1}{2}kr^2\right)$$

$$\Rightarrow \quad F = kr$$

This force is acting like centripetal force.

$$\therefore \quad \frac{mv^2}{r} = kr \qquad \ldots\text{(i)}$$

So, for n^{th} orbit,

$$\Rightarrow \quad m^2 v_n^2 = mkr_n^2$$

$$\Rightarrow \quad \frac{n^2h^2}{4\pi^2 r^2} = mkr_n^2 \qquad \left[\because v_n = \frac{nh}{2\pi mr}\right]$$

Therefore, $$r_n^4 \propto n^2$$

$$\Rightarrow \quad r_n^2 \propto n$$

So, $$r_n \propto \sqrt{n}$$

Energy of particle is

$$E_n = \text{PE} + \text{KE} = \frac{1}{2}kr_n^2 + \frac{1}{2}mv_n^2$$

$$= \frac{1}{2}kr_n^2 + \frac{1}{2}kr_n^2 \qquad \text{[using Eq. (i)]}$$

$$= kr_n^2$$

So, energy, $E_n \propto r_n^2$

$$\Rightarrow \quad E_n \propto n$$

8. For hydrogen or hydrogen like atoms, we know that

$$\frac{1}{\lambda} = RZ^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] \qquad \ldots\text{(i)}$$

where, R is Rydberg constant and Z is atomic number.
When electron jumps from M - shell to the L - shell, then

$n_1 = 2$ (for L - shell)

$n_2 = 3$ (for M - shell)

$\therefore$ Eq (i) becomes

$$\frac{1}{\lambda} = RZ^2\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = \frac{5}{36}RZ^2] \qquad \ldots\text{(ii)}$$

Now, electron jumps from N-shell to the L - shell, for this

$$n_1 = 2 \quad \text{(for } L \text{ - shell)}$$
$$n_2 = 4 \quad \text{(for } N \text{ - shell)}$$

$\therefore$ Eq. (i) becomes

$$\frac{1}{\lambda'} = RZ^2\left[\frac{1}{2^2} - \frac{1}{4^2}\right] = \frac{3}{16}RZ^2 \quad \ldots\text{(iii)}$$

Now, we divide Eq (ii) by Eq (iii)

$$\frac{\lambda'}{\lambda} = \left(\frac{5}{36}RZ^2\right) \div \left(\frac{3}{16}RZ^2\right) = \frac{20}{27}$$

or $$\lambda' = \frac{20}{27}\lambda$$

9. We know that net change in energy of a photon in a transition with wavelength λ is $\Delta E = hc/\lambda$.

Here, $hc = 12500\,\text{eV Å}$ and $\lambda = 980\,\text{Å}$

$$\Delta E = 12500/980 = 12.76\,\text{eV}$$
$$\Rightarrow \quad E_n - E_1 = 12.76\,\text{eV}$$

Since, the energy associated with an electron in n^{th} Bohr's orbit is given as,

$$E_n = \frac{-13.6}{n^2}\,\text{eV} \quad \ldots\text{(i)}$$
$$\Rightarrow \quad E_n = E_1 + 12.76\,\text{eV} \quad \ldots\text{(ii)}$$
$$= \frac{-13.6}{(1)^2} + 12.76 = -0.84$$

Putting this value in Eq. (i)

$$\Rightarrow \quad n^2 = \frac{-13.6}{-0.84} = 16 \Rightarrow n = 4$$

and radius of n^{th} orbit, $r_n = n^2 a_0 \Rightarrow r_n = 16\,a_0$

10. Lyman series ends at $n=1$

$\therefore$ Series limit frequency of the Lyman series is given by,

$$\nu_L = RcZ^2\left(\frac{1}{1^2} - \frac{1}{\infty^2}\right) \Rightarrow \nu_L = RcZ^2$$

Pfund series ends at $n=5$.

$\therefore$ Series limit frequency of the Pfund series,

$$\nu_P = RcZ^2\left(\frac{1}{5^2} - \frac{1}{\infty^2}\right) = \frac{RcZ^2}{25} \text{ or } \nu_p = \frac{\nu_L}{25}$$

11. The expressions of kinetic energy, potential energy and total energy are

$$K_n = \frac{me^4}{8\varepsilon_0^2 n^2 h^2} \quad \Rightarrow \quad K_n \propto \frac{1}{n^2}$$
$$U_n = \frac{-me^4}{4\varepsilon_0^2 n^2 h^2} \quad \Rightarrow \quad U_n \propto -\frac{1}{n^2}$$
and $$E_n = \frac{-me^4}{8\varepsilon_0^2 n^2 h^2} \quad \Rightarrow \quad E_n \propto -\frac{1}{n^2}$$

In the transition from some excited state to ground state value of n decreases, therefore kinetic energy increases, but potential and total energy decreases.

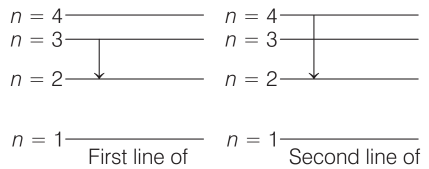
12. For hydrogen atom, we get

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right) \qquad \Rightarrow \frac{1}{\lambda_1} = R(1)^2\left(\frac{3}{4}\right)$$
$$\Rightarrow \frac{1}{\lambda_2} = R(1)^2\left(\frac{3}{4}\right) \qquad \Rightarrow \frac{1}{\lambda_3} = R(2)^2\left(\frac{3}{4}\right)$$
$$\Rightarrow \frac{1}{\lambda_4} = R(3)^2\left(\frac{3}{4}\right) \qquad \Rightarrow \frac{1}{\lambda_1} = \frac{1}{4\lambda_3} = \frac{1}{9\lambda_4} = \frac{1}{\lambda_2}$$

13. $\Delta E = h\nu$

$$\nu = \frac{\Delta E}{h} = k\left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right]$$
$$= \frac{k\,2n}{n^2(n-1)^2} \approx \frac{2k}{n^3} \propto \frac{1}{n^3}$$

14.



First line of Balmer series — Second line of Balmer series

For hydrogen or hydrogen type atoms,

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

In the transition from $n_i \longrightarrow n_f$

$$\therefore \quad \lambda \propto \frac{1}{Z^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)}$$

$$\therefore \quad \frac{\lambda_2}{\lambda_1} = \frac{Z_1^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)_1}{Z_2^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)_2}$$

$$\lambda_2 = \frac{\lambda_1 Z_1^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)_1}{Z_2^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)_2}$$

Substituting the values, we have

$$= \frac{(6561\,\text{Å})(1)^2\left(\frac{1}{2^2} - \frac{1}{3^2}\right)}{(2)^2\left(\frac{1}{2^2} - \frac{1}{4^2}\right)} = 1215\,\text{Å}$$

15. The series in U-V region is Lymen series. Longest wavelength corresponds to minimum energy which occurs in transition from $n = 2$ to $n = 1$.

$$\therefore \quad 122 = \frac{\frac{1}{R}}{\left(\frac{1}{1^2} - \frac{1}{2^2}\right)} \quad \ldots(i)$$

The smallest wavelength in the infrared region corresponds to maximum energy of Paschen series.

$$\therefore \quad \lambda = \frac{\frac{1}{R}}{\left(\frac{1}{3^2} - \frac{1}{\infty}\right)} \quad \ldots(ii)$$

Solving Eqs. (i) and (ii), we get

$$\lambda = 823.5 \text{ nm}$$

16. The first photon will excite the hydrogen atom (in ground state) to first excited state (as $E_2 - E_1 = 10.2$ eV). Hence, during de-excitation a photon of 10.2 eV will be released. The second photon of energy 15 eV can ionise the atom. Hence, the balance energy i.e.$(15 - 13.6)$ eV $= 1.4$ eV is retained by the electron. Therefore, by the second photon an electron of energy 1.4 eV will be released.

17. $$(r_m) = \left(\frac{m^2}{z}\right)(0.53 \text{ Å}) = (n \times 0.53) \text{ Å}$$

$$\therefore \quad \frac{m^2}{z} = n$$

$m = 5$ for ${}_{100}\text{Fm}^{257}$ (the outermost shell) and $z = 100$

$$\therefore \quad n = \frac{(5)^2}{100} = \frac{1}{4}$$

18. $$U = eV = eV_0 \ln\left(\frac{r}{r_0}\right)$$

$$|F| = \left|-\frac{dU}{dr}\right| = \frac{eV_0}{r}$$

This force will provide the necessary centripetal force. Hence, $\frac{mv^2}{r} = \frac{eV_0}{r}$

$$\text{or} \quad v = \sqrt{\frac{eV_0}{m}} \quad \ldots(i)$$

$$\text{Moreover,} \quad mvr = \frac{nh}{2\pi} \quad \ldots(ii)$$

Dividing Eq. (ii) by (i), we have

$$mr = \left(\frac{nh}{2\pi}\right)\sqrt{\frac{m}{eV_0}}$$

$$\text{or} \quad r_n \propto n$$

19. In second excited state $n = 3$,

$$\text{So,} \quad l_H = l_{Li} = 3\left(\frac{h}{2\pi}\right)$$

$$\text{while} \quad E \propto Z^2$$

$$\text{and} \quad Z_H = 1, Z_{Li} = 3$$

$$\text{So,} \quad |E_{Li}| = 9|E_H|$$

$$\text{or} \quad |E_H| < |E_{Li}|$$

20. Energy of infrared radiation is less than the energy of ultraviolet radiation. In options (a), (b) and (c), energy released will be more, while in option (d) only, energy released will be less.

21. $v_n \propto \frac{1}{n}$ $\therefore$ KE $\propto \frac{1}{n^2}$ (with positive sign)

Potential energy U is negative and $U_n \propto \frac{1}{r_n}$

$$\left[U_n = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{Ze^2}{r_n}\right]$$

$$E_n \propto \frac{1}{n^2} \qquad \text{(because } r_n \propto n^2\text{)}$$

Similarly, total energy $E_n \propto \frac{1}{n^2}$. (with negative sign)

Therefore, when an electron jumps from some excited state to the ground state, value of n will decrease. Therefore, kinetic energy will increase (with positive sign), potential energy and total energy will also increase but with negative sign.
Thus, finally kinetic energy will increase, while potential and total energies will decrease.

NOTE

- For hydrogen and hydrogen-like atoms

$$E_n = -13.6\frac{Z^2}{n^2} \text{ eV}$$

$$U_n = 2E_n = -27.2\frac{Z^2}{n^2} \text{ eV} \textit{ and } K_n = |E_n| = 13.6\frac{Z^2}{n^2} \text{ eV}$$

From these three relations we can see that as n decreases, K_n will increase but E_n and U_n will decrease.

- As an electron comes closer to the nucleus, the electrostatic force (which provides the necessary centripetal force) increases or speed (or KE) of the electron increases.

22. In hydrogen atom, $E_n = -\frac{Rhc}{n^2}$. Also, $E_n \propto m$

where, m is the mass of the electron.

Here, the electron has been replaced by a particle whose mass is double of an electron. Therefore, for this hypothetical atom energy in n^{th} orbit will be given by

$$E_n = -\frac{2Rhc}{n^2}$$

The longest wavelength $\lambda_{\max}$ (or minimum energy) photon will correspond to the transition of particle from $n = 3$ to $n = 2$.

$$\therefore \quad \frac{hc}{\lambda_{\max}} = E_3 - E_2 = 2Rhc\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

This gives, $\lambda_{\max} = 18/5R$

23. Since, the wavelength (λ) is increasing, we can say that the galaxy is receding. Doppler effect can be given by

$$\lambda' = \lambda\sqrt{\frac{c+v}{c-v}} \quad \ldots(i)$$

or $$706 = 656\sqrt{\frac{c+v}{c-v}}$$

or $$\frac{c+v}{c-v} = \left(\frac{706}{656}\right)^2 = 1.16$$

$\therefore$ $$c + v = 1.16\,c - 1.16v$$

$\therefore$ $$v = \frac{0.16c}{2.16} = \frac{0.16 \times 3.0 \times 10^8}{2.16}$$
$$= 0.22 \times 10^8 \text{ m/s}$$
$$v \approx 2.2 \times 10^7 \text{ m/s}$$

If we take the approximation then Eq. (i) can be written as

$$\Delta\lambda = \lambda\left(\frac{v}{c}\right) \qquad \ldots\text{(ii)}$$

From here $v = \left(\frac{\Delta\lambda}{\lambda}\right)\cdot c = \left(\frac{706-656}{656}\right)(3 \times 10^8)$

$$v = 0.23 \times 10^8 \text{ m/s}$$

which is almost equal to the previous answer. So, we may use Eq. (ii) also.

24. For hydrogen and hydrogen like atoms

$$E_n = -13.6\frac{(Z^2)}{(n^2)}\text{ eV}$$

Therefore, ground state energy of doubly ionised lithium atom ($Z = 3$, $n = 1$) will be

$$E_1 = (-13.6)\frac{(3)^2}{(1)^2} = -122.4 \text{ eV}$$

$\therefore$ Ionisation energy of an electron in ground state of doubly ionised lithium atom will be 122.4 eV.

25. Shortest wavelength will correspond to maximum energy. As value of atomic number (Z) increases, the magnitude of energy in different energy states gets increased. Value of Z is maximum for doubly ionised lithium atom ($Z = 3$) among the given elements. Hence, wavelength corresponding to this will be least.

26. $L = I\omega = \frac{nh}{2\pi}$

$\therefore$ $$\omega = \frac{nh}{2\pi I}$$

$$K = \frac{1}{2}I\omega^2 = \frac{1}{2}I\left(\frac{nh}{2\pi I}\right)^2 = \frac{n^2h^2}{8\pi^2 I}$$

27. $h\nu = K_2 - K_1 = \frac{3h^2}{8\pi^2 I}$

$\therefore$ $$I = \frac{3h}{8\pi^2 f} = \frac{3 \times 2\pi \times 10^{-34} \times \pi}{8 \times \pi^2 \times 4 \times 10^{11}}$$
$$= 1.87 \times 10^{-46} \text{ kgm}^2$$

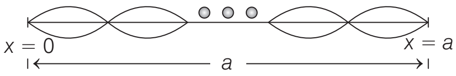
28. $I = \mu r^2$ (where, μ = reduced mass)

$$\mu = \frac{m_1 m_2}{m_1 + m_2} = \frac{48}{7}\text{ amu} = 11.43 \times 10^{-27} \text{ kg}$$

Substituting in $I = \mu r^2$ we get,

$$r = \sqrt{\frac{I}{\mu}} = \sqrt{\frac{1.87 \times 10^{-46}}{11.43 \times 10^{-27}}}$$
$$= 1.28 \times 10^{-10} \text{ m}$$

29.



$$a = \frac{n\lambda}{2}$$

$\therefore$ $$\lambda = \frac{2a}{n} = \frac{h}{p} = \frac{h}{\sqrt{2Em}} \qquad \ldots\text{(i)}$$

or $\sqrt{E} \propto \frac{1}{a}$ $\Rightarrow$ $\therefore$ $E \propto \frac{1}{a^2}$

30. From Eq. (i) $E = \frac{n^2h^2}{8a^2m}$

In ground state $n = 1$

$\therefore$ $$E_1 = \frac{h^2}{8ma^2}$$

Substituting the values, we get

$$E_1 = 8 \text{ meV}$$

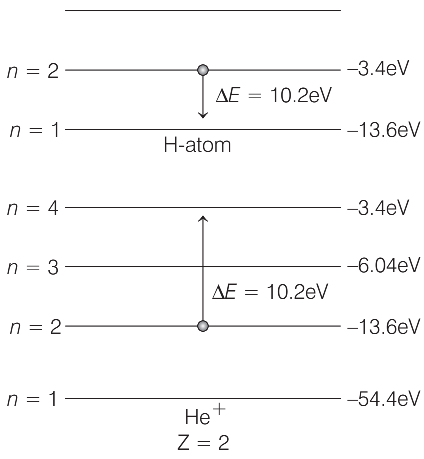
31. From Eq. (i)

$$p \propto n$$

$\therefore$ $$mv \propto n$$

or $$v \propto n$$

32.



Energy given by H-atom in transition from $n = 2$ to $n = 1$ is equal to energy taken by He^+ atom in transition from $n = 2$ to $n = 4$.

33. Visible light lies in the range, $\lambda_1 = 4000\text{Å}$ to $\lambda_2 = 7000\text{Å}$. Energy of photons corresponding to these wavelengths (in eV) would be :

$$E_1 = \frac{12375}{4000} = 3.09 \text{ eV}$$

$$E_2 = \frac{12375}{7000} = 1.77 \text{ eV}$$

From energy level diagram of He^+ atom, we can see that in transition from $n = 4$ to $n = 3$, energy of photon released will lie between E_1 and E_2.

$$\Delta E_{43} = -3.4 - (-6.04)$$
$$= 2.64 \text{ eV}$$

Wavelength of photon corresponding to this energy,

$$\lambda = \frac{12375}{2.64} \text{Å} = 4687.5 \text{ Å}$$
$$= 4.68 \times 10^{-7} \text{m}$$

34. Kinetic energy $K \propto Z^2$

$$\therefore \quad \frac{K_H}{K_{He^+}} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

35. As radius $\quad r \propto \dfrac{n^2}{z}$

$$\Rightarrow \quad \frac{\Delta r}{r} = \frac{\left(\frac{n+1}{z}\right)^2 - \left(\frac{n}{z}\right)^2}{\left(\frac{n}{z}\right)^2} = \frac{2n+1}{n^2} \approx \frac{2}{n} \propto \frac{1}{n}$$

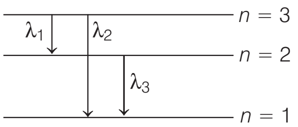
Energy, $\quad E \propto \dfrac{z^2}{n^2}$

$$\Rightarrow \quad \frac{\Delta E}{E} = \frac{\frac{z^2}{n^2} - \frac{z^2}{(n-1)^2}}{\frac{z^2}{(n+1)^2}}$$
$$= \frac{(n+1)^2 - n^2}{n^2 \cdot (n+1)^2} \cdot (n+1)^2$$
$$\Rightarrow \quad \frac{\Delta E}{E} = \frac{2n+1}{n^2} \simeq \frac{2n}{n^2} \propto \frac{1}{n}$$

Angular momentum, $L = \dfrac{nh}{2\pi}$

$$\Rightarrow \quad \frac{\Delta L}{L} = \frac{\frac{(n+1)}{2\pi} - \frac{nh}{2\pi}}{\frac{nh}{2\pi}} = \frac{1}{n} \propto \frac{1}{n}$$

36. $L = 3\left(\dfrac{h}{2\pi}\right)$



$$\therefore \quad n = 3, \text{as } L = n\left(\frac{h}{2\pi}\right)$$
$$r_n \propto \frac{n^2}{z}$$
$$r_3 = 4.5 a_0$$
$$\therefore \quad z = 2$$

$$\frac{1}{\lambda_1} = Rz^2\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = 4R\left(\frac{1}{4} - \frac{1}{9}\right)$$
$$\therefore \quad \lambda_1 = \frac{9}{5R}$$
$$\frac{1}{\lambda_2} = Rz^2\left(\frac{1}{1^2} - \frac{1}{3^2}\right) = 4R\left(1 - \frac{1}{9}\right)$$
$$\Rightarrow \quad \lambda_2 = \frac{9}{32R}$$
$$\frac{1}{\lambda_3} = Rz^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = 4R\left(1 - \frac{1}{4}\right)$$
$$\Rightarrow \quad \lambda_3 = \frac{1}{3R}$$

37. Time period, $T_n = \dfrac{2\pi r_n}{v_n}$ (in nth state)

i.e. $\quad T_n \propto \dfrac{r_n}{v_n}$

But $\quad r_n \propto n^2$

and $\quad v_n \propto \dfrac{1}{n}$

Therefore, $\quad T_n \propto n^3$

Given $\quad T_{n_1} = 8 T_{n_2}$

Hence, $\quad n_1 = 2n_2$

38. $r_n \propto \dfrac{n^2}{Z}$ and $|PE| = 2\,(KE)$

39.
$$\Delta E_{2-1} = 13.6 \times Z^2 \left[1 - \frac{1}{4}\right] = 13.6 \times Z^2 \left[\frac{3}{4}\right]$$
$$\Delta E_{3-2} = 13.6 \times Z^2 \left[\frac{1}{4} - \frac{1}{9}\right] = 13.6 \times Z^2 \left[\frac{5}{36}\right]$$
$$\therefore \quad \Delta E_2 = \Delta E_{3-2} + 74.8$$
$$13.6 \times Z^2 \left[\frac{3}{4}\right] = 13.6 \times Z^2 \left[\frac{5}{36}\right] + 74.8$$
$$13.6 \times Z^2 \left[\frac{3}{4} - \frac{5}{36}\right] = 74.8$$
$$Z^2 = 9$$
$$\therefore \quad Z = 3$$

40. From conservation of linear momentum,

|Momentum of recoil hydrogen atom| = |Momentum of emitted photon|

or $\quad mv = \dfrac{\Delta E}{c}$

Here, $\Delta E = E_5 - E_1 = -13.6\left[\dfrac{1}{5^2} - \dfrac{1}{1^2}\right]$ eV

$$= (13.6)\,(24/25) \text{ eV} = 13.056 \text{ eV}$$
$$= 13.056 \times 1.6 \times 10^{-19} \text{ J} = 2.09 \times 10^{-18} \text{ J}$$

and m = mass of hydrogen atom $= 1.67 \times 10^{-27}$ kg

$$\therefore \quad v = \frac{\Delta E}{mc} = \frac{2.09 \times 10^{-18}}{(1.67 \times 10^{-27})(3 \times 10^{8})},$$

$$v \approx 4.17 \text{ m/s}$$

41. Kinetic energy of an electron in nth orbit of hydrogen atom is $K = \frac{me^4}{8\varepsilon_0^2 h^2 n^2}$ and total energy of electron in nth orbit is

$$E = -\frac{me^4}{8\varepsilon_0^2 h^2 n^2}, \quad \frac{K}{E} = -1$$

or $\quad K = -E$

42. Potential energy of hydrogen atom ($Z = 1$) in nth orbit (in eV)

$$\text{PE} = -\frac{27.2}{n^2}$$

$$\frac{v_f}{v_i} = \frac{-\frac{27.2}{n_f^2}}{-\frac{27.2}{n_i^2}} = \frac{1}{6.25}$$

$$6.25 = \frac{n_f^2}{n_i^2}$$

$$\frac{n_f}{n_i} = 2.5 = \frac{5}{2}$$

Hence the answer is 5.

43. Energy of incident light (in eV)

$$E = \frac{12375}{970} = 12.7 \text{eV}$$

After excitation, let the electron jumps to nth state, then

$$\frac{-13.6}{n^2} = -13.6 + 12.7$$

Solving this equation, we get

$$n = 4$$

$\therefore$ Total number of lines in emission spectrum,

$$= \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$$

44. Kinetic energy of ejected electron

= Energy of incident photon − energy required to ionize the electron from nth orbit (all in eV)

$$\therefore \quad 10.4 = \frac{1242}{90} - |E_n|$$

$$= \frac{1242}{90} - \frac{13.6}{n^2} \qquad \left(\text{as } E_n \propto \frac{1}{n^2} \text{ and } E_1 = -13.6 \text{ eV}\right)$$

Solving this equation, we get

$$n = 2$$

45. Wavelengths corresponding to minimum wavelength ($\lambda_{\min}$) or maximum energy will emit photoelectrons having maximum kinetic energy. ($\lambda_{\min}$) belonging to Balmer series and lying in the given range (450 nm to 750 nm) corresponds to transition from ($n = 4$ to $n = 2$). Here,

$$E_4 = -\frac{13.6}{(4)^2} = -0.85 \text{ eV}$$

and $$E_2 = -\frac{13.6}{(2)^2} = -3.4 \text{ eV}$$

$$\therefore \quad \Delta E = E_4 - E_2 = 2.55 \text{ eV}$$

$$K_{\max} = \text{Energy of photon} - \text{work function}$$

$$= 2.55 - 2.0 = 0.55 \text{ eV}$$

46. (a) Total 6 lines are emitted. Therefore,

$$\frac{n(n-1)}{2} = 6 \quad \text{or} \quad n = 4$$

So, transition is taking place between m^{th} energy state and $(m+3)^{\text{th}}$ energy state.

$$E_m = -0.85 \text{ eV}$$

or $$-13.6\left(\frac{z^2}{m^2}\right) = -0.85$$

or $$\frac{z}{m} = 0.25 \qquad \ldots\text{(i)}$$

Similarly, $$E_{m+3} = -0.544 \text{ eV}$$

or $$-13.6 \frac{z^2}{(m+3)^2} = -0.544$$

or $$\frac{z}{(m+3)} = 0.2 \qquad \ldots\text{(ii)}$$

Solving Eqs. (i) and (ii) for z and m, we get

$$m = 12 \text{ and } z = 3$$

(b) Smallest wavelength corresponds to maximum difference of energies which is obviously $E_{m+3} - E_m$

$$\therefore \quad \Delta E_{\max} = -0.544 - (-0.85) = 0.306 \text{ eV}$$

$$\therefore \quad \lambda_{\min} = \frac{hc}{\Delta E_{\max}}$$

$$= \frac{1240}{0.306} = 4052.3 \text{ nm.}$$

47. Let ground state energy (in eV) be E_1.

Then, from the given condition

$$E_{2n} - E_1 = 204 \text{ eV}$$

or $$\frac{E_1}{4n^2} - E_1 = 204 \text{ eV}$$

or $$E_1\left(\frac{1}{4n^2} - 1\right) = 204 \text{ eV} \qquad \ldots\text{(i)}$$

and $$E_{2n} - E_n = 40.8 \text{ eV}$$

or $$\frac{E_1}{4n^2} - \frac{E_1}{n^2} = 40.8 \text{ eV}$$

or $$E_1\left(\frac{-3}{4n^2}\right) = 40.8 \text{ eV} \qquad \ldots\text{(ii)}$$

From Eqs. (i) and (ii),

$$\frac{1-\frac{1}{4n^2}}{\frac{3}{4n^2}} = 5$$

or $$1 = \frac{1}{4n^2} + \frac{15}{4n^2} \quad \text{or} \quad \frac{4}{n^2} = 1$$

or $$n = 2$$

From Eq. (ii),

$$E_1 = -\frac{4}{3} n^2 \,(40.8)\,\text{eV}$$

$$= -\frac{4}{3} (2)^2 \,(40.8)\,\text{eV}$$

or $$E_1 = -217.6\,\text{eV}$$

$$E_1 = -(13.6)\,Z^2$$

$\therefore$ $$Z^2 = \frac{E_1}{-13.6} = \frac{-217.6}{-13.6} = 16$$

$\therefore$ $$Z = 4$$

$$E_{\min} = E_{2n} - E_{2n-1}$$

$$= \frac{E_1}{4n^2} - \frac{E_1}{(2n-1)^2}$$

$$= \frac{E_1}{16} - \frac{E_1}{9} = -\frac{7}{144} E_1$$

$$= -\left(\frac{7}{144}\right)(-217.6)\,\text{eV}$$

$\therefore$ $$E_{\min} = 10.58\,\text{eV}$$

48. (a) Kinetic energy of electron in the orbits of hydrogen and hydrogen like atoms $= |\text{Total energy}|$

$\therefore$ Kinetic energy = 3.4 eV

(b) The de-Broglie wavelength is given by

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2Km}}$$

Here, K = kinetic energy of electron

Substituting the values, we have

$$\lambda = \frac{(6.6 \times 10^{-34}\,\text{J-s})}{\sqrt{2(3.4 \times 1.6 \times 10^{-19}\,\text{J})\,(9.1 \times 10^{-31}\,\text{kg})}}$$

$$\lambda = 6.63 \times 10^{-10}\,\text{m}$$

or $$\lambda = 6.63\,\text{Å}$$

49. From the given conditions

$$E_n - E_2 = (10.2 + 17)\,\text{eV} = 27.2\,\text{eV} \quad \text{...(i)}$$

and $$E_n - E_3 = (4.25 + 5.95)\,\text{eV} = 10.2\,\text{eV} \quad \text{...(ii)}$$

Eq. (i) – Eq. (ii) gives

$$E_3 - E_2 = 17.0\,\text{eV}$$

or $$Z^2\,(13.6)\left(\frac{1}{4} - \frac{1}{9}\right) = 17.0$$

$\Rightarrow$ $$Z^2\,(13.6)\,(5/36) = 17.0$$

$\Rightarrow$ $$Z^2 = 9 \quad \text{or} \quad Z = 3$$

From Eq. (i) $$Z^2\,(13.6)\left(\frac{1}{4} - \frac{1}{n^2}\right) = 27.2$$

or $$(3)^2\,(13.6)\left(\frac{1}{4} - \frac{1}{n^2}\right) = 27.2$$

or $$\frac{1}{4} - \frac{1}{n^2} = 0.222$$

or $$1/n^2 = 0.0278 \quad \text{or} \quad n^2 = 36$$

$\therefore$ $$n = 6$$

50. If we assume that mass of nucleus >> mass of mu-meson, then nucleus will be assumed to be at rest, only mu-meson is revolving round it.

(a) In nth orbit the necessary centripetal force to the mu-meson will be provided by the electrostatic force between the nucleus and the mu-meson.

Hence,

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0} \frac{(Ze)\,(e)}{r^2} \quad \text{...(i)}$$

Further, it is given that Bohr model is applicable to this system also. Hence

$$\text{Angular momentum in } n^{\text{th}} \text{ orbit} = \frac{nh}{2\pi}$$

or $$mvr = n\frac{h}{2\pi} \quad \text{...(ii)}$$

We have two unknowns v and r (in n^{th} orbit). After solving these two equations, we get

$$r = \frac{n^2 h^2 \varepsilon_0}{Z\pi m e^2}$$

Substituting $Z = 3$ and $m = 208\,m_e$, we get

$$r_n = \frac{n^2 h^2 \varepsilon_0}{624\pi m_e e^2}$$

(b) The radius of the first Bohrs orbit for the hydrogen atom is $\frac{h^2 \varepsilon_0}{\pi m_e e^2}$.

Equating this with the radius calculated in part (a), we get

$$n^2 \approx 624 \quad \text{or} \quad n \approx 25$$

(c) Kinetic energy of atom $= \frac{mv^2}{2} = \frac{Ze^2}{8\pi\varepsilon_0 r}$

and potential energy $= -\frac{Ze^2}{4\pi\varepsilon_0 r}$

$\therefore$ Total energy $E_n = \frac{-Ze^2}{8\pi\varepsilon_0 r}$

Substituting value of r, calculated in part (a),

$$E_n = \frac{1872}{n^2}\left[-\frac{m_e e^4}{8\varepsilon_0^2 h^2}\right]$$

But $\left[-\frac{m_e e^4}{8\varepsilon_0^2 h^2}\right]$ is the ground state energy of hydrogen atom and hence is equal to – 13.6 eV.

$$\therefore \quad E_n = \frac{-1872}{n^2}(13.6)\,\text{eV} = -\frac{25459.2}{n^2}\text{eV}$$

$$\therefore \quad E_3 - E_1 = -25459.2\left[\frac{1}{9}-\frac{1}{1}\right] = 22630.4 \text{ eV}$$

∴ The corresponding wavelength,

$$\lambda\,(\text{in Å}) = \frac{12375}{22630.4} = 0.546 \text{ Å}$$

51. Given $Z = 3$, $E_n \propto \frac{Z^2}{n^2}$

(a) To excite the atom from $n = 1$ to $n = 3$, energy of photon required is

$$E_{1-3} = E_3 - E_1 = \frac{(-13.6)(3)^2}{(3)^2} - \left[\frac{(-13.6)(3)^2}{(1)^2}\right]$$
$$= 108.8 \text{ eV}$$

Corresponding wavelength will be,

$$\lambda\,(\text{in Å}) = \frac{12375}{E(\text{in eV})} = \frac{12375}{108.8} = 113.74 \text{ Å}$$

(b) From n^{th} orbit total number of emission lines can be $\frac{n(n-1)}{2}$.

$$\therefore \text{ Number of emission lines} = \frac{3(3-1)}{2} = 3$$

52. (a) 1 rydberg $= 2.2 \times 10^{-18}$ J $= Rhc$

Ionisation energy is given as 4 rydberg

$$= 8.8 \times 10^{-18}\text{J} = \frac{8.8\times 10^{-18}}{1.6 \times 10^{-19}} = 55 \text{ eV}$$

∴ Energy in first orbit $E_1 = -55$ eV

Energy of radiation emitted when electron jumps from first excited state ($n = 2$) to ground state ($n = 1$):

$$E_{21} = \frac{E_1}{(2)^2} - E_1 = -\frac{3}{4}E_1 = 41.25 \text{ eV}$$

∴ Wavelength of photon emitted in this transition would be

$$\lambda = \frac{12375}{41.25} = 300 \text{ Å}$$

(b) Let Z be the atomic number of given element. Then

$$E_1 = (-13.6)(Z^2)$$

or $\quad -55 = (-13.6)(Z^2)$

or $\quad Z \approx 2$

Now, as $\quad r \propto \frac{1}{Z}$

Radius of first orbit of this atom,

$$r_1 = \frac{r_{H_1}}{Z} = \frac{0.529}{2} = 0.2645 \text{ Å}$$

53. When 800 Å wavelength falls on hydrogen atom (in ground state) 13.6 eV energy is used in liberating the electron. The rest goes to kinetic energy of electron.

Hence, $K = E - 13.6$ (in eV) or

$$(1.8 \times 1.6 \times 10^{-19}) = \frac{hc}{800 \times 10^{-10}} - 13.6 \times 1.6 \times 10^{-19} \quad \ldots\text{(i)}$$

Similarly, for the second wavelength :

$$(4.0 \times 1.6 \times 10^{-19}) = \frac{hc}{700 \times 10^{-10}} - 13.6 \times 1.6 \times 10^{-19} \quad \ldots\text{(ii)}$$

Solving these two equations, we get

$$h \approx 6.6 \times 10^{-34} \text{ J-s}$$

54. Energy corresponding to given wavelength

$$E\,(\text{in eV}) = \frac{12375}{\lambda\,(\text{in Å})} = \frac{12375}{975} = 12.69 \text{ eV}$$

Now, let the electron excites to n^{th} energy state. Then,

$$E_n - E_1 = 12.69 \text{ or } \frac{(-13.6)}{(n^2)} - (-13.6) = 12.69$$

$$\therefore \quad n \approx 4$$

i.e. electron excites to 4th energy state. Total number of lines in emission spectrum would be

$$\frac{n(n-1)}{2} = \frac{4 \times 3}{2} = 6$$

Longest wavelength will correspond to the minimum energy and minimum energy is released in transition from $n = 4$ to $n = 3$.

$$E_{4-3} = E_4 - E_3 = \frac{-13.6}{(4^2)} - \left[\frac{-13.6}{(3)^2}\right] = 0.66 \text{ eV}$$

∴ Longest wavelength will be,

$$\lambda_{\text{max}} = \frac{12375}{E\,(\text{in eV})}$$
$$= \frac{12375}{0.66}\text{Å} = 1.875 \times 10^{-6}\text{m} = 1.875\,\mu\text{m}$$

55. (a) Given, $E_3 - E_2 = 47.2$ eV

Since $\quad E_n \propto \frac{Z^2}{n^2}$ (for hydrogen like atoms)

or $\quad (-13.6)\left(\frac{Z^2}{9}\right) - \left[(-13.6)\left(\frac{Z^2}{4}\right)\right] = 47.2$

Solving this equation, we get

$$Z = 5$$

(b) Energy required to excite the electron from 3rd to 4th orbit :

$$E_{3-4} = E_4 - E_3$$
$$= (-13.6)\left(\frac{25}{16}\right) - \left[(-13.6)\left(\frac{25}{9}\right)\right] = 16.53 \text{ eV}$$

(c) Energy required to remove the electron from first orbit to infinity (or the ionisation energy) will be

$$E = (13.6)(5)^2 = 340 \text{ eV}$$

The corresponding wavelength would be,

$$\lambda = \frac{hc}{E} = \frac{6.6\times10^{-34}\times3\times10^{8}}{340\times1.6\times10^{-19}}$$

$$= 0.0364\times10^{-7} \text{ m} = 36.4 \text{ Å}$$

(d) In first orbit, total energy $= -340$ eV

kinetic energy $= +340$ eV

Potential energy $= -2\times340$ eV $= -680$ eV and

$$\text{angular momentum} = \frac{h}{2\pi} = \frac{6.6\times10^{-34}}{2\pi}$$

$$= 1.05\times10^{-34} \text{ kg-m}^2\text{/s}$$

(e) $r_n \propto \frac{n^2}{Z}$

Radius of first Bohrs orbit

$$r_1 = \frac{r_1^H}{Z} = \frac{5.3\times10^{-11}}{5}$$

$$= 1.06\times10^{-11} \text{ m}$$

Topic 2 Photo Electric Effect

1. Given,

Planck's constant,

$$h = 6.63\times10^{-34} \text{ J-s}$$

$$e = 1.6\times10^{-19} \text{ C}$$

and there is a graph between stopping potential and frequency.

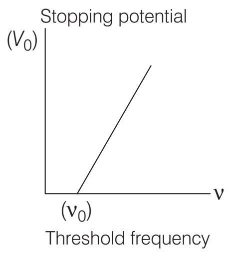
We need to determine work function W.

Using Einstein's relation of photoelectric effect,

$$(\text{KE})_{\max} = eV_0 = h\nu - h\nu_0 = h\nu - W \quad [\because W = h\nu_0]$$

or $$V_0 = \frac{h}{e}\nu - \frac{W}{e}$$

From graph at $V_0 = 0$ and $\nu = 4\times10^{14}$ Hz



$\therefore$ $$0 = \frac{6.63\times10^{-34}}{e}\times4\times10^{14} - \frac{W}{e}$$

$\Rightarrow$ $$\frac{W}{e} = \frac{6.63\times10^{-34}\times4\times10^{14}}{e} \text{ J}$$

or $$W = 6.63\times4\times10^{-20} \text{ J}$$

or $$W = \frac{6.63\times4\times10^{-20}}{1.6\times10^{-19}} \text{ eV} = 1.657 \text{ eV}$$

$\therefore$ $$W = 1.66 \text{ eV}$$

Alternate Solution

From graph, threshold frequency,

$$\nu_0 = 4\times10^{14} \text{ Hz (where, } V_0 = 0)$$

$\therefore$ Work function, $W = h\nu_0$

$\Rightarrow$ $$W = 6.63\times10^{-34}\times4\times10^{14} \text{ J}$$

$\Rightarrow$ $$W = \frac{6.63\times4\times10^{-20}}{1.6\times10^{-19}} \text{ eV} = 1.657 \text{ eV} \approx 1.66 \text{ eV}$$

2. Power of laser is given as

$$P = \frac{\text{Energy}}{\text{Time}}$$

$$= \frac{\text{Number of photons emitted} \times \text{Energy of one photon}}{\text{Time}}$$

$\Rightarrow$ $$P = \frac{NE}{t} = \left(\frac{N}{t}\right)\cdot E$$

So, number of photons emitted per second

$$= \frac{N}{t} = \frac{P}{E}$$

$$= \frac{P}{hc/\lambda} = \frac{P\lambda}{hc} \qquad \left[\because E = h\nu = \frac{hc}{\lambda}\right]$$

Here, $h = 6.6\times10^{-34}$ J-s, $\lambda = 500 \text{ nm} = 500\times10^{-9}$ m

$$c = 3\times10^{8} \text{ ms}^{-1}$$

$$P = 2 \text{ mW} = 2\times10^{-3} \text{ W}$$

$\therefore$ $$\frac{N}{t} = \frac{2\times10^{-3}\times500\times10^{-9}}{6.6\times10^{-34}\times3\times10^{8}}$$

$$= 5.56\times10^{15}$$

$$\approx 5\times10^{15} \text{ photons per second}$$

3. Given, threshold wavelength, $\lambda_0 = 380$ nm

Wavelength of incident light, $\lambda = 260$ nm

Using Einstein's relation of photoelectric effect,

$$(\text{KE})_{\max} = eV_0 = h\nu - h\nu_0 \quad \text{...(i)}$$

But $$h\nu = E = \frac{1237}{\lambda(\text{nm})} \text{ eV}$$

$\therefore$ $$E_0 = \frac{1237}{\lambda_0(\text{nm})} \text{ eV} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$(\text{KE})_{\max} = E - E_0 \left(\frac{1237}{\lambda} - \frac{1237}{\lambda_0}\right) \text{ eV}$$

$$= 1237\left[\frac{1}{\lambda} - \frac{1}{\lambda_0}\right] \text{ eV } (\lambda \text{ in nm}) \quad \text{...(iii)}$$

By putting values of λ and λ_0 in Eq. (iii), we get

$$(\text{KE})_{\max} = 1237\left(\frac{1}{260}-\frac{1}{380}\right)\text{eV}$$

$$= 1237\times\left[\frac{380-260}{380\times 260}\right]\text{eV}$$

$\Rightarrow (\text{KE})_{\max} = 1.5$ eV

4. Given, $\mathbf{E} = 10^{-3}\cos\left(\dfrac{2\pi x}{5\times10^{-7}} - 2\pi\times6\times10^{14}t\right)\hat{\mathbf{x}}\text{NC}^{-1}$

By comparing it with the general equation of electric field of light, i.e.

$E = E_0\cos(kx-\omega t)\,\hat{\mathbf{x}}$, we get

$$k = \frac{2\pi}{5\times10^{-7}} = 2\pi/\lambda$$

(from definition, $k = 2\pi/\lambda$)

$$\Rightarrow \quad \lambda = 5\times10^{-7}\text{ m} = 5000\text{ Å} \quad \ldots\text{(i)}$$

Or

The value of λ can also be calculated as, after comparing the given equation of $\mathbf{E}$ with standard equation, we get

$$\omega = 6\times10^{14}\times2\pi$$

$$\Rightarrow \quad \nu = 6\times10^{14} \quad [\because 2\pi\nu=\omega]$$

As, $\quad c = \nu\lambda$

$$\Rightarrow \quad \lambda = \frac{c}{\nu} = \frac{3\times10^8}{6\times10^{14}} = 5\times10^{-7}\text{ m} = 5000\text{ Å}$$

According to Einstein's equation for photoelectric effect, i.e.,

$$\frac{hc}{\lambda} - \phi = (\text{KE})_{\max} = eV_0 \quad \ldots\text{(ii)}$$

For photon, substituting the given values,

$$E = \frac{hc}{\lambda} = \frac{12375\text{ eV}}{\lambda} \quad \text{[given]}$$

or $$\frac{hc}{\lambda} = \frac{12375}{5000}\text{ eV} \quad \text{[using Eq. (i)]} \ldots\text{(iii)}$$

Now, substituting the values from Eq. (iii) in Eq. (ii), we get

$$\frac{12375}{5000}\text{ eV} - 2\text{eV} = eV_0$$

$$\Rightarrow \quad 2.475\text{ eV} - 2\text{eV} = eV_0$$

or $$V_0 = 2.475\text{V} - 2\text{V}$$

$$= 0.475\text{ V} \Rightarrow V_0 \approx 0.48\text{ V}$$

5. Relation between stopping potential and incident light's frequency is $eV_0 = hf - \phi_0$.

where, V_0 is the stopping potential and ϕ_0 is the the work function of the photosensitive surface.

So, from given data, we have,

$$-e\frac{V_0}{2} = h\nu - \phi_0 \quad \ldots\text{(i)}$$

and $$-eV_0 = \frac{h\nu}{2} - \phi_0 \quad \ldots\text{(ii)}$$

Subtracting Eqs. (i) from (ii), we have

$$-eV_0 - \left(-\frac{eV_0}{2}\right) = \frac{h\nu}{2} - h\nu \Rightarrow -\frac{eV_0}{2} = -\frac{h\nu}{2}$$

$$\Rightarrow \quad eV_0 = h\nu$$

Substituting this in Eq. (i), we get

$$-\frac{eV_0}{2} = eV_0 - \phi_0$$

$$\Rightarrow \quad -\left(\frac{3}{2}eV_0\right) = -\phi_0 \quad \text{or} \quad \frac{3}{2}h\nu = \phi_0$$

If threshold frequency is ν_0 then

$$h\nu_0 = \frac{3}{2}h\nu \Rightarrow \nu_0 = \frac{3}{2}\nu$$

6. Given, $\lambda_1 = 300$ nm

$$\lambda_2 = 400\text{ nm}$$

$$\frac{hc}{e} = 1240\text{ nm}$$

Using Einstein equation for photoelectric effect,

$$E = h\nu = \phi + \text{eV}_0 \quad \ldots\text{(i)}$$

(here, ϕ is work function of the metal and V_0 is stopping potential)

For λ_1 wavelength's wave,

$$E_1 = h\nu_1 = \phi + \text{eV}_{01}$$

or $$\frac{hc}{\lambda_1} = \phi + \text{eV}_{01} \quad \ldots\text{(ii)}$$

Similarly, $$\frac{hc}{\lambda_2} = \phi + \text{eV}_{02} \quad \ldots\text{(iii)}$$

From Eqs. (ii) and (iii), we get

$$hc\left[\frac{1}{\lambda_1}-\frac{1}{\lambda_2}\right] = e(V_{01}-V_{02}) \text{ or } \frac{hc}{e}\left[\frac{1}{\lambda_1}-\frac{1}{\lambda_2}\right] = \Delta V$$

By using given values,

$$\Delta V = 1240\left[\frac{1}{300}-\frac{1}{400}\right]\frac{\text{nmV}}{\text{nm}}$$

$$= 1240\times\frac{1}{1200}\text{ V}$$

$$\Rightarrow \quad \Delta V = 1.03.\text{ V} \approx 1\text{V}$$

7. We know that, intensity of a radiation I with energy 'E' incident on a plate per second per unit area is given as

$$\Rightarrow \quad I = \frac{dE}{dA\times dt} \Rightarrow \frac{dE}{dt} = IdA \text{ or } IA$$

i.e., energy incident per unit time $= IA$

Substituting the given values, we get

$$\frac{dE}{dt} = 16\times10^{-3}\times1\times10^{-4}$$

$$\frac{dE}{dt} = 16\times10^{-7}\text{ W} \quad \ldots\text{(i)}$$

Using Einstein's photoelectric equation, we can find kinetic energy of the incident radiation as

$$E = \frac{1}{2}mv^2 + \phi$$

(Here, ϕ is work function of metal)

or $E = \text{KE} + \phi$

$\text{KE} = E - \phi = 10 \text{ eV} - 5 \text{ eV}$

$\Rightarrow$ $\text{KE} = 5 \text{ eV}$ ...(ii)

Now, energy per unit time for incident photons will be

$\because$ $E = Nh\nu$

$\therefore$ $\frac{dE}{dt} = h\nu \frac{dN}{dt}$ or $h\nu \dot{N}$ ...(iii)

From Eqs. (i) and (iii), we get

$$h\nu \dot{N} = 16 \times 10^{-7} \text{ or } E\dot{N} = 16 \times 10^{-7}$$

But $E = 10$ eV, so

$$\dot{N}(10 \times 1.6 \times 10^{-19}) = 16 \times 10^{-7} \Rightarrow \dot{N} = 10^{12}$$

$\because$ Only 10% of incident photons emit electrons.

So, emitted electrons per second are

$$\frac{10}{100} \times 10^{12} = 10^{11}$$

8. According to question, the wave equation of the magnetic field which produce photoelectric effect

$$B = B_0[(\sin(3.14 \times 10^7 ct) + \sin(6.28 \times 10^7 ct)]$$

Here, the photoelectric effect produced by the angular frequency $(\omega) = 6.28 \times 10^7$ c

$\Rightarrow$ $\omega = 6.28 \times 10^7 \times 3 \times 10^8$

$\omega = 2\pi \times 10^7 \times 3 \times 10^8$ rad/s ...(i)

Using Eqs. (i)

$$h\nu = \frac{h\omega}{2\pi} = \frac{h \times 2\pi \times 10^7 \times 3 \times 10^8}{2\pi}$$

$$h\nu = 12.4 \text{ eV}$$

Therefore, according to Einstein equation for photoelectric effect

$$E = h\nu = \phi + \text{KE}_{\text{max}}$$

$\Rightarrow$ $KE_{\text{max}} = E - \phi$

(where, ϕ = work-function = 4.7 eV)

$\text{KE}_{\text{max}} = 12.4 - 4.7 = 7.7 \text{ eV}$

or $\text{KE}_{\text{max}} = 7.7 \text{ eV}$

9. Let maximum speed of photo electrons in first case is v_1 and maximum speed of photo electrons in second case is v_2

Assumption I if we assume difference in maximum speed in two cases is 2 then $v_1 = v$ and $v_2 = 3v$

According to Einstein's photo electron equation

Energy of incident photon = work function + KE

i.e. $\frac{hc}{\lambda} = \phi_0 + \frac{1}{2}mv^2$

Where $hc = 1240$ eV, λ is wavelength of light incident, ϕ_0 is work function and v is speed of photo electrons.

When $\lambda_1 = 350$ nm

$\therefore$ $\frac{hc}{350} = \phi_0 + \frac{1}{2}mv^2$

or $\frac{hc}{350} - \phi_0 = \frac{1}{2}mv^2$ ...(i)

when $\lambda_2 = 540$ nm

$\therefore$ $\frac{hc}{540} = \phi_0 + \frac{1}{2}m(3v^2)$

$\therefore$ $\frac{hc}{540} - \phi_0 = (\frac{1}{2}mv^2) \times 9$ ...(ii)

Now, we divide Eq. (i) by Eq. (ii), we get

$$\frac{\frac{hc}{350} - \phi_0}{\frac{hc}{540} - \phi_0} = \frac{\frac{1}{2}mv^2}{(\frac{1}{2}mv^2) \times 9} = \frac{1}{9}$$

or $9\left(\frac{hc}{350} - \phi_0\right) = \frac{hc}{540} - \phi_0$

or $8\phi_0 = hc\left[\frac{9}{350} - \frac{1}{540}\right]$

or $\phi_0 = \frac{1}{8} \times 1240\left[\frac{9 \times 540 - 350}{350 \times 540}\right] = 3.7 \text{ eV}.$

No option given is correct.

Alternate Method

Assumption II If we assume velocity of one is twice in factor with second, then.

Let $v_1 = 2v$ and $v_2 = v$

We know that from Einstein's photoelectric equation, energy of incident radiation = work function + KE

or $\frac{hc}{\lambda} = \phi + \frac{1}{2}mv^2$

Let when $\lambda_1 = 350$ nm then $v_1 = 2v$

and when $\lambda_1 = 540$ nm then $v_2 = v$

$\therefore$ Above Eq. becomes

$$\frac{hc}{\lambda_1} = \phi + \frac{1}{2}m(2v^2)$$

or $\frac{hc}{\lambda_1} - \phi = \frac{1}{2}m \times 4v^2$ ...(i)

and $\frac{hc}{\lambda_2} = \phi + \frac{1}{2}mv^2$

or $\frac{hc}{\lambda_2} - \phi = \frac{1}{2}mv^2$ ...(ii)

Now, we divide Eq. (i) by (ii) Eq.

$$\frac{\frac{hc}{\lambda_1} - \phi}{\frac{hc}{\lambda_2} - \phi} = \frac{\frac{1}{2}m \times 4v^2}{\frac{1}{2}mv^2} = 4$$

or $\frac{hc}{\lambda_1} - \phi = \frac{4hc}{\lambda_2} - 4\phi$

or $\phi = \frac{1}{3}hc\left(\frac{4}{\lambda_2} - \frac{1}{\lambda_1}\right) = \frac{1}{3} \times 1240\left(\frac{4 \times 350 - 540}{350 \times 540}\right)$

or $\phi = 1.8\,\text{eV}$

According the assumption II, correct option is (c).

10. We have, $\lambda = \frac{hc}{\Delta E}$

$$\therefore \quad \frac{\lambda_1}{\lambda_2} = \frac{hc/\Delta E_1}{hc/\Delta E_2} = \frac{\Delta E_2}{\Delta E_1}$$

$$= \frac{\left(\frac{4}{3}E - E\right)}{2E - E} = \frac{1}{3}$$

11. $\lambda_{\text{min}} = \frac{hc}{eV}$

$$\log(\lambda_{\text{min}}) = \log\left(\frac{hc}{e}\right) - \log V$$

$$y = c - x$$

12. According to the law of conservation of energy, i.e. Energy of a photon $(h\nu)$ = Work function (ϕ) + Kinetic energy of the photoelectron $\left(\frac{1}{2}mv_{\text{max}}^2\right)$

According to Einstein's photoelectric emission of light

i.e. $E = (\text{KE})_{\text{max}} + \phi$

As, $\frac{hc}{\lambda} = (\text{KE})_{\text{max}} + \phi$

If the wavelength of radiation is changed to $\frac{3\lambda}{4}$, then

$$\Rightarrow \quad \frac{4}{3}\frac{hc}{\lambda} = \left(\frac{4}{3}(\text{KE})_{\text{max}} + \frac{\phi}{3}\right) + \phi$$

$(\text{KE})_{\text{max}}$ for fastest emitted electron $= \frac{1}{2}mv'^2 + \phi$

$$\Rightarrow \quad \frac{1}{2}mv'^2 = \frac{4}{3}\left(\frac{1}{2}mv^2\right) + \frac{\phi}{3}$$

i.e. $v' > v\left(\frac{4}{3}\right)^{1/2}$

13.
$$\frac{hc}{\lambda} - \phi = eV_0$$

$$\frac{hc}{0.3 \times 10^{-6}} - \phi = 2e \qquad \text{...(i)}$$

$$\frac{hc}{0.4 \times 10^{-6}} - \phi = 1e \qquad \text{...(ii)}$$

Subtracting Eq. (ii) from Eq. (i)

$$hc\left(\frac{1}{0.3} - \frac{1}{0.4}\right)10^6 = e$$

$$hc\left(\frac{0.1}{0.12} \times 10^6\right) = e$$

$$h = 0.64 \times 10^{-33} = 6.4 \times 10^{-34}\,\text{J-s}$$

14. Energy corresponding to 248 nm wavelength

$$= \frac{1240}{248}\,\text{eV} = 5\,\text{eV}$$

Energy corresponding to 310 nm wavelength

$$= \frac{1240}{310}\,\text{eV} = 4\,\text{eV}$$

$$\frac{\text{KE}_1}{\text{KE}_2} = \frac{u_1^2}{u_2^2} = \frac{4}{1} = \frac{5\,\text{eV} - W}{4\,\text{eV} - W}$$

$$\Rightarrow \quad 16 - 4W = 5 - W \Rightarrow 11 = 3W$$

$$\Rightarrow \quad W = \frac{11}{3} = 3.67\,\text{eV} \cong 3.7\,\text{eV}$$

15. **Key Idea** The problem is based on frequency dependence of photoelectric emission. When incident light with certain frequency (greater than on the threshold frequency is focus on a metal surface) then some electrons are emitted from the metal with substantial initial speed.

When an electron moves in a circular path, then

$$r = \frac{mv}{eB} \quad \Rightarrow \quad \frac{r^2e^2B^2}{2} = \frac{m^2v^2}{2}$$

$$\text{KE}_{\text{max}} = \frac{(mv)^2}{2m} \quad \Rightarrow \quad \frac{r^2e^2B^2}{2m} = (\text{KE})_{\text{max}}$$

Work function of the metal (W), i.e. $W = h\nu - \text{KE}_{\text{max}}$

$$1.89 - \phi = \frac{r^2e^2B^2}{2m}\frac{1}{2}\text{eV} = \frac{r^2eB^2}{2m}\text{eV}$$

[$h\nu \to 1.89$ eV, for the transition on from third to second orbit of H-atom]

$$= \frac{100 \times 10^{-6} \times 1.6 \times 10^{-19} \times 9 \times 10^{-8}}{2 \times 9.1 \times 10^{-31}}$$

$$\phi = 1.89 - \frac{1.6 \times 9}{2 \times 9.1} = 1.89 - 0.79 = 1.1\,\text{eV}$$

16. $E_1 = \frac{1240}{550} = 2.25\,\text{eV}$, $\quad E_2 = \frac{1240}{450} = 2.75\,\text{eV}$

$E_3 = \frac{1240}{350} = 3.54\,\text{eV}$

E_1 cannot emit photoelectrons from q and r plates. E_2 can not emit photoelectrons from r.

Further, work function of p is least and it can emit photoelectrons from all three wavelengths. Hence magnitude of its stopping potential and saturation current both will be maximum.

17. Saturation current is proportional to intensity while stopping potential increases with increase in frequency.

Hence, $f_a = f_b$ while $I_a < I_b$

18. $\lambda\,(\text{in Å}) = \frac{12375}{W\,(\text{eV})} = \frac{12375}{4.0}\,\text{Å} \approx 3093\,\text{Å}$

or $\lambda \approx 309.3\,\text{nm} \approx 310\,\text{nm}$

NOTE $\lambda(\text{in Å}) = \frac{12375}{W\,(\text{eV})}$ comes from $W = \frac{hc}{\lambda}$

19. Stopping potential is the negative potential applied to stop the electrons having maximum kinetic energy. Therefore, stopping potential will be 4 V.

20. $$eV_0 = \frac{hc}{\lambda} - W$$

$$V_0 = \left(\frac{hc}{e}\right)\left(\frac{1}{\lambda}\right) - \frac{W}{e}$$

V_0 *versus* $\frac{1}{\lambda}$ graph is in the form $y = mx - c$

Therefore option (c) is correct.

Clearly, V_0 *versus* λ graph is not a straight line but V_0 decreases with increase in λ and V_0 becomes zero when $\frac{hc}{\lambda} = W$.

i.e. $\lambda = \lambda_0$ (Threshold wavelength)

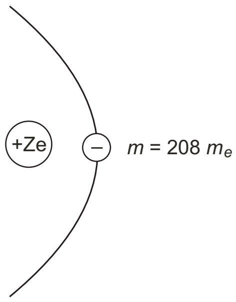
$\therefore$ Option (a) is also correct.

21. From the relation,

$$eV = \frac{hc}{\lambda} - \phi$$

or $$V = \left(\frac{hc}{e}\right)\left(\frac{1}{\lambda}\right) - \frac{\phi}{e}$$

This is equation of straight line.



Slope is $\tan\theta = \frac{hc}{e}$

$$\phi_1 : \phi_2 : \phi_3 = \frac{hc}{\lambda_{01}} : \frac{hc}{\lambda_{02}} : \frac{hc}{\lambda_{03}}$$

$$= \frac{1}{\lambda_{01}} : \frac{1}{\lambda_{02}} : \frac{1}{\lambda_{03}} = 1 : 2 : 4$$

$\frac{1}{\lambda_{01}} = 0.001\ \text{nm}^{-1}$ or $\lambda_{01} = 10000\ \text{Å}$

$\frac{1}{\lambda_{02}} = 0.002\ \text{nm}^{-1}$ or $\lambda_{02} = 5000\ \text{Å}$

$\frac{1}{\lambda_{03}} = 0.004\ \text{nm}^{-1}$ or $\lambda_{03} = 2500\ \text{Å}$

Violet colour has wavelength 4000 Å.

So, violet colour can eject photoelectrons from metal-1 and metal-2.

22. $K_{\max} = E - W$

Therefore,

$$T_A = 4.25 - W_A \quad \text{...(i)}$$

$$T_B = (T_A - 1.50) = 4.70 - W_B \quad \text{...(ii)}$$

From Eqs. (i) and (ii),

$$W_B - W_A = 1.95\ \text{eV} \quad \text{...(iii)}$$

de-Broglie wavelength is given by

$$\lambda = \frac{h}{\sqrt{2Km}} \quad \text{or} \quad \lambda \propto \frac{1}{\sqrt{K}} \qquad K = \text{KE of electron}$$

$$\therefore \quad \frac{\lambda_B}{\lambda_A} = \sqrt{\frac{K_A}{K_B}} \quad \text{or} \quad 2 = \sqrt{\frac{T_A}{T_A - 1.5}}$$

This gives, $T_A = 2\ \text{eV}$

From Eq. (i) $W_A = 4.25 - T_A = 2.25\ \text{eV}$

From Eq. (iii) $W_B = W_A + 1.95\ \text{eV} = (2.25 + 1.95)\ \text{eV}$

or $W_B = 4.20\ \text{eV}$

$$T_B = 4.70 - W_B = 4.70 - 4.20$$
$$= 0.50\ \text{eV}$$

23. (b) Stopping potential depends on two factors – one the energy of incident light and the other the work function of the metal. By increasing the distance of source from the cell, neither of the two change. Therefore, stopping potential remains the same.

(d) Saturation current is directly proportional to the intensity of light incident on cell and for a point source, intensity $I \propto 1/r^2$

When distance is increased from 0.2 m to 0.6 m (three times), the intensity and hence the saturation current will decrease 9 times, i.e. the saturation current will be reduced to 2.0 mA.

24. No solution is required.

25. For photoemission to take place, wavelength of incident light should be less than the threshold wavelength. Wavelength of ultraviolet light < 5200 Å while that of infrared radiation > 5200 Å.

26. $$eV_0 = hf - W$$

$$\therefore \quad V_0 = \left(\frac{h}{e}\right)f - \frac{W}{e}$$

V_0 *versus* f graph is a straight line with slope $= \frac{h}{e} =$ a universal constant. Therefore, the ratio of two slopes should be 1.

27. Photo emission will stop when potential on silver sphere becomes equal to the stopping potential.

$$\therefore \quad \frac{hc}{\lambda} - W = eV_0$$

Here, $$V_0 = \frac{1}{4\pi\varepsilon_0}\frac{ze}{r}$$

$$\therefore \quad \left(\frac{1240}{200}\ \text{eV}\right) - (4.7\ \text{eV}) = \frac{9 \times 10^9 \times n \times 1.6 \times 10^{-19}}{10^{-2}}$$

$$(6.2 - 4.7) = \frac{9 \times 10^9 \times n \times 1.6 \times 10^{-19}}{10^{-2}}$$

or $$z = \frac{1.5 \times 10^{-2}}{9 \times 1.6 \times 10^{-10}}$$

$$= 1.04 \times 10^7$$

$\therefore$ Answer is 7.

28. $K_{max} = h\nu - W$

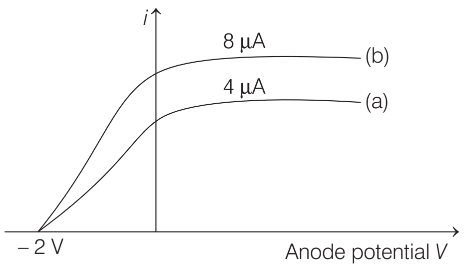
Therefore, K_{max} is linearly dependent on frequency of incident radiation.

29. Maximum energy of photoelectrons increases with increase in frequency of incident light. So, if intensity is increased by increasing frequency of incident light, maximum energy will increase. If intensity is increased merely by increasing number of photons incident per second, maximum energy of photoelectrons will not change. So, question is incomplete because it is not mentioned whether how the intensity is increased ?

30. Kinetic energy of photoelectrons depends on frequency of incident radiation.

31. Maximum kinetic energy of the photoelectrons would be

$$K_{max} = E - W = (5 - 3)\text{eV} = 2\,\text{eV}$$



Therefore, the stopping potential is 2 V. Saturation current depends on the intensity of light incident. When the intensity is doubled the saturation current will also become two fold. The corresponding graphs are shown in above figure.

32. Area of plates $A = 5 \times 10^{-4}\ \text{m}^2$

Distance between the plates $d = 1$ cm $= 10^{-2}$ m

(a) Number of photoelectrons emitted upto $t = 10$ s are

$$n = \frac{(\text{number of photons falling in unit area in unit time}) \times (\text{area} \times \text{time})}{10^6}$$

$$= \frac{1}{10^6}\,[(10)^{16} \times (5 \times 10^{-4}) \times (10)] = 5.0 \times 10^7$$

(b) At time, $t = 10$ s

Charge on plate A, $q_A = + ne = (5.0 \times 10^7)(1.6 \times 10^{-19})$

$$= 8.0 \times 10^{-12}\ \text{C}$$

and charge on plate B,

$$q_B = (33.7 \times 10^{-12} - 8.0 \times 10^{-12})$$

$$= 25.7 \times 10^{-12}\ \text{C}$$

$\therefore$ Electric field between the plates, $E = \dfrac{(q_B - q_A)}{2A\varepsilon_0}$

or $E = \dfrac{(25.7 - 8.0) \times 10^{-12}}{2 \times (5 \times 10^{-4})(8.85 \times 10^{-12})} = 2 \times 10^3$ N/C

(c) Energy of photoelectrons at plate A

$$= E - W = (5 - 2)\,\text{eV} = 3\,\text{eV}$$

Increase in energy of photoelectrons

$$= (eEd)\ \text{joule} = (Ed)\ \text{eV}$$

$$= (2 \times 10^3)(10^{-2})\ \text{eV} = 20\ \text{eV}$$

Energy of photoelectrons at plate B

$$= (20 + 3)\,\text{eV} = 23\ \text{eV}$$

33. Given work function, $W = 1.9$ eV

Wavelength of incident light, $\lambda = 400$ nm

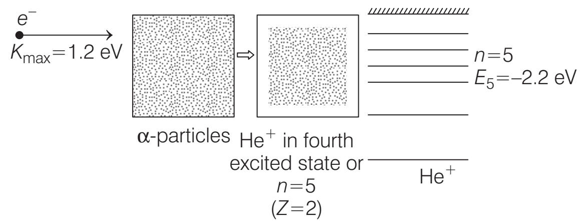
$\therefore$ Energy of incident light, $E = \dfrac{hc}{\lambda} = 3.1$ eV

(Substituting the values of h, c and λ)

Therefore, maximum kinetic energy of photoelectrons

$$K_{max} = E - W = (3.1 - 1.9) = 1.2\,\text{eV}$$

Now the situation is as shown below :



Energy of electron in 4^{th} excited state of He^+ $(n = 5)$ will be

$$E_5 = -13.6\,\frac{Z^2}{n^2}\ \text{eV} \quad \Rightarrow \quad E_5 = -(13.6)\,\frac{(2)^2}{(5)^2} = -2.2\,\text{eV}$$

Therefore, energy released during the combination

$$= 1.2 - (-2.1) = 3.4\ \text{eV}$$

Similarly, energies in other energy states of He^+ will be

$$E_4 = -13.6\,\frac{(2)^2}{(4)^2} = -3.4\ \text{eV}$$

$$E_3 = -13.6\,\frac{(2)^2}{(3)^2} = -6.04\ \text{eV}$$

$$E_2 = -13.6\,\frac{(2)^2}{2^2} = -13.6\,\text{eV}$$

The possible transitions are

$\Delta E_{5\to 4} = E_5 - E_4 = 1.2\,\text{eV} < 2$ eV

$\Delta E_{5\to 3} = E_5 - E_3 = 3.84$ eV

$\Delta E_{5\to 2} = E_5 - E_2 = 11.4\ \text{eV} > 4\text{eV}$

$\Delta E_{4\to 3} = E_4 - E_3 = 2.64$ eV

$\Delta E_{4\to 2} = E_4 - E_2 = 10.2\,\text{eV} > 4$ eV

Hence, the energy of emitted photons in the range of 2 eV and 4 eV are

3.4 eV during combination and

3.84 eV and 2.64 after combination.

34. (a) Energy of emitted photons

$$E_1 = 5.0\ \text{eV} = 5.0 \times 1.6 \times 10^{-19}\,\text{J} = 8.0 \times 10^{-19}\,\text{J}$$

Power of the point source is 3.2×10^{-3} W or 3.2×10^{-3} J/s.

Therefore, energy emitted per second,

$$E_2 = 3.2 \times 10^{-3} \text{ J}$$

Hence, number of photons emitted per second $n_1 = \dfrac{E_2}{E_1}$

or $$n_1 = \frac{3.2 \times 10^{-3}}{8.0 \times 10^{-19}}$$

$$n_1 = 4.0 \times 10^{15} \text{ photon/s}$$

Number of photons incident on unit area at a distance of 0.8 m from the source S will be

$$n_2 = \frac{n_1}{4\pi\,(0.8)^2} = \frac{4.0 \times 10^{15}}{4\pi\,(0.64)}$$

$$\approx 5 \times 10^{14} \text{ photon/s -m}^2$$

The area of metallic sphere over which photons will fall is

$$A = \pi r^2 = \pi\,(8 \times 10^{-3})^2 \text{ m}^2 \approx 2.01 \times 10^{-4} \text{ m}^2$$

Therefore, number of photons incident on the sphere per second are

$$n_3 = n_2\,A = (5.0 \times 10^{14} \times 2.01 \times 10^{-4}) \approx 10^{11}/\text{s}$$

But since, one photoelectron is emitted for every 10^6 photons, hence number of photoelectrons emitted per second.

$$n = \frac{n_3}{10^6} = \frac{10^{11}}{10^6} = 10^5/\text{s} \quad \text{or} \quad n = 10^5/\text{s}$$

(b) Maximum kinetic energy of photoelectrons

K_{max} = Energy of incident photons − work function

$$= (5.0 - 3.0)\text{ eV} = 2.0 \text{ eV}$$

$$= 2.0 \times 1.6 \times 10^{-19} \text{ J}$$

$$K_{\text{max}} = 3.2 \times 10^{-19} \text{ J}$$

The de-Broglie wavelength of these photoelectrons will be

$$\lambda_1 = \frac{h}{p} = \frac{h}{\sqrt{2K_{\text{max}}m}}$$

Here, h = Planck's constant and m = mass of electron

$$\lambda_1 = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 3.2 \times 10^{-19} \times 9.1 \times 10^{-31}}}$$

$$= 8.68 \times 10^{-10} = 8.68 \text{ Å}$$

Wavelength of incident light λ_2 (in Å) $= \dfrac{12375}{E_1 \text{ (in eV)}}$

or $$\lambda_2 = \frac{12375}{5} = 2475 \text{Å}$$

Therefore, the desired ratio is

$$\frac{\lambda_2}{\lambda_1} = \frac{2475}{8.68} = 285.1$$

(c) As soon as electrons are emitted from the metal sphere, it gets positively charged and acquires positive potential. The positive potential gradually increases as more and more photoelectrons are emitted from its surface. Emission of photoelectrons is stopped when its potential is equal to the stopping potential required for fastest moving electrons.

(d) As discussed in part (c), emission of photoelectrons is stopped when potential on the metal sphere is equal to the stopping potential of fastest moving electrons.

Since, $K_{\text{max}} = 2.0 \text{ eV}$

Therefore, stopping potential $V_0 = 2$ V. Let q be the charge required for the potential on the sphere to be equal to stopping potential or 2 V. Then

$$2 = \frac{1}{4\pi\,\varepsilon_0} \cdot \frac{q}{r} = (9.0 \times 10^9)\,\frac{q}{8.0 \times 10^{-3}}$$

$$\therefore \quad q = 1.78 \times 10^{-12} \text{ C}$$

Photoelectrons emitted per second = 10^5 [Part (a)]

or charge emitted per second = $(1.6 \times 10^{-19}) \times 10^5$ C

$$= 1.6 \times 10^{-14} \text{ C}$$

Therefore, time required to acquire the charge q will be

$$t = \frac{q}{1.6 \times 10^{-14}}\text{s} = \frac{1.78 \times 10^{-12}}{1.6 \times 10^{-14}}\text{ s} \quad \text{or} \quad t \approx 111 \text{ s}$$

35. (a) From Einstein's equation of photoelectric effect,

Energy of photons causing the photoelectric emission = Maximum kinetic energy of emitted photons + work function

or $$E = K_{\text{max}} + W = (0.73 + 1.82)\text{ eV}$$

or $$E = 2.55 \text{ eV}$$

(b) In case of a hydrogen atom,

$$E_1 = -13.6 \text{ eV}, E_2 = -3.4 \text{ eV}, E_3 = -1.5 \text{ eV},$$
$$E_4 = -0.85 \text{ eV}$$

Since, $E_4 - E_2 = 2.55 \text{ eV}$

Therefore, quantum numbers of the two levels involved in the emission of these photons are 4 and 2($4 \to 2$).

(c) Change in angular momentum in transition from 4 to 2 will be

$$\Delta L = L_2 - L_4 = 2\left(\frac{h}{2\pi}\right) - 4\left(\frac{h}{2\pi}\right) \quad \text{or} \quad \Delta L = -\frac{h}{\pi}$$

(d) From conservation of linear momentum

| Momentum of hydrogen atom | = | Momentum of emitted photon |

or $$mv = \frac{E}{c} \qquad (m = \text{mass of hydrogen atom})$$

or $$v = \frac{E}{mc} = \frac{(2.55 \times 1.6 \times 10^{-19} \text{ J})}{(1.67 \times 10^{-27} \text{kg})\,(3.0 \times 10^8 \text{m/s})}$$

$$v = 0.814 \text{ m/s}$$

36. Energy of photon having wavelength 4144 Å,

$$E_1 = \frac{12375}{4144}\text{ eV} = 2.99\text{ eV}$$

Similarly, $$E_2 = \frac{12375}{4972}\text{ eV} = 2.49\text{ eV and}$$

$$E_3 = \frac{12375}{6216}\text{ eV} = 1.99\text{ eV}$$

Since, only E_1 and E_2 are greater than the work function $W = 2.3$ eV, only first two wavelengths are capable for ejecting photoelectrons. Given intensity is equally distributed in all wavelengths. Therefore, intensity corresponding to each wavelength is

$$\frac{3.6\times10^{-3}}{3} = 1.2\times10^{-3}\text{ W/m}^2$$

Or energy incident per second in the given area $(A = 1.0\text{ cm}^2 = 10^{-4}\text{ m}^2)$ is

$$\rho = 1.2\times10^{-3}\times10^{-4}$$
$$= 1.2\times10^{-7}\text{ J/s}$$

Let n_1 be the number of photons incident per unit time in the given area corresponding to first wavelength. Then

$$n_1 = \frac{\rho}{E_1} = \frac{1.2\times10^{-7}}{2.99\times1.6\times10^{-19}}$$
$$= 2.5\times10^{11}$$

Similarly, $$n_2 = \frac{\rho}{E_2} = \frac{1.2\times10^{-7}}{2.49\times1.6\times10^{-19}}$$
$$= 3.0\times10^{11}$$

Since, each energetically capable photon ejects electron, total number of photoelectrons liberated in 2 s.

$$= 2(n_1 + n_2) = 2\,(2.5 + 3.0)\times10^{11}$$
$$= 1.1\times10^{12}$$

Topic 3 Radioactivity

1. An α-particle decay $({}^4_2\text{He})$ reduces, mass number by 4 and atomic number by 2.

$\therefore$ Decay of 6α-particles results

$$^{232}_{90}\text{Th} \xrightarrow{6\alpha} {}^{232-24}_{90-12}\text{Y} = {}^{208}_{78}\text{Y}$$

A β-decay does not produces any change in mass number but it increases atomic number by 1.

$\therefore$ Decay of 4β-particles results

$$^{208}_{78}\text{Y} \xrightarrow{4\beta} {}^{208}_{82}\text{X}$$

$\therefore$ In the end nucleus $A = 208$, $Z = 82$

2. Here given, at $t = 0$, count rate or initial activity is

$$A_0 = 1600\text{ s}^{-1}.$$

At $t = 8$ s, count rate or activity is

$$A = 100\text{ s}^{-1}$$

So, decay scheme for given sample is

$$1600 \xrightarrow{T_{1/2}} 800 \xrightarrow{T_{1/2}} 400 \xrightarrow{T_{1/2}} 200 \xrightarrow{T_{1/2}} 100$$

So, $$8\text{ s} = 4T_{1/2}$$

where, $T_{1/2}$ = Half-life time.

$$\Rightarrow \quad T_{1/2} = 2\text{ s}$$

$\therefore$ From above decay scheme, we see that activity after 6 s is 200 counts per second.

3. Activity of radioactive material is given as

$$R = \lambda N$$

where, λ is the decay constant N is the number of nuclei in the radioactive material.

For substance A,

$$R_A = \lambda_A N_A = \lambda_A N_{0A} \text{ (initially } N_A = N_{0A})$$

For substance B,

$$R_B = \lambda_B N_B = \lambda_B N_{0B} \text{ (initially } N_B = N_{0B})$$

At $t = 0$, activity is equal, therefore

$$\lambda_A N_{0A} = \lambda_B N_{0B} \quad \ldots\text{(i)}$$

The half-life is given by

$$T_{1/2} = \frac{0.693}{\lambda} = \frac{\ln 2}{\lambda}$$

So, for substance A,

$$(T_{1/2})_A = \frac{\ln 2}{\lambda_A} \Rightarrow \ln 2 = \frac{\ln 2}{\lambda_A}$$

$$\lambda_A = 1 \quad \ldots\text{(ii)}$$

According to the given question,

at time t,

$$\frac{R_B}{R_A} = e^{-3t} \quad \ldots\text{(iii)}$$

Using Eqs. (i), (ii) and (iii)

$$\frac{R_B}{R_A} = e^{-3t} = \frac{\lambda_B N_{0B} e^{-\lambda_B t}}{\lambda_A N_{0A} e^{-\lambda_A t}}$$

$$\Rightarrow \quad e^{-3t} = e^{(\lambda_A - \lambda_B)t}$$

$$-3 = \lambda_A - \lambda_B$$
$$\Rightarrow \quad \lambda_B = \lambda_A + 3$$
$$\lambda_B = 1 + 3 = 4 \quad \ldots\text{(iv)}$$

The half-life of substance B is

$$(T_{1/2})_B = \frac{\ln 2}{\lambda_B} = \frac{\ln 2}{4}$$

4. Activity of a radioactive material is given as

$$R = \lambda N$$

where, λ is the decay constant and N is the number of nuclei in the radioactive material.

For substance A,

$$R_A = \lambda_A N_A = 10\text{ mCi}$$

For substance B,

$$R_B = \lambda_B N_B = 20\text{ mCi} \quad \ldots\text{(i)}$$

As given in the question,

$$N_A = 2N_B$$

$\Rightarrow \quad R_A = \lambda_A (2N_B) = 10 \text{ mCi}$ ...(ii)

$\therefore$ Dividing Eq. (ii) and Eq.(i), we get

$$\frac{R_A}{R_B} = \frac{\lambda_A (2N_B)}{\lambda_B (N_B)} = \frac{10}{20}$$

or $\quad \frac{\lambda_A}{\lambda_B} = \frac{1}{4}$ ...(iii)

As, half-life of a radioactive material is given as

$$T_{1/2} = \frac{0.693}{\lambda}$$

$\therefore$ For material A and B, we can write

$$\frac{(T_{1/2})_A}{(T_{1/2})_B} = \frac{\dfrac{0.693}{\lambda_A}}{\dfrac{0.693}{\lambda_B}} = \frac{\lambda_B}{\lambda_A}$$

Using Eq. (iii), we get

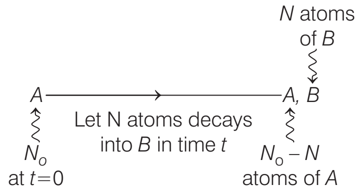
$$\frac{(T_{1/2})_A}{(T_{1/2})_B} = \frac{4}{1}$$

Hence, from the given options, only option (d) satisfies this ratio.

Therefore, $(T_{1/2})_A = 20$ days

and $\quad (T_{1/2})_B = 5$ days

5. Decay scheme is ,



Given, $\frac{N_B}{N_A} = 0.3 = \frac{3}{10}$

$$\Rightarrow \quad \frac{N_B}{N_A} = \frac{30}{100}$$

So, $\quad N_0 = 100 + 30 = 130$ atoms

By using $\quad N = N_0 e^{-\lambda t}$

We have, $\quad 100 = 130 e^{-\lambda t}$

$$\Rightarrow \quad \frac{1}{1.3} = e^{-\lambda t} \Rightarrow \log 1.3 = \lambda t$$

$$\Rightarrow \quad \log 1.3 = \frac{\log 2}{T} \cdot t$$

$$\therefore \quad T = \frac{T \cdot \log (1.3)}{\log 2}$$

6. **A :** Numbers left : $N \rightarrow \frac{N}{2} \rightarrow \frac{N}{4} \rightarrow \frac{N}{8} \rightarrow \frac{N}{16}$

$\therefore$ Number decayed, $N_A = N - \frac{N}{16} = \frac{15}{16} N$

B : Numbers left : $N \rightarrow \frac{N}{2} \rightarrow \frac{N}{4}$

$\therefore$ Numbers decayed, $N_B = N - \frac{N}{4} = \frac{3}{4} N$

Ratio : $\quad \frac{N_A}{N_B} = \frac{(15/16)N}{(3/4)N} = \frac{5}{4}$

7. Using the relation

$$R = R_0 \left(\frac{1}{2}\right)^n$$

Here, R is activity of radioactive substance, R_0 initial activity and n is number of half lives.

$$1 = 64 \left(\frac{1}{2}\right)^n$$

Solving we get, $n = 6$

Now, $\quad t = n(t_{12})$

$= 6(18 \text{ days})$

$= 108$ days

8. Activity of $S_1 = \frac{1}{2}$(activity of S_2)

or $\quad \lambda_1 N_1 = \frac{1}{2}(\lambda_2 N_2)$

or $\quad \frac{\lambda_1}{\lambda_2} = \frac{N_2}{2N_1}$

or $\quad \frac{T_1}{T_2} = \frac{2N_1}{N_2} \qquad \left(T = \text{half-life} = \frac{\ln 2}{\lambda}\right)$

Given $\quad N_1 = 2N_2$

$\therefore \quad \frac{T_1}{T_2} = 4$

9. After two half lives $\frac{1}{4}$ th fraction of nuclei will remain undecayed. Or, $\frac{3}{4}$ th fraction will decay. Hence, the probability that a nucleus decays in two half lives is $\frac{3}{4}$.

10. Activity reduces from 6000 dps to 3000 dps in 140 days. It implies that half-life of the radioactive sample is 140 days. In 280 days (or two half-lives) activity will remain $\frac{1}{4}$ th of the initial activity. Hence, the initial activity of the sample is

$$4 \times 6000 \text{ dps} = 24000 \text{ dps}$$

11. During γ-decay atomic number (Z) and mass number (A) does not change. So, the correct option is (c) because in all other options either Z, A or both is/are changing.

12. $$R = R_0 \left(\frac{1}{2}\right)^n \quad \text{...(i)}$$

Here R = activity of radioactive substance after n half-lives

$= \frac{R_0}{16}$ (given)

Substituting in Eq. (i), we get $n = 4$

$\therefore \quad t = (n) t_{1/2} = (4)(100 \mu\text{s}) = 400 \mu\text{s}$

13. The total number of atoms can neither remain constant (as in option a) nor can ever increase (as in options b and c). They will continuously decrease with time. Therefore, (d) is the appropriate option.

14. During β -decay, a neutron is transformed into a proton and an electron. This is why atomic number (Z = number of protons) increases by one and mass number (A = number of protons + neutrons) remains unchanged during beta decay.

15. $\dfrac{N_{x_1}(t)}{N_{x_2}(t)} = \dfrac{1}{e}$ or $\dfrac{N_0 e^{-10\lambda t}}{N_0 e^{-\lambda t}} = \dfrac{1}{e}$

(Initially, both have same number of nuclei say N_0)

or $e = e^{-\lambda t}/e^{-10\lambda t}$

or $e = e^{9\lambda t}$

or $9\lambda t = 1$

or $t = \dfrac{1}{9\lambda}$

16. $(t_{1/2})_x = (t_{\text{mean}})_y$

or $\dfrac{0.693}{\lambda_x} = \dfrac{1}{\lambda_y}$

$\therefore$ $\lambda_x = 0.693\,\lambda_y$

$\lambda_x < \lambda_y$

or Rate of decay = λN

Initially number of atoms (N) of both are equal but since $\lambda_y > \lambda_x$, therefore, y will decay at a faster rate than x.

17. Both the beta rays and the cathode rays are made up of electrons. So, only option (a) is correct.
(b) Gamma rays are electromagnetic waves.
(c) Alpha particles are doubly ionized helium atoms and
(d) Protons and neutrons have approximately the same mass.
Therefore, (b), (c) and (d) are wrong options.

18. Number of nuclei decreases exponentially

$$N = N_0\, e^{-\lambda t}$$

and rate of decay $\left(-\dfrac{dN}{dt}\right) = \lambda N$

Therefore, decay process lasts upto $t = \infty$. Therefore, a given nucleus may decay at any time after $t = 0$.

19. In beta decay, atomic number increases by 1 whereas the mass number remains the same.

Therefore, following equation can be possible

$$^{64}_{29}\text{Cu} \longrightarrow {}^{64}_{30}\text{Zn} + {}_{-1}e^0$$

20. Penetrating power is maximum for γ-rays, then of β -particles and then α-particles because basically it depends on the velocity. However, ionization power is in reverse order.

21. As we know, $T_{y2} = \dfrac{\ln 2}{\lambda}$ and $\tau = \dfrac{1}{\lambda}$.

22. From $R = R_0\left(\dfrac{1}{2}\right)^n$

we have, $1 = 64\left(\dfrac{1}{2}\right)^n$

or $n = 6$ = number of half lives

$\therefore$ $t = n \times t_{t_{1/2}} = 6 \times 2 = 12\,\text{h}$

24. Beta particles are fast moving electrons which are emitted by the nucleus.

25. Using $N = N_0 e^{-\lambda t}$

where $\lambda = \dfrac{\ln 2}{t_{1/2}} = \dfrac{\ln(2)}{3.8}$

$\therefore$ $\dfrac{N_0}{20} = N_0 e^{-\frac{\ln(2)}{3.8}t}$

Solving this equation with the help of given data we find :

$$t = 16.5\ \text{days}$$

26. $^{232}_{90}\text{Th}$ is converting into $^{212}_{82}\text{Pb}$.

Change in mass number (A) = 20

$\therefore$ Number of α-particle emitted = $\dfrac{20}{4} = 5$

Due to 5 α-particles, Z will change by 10 units.

Since, given change is 8, therefore number of β-particles emitted is 2.

27. $\text{I}^{131} \xrightarrow[T_{1/2} = 8\ \text{Days}]{} \text{Xe}^{131} + \beta$

$A_0 = 2.4 \times 10^5\ \text{Bq} = \lambda N_0$

Let the volume is V,

$t = 0$ $\quad A_0 = \lambda N_0$

$t = 11.5\ \text{h}$ $\quad A = \lambda N$

$$115 = \lambda\left(\frac{N}{V} \times 2.5\right)$$

$$115 = \frac{\lambda}{V} \times 2.5 \times (N_0 e^{-\lambda t})$$

$$115 = \frac{(N_0\lambda)}{V} \times (2.5) \times e^{-\frac{\ln 2}{8\ \text{day}}(11.5\ \text{h})}$$

$$115 = \frac{(2.4 \times 10^5)}{V} \times (2.5) \times e^{-1/24}$$

$$V = \frac{2.4 \times 10^5}{115} \times 2.5\left[1 - \frac{1}{24}\right]$$

$$= \frac{2.4 \times 10^5}{115} \times 2.5\left[\frac{23}{24}\right]$$

$$= \frac{10^5 \times 23 \times 25}{115 \times 10^2} = 5 \times 10^3\ \text{ml} = 5\ \text{L}$$

28. Let initial numbers are N_1 and N_2.

$$\frac{\lambda_1}{\lambda_2} = \frac{\tau_2}{\tau_1} = \frac{2\tau}{\tau} = 2 = \frac{T_2}{T_1} \qquad (T = \text{Half life})$$

$$A = \frac{-dN}{dt} = \lambda N$$

Initial activity is same

$\therefore\ \lambda_1 N_1 = \lambda_2 N_2$ ...(i)

Activity at time t,

$$A = \lambda N = \lambda N_0 e^{-\lambda t}$$

$$A_1 = \lambda_1 N_1 e^{-\lambda_1 t}$$

$$\Rightarrow \quad R_1 - = \frac{dA_1}{dt} = \lambda_1^2 N_1 e^{-\lambda_1 t}$$

Similarly, $R_2 = \lambda_2^2 N_2 e^{-\lambda_2 t}$

After $t = 2\tau$

$$\lambda_1 t = \frac{1}{\tau_1}(t) = \frac{1}{\tau}(2\tau) = 2$$

$$\lambda_2 t = \frac{1}{\tau_2}(t) = 1 = \frac{1}{2\tau}(2\tau) = 1$$

$$\frac{R_P}{R_Q} = \frac{\lambda_1^2 N_1 e^{-\lambda_1 t}}{\lambda_2^2 N_2 e^{-\lambda_2 t}}$$

$$\frac{R_P}{R_Q} = \frac{\lambda_1}{\lambda_2}\left(\frac{e^{-2}}{e^{-1}}\right) = \frac{2}{e}$$

29. Number of nuclei decayed in time t,

$$N_d = N_0(1 - e^{-\lambda t})$$

$$\therefore \quad \%\text{ decayed} = \left(\frac{N_d}{N_0}\right) \times 100$$

$$= (1 - e^{-\lambda t} d) \times 100 \quad \text{...(i)}$$

Here, $\lambda = \frac{0.693}{1386} = 5 \times 10^{-4}\ \text{s}^{-1}$

$$\therefore \quad \%\text{ decayed} \approx (\lambda t) \times 100$$

$$= (5 \times 10^{-4})(80)(100) = 4$$

30. Activity $\left(-\frac{dN}{dt}\right) = \lambda N = \left(\frac{1}{t_{\text{mean}}}\right) \times N$

$\therefore\ N = \left(-\frac{dN}{dt}\right) \times t_{\text{mean}}$ = Total number of atoms

Mass of one atom is 10^{-25} kg = m (say)

$\therefore$ Total mass of radioactive substance

= (number of atoms) × (mass of one atom)

$$= \left(-\frac{dN}{dt}\right)(t_{\text{mean}})(m)$$

Substituting the values, we get

Total mass of radioactive substance = 1 mg

$\therefore$ Answer is 1.

31. ${}^{11}_{6}\text{C} \rightarrow {}^{11}_{5}\text{B} + \beta^+ + \gamma$ (neutrino)

32. ${}_5\text{B}^{10} + {}_0n^1 \longrightarrow {}_2\text{He}^4 + {}_3\text{Li}^7$

Therefore, resulting nucleus is lithium and its mass number is 7.

33. Number of α-particles emitted, $n_1 = \frac{238 - 206}{4} = 8$

and number of β-particles emitted are say n_2, then

$$92 - 8 \times 2 + n_2 = 82$$

$$\therefore \quad n_2 = 6$$

34. $R = R_0\left(\frac{1}{2}\right)^n$

Here R_0 = initial activity = 1000 disintegration/s

and n = number of half-lives.

At $t = 1$ s, $n = 1$

$$\therefore \quad R = 10^3\left(\frac{1}{2}\right) = 500 \text{ disintegration/s}$$

At $t = 3$ s, $n = 3$

$$R = 10^3\left(\frac{1}{2}\right)^3 = 125 \text{ disintegration/s}$$

35. Let N_0 be the initial number of nuclei of ^{238}U.

After time t, $N_{\text{U}} = N_0\left(\frac{1}{2}\right)^n$

Here n = number of half-lives $= \frac{t}{t_{1/2}} = \frac{1.5 \times 10^9}{4.5 \times 10^9} = \frac{1}{3}$

$$N_{\text{U}} = N_0\left(\frac{1}{2}\right)^{\frac{1}{3}} \text{ and } N_{\text{Pb}} = N_0 - N_{\text{U}} = N_0\left[1 - \left(\frac{1}{2}\right)^{1/3}\right]$$

$$\therefore \quad \frac{N_{\text{U}}}{N_{\text{Pb}}} = \frac{\left(\frac{1}{2}\right)^{1/3}}{1 - \left(\frac{1}{2}\right)^3} = 3.861$$

36. Let n_0 be the number of radioactive nuclei at time $t = 0$. Number of nuclei decayed in time t are given by $n_0(1 - e^{-\lambda t})$, which is also equal to the number of beta particles emitted during the same interval of time. For the given condition,

$$n = n_0(1 - e^{-2\lambda}) \quad \text{...(i)}$$

$$(n + 0.75n) = n_0(1 - e^{-4\lambda}) \quad \text{...(ii)}$$

Dividing Eq. (ii) by (i), we get

$$1.75 = \frac{1 - e^{-4\lambda}}{1 - e^{-2\lambda}}$$

or $\quad 1.75 - 1.75\, e^{-2\lambda} = 1 - e^{-4\lambda}$

$$\therefore \quad 1.75\, e^{-2\lambda} - e^{-4\lambda} = \frac{3}{4} \quad \text{...(iii)}$$

Let us take $e^{-2\lambda} = x$

Then, the above equation is

$$x^2 - 1.75x + 0.75 = 0$$

or $$x = \frac{1.75 \pm \sqrt{(1.75)^2 - (4)(0.75)}}{2}$$

or $$x = 1 \text{ and } \frac{3}{4}$$

$\therefore$ From Eq. (iii) either

$$e^{-2\lambda} = 1 \text{ or } e^{-2\lambda} = \frac{3}{4}$$

but $e^{-2\lambda} = 1$ is not accepted because which means $\lambda = 0$.

Hence, $e^{-2\lambda} = \frac{3}{4}$

or $-2\lambda \ln(e) = \ln(3) - \ln(4) = \ln(3) - 2\ln(2)$

$\therefore$ $$\lambda = \ln(2) - \frac{1}{2}\ln(3)$$

Substituting the given values,

$$\lambda = 0.6931 - \frac{1}{2} \times (1.0986) = 0.14395\,\text{s}^{-1}$$

$\therefore$ Mean-life $t_{\text{means}} = \frac{1}{\lambda} = 6.947$ s

37. (a) Let at time $t = t$, number of nuclei of Y and Z are N_Y and N_Z. Then,

Rate equations of the populations of X, Y and Z are

$$\left(\frac{dN_X}{dt}\right) = -\lambda_X N_X \quad \text{...(i)}$$

$$\left(\frac{dN_Y}{dt}\right) = \lambda_X N_X - \lambda_Y N_Y \quad \text{...(ii)}$$

and $$\left(\frac{dN_Z}{dt}\right) = \lambda_Y N_Y \quad \text{...(iii)}$$

(b) Given $N_Y(t) = \frac{N_0 \lambda_X}{\lambda_X - \lambda_Y}[e^{-\lambda_Y t} - e^{-\lambda_X t}]$

For N_Y to be maximum

$$\frac{dN_Y(t)}{dt} = 0$$

i.e $\lambda_X N_X = \lambda_Y N_Y$ ...(iv) [from Eq. (ii)]

or $\lambda_X (N_0 e^{-\lambda_X t}) = \lambda_Y \frac{N_0 \lambda_X}{\lambda_X - \lambda_Y}[e^{-\lambda_Y t} - e^{-\lambda_X t}]$

or $$\frac{\lambda_X - \lambda_Y}{\lambda_Y} = \frac{e^{-\lambda_Y t}}{e^{-\lambda_X t}} - 1$$

$$\frac{\lambda_X}{\lambda_Y} = e^{(\lambda_X - \lambda_Y)t}$$

or $(\lambda_X - \lambda_y)\, t \ln(e) = \ln\left(\frac{\lambda_X}{\lambda_Y}\right)$

or $$t = \frac{1}{\lambda_X - \lambda_Y} \ln\left(\frac{\lambda_X}{\lambda_Y}\right)$$

Substituting the values of λ_X and λ_Y, we have

$$t = \frac{1}{(0.1 - 1/30)} \ln\left(\frac{0.1}{1/30}\right) = 15\ln(3)$$

or $t = 16.48$ s.

(c) The population of X at this moment,

$$N_X = N_0 e^{-\lambda_X t} = (10^{20})\, e^{-(0.1)(16.48)}$$

$$N_X = 1.92 \times 10^{19}$$

$$N_Y = \frac{N_X \lambda_X}{\lambda_Y} \text{ [From Eq. (iv)]}$$

$$= (1.92 \times 10^{19})\left(\frac{0.1)}{(1/30)}\right. = 5.76 \times 10^{19}$$

$$N_Z = N_0 - N_X - N_Y$$

$$= 10^{20} - 1.92 \times 10^{19} - 5.76 \times 10^{19}$$

or $N_Z = 2.32 \times 10^{19}$

38. (a) Let at time t, number of radioactive nuclei are N.

Net rate of formation of nuclei of A

$$\frac{dN}{dt} = \alpha - \lambda N$$

or $$\frac{dN}{\alpha - \lambda N} = dt$$

or $$\int_{N_0}^{N} \frac{dN}{\alpha - \lambda N} = \int_0^t dt$$

Solving this equation, we get

$$N = \frac{1}{\lambda}[\alpha - (\alpha - \lambda N_0)e^{-\lambda t}] \quad \text{...(i)}$$

(b) (i) Substituting $\alpha = 2\lambda N_0$ and $t = t_{1/2} = \frac{\ln(2)}{\lambda}$ in

Eq. (i) we get, $N = \frac{3}{2}N_0$

(ii) Substituting $\alpha = 2\lambda N_0$ and $t \to \infty$ in Eq. (i), we get

$$N = \frac{\alpha}{\lambda} = 2N_0 \text{ or } N = 2N_0$$

39. (a) In 10 s, number of nuclei has been reduced to half (25% to 12.5%).

Therefore, its half-life is $t_{1/2} = 10\,\text{s}$

Relation between half-life and mean life is

$$t_{\text{mean}} = \frac{t_{1/2}}{\ln(2)} = \frac{10}{0.693}\text{s}$$

$$t_{\text{mean}} = 14.43 \text{ s}$$

(b) From initial 100% to reduction till 6.25%, it takes four half lives.

$$100\% \xrightarrow{t_{1/2}} 50\% \xrightarrow{t_{1/2}} 25\% \xrightarrow{t_{1/2}} 12.5\% \xrightarrow{t_{1/2}} 6.25\%$$

$\therefore$ $t = 4\, t_{1/2} = 4\,(10)$ s $= 40$ s

$t = 40$ s

40. λ = Disintegration constant

$$\frac{0.693}{t_{1/2}} = \frac{0.693}{15}\,\text{h}^{-1} = 0.0462\,\text{h}^{-1}$$

Let R_0 = initial activity = 1 microcurie

$= 3.7 \times 10^4$ disintegrations per second.

r = Activity in $1\,\text{cm}^3$ of blood at $t = 5\text{h}$

$= \frac{296}{60}$ disintegration per second

$= 4.93$ disintegration per second, and

R = Activity of whole blood at time $t = 5\text{h}$

Total volume of blood should be

$$V = \frac{R}{r} = \frac{R_0 e^{-\lambda t}}{r}$$

Substituting the values, we have

$$V = \left(\frac{3.7 \times 10^4}{4.93}\right) e^{-(0.0462)(5)}\,\text{cm}^3$$

$$V = 5.95 \times 10^3\,\text{cm}^3 \quad \text{or} \quad V = 5.95\,\text{L}$$

41. Speed of neutrons $= \sqrt{\frac{2K}{m}}$ $\left(\text{From } K = \frac{1}{2}mv^2\right)$

or $$v = \sqrt{\frac{2 \times 0.0327 \times 1.6 \times 10^{-19}}{1.675 \times 10^{-27}}}$$

$$\approx 2.5 \times 10^3\,\text{m/s}$$

Time taken by the neutrons to travel a distance of 10 m :

$$t = \frac{d}{v} = \frac{10}{2.5 \times 10^3} = 4.0 \times 10^{-3}$$

Number of neutrons decayed after time

$$t : N = N_0(1 - e^{-\lambda t})$$

$\therefore$ Fraction of neutrons that will decay in this time interval

$$= \frac{N}{N_0} = (1 - e^{-\lambda t}) = 1 - e^{-\frac{\ln(2)}{700} \times 4.0 \times 10^{-3}}$$

$$= 3.96 \times 10^{-6}$$

42. $${}_{92}\text{U}^{238} \xrightarrow{\alpha\text{-decay}} {}_{90}X^{234} \xrightarrow{\beta\text{ - decay}} {}_{91}Y^{234}$$

During an α-decay atomic number decreases by 2 and mass number by 4. During a β-decay, atomic number increases by 1 while mass number remains unchanged.

Topic 4 X-Rays and de-Broglie Wavelength

1. By Bohr's IInd postulate, for revolving electron,

$$\text{Angular momentum} = \frac{nh}{2\pi} \Rightarrow mvr_n = \frac{nh}{2\pi}$$

$\Rightarrow$ Momentum of electron, $p = mv = \frac{nh}{2\pi r_n}$

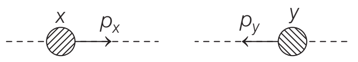
de-Broglie wavelength associated with electron is

$$\lambda_n = \frac{h}{p} = \frac{2\pi r_n}{n}$$

Given, $n = 3$, $r_n = 4.65\,\text{Å}$

$\therefore$ $$\lambda_n = \frac{(2 \times \pi \times 4.65)}{3} \approx 9.7\,\text{Å}$$

2. Initially,

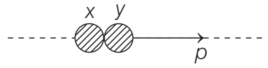


We have, de-Broglie wavelengths associated with particles are

$$\lambda_x = \frac{h}{p_x} \text{ and } \lambda_y = \frac{h}{p_y}$$

$\Rightarrow$ $$p_x = \frac{h}{\lambda_x} \text{ and } p_y = \frac{h}{\lambda_y}$$

Finally, particles collided to form a single particle.



As we know that linear momentum is conserved in collision, so

$$\mathbf{p}_p = |\,\mathbf{p}_x - \mathbf{p}_y\,| \Rightarrow \mathbf{p}_p = \left|\frac{h}{\lambda_x} - \frac{h}{\lambda_y}\right|$$

So, de-Broglie wavelength of combined particle is

$$\lambda_p = \frac{h}{|\,\mathbf{p}_p\,|} = \frac{h}{\left|\frac{h}{\lambda_x} - \frac{h}{\lambda_y}\right|} = \frac{h}{\left|\frac{h\lambda_y - h\lambda_x}{\lambda_x\lambda_y}\right|} = \frac{\lambda_x\lambda_y}{|\,\lambda_x - \lambda_y\,|}$$

3. Given, de-Broglie wavelengths for particles are λ_1 and λ_2.

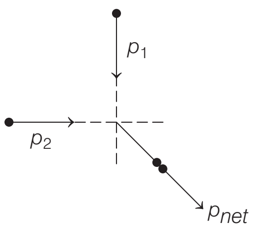
So, $\lambda_1 = \frac{h}{p_1}$ and $\lambda_2 = \frac{h}{p_2}$

and momentum of particles are

$$p_1 = \frac{h}{\lambda_1} \text{ and } p_2 = \frac{h}{\lambda_2}$$

Given that, particles are moving perpendicular to each other and collide inelastically.

So, they move as a single particle.



So, by conservation of momentum and vector addition law, net momentum after collision,

$$p_{\text{net}} = \sqrt{p_1^2 + p_2^2 + 2p_1p_2\cos 90^\circ} = \sqrt{p_1^2 + p_2^2}$$

Since, $p_1 = \dfrac{h}{\lambda_1}$ and $p_2 = \dfrac{h}{\lambda_2}$

So, $$p_{\text{net}} = \sqrt{\frac{h^2}{\lambda_1^2} + \frac{h^2}{\lambda_2^2}} \quad \text{...(i)}$$

Let the de-Broglie wavelength after the collision is λ_{net}, then

$$p_{\text{net}} = \frac{h}{\lambda_{\text{net}}} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{h}{\lambda_{\text{net}}} = \sqrt{\frac{h^2}{\lambda_1^2} + \frac{h^2}{\lambda_2^2}} \Rightarrow \frac{1}{\lambda_{\text{net}}^2} = \frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2}$$

4. de-Broglie wavelength associated with a moving charged particle of charge q is

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mqV}}$$

where, $V =$ accelerating potential.

Ratio of de-Broglie wavelength for particle A and B is,

$$\frac{\lambda_A}{\lambda_B} = \frac{\sqrt{m_B q_B V_B}}{\sqrt{m_A q_A V_A}} = \sqrt{\frac{m_B}{m_A}} \cdot \sqrt{\frac{q_B}{q_A}} \cdot \sqrt{\frac{V_B}{V_A}}$$

Substituting the given values, we get.

$$= \sqrt{\frac{4m}{m}} \cdot \sqrt{\frac{q}{q}} \cdot \sqrt{\frac{2500}{50}}$$

$$= 2 \times 1 \times 5 \times 1.414 = 14.14$$

5. Wavelength of the given photon is given as,

$$\lambda_p = \frac{c}{\nu_p} = \frac{3 \times 10^8}{6 \times 10^{14}} \text{ m}$$

$$= 5 \times 10^{-7} \text{m} \quad \text{...(i)}$$

As, it is given that, de-Broglie wavelength of the electron is

$$\lambda_e = 10^{-3} \times \lambda_p \quad [\because \text{using Eq. (i)}]$$

$$= 5 \times 10^{-10} \text{m}$$

Also, the de-Broglie wavelength of an electron is given as,

$$\lambda_e = \frac{h}{p} = \frac{h}{mv_e} \Rightarrow v_e = \frac{h}{\lambda_e m_e}$$

Substituting the given values, we get

$$= \frac{6.63 \times 10^{-34}}{5 \times 10^{-10} \times 9.1 \times 10^{-31}} \text{ m/s}$$

$$= 1.45 \times 10^6 \text{ m/s}$$

6. $$2\pi r = n\lambda_n$$

$$\lambda_n = \frac{2\pi r}{n} = \frac{2\pi r_0 n^2}{n} = 2\pi r_0 n \quad \text{...(i)}$$

$$\frac{1}{\Lambda_n} = R\left\{\frac{1}{1^2} - \frac{1}{n^2}\right\}$$

$$\Lambda_n = \frac{1}{R}\left\{1 + \frac{1}{n^2 - 1}\right\}$$

$$\Lambda_n = \frac{1}{R}\left\{1 + \frac{1}{n^2}\right\} \quad (n >> 1) \quad \text{...(ii)}$$

From Eqs. (i) and (ii),

$$\Lambda_n = \frac{1}{R}\left\{1 + \frac{4\pi^2 r_0^2}{\lambda_n^2}\right\} = A + \frac{B}{\lambda_n^2}$$

7. For elastic collision,

$$p_{\textbf{before collision}} = p_{\textbf{after collision}}.$$

$$mv = mv_A + \frac{m}{2}v_B$$

$$2v = 2v_A + v_B \quad \text{...(i)}$$

Now, coefficient of restitution,

$$e = \frac{v_B - v_A}{u_A - v_B}$$

Here, $u_B = 0$ (Particle at rest) and for elastic collision $e = 1$

$$\therefore \quad 1 = \frac{v_B - v_A}{v}$$

$$\Rightarrow \quad v = v_B - v_A \quad \text{...(ii)}$$

From Eq. (i) and Eq. (ii)

$$v_A = \frac{v}{3} \text{ and } v_B = \frac{4v}{3}$$

Hence, $$\frac{\lambda_A}{\lambda_B} = \frac{\left(\dfrac{h}{mV_A}\right)}{\dfrac{h}{\dfrac{m}{2}.V_B}} = \frac{V_B}{2V_A} = \frac{4/3}{2/3} = 2$$

8. According to photoelectric effect equation

$$\text{KE}_{\text{max}} = \frac{hc}{\lambda} - \phi_0$$

$$\frac{p^2}{2m} = \frac{hc}{\lambda} - \phi_0 \quad [\text{KE} = p^2/2m]$$

$$\frac{(h/\lambda_d)^2}{2m} = \frac{hc}{\lambda} - \phi_0 \quad [p = h/\lambda]$$

Assuming small changes, differentiating both sides,

$$\frac{h^2}{2m}\left(-\frac{2d\lambda_d}{\lambda_d^3}\right) = -\frac{hc}{\lambda^2}d\lambda, \quad \frac{d\lambda_d}{d\lambda} \propto \frac{\lambda_d^3}{\lambda^2}$$

9. $$K_{\text{max}} = \frac{hc}{\lambda_{\text{ph}}} - \phi$$

Kinetic energy of electron reaching the anode will be

$$K = \frac{hc}{\lambda_{\text{ph}}} - \phi + eV$$

Now, $$\lambda_e = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2m\left(\dfrac{hc}{\lambda_{\text{ph}}} - \phi + eV\right)}}$$

If eV >> ϕ then, $\lambda_e = \dfrac{h}{\sqrt{2m\left(\dfrac{hc}{\lambda_{ph}} + eV\right)}}$

If $V_f = 4V_i$ then, $(\lambda_e)_f \simeq \dfrac{(\lambda_e)_i}{2}$

10. K_α transition takes place from $n_1 = 2$ to $n_2 = 1$

$$\therefore \quad \frac{1}{\lambda} = R(Z-b)^2\left[\frac{1}{(1)^2} - \frac{1}{(2)^2}\right]$$

For K-series, $b = 1$

$$\therefore \quad \frac{1}{\lambda} \propto (Z-1)^2$$

$$\Rightarrow \quad \frac{\lambda_{Cu}}{\lambda_{Mo}} = \frac{(z_{Mo}-1)^2}{(z_{Cu}-1)^2} = \frac{(42-1)^2}{(29-1)^2}$$

$$= \frac{41\times 41}{28\times 28} = \frac{1681}{784} = 2.144$$

11. Cut-off wavelength depends on the applied voltage not on the atomic number of the target. Characteristic wavelengths depend on the atomic number of target.

12. Momentum of striking electrons

$$p = \frac{h}{\lambda}$$

$\therefore$ Kinetic energy of striking electrons

$$K = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}$$

This is also, maximum energy of X-ray photons.

Therefore, $\dfrac{hc}{\lambda_0} = \dfrac{h^2}{2m\lambda^2}$ or $\lambda_0 = \dfrac{2m\lambda^2 c}{h}$

13. $$\frac{1}{\lambda} \propto (Z-1)^2$$

$$\therefore \quad \frac{\lambda_1}{\lambda_2} = \left(\frac{Z_2-1}{Z_1-1}\right)^2 \text{ or } \frac{1}{4} = \left(\frac{Z_2-1}{11-1}\right)^2$$

Solving this, we get $Z_2 = 6$

14. $$\frac{\lambda_1}{\lambda_2} = \frac{\dfrac{h}{\sqrt{2mE}}}{\dfrac{hc}{E}} \text{ or } \frac{\lambda_1}{\lambda_2} \propto E^{1/2}$$

15. $$i = \frac{q}{t} = \frac{ne}{t} \quad \therefore \quad n = \frac{it}{e}$$

Substituting $i = 3.2\times 10^{-3}$ A,

$$e = 1.6\times 10^{-19} \text{ C and } t = 1\text{s}$$

we get, $n = 2\times 10^{16}$

16. Wavelength λ_k is independent of the accelerating voltage (V), while the minimum wavelength λ_c is inversely proportional to V. Therefore, as V is increased, λ_k remains unchanged whereas λ_c decreases or $\lambda_k - \lambda_c$ will increase.

17. Minimum wavelength of continuous X-ray spectrum is given by $\lambda_{min}\text{ (in Å)} = \dfrac{12375}{E\text{ (in eV)}}$

Here E = energy of incident electrons (in eV)

= energy corresponding to minimum wavelength λ_{min} of X-rays.

$$E = 80\text{ keV} = 80\times 10^3 \text{ eV}$$

$$\therefore \quad \lambda_{min}\text{ (in Å)} = \frac{12375}{80\times 10^3} \approx 0.155$$

Also the energy of the incident electrons (80 keV) is more than the ionization energy of the K-shell electrons (*i.e.*, 72.5 keV). Therefore, characteristic X-ray spectrum will also be obtained because energy of incident electron is high enough to knock out the electron from K or L-shells.

18. From law of conservation of momentum,

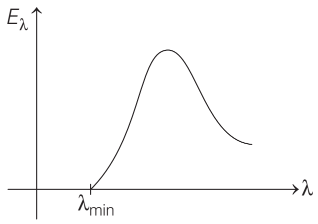
$$p_1 = p_2 \quad \text{(in opposite directions)}$$

Now de-Broglie wavelength is given by

$$\lambda = \frac{h}{p}, \text{ where } h = \text{Planck's constant}$$

Since magnitude of momentum (p) of both the particles is equal, therefore $\lambda_1 = \lambda_2$ or $\lambda_1/\lambda_2 = 1$

19. The continuous X-ray spectrum is shown in figure.



All wavelengths >λ_{min} are found, where

$$\lambda_{min} = \frac{12375}{V\text{ (in volt)}}\text{ Å}$$

Here, V is the applied voltage.

20. $\lambda_{k_\alpha} = 0.021\text{nm} = 0.21\text{Å}$

Since, λ_{k_α} corresponds to the transition of an electron from L-shell to K-shell, therefore

$$E_L - E_K = \text{(in eV)} = \frac{12375}{\lambda\text{ (in Å)}}$$

$$= \frac{12375}{0.21} \approx 58928\text{ eV}$$

or $\Delta E \approx 59$ keV

22. Shortest wavelength or cut-off wavelength depends only upon the voltage applied in the coolidge tube.

23. Cut-off wavelength depends on the accelerating voltage, not the characteristic wavelengths. Further, approximately 2% kinetic energy of the electrons is utilised in producing X-rays. Rest 98% is lost in heat.

24. $\lambda_m \text{ (in Å)} = \dfrac{12375}{V \text{ (in volt)}}$

With increase in V, λ_m will decrease. With decrease in λ_m energy of emitted photons will increase. And hence intensity will increase even if number of photons emitted per second are constant. Because intensity is basically energy per unit area per unit time.

25. Angular momentum $= n\left(\dfrac{h}{2\pi}\right) = 3\left(\dfrac{h}{2\pi}\right)$

$\therefore \quad n = 3$

Now, $\quad r_n \propto \dfrac{n^2}{z}$

$\therefore \quad r_3 = \dfrac{(3)^2}{3}(a_0) = 3a_0$

Now, $mv_3 r_3 = 3\left(\dfrac{h}{2\pi}\right)$

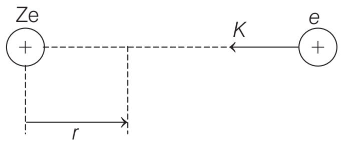
$\therefore \quad mv_3(3a_0) = 3\left(\dfrac{h}{2\pi}\right)$

or $\quad \dfrac{h}{mv_3} = 2\pi a_0$ or $\dfrac{h}{P_3} = 2\pi a_0 \qquad (\because P = mv)$

or $\quad \lambda_3 = 2\pi a_0 \qquad \left(\lambda = \dfrac{h}{P}\right)$

$\therefore$ Answer is 2.

26.



r = closest distance = 10 fm.

From energy conservation, we have

$$K_i + U_i = K_f + U_f$$

or $$K + 0 = 0 + \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r}$$

or $$K = \frac{1}{4\pi\varepsilon_0} \cdot \frac{(120\,e)(e)}{r} \qquad \text{...(i)}$$

de-Broglie wavelength

$$\lambda = \frac{h}{\sqrt{2Km}} \qquad \text{...(ii)}$$

Substituting the given values in above two equations, we get

$$\lambda = 7 \times 10^{-15} \text{ m} = 7 \text{ fm}$$

27. $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2qqVm}}$ or $\lambda \propto \dfrac{1}{\sqrt{qm}}$

$$\frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{q_\alpha}{q_p} \cdot \frac{m_\alpha}{m_p}} = \sqrt{\frac{(2)(4)}{(1)(1)}} = 2.828$$

The nearest integer is 3.

$\therefore$ Answer is 3.

28. $\dfrac{1}{\lambda_{K_\alpha}} = R(Z-1)^2\left(\dfrac{1}{1^2} - \dfrac{1}{2^2}\right) = \dfrac{3}{4}R(Z-1)^2$

$\therefore \quad (Z-1) = \dfrac{2}{\sqrt{3R\lambda_{K_\alpha}}}$

Here, R = Rydberg's constant

$= 1.097 \times 10^7 \text{ m}^{-1}$ and

$\lambda_{K_\alpha} = 0.76 \text{ Å} = 0.76 \times 10^{-10}$ m

Substituting the values, we have

or $\quad Z = 39$

29. $\lambda_{\min} \text{(in Å)} = \dfrac{12375}{V \text{ (in volts)}}$

$$\lambda_{\min} = \frac{12375}{20 \times 10^3} = 0.62 \text{ Å}$$

30. X-rays are electromagnetic waves and electromagnetic waves travel with the speed of light i.e. 3.0×10^8 m/s.

31. K_α corresponds to $n = 2$ to $n = 1$

and K_β corresponds to $n = 3$ to $n = 1$

Since, $E_{31} > E_{21}$

$\therefore \quad \lambda_{K_\beta} < \lambda_{K_\alpha}$ or $\lambda_{K_\beta} = \lambda_{K_\alpha}\left[\dfrac{\dfrac{1}{1^2} - \dfrac{1}{2^2}}{\dfrac{1}{1^2} - \dfrac{1}{3^2}}\right]$

$$= 0.32\left(\frac{3/4}{8/9}\right) = 0.27 \text{Å}$$

32. Cut-off wavelength is given by

$$\lambda_{\min} = \left\{\frac{12375}{V \text{(in volts)}}\right\} \text{(in Å)}$$

with increase in applied voltage V, speed of electrons striking the anode is increased or cut-off wavelength of the emitted X-rays decreases.

Further, with increase in number of electrons striking the anode more number of photons of X-rays will be emitted. Therefore, intensity of X-rays will increase.

33. Minimum voltage required is corresponding to $n = 1$ to $n = 2$.

Binding energy of the innermost electron is given as 40 keV i.e. ionisation potential is 40 kV. Therefore,

$$V_{\min} = \frac{40 \times 10^3 \left(\dfrac{1}{1^2} - \dfrac{1}{2^2}\right)}{\left(\dfrac{1}{1^2} - \dfrac{1}{\infty}\right)} = 30 \times 10^3 \text{ V.}$$

The energy of the characteristic radiation will be 30×10^3 eV.

34. nth line of Lymen series means transition from $(n+1)^{\text{th}}$ state to first state.

$$\frac{1}{\lambda} = RZ^2\left[1 - \frac{1}{(n+1)^2}\right] \qquad \text{...(i)}$$

de-Broglie wavelength in $(n+1)^{\text{th}}$ orbit :

$$\lambda = \frac{h}{mv} = \frac{hr}{mvr} = \frac{(2\pi)(hr)}{(n+1)h} = \frac{2\pi r}{(n+1)}$$

or $$\frac{1}{\lambda} = \frac{(n+1)}{2\pi r} \qquad \text{...(ii)}$$

Equating Eqs. (i) and (ii), we get

$$\left(\frac{n+1}{2\pi r}\right) = RZ^2\left[\frac{n(n+2)}{(n+1)^2}\right] \qquad \text{...(iii)}$$

Now, as $$r \propto \frac{n^2}{Z}$$

$\therefore$ $$r = \frac{(n+1)^2}{11} r_o$$

Substituting in Eq. (iii), we get

$$\frac{11}{2\pi r_o} = \frac{R(11)^2(n)(n+2)}{(n+1)}$$

or $$(n+1) = (1.09\times 10^7)(11)(2\pi)\times (0.529\times 10^{-10})(n^2+2n)$$

Solving this equation,

We get, $n = 24$

35. (a) From the relation $r \propto A^{1/3}$,

we have, $$\frac{r_2}{r_1} = \left(\frac{A_2}{A_1}\right)^{1/3} \text{ or } \left(\frac{A_2}{4}\right)^{1/3} = (14)^{1/3}$$

$\therefore$ $A_2 = 56$

(b) $Z_2 = A_2 -$ number of neutrons

$= 56 - 30 = 26$

$$\therefore\ fk_\alpha = Rc(Z-1)^2\left(\frac{1}{1^2}-\frac{1}{2^2}\right) = \frac{3Rc}{4}(Z-1)^2$$

Substituting the given values of R, c and Z, we get

$$fk_\alpha = 1.55\times 10^{18} \text{ Hz}$$

36. For $0 \le x \le 1$, PE $= E_0$

$\therefore$ Kinetic energy $K_1 =$ Total energy – PE

$= 2E_0 - E_0 = E_0$

$\therefore$ $$\lambda_1 = \frac{h}{\sqrt{2mE_0}} \qquad \text{...(i)}$$

For $x > 1$, PE $= 0$

$\therefore$ Kinetic energy $K_2 =$ Total energy $= 2E_0$

$\therefore$ $$\lambda_2 = \frac{h}{\sqrt{2m(E_0)}} \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we have

$$\frac{\lambda_1}{\lambda_2} = \sqrt{2}$$

37. $$\Delta E = h\nu = Rhc(Z-b)^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$$

For K-series, $b = 1$

$\therefore$ $$\nu = Rc(Z-1)^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$$

Substituting the values,

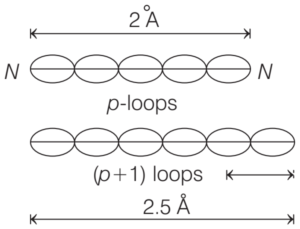
$$4.2\times 10^{18} = (1.1\times 10^7)(3\times 10^8)(Z-1)^2\left(\frac{1}{1}-\frac{1}{4}\right)$$

$\therefore$ $(Z-1)^2 = 1697$

or $Z - 1 \approx 41$

or $Z = 42$

38. From the figure it is clear that



$$p\cdot(\lambda/2) = 2\text{ Å}$$

$$(p+1)\cdot\lambda/2 = 2.5\text{ Å}$$

$\therefore$ $\lambda/2 = (2.5-2.0)\text{Å} = 0.5\text{ Å}$

or $\lambda = 1\text{ Å} = 10^{-10}$ m.

(a) de-Broglie wavelength is given by

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2Km}}$$ Here, $K =$ kinetic energy of electron

$$\therefore\ K = \frac{h^2}{2m\lambda^2} = \frac{(6.63\times 10^{-34})^2}{2(9.1\times 10^{-31})(10^{-10})^2}$$

$$= 2.415\times 10^{-17}\text{ J} = \left(\frac{2.415\times 10^{-17}}{1.6\times 10^{-19}}\right)\text{eV}$$

$\therefore$ $K = 150.8$ eV

(b) The least value of d will be, when only one loop is formed .

$\therefore$ $d_{\min} = \lambda/2$ or $d_{\min} = 0.5$Å

Topic 5 Nuclear Physics

1. For substance A, half-life is 10 min, so it decays as

$$N_{0_A} \xrightarrow{10\text{ min}} \frac{N_{0_A}}{2} \xrightarrow{10\text{ min}} \frac{N_{0_A}}{4} \xrightarrow{10\text{ min}} \frac{N_{0_A}}{8}$$

(initial number of nuclei at $t=0$) (Active nuclei remained after 10 min)

$$\xrightarrow{10\text{ min}} \frac{N_{0_A}}{16} \xrightarrow{10\text{ min}} \frac{N_{0_A}}{32} \xrightarrow{10\text{ min}} \frac{N_{0_A}}{64}$$

$\therefore$ For substance A, number of nuclei decayed in 60 min is

$$N_{0_A} - \frac{N_{0_A}}{64} = \frac{63N_{0_A}}{64}$$

Similarly, for substance B, half-life is 20 min, so it's decay scheme is

$$N_{0_B} \xrightarrow{20\text{ min}} \frac{N_{0_B}}{2} \xrightarrow{20\text{ min}} \frac{N_{0_B}}{4} \xrightarrow{20\text{ min}} \frac{N_{0_B}}{8}$$

So, number of nuclei of B decayed in 60 min is

$$N_{0_B} - \frac{N_{0_B}}{8} = \frac{7}{8}N_{0_B}$$

Hence, ratio of decayed nuclei of A and B in 60 min is

$$\frac{\frac{63}{64}N_{0_A}}{\frac{7}{8}N_{0_B}} = \frac{9}{8} \qquad [\because N_{0_A} = N_{0_B}]$$

Alternate Solution

Number of active nuclei remained after 60 min can also be calculated as

$$N' = \frac{N}{2^{T/t_{1/2}}}$$

where, $T = 60$ min

So, for nuclei A,

$$N'_{0_A} = \frac{N_{0_A}}{2^{\frac{60}{10}}} = \frac{N_{0_A}}{2^6} = \frac{N_{0_A}}{64}$$

Similarly, for nuclei B,

$$N'_{0_B} = \frac{N_{0_B}}{2^{\frac{60}{20}}} = \frac{N_{0_B}}{2^3} = \frac{N_{0_B}}{8}$$

2. Number of active nuclei remained after time t in a sample of radioactive substance is given by

$$N = N_0 e^{-\lambda t}$$

where, N_0 = initial number of nuclei at $t = 0$ and λ = decay constant.

Here, at $t = 0$,

Number of nuclei in sample A and B are equal, i.e.

$$N_{0_A} = N_{0_B} = N_0$$

Also, $\lambda_A = 5\lambda$ and $\lambda_B = \lambda$

So, after time t, number of active nuclei of A and B are

$$N_A = N_0 e^{-5\lambda t} \text{ and } N_B = N_0 e^{-\lambda t}$$

If $\frac{N_A}{N_B} = \frac{1}{e^2}$, then

$$\frac{N_A}{N_B} = \frac{N_0 e^{-5\lambda t}}{N_0 e^{-\lambda t}} = \frac{1}{e^2} \Rightarrow e^{(\lambda - 5\lambda)t} = e^{-2}$$

Comparing the power of e on both sides, we get

$$4\lambda t = 2$$

$$\Rightarrow \qquad t = \frac{1}{2\lambda}$$

3. Given, $\lambda_A = 10\lambda$ and $\lambda_B = \lambda$

Number of nuclei (at any instant) present in material is

$$N = N_0 e^{-\lambda t}$$

So, for materials A and B, we can write

$$\frac{N_A}{N_B} = \frac{e^{-\lambda_A t}}{e^{-\lambda_B t}} = e^{-(\lambda_A - \lambda_B)t} \qquad \ldots \text{(i)}$$

Given,

$$\frac{N_A}{N_B} = \frac{1}{e} \qquad \ldots \text{(ii)}$$

Equating Eqs. (i) and (ii), we get

$$\frac{1}{e} = e^{-(\lambda_A - \lambda_B)t}$$

$$\Rightarrow \qquad e^{-1} = e^{-(\lambda_A - \lambda_B)t}$$

Comparing the power of 'e' on both sides, we get

or $\qquad (\lambda_A - \lambda_B)t = 1$

$$\Rightarrow \qquad t = \frac{1}{\lambda_A - \lambda_B}$$

By putting values of λ_A and λ_B in the above equation, we get

$$t = \frac{1}{10\lambda - \lambda} \Rightarrow t = \frac{1}{9\lambda}$$

4. Mass density of nuclear matter is a constant quantity for all elements. It does not depends on element's mass number or atomic radius.

$\therefore$ The ratio of mass densities of ^{40}Ca and ^{16}O is 1 : 1.

5. Energy absorbed or released in a nuclear reaction is given by

ΔQ = Binding energy of products − Binding energy of reactants.

If energy is absorbed, ΔQ is negative and if it is positive then energy is released. Also, Binding energy = Binding energy per nucleon $\times$ Number of nucleons.

Here, binding energy of products

$$= 2 \times (\text{B.E. of He}^4) + (\text{B.E. of C}^{12})$$

$$= 2(4 \times 7.07) + (12 \times 7.86) = 150.88 \text{ MeV}$$

and binding energy of reactants $= 20 \times 8.03 = 160.6$ MeV

So, $\Delta Q = (\text{B.E.})_{\text{Products}} - (\text{B.E.})_{\text{reactants}}$

$$= 150.88 - 160.6 = -9.72 \text{ MeV}.$$

As ΔQ is negative

$\therefore$ energy of 9.72 Mev is absorbed in the reaction.

$\therefore$ No option is correct.

6. Electrostatic energy

= Binding energy of N − Binding energy of O

$$= [[7M_H + 8M_n - M_N] - [8M_H + 7M_n - M_O]] \times C^2$$

$$= [-M_H + M_n + M_O\ M_N]C^2$$

$$= [-1.007825 + 1.008665 + 15.003065 - 15.000109] \times 93.15$$

$$= +3.5359 \text{ MeV}$$

$$\Delta E = \frac{3}{5} \times \frac{1.44 \times 8 \times 7}{R} = 3.5359$$

$$R = \frac{3 \times 1.44 \times 14}{5 \times 3.5359} = 3.42 \text{ fm}$$

7. From conservation laws of mass number and atomic number, we can say that $x = n$, $y = n$

$$(x = {}_0^1n,\ y = {}_0^1n)$$

$\therefore$ Only (a) and (d) options may be correct.

From conservation of momentum, $|P_{xe}| = |P_{sr}|$

From $$K = \frac{P^2}{2m} \Rightarrow K \propto \frac{1}{m}$$

$$\frac{K_{sr}}{K_{xe}} = \frac{m_{xe}}{m_{sr}}$$

$\therefore$ $$K_{sr} = 129\ \text{MeV},\ K_{xe} = 86\ \text{MeV}$$

NOTE There is no need of finding total energy released in the process.

8. Rest mass of parent nucleus should be greater than the rest mass of daughter nuclei. Therefore, option (a) will be correct.

9. $4\,({}_2\text{He}^4) = {}_8\text{O}^{16}$

Mass defect, $\Delta m = \{4\,(4.0026) - 15.9994\} = 0.011$ amu

$\therefore$ Energy released per oxygen nuclei

$= (0.011)\,(931.48)$ MeV $= 10.24$ MeV

10. Given that $K_1 + K_2 = 5.5$ MeV …(i)

From conservation of linear momentum,

$$p_1 = p_2$$

or $\sqrt{2K_1\,(216m)} = \sqrt{2K_2\,(4m)}$ as $p = \sqrt{2Km}$

$\therefore$ $K_2 = 54\,K_1$ …(ii)

Solving Eqs. (i) and (ii), we get

K_2 = KE of α-particle = 5.4 MeV.

11. Nuclear density is constant hence, mass $\propto$ volume

or $m \propto V$

12. Radius of a nucleus is given by

$$R = R_0\,A^{1/3} \quad (\text{where } R_0 = 1.25 \times 10^{-15}\ \text{m})$$

$$= 1.25\,A^{1/3} \times 10^{-15}\ \text{m}$$

Here A is the mass number and mass of the uranium nucleus will be

$m \approx Am_p$ where m_p = mass of proton

$$= A\,(1.67 \times 10^{-27}\ \text{kg})$$

$$\therefore \text{ Density } \rho = \frac{\text{mass}}{\text{volume}} = \frac{m}{\frac{4}{3}\pi R^3}$$

$$= \frac{A\,(1.67 \times 10^{-27}\ \text{kg})}{A\,(1.25 \times 10^{-15}\ \text{m})^3} \text{ or } \rho \approx 2.0 \times 10^{17}\ \text{kg/m}^3$$

13. Energy is released in a process when total binding energy of the nucleus (= binding energy per nucleon × number of nucleons) is increased or we can say, when total binding energy of products is more than the reactants. By calculation we can see that only in option (c), this happens.

Given : $W \to 2Y$

Binding energy of reactants $= 120 \times 7.5 = 900$ MeV

and binding energy of products $= 2\,(60 \times 8.5)$

$= 1020$ MeV > 900 MeV

14. Heavy water is used as moderators in nuclear reactors to slow down the neutrons.

15. The given reactions are :

$${}_1\text{H}^2 + {}_1\text{H}^2 \longrightarrow {}_1\text{H}^3 + p$$
$${}_1\text{H}^2 + {}_1\text{H}^3 \longrightarrow {}_2\text{He}^4 + n$$
$$3\,{}_1\text{H}^2 \longrightarrow {}_2\text{He}^4 + n + p$$

Mass defect

$\Delta m = (3 \times 2.014 - 4.001 - 1.007 - 1.008)$ amu

$= 0.026$ amu

Energy released $= 0.026 \times 931$ MeV

$= 0.026 \times 931 \times 1.6 \times 10^{-13}\ \text{J} = 3.87 \times 10^{-12}$ J

This is the energy produced by the consumption of three deuteron atoms.

$\therefore$ Total energy released by 10^{40} deuterons

$$= \frac{10^{40}}{3} \times 3.87 \times 10^{-12}\ \text{J} = 1.29 \times 10^{28}\ \text{J}$$

The average power radiated is $P = 10^{16}$ W or 10^{16} J/s.

Therefore, total time to exhaust all deuterons of the star will be $t = \frac{1.29 \times 10^{28}}{10^{16}} = 1.29 \times 10^{12}\ \text{s} \approx 10^{12}\ \text{s}$

17. During fusion process two or more lighter nuclei combine to form a heavy nucleus.

18. (a) ${}_3\text{Li}^7 \to {}_2\text{He}^4 + {}_1\text{H}^3$

$\Delta m = [M_{Li} - M_{He} - M_{H^3}]$

$= [6.01513 - 4.002603 - 3.016050]$

$= -1.003523$ u

Δm is negative so reaction is not possible.

(b) ${}_{84}\text{Po}^{210} \to {}_{83}\text{Bi}^{209} + {}_1\text{P}^1$

Δm is negative so reaction is not possible.

(c) ${}_1\text{H}^2 \to {}_2\text{He}^4 + {}_3\text{Li}^6$

Δm is positive so reaction is possible.

(d) ${}_{30}\text{Zn}^{70} + {}_{34}\text{Se}^{82} \to {}_{64}\text{Gd}^{152}$

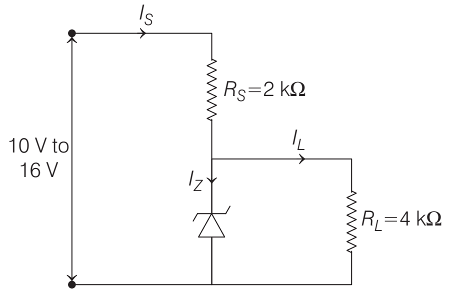
Δm is positive so reaction is not possible.

19. ${}_{84}\text{Po}^{210} \longrightarrow {}_2\text{He}^4 + {}_{82}\text{Pb}^{206}$

Mass defect $\Delta m = (m_{Po} - M_{He} - m_{Pb}) = 0.005818$ u

$\therefore$ $Q = (\Delta m)\,(931.48)$ MeV $= 5.4193$ MeV

$= 5419$ keV



From conservation of linear momentum,

$$p_{Pb} = p_\alpha$$

$$\therefore \quad \sqrt{2m_{Pb}\,k_{Pb}} = \sqrt{2m_\alpha\,k_\alpha} \quad \text{or} \quad \frac{k_\alpha}{k_{Pb}} = \frac{m_{Pb}}{m_\alpha} = \frac{206}{4}$$

$$\therefore \quad k_\alpha = \left(\frac{206}{206+4}\right)(k_{\text{total}})$$

$$= \left(\frac{206}{210}\right)(5419) = 5316\ \text{keV}$$

21. From conservation of mechanical energy, we have

$$U_i + K_i = U_f + U_f$$

$$0 + 2(1.5kT) = \frac{1}{4\pi\varepsilon_0}\cdot\frac{(e)(e)}{d} + 0$$

Substituting the values, we get

$$T = 1.4 \times 10^9\ \text{K}$$

22. As given in the paragraph, a reactor is termed successful, if

$$nt_0 > 5 \times 10^{14}\ \text{s cm}^{-3}$$

24. In fusion, two or more lighter nuclei combine to make a comparatively heavier nucleus.

In fission, a heavy nucleus breaks into two or more comparatively lighter nuclei.

Further, energy will be released in a nuclear process if total binding energy increases.

$\therefore$ Correct options are (b) and (d).

25. Due to mass defect (which is finally responsible for the binding energy of the nucleus), mass of a nucleus is always less than the sum of masses of its constituent particles.

${}^{20}_{10}$Ne is made up of 10 protons plus 10 neutrons. Therefore, mass of ${}^{20}_{10}$Ne nucleus, $M_1 < 10\,(m_p + m_n)$.

Also, heavier the nucleus, more is the mass defect.

Thus, $20(m_n + m_p) - M_2 > 10\,(m_p + m_n) - M_1$

or $10\,(m_p + m_n) > M_2 - M_1$

or $M_2 < M_1 + 10\,(m_p + m_n)$

Now since $M_1 < 10\,(m_p + m_n)$

$\therefore$ $M_2 < 2M_1$

26. In nuclear fusion, two or more lighter nuclei are combined to form a relatively heavy nucleus and thus, releasing the energy.

27. In case of ${}_1H^1$, mass number and atomic number are equal and in case of ${}_1H^2$, mass number is greater than its atomic number.

28. In fusion reaction, two or more lighter nuclei combine to form a comparatively heavier nucleus.

29. $Q = (\Delta\ m \text{ in atomic mass unit}) \times 931.4$ MeV

$= (2 \times \text{mass of } {}_1H^2 - \text{mass of } {}_2He^4) \times 931.4$ MeV

$= (2 \times 2.0141 - 4.0024) \times 931.4$ MeV

$Q \approx 24$ MeV (fusion)

30. $2\,{}_1H^2 \longrightarrow {}_2He^4$

Binding energy of two deuterons,

$$E_1 = 2\,[2 \times 1.1] = 4.4\ \text{MeV}$$

Binding energy of helium nucleus,

$$E_2 = 4\,(7.0) = 28.0\ \text{MeV}$$

$\therefore$ Energy released $\Delta E = E_2 - E_1$

$= (28 - 4.4)$ MeV $= 23.6$ MeV

31. Order of magnitude of nuclear density is $10^{17}\ \text{kg/m}^3$.

32. ${}^{12}_5B \longrightarrow {}^{13}_6C + {}^0_{-1}e + \bar{\nu}$

Mass of ${}^{12}_6$C = 12.000 u (by definition of 1 a.m.u.)

Q-value of reaction,

$Q = (M_B - M_C) \times c^2 = (12.014 - 12.000) \times 931.5$

$= 13.041$ MeV

4.041MeV of energy is taken by ${}^{12}_{65}$C*

$\Rightarrow$ Maximum KE of β-particle is $(13.041 - 4.041) = 9$ MeV

33. Let initial power available from the plant is P_0. After time $t = nT$ or n half lives, this will become $\left(\frac{1}{2}\right)^n P_0$. Now, it is given that, $\left(\frac{1}{2}\right)^n P_0 = 12.5\%$ of $P_0 = (0.125)\,P_0$

Solving this equation we get, $n = 3$

34. The reactor produces 1000 MW power or 10^9 J/s. The reactor is to function for 10 yr. Therefore, total energy which the reactor will supply in 10 yr is

E = (power) (time)

$= (10^9\ \text{J/s})\,(10 \times 365 \times 24 \times 3600\text{s})$

$= 3.1536 \times 10^{17}$ J

But since the efficiency of the reactor is only 10%, therefore actual energy needed is 10 times of it or 3.1536×10^{18}J. One uranium atom liberates 200 MeV of energy or $200 \times 1.6 \times 10^{-13}$ J or 3.2×10^{-11} J of energy. So, number of uranium atoms needed are

$$\frac{3.1536 \times 10^{18}}{3.2 \times 10^{-11}} = 0.9855 \times 10^{29}$$

or number of kg-moles of uranium needed are

$$n = \frac{0.9855 \times 10^{29}}{6.02 \times 10^{26}} = 163.7$$

Hence, total mass of uranium required is

$m = (n)M = (163.7)\,(235)$ kg

or $m \approx 38470$ kg

or $m = 3.847 \times 10^4$ kg

35. The reaction involved in α-decay is

$${}^{248}_{96}\text{Cm} \rightarrow {}^{244}_{94}\text{Pu} + {}^4_2\text{He}$$

Mass defect

$\Delta m = \text{mass of } {}^{248}_{96}\text{Cm} - \text{ mass of } {}^{244}_{94}\text{Pu} - \text{mass of } {}^{4}_{2}\text{He}$

$= (248.072220 - 244.064100 - 4.002603)\text{u}$

$= 0.005517\text{u}$

Therefore, energy released in α-decay will be

$$E_\alpha = (0.005517 \times 931)\text{MeV} = 5.136 \text{ MeV}$$

Similarly, $E_{\text{fission}} = 200\text{MeV}$ (given)

Mean life is given as $t_{\text{mean}} = 10^{13}$ s $= 1/\lambda$

$\therefore$ Disintegration constant $\lambda = 10^{-13}$ s^{-1}

Rate of decay at the moment when number of nuclei are 10^{20}

$$= \lambda N = (10^{-13})(10^{20})$$

$$= 10^7 \text{ disintegration per second}$$

Of these disintegrations, 8% are in fission and 92% are in α-decay.

Therefore, energy released per second

$$= (0.08 \times 10^7 \times 200 + 0.92 \times 10^7 \times 5.136) \text{ MeV}$$

$$= 2.074 \times 10^8 \text{ MeV}$$

$\therefore$ Power output (in watt)

$$= \text{energy released per second (J/s)}$$

$$= (2.074 \times 10^8)(1.6 \times 10^{-13})$$

$\therefore$ Power output $= 3.32 \times 10^{-5}$ W

36. (a) $A - 4 = 228$

$\therefore$ $A = 232$

$92 - 2 = Z$ or $Z = 90$

(b) From the relation, $r = \dfrac{\sqrt{2Km}}{Bq}$

$$K_\alpha = \frac{r^2 B^2 q^2}{2m} = \frac{(0.11)^2 (3)^2 (2 \times 1.6 \times 10^{-19})^2}{2 \times 4.003 \times 1.67 \times 10^{-27} \times 1.6 \times 10^{-13}}$$

$= 5.21$ MeV

From the conservation of momentum,

$$p_\gamma = p_\alpha \quad \text{or} \quad \sqrt{2K_\gamma m_\gamma} = \sqrt{2K_\alpha m_\alpha}$$

$$\therefore \quad K_\gamma = \left(\frac{m_\alpha}{m_\gamma}\right) K_\alpha = \frac{4.003}{228.03} \times 5.21$$

$$= 0.09\text{MeV}$$

$\therefore$ Total energy released $= K_\alpha + K_\gamma = 5.3\text{MeV}$

Total binding energy of daugther products

$= [92 \times (\text{mass of proton}) + (232 - 92)(\text{mass of neutron}) - (m_\gamma) - (m_\alpha)] \times 931.48$ MeV

$= [(92 \times 1.008) + (140)(1.009) - 228.03 - 4.003]\ 931.48$ MeV

$= 1828.5$ MeV

$\therefore$ Binding energy of parent nucleus

= binding energy of daughter products – energy released

$= (1828.5 - 5.3)$ MeV $= 1823.2$ MeV

37. Mass defect in the given nuclear reaction :

$$\Delta m = 2(\text{mass of deuterium}) - (\text{mass of helium})$$

$$= 2\,(2.0141) - (4.0026) = 0.0256$$

Therefore, energy released

$$\Delta E = (\Delta m)(931.48) \text{ MeV} = 23.85 \text{ MeV}$$

$$= 23.85 \times 1.6 \times 10^{-13} \text{ J} = 3.82 \times 10^{-12} \text{ J}$$

Efficiency is only 25%, therefore,

$$25\% \text{ of } \Delta E = \left(\frac{25}{100}\right)(3.82 \times 10^{-12}) \text{ J}$$

$$= 9.55 \times 10^{-13} \text{ J}$$

i.e, by the fusion of two deuterium nuclei, 9.55×10^{-13} J energy is available to the nuclear reactor.

Total energy required in one day to run the reactor with a given power of 200 MW :

$$E_{\text{Total}} = 200 \times 10^6 \times 24 \times 3600 = 1.728 \times 10^{13} \text{J}$$

$\therefore$ Total number of deuterium nuclei required for this purpose

$$n = \frac{E_{\text{Total}}}{\Delta E / 2} = \frac{2 \times 1.728 \times 10^{13}}{9.55 \times 10^{-13}}$$

$$= 0.362 \times 10^{26}$$

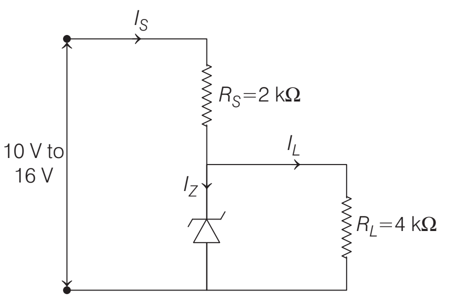
$\therefore$ Mass of deuterium required

= (Number of g-moles of deuterium required) × 2 g

$$= \left(\frac{0.362 \times 10^{26}}{6.02 \times 10^{23}}\right) \times 2 = 120.26 \text{ g.}$$

Topic 6 Semiconductor Devices, Diodes and Triodes

1. In given voltage regulator circuit breakdown of Zener occurs at 6 V.



After breakdown, voltage across load resistance ($R_L = 4\text{k}\Omega$) is,

$$V = V_Z = 6\text{V}$$

$\therefore$ Load current after breakdown,

$$I_L = \frac{V_Z}{R_L} = \frac{6}{4000} = 1.5 \times 10^{-3} \text{ A}$$

When unregulated supply is of 16 V, potential drop occurring across series resistance ($R_S = 2 \text{ k}\Omega$) is;

$$V_S = V - V_Z = 16 - 6 = 10 \text{ V}$$

So, current across series resistance is

$$I_S = \frac{V_S}{R_S} = \frac{10}{2 \times 10^3} = 5 \times 10^{-3} \text{ A}$$

So, current across Zener diode is

$$I_Z = I_S - I_L = 5 \times 10^{-3} - 1.5 \times 10^{-3} = 3.5 \times 10^{-3} \text{ A}$$

2. Given curve is between I_c and I_b as output and input currents, respectively.

So, it is transfer characteristics curve of a common emitter (CE) configuration.

In CE configuration,

$$\text{Current gain, } \beta = \frac{I_{\text{out}}}{I_{\text{in}}} = \frac{I_c}{I_b} \quad \ldots\text{(i)}$$

Voltage gain,

$$A_V = \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{I_c \times R_{\text{out}}}{I_b \times R_{\text{in}}} = \beta \times \frac{R_{\text{out}}}{R_{\text{in}}} \quad \ldots\text{(ii)}$$

and power gain,

$$A_P = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{I_c^2 \times R_{\text{out}}}{I_b^2 \times R_{\text{in}}} = \beta^2 \times \frac{R_{\text{out}}}{R_{\text{in}}} \quad \ldots\text{(iii)}$$

Given, $R_{\text{in}} = 100\ \Omega$ and $R_{\text{out}} = 100 \times 10^3\ \Omega$

From Eq. (i), we get

$$\beta = \frac{5 \text{ mA}}{100\,\mu\text{A}}\left(\text{or } \frac{10 \text{ mA}}{200\,\mu\text{A}} \text{ or } \frac{15 \text{ mA}}{300\,\mu\text{A}} \text{ or } \frac{20 \text{ mA}}{400\,\mu\text{A}}\right)$$

$$\Rightarrow \quad \beta = \frac{5 \times 10^{-3}}{100 \times 10^{-6}} = 50 \ldots\text{(iv)}$$

From Eqs. (ii) and (iv), we get

$$\text{Voltage gain, } A_V = \beta \times \frac{R_{\text{out}}}{R_{\text{in}}} = 50 \times \frac{100 \times 10^3}{100}$$

$$\Rightarrow \quad A_V = 50000 = 5 \times 10^4 \quad \ldots\text{(v)}$$

From Eqs. (iii) and (iv), we get

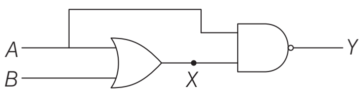
$$\text{Power gain, } A_P = \beta^2 \times \frac{R_{\text{out}}}{R_{\text{in}}}$$

$$= (50)^2 \times \frac{100 \times 10^3}{100}$$

$$= 2500 \times 1000$$

$$\Rightarrow \quad A_P = 2.5 \times 10^6$$

3. Given circuit is



Let the intermediate state X of OR gate is shown in figure.

Clearly, $Y = \overline{AX}$ $\quad \ldots$(i)

Here, $X = A + B$ $\quad \ldots$(ii)

$$\therefore \quad Y = \overline{A\,(A + B)} = \overline{AA + AB}$$

$$= \overline{A + AB} \quad (\because AA = A)$$

$$= \overline{A\,(1 + B)}$$

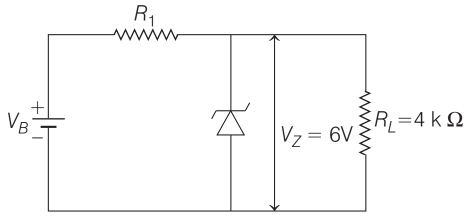
$$\Rightarrow \quad Y = \overline{A} \quad (\because 1 + B = 1)$$

So, truth table shown in option (c) is correct.

Alternate Solution We can solve it using truth table

A	B	$X = A + B$	$Y = \overline{AX}$
0	0	0	1
0	1	1	1
1	0	1	0
1	1	1	0

4. In given voltage regulator circuit,



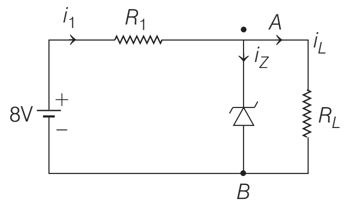
Zener breakdown voltage, $V_Z = 6$ V

So, across R_L, potential drop is always 6 V.

So, current through load resistance is

$$i_L = \frac{V_Z}{R_L} = \frac{6}{4 \times 10^3} = 1.5 \times 10^{-3} \text{ A}$$

Now, when $V_B = 8$ V



Potential drop across $R_1 = 8 - 6 = 2$ V

So, current through R_1 is $i_1 = \dfrac{V}{R_1} = \dfrac{2}{1 \times 10^3} = 2 \times 10^{-3}$ A

So, current through Zener diode is

$$i_Z = i_1 - i_L = 2 \times 10^{-3} - 1.5 \times 10^{-3}$$

$$= 0.5 \times 10^{-3} \text{ A} = 0.5 \text{ mA}$$

Similarly, when $V_B = 16$ V

$$V_{R_1} = 16 - 6 = 10 \text{ V}$$

$$\therefore \quad i_1 = \frac{10}{1 \times 10^3} = 10 \times 10^{-3} \text{ A}$$

Hence,

$$i_Z = i_1 - i_L = 10 \times 10^{-3} - 1.5 \times 10^{-3}$$
$$= 8.5 \times 10^{-3}\ \text{A} = 8.5\ \text{mA}$$

5. Given, $A_P = 60\ \text{dB}$ (in decibel)

Power gain in decibel can be given as

$$A_P = 10 \log_{10}\left(\frac{\text{Output power}}{\text{Input power}}\right)$$

$$\Rightarrow \quad 60 = 10 \log_{10}\left(\frac{P_{\text{out}}}{P_{\text{in}}}\right) \Rightarrow \log_{10}\left(\frac{P_{\text{out}}}{P_{\text{in}}}\right) = 6$$

$$\Rightarrow \quad \frac{P_{\text{out}}}{P_{\text{in}}} = 10^6 = A_P \qquad \ldots \text{(i)}$$

Also, given $R_{\text{out}} = 10\ \text{k}\Omega$, $R_{\text{in}} = 100\ \Omega$

$\therefore$ Power gain of a transistor is given by

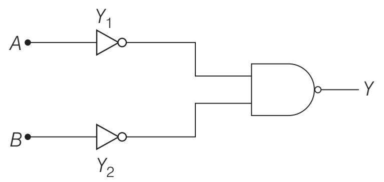
$$A_P = \beta^2 \left(\frac{R_{\text{out}}}{R_{\text{in}}}\right)$$

where, β is current gain.

$$\Rightarrow \beta^2 = A_P \times \frac{R_{\text{in}}}{R_{\text{out}}} = 10^6 \times \frac{100}{10 \times 10^3}$$

$$\Rightarrow \quad \beta^2 = 10^4 \text{ or } \beta = 10^2$$

6. Truth table for given combination of logic gates is

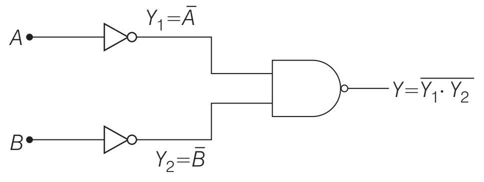


A	B	$Y_1 = \overline{A}$	$Y_2 = \overline{B}$	$Y = \overline{Y_1 \cdot Y_2}$
0	0	1	1	0
1	0	0	1	1
0	1	1	0	1
1	1	0	0	1

Output Y resembles output of an OR gate. So, given combination acts like an OR gate.

Alternate Solution

The given logic gate circuit can be drawn as shown below



Here, $\quad Y = \overline{Y_1 \cdot Y_2} = \overline{\overline{A} \cdot \overline{B}}$

Using de-Morgan's theorem, i.e.

$$\overline{x \cdot y} = \overline{x} + \overline{y}$$

$$\therefore \quad Y = \overline{\overline{A}} + \overline{\overline{B}} = A + B \qquad [\because \overline{\overline{x}} = x]$$

This represents the boolean expression for OR gate.

7. Given, load resistance, $R_L = 1\ \text{k}\Omega$

Input voltage, $V_{\text{in}} = 10\ \text{mV} = 10 \times 10^{-3}\ \text{V}$

Base current, $\Delta I_B = 15\ \mu\text{A} = 15 \times 10^{-6}\ \text{A}$

Collector current, $\Delta I_C = 3\ \text{mA}$

Input resistance,

$$R_{\text{in}} = \frac{V_{\text{in}}}{\Delta I_B} = \frac{10 \times 10^{-3}}{15 \times 10^{-6}} = 0.67\ \text{k}\Omega$$

and voltage gain $= \beta \times \dfrac{R_L}{R_{\text{in}}} = \dfrac{\Delta I_C \times R_L}{\Delta I_B \times R_{\text{in}}}$

$$= \left(\frac{3\ \text{mA}}{15\ \mu\text{A}}\right) \times \left(\frac{1\ \text{k}\Omega}{0.67\ \text{k}\Omega}\right)$$

$$= \left(\frac{3 \times 10^{-3}}{15 \times 10^{-6}}\right)\left(\frac{1 \times 10^3}{0.67 \times 10^3}\right)$$

$$= \frac{1000 \times 3 \times 3}{15 \times 2} = 300 \qquad (\because 0.67 \cong 2/3)$$

Alternate Solution

$$\therefore \text{Voltage gain} = \frac{V_{\text{output}}}{V_{\text{input}}} = \frac{R_L \times \Delta I_C}{V_{\text{in}}}$$

$$= \frac{1 \times 10^3 \times 3 \times 10^{-3}}{10 \times 10^{-3}} = 300$$

8. For a common emitter *n-p-n* transistor, DC current gain is

$$\beta_{\text{DC}} = \frac{I_C}{I_B}$$

At saturation state, V_{CE} becomes zero.

$$\therefore \quad V_{CC} - I_C R_C = 0$$

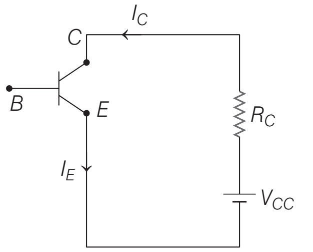
$$I_C \approx \frac{V_{CC}}{R_C} = \frac{10}{1000} = 10^{-2}\ \text{A}$$

Hence, saturation base current,

$$I_B = \frac{I_C}{\beta_{\text{DC}}} = \frac{10^{-2}}{250} = 40\ \mu\text{A}$$

9. Transistor saturation occurs when $V_{CE} = 0$.

Now, for closed loop of collector and emitter by Kirchhoff's voltage rule, we have



$$V_{CE} = V_{CC} - I_C R_C \Rightarrow 0 = V_{CC} - I_C R_c$$

$$\Rightarrow \quad I_C = \frac{V_{CC}}{R_C} = \frac{5}{1 \times 10^3} = 5 \times 10^{-3}\ \text{A}$$

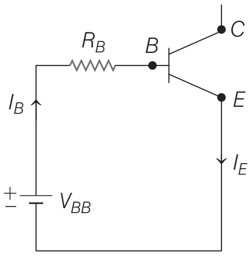
Now, $\beta_{DC} = 200 \text{ (given)} \Rightarrow \dfrac{I_C}{I_B} = \beta_{DC} = 200$

$$\Rightarrow \quad I_B = \frac{I_C}{200} = \frac{5 \times 10^{-3}}{200}$$

$$\Rightarrow \quad I_B = 2.5 \times 10^{-5} = 25\,\mu\text{A}$$

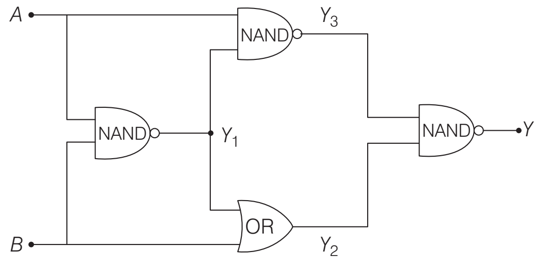
Now, we apply Kirchhoff's voltage rule in base-emitter closed loop, we get

$$V_{BB} = I_B R_B + V_{BE}$$



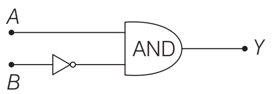
$$\Rightarrow V_{BB} = (25 \times 10^{-6} \times 100 \times 1000) + 1 = 3.5\text{ V}$$

10. Truth table for given circuit is



A	B	Y_1	Y_2	Y_3	Y
0	0	1	1	1	0
1	0	1	1	0	1
0	1	1	1	1	0
1	1	0	1	1	0

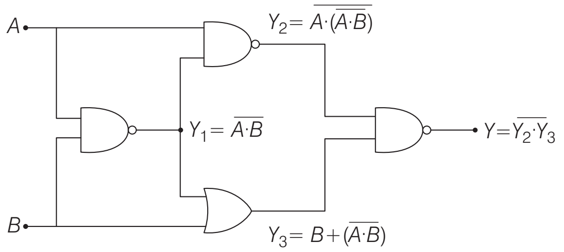
This is the same output produced by $A \cdot \overline{B}$ gate or



So, given circuit is equivalent to Boolean expression $A \cdot \overline{B}$.

Alternate Method

Using the Boolean algebra, output of the given logic circuit can be given as



Here $\quad Y_2 = \overline{A \cdot (\overline{A \cdot B})}$

Using de-Morgan's principle,

$$\overline{x \cdot y} = \overline{x} + \overline{y} \text{ and } \overline{x + y} = \overline{x} \cdot \overline{y}$$

$$\Rightarrow \quad Y_2 = \overline{A} + (\overline{\overline{A \cdot B}}) \qquad [\because \overline{\overline{x}} = x]$$

$$= \overline{A} + (A.B) \qquad \ldots\text{(i)}$$

Similarly, $Y_3 = B + \overline{A} + \overline{B} = 1 + \overline{A} \qquad [\because x + \overline{x} = 1]$

$$\Rightarrow \quad Y_3 = 1 \qquad \text{(ii)} \ [\because \overline{x} + 1 = 1]$$

As, $\quad Y = \overline{Y_2 \cdot Y_3}$

Using Eqs. (i) and (ii), we get

$$Y = \overline{(\overline{A} + A \cdot B)(1)}$$

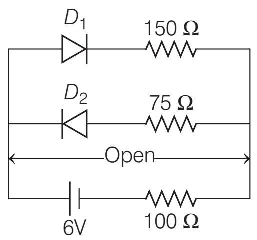
$$= \overline{(\overline{A} + A \cdot B)} + \overline{1} = \overline{\overline{A}} \cdot (\overline{A \cdot B}) + 0$$

$$= A \cdot (\overline{A} + \overline{B}) \qquad [\because x + 0 = x]$$

$$= A \cdot \overline{A} + A \cdot \overline{B} \qquad [\because x\overline{x} = 0]$$

$$= A \cdot \overline{B}$$

11. In this circuit, D_1 is forward biased and D_2 is reversed biased.



Resistance of D_1 is 50Ω.

$\therefore$ Net resistance of the circuit,

$$R_{net} = 50 + 150 + 100 = 300\Omega$$

$\therefore$ Current through the 100Ω resistance

$$= \frac{V}{R_{net}} = \frac{6}{300} = 0.020\text{ A}$$

12. **Key Idea** When the applied reverse voltage (V) reaches the breakdown voltage of the Zener diode, then only a large amount of current is flown through it, otherwise it is approximately zero.

In the given situation, if we consider that Zener diode is at breakdown. Then, potential drop across 1500 Ω resistances will be 10 V. So potential drop at 500 Ω resistor will be 2 V.

$$\therefore \text{ Current in } R_1 = \frac{2}{500} = 4\text{mA} = I_1 \text{ (say)}$$

Current in each

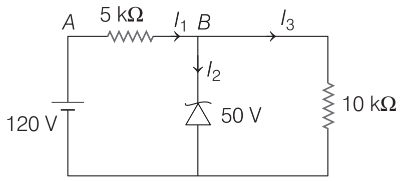
$$R_2 = \frac{10}{750} = \frac{2}{150} = 13.33\text{ mA} = I_2 \text{ (say)}$$

$\Rightarrow \quad I_1 < I_2$ which is not possible.

So, Zener diode will never reach to its breakdown.

$\therefore$ Current flowing through a reverse biased Zener diode $= 0$.

13. In the circuit, let the current in branches is as shown in figure below



By Kirchhoff's node law,

$$I_1 = I_2 + I_3 \qquad \ldots \text{(i)}$$

Now, when diode conducts, voltage difference between points A and B will be

$$V_{AB} = 120 - 50 = 70 \text{ V}$$

So, current $I_1 = \dfrac{V_{AB}}{5\text{ k}\Omega} = \dfrac{70}{5 \times 10^3}$

$$I_1 = 14 \text{ mA} \qquad \ldots \text{(ii)}$$

Since, diode and 10 kΩ resistor are in parallel combination, so voltage across 10 k Ω resistor will be 50 V only.

$$\Rightarrow \qquad I_3 = \frac{50}{10\text{ k}\Omega} = \frac{50}{10 \times 10^3}$$

$$\Rightarrow \qquad I_3 = 5 \text{ mA} \qquad \ldots \text{(iii)}$$

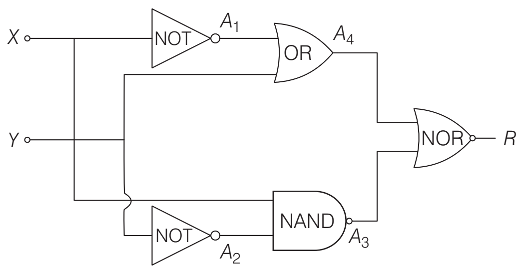
∴ From Eqs. (i), (ii) and (iii), we get

$$14 \text{ mA} = I_2 + 5\text{mA}$$

or current through diode,

$$I_2 = 14 \text{ mA} - 5\text{mA} = 9 \text{ mA}$$

14. The given circuit can be drawn as shown in the figure given below

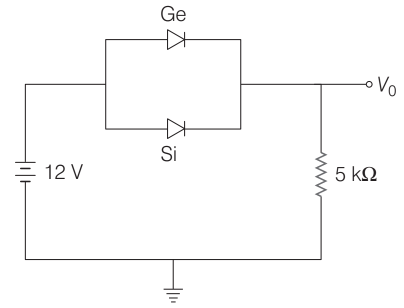


Truth table for this given logic gate is

Given inputs in the options		A_1	A_2	A_3	A_4	R
X	Y					
0	0	1	1	1	1	0
1	0	0	1	0	0	1
1	1	0	0	1	1	0
0	1	1	0	1	1	0

So to get output $R = 1$, inputs must be $X = 1$ and $Y = 0$.

15. Initially Ge and Si are both forward biased. So, current will effectively pass through Ge diode with a voltage drop of 0.3 V.

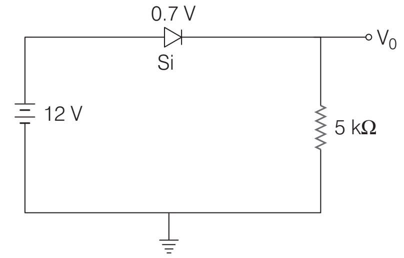


∴ Initial output voltage, $V_0 = 12 - 0.3 = 11.7$ V

If Ge is reversed biased, then only Si diode will work.

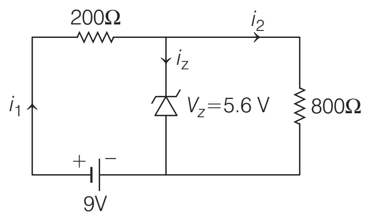
In this condition, output voltage

$$V_0 = 12 - 0.7 = 11.3 \text{ V}$$



∴ Change in output voltage = 11.7 − 11.3 = 0.4 V

16. Given circuit is Zener diode circuit



where, potential drop across 800 Ω resistance = potential drop across Zener diode = 5.6 V

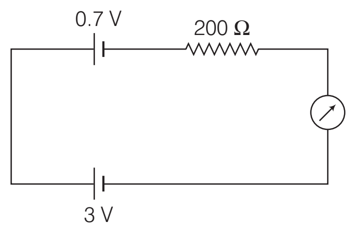
So, current, $i_2 = \dfrac{V}{R} = \dfrac{5.6}{800} = 7\text{mA}$

Now, potential drop across 200 Ω resistance
 = 9 − 5.6 = 3.4 V

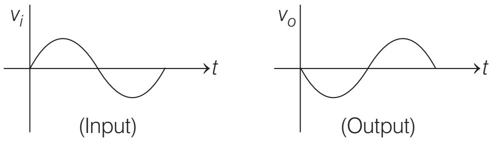
Current, $i_1 = \dfrac{V}{R} = \dfrac{3.4}{200} = 17 \text{ mA}$

So, current, $i_z = i_1 - i_2 = 17 - 7 = 10 \text{ mA}$

17. For silicon diode, potential barrier is 0.7 V. Therefore, the given circuit is as shown in figure.



$$I = \frac{\text{net emf}}{\text{net resistance}} = \frac{3-0.7}{200}\text{A} = 11.5\,\text{mA}$$

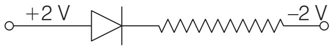
18. 

In a CE n-p-n transistor amplifier, output is 180° out of phase with input.

19. As, we know Cu is conductor, so increase in temperature, resistance will increase. Then, Si is semiconductor, so with increase in temperature, resistance will decrease.

20. Theoretical question. Therefore, no solution is required.

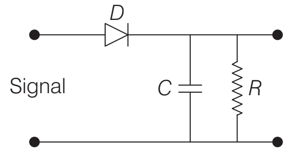
21. For forward bias, *p*-side must be a higher potential than *n*-side.



So, is forward biased.

22. For same value of current higher value of voltage is required for higher frequency.

23. As λ is increased, there will be a value of λ above which photoelectron will cease to come out. So, photocurrent will be zero.

24. 

$$\tau = RC = 100\times 10^3 \times 250\times 10^{-12}\ \text{s}$$
$$= 2.5\times 10^{-5}\ \text{s}$$

The higher frequency which can be detected with tolerable distortion is

$$f = \frac{1}{2\pi m_a RC} = \frac{1}{2\pi \times 0.6\times 2.5\times 10^{-5}}\ \text{Hz}$$
$$= \frac{100\times 10^4}{25\times 1.2\,\pi}\ \text{Hz} = \frac{4}{1.2\,\pi}\times 10^4\ \text{Hz} = 10.61\,\text{kHz}$$

25. At junction a potential barrier/depletion layer is formed, with *n*-side at higher potential and *p*-side at lower potential. Therefore, there is an electric field at the junction directed from the *n*-side to *p*-side.

26. In *n*-type semiconductors, electrons are the majority charge carriers.

27. In the circuit, diode D_1 is forward biased, while D_2 is reverse biased. Therefore, current i (through D_1 and 100 Ω resistance) will be

$$i = \frac{6}{50+100+150} = 0.02\,\text{A}$$

Here 50 Ω is the resistance of D_1 in forward biasing.

29. $\lambda_{\min} = 2480\text{nm} = 24800\text{Å}$

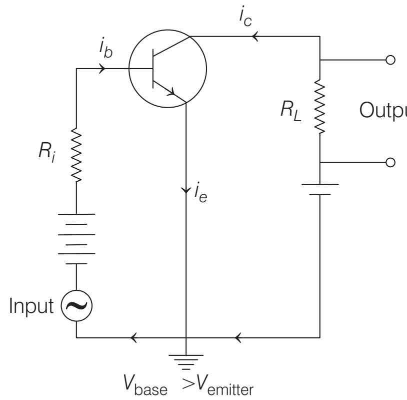
$$\text{Energy (in eV)} = \frac{12375}{\lambda}(\text{in Å})$$
$$E = \frac{12375}{24800}\text{eV},\ E \approx 0.5\ \text{eV}$$

30. In circuit 2 both the diodes are forward biased and in circuit 3 both the diodes are reverse biased.

33. $$I_b + I_c = I_e$$

$$\Rightarrow \quad \frac{I_b}{I_c} + 1 = \frac{I_e}{I_c}$$
$$\Rightarrow \quad \frac{1}{\beta} + 1 = \frac{1}{\alpha}$$
$$\Rightarrow \quad \frac{1}{\alpha} = \frac{1+\beta}{\beta} \quad \Rightarrow \alpha = \frac{\beta}{1+\beta}$$

34. The circuit of a common emitter amplifier is as shown.



This has been shown an *n-p-n* transistor. Therefore, base emitter are forward biased and input signal is connected between base and emitter.

35. In intrinsic semiconductors, number of holes = number of free electrons while in case of a *p*-type semiconductor, number of holes > number of free electrons.

36. For half cycle diode 1 is forward biased and for the rest half it is reverse biased. Therefore, it will conduct only for one half cycle. Therefore, choice (b) or (c) is correct. (Not both)

37. Given : $i_c = 10\text{mA} = (0.9)\,i_e$ [Given that i_c is 90% of i_e]

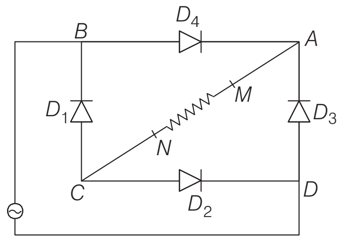
$$\therefore \quad i_e = \frac{10}{0.9}\ \text{mA} \approx 11\text{mA}$$

and $$i_b = i_e - i_c = (11-10)\ \text{mA} = 1\,\text{mA}$$

38. To make a *p*-type semiconductor, a trivalent impurity should be added to pure tetravalent compounds.

39. Reverse

40. In one half cycle when B is at higher potential then, D_4 and D_2 are forward biased and current follows the path BD_4MND_2DB.



In the second half cycle when D is at higher potential, D_3 and D_1 are forward biased. Hence, current follows the path DD_3MND_1BD. In both the cycles we see that current through the resistance is from M to N, *i.e.,* the given circuit behaves as full wave rectifier.

41. No solution is required.

42. No solution is required.

43. With increase in temperature saturation current in diode valve gets increased. Hence, $T_2 > T_1$.

44. Given, at $V_g = -1\text{V}$

$$I_p = (0.125V_p - 7.5) \times 10^{-3} \text{ A}$$

$$\left.\frac{dI_p}{dV_p}\right|_{V_g = -1 \text{ volt}} = 0.125 \times 10^{-3} \text{ A/V}$$

$$\therefore \quad r_p = \left.\frac{dV_p}{dI_p}\right|_{V_g = -1 \text{ volt}} = \frac{1}{0.125 \times 10^{-3}} \Omega$$

$$= 8 \times 10^3 \ \Omega = 8 \text{ k}\Omega$$

From the given equation,

$$I_p = (0.125 \times 300 - 7.5) \text{ mA}$$

$$(\text{at } V_g = -1\text{V and } V_p = 300\text{V}) = 30 \text{ mA}$$

$$\text{Now,} \quad g_m = \left.\frac{\Delta I_p}{\Delta V_g}\right|_{V_p = \text{constant}}$$

$$= \left.\frac{(30-5)}{(-1)-(-3)}\right|_{V_p = 300 \text{ volt}}$$

$$= 12.5 \times 10^{-3} \text{ A/V}$$

Amplification factor $\mu = r_p \times g_m = 100$

Topic 7 Miscellaneous Problems

1. In an electromagnetic wave, magnetic field and electric field are perpendicular to each other and both are also perpendicular to the direction of propagation of wave.

Now, given direction of propagation is along z-direction. So, magnetic field is in either x or y direction.

Also, angular wave number for wave is

$$k = \frac{2\pi}{\lambda} = \frac{2\pi\nu}{c} = \frac{2\pi \times 23.9 \times 10^9}{3 \times 10^8} \approx 0.5 \times 10^3 \text{ m}^{-1}$$

and angular frequency ω for wave is

$$\omega = 2\pi\nu = 2\pi \times 23.9 \times 10^9 \text{ Hz} = 1.5 \times 10^{11} \text{ Hz}$$

Magnitude of magnetic field is

$$B_0 = \frac{E_0}{c} = \frac{60}{3 \times 10^8} = 2 \times 10^{-7}\text{T}$$

As the general equation of magnetic field of an electromagnetic wave propagating in $+z$- direction is given as,

$$\mathbf{B} = B_0 \sin(kz - \omega t)\hat{\mathbf{i}} \text{ or } \hat{\mathbf{j}}$$

Thus, substituting the values of B_0, k and ω, we get

$$\Rightarrow \mathbf{B} = 2 \times 10^{-7} \sin(0.5 \times 10^3 z - 1.5 \times 10^{11} t)\hat{\mathbf{i}} \text{ or } \hat{\mathbf{j}}$$

2. Given, carrier wave,

$$C(t) = 4\sin(20000\,\pi t)$$

Modulating signal,

$$m(t) = 2\sin(2000\,\pi t)$$

So, carrier wave's amplitude and frequency are

$$A_c = 4 \text{ V}, \ \omega_c = 20000\pi = 2\pi \times 10^4 \text{ rad/s}$$

$$\Rightarrow \quad f_c = \frac{\omega_c}{2\pi} = 10^4 \text{ Hz} = 10 \text{ kHz}$$

and modulating signal's amplitude and frequency are

$$A_m = 2\text{V}, \omega_m = 2000\pi = 2\pi \times 10^3 \text{ rad/s}$$

$$\Rightarrow f_m = \frac{\omega_m}{2\pi} = 10^3 \text{ Hz} = 1 \text{ kHz}$$

So, modulating index is $m = \dfrac{A_m}{A_c} = \dfrac{2}{4} = 0.5$

and lower side band frequency is,

$$f_{\text{LSB}} = f_c - f_m = 10 - 1 = 9 \text{ kHz}$$

3. Standard expression of electromagnetic wave is given by

$$\mathbf{E} = E_0\hat{\mathbf{n}} [\sin(\omega t - \mathbf{k} \cdot \hat{\mathbf{r}})] \quad \ldots\text{(i)}$$

Here, $\mathbf{k}$ is the propagation vector.

Direction of propagation in this case is $\hat{\mathbf{k}}$.

Given expression of electromagnetic wave,

$$\mathbf{E} = E_0\hat{\mathbf{n}} \sin[\omega t + (6y - 8z)]$$

$$\Rightarrow \quad \mathbf{E} = E_0\hat{\mathbf{n}} \sin[\omega t - (8z - 6y)] \quad \ldots\text{(ii)}$$

Comparing Eq. (ii) with Eq. (i), we get

$$\mathbf{k} \cdot \hat{\mathbf{r}} = 8z - 6y \quad \ldots\text{(iii)}$$

Here, $\quad \hat{\mathbf{r}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$

and $\quad \mathbf{k} = k_x\hat{\mathbf{i}} + k_y\hat{\mathbf{j}} + k_z\hat{\mathbf{k}}$

$$\therefore \quad \mathbf{k} \cdot \hat{\mathbf{r}} = xk_x + yk_y + zk_z \quad \ldots\text{(iv)}$$

From Eqs. (iii) and (iv), we get

$$xk_x = \text{zero} \Rightarrow k_x = 0$$
$$yk_y = -6y \Rightarrow k_y = -6$$
$$zk_z = 8z \Rightarrow k_z = 8$$

Hence, $\mathbf{k} = -6\hat{\mathbf{j}} + 8\hat{\mathbf{k}}$

So, direction of propagation,

$$\hat{\mathbf{s}} = \hat{\mathbf{k}} = \frac{\mathbf{k}}{|\mathbf{k}|} = \frac{-6\hat{\mathbf{j}} + 8\hat{\mathbf{k}}}{\sqrt{6^2 + 8^2}} = \frac{-6\hat{\mathbf{j}} + 8\hat{\mathbf{k}}}{10} = \frac{-3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}}{5}$$

4. Radiation pressure over an absorbing surface is, $p = \frac{I}{c}$

where, I = intensity or energy flux

and c = speed of light.

If A = area of surface, then force due to radiation on the surface is

$$F = p \times A = \frac{IA}{c}$$

If force F acts for a duration of Δt seconds, then momentum transferred to the surface is

$$\Delta p = F \times \Delta t = \frac{IA}{c} \times \Delta t$$

Here, $I = 25 \text{ W cm}^{-2}$, $A = 25 \text{ cm}^2$,

$c = 3 \times 10^8 \text{ ms}^{-1}$, $\Delta t = 40 \text{ min} = 2400 \text{ s}$

So, momentum transferred to the surface,

$$\Delta p = \frac{25 \times 25 \times 2400}{3 \times 10^8} = 5 \times 10^{-3} \text{ N-s}$$

5. In optical fibre communication, infrared light is used to transmit information from one point to another.

RADAR (Radio detection and ranging) is a detection system that uses radio waves to determine range, angle or velocity of objects.

SONAR (Sound navigation and ranging) is also a detection system that uses ultrasound to detect under water objects, submarines, etc.

Mobile phone is a portable telephone that can make and receive calls, which make use of microwave.

$\therefore$ Correct sequence is

$$A \rightarrow Q,\ B \rightarrow S,\ C \rightarrow P \text{ and } D \rightarrow R$$

6. **Key Idea** For an electromagnetic wave, its electric field vector (**E**) and magnetic field vector (**B**) is mutually perpendicular to each other and also to its direction of propagation.

We know that, $\mathbf{E} \times \mathbf{B}$ represents direction of propagation of an electromagnetic wave

$$\Rightarrow \qquad (\mathbf{E} \times \mathbf{B}) \parallel v$$

$\therefore$ From the given electric field, we can state that direction of propagation is along Z-axis and direction of $\mathbf{E}$ is along X-axis.

Thus, from the above discussion, direction of $\mathbf{B}$ must be Y-axis.

From Maxwell's equation,

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Here, $$\frac{\partial \mathbf{E}}{\partial Z} = -\frac{\partial B}{\partial t} \qquad \ldots \text{(i)}$$

and $$B_0 = E_0 / C \qquad \ldots \text{(ii)}$$

Given, $$\mathbf{E} = E_0\, \hat{\mathbf{i}} \cos kz \cos \omega t$$

$$\Rightarrow \qquad \frac{-\partial \mathbf{E}}{\partial Z} = kE_0 \sin kz \cos \omega t$$

$\therefore$ Using Eq. (i), we get

$$\frac{\partial \mathbf{B}}{\partial t} = kE_0 \sin kz \cos \omega t$$

Integrating both sides of the above equation w.r.t. t, we get

$$\Rightarrow \quad \mathbf{B} = \frac{k}{\omega} E_0 \sin kz \sin \omega t = \frac{E_0}{C} \sin kz \sin \omega t$$

$$\Rightarrow \quad \mathbf{B} = \frac{E_0}{C} \sin(kz) \sin(\omega t)\, \hat{\mathbf{j}}$$

7. Given, $f_m = 100 \text{ MHz} = 100 \times 10^6 \text{ Hz}$,

$$A_m = 100 \text{ V},$$
$$A_c = 400 \text{ V}$$

Range of frequency in case of amplitude modulation is $(f_c - f_m)$ to $(f_c + f_m)$.

$\therefore$ Bandwidth $= 2 f_m = 2 \times 100 \times 10^6$ Hz

$$= 2 \times 10^8 \text{Hz}$$

and modulation index,

$$\text{MI} = \frac{A_m}{A_c} = \frac{100}{400} = 0.25$$

8. Size of antenna is directly proportional to the wavelength of the signal.

Also, the speed at which signal moves = carrier frequency $\times$ wavelength

$$\Rightarrow \qquad f\lambda = c \Rightarrow \lambda \propto \frac{1}{f}$$

$\therefore$ Size of antenna $\propto \frac{1}{f}$.

NOTE Minimum size of the antenna is $\lambda / 4$.

9. Radiation pressure or momentum imparted per second per unit area when light falls is

$$p = \begin{cases} \frac{2I}{c} & ; \text{ for reflection of radiation} \\ \frac{I}{c} & ; \text{ for absorption of radiation} \end{cases}$$

where, I is the intensity of the light.

In given case, there is 25% reflection and 75% absorption, so radiation pressure = force per unit area

$$= \frac{25}{100} \times \frac{2I}{c} + \frac{75}{100} \times \frac{I}{c}$$

$$= \frac{1}{2} \times \frac{I}{c} + \frac{3}{4} \times \frac{I}{c} = \frac{5}{4} \times \frac{I}{c} = \frac{5}{4} \times \frac{50}{3 \times 10^8}$$

$$= 20.83 \times 10^{-8} \text{N/m}^2 \approx 20 \times 10^{-8} \text{ N/m}^2$$

10. Given, modulating signal,

$$A_m = A \cos \omega t$$

Carrier wave, $A_c = v_0 \sin \omega_0 t$

In amplitude modulation, modulated wave is given by

$$Y_m = [A_0 + A_m] \sin \omega_0 t$$

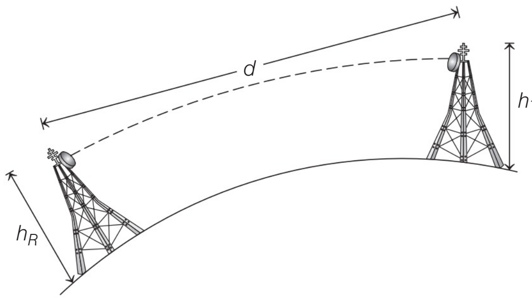
where, A_0 is amplitude of the carrier wave (given as v_0)

$$\therefore\ Y_m = [v_0 + A\cos\omega t]\sin\omega_0 t$$
$$= v_0 \sin\omega_0 t + A\sin\omega_0 t\cos\omega t$$
$$= v_0\sin\omega_0 t + \frac{A}{2}[\sin(\omega_0+\omega)t + \sin(\omega_0-\omega)t]$$
$$= v_0\sin\omega_0 t + \frac{A}{2}\sin(\omega_0-\omega)t + \frac{A}{2}\sin(\omega_0+\omega)t$$

11. **Key Idea** In line of sight communication, distance d between transmitting antenna and receiving antenna is given by

$$d = \sqrt{2Rh_T} + \sqrt{2Rh_R}$$

Here in figure, h_R and h_T is the height of receiving and transmitting antenna, respectively.



Given, $d = 50\,\text{km} = 50\times10^3$ m

$h_R = 70\,\text{m},\ R = 6.4\times10^6$ m

Then, distance between transmitting and receiving antenna, i.e. $d = \sqrt{2Rh_T} + \sqrt{2Rh_R}$

$$50\times10^3 = \sqrt{2R}(\sqrt{h_T}+\sqrt{h_R})$$
$$= \sqrt{2\times6.4\times10^6}(\sqrt{h_T}+\sqrt{70})$$
$$\Rightarrow \quad \sqrt{h_T} \approx \frac{50\times10^3}{3577.7} - 8.37$$
$$= 13.98 - 8.37 = 5.61$$

or $\quad h_T = 31.5 \approx 32$ m

12. Given,

$$\mathbf{B} = 1.6\times10^{-6}\cos(2\times10^7 z + 6\times10^{15}t)(2\hat{\mathbf{i}}+\hat{\mathbf{j}})\,\text{Wbm}^{-2}$$

From the given equation, it can be said that the electromagnetic wave is propagating negative z-direction, i.e. $-\hat{\mathbf{k}}$.

Equation of associated electric field will be

$$\mathbf{E} = (|\mathbf{B}|c)\cos(kz+\omega t)\cdot\hat{\mathbf{n}}$$

where, $\hat{\mathbf{n}}$ = a vector perpendicular to $\mathbf{B}$.

So, $|\mathbf{E}| = |\mathbf{B}|c$

$$= 1.6\times10^{-6}\times3\times10^8 = 4.8\times10^2\ \text{V/m}$$

Since, we know that for an electromagnetic wave, $\mathbf{E}$ and $\mathbf{B}$ are mutually perpendicular to each other.

So, $\quad \mathbf{E}\cdot\mathbf{B} = 0$

From the given options, when $\hat{\mathbf{n}} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}}$

$$\mathbf{E}\cdot\mathbf{B} = (2\hat{\mathbf{i}}+\hat{\mathbf{j}})\cdot(\hat{\mathbf{i}}-2\hat{\mathbf{j}}) = 0$$

Also, when $\hat{\mathbf{n}} = -\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$

$$\mathbf{E}\cdot\mathbf{B} = (2\hat{\mathbf{i}}+\hat{\mathbf{j}})\cdot(-\hat{\mathbf{i}}+2\hat{\mathbf{j}}) = 0$$

But, we also know that the direction of propagation of electromagnetic wave is perpendicular to both $\mathbf{E}$ and $\mathbf{B}$, i.e. it is in the direction of $\mathbf{E}\times\mathbf{B}$.

Again, when $\hat{\mathbf{n}} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}}$

$$\mathbf{E}\times\mathbf{B} = (2\hat{\mathbf{i}}+\hat{\mathbf{j}})\times(\hat{\mathbf{i}}-2\hat{\mathbf{j}}) = -\hat{\mathbf{k}}$$

and when $\hat{\mathbf{n}} = -\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$

$$\mathbf{E}\times\mathbf{B} = (2\hat{\mathbf{i}}+\hat{\mathbf{j}})\times(-\hat{\mathbf{i}}+2\hat{\mathbf{j}}) = \hat{\mathbf{k}}$$

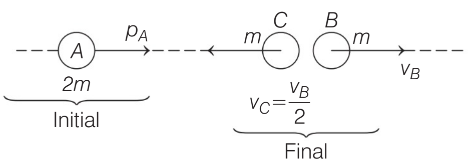
But, it is been given in the question that the direction of propagation of wave is in $-\hat{\mathbf{k}}$.

Thus, associated electric field will be

$$\mathbf{E} = 4.8\times10^2\cos(2\times10^7 z + 6\times10^{15}t)(\hat{\mathbf{i}}-2\hat{\mathbf{j}})\,\text{Vm}^{-1}$$

13. Let m be the mass of nuclei B and C.

So, the given situation can be shown in the figure below



Now, according to the conservation of linear momentum,

Initial momentum = Final momentum

$$\Rightarrow \quad p_A = p_B + p_C \quad \text{or} \quad 2mv_A = mv_B + mv_C$$
$$2mv_A = mv_B - \frac{mv_B}{2}$$
$$\Rightarrow \quad 2v_A = \frac{1}{2}v_B \Rightarrow v_B = 4v_A \qquad \ldots\text{(i)}$$

and $\quad v_C = \frac{v_B}{2} = 2v_A \qquad \ldots$(ii)

So, momentum of B and C respectively, can now be given as

$p_B = m_B v_B = m4v_A = 2(2mv_A)$ [using Eq. (i)]

or $p_B = 2p_A \qquad \ldots$(iii)

and $p_C = m_C v_C = m2v_A$ [using Eq. (ii)]

or $\quad p_C = p_A \qquad \ldots$(iv)

From the relation of de-Broglie wavelength, i.e. $\lambda = \frac{h}{p}$

where, p is momentum and h is Planck's constant.

So, for A, $\lambda_A = \frac{h}{p_A}$ or $p_A = \frac{h}{\lambda_A} \qquad \ldots$(v)

From Eq. (v), λ_B can be written as

$$\lambda_B = \frac{h}{2\times\frac{h}{\lambda_A}} = \frac{\lambda_A}{2}$$

Similarly, for C, $\lambda_C = \frac{h}{p_C} = \frac{h}{p_A}$ [using Eq. (iv)]

Similarly, from Eq. (v), λ_C can be written as

$$\lambda_C = \frac{h}{\frac{h}{\lambda_A}} = \lambda_A$$

14. In optical fibre communication network, the signals are transmitted by laser light operating in range of 1310 nm-1550 nm.

So, the closest value is 1500 nm.

15. **Key Idea** For an electromagnetic wave, ratio of magnitudes of electric and magnetic field is

$$\frac{E}{B} = c$$

where, c is the speed of electromagnetic wave in vacuum.

Given, $E = 6\,\text{V/m}, c = 3 \times 10^8\ \text{ms}^{-1}$

So, $B = \frac{E}{c} = \frac{6}{3 \times 10^8} = 2 \times 10^{-8}\ \text{T}$

Also, direction of propagation of electromagnetic wave is given by

$$\hat{\mathbf{n}} = \mathbf{E} \times \mathbf{B}$$

Here, $\hat{\mathbf{n}} = \hat{\mathbf{i}}$ and $\mathbf{E}$ = Unit vector of electric field $(\hat{\mathbf{j}})$

$\mathbf{B}$ = unit vector of magnetic field.

$\Rightarrow \quad \hat{\mathbf{i}} = \hat{\mathbf{j}} \times \mathbf{B} \Rightarrow \mathbf{B} = \hat{\mathbf{k}}$

Hence, magnetic field components,

$$\mathbf{B} = 2 \times 10^{-8}\,\hat{\mathbf{k}}\,\text{T} = 2 \times 10^{-8}\,\text{T} \qquad \text{(along } z\text{-direction)}$$

16. Range of TV transmitting tower $d = \sqrt{2hR}$

where, h is the height of the transmission tower. when range is doubled,

$$\therefore \quad 2d = 2\sqrt{2hR} = \sqrt{2(4h)R}$$

So, height must be multiplied with 4.

17. Minimum wavelength occurs when mercury atom deexcites from highest energy level.

$\therefore$ Maximum possible energy absorbed by mercury atom

$$= \Delta E = 5.6 - 0.7 = 4.9\,\text{eV}$$

Wavelength of photon emitted in deexcitation is

$$\lambda = \frac{hc}{E} \approx \frac{1240\ \text{eVnm}}{4.9\ \text{eV}} \approx 250\,\text{nm}$$

NOTE

Frank-Hertz experiment was the first electrical measurement to show quantum nature of atoms. In a vacuum tube energatic electrons are passed through thin mercury vapour film. It was discovered that when an electron collided with a mercury atom, it loses only a specific quantity (4.9 eV) of it's kinetic energy. This experiment shows existence of quantum energy levels.

18. Modulation index is given by

$$\mu = \frac{A_{\max} - A_{\min}}{A_{\max} + A_{\min}} = \frac{V_{\max} - V_{\min}}{V_{\max} + V_{\min}}$$

$$= \frac{160 - 40}{160 + 40} = \frac{120}{200} = 0.6$$

19. Given,

Power of laser beam $(P) = 27\text{mW} = 27 \times 10^{-3}$ W

Area of cros- section $(A) = 10\text{m m}^2 = 10 \times 10^{-6}\text{m}^2$

Permittivity of free space $(\varepsilon_0) = 9 \times 10^{-12}$ SI unit

Speed of light $(c) = 3 \times 10^8$ m/s

Intensity of electromagnetic wave is given by the relation

$$I = \frac{1}{2} nc\varepsilon_0 E^2$$

where, n is refractive index, for air $n = 1$.

$$\therefore \quad I = \frac{1}{2} c \cdot \varepsilon_0 E^2 \qquad \text{...(i)}$$

Also, $$I = \frac{P}{A} \qquad \text{...(ii)}$$

From Eq. (i) and (ii), we get

$$\frac{1}{2} c\varepsilon_0 E^2 = \frac{P}{A} \text{ or } E^2 = \frac{2P}{Ac\varepsilon_0}$$

or $$E = \sqrt{\frac{2 \times 27 \times 10^{-3}}{10 \times 10^{-6} \times 3 \times 10^8 \times 9 \times 10^{-12}}}$$

$$\simeq 1.4 \times 10^3\ \text{V/m} = 1.4\ \text{kV/m}$$

20 Equation of an amplitude modulated wave is given by the relation,

$$C_m = (A_c + A_m \sin \omega_m t) \cdot \sin \omega_c \cdot t \qquad \text{...(i)}$$

For the given graph, maximum amplitude,

$$A_c + A_m = 10 \qquad \text{...(ii)}$$

and minimum amplitude, $A_c - A_m = 8$ ...(iii)

From Eqs. (ii) and (iii), we get

$$A_c = 9\text{V}$$

and $$A_m = 1\,\text{V} \qquad \text{...(iv)}$$

$\because$ For angular frequency of message signal and carrier wave, we use a relation

$$\omega_c = \frac{2\pi}{T_c} = \frac{2\pi}{8 \times 10^{-6}}$$

(as from given graph, $T_c = 8 \times 10^{-6}$ s)

$$= 2.5\pi \times 10^5\,\text{s}^{-1} \qquad \text{...(v)}$$

and $$\omega_m = \frac{2\pi}{T_m} = \frac{2\pi}{100 \times 10^{-6}}$$

(as from given graph, $T_m = 100 \times 10^{-6}$s)

$$= 2\pi \times 10^4\,\text{s}^{-1}$$

When we put values of A_c, A_m, ω_c and ω_m in Eq. (i), we get

$$C_m = [9 + \sin(2\pi \times 10^4 t)]\sin(2.5\pi \times 10^5 t)\ \text{V}$$

21. $v(t) = 10\,[1 + 0.3\cos(2.2 \times 10^4 t)] \qquad [\sin(5.5 \times 10^5 t)]$

Upper band angular frequency

$$\omega_v = (2.2 \times 10^4 + 5.5 \times 10^5)\ \text{rad / s}$$

$$= 572 \times 10^3\ \text{rad / s}$$

Similarly, lower band angular frequency.

$$\omega_L = (5.5\times 10^5 - 2.2\times 10^4)\text{ rad / s}$$

$$= 528\times 10^3 \text{ rad / s}$$

$\therefore$ Side band frequency are,

$$f_u = \frac{\omega_u}{2\pi} = \frac{572}{2\pi}\text{ kHz} \simeq 91\text{ kHz}$$

and $$f_L = \frac{\omega_L}{2\pi} = \frac{528}{2\pi}\text{ kHz} \simeq 84\text{ kHz}$$

22. In the free space, the speed of electromagnetic wave is given as,

$$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = \frac{E_0}{B_0} \quad \ldots\text{(i)}$$

where, E_0 and B_0 are the amplitudes of varying electric and magnetic fields, respectively.

Now, when it enters in a medium of refractive index 'n', its speed is given as,

$$v = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{K\varepsilon_0\mu}} = \frac{E}{B} \quad \ldots\text{(ii)}$$

where, K is dielectric strength of the medium.

Using Eqs. (i) and (ii), we get

$$\frac{v}{c} = \frac{1}{\sqrt{K}} \quad \ldots\text{(iii)}$$

($\because$ For a transparent medium, $\mu_0 \approx \mu$)

Also, refractive index of medium is 'n' and is given as

$$\frac{c}{v} = n \quad \text{or} \quad \frac{v}{c} = \frac{1}{n} \quad \ldots\text{(iv)}$$

$\therefore$ From Eqs. (iii) and (iv), we get

$$n = \sqrt{K} \quad \text{or} \quad K = n^2 \quad \ldots\text{(v)}$$

The intensity of a EM wave is given as,

$$I = \frac{1}{2}\varepsilon_0 E_0^2 c$$

and in the medium, it is given as $I' = \frac{1}{2}K\varepsilon_0 E^2 v$

It is given that, $I = I'$

$$\Rightarrow \quad \frac{1}{2}\varepsilon_0 E_0^2 c = \frac{1}{2}K\varepsilon_0 E^2 v \quad \text{or} \quad \left(\frac{E_0}{E}\right)^2 = \frac{Kv}{c} \quad \ldots\text{(vi)}$$

From Eqs. (iv), (v) and (vi), we get

$$\left(\frac{E_0}{E}\right)^2 = \frac{n^2}{n} = n \quad \text{or} \quad E_0 / E = \sqrt{n}$$

Similarly, $\frac{1}{2}\cdot\frac{B_0^2}{\mu_0}c = \frac{1}{2}\cdot\frac{B^2}{\mu_0}v \Rightarrow \frac{B_0}{B} = \frac{1}{\sqrt{n}}$

Alternate method

We know that,

$$\left(\frac{E_0}{B_0}\right)_{\text{air/ vacuum}} = c \quad \text{and} \quad \left(\frac{E}{B}\right)_{\text{medium}} = v$$

Also, $$n = \frac{c}{v}$$

$$\frac{E_0 / B_0}{E / B} = \frac{c}{v} = n$$

$$\Rightarrow \quad \frac{E_0 / E}{B_0 / B} = n$$

This is possible only if $\frac{E_0}{E} = \sqrt{n}$ and $\frac{B_0}{B} = \frac{1}{\sqrt{n}}$.

23. We are given electric field as

$$\mathbf{E} = 10\,\hat{\mathbf{j}}\cos(6x + 8z) \quad \ldots\text{(i)}$$

where, phase angle is independent of time,
i.e., phase angle at $t = 0$ is $\phi = 6x + 8z$.

Phase angle for $\mathbf{B}$ will also be $6x + 8z$ because for an electromagnetic wave $\mathbf{E}$ and $\mathbf{B}$ oscillate in same phase.

Thus, direction of wave propagation

$$= \frac{6\hat{\mathbf{i}} + 8\hat{\mathbf{k}}}{\sqrt{6^2 + 8^2}} = \frac{6\hat{\mathbf{i}} + 8\hat{\mathbf{k}}}{10} \quad \ldots\text{(ii)}$$

Let magnetic field vector,

$\mathbf{B} = a\hat{\mathbf{i}} + b\hat{\mathbf{j}} + d\hat{\mathbf{k}}$, then direction of wave propagation is given by

$$\frac{\mathbf{E}\times\mathbf{B}}{|\mathbf{E}'||\mathbf{B}|} = \frac{10\hat{\mathbf{j}}\times(a\hat{\mathbf{i}} + b\hat{\mathbf{j}} + d\hat{\mathbf{k}})}{10\times(a^2 + b^2 + d^2)^{1/2}}$$

$$= \frac{-10a\hat{\mathbf{k}} + 10d\hat{\mathbf{i}}}{10(a^2 + b^2 + d^2)^{1/2}} \quad \ldots\text{(iii)}$$

As, $$|\mathbf{B}| = \frac{|\mathbf{E}|}{c} = \frac{10}{c}$$

We get, $$|\mathbf{B}| = \sqrt{a^2 + b^2 + d^2} = 10/c$$

By putting this value in Eqs. (iii) and (ii), we get direction of propagation

$$\Rightarrow \quad \frac{c10(d\,\hat{\mathbf{i}} - a\hat{\mathbf{k}})}{10\times 10} = \frac{6\hat{\mathbf{i}}\times 8\hat{\mathbf{k}}}{10}$$

$$\Rightarrow \quad d = 6/c \text{ and } a = -8/c$$

Hence, $\mathbf{B} = \frac{6}{c}\hat{\mathbf{k}} - \frac{8}{c}\hat{\mathbf{i}} = \frac{1}{c}(6\hat{\mathbf{k}} - 8\hat{\mathbf{i}})$

As the general equation of magnetic field of an EM wave propagating in positive y-direction is given as,

$$B = B_0\cos(Ry - \omega t))$$

$$\therefore \quad \mathbf{B} = \frac{1}{c}(6\hat{\mathbf{k}} - 8\hat{\mathbf{i}})\cos(6x + 8z - 10ct)$$

Alternate method Given, electric field is $E(x, y)$, i.e. electric field is in xy-plane which is given as

$\because$ $\mathbf{E} = 10\hat{\mathbf{j}}\cos(6x + 8z)$

Since, the magnetic field given is $\mathbf{B}(x, z, t)$, this means $\mathbf{B}$ is in xz-plane.

$\therefore$ Propagation of wave is in y-direction.

[$\because$ for an electromagnetic wave,
$\mathbf{E}\perp\mathbf{B}\perp$ propagation direction]

As Poynting vector suggests that $\mathbf{E}\times\mathbf{B}$ is parallel to $(6\hat{\mathbf{i}}+8\hat{\mathbf{k}})$.

Let $\qquad \mathbf{B}=(x\hat{\mathbf{i}}+z\hat{\mathbf{k}})$,

then $\qquad \mathbf{E}\times\mathbf{B}=\hat{\mathbf{j}}\times(x\hat{\mathbf{i}}+z\hat{\mathbf{k}})=6\hat{\mathbf{i}}+8\hat{\mathbf{k}}$

or $\qquad -x\hat{\mathbf{k}}+z\hat{\mathbf{i}}=6\hat{\mathbf{i}}+8\hat{\mathbf{k}}$

or $\qquad x=-8$ and $z=6$

$$\therefore \quad \mathbf{B}=\frac{1}{c}(6\hat{\mathbf{k}}-8\hat{\mathbf{i}})\cos(6x+8z-10ct) \qquad \left[\because \frac{|\mathbf{E}|}{|\mathbf{B}|}=c\right]$$

24. For a given carrier wave of frequency f_c with modulation frequency f_m, the bandwidth is calculated by

$$f_{\text{upper}}=f_c+f_m$$
$$f_{\text{lower}}=f_c-f_m \qquad \ldots \text{(i)}$$

To avoid overlapping of bandwidths, next broadcast frequencies can be

$$f_1=f_c\pm 2f_m,\ f_2=f_c\pm 3f_m$$

So, next immediate available broadcast frequency is

$$f_1=f_c+2f_m \text{ and } f_1'=f_c-2f_m$$

Given, $f_m=250$ kHz

and also that f_m is 10% of f_c, i.e. $f_c=2500$ kHz.

So, $\quad f_1=2500+(2\times 250)=3000$ kHz

and $\ f'_1=2500-(2\times 250)=2000$ kHz

25. Given, instantaneous value of magnetic field

$$B=100\times 10^{-6}\sin\left[2\pi\times 2\times 10^{15}\left(t-\frac{x}{c}\right)\right]$$

and speed of light, $c=3\times 10^8\ \text{ms}^{-1}$

For an electromagnetic wave,

$$E_{\max}=B_{\max}\times c$$

where, $E_{\max}$ = maximum value of the electric field.

We get, $E_{\max}=100\times 10^{-6}\times 3\times 10^8=3\times 10^4\ \dfrac{\text{N}}{\text{C}}$

26. Maximum distance of transmission is given by

$$d=\sqrt{2Rh_T}+\sqrt{2Rh_R}$$

where, $\quad h_T$ = height of transmitter, = 140 m,

h_R = height of receiver = 40 m and

R = radius of earth = 6.4×10^6 m.

Substituting values, we get

$$d=\sqrt{2\times 6.4\times 10^6}\ (\sqrt{140}+\sqrt{40})=65\text{ km}$$

27. Given, resolution achieved in electron microscope is of the order of wavelength.

So, to resolve 7.5×10^{-12} m seperation wavelength associated with electrons is

$$\lambda=7.5\times 10^{-12}\text{ m}$$

$\therefore$ Momentum of electrons required is

$$p=\frac{h}{\lambda}$$

or kinetic energy of electron must be

$$\text{KE}=\frac{p^2}{2m}=\frac{(h/\lambda)^2}{2m}$$

Substituting the given values, we get

$$=\frac{\left(\dfrac{6.6\times 10^{-34}}{7.5\times 10^{-12}}\right)^2}{2\times 9.1\times 10^{-31}}\text{ J}$$

$$=\frac{(6.6\times 10^{-34})^2}{2\times 9.1\times 10^{-31}\times(7.5\times 10^{-12})^2\times(1.6\times 10^{-19})}\text{eV}$$

$$(\because 1\text{ eV}=1.6\times 10^{-19}\text{ J})$$

$=26593.4\simeq 26.6\times 10^3$ eV $\simeq 26$ keV

which is nearest to 25 keV.

28. Energy density of an electomagnetic wave in electric field,

$$U_E=\frac{1}{2}\varepsilon_0\cdot E^2 \qquad \ldots\text{(i)}$$

Energy density of an electromagnetic wave in magnetic field,

$$U_B=\frac{B^2}{2\mu_0} \qquad \ldots\text{(ii)}$$

where, E = electric field,

B = magnetic field,

ε_0 = permittivity of medium and

μ_0 = magnetic permeability of medium.

From the theory of electro-magnetic waves, the relation between μ_0 and ε_0 is

$$c=\frac{1}{\sqrt{\mu_0\varepsilon_0}} \qquad \ldots\text{(iii)}$$

where, c = velocity of light $=3\times 10^8$ m/s

and $$\frac{E}{B}=c \qquad \ldots\text{(iv)}$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{U_E}{U_B}=\frac{\frac{1}{2}\varepsilon_0E^2}{\frac{1}{2}B^2\times\frac{1}{\mu_0}}=\frac{\mu_0\varepsilon_0E^2}{B^2} \qquad \ldots\text{(v)}$$

Using Eqs. (iii), (iv) and (v), we get

$$\frac{U_E}{U_B}=\frac{c^2}{c^2}=1$$

Therefore, $\qquad U_E=U_B$

29. Here,

Signal wavelength, $\lambda=800$ n-m $=8\times 10^{-7}$ m

Frequency of source is

As, $$f=\frac{c}{\lambda}=\frac{3\times 10^8}{8\times 10^{-7}}$$

$$=3.75\times 10^{14}\text{ Hz}$$

$\therefore$ Total bandwidth used for communication

$$= 1\% \text{ of } 3.75 \times 10^{14}$$

$$= 3.75 \times 10^{12} \text{ Hz} \quad \ldots\text{(i)}$$

So, number of channel for signals

$$= \frac{\text{total bandwidth available for communication}}{\text{bandwidth of TV signal}}$$

$$= \frac{3.75 \times 10^{12}}{6 \times 10^{6}} = 0.625 \times 10^{6} = 6.25 \times 10^{5}$$

30. $v = \dfrac{\text{coefficient of } t}{\text{coefficient of } z} = \dfrac{1}{\sqrt{\varepsilon\mu}}$

where, $\varepsilon = \varepsilon_0 \varepsilon_r$ and $\mu = \mu_0 \mu_r$.

$$\frac{v_{\text{air}}}{v_{\text{med}}} = \frac{c}{c/2} = 2 = \frac{\sqrt{\mu_0 \varepsilon_0 \mu_{r_2} \varepsilon_{r_2}}}{\sqrt{\mu_0 \varepsilon_0 \mu_{r_1} \varepsilon_{r_1}}} \Rightarrow \frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = \frac{1}{4}$$

Note Medium is non-magnetic.

$$\therefore \qquad \mu_{r_1} = \mu_{r_2}$$

31. $N = \dfrac{1}{10} \cdot \dfrac{(10 \text{ kHz})}{(5\text{kHz})} = \dfrac{10^9}{5 \times 10^3} = \dfrac{10^6}{5} = 2 \times 10^5$

32. $f_c = 2\,\text{MHz} = 2000\,\text{kHz}, \quad f_m = 5\,\text{kHz}$

Resultant frequencies are,

$$f_c + f_m,\ f_c \text{ and } f_c - f_m$$

or, 2005 kHz, 2000 kHz and 1995 kHz

33. Intensity at a distance r from a point source of power P is given by

$$I = \frac{P}{4\pi r^2} \quad \text{...(i)}$$

Also, $$I = \frac{1}{2}\varepsilon_0 E_0^2 c \quad \text{...(ii)}$$

where, E_0 is amplitude of electric field and c the speed of light. Eqs. (i) and (ii) we get

$$E_0 = \sqrt{\frac{2P}{4\pi\varepsilon_0 r^2 c}}$$

$$= \sqrt{\frac{2 \times 9 \times 10^9 \times 0.1}{(1)^2 \times 3 \times 10^8}}$$

$$= \sqrt{6} = 2.45 \text{ V/m}$$

34. Both the energy densities are equal i.e. energy is equally divided between electric and magnetic fields.

35. Peak value of electric field

$$E_0 = B_0 c = 20 \times 10^{-9} \times 3 \times 10^8 = 6 \text{ V/m}$$

36. As velocity (or momentum) of electron is increased, the wavelength $\left(\lambda = \dfrac{h}{p}\right)$ will decrease. Hence, fringe width will decrease ($\omega \propto \lambda$).

37. Atomic number of neon is 10.

By the emission of two α-particles, atomic number will be reduced by 4. Therefore, atomic number of the unknown element will be $Z = 10 - 4 = 6$

Similarly, mass number of the unknown element will be

$$A = 22 - 2 \times 4 = 14$$

Unknown nucleus is carbon ($A = 14, Z = 6$).

39. The maximum number of electrons in an orbit are $2n^2$. If $n > 4$, is not allowed, then the number of maximum electrons that can be in first four orbit are :

$$2(1)^2 + 2(2)^2 + 2(3)^2 + 2(4)^2 = 2 + 8 + 18 + 32 = 60$$

Therefore, possible elements can be 60.

40. Given, $r_p = 3 \times 10^3\ \Omega,\ g_m = 1.5 \times 10^{-3}$ A/V

$\therefore$ Amplification factor, $\mu = g_m \times r_p = 4.5$

41. $\therefore$ Power $= nhf$

(where, n = number of photons incident per second)

Since, KE = 0, hf = work-funcition W

$$200 = nW = n\,[6.25 \times 1.6 \times 10^{-19}]$$

$$\Rightarrow \qquad n = \frac{200}{1.6 \times 10^{-19} \times 6.25}$$

As photon is just above threshold frequency $KE_{\max}$ is zero and they are accelerated by potential difference of 500 V.

$$\therefore \qquad \text{KE}_f = q\Delta V$$

$$\frac{P^2}{2m} = q\Delta V \Rightarrow P = \sqrt{2\,mq\,\Delta V}$$

Since, efficiency is 100%, number of electrons emitted per second = number of photons incident per second.

As, photon is completely absorbed, force exerted

$$= n(mV) = nP = n\sqrt{2mq\Delta V}$$

$$= \frac{200}{6.25 \times 1.6 \times 10^{-19}} \times \sqrt{2\,(9 \times 10^{-31}) \times 1.6 \times 10^{-19} \times 500}$$

$$= 24$$

42-43. Maximum kinetic energy of anti-neutrino is nearly (0.8×10^6) eV.

44. (A) → (R or RT); (B) → (P, S); (C) → (Q, T); (D) → (R,)

No solution is required.

45. No Solution is required

46. (a) Infrared rays are used to treat muscular strain.

(b) Radiowaves are used for broadcasting purposes.

(c) X-rays are used to detect fracture of bones.

(d) Ultraviolet rays are absorbed by ozone.

47. (p) In α-decay mass number decreases by 4 and atomic number decreases by 2.

(q) In β^+-decay mass number remains unchanged while atomic number decreases by 1.

(r) In fission, parent nucleus breaks into all most two equal fragments.

(s) In proton emission both mass number and atomic number decreases by 1.

49. $\left|\frac{dN}{dt}\right| = |\text{Activity of radioactive substance}|$

$$= \lambda N = \lambda N_0 \, e^{-\lambda t}$$

Taking log both sides

$$\ln \left|\frac{dN}{dt}\right| = \ln\,(\lambda N_0) - \lambda t$$

Hence, $\ln \left|\frac{dN}{dt}\right|$ *versus* t graph is a straight line with slope $-\lambda$.

From the graph we can see that, $\lambda = \frac{1}{2} = 0.5\ \text{yr}^{-1}$

Now applying the equation,

$$N = N_0 e^{-\lambda t} = N_0\, e^{-0.5 \times 4.16}$$

$$= N_0\, e^{-2.08} = 0.125\, N_0 = \frac{N_0}{8}$$

i.e, nuclei decreases by a factor of 8.

Hence, the answer is 8.

50. No solution is required.

51. (a) Given mass of α-particle, $m = 4.002$ amu and mass of daughter nucleus,

$$M = 223.610 \text{ amu},$$

de-Broglie wavelength of α-particle,

$$\lambda = 5.76 \times 10^{-15} \text{m}$$

So, momentum of α-particle would be

$$p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{5.76 \times 10^{-15}} \text{ kg-m/s}$$

or $\quad p = 1.151 \times 10^{-19}$ kg-m/s

From law of conservation of linear momentum, this should also be equal to the linear momentum of the daughter nucleus (in opposite direction).

Let K_1 and K_2 be the kinetic energies of α-particle and daughter nucleus. Then total kinetic energy in the final state is

$$K = K_1 + K_2 = \frac{p^2}{2m} + \frac{p^2}{2M} = \frac{p^2}{2}\left(\frac{1}{m} + \frac{1}{M}\right)$$

$$K = \frac{p^2}{2}\left(\frac{M+m}{Mm}\right)$$

$$1 \text{ amu} = 1.67 \times 10^{-27} \text{ kg}$$

Substituting the values, we get

$$K = 10^{-12} \text{ J}$$

$$K = \frac{10^{-12}}{1.6 \times 10^{-13}} = 6.25 \text{ MeV}$$

or $\quad K = 6.25$ MeV.

(b) Mass defect, $\Delta m = \frac{6.25}{931.470} = 0.0067$ amu

Therefore, mass of parent nucleus = mass of α-particle + mass of daughter nucleus + mass defect (Δm)

$$= (4.002 + 223.610 + 0.0067) \text{ amu}$$

$$= 227.62 \text{ amu}$$

Hence, mass of parent nucleus is 227.62 amu.

52. Energy of incident photons,

$$E_i = 10.6 \text{ eV} = 10.6 \times 1.6 \times 10^{-19} \text{ J}$$

$$= 16.96 \times 10^{-19} \text{ J}$$

Energy incident per unit area per unit time (intensity) = 2 J

$\therefore$ Number of photons incident on unit area in unit time

$$= \frac{2}{16.96 \times 10^{-19}} = 1.18 \times 10^{18}$$

Therefore, number of photons incident per unit time on given area $(1.0 \times 10^{-4} \text{ m}^2)$

$$= (1.18 \times 10^{18})(1.0 \times 10^{-4}) = 1.18 \times 10^{14}$$

But only 0.53% of incident photons emit photoelectrons

$\therefore$ Number of photoelectrons emitted per second (n)

$$n = \left(\frac{0.53}{100}\right)(1.18 \times 10^{14})$$

$$n = 6.25 \times 10^{11}$$

$$K_{\text{min}} = 0$$

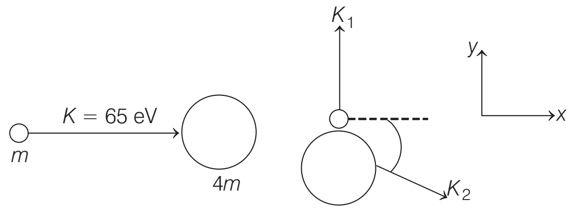
and $\quad K_{\text{max}} = E_i - \text{work function}$

$$= (10.6 - 5.6)\text{eV} = 5.0 \text{ eV}$$

$\therefore \quad K_{\text{max}} = 5.0$ eV

and $\quad K_{\text{min}} = 0$

54. (a) Let K_1 and K_2 be the kinetic energies of neutron and helium atom after collision and ΔE be the excitation energy.



From conservation of linear momentum along x-direction.

$$p_i = p_f$$

$\Rightarrow \quad \sqrt{2Km} = \sqrt{2(4m)K_2}\cos\theta \quad$ ....(i)

Similarly, applying conservation of linear momentum in y-direction, we have

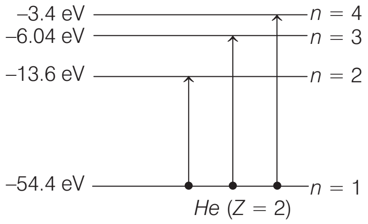
$$\sqrt{2K_1 m} = \sqrt{2(4m)K_2}\sin\theta \quad \text{...(ii)}$$

Squaring and adding Eqs. (i) and (ii), we get

$$K + K_1 = 4K_2 \quad \text{...(iii)}$$

or $\quad 4K_2 - K_1 = K = 65$ eV $\quad$ ...(iv)

Now, during collision, electron can be excited to any higher energy state. Applying conservation of energy, we get $K = K_1 + K_2 + \Delta E$



or $\quad 65 = K_1 + K_2 + \Delta E \quad \ldots(v)$

ΔE can have the following values,

$$\Delta E_1 = \{-13.6 - (-54.4)\}\text{ eV} = 40.8\text{ eV}$$

Substituting in (v), we get

$$K_1 + K_2 = 24.2\text{ eV} \quad \ldots(vi)$$

Solving (iv) and (vi), we get

$$K_1 = 6.36\text{ eV}$$

and $\quad K_2 = 17.84$ eV

Similarly, when we put $\Delta E = \Delta E_2$

$$= \{-6.04 - (-54.4)\}\text{eV}$$
$$= 48.36\text{ eV}$$

Put in Eq. (v), we get

$$K_1 + K_2 = 16.64\text{ eV} \quad \ldots(vii)$$

Solving Eqs. (iv) and (vii), we get

$$K_1 = 0.312\text{ eV} \text{ and } K_2 = 16.328\text{ eV}$$

Similarly, when we put

$$\Delta E = \Delta E_3 = \{-3.4 - (-54.4)\} = 51\text{ eV}$$

Put in Eq. (v), we get

$$K_1 + K_2 = 14\text{ eV} \quad \ldots(viii)$$

Now, solving Eqs. (iv) and (viii), we get

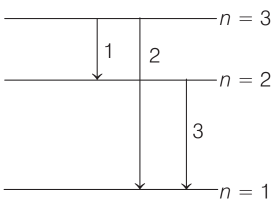
$$K_1 = -1.8\text{ eV and } K_2 = 15.8\text{ eV}$$

But since the kinetic energy cannot have the negative values, the electron will not jump to third excited state or $n = 4$.

Therefore, the allowed values of K_1(KE of neutron) are 6.36 eV and 0.312 eV and of K_2 (KE of the atom) are 17.84 eV and 16.328 eV and the electron can jump upto second excited state only ($n = 3$).

(b) Possible emission lines are only three as shown in figure. The corresponding frequencies are

$$\nu_1 = \frac{(E_3 - E_2)}{h}$$



$$= \frac{\{-6.04 - (-13.6)\} \times 1.6 \times 10^{-19}}{6.63 \times 10^{-34}}$$

$$= 1.82 \times 10^{15}\text{ Hz}$$

$$\nu_2 = \frac{E_3 - E_1}{h} = \frac{\{-6.04 - (-54.4)\} \times 1.6 \times 10^{-19}}{6.63 \times 10^{-34}}$$

$$= 11.67 \times 10^{15}\text{ Hz}$$

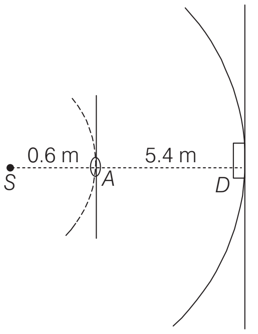
and $\quad \nu_3 = \dfrac{E_2 - E_1}{h}$

$$= \frac{\{-13.6 - (-54.4)\} \times 1.6 \times 10^{-19}}{6.63 \times 10^{-34}}$$

$$= 9.84 \times 10^{15}\text{ Hz}$$

Hence, the frequencies of emitted radiations are 1.82×10^{15} Hz, 11.67×10^{15} Hz and 9.84×10^{15} Hz.

55. (a) Energy of one photon,



$$E = \frac{hc}{\lambda}$$

$$= \frac{(6.6 \times 10^{-34})(3.0 \times 10^8)}{6000 \times 10^{-10}}$$

$$= 3.3 \times 10^{-19}\text{ J}$$

Power of the source is 2 W or 2 J/s. Therefore, number of photons emitting per second,

$$n_1 = \frac{2}{3.3 \times 10^{-19}} = 6.06 \times 10^{18}/\text{s}$$

At distance 0.6 m, number of photons incident per unit area per unit time :

$$n_2 = \frac{n_1}{4\pi(0.6)^2} = 1.34 \times 10^{18}/\text{m}^2/\text{s}$$

Area of aperture is,

$$S_1 = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.1)^2 = 7.85 \times 10^{-3}\text{ m}^2$$

$\therefore$Total number of photons incident per unit time on the aperture,

$$n_3 = n_2 S_1$$
$$= (1.34 \times 10^{18})(7.85 \times 10^{-3})/\text{s}$$
$$= 1.052 \times 10^{16}/\text{s}$$

The aperture will become new source of light.

Now, these photons are further distributed in all directions. Hence, at the location of detector, photons incident per unit area per unit time

$$n_4 = \frac{n_3}{4\pi(6-0.6)^2} = \frac{1.052 \times 10^{16}}{4\pi\,(5.4)^2}$$

$$= 2.87 \times 10^{13}\ \text{s}^{-1}\text{m}^{-2}$$

This is the photon flux at the centre of the screen. Area of detector is 0.5 cm^{2}or $0.5 \times 10^{-4}\,\text{m}^2$. Therefore, total number of photons incident on the detector per unit time

$$n_5 = (0.5 \times 10^{-4})\,(2.87 \times 10^{13}\,d)$$

$$= 1.435 \times 10^{9}\ \text{s}^{-1}$$

The efficiency of photoelectron generation is 0.9. Hence, total photoelectrons generated per unit time

$$n_6 = 0.9 n_5$$

$$= 1.2915 \times 10^{9}\text{s}^{-1}$$

or, photocurrent in the detector

$$i = (e)n_6 = (1.6 \times 10^{-19})(1.2915 \times 10^{9})$$

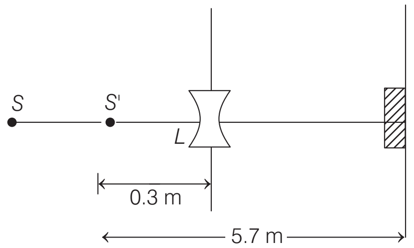
$$= 2.07 \times 10^{-10}\ \text{A}$$

(b) Using the lens formula :

$$\frac{1}{v} - \frac{1}{-0.6} = \frac{1}{-0.6}$$

or $\qquad v = -\,0.3\ \text{m}$

i.e. image of source (say S', is formed at 0.3 m) from the lens.



Total number of photons incident per unit time on the lens are still n_3 or 1.052×10^{16}/s. 80% of it transmits to second medium. Therefore, at a distance of 5.7 m from S' number of photons incident per unit area per unit time will be

$$n_7 = \frac{(80/100)\,(1.052 \times 10^{16})}{(4\pi)\,(5.7)^2}$$

$$= 2.06 \times 10^{13}\ \text{s}^{-1}\text{m}^{-2}$$

This is the photon flux at the detector

New, value of photocurrent is

$$i' = (2.06 \times 10^{13})\,(0.5 \times 10^{-4})\,(0.9)\,(1.6 \times 10^{-19})$$

$$= 1.483 \times 10^{-10}\,\text{A}$$

(c) Energy of incident photons (in both the cases) :

$$E = \frac{12375\,\text{eV-Å}}{6000\ \text{Å}} = 2.06\,\text{eV}$$

Work function $W = 1.0\,\text{eV}$

$\therefore$ Maximum kinetic energy of photoelectrons in both cases,

$$K_{\max} = E - W = 1.06\,\text{eV}$$

or the stopping potential will be 1.06 V.

56. The stopping potential for shorter wavelength is 3.95 V i.e. maximum kinetic energy of photoelectrons corresponding to shorter wavelength will be 3.95 eV. Further energy of incident photons corresponding to shorter wavelength will be in transition from $n = 4$ to $n = 3$.

$$E_{4-3} = E_4 - E_3 = \frac{-(13.6)\,(3)^2}{(4)^2} - \left[\frac{-(13.6)\,(3)^2}{(3)^2}\right]$$

$$= 5.95\ \text{eV}$$

Now, from the equation,

$$K_{\max} = E - W$$

we have $\quad W = E - K_{\max} = E_{4-3} - K_{\max}$

$$= (5.95 - 3.95)\,\text{eV} = 2\,\text{eV}$$

Longer wavelength will correspond to transition from $n = 5$ to $n = 4$. From the relation,

$$\frac{1}{\lambda} = Rz^2\left(\frac{1}{N_f{}^2} - \frac{1}{N_i{}^2}\right)$$

The longer wavelength,

$$\frac{1}{\lambda} = (1.094 \times 10^{7})\,(3)^3\left(\frac{1}{16} - \frac{1}{25}\right)$$

or $\qquad \lambda = 4.514 \times 10^{-7}\ \text{m} = 4514\ \text{Å}$

Energy corresponding to this wavelength,

$$E = \frac{12375\,\text{eV-Å}}{4514\,\text{Å}} = 2.74\ \text{eV}$$

$\therefore$ Maximum kinetic energy of photo-electrons

$$K_{\max} = E - W = (2.74 - 2)\,\text{eV}$$

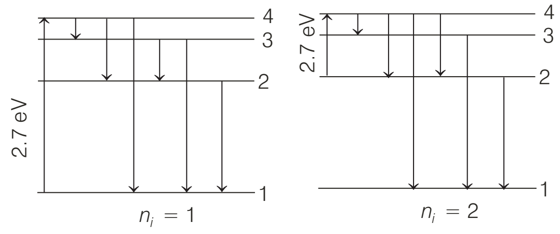
$$= 0.74\ \text{eV}$$

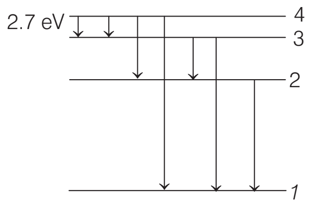
or the stoping potential is 0.74 V.

57. (a) In emission spectrum total six lines are obtained. Hence, after excitation if n_f be the final principal quantum number then,

$$\frac{n_f\,(n_f - 1)}{2} = 6 \quad \text{or} \quad n_f = 4\text{z}$$

i.e. after excitation atom goes to 4$^{\text{th}}$ energy state. Hence, n_i can be either 1, 2 or 3.





Absorption and emission spectrum corresponding to $n_i = 1, n_i = 2$ and $n_i = 3$ are shown in figure.

For $n_i = 1$, energy of emitted photons ≤ 2.7 eV

For $n_i = 2$, energy of emitted photons >=< 2.7 eV and

For $n_i = 3$, energy of emitted photons ≥ 2.7 eV.

As per the given condition $n_i = 2$.

(b) $$E_4 - E_2 = 2.7 \text{ eV}$$

or $$\frac{E_1}{16} - \frac{E_1}{4} = 2.7$$

or $$-\frac{3}{16}E_1 = 2.7$$

$$E_1 = -14.4 \text{ eV}$$

Therefore, ionization energy for the gas atoms is $|E_1|$ or 14.4 eV.

(c) Maximum energy of the emitted photons is corresponding to transition from $n = 4$ to $n = 1$.

$\therefore$ $$E_{max} = E_4 - E_1$$

$$= \frac{E_1}{16} - E_1 = \frac{-15E_1}{16}$$

$$= \left(-\frac{15}{16}\right)(14.4) = 13.5 \text{ eV}$$

Similarly, minimum energy of the emitted photons is corresponding to transition from $n = 4$ to $n = 3$.

$\therefore$ $$E_{min} = E_4 - E_3$$

$$= \frac{E_1}{16} - \frac{E_1}{9} = -\frac{7E_1}{16 \times 9}$$

$$= \frac{7 \times 14.4}{16 \times 9} = 0.7 \text{ eV}$$

58. Number of proton = atomic number = 11

Number of neutron = mass number – atomic number = 13

But note that in the nucleus number of electrons will be zero.