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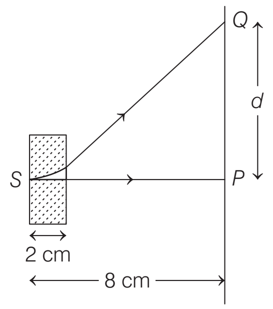
Magnetics

Topic 1 Magnetic Force and Path of Charged Particle in Uniform Fields

Objective Questions I (Only one correct option)

1. An electron moving along the X-axis with an initial energy of 100 eV, enters a region of magnetic field $\mathbf{B} = (1.5 \times 10^{-3}\,\text{T})\,\hat{\mathbf{k}}$ at S (see figure). The field extends between $x = 0$ and $x = 2$ cm. The electron is detected at the point Q on a screen placed 8 cm away from the point S. The distance d between P and Q (on the screen) is

(Take, electron's charge $= 1.6 \times 10^{-19}$ C, mass of electron $= 9.1 \times 10^{-31}$ kg) **(2019 Main, 12 April II)**



(a) 11.65 cm (b) 12.87 cm
(c) 1.22 cm (d) 2.25 cm

2. A proton, an electron and a helium nucleus, have the same energy. They are in circular orbits in a plane due to magnetic field perpendicular to the plane.

Let r_p, r_e and r_{He} be their respective radii, then **(2019 Main, 10 April I)**

(a) $r_e < r_p = r_{He}$ (b) $r_e > r_p = r_{He}$
(c) $r_e < r_p < r_{He}$ (d) $r_e > r_p > r_{He}$

3. A proton and an α-particle (with their masses in the ratio of 1 : 4 and charges in the ratio of 1 : 2) are accelerated from rest through a potential difference V. If a uniform magnetic field B is set up perpendicular to their velocities, the ratio of the radii $r_p : r_\alpha$ of the circular paths described by them will be **(2019 Main, 12 Jan I)**

(a) $1 : \sqrt{2}$ (b) $1 : \sqrt{3}$ (c) $1 : 3$ (d) $1 : 2$

4. A particle of mass m and charge q is in an electric and magnetic field is given by

$$\mathbf{E} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}},\ \mathbf{B} = 4\hat{\mathbf{j}} + 6\hat{\mathbf{k}}.$$

The charged particle is shifted from the origin to the point $P(x = 1;\ y = 1)$ along a straight path. The magnitude of the total work done is **(Main 2019, 11 Jan II)**

(a) $(0.35)\,q$ (b) $(0.15)\,q$ (c) $(2.5)\,q$ (d) $5\,q$

5. In an experiment, electrons are accelerated, from rest by applying a voltage of 500 V. Calculate the radius of the path, if a magnetic field 100 mT is then applied.
(Take, charge of the electron $= 1.6 \times 10^{-19}$ C and mass of the electron $= 9.1 \times 10^{-31}$ kg) **(2019 Main, 11 Jan I)**

(a) 7.5×10^{-2} m (b) 7.5×10^{-4} m
(c) 7.5×10^{-3} m (d) 7.5 m

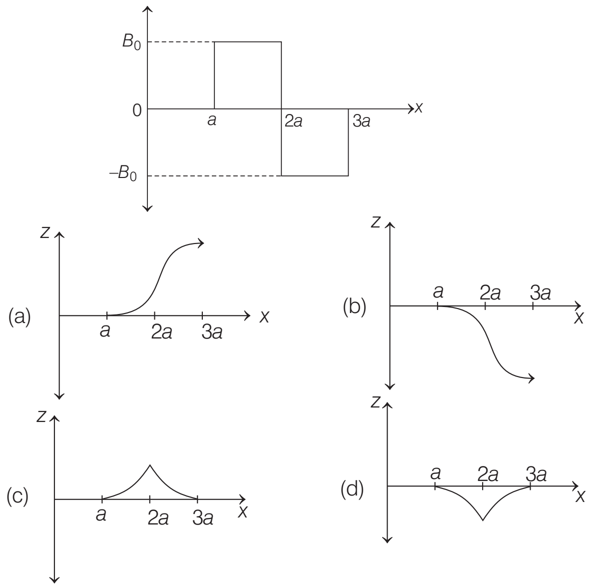
6. A particle having the same charge as of electron moves in a circular path of radius 0.5 cm under the influence of a magnetic field of 0.5 T. If an electric field of 100 V/m makes it to move in a straight path, then the mass of the particle is (Take, charge of electron $= 1.6 \times 10^{-19}$ C) **(2019 Main, 9 Jan II)**

(a) 1.6×10^{-19} kg (b) 1.6×10^{-27} kg
(c) 9.1×10^{-31} kg (d) 2.0×10^{-24} kg

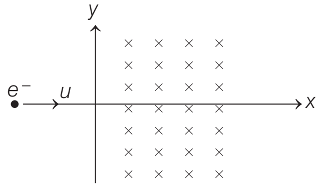
7. An electron, a proton and an alpha particle having the same kinetic energy are moving in circular orbits of radii r_e, r_p, r_α respectively, in a uniform magnetic field B. The relation between r_e, r_p, r_α is **(2018 Main)**

(a) $r_e < r_\alpha < r_p$ (b) $r_\alpha > r_e = r_p$
(c) $r_e < r_p = r_\alpha$ (d) $r_e < r_p < r_\alpha$

8. A magnetic field $\mathbf{B} = B_0\hat{\mathbf{j}}$ exists in the region $a < x < 2a$ and $\mathbf{B} = -B_0\hat{\mathbf{j}}$, in the region $2a < x < 3a$, where B_0 is a positive constant. A positive point charge moving with a velocity $\mathbf{v} = v_0\hat{\mathbf{i}}$, where v_0 is a positive constant, enters the magnetic field at $x = a$. The trajectory of the charge in this region can be like **(2007, 3M)**

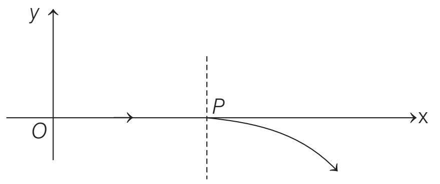


9. An electron moving with a speed u along the positive x-axis at $y = 0$ enters a region of uniform magnetic field $\mathbf{B} = -B_0\hat{\mathbf{k}}$ which exists to the right of y-axis. The electron exits from the region after sometime with the speed v at coordinate y, then **(2004, 2M)**



(a) $v > u, y < 0$ (b) $v = u, y > 0$
(c) $v > u, y > 0$ (d) $v = u, y < 0$

10. For a positively charged particle moving in a x-y plane initially along the x-axis, there is a sudden change in its path due to the presence of electric and/or magnetic fields beyond P. The curved path is shown in the x-y plane and is found to be non-circular. **(2003, 2M)**

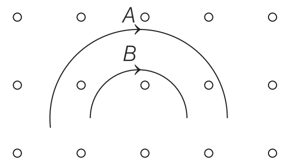


Which one of the following combinations is possible?
(a) $\mathbf{E} = 0; \mathbf{B} = b\hat{\mathbf{j}} + c\hat{\mathbf{k}}$ (b) $\mathbf{E} = a\hat{\mathbf{i}}; \mathbf{B} = c\hat{\mathbf{k}} + a\hat{\mathbf{i}}$
(c) $\mathbf{E} = 0; \mathbf{B} = c\hat{\mathbf{j}} + b\hat{\mathbf{k}}$ (d) $\mathbf{E} = a\hat{\mathbf{i}}; \mathbf{B} = c\hat{\mathbf{k}} + b\hat{\mathbf{j}}$

11. A particle of mass m and charge q moves with a constant velocity v along the positive x-direction. It enters a region containing a uniform magnetic field B directed along the negative z-direction, extending from $x = a$ to $x = b$. The minimum value of v required so that the particle can just enter the region $x > b$ is **(2002, 2M)**

(a) $\frac{qbB}{m}$ (b) $\frac{q(b-a)B}{m}$ (c) $\frac{qaB}{m}$ (d) $\frac{q(b+a)B}{2m}$

12. Two particles A and B of masses m_A and m_B respectively and having the same charge are moving in a plane. A uniform magnetic field exists perpendicular to this plane. The speeds of the particles are v_A and v_B, respectively and the trajectories are as shown in the figure. Then **(2001, 2M)**



(a) $m_A v_A < m_B v_B$ (b) $m_A v_A > m_B v_B$
(c) $m_A < m_B$ and $v_A < v_B$ (d) $m_A = m_B$ and $v_A = v_B$

13. An ionized gas contains both positive and negative ions. If it is subjected simultaneously to an electric field along the $+x$-direction and a magnetic field along the $+z$-direction, then **(2000, 2M)**
(a) positive ions deflect towards $+y$-direction and negative ions towards $-y$-direction
(b) all ions deflect towards $+y$-direction
(c) all ions deflect towards $-y$-direction
(d) positive ions deflect towards $-y$-direction and negative ions towards $-y$-direction

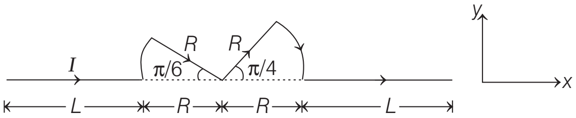
14. A charged particle is released from rest in a region of steady and uniform electric and magnetic fields which are parallel to each other. The particle will move in a **(1999, 2M)**
(a) straight line (b) circle
(c) helix (d) cycloid

15. A proton, a deutron and an α-particle having the same kinetic energy are moving in circular trajectories in a constant magnetic field. If r_p, r_d and r_α denote, respectively the radii of the trajectories of these particles, then **(1997, 1M)**
(a) $r_\alpha = r_p < r_d$ (b) $r_\alpha > r_d > r_p$
(c) $r_\alpha = r_d > r_p$ (d) $r_p = r_d = r_\alpha$

16. Two particles X and Y having equal charges, after being accelerated through the same potential difference, enter a region of uniform magnetic field and describe circular paths of radii R_1 and R_2 respectively. The ratio of the mass of X to that of Y is **(1988, 2M)**
(a) $(R_1/R_2)^{1/2}$ (b) R_2/R_1
(c) $(R_1/R_2)^2$ (d) R_1/R_2

Objective Questions II (One or more correct option)

17. A conductor (shown in the figure) carrying constant current I is kept in the x-y plane in a uniform magnetic field $\mathbf{B}$. If F is the magnitude of the total magnetic force acting on the conductor, then the correct statements is/are **(2015 Adv.)**



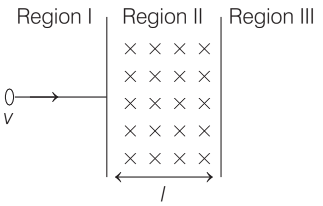
(a) if **B** is along $\hat{\mathbf{z}}$, $F \propto (L+R)$
(b) if **B** is along $\hat{\mathbf{x}}$, $F = 0$
(c) if **B** is along $\hat{\mathbf{y}}$, $F \propto (L+R)$
(d) if **B** is along $\hat{\mathbf{z}}$, $F = 0$

18. A particle of mass M and positive charge Q, moving with a constant velocity $\mathbf{u}_1 = 4\hat{\mathbf{i}}\ \mathrm{ms}^{-1}$, enters a region of uniform static magnetic field normal to the x-y plane. The region of the magnetic field extends from $x = 0$ to $x = L$ for all values of y. After passing through this region, the particle emerges on the other side after 10 milliseconds with a velocity $\mathbf{u}_2 = 2(\sqrt{3}\hat{\mathbf{i}} + \hat{\mathbf{j}})\ \mathrm{ms}^{-1}$. The correct statement(s) is (are)
(a) the direction of the magnetic field is $-z$ direction.
(b) the direction of the magnetic field is $+z$ direction
(c) the magnitude of the magnetic field is $\frac{50\pi M}{3Q}$ units.
(d) the magnitude of the magnetic field is $\frac{100\pi M}{3Q}$ units.
(2013 Adv.)

19. Consider the motion of a positive point charge in a region where there are simultaneous uniform electric and magnetic fields $\mathbf{E} = E_0\hat{\mathbf{j}}$ and $\mathbf{B} = B_0\hat{\mathbf{j}}$. At time $t = 0$, this charge has velocity $\mathbf{v}$ in the x-y plane, making an angle θ with the x-axis. Which of the following option(s) is(are) correct for time $t > 0$? **(2012)**
(a) If $\theta = 0°$, the charge moves in a circular path in the x-z plane.
(b) If $\theta = 0°$, the charge undergoes helical motion with constant pitch along the y-axis
(c) If $\theta = 10°$, the charge undergoes helical motion with its pitch increasing with time, along the y-axis.
(d) If $\theta = 90°$, the charge undergoes linear but accelerated motion along the y-axis.

20. An electron and a proton are moving on straight parallel paths with same velocity. They enter a semi-infinite region of uniform magnetic field perpendicular to the velocity. Which of the following statement(s) is/are true? **(2011)**
(a) They will never come out of the magnetic field region
(b) They will come out travelling along parallel paths
(c) They will come out at the same time
(d) They will come out at different times

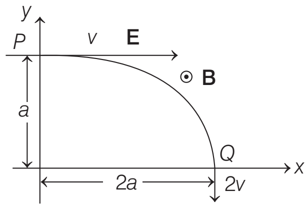
21. A particle of mass m and charge q, moving with velocity v enters Region II normal to the boundary as shown in the figure. Region II has a uniform magnetic field B perpendicular to the plane of the paper. The length of the Region II is l. Choose the correct choice (s). **(2008, 4M)**



(a) The particle enters Region III only if its velocity $v > \frac{qlB}{m}$
(b) The particle enters Region III only if its velocity $v < \frac{qlB}{m}$
(c) Path length of the particle in Region II is maximum when velocity $v = qlB/m$
(d) Time spent in Region II is same for any velocity v as long as the particle returns to Region I.

22. H^+, He^+ and O^{2+} all having the same kinetic energy pass through a region in which there is a uniform magnetic field perpendicular to their velocity. The masses of H^+, He^+ and O^{2+} are 1 amu, 4 amu and 16 amu respectively. Then
(a) H^+ will be deflected most **(1994, 2M)**
(b) O^{2+} will be deflected most
(c) He^+ and O^{2+} will be deflected equally
(d) all will be deflected equally

23. A particle of charge $+q$ and mass m moving under the influence of a uniform electric field $E\hat{\mathbf{i}}$ and uniform magnetic field $B\hat{\mathbf{k}}$ follows a trajectory from P to Q as shown in figure. The velocities at P and Q are $v\hat{\mathbf{i}}$ and $-2\hat{\mathbf{j}}$. Which of the following statement(s) is/are correct ? **(1991, 2M)**



(a) $E = \frac{3}{4}\left[\frac{mv^2}{qa}\right]$
(b) Rate of work done by the electric field at P is $\frac{3}{4}\left[\frac{mv^3}{a}\right]$
(c) Rate of work done by the electric field at P is zero
(d) Rate of work done by both the fields at Q is zero

24. A proton moving with a constant velocity passes through a region of space without any change in its velocity. If E and B represent the electric and magnetic fields respectively. Then, this region of space may have **(1985, 2M)**
(a) $E = 0, B = 0$
(b) $E = 0, B \neq 0$
(c) $E \neq 0, B = 0$
(d) $E \neq 0, B \neq 0$

Match the Columns

Directions (Q.Nos. 25-27) Matching the information given in the three columns of the following table.

A charged particle (electron or proton) is introduced at the origin ($x = 0$, $y = 0$, $z = 0$) with a given initial velocity $\mathbf{v}$. A uniform electric field $\mathbf{E}$ and a uniform magnetic field $\mathbf{B}$ exist everywhere. The velocity $\mathbf{v}$, electric field $\mathbf{E}$ and magnetic field $\mathbf{B}$ are given in columns 1, 2 and 3, respectively. The quantities E_0, B_0 are positive in magnitude.

Column 1	Column 2	Column 3
(I) Electron with $v = 2\frac{E_0}{B_0}\hat{x}$	(i) $E = E_0\hat{z}$	(P) $B = -B_0\hat{x}$
(II) Election with $v = \frac{E_0}{B_0}\hat{y}$	(ii) $E = -E_0\hat{y}$	(Q) $B = B_0\hat{x}$
(III) Proton with $v = 0$	(iii) $E = -E_0\hat{x}$	(R) $B = B_0\hat{y}$
(IV) Proton with $v = 2\frac{E_0}{B_0}\hat{x}$	(iv) $E = E_0\hat{x}$	(S) $B = B_0\hat{z}$

(2017 Adv.)

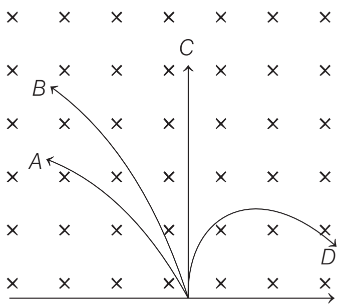
25. In which case would the particle move in a straight line along the negative direction of Y-axis (i.e. move along $-\hat{y}$)?
(a) (IV) (ii) (S) (b) (II) (iii) (Q)
(c) (III), (ii) (R (d) (III) (ii) (P)

26. In which case will the particle move in a straight line with constant velocity?
(a) (II) (iii) (S) (b) (III) (iii) (P)
(c) (IV) (i) (S) (d) (III) (ii) (R)

27. In which case will the particle describe a helical path with axis along the positive z-direction?
(a) (II) (ii) (R) (b) (III) (iii) (P)
(c) (IV) (i) (S) (d) (IV) (ii) (R)

Fill in the Blanks

28. A neutron, a proton, an electron and an alpha particle enter a region of constant magnetic field with equal velocities. The magnetic field is along the inward normal to the plane of the paper. The tracks of the particles are labelled in figure. The electron follows track…… and the alpha particle follows track…… **(1984, 2M)**



True/False

29. An electron and a proton are moving with the same kinetic energy along the same direction. When they pass through a uniform magnetic field perpendicular to the direction of their motion, they describe circular path of the same radius. **(1985, 3M)**

30. A charged particle enters a region of uniform magnetic field at an angle of 85° to the magnetic line of force. The path of the particle is a circle. **(1983, 2M)**

31. There is no change in the energy of a charged particle moving in magnetic field although a magnetic force is acting on it. **(1983, 2M)**

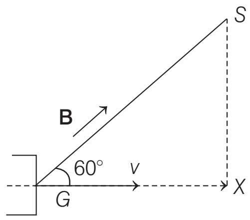
Analytical & Descriptive Questions

32. A proton and an alpha particle, after being accelerated through same potential difference, enter uniform magnetic field, the direction of which is perpendicular to their velocities. Find the ratio of radii of the circular paths of the two particles. **(2004, 2M)**

33. The region between $x = 0$ and $x = L$ is filled with uniform steady magnetic field $-B_0\hat{\mathbf{k}}$. A particle of mass m, positive charge q and velocity $v_0\hat{\mathbf{i}}$ travels along x-axis and enters the region of the magnetic field. **(1999, 10M)**
Neglect the gravity throughout the question.

(a) Find the value of L if the particle emerges from the region of magnetic field with its final velocity at an angle 30° to its initial velocity.
(b) Find the final velocity of the particle and the time spent by it in the magnetic field, if the magnetic field now expands upto $2.1\,L$.

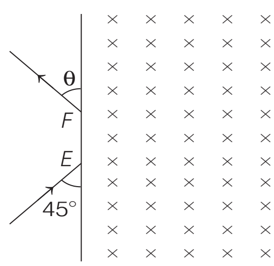
34. An electron gun G emits electrons of energy 2 keV travelling in the positive x-direction. The electrons are required to hit the spot S where $GS = 0.1$ m, and the line GS make an angle of 60° with the x-axis as shown in figure. A uniform magnetic field **B** parallel to GS exists in the region outside the electron gun.



Find the minimum value of B needed to make the electrons hit S. **(1993, 7M)**

35. A beam of protons with a velocity 4×10^5 m/s enters a uniform magnetic field of 0.3 T at an angle of 60° to the magnetic field. Find the radius of the helical path taken by the protons beam. Also find the pitch of the helix (which is the distance travelled by a proton in the beam parallel to the magnetic field during one period of rotation). **(1986, 6M)**

36. A particle of mass $m = 1.6 \times 10^{-27}$ kg and charge $q = 1.6 \times 10^{-19}$C enters a region of uniform magnetic field of strength 1 T along the direction shown in figure. The speed of the particle is 10^7m/s. **(1984, 8M)**

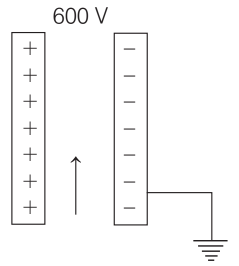


(a) The magnetic field is directed along the inward normal to the plane of the paper. The particle leaves the region of the field at the point F. Find the distance EF and the angle θ.

(b) If the direction of the field is along the outward normal to the plane of the paper, find the time spent by the particle in the region of the magnetic field after entering it at E.

37. A particle of mass 1×10^{-26} kg and charge $+1.6\times10^{-19}$ C travelling with a velocity 1.28×10^{6} m/s in the $+x$ direction enters a region in which a uniform electric field E and a uniform magnetic field of induction B are present such that $E_x = E_y = 0, E_z = -102.4$ kV/m and $B_x = B_z = 0$, $B_y = 8\times10^{-2}$ T. The particle enters this region at the origin at time $t = 0$. Determine the location (x, y and z coordinates) of the particle at $t = 5\times10^{-6}$ s. If the electric field is switched off at this instant (with the magnetic field still present), what will be the position of the particle at $t = 7.45\times10^{-6}$ s ? **(1982, 7M)**

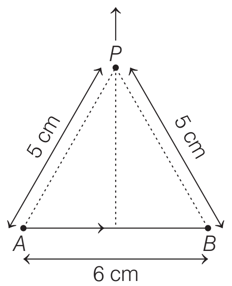
38. A potential difference of 600 V is applied across the plates of a parallel plate condenser. The separation between the plates is 3 mm. An electron projected vertically, parallel to the plates, with a velocity of 2×10^{6} m/s moves undeflected between the plates. Find the magnitude and direction of the magnetic field in the region between the condenser plates. (Neglect the edge effects).
(Charge of the electron $= 1.6\times10^{-19}$ C) **(1981, 3M)**



Topic 2 Biot-Savart Law and Ampere's Circuital Law

Objective Questions I (Only one correct option)

1. Find the magnetic field at point P due to a straight line segment AB of length 6 cm carrying a current of 5 A (See figure). (Take, $\mu_0 = 4\pi\times10^{-7}$ N-A^{-2}) **(2019 Main, 12 April II)**



(a) 2.0×10^{-5} T (b) 1.5×10^{-5} T
(c) 3.0×10^{-5} T (d) 2.5×10^{-5} T

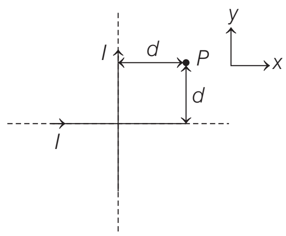
2. A thin ring of 10 cm radius carries a uniformly distributed charge. The ring rotates at a constant angular speed of 40π rad s^{-1} about its axis, perpendicular to its plane. If the magnetic field at its centre is 3.8×10^{-9} T, then the charge carried by the ring is close to ($\mu_0 = 4\pi\times10^{-7}$ N/A^2). **(2019 Main, 12 April I)**

(a) 2×10^{-6} C (b) 3×10^{-5} C
(c) 4×10^{-5} C (d) 7×10^{-6} C

3. The magnitude of the magnetic field at the centre of an equilateral triangular loop of side 1 m which is carrying a current of 10 A is
[Take, $\mu_0 = 4\pi\times10^{-7}$NA^{-2}] **(2019 Main, 10 April II)**

(a) 9 μT (b) 1 μT
(c) 3 μT (d) 18 μT

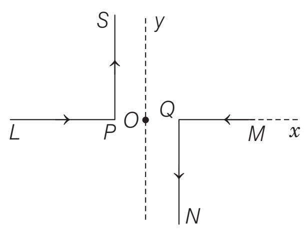
4. Two very long, straight and insulated wires are kept at 90° angle from each other in xy-plane as shown in the figure



These wires carry currents of equal magnitude I, whose directions are shown in the figure. The net magnetic field at point P will be **(2019 Main, 8 April II)**

(a) zero (b) $\frac{+\mu_0 I}{\pi d}(\hat{\mathbf{z}})$
(c) $-\frac{\mu_0 I}{2\pi d}(\hat{\mathbf{x}}+\hat{\mathbf{y}})$ (d) $\frac{\mu_0 I}{2\pi d}(\hat{\mathbf{x}}+\hat{\mathbf{y}})$

5. As shown in the figure, two infinitely long, identical wires are bent by 90° and placed in such a way that the segments LP and QM are along the X-axis, while segments PS and QN are parallel to the Y-axis. If $OP = OQ = 4$ cm and the magnitude of the magnetic field at O is 10^{-4} T and the two wires carry equal currents (see figure), the magnitude of the current in each wire and the direction of the magnetic field at O will be (Take, $\mu_0 = 4\pi\times10^{-7}$ NA^{-2}) **(2019 Main, 12 Jan I)**

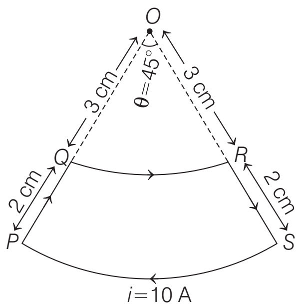


(a) 40 A, perpendicular out of the page
(b) 20 A, perpendicular into the page
(c) 20 A, perpendicular out of the page
(d) 40 A, perpendicular into the page

6. One of the two identical conducting wires of length L is bent in the form of a circular loop and the other one into a circular coil of N identical turns. If the same current is passed in both, the ratio of the magnetic field at the centre of the loop (B_L) to that at the centre of the coil (B_C), i.e. $\frac{B_L}{B_C}$ will be **(2019 Main, 9 Jan II)**

(a) $\frac{1}{N}$ (b) N (c) $\frac{1}{N^2}$ (d) N^2

7. A current loop, having two circular arcs joined by two radial lines as shown in the figure. It carries a current of 10 A. The magnetic field at point O will be close to **(2019 Main, 9 Jan I)**

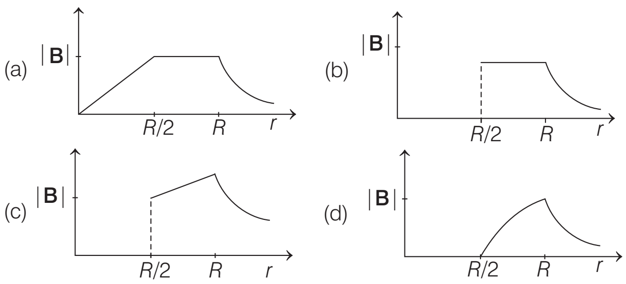


(a) 1.0×10^{-5} T (b) 1.0×10^{-7} T
(c) 1.5×10^{-7} T (d) 1.5×10^{-5} T

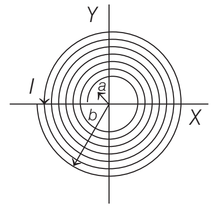
8. Two identical wires A and B, each of length l, carry the same current I. Wire A is bent into a circle of radius R and wire B is bent to form a square of side a. If B_A and B_B are the values of magnetic field at the centres of the circle and square respectively, then the ratio $\frac{B_A}{B_B}$ is **(2016 Main)**

(a) $\frac{\pi^2}{8}$ (b) $\frac{\pi^2}{16\sqrt{2}}$ (c) $\frac{\pi^2}{16}$ (d) $\frac{\pi^2}{8\sqrt{2}}$

9. An infinitely long hollow conducting cylinder with inner radius $R/2$ and outer radius R carries a uniform current density along its length. The magnitude of the magnetic field, $|\mathbf{B}|$ as a function of the radial distance r from the axis is best represented by **(2012)**



10. A long insulated copper wire is closely wound as a spiral of N turns. The spiral has inner radius a and outer radius b. The spiral lies in the X-Y plane and a steady current I flows through the wire. The Z-component of the magnetic field at the centre of the spiral is **(2011)**



(a) $\frac{\mu_0 NI}{2(b-a)}\ln\left(\frac{b}{a}\right)$ (b) $\frac{\mu_0 NI}{2(b-a)}\ln\left(\frac{b+a}{b-a}\right)$
(c) $\frac{\mu_0 NI}{2b}\ln\left(\frac{b}{a}\right)$ (d) $\frac{\mu_0 NI}{2b}\ln\left(\frac{b+a}{b-a}\right)$

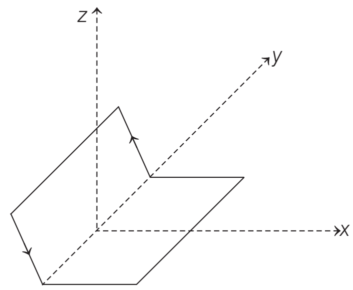
11. A long straight wire along the z-axis carries a current I in the negative z-direction. The magnetic vector field $\mathbf{B}$ at a point having coordinate (x, y) on the $z = 0$ plane is **(2002, 2M)**

(a) $\frac{\mu_0 I(y\hat{\mathbf{i}} - x\hat{\mathbf{j}})}{2\pi(x^2+y^2)}$ (b) $\frac{\mu_0 I(x\hat{\mathbf{i}} + y\hat{\mathbf{j}})}{2\pi(x^2+y^2)}$
(c) $\frac{\mu_0 I(x\hat{\mathbf{j}} - y\hat{\mathbf{i}})}{2\pi(x^2+y^2)}$ (d) $\frac{\mu_0 I(x\hat{\mathbf{i}} - y\hat{\mathbf{j}})}{2\pi(x^2+y^2)}$

12. A coil having N turns is wound tightly in the form of a spiral with inner and outer radii a and b respectively. When a current I passes through the coil, the magnetic field at the centre is **(2001, 2M)**

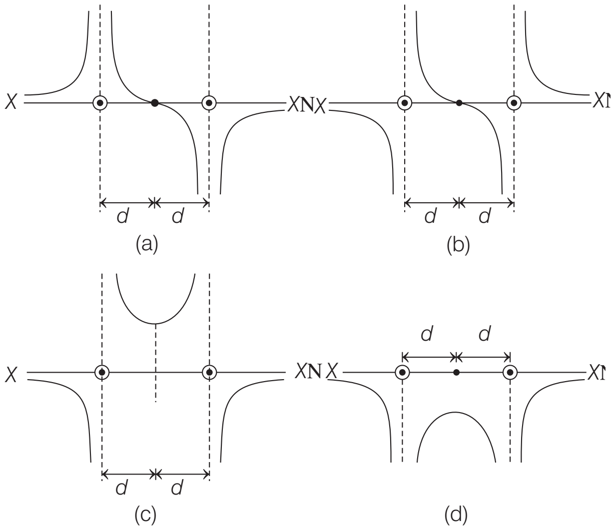
(a) $\frac{\mu_0 NI}{b}$ (b) $\frac{2\mu_0 NI}{a}$
(c) $\frac{\mu_0 NI}{2(b-a)}\ln\frac{b}{a}$ (d) $\frac{\mu_0 I^N}{2(b-a)}\ln\frac{b}{a}$

13. A non-planar loop of conducting wire carrying a current I is placed as shown in the figure. Each of the straight sections of the loop is of length $2a$. The magnetic field due to this loop at the point P $(a, 0, a)$ points in the direction **(2001, 2M)**

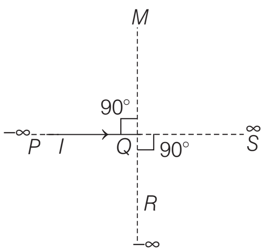


(a) $\frac{1}{\sqrt{2}}(-\hat{\mathbf{j}} + \hat{\mathbf{k}})$ (b) $\frac{1}{\sqrt{3}}(-\hat{\mathbf{j}} + \hat{\mathbf{k}} + \hat{\mathbf{i}})$
(c) $\frac{1}{\sqrt{3}}(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$ (d) $\frac{1}{\sqrt{2}}(\hat{\mathbf{i}} + \hat{\mathbf{k}})$

14. Two long parallel wires are at a distance $2d$ apart. They carry steady equal currents flowing out of the plane of the paper as shown. The variation of the magnetic field B along the line XX' is given by **(2000, 2M)**



15. An infinitely long conductor PQR is bent to form a right angle as shown in figure. A current I flows through PQR. The magnetic field due to this current at the point M is H_1. Now, another infinitely long straight conductor QS is connected at Q, so that current is $I/2$ in QR as well as in QS, the current in PQ remaining unchanged. The magnetic field at M is now H_2. The ratio H_1/H_2 is given by **(2000, 2M)**



(a) 1/2 (b) 1 (c) 2/3 (d) 2

16. A battery is connected between two points A and B on the circumference of a uniform conducting ring of radius r and resistance R. One of the arcs AB of the ring subtends an angle θ at the centre. The value of the magnetic induction at the centre due to the current in the ring is **(1995, 2M)**

(a) proportional to $(180° - \theta)$ (b) inversely proportional to r
(c) zero, only if $(\theta = 180°)$ (d) zero for all values of θ

17. A current I flows along the length of an infinitely long, straight, thin-walled pipe. Then **(1993, 2M)**

(a) the magnetic field at all points inside the pipe is the same, but not zero
(b) the magnetic field at any point inside the pipe is zero
(c) the magnetic field is zero only on the axis of the pipe
(d) the magnetic field is different at different points inside the pipe

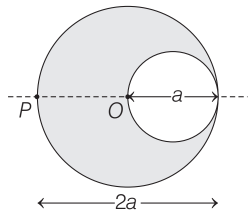
Objective Questions II (One or more correct option)

18. A steady current I flows along an infinitely long hollow cylindrical conductor of radius R. This cylinder is placed coaxially inside an infinite solenoid of radius $2R$. The solenoid has n turns per unit length and carries a steady current I. Consider a point P at a distance r from the common axis. The correct statement(s) is (are) **(2013 Adv.)**

(a) In the region $0 < r < R$, the magnetic field is non-zero
(b) In the region $R < r < 2R$, the magnetic field is along the common axis
(c) In the region $R < r < 2R$, the magnetic field is tangential to the circle of radius r, centered on the axis
(d) In the region $r > 2R$, the magnetic field is non-zero

Integer Answer Type Questions

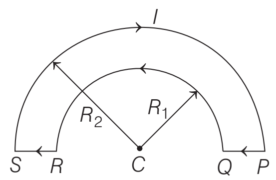
19. A cylindrical cavity of diameter a exists inside a cylinder of diameter $2a$ as shown in the figure. Both the cylinder and the cavity are infinitely long. A uniform current density J flows along the length. If the magnitude of the magnetic field at the point P is given by $\frac{N}{12}\mu_0 aJ$, then the value of N is **(2012)**



20. Two parallel wires in the plane of the paper are distance x_0 apart. A point charge is moving with speed u between the wires in the same plane at a distance x_1 from one of the wires. When the wires carry current of magnitude I in the same direction, the radius of curvature of the path of the point charge is R_1. In contrast, if the currents I in the two wires have directions opposite to each other, the radius of curvature of the path is R_2. If $\frac{x_0}{x_1} = 3$, and value of $\frac{R_1}{R_2}$ is **(2014 Adv.)**

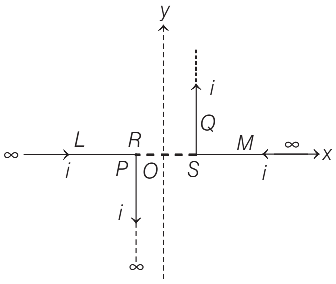
Fill in the Blank

21. The wire loop $PQRSP$ formed by joining two semicircular wires of radii R_1 and R_2 carries a current I as shown. The magnitude of the magnetic induction at the centre C is …… **(1988, 2M)**



Analytical & Descriptive Questions

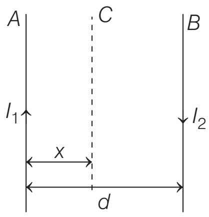
22. A pair of stationary and infinitely long bent wires are placed in the xy plane as shown in figure. The wires carry current of $i = 10$ A each as shown. The segments L and M are along the x-axis. The segments P and Q are parallel to the y-axis such that $OS = OR = 0.02$ m. Find the magnitude and direction of the magnetic induction at the origin O. **(1989, 6M)**



Topic 3 Magnetic Force on Current Carrying Wires

Objective Questions I (Only one correct option)

1. Two wires A and B are carrying currents I_1 and I_2 as shown in the figure. The separation between them is d. A third wire C carrying a current I is to be kept parallel to them at a distance x from A such that the net force acting on it is zero. The possible values of x are **(2019 Main, 10 April I)**



(a) $x = \left(\dfrac{I_2}{I_1 + I_2}\right) d$ and $x = \left(\dfrac{I_2}{I_1 - I_2}\right) d$

(b) $x = \left(\dfrac{I_1}{I_1 - I_2}\right) d$ and $x = \left(\dfrac{I_2}{I_1 + I_2}\right) d$

(c) $x = \left(\dfrac{I_1}{I_1 + I_2}\right) d$ and $x = \left(\dfrac{I_2}{I_1 - I_2}\right) d$

(d) $x = \pm \dfrac{I_1 d}{(I_1 - I_2)}$

2. A circular coil having N turns and radius r carries a current I. It is held in the XZ-plane in a magnetic field $B\hat{\mathbf{i}}$. The torque on the coil due to the magnetic field (in N-m) is **(2019 Main, 8 April I)**

(a) $\dfrac{Br^2 I}{\pi N}$ (b) $B\pi r^2 IN$ (c) $\dfrac{B\pi r^2 I}{N}$ (d) Zero

3. Two coaxial solenoids of different radii carry current I in the same direction. Let $\mathbf{F}_1$ be the magnetic force on the inner solenoid due to the outer one and $\mathbf{F}_2$ be the magnetic force on the outer solenoid due to the inner one. Then, **(2015 Main)**

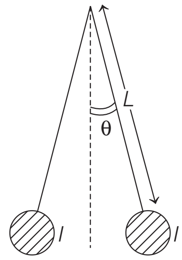
(a) $\mathbf{F}_1$ is radially outwards and $\mathbf{F}_2 = 0$

(b) $\mathbf{F}_1$ is radially inwards and $\mathbf{F}_2$ is radially outwards

(c) $\mathbf{F}_1$ is radially inwards and $\mathbf{F}_2 = 0$

(d) $\mathbf{F}_1 = \mathbf{F}_2 = 0$

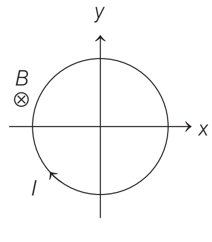
4. Two long current carrying thin wires, both with current I, are held by insulating threads of length L and are in equilibrium as shown in the figure, with threads making an angle θ with the vertical. If wires have mass λ per unit length then, the value of I is (g = gravitational acceleration) **(2015 Main)**



(a) $2\sin\theta \sqrt{\dfrac{\pi\lambda g L}{\mu_0 \cos\theta}}$ (b) $\sin\theta \sqrt{\dfrac{\pi\lambda g L}{\mu_0 \cos\theta}}$

(c) $2\sqrt{\dfrac{\pi g L}{\mu_0}\tan\theta}$ (d) $\sqrt{\dfrac{\pi\lambda g L}{\mu_0}\tan\theta}$

5. A conducting loop carrying a current I is placed in a uniform magnetic field pointing into the plane of the paper as shown. The loop will have a tendency to **(2003, 2M)**



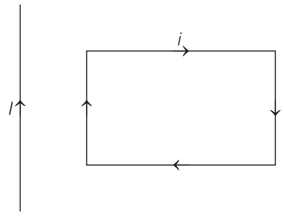
(a) contract
(b) expand
(c) move towards + ve x-axis
(d) move towards –ve x-axis

6. Two thin long parallel wires separated by a distance b are carrying a current i ampere each. The magnitude of the force per unit length exerted by one wire on the other is **(1986, 2M)**

(a) $\dfrac{\mu_0 i^2}{b^2}$ (b) $\dfrac{\mu_0 i^2}{2\pi b}$

(c) $\dfrac{\mu_0 i}{2\pi b}$ (d) $\dfrac{\mu_0 i}{2\pi b^2}$

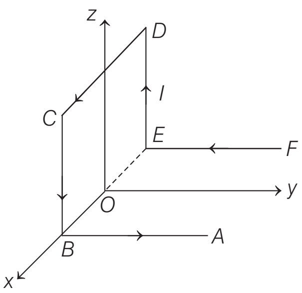
7. A rectangular loop carrying a current i is situated near a long straight wire such that the wire is parallel to one of the sides of the loop and is in the plane of the loop. If steady current I is established in the wire as shown in the figure, the loop will **(1985, 2M)**



(a) rotate about an axis parallel to the wire
(b) move away from the wire
(c) move towards the wire
(d) remain stationary

Fill in the Blank

8. A wire $ABCDEF$ (with each side of length L) bent as shown in figure and carrying a current I is placed in a uniform magnetic induction B parallel to the positive y-direction. The force experienced by the wire is ……in the …… direction. **(1990, 2M)**



True/False

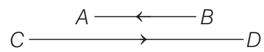
9. No net force acts on a rectangular coil carrying a steady current when suspended freely in a uniform magnetic field. **(1981, 2M)**

Analytical & Descriptive Questions

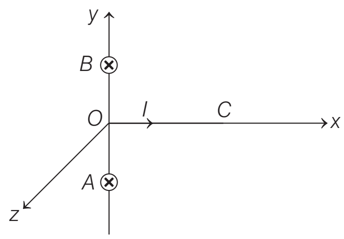
10. Three infinitely long thin wires, each carrying current i in the same direction, are in the x-y plane of a gravity free space. The central wire is along the y-axis while the other two are along $x = \pm d$. **(1997, 5M)**

(a) Find the locus of the points for which the magnetic field B is zero.
(b) If the central wire is displaced along the z-direction by a small amount and released, show that it will execute simple harmonic motion. If the linear density of the wires is λ, find the frequency of oscillation.

11. A long horizontal wire AB, which is free to move in a vertical plane and carries a steady current of 20 A, is in equilibrium at a height of 0.01 m over another parallel long wire CD which is fixed in a horizontal plane and carries a steady current of 30 A, as shown in figure. Show that when AB is slightly depressed, it executes simple harmonic motion. Find the period of oscillations. **(1994, 6M)**

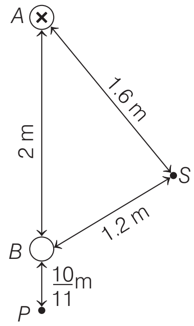


12. A straight segment OC (of length L) of a circuit carrying a current I is placed along the x-axis . Two infinitely long straight wires A and B, each extending from $z = -\infty$ to $+\infty$, are fixed at $y = -a$ and $y = +a$ respectively, as shown in the figure. If the wires A and B each carry a current I into the plane of the paper, obtain the expression for the force acting on the segment OC. What will be the force on OC if the current in the wire B is reversed? **(1992, 10M)**



13. Two long straight parallel wires are 2 m apart, perpendicular to the plane of the paper.

The wire A carries a current of 9.6 A, directed into the plane of the paper. The wire B carries a current such that the magnetic field of induction at the point P, at a distance of $10/11$ m from the wire B, is zero. **(1987, 7M)**
Find

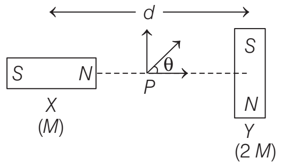


(a) the magnitude and direction of the current in B.
(b) the magnitude of the magnetic field of induction at the point S.
(c) the force per unit length on the wire B.

Topic 4 Magnetic Dipole

Objective Questions I (Only one correct option)

1. Two magnetic dipoles X and Y are placed at a separation d, with their axes perpendicular to each other. The dipole moment of Y is twice that of X. A particle of charge q is passing through their mid-point P, at angle $\theta = 45°$ with the horizontal line, as shown in figure. What would be the magnitude of force on the particle at that instant? (d is much larger than the dimensions of the dipole) **(2019 Main, 8 April II)**



(a) $\left(\frac{\mu_0}{4\pi}\right)\frac{M}{\left(\frac{d}{2}\right)^3} \times qv$ (b) 0

(c) $\sqrt{2}\left(\frac{\mu_0}{4\pi}\right)\frac{M}{\left(\frac{d}{2}\right)^3} \times qv$ (d) $\left(\frac{\mu_0}{4\pi}\right)\frac{2M}{\left(\frac{d}{2}\right)^3} \times qv$

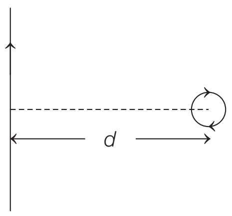
2. A square loop is carrying a steady current I and the magnitude of its magnetic dipole moment is m. If this square loop is changed to a circular loop and it carries the same current, the magnitude of the magnetic dipole moment of circular loop (in A-m) will be **(2019 Main, 10 April II)**

(a) $\frac{4m}{\pi}$ (b) $\frac{3m}{\pi}$ (c) $\frac{2m}{\pi}$ (d) $\frac{m}{\pi}$

3. An insulating thin rod of length l has a linear charge density $\rho(x) = \rho_0 \frac{x}{l}$ on it. The rod is rotated about an axis passing through the origin $(x = 0)$ and perpendicular to the rod. If the rod makes n rotations per second, then the time averaged magnetic moment of the rod is **(2019 Main, 10 Jan I)**

(a) $n\rho l^3$ (b) $\pi n \rho l^3$ (c) $\frac{\pi}{3} n \rho l^3$ (d) $\frac{\pi}{4} n \rho l^3$

4. An infinitely long current-carrying wire and a small current-carrying loop are in the plane of the paper as shown. The radius of the loop is a and distance of its centre from the wire is $d\,(d >> a)$. If the loop applies a force F on the wire, then **(2019 Main, 9 Jan I)**

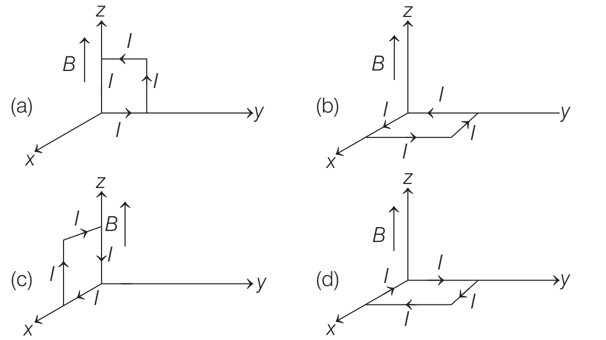


(a) $F \propto \left(\frac{a^2}{d^3}\right)$ (b) $F = 0$ (c) $F \propto \left(\frac{a}{d}\right)$ (d) $F \propto \left(\frac{a}{d}\right)^2$

5. The dipole moment of a circular loop carrying a current I is m and the magnetic field at the centre of the loop is B_1. When the dipole moment is doubled by keeping the current constant, the magnetic field at the centre of the loop is B_2. The ratio $\frac{B_1}{B_2}$ is **(2018 Main)**

(a) $\frac{1}{\sqrt{2}}$ (b) 2 (c) $\sqrt{3}$ (d) $\sqrt{2}$

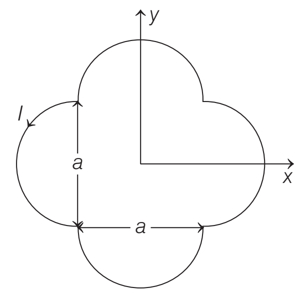
6. A rectangular loop of sides 10 cm and 5 cm carrying a current I of 12 A is placed in different orientations as shown in the figures below. **(2015 Main)**



If there is a uniform magnetic field of 0.3 T in the positive z-direction, in which orientations the loop would be in (i) stable equilibrium and (ii) unstable equilibrium?

(a) (a) and (b) respectively (b) (b) and (d) respectively
(c) (a) and (c) respectively (d) (b) and (c) respectively

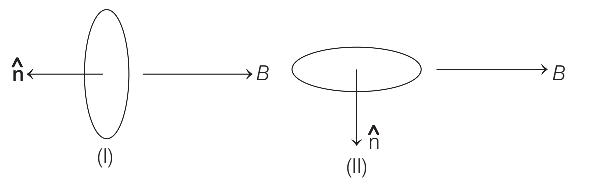
7. A loop carrying current I lies in the x-y plane as shown in the figure. The unit vector $\hat{\mathbf{k}}$ is coming out of the plane of the paper. The magnetic moment of the current loop is **(2012)**

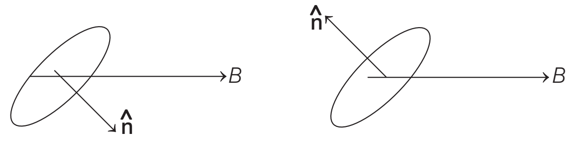


(a) $a^2 I\,\hat{\mathbf{k}}$ (b) $\left(\frac{\pi}{2}+1\right)a^2\,I\,\hat{\mathbf{k}}$

(c) $-\left(\frac{\pi}{2}+1\right)a^2\,I\,\hat{\mathbf{k}}$ (d) $(2\pi+1)\,a^2\,I\,\hat{\mathbf{k}}$

8. A current carrying loop is placed in a uniform magnetic field in four different orientations, I, II, III and IV. Arrange them in the decreasing order of potential energy. **(2003, 2M)**





(III) (IV)

(a) I > III > II > IV
(b) I > II > III > IV
(c) I > IV > II > III
(d) III > IV > I > II

9. A particle of charge q and mass m moves in a circular orbit of radius r with angular speed ω. The ratio of the magnitude of its magnetic moment to that of its angular momentum depends on **(2000, 2M)**

(a) ω and q
(b) ω, q and m
(c) q and m
(d) ω and m

10. Two particles, each of mass m and charge q, are attached to the two ends of a light rigid rod of length $2R$. The rod is rotated at constant angular speed about a perpendicular axis passing through its centre. The ratio of the magnitudes of the magnetic moment of the system and its angular momentum about the centre of the rod is **(1998, 2M)**

(a) $q/2m$
(b) q/m
(c) $2q/m$
(d) $q/\pi m$

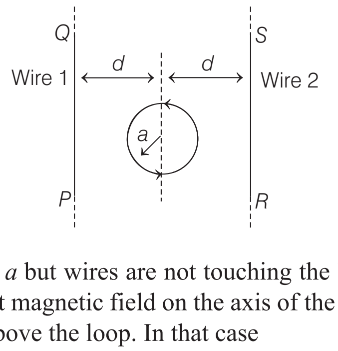
11. A conducting circular loop of radius r carries a constant current i. It is placed in a uniform magnetic field $\mathbf{B}_0$ such that $\mathbf{B}_0$ is perpendicular to the plane of the loop. The magnetic force acting on the loop is **(1983, 1M)**

(a) $ir\mathbf{B}_0$ (b) $2\pi ir\mathbf{B}_0$ (c) zero (d) $\pi ir\mathbf{B}_0$

Passage Based Questions

Passage

The figure shows a circular loop of radius a with two long parallel wires (numbered 1 and 2) all in the plane of the paper. The distance of each wire from the centre of the loop is d. The loop and the wires are carrying the same current I. The current in the loop is in the counter-clockwise direction if seen from above. **(2014 Adv.)**



12. When $d \approx a$ but wires are not touching the loop, it is found that the net magnetic field on the axis of the loop is zero at a height h above the loop. In that case

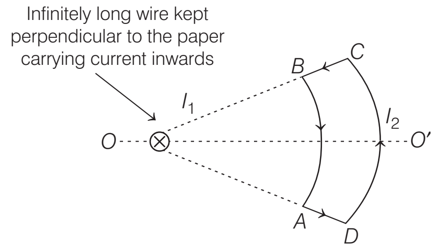
(a) current in wire 1 and wire 2 is the direction PQ and RS, respectively and $h \approx a$
(b) current in wire 1 and wire 2 is the direction PQ and SR, respectively and $h \approx a$
(c) current in wire 1 and wire 2 is the direction PQ and SR, respectively and $h \approx 1.2a$
(d) current in wire 1 and wire 2 is the direction PQ and RS, respectively and $h \approx 1.2a$

13. Consider $d >> a$, and the loop is rotated about its diameter parallel to the wires by 30° from the position shown in the figure. If the currents in the wires are in the opposite directions, the torque on the loop at its new position will be (assume that the net field due to the wires is constant over the loop)

(a) $\dfrac{\mu_0 I^2 a^2}{d}$
(b) $\dfrac{\mu_0 I^2 a^2}{2d}$
(c) $\dfrac{\sqrt{3}\,\mu_0 I^2 a^2}{d}$
(d) $\dfrac{\sqrt{3}\,\mu_0 I^2 a^2}{2d}$

Questions II (One or more correct option)

14. Which of the following statement is (are) correct in the given figure ? **(2006, 5M)**



(a) Net force on the loop is zero
(b) Net torque on the loop is zero
(c) Loop will rotate clockwise about axis OO' when seen from O
(d) Loop will rotate anticlockwise about OO' when seen from O

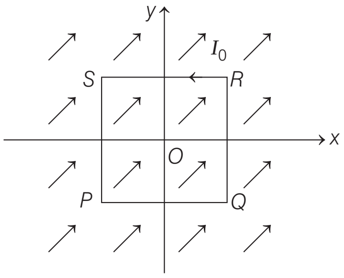
Fill in the Blanks

15. In a hydrogen atom, the electron moves in an orbit of radius 0.5 Å making 10^{16} revolutions per second. The magnetic moment associated with the orbital motion of the electron is …… **(1988, 2M)**

16. A wire of length L metre carrying a current i ampere is bent in the form of circle. The magnitude of its magnetic moment is …… in MKS units. **(1987, 2M)**

Analytical & Descriptive Questions

17. A uniform constant magnetic field $\mathbf{B}$ is directed at an angle of 45° to the x-axis in x-y plane. $PQRS$ is rigid square wire frame carrying a steady current I_0, with its centre at the origin O. At time $t = 0$, the frame is at rest in the position shown in the figure with its sides parallel to x and y-axes. Each side of the frame is of mass M and length L **(1998, 8M)**

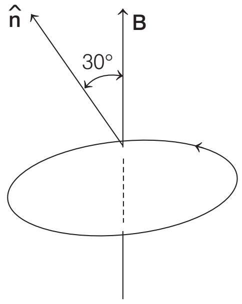


(a) What is the magnitude of torque τ acting on the frame due to the magnetic field ?

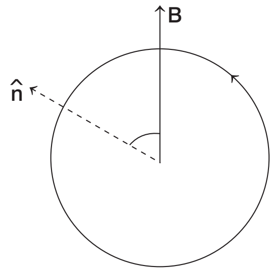
(b) Find the angle by which the frame rotates under the action of this torque in a short interval of time Δt and the axis about which this rotation occurs (Δt is so short that any variation in the torque during this interval may be neglected). Given : the moment of inertia of the frame about an axis through its centre perpendicular to its plane is $\frac{4}{3}ML^2$.

18. An electron in the ground state of hydrogen atom is revolving in anti-clockwise direction in a circular orbit of radius R . **(1996, 5M)**

(a) Obtain an expression for the orbital magnetic moment of the electron.



(b) The atom is placed in a uniform magnetic induction **B** such that the normal to the plane of electron's orbit makes an angle of 30° with the magnetic induction. Find the torque experienced by the orbiting electron.



Topic 5 Magnetism

Objective Questions I (Only one correct option)

1. A magnetic compass needle oscillates 30 times per minute at a place, where the dip is 45° and 40 times per minute, where the dip is 30°. If B_1 and B_2 are respectively, the total magnetic field due to the earth at the two places, then the ratio $\frac{B_1}{B_2}$ is best given by **(2019 Main, 12 April I)**

(a) 1.8 (b) 0.7 (c) 3.6 (d) 2.2

2. A moving coil galvanometer has a coil with 175 turns and area 1 cm^2. It uses a torsion band of torsion constant 10^{-6} N-m/rad. The coil is placed in a magnetic field B parallel to its plane. The coil deflects by 1º for a current of 1 mA. The value of B (in Tesla) is approximately **(2019 Main, 9 April II)**

(a) 10^{-3} (b) 10^{-4} (c) 10^{-1} (d) 10^{-2}

3. A paramagnetic material has 10^{28} atoms/m^3. Its magnetic susceptibility at temperature 350 K is 2.8×10^{-4}. Its susceptibility at 300 K is **(2019 Main, 12 Jan II)**

(a) 3.726×10^{-4} (b) 3.672×10^{-4}

(c) 2.672×10^{-4} (d) 3.267×10^{-4}

4. A paramagnetic substance in the form of a cube with sides 1 cm has a magnetic dipole moment of 20×10^{-6} J/T when a magnetic intensity of 60×10^3 A/m is applied. Its magnetic susceptibility is **(2019 Main, 11 Jan II)**

(a) 3.3×10^{-4} (b) 3.3×10^{-2}

(c) 4.3×10^{-2} (d) 2.3×10^{-2}

5. At some location on earth, the horizontal component of earth's magnetic field is 18×10^{-6} T. At this location, magnetic needle of length 0.12 m and pole strength 1.8 A-m is suspended from its mid point using a thread, it makes 45° angle with horizontal in equilibrium. To keep this needle horizontal, the vertical force that should be applied at one of its ends is **(2019 Main, 10 Jan II)**

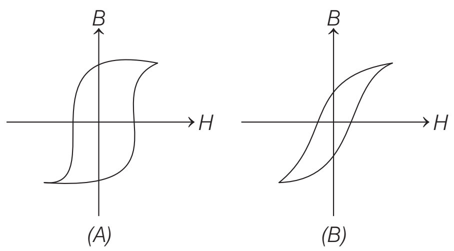
(a) 6.5×10^{-5} N (b) 3.6×10^{-5} N

(c) 1.3×10^{-5} N (d) 1.8×10^{-5} N

6. A bar magnet is demagnetised by inserting it inside a solenoid of length 0.2 m, 100 turns and carrying a current of 5.2 A. The coercivity of the bar magnet is **(Main 2019, 9 Jan I)**

(a) 1200 A/m (b) 285 A/m (c) 2600A/m (d) 520A/m

7. Hysteresis loops for two magnetic materials A and B are as given below: **(2016 Main)**



These materials are used to make magnets for electric generators, transformer core and electromagnet core. Then, it is proper to use

(a) A for electric generators and transformers
(b) A for electromagnets and B for electric generators
(c) A for transformers and B for electric generators
(d) B for electromagnets and transformers

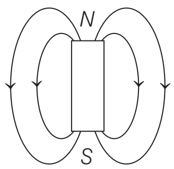
8. The coercivity of a small magnet where the ferromagnet gets demagnetised is 3×10^3 Am^{-1}. The current required to be passed in a solenoid of length 10 cm and number of turns 100, so that the magnet gets demagnetised when inside the solenoid is **(2014 Main)**

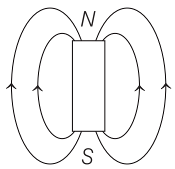
(a) 30 mA (b) 60 mA (c) 3 A (d) 6 A

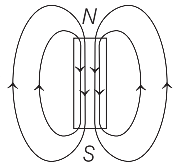
9. Two short bar magnets of length 1 cm each have magnetic moments 1.20 Am^2 and 1.00 Am^2, respectively. They are placed on a horizontal table parallel to each other with their N poles pointing towards the South. They have a common magnetic equator and are separated by a distance of 20.0 cm. The value of the resultant horizontal magnetic induction at the mid-point O of the line joining their centres is close to (Horizontal component of the earth's magnetic induction is 3.6×10^{-5} Wb/m^2) **(2013 Main)**

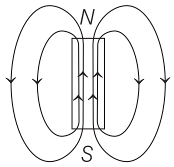
(a) 3.6×10^{-5} Wb / m^2
(b) 2.56×10^{-4} Wb / m^2
(c) 3.50×10^{-4} Wb / m^2
(d) 5.80×10^{-4} Wb / m^2

10. The magnetic field lines due to a bar magnet are correctly shown in **(2002, 2M)**

(a)


(b)


(c)


(d)


11. A magnetic needle is kept in a non-uniform magnetic field. It experiences **(1982, 3M)**

(a) a force and a torque (b) a force but not a torque
(c) a torque but not a force (d) neither a force nor a torque

Assertion and Reason

Mark your answer as

(a) If Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) If Statement I is true, Statement II is true; Statement II is not a correct explanation for Statement I
(c) If Statement I is true; Statement II is false
(d) If Statement I is false; Statement II is true

12. **Statement I** The sensitivity of a moving coil galvanometer is increased by placing a suitable magnetic material as a core inside the coil. **(2008, 3M)**

Statement II Soft iron has a high magnetic permeability and cannot be easily magnetised or demagnetised.

Analytical & Descriptive Questions

13. A moving coil galvanometer experiences torque $= ki$, where i is current. If N coils of area A each and moment of inertia I is kept in magnetic field B. **(2005, 6M)**

(a) Find k in terms of given parameters.
(b) If for current i deflection is $\frac{\pi}{2}$, find out torsional constant of spring.
(c) If a charge Q is passed suddenly through the galvanometer, find out maximum angle of deflection.

Topic 6 Miscellaneous Problems

Objective Questions I (Only one correct option)

1. A moving coil galvanometer, having a resistance G, produces full scale deflection when a current I_g flows through it. This galvanometer can be converted into (i) an ammeter of range 0 to $I_0 (I_0 > I_g)$ by connecting a shunt resistance R_A to it and (ii) into a voltmeter of range 0 to V $(V = GI_0)$ by connecting a series resistance R_V to it. Then, **(2019 Main, 12 April II)**

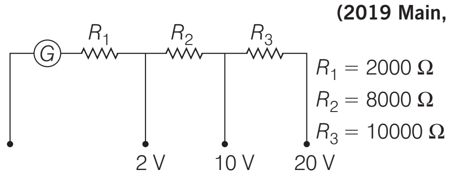
(a) $R_A R_V = G^2 \left(\frac{I_0 - I_g}{I_g}\right)$ and $\frac{R_A}{R_V} = \left(\frac{I_g}{(I_0 - I_g)}\right)^2$

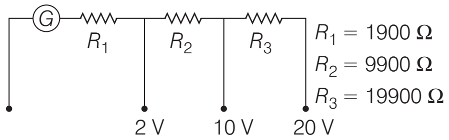
(b) $R_A R_V = G^2$ and $\frac{R_A}{R_V} = \left(\frac{I_g}{I_0 - I_g}\right)^2$

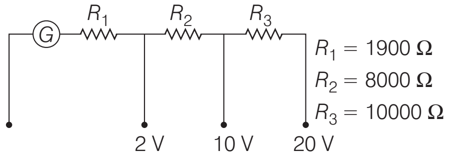
(c) $R_A R_V = G^2 \left(\frac{I_g}{I_0 - I_g}\right)$ and $\frac{R_A}{R_V} = \left(\frac{I_0 - I_g}{I_g}\right)^2$

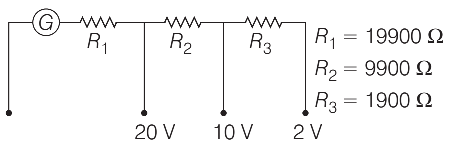
(d) $R_A R_V = G^2$ and $\frac{R_A}{R_V} = \frac{I_g}{(I_0 - I_g)}$

2. A galvanometer of resistance 100 Ω has 50 divisions on its scale and has sensitivity of 20 µA/division. It is to be converted to a voltmeter with three ranges of 0-2 V, 0-10 V and 0-20 V. The appropriate circuit to do so is **(2019 Main, 12 April I)**

(a)


(b)


(c)


(d)


3. A moving coil galvanometer allows a full scale current of 10^{-4} A. A series resistance of 2 MΩ is required to convert the above galvanometer into a voltmeter of range 0-5 V. Therefore, the value of shunt resistance required to convert the above galvanometer into an ammeter of range 0.10 mA is **(2019 Main, 10 April I)**

(a) 100 Ω (b) 500 Ω (c) 200 Ω (d) 10 Ω

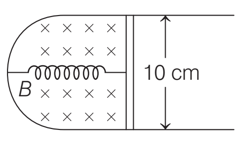
4. The resistance of a galvanometer is 50 ohm and the maximum current which can be passed through it is 0.002 A. What resistance must be connected to it in order to convert it into an ammeter of range 0-0.5 A? **(2019 Main, 9 April II)**

(a) 0.2 ohm (b) 0.5 ohm
(c) 0.002 ohm (d) 0.02 ohm

5. A moving coil galvanometer has resistance 50 Ω and it indicates full deflection at 4 mA current. A voltmeter is made using this galvanometer and a 5 kΩ resistance. The maximum voltage, that can be measured using this voltmeter, will be close to **(2019 Main, 9 April I)**

(a) 40 V (b) 10 V (c) 15 V (d) 20 V

6. A thin strip 10 cm long is on an U-shaped wire of negligible resistance and it is connected to a spring of spring constant $0.5\ \text{Nm}^{-1}$ (see figure). The assembly is kept in a uniform magnetic field of 0.1 T. If the strip is pulled from its equilibrium position and released, the number of oscillations it performs before its amplitude decreases by a factor of e is N. If the mass of the strip is 50 grams, its resistance 10 Ω and air drag negligible, N will be close to **(2019 Main, 8 April I)**



(a) 1000 (b) 50000 (c) 5000 (d) 10000

7. The mean intensity of radiation on the surface of the sun is about $10^8\ \text{W/m}^2$. The rms value of the corresponding magnetic field is closest to **(Main 2019, 12 Jan II)**

(a) 1 T (b) 10^2 T (c) 10^{-4} T (d) 10^{-2} T

8. A hoop and a solid cylinder of same mass and radius are made of a permanent magnetic material with their magnetic moment parallel to their respective axes. But the magnetic moment of hoop is twice of solid cylinder. They are placed in a uniform magnetic field in such a manner that their magnetic moments make a small angle with the field. If the oscillation periods of hoop and cylinder are T_h and T_c respectively, then **(2019 Main, 10 Jan II)**

(a) $T_h = 0.5\,T_c$ (b) $T_h = T_c$ (c) $T_h = 2\,T_c$ (d) $T_h = 1.5\,T_c$

9. A magnet of total magnetic moment $10^{-2}\,\hat{\mathbf{i}}$ A-m^2 is placed in a time varying magnetic field, $B\hat{\mathbf{i}}$ $(\cos\omega t)$, where $B = 1$ T and $\omega = 0.125$ rad/s. The work done for reversing the direction of the magnetic moment at $t = 1$ s is **(2019 Main, 10 Jan I)**

(a) 0.01 J (b) 0.007 J (c) 0.014 J (d) 0.028 J

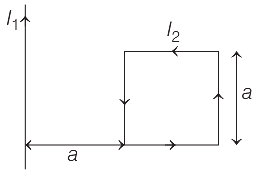
10. A conducting circular loop is made of a thin wire has area $3.5\times10^{-3}\ \text{m}^2$ and resistance 10 Ω. It is placed perpendicular to a time dependent magnetic field $B(t) = (0.4\text{T})\sin(0.5\pi t)$. The field is uniform in space. Then the net charge flowing through the loop during $t = 0$ s and $t = 10$ ms is close to **(2019 Main, 9 Jan I)**

(a) 6 mC (b) 21 mC (c) 7 mC (d) 14 mC

11. A rectangular coil (dimension 5 cm $\times$ 2.5 cm) with 100 turns, carrying a current of 3A in the clockwise direction, is kept centred at the origin and in the X-Z plane. A magnetic field of 1 T is applied along X-axis. If the coil is tilted through 45° about Z-axis, then the torque on the coil is **(2019 Main, 9 April I)**

(a) 0.27 N-m (b) 0.38 N-m (c) 0.42 N-m (d) 0.55 N-m

12. A rigid square loop of side a and carrying current I_2 is lying on a horizontal surface near a long current I_1 carrying wire in the same plane as shown in figure. The net force on the loop due to the wire will be **(2019 Main, 9 April I)**



(a) repulsive and equal to $\dfrac{\mu_0 I_1 I_2}{2\pi}$

(b) attractive and equal to $\dfrac{\mu_0 I_1 I_2}{3\pi}$

(c) zero

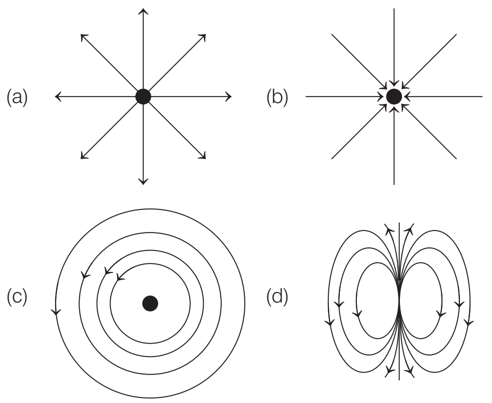
(d) repulsive and equal to $\dfrac{\mu_0 I_1 I_2}{4\pi}$

13. The region between $y = 0$ and $y = d$ contains a magnetic field $\mathbf{B} = B\hat{\mathbf{k}}$. A particle of mass m and charge q enters the region with a velocity $\mathbf{v} = v\hat{\mathbf{i}}$. If $d = \dfrac{mv}{2qB}$ then the acceleration of the charged particle at the point of its emergence at the other side is **(2019 Main, 11 Jan II)**

(a) $\frac{qvB}{m}\left(\frac{\sqrt{3}}{2}\hat{\mathbf{i}}+\frac{1}{2}\hat{\mathbf{j}}\right)$ (b) $\frac{qvB}{m}\left(\frac{1}{2}\hat{\mathbf{i}}-\frac{\sqrt{3}}{2}\hat{\mathbf{j}}\right)$

(c) $\frac{qvB}{m}\left(\frac{-\hat{\mathbf{j}}+\hat{\mathbf{i}}}{\sqrt{2}}\right)$ (d) $\frac{qvB}{m}\left(\frac{\hat{\mathbf{i}}+\hat{\mathbf{j}}}{\sqrt{2}}\right)$

14. Which of the field patterns given in the figure is valid for electric field as well as for magnetic field? **(2011)**



15. Two very long straight parallel wires carry steady currents I and $-I$ respectively. The distance between the wires is d. At a certain instant of time, a point charge q is at a point equidistant from the two wires in the plane of the wires. Its instantaneous velocity $\mathbf{v}$ is perpendicular to this plane. The magnitude of the force due to the magnetic field acting on the charge at this instant is **(1998, 2M)**

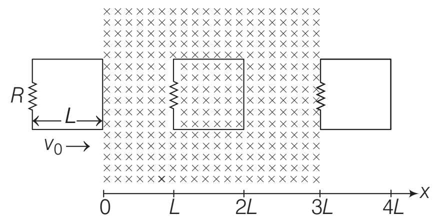
(a) $\frac{\mu_0 Iqv}{2\pi d}$ (b) $\frac{\mu_0 Iqv}{\pi d}$ (c) $\frac{2\mu_0 Iqv}{\pi d}$ (d) zero

Objective Questions II (One or more correct option)

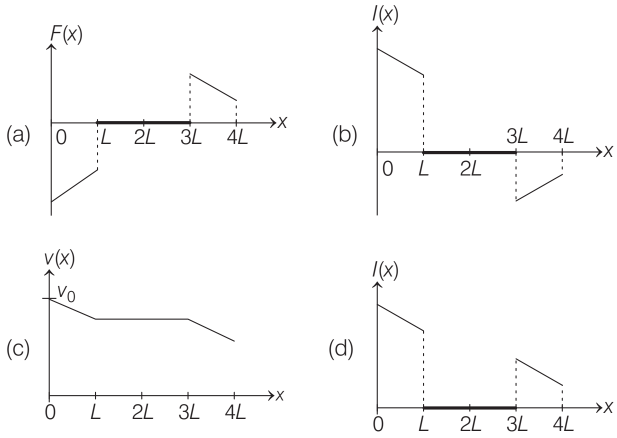
16. Two infinitely long straight wires lie in the xy-plane along the lines $x = \pm R$. The wire located at $x = +R$ carries a constant current I_1 and the wire located at $x = -R$ carries a constant current I_2. A circular loop of radius R is suspended with its centre at $(0, 0, \sqrt{3}R)$ and in a plane parallel to the xy-plane. This loop carries a constant current I in the clockwise direction as seen from above the loop. The current in the wire is taken to be positive, if it is in the $+\hat{\mathrm{j}}$-direction. Which of the following statements regarding the magnetic field B is (are) true? **(2018 Adv.)**

(a) If $I_1 = I_2$, then B cannot be equal to zero at the origin (0,0,0)
(b) If $I_1 > 0$ and $I_2 < 0$, then B can be equal to zero at the origin (0,0,0)
(c) If $I_1 < 0$ and $I_2 > 0$, then B can be equal to zero at the origin (0,0,0)
(d) If $I_1 = I_2$, then the z-component of the magnetic field at the centre of the loop is $\left(-\frac{\mu_o I}{2R}\right)$

17. A rigid wire loop of square shape having side of length L and resistance R is moving along the x-axis with a constant velocity v_0 in the plane of the paper. At $t = 0$, the right edge of the loop enters a region of length $3L$, where there is a uniform magnetic field B_0 into the plane of the paper, as shown in the figure. For sufficiently large v_0, the loop eventually crosses the region. Let x be the location of the right edge of the loop. Let $v(x)$, $I(x)$ and $F(x)$ represent the velocity of the loop, current in the loop, and force on the loop, respectively, as a function of x. Counter-clockwise current is taken as positive. **(2016 Adv.)**

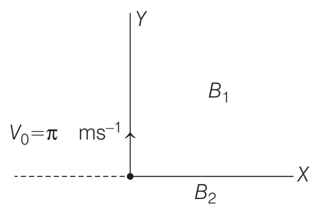


Which of the following schematic plot(s) is (are) correct? (Ignore gravity)



Numerical Value

18. In the xy-plane, the region $y > 0$ has a uniform magnetic field $B_1\hat{\mathrm{k}}$ and the region $y < 0$ has another uniform magnetic field $B_2\hat{\mathrm{k}}$. A positively charged particle is projected from the origin along the positive Y-axis with speed $v_0 = \pi\ \mathrm{ms^{-1}}$ at $t = 0$, as shown in figure. Neglect gravity in this problem. Let $t = T$ be the time when the particle crosses the X-axis from below for the first time. If $B_2 = 4B_1$, the average speed of the particle, in $\mathrm{ms^{-1}}$, along the X-axis in the time interval T is........... . **(2018 Adv.)**

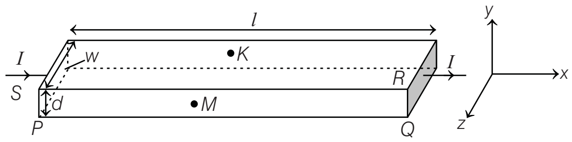


Passage Based Questions

Passage 1

In a thin rectangular metallic strip a constant current I flows along the positive x-direction, as shown in the figure. The length, width and thickness of the strip are l, w and d, respectively. A uniform magnetic field $\mathbf{B}$ is applied on the strip along the positive y-direction. Due to this, the charge carriers experience a net deflection along the z-direction.

This results in accumulation of charge carriers on the surface $PQRS$ and appearance of equal and opposite charges on the face opposite to $PQRS$. A potential difference along the z-direction is thus developed. Charge accumulation continues until the magnetic force is balanced by the electric force. The current is assumed to be uniformly distributed on the cross section of the strip and carried by electrons.



19. Consider two different metallic strips (1 and 2) of the same material. Their lengths are the same, widths are w_1 and w_2 and thicknesses are d_1 and d_2, respectively. Two points K and M are symmetrically located on the opposite faces parallel to the x-y plane (see figure). V_1 and V_2 are the potential differences between K and M in strips 1 and 2, respectively. Then, for a given current I flowing through them in a given magnetic field strength B, the correct statements is/are **(2015 Adv.)**

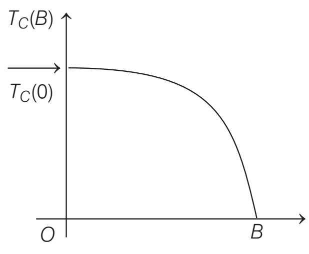
(a) If $w_1 = w_2$ and $d_1 = 2d_2$, then $V_2 = 2V_1$
(b) If $w_1 = w_2$ and $d_1 = 2d_2$, then $V_2 = V_1$
(c) If $w_1 = 2w_2$ and $d_1 = d_2$, then $V_2 = 2V_1$
(d) If $w_1 = 2w_2$ and $d_1 = d_2$, then $V_2 = V_1$

20. Consider two different metallic strips (1 and 2) of same dimensions (length l, width w and thickness d) with carrier densities n_1 and n_2, respectively. Strip 1 is placed in magnetic field B_1 and strip 2 is placed in magnetic field B_2, both along positive y-directions. Then V_1 and V_2 are the potential differences developed between K and M in strips 1 and 2, respectively. Assuming that the current I is the same for both the strips, the correct options is/are **(2015 Adv.)**

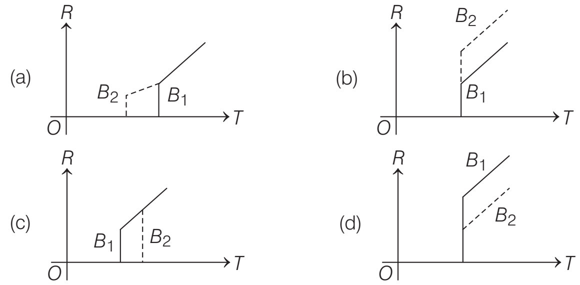
(a) If $B_1 = B_2$ and $n_1 = 2n_2$, then $V_2 = 2V_1$
(b) If $B_1 = B_2$ and $n_1 = 2n_2$, then $V_2 = V_1$
(c) If $B_1 = 2B_2$ and $n_1 = n_2$, then $V_2 = 0.5V_1$
(d) If $B_1 = 2B_2$ and $n_1 = n_2$, then $V_2 = V_1$

Passage 2

Electrical resistance of certain materials, known as superconductors, changes abruptly from a non-zero value to zero as their temperature is lowered below a critical temperature $T_C(0)$. An interesting property of superconductors is that their critical temperature becomes smaller than $T_C(0)$ if they are placed in a magnetic field i.e. the critical temperature $T_C(B)$ is a function of the magnetic field strength B. The dependence of $T_C(B)$ on B is shown in the figure. **(2010)**



21. In the graphs below, the resistance R of a superconductor is shown as a function of its temperature T for two different magnetic fields B_1 (*solid line*) and B_2 (*dashed line*). If B_2 is larger than B_1, which of the following graphs shows the correct variation of R with T in these fields?

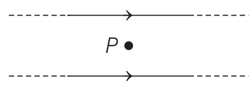
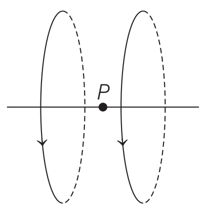
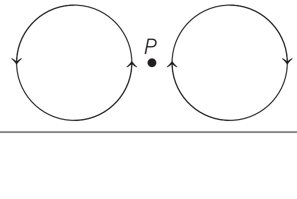


22. A superconductor has $T_C(0) = 100\,\text{K}$. When a magnetic field of 7.5 T is applied, its T_C decreases to 75 K. For this material one can definitely say that when **(Note** T = Tesla**)**

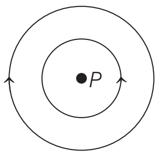
(a) $B = 5\,\text{T}, T_C(B) = 80\,\text{K}$
(b) $B = 5\,\text{T}, 75\,\text{K} < T_C(B) < 100\,\text{K}$
(c) $B = 10\,\text{T}, 75\,\text{K} < T_C(B) < 100\,\text{K}$
(d) $B = 10\,\text{T}, T_C(B) = 70\,\text{K}$

Match the Columns

23. Two wires each carrying a steady current I are shown in four configurations in Column I. Some of the resulting effects are described in Column II. Match the satements in Column I with the statements in Column II. **(2007, 6M)**

	Column I		Column II
(A)	Point P is situated midway between the wires. 	(p)	The magnetic fields (B) at P due to the currents in the wires are in the same direction.
(B)	Point P is situated at the mid-point of the line joining the centres of the circular wires, which have same radii. 	(q)	The magnetic fields (B) at P due to the currents in the wires are in opposite directions.
(C)	Point P is situated at the mid-point of the line joining the centres of the circular wires, which have same radii. 	(r)	There is no magnetic field at P.

Column I		Column II	
(D)	Point P is situated at the common centre of the wires.	(s)	The wires repel each other.



24. **Column I** gives certain situations in which a straight metallic wire of resistance R is used and **Column II** gives some resulting effects. Match the statements in **Column I** with the statements in **Column II.** **(2007, 6M)**

	Column I		Column II
(A)	A charged capacitor is connected to the ends of the wire	(p)	A constant current flows through the wire
(B)	The wire is moved perpendicular to its length with a constant velocity in a uniform magnetic field perpendicular to the plane of motion	(q)	Thermal energy is generated in the wire
(C)	The wire is placed in a constant electric field that has a direction along the length of the wire	(r)	A constant potential difference develops between the ends of the wire
(D)	A battery of constant emf is connected to the ends of the wire	(s)	Charges of constant magnitude appear at the ends of the wire

25. Some laws/processes are given in **Column I.** Match these with the physical phenomena given in **Column II.** **(2006, 4M)**

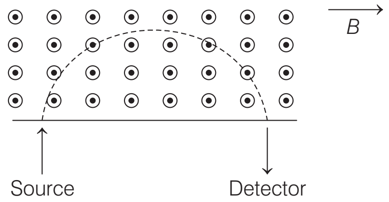
	Column I		Column II
(A)	Dielectric ring uniformly charged	(p)	Time independent electrostatic field out of system
(B)	Dielectric ring uniformly charged rotating with angular velocity ω	(q)	Magnetic field
(C)	Constant current in ring i_o	(r)	Induced electric field
(D)	$i = i_0 \cos \omega t$	(s)	Magnetic moment

Integer Answer Type Question

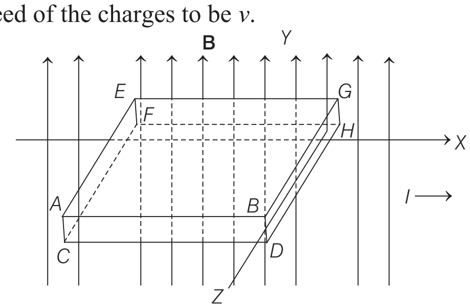
26. A steady current I goes through a wire loop PQR having shape of a right angle triangle with $PQ = 3x$, $PR = 4x$ and $QR = 5x$. If the magnitude of the magnetic field at P due to this loop is $k\left(\frac{\mu_0 I}{48\pi x}\right)$, find the value of k. **(2009)**

Fill in the Blanks

27. A uniform magnetic field with a slit system as shown in figure is to be used as momentum filter for high-energy charged particles. With a field B Tesla, it is found that the filter transmits α-particles each of energy 5.3 MeV. The magnetic field is increased to 2.3 B Tesla and deuterons are passed into the filter. The energy of each deuteron transmitted by the filter is ……MeV. **(1997C, 1M)**

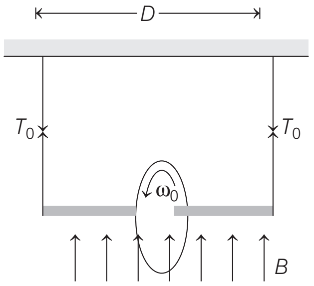


28. A metallic block carrying current I is subjected to a uniform magnetic induction **B** as shown in figure. The moving charges experience a force **F** given by ……which results in the lowering of the potential of the face……. Assume the speed of the charges to be v. **(1996, 2M)**

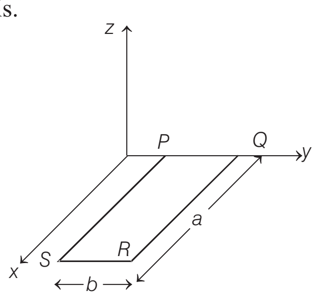


Analytical & Descriptive Questions

29. A ring of radius R having uniformly distributed charge Q is mounted on a rod suspended by two identical strings. The tension in strings in equilibrium is T_0. Now, a vertical magnetic field is switched on and ring is rotated at constant angular velocity ω. Find the maximum ω with which the ring can be rotated if the strings can withstand a maximum tension of $3T_0/2$. **(2003, 4M)**



30. A rectangular loop $PQRS$ made from a uniform wire has length a, width b and mass m. It is free to rotate about the arm PQ, which remains hinged along a horizontal line taken as the y-axis (see figure). Take the vertically upward direction as the z-axis.



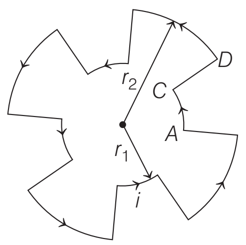
A uniform magnetic field $\mathbf{B} = (3\hat{\mathbf{i}} + 4\hat{\mathbf{k}})B_0$ exists in the region. The loop is held in the x-y plane and a current I is passed through it. The loop is now released and is found to stay in the horizontal position in equilibrium. **(2002, 5M)**

(a) What is the direction of the current I in PQ ?

(b) Find the magnetic force on the arm RS.

(c) Find the expression for I in terms of B_0, a, b and m.

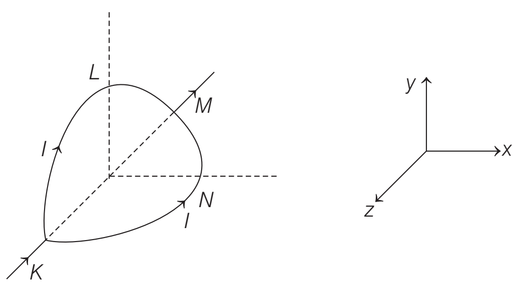
31. A current of 10 A flows around a closed path in a circuit which is in the horizontal plane as shown in the figure. The circuit consists of eight alternating arcs of radii $r_1 = 0.08$ m and $r_2 = 0.12$ m. Each subtends the same angle at the centre. **(2001, 10M)**



(a) Find the magnetic field produced by this circuit at the centre.

(b) An infinitely long straight wire carrying a current of 10 A is passing through the centre of the above circuit vertically with the direction of the current being into the plane of the circuit. What is the force acting on the wire at the centre due to the current in the circuit? What is the force acting on the arc AC and the straight segment CD due to the current at the centre ?

32. A circular loop of radius R is bent along a diameter and given a shape as shown in figure. One of the semicircles (KNM) lies in the x-z plane and the other one (KLM) in the y-z plane with their centres at origin. Current I is flowing through each of the semicircles as shown in figure. **(2000, 10M)**

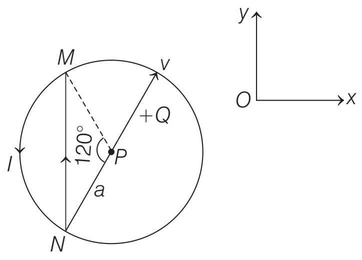


(a) A particle of charge q is released at the origin with a velocity $\mathbf{v} = -v_0\hat{\mathbf{i}}$. Find the instantaneous force $\mathbf{F}$ on the particle. Assume that space is gravity free.

(b) If an external uniform magnetic field $B_0\hat{\mathbf{j}}$ is applied determine the force $\mathbf{F}_1$ and $\mathbf{F}_2$ on the semicircles KLM and KNM due to the field and the net force $\mathbf{F}$ on the loop.

33. A particle of mass m and charge q is moving in a region where uniform, constant electric and magnetic fields $\mathbf{E}$ and $\mathbf{B}$ are present. $\mathbf{E}$ and $\mathbf{B}$ are parallel to each other. At time $t = 0$, the velocity $\mathbf{v}_0$ of the particle is perpendicular to $\mathbf{E}$ (Assume that its speed is always << c, the speed of light in vacuum). Find the velocity $\mathbf{v}$ of the particle at time t. You must express your answer in terms of t, q, m, the vector $\mathbf{v}_0$, $\mathbf{E}$ and $\mathbf{B}$ and their magnitudes v_0, E and B. **(1998, 8M)**

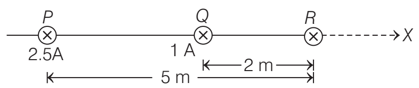
34. A wire loop carrying a current I is placed in the x-y plane as shown in figure. **(1991, 4+4M)**



(a) If a particle with charge $+Q$ and mass m is placed at the centre P and given a velocity $\mathbf{v}$ along NP (see figure), find its instantaneous acceleration.

(b) If an external uniform magnetic induction field $\mathbf{B} = B\hat{\mathbf{i}}$ is applied, find the force and the torque acting on the loop due to this field.

35. Two long parallel wires carrying currents 2.5 A and I (ampere) in the same direction (directed into the plane of the paper) are held at P and Q respectively such that they are perpendicular to the plane of paper. The points P and Q are located at a distance of 5 m and 2 m respectively from a collinear point R (see figure). **(1990, 8M)**



(a) An electron moving with a velocity of 4×10^5 m/s along the positive x-direction experiences a force of magnitude 3.2×10^{-20} N at the point R. Find the value of I.

(b) Find all the positions at which a third long parallel wire carrying a current of magnitude 2.5 A may be placed, so that the magnetic induction at R is zero.

Answers

Topic 1

1. (*) **2.** (c) **3.** (a) **4.** (d)

5. (b) **6.** (d) **7.** (c)

8. (a) **9.** (d) **10.** (b) **11.** (b)

12. (b) **13.** (c) **14.** (a) **15.** (a)

16. (c) **17.** (a,b.c) **18.** (a, c) **19.** (c, d)

20. (b, d) **21.** (a, c, d) **22.** (a, c) **23.** (a, b, d)

24. (a, b, d) **25.** (c) **26.** (a) **27.** (c)

28. D, B **29.** F **30.** F **31.** T

32. $\frac{1}{\sqrt{2}}$ **33.** (a) $L = \frac{mv_0}{2B_0q}$ (b) $\mathbf{v}_f = -v_0\hat{\mathbf{i}}$, $t = \frac{\pi m}{B_0 q}$

34. 4.73×10^{-3} T **35.** 1.2×10^{-2}m, 4.37×10^{-2}m

36. (a) 0.14 m, 45° (b) 4.712×10^{-8} s

37. (6.4 m, 0, 0), (6.4 m, 0, 2 m)

38. 0.1 T (perpendicular to paper inwards)

Topic 2

1. (b) **2.** (b) **3.** (d) **4.** (a)

5. (b) **6.** (c) **7.** (a) **8.** (d)

9. (d) **10.** (a) **11.** (a) **12.** (c)

13. (d) **14.** (b) **15.** (c) **16.** (d)

17. (b) **18.** (a, d) **19.** 5 **20.** 3

21. $\frac{\mu_0 I}{4}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ (perpendicular to paper outwards)

22. 10^{-4} T, perpendicular to paper outwards

Topic 3

1. (d) **2.** (b) **3.** (d) **4.** (a)

5. (b) **6.** (b) **7.** (c) **8.** ILB, positive-z

9. T

10. (a) $x = 0 = z$ and $z = 0, x = \pm \frac{d}{\sqrt{3}}$ (b) $f = \frac{i}{2\pi d}\sqrt{\frac{\mu_0}{\pi\lambda}}$

11. 0.2 s **12.** $\mathbf{F} = \frac{-\mu_0 I^2}{2\pi} \ln\left(\frac{L^2 + a^2}{a^2}\right)\hat{\mathbf{k}}$, zero

13. (a) 3 A, perpendicular to paper outwards
(b) 13×10^{-7} T
(c) 2.88×10^{-6} N / m

Topic 4

1. (b) **2.** (a) **3.** (d) **4.** (d)

5. (d) **6.** (b) **7.** (b) **8.** (c)

9. (c) **10.** (a) **11.** (c) **12.** (c)

13. (b) **14.** (a, c) **15.** 1.26×10^{-23} A-m^2

16. $\frac{L^2 i}{4\pi}$ **17.** (a) $|\tau| = I_0 L^2 B$ (b) $\theta = \frac{3}{4}\frac{I_0 B}{M}(\Delta t)^2$

18. (a) $M = \frac{eh}{4\pi m}$ (b) $\tau = \frac{ehB}{8\pi m}$, perpendicular to both **M** and **B**.

Topic 5

1. (b) **2.** (a) **3.** (d) **4.** (a)

5. (a) **6.** (c) **7.** (d) **8.** (c)

9. (b) **10.** (d) **11.** (a) **12.** (c)

13. (a) $k = BNA$ (b) $k = \frac{2BiNA}{\pi}$ (c) $Q\sqrt{\frac{BNA\pi}{2I}}$

Topic 6

1. (b) **2.** (c) **3.** (*) **4.** (a)

5. (d) **6.** (c) **7.** (c) **8.** (b)

9. (c) **10.** (d) **11.** (a) **12.** (d)

13. (*) **14.** (c) **15.** (d) **16.** (a,b,d)

17. (b,c) **18.** (2) **19.** (a,d)

20. (a,c) **21.** (a) **22.** (b)

23. (A) $\to$ q, r (B) $\to$ p (C) $\to$ q, r (D) $\to$ q, s or q

24. (A) $\to$ q (B) $\to$ r, s (C) $\to$ s (D) $\to$ p, q, r

25. (A) $\to$ p (B) $\to$ p, q, s (C) $\to$ q, s (D) $\to$ q, r

26. 7 **27.** 14.0185 **28.** $evB\hat{\mathbf{k}}$, ABCD **29.** $\omega_{\max} = \frac{DT_0}{BQR^2}$

30. (a) P to Q (b) $IbB_0\,(3\hat{\mathbf{k}} - 4\hat{\mathbf{i}})$ (c) $\frac{mg}{6bB_0}$

31. (a) 6.54×10^{-5} T (vertically upward or outward normal to the paper) (b) Zero, Zero, 8.1×10^{-6} N (inwards)

32. (a) $\mathbf{F} = -\frac{\mu_0 q v_0 I}{4R}\hat{\mathbf{k}}$ (b) $\mathbf{F}_1 = \mathbf{F}_2 = 2BIR\hat{\mathbf{i}}$, $\mathbf{F} = 4BIR\hat{\mathbf{i}}$

33. $\mathbf{v} = \cos\left(\frac{qB}{m}t\right)(\mathbf{v}_0) + \left(\frac{q}{m}t\right)(\mathbf{E}) + \sin\left(\frac{qB}{m}t\right)\left(\frac{\mathbf{v}_0 \times \mathbf{B}}{B}\right)$

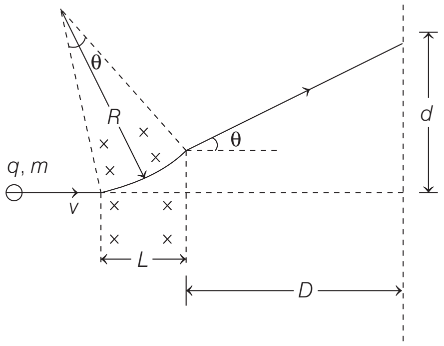
34. (a) $\frac{0.11\mu_0 IQv}{2am}(\hat{\mathbf{j}} - \sqrt{3}\,\hat{\mathbf{i}})$ (b) zero, $(0.61 Ia^2 B)\hat{\mathbf{j}}$

35. (a) 4A (b) At distance 1m from R to the left or right of it, current is outwards if placed to the left and inwards if placed to the right of 6R.

Hints & Solutions

Topic 1 Magnetic Force and Path of Charged Particle in Uniform Fields

1.



When electron enters the region of magnetic field, it experiences a Lorentz force which rotates electron in a circular path of radius R.

So, Lorentz force acts like a centripetal force and we have

$$\frac{mv^2}{R} = Bqv$$

where, m = mass of electron,

q = charge of electron, v = speed of electron,

R = radius of path,

and B = magnetic field intensity.

Radius of path of electron,

$$R = \frac{mv}{Bq}$$

Now, from geometry of given arrangement, comparing values of $\tan\theta$, we have

$$\tan\theta = \frac{L}{R} = \frac{d}{D} \Rightarrow d = \frac{LD}{R} = \frac{Bq\,LD}{mv}$$

$$\Rightarrow \quad d = \frac{BqLD}{\sqrt{2mk}} \qquad [\because mv = \sqrt{2mk}]$$

where, k = kinetic energy of electron

Here, $B = 1.5 \times 10^{-3}$ T,

$q = 1.6 \times 10^{-19}$ C, $L = 2 \times 10^{-2}$ m, $D = 6 \times 10^{-2}$ m,

$m = 9.1 \times 10^{-31}$ kg, $k = 100 \times 1.6 \times 10^{-19}$ J

$$\text{So, } d = \frac{(1.5 \times 10^{-3} \times 1.6 \times 10^{-19} \times 2 \times 10^{-2} \times 6 \times 10^{-2})}{\sqrt{(2 \times 9.1 \times 10^{-31} \times 100 \times 1.6 \times 10^{-19})}}$$

$$= \frac{28.8 \times 10^{-26}}{\sqrt{29.12 \times 10^{-48}}} = \frac{28.8 \times 10^{-26}}{5.39 \times 10^{-24}} = 5.34 \times 10^{-2} \text{ m}$$

$= 5.34$ cm

No option is matching.

2. When a moving charged particle is placed in a magnetic field **B**.

Then, the net magnetic force acting on it is

$$\mathbf{F}_m = q(\mathbf{v} \times \mathbf{B})$$
$$\mathbf{F}_m = q\,vB\sin\theta$$

Here, $\theta = 90°$

$$\mathbf{F}_m = q\,vB$$

Also, due to this net force, the particle transverses a circular path, whose necessary centripetal force is being provided by F_m, i.e.

$$\frac{mv^2}{r} = q\,vB$$

$$\Rightarrow \quad r = \frac{mv^2}{qvB} = \frac{mv}{qB}$$

$$\Rightarrow \quad r \propto m$$

So, for electron,

$$r_e = \frac{m_e v}{eB}$$

or $\quad r_e \propto m_e$

For proton,

$$r_p = \frac{m_p v}{eB}$$

or $\quad r_p \propto m_p$

For He-particle,

$$r_{\text{He}} = \frac{4m_p v}{2eB} = \frac{2m_p v}{eB}$$

Clearly, $\quad r_{\text{He}} > r_p \qquad (\because r_{\text{He}} = 2r_p)$

and we know that, $m_p > m_e \qquad \left[\begin{array}{l}\because m_p \approx 10^{-27}\text{ kg,} \\ m_e \approx 10^{-31}\text{ kg}\end{array}\right]$

$\therefore \quad r_p > r_e$

$\Rightarrow \quad r_{\text{He}} > r_p > r_e$

3. Radius of path of charged particle q in a uniform magnetic field B of mass 'm' moving with velocity v is

$$r = \frac{mv}{Bq} = \frac{m\sqrt{(2qV/m)}}{Bq}$$

$$\Rightarrow \quad r \propto \frac{\sqrt{m}}{\sqrt{q}} \text{ so, required ratio is}$$

$$\Rightarrow \quad \frac{r_p}{r_\alpha} = \sqrt{\frac{m_p}{m_\alpha}} \times \sqrt{\frac{q_\alpha}{q_p}}$$

$$= \sqrt{\frac{1}{4}} \times \sqrt{\frac{2}{1}} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

4. Here, $\mathrm{E} = 2\hat{\mathrm{i}} + 3\hat{\mathrm{j}}$, $\mathrm{B} = 4\hat{\mathrm{j}} + 6\hat{\mathrm{k}}$,

q = charge on a particle.

Initial position, $r_1 = (0,0)$

Final position, $r_2 = (1,1)$

Net force experienced by charge particle inside electromagnetic field is

$$\mathbf{F}_{net} = q\mathbf{E} + q(\mathbf{v} \times \mathbf{B})$$
$$= q(2\hat{i} + 3\hat{j}) \quad [\text{Here, } \mathbf{v} \times \mathbf{B} = 0]$$
$$= (2q\,\hat{i} + 3q\,\hat{j})$$

$\therefore \quad dW = \mathbf{F}_{net}.\mathbf{dr}$

$$\Rightarrow \quad \int dW = \int_{r_1}^{r_2} (2q\,\hat{i} + 3q\,\hat{j})\cdot(dx\,\hat{i} + dy\,\hat{j})$$

$$[\text{Here, } dr = dx\,\hat{i} + dy\,\hat{j}]$$

$$\Rightarrow \quad W = 2q\int_0^1 dx + 3q\int_0^1 dy$$

or $\quad W = 2q + 3q$ or $W = 5q$

5. During the circular motion of accelerated electron in the presence of magnetic field, its radius is given by

$$r = \frac{mv}{Be} = \frac{\sqrt{2meV}}{eB}$$

where, v is velocity and V is voltage.

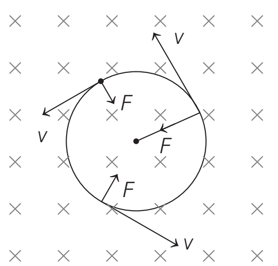
After substituting the given values, we get

$$= \frac{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 500}}{1.6 \times 10^{-19} \times 100 \times 10^{-3}}$$

$$= 10\left[\frac{2 \times 9.1 \times 500}{1.6} \times 10^{-12}\right]^{1/2}$$

$$r = 7.5 \times 10^{-4} \text{ m}$$

6. According to given condition, when a particle having charge same as electron move in a magnetic field on circular path, then the force always acts towards the centre and perpendicular to the velocity.



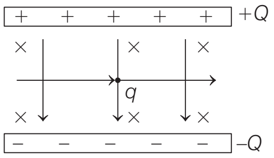
Here, $\quad B = 0.5\,\text{T}$

R = radius of circular path = 5 cm

Now, the magnetic force is

$$F_m = q(v \times B) = q\,vB \sin 90°$$
$$F_m = qvB \quad \ldots(\text{i})$$

When the electric field applied, then the particle moves in a straight path, then this is the case of velocity selector.



Here, the electric force on charge,

$$F_e = qE \quad \ldots(\text{ii})$$

In velocity selector, $\quad F_m = F_e$

$$\Rightarrow \quad qvB = qE \quad \ldots(\text{iii})$$

Initially particle moves under the magnetic field, So the radius of circular path taken by the particle is

$$R = \frac{mv}{qB} \quad \ldots(\text{iv})$$

From Eqs. (iii) and (iv),

$$m = \frac{qB^2R}{E}$$

$$m = \frac{1.6 \times 10^{-19} \times 0.25 \times 0.5 \times 10^{-2}}{10^2}$$

$$m = 2 \times 10^{-24} \text{ kg}$$

7. **For circular path in magnetic field,**

$$r = \frac{\sqrt{2mK}}{qB}$$

where, K = kinetic energy.

$$\Rightarrow \quad r \propto \frac{\sqrt{m}}{q}$$

Further,

	e	p	α
m	1/1836	1	14
q	$-e$	$+e$	$2e$

$\therefore \quad r_p = r_\alpha > r_e$

8. $\mathbf{F}_m = q(\mathbf{v} \times \mathbf{B})$

9. Magnetic force does not change the speed of charged particle. Hence, $v = u$. Further magnetic field on the electron in the given condition is along negative y-axis in the starting. Or it describes a circular path in clockwise direction. Hence, when it exits from the field, $y < 0$.

10. Electric field can deviate the path of the particle in the shown direction only when it is along negative y-direction. In the given options $\mathbf{E}$ is either zero or along x-direction. Hence, it is the magnetic field which is really responsible for its curved path. Options (a) and (c) cannot be accepted as the path will be circular in that case. Option (d) is wrong because in that case component of net force on the particle also comes in $\hat{\mathbf{k}}$ direction which is not acceptable as the particle is moving in x-y plane. Only in option (b), the particle can move in x-y plane.

In option (d)

$$\mathbf{F}_{net} = q\,\mathbf{E} + q(\mathbf{v} \times \mathbf{B})$$

Initial velocity is along x-direction. So, let $\mathbf{v} = v\hat{\mathbf{i}}$

$\therefore \quad \mathbf{F}_{net} = qa\hat{\mathbf{i}} + q[(v\hat{\mathbf{i}}) \times (c\hat{\mathbf{k}} + b\hat{\mathbf{j}})]$

$$= qa\hat{\mathbf{i}} - qvc\hat{\mathbf{j}} + qvb\hat{\mathbf{k}}$$

In option (b)

$$\mathbf{F}_{net} = q(a\hat{\mathbf{i}}) + q[(v\hat{\mathbf{i}}) \times (c\hat{\mathbf{k}} + a\,\hat{\mathbf{i}})] = qa\hat{\mathbf{i}} - qvc\hat{\mathbf{j}}$$

11. If $(b - a) \geq r$

(r = radius of circular path of particle)

The particle cannot enter the region $x > b$.

So, to enter in the region $x > b$

$$r > (b - a) \text{ or } \frac{mv}{Bq} > (b - a)$$

or $v > \dfrac{q\,(b - a)\,B}{m}$

12. Radius of the circle $= \dfrac{mv}{Bq}$

or radius $\propto mv$ if B and q are same.

$$(\text{Radius})_A > (\text{Radius})_B$$

$\therefore \quad m_A v_A > m_B v_B$

13. We can write $\mathbf{E} = E \cdot \hat{\mathbf{i}}$ and $\mathbf{B} = B\hat{\mathbf{k}}$

Velocity of the particle will be along q. $\mathbf{E}$ direction. Therefore, we can write

$$\mathbf{v} = Aq\,E\hat{\mathbf{i}}$$

In $\mathbf{E}$, $\mathbf{B}$ and $\mathbf{v}$, A, E and B are positive constants while q can be positive or negative.

Now, magnetic force on the particle will be

$$\mathbf{F}_m = q(\mathbf{v} \times \mathbf{B}) = q\,\{AqE\hat{\mathbf{i}}\} \times \{B\hat{\mathbf{k}}\}$$

$$= q^2 AEB\,(\hat{\mathbf{i}} \times \hat{\mathbf{k}})$$

$$\mathbf{F}_m = q^2 AEB\,(-\hat{\mathbf{j}})$$

Since, $\mathbf{F}_m$ is along negative y-axis, no matter what is the sign of charge q. Therefore, all ions will deflect towards negative y-direction.

14. The charged particle will be accelerated parallel (if it is a positive charge) or antiparallel (if it is a negative charge) to the electric field, i.e. the charged particle will move parallel or antiparallel to electric and magnetic field. Therefore, net magnetic force on it will be zero and its path will be a straight line.

15. Radius of the circular path is given by

$$r = \frac{mv}{Bq} = \frac{\sqrt{2Km}}{Bq}$$

Here, K is the kinetic energy to the particle.

Therefore, $r \propto \dfrac{\sqrt{m}}{q}$ if K and B are same.

$$\therefore \quad r_p : r_d : r_\alpha = \frac{\sqrt{1}}{1} : \frac{\sqrt{2}}{1} : \frac{\sqrt{4}}{2} = 1 : \sqrt{2} : 1$$

Hence, $\quad r_\alpha = r_p < r_d$

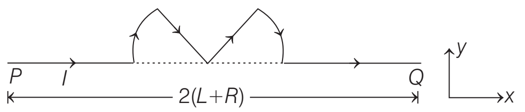
16. $R = \dfrac{\sqrt{2qVm}}{Bq}$

or $\quad R \propto \sqrt{m}$

or $\quad \dfrac{R_1}{R_2} = \sqrt{\dfrac{m_X}{m_Y}}$

or $\quad \dfrac{m_X}{m_Y} = \left(\dfrac{R_1}{R_2}\right)^2$

17.



Force on the complete wire = force on straight wire PQ carrying a current I.

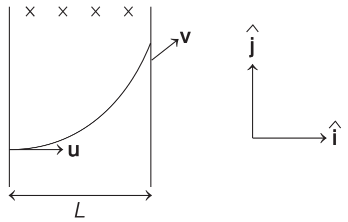
$$\mathbf{F} = I(\mathbf{PQ} \times \mathbf{B}) = I[\{2(L + R)\hat{\mathbf{i}}\} \times \mathbf{B}]$$

This force is zero if $\mathbf{B}$ is along $\hat{\mathbf{i}}$ direction or x-direction. If magnetic field is along $\hat{\mathbf{j}}$ direction or $\hat{\mathbf{k}}$ direction,

$$|\mathbf{F}| = F = (I)(2)(L + R)B \sin 90°$$

or $\quad F = 2I(L + R)B \quad$ or $\quad F \propto (L + R)$

18. $\mathbf{u} = 4\hat{\mathbf{i}}$; $\mathbf{v} = 2(\sqrt{3}\,\hat{\mathbf{i}} + \hat{\mathbf{j}})$



According to the figure, magnetic field should be in $\otimes$ direction, or along $-z$ direction.

Further, $\quad \tan\theta = \dfrac{v_y}{v_x} = \dfrac{2}{2\sqrt{3}} = \dfrac{1}{\sqrt{3}}$

$\therefore \quad \theta = 30°$

or $\quad \dfrac{\pi}{6}$ = angle of $\mathbf{v}$ with x-axis

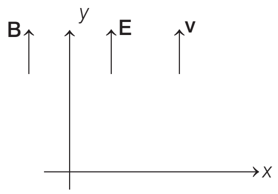
$= $ angle rotated by the particle $= Wt = \left(\dfrac{BQ}{M}\right)t$

$\therefore \quad B = \dfrac{\pi M}{6Qt} = \dfrac{50\pi M}{3Q}$ units $\qquad$ (as $t = 10^{-3}$ s)

19. and Magnetic field will rotate the particle in a circular path (in x-z plane or perpendicular to B). Electric field will exert a constant force on the particle in positive y-direction. Therefore, resultant path is neither purely circular nor helical or the options (a) and (b) both are wrong.

$v_\perp$ and $\mathbf{B}$ will rotate the particle in a circular path in x-z plane (or perpendicular to $\mathbf{B}$). Further $v_\parallel$ and $\mathbf{E}$ will move the particle (with increasing speed) along positive y-axis (or along the axis of above circular path). Therefore, the resultant path is helical with increasing pitch, along the y-axis (or along **B** and **E**). Therefore option (c) is correct.

(d)



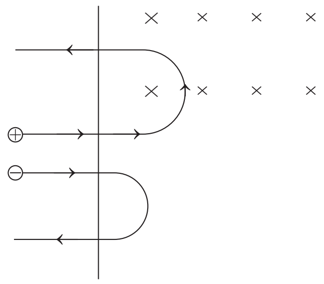
Magnetic force is zero, as θ between **B** and **v** is zero. But electric force will act in y-direction. Therefore, motion is 1-D and uniformly accelerated (towards positive y-direction).

20. $r = \frac{mv}{Bq}$ or $r \propto m$

$\therefore$ $r_e < r_p$ as $m_e < m_p$

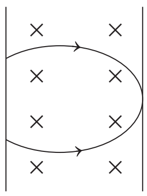
Further, $T = \frac{2\pi m}{Bq}$

or $T \propto m$



$\therefore$ $T_e < T_p$, $t_e = \frac{T_e}{2}$ and $t_p = \frac{T_p}{2}$ or $t_e < t_p$

21. $\mathbf{v} \perp \mathbf{B}$ in region II. Therefore, path of particle is circle in region II.



$v = \frac{Bql}{m}$

Particle enters in region III if,

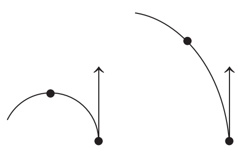
radius of circular path, $r > l$

or $\frac{mv}{Bq} > l$ or $v > \frac{Bql}{m}$

If $v = \frac{Bql}{m}, r = \frac{mv}{Bq} = l$, particle will turn back and path length will be maximum. If particle returns to region I, time spent in region II will be

$$t = \frac{T}{2} = \frac{\pi m}{Bq}, \text{ which is independent of } v.$$

22. $r = \frac{mv}{Bq} = \frac{p}{Bq} = \frac{\sqrt{2Km}}{Bq}$ i.e. $r \propto \frac{\sqrt{m}}{q}$



If K and B are same.

i.e., $r_{H^+} : r_{He^+} : r_{O^{2+}} = \frac{\sqrt{1}}{1} : \frac{\sqrt{4}}{1} : \frac{\sqrt{16}}{2} = 1 : 2 : 2$

Therefore, He^+ and O^{2+} will be deflected equally but H^+ having the least radius will be deflected most.

23. Magnetic force does not do work. From work-energy theorem :

$$W_{F_e} = \Delta \text{KE} \quad \text{or} \quad (qE)(2a) = \frac{1}{2}m[4v^2 - v^2]$$

or $$E = \frac{3}{4}\left(\frac{mv^2}{qa}\right)$$

At P, rate of work done by electric field

$$= \mathbf{F}_e \cdot \mathbf{v} = (qE)(v)\cos 0°$$
$$= q\left(\frac{3}{4}\frac{mv^2}{qa}\right)v = \frac{3}{4}\left(\frac{mv^3}{a}\right)$$

Therefore, option (b) is also correct.

Rate of work done at Q :

of electric field $= \mathbf{F}_e \cdot \mathbf{v} = (qE)(2v)\cos 90° = 0$

and of magnetic field is always zero. Therefore, option (d) is also correct.

Note that $\mathbf{F}_e = qE\hat{\mathbf{i}}$.

24. If both E and B are zero, then $\mathbf{F}_e$ and $\mathbf{F}_m$ both are zero. Hence, velocity may remain constant. Therefore, option (a) is correct.

If $E = 0$, $B \neq 0$ but velocity is parallel or antiparallel to magnetic field, then also $\mathbf{F}_e$ and $\mathbf{F}_m$ both are zero. Hence, option (b) is also correct.

If $E \neq 0$, $B \neq 0$ but $\mathbf{F}_e + \mathbf{F}_m = 0$, then also velocity may remain constant or option (d) is also correct.

25. For particle to move in negative y-direction, either its velocity must be in negative y-direction (if initial velocity $\neq 0$) and force should be parallel to velocity or it must experience a net force in negative y-direction only (if initial velocity = 0)

26. $\mathbf{F}_{\text{net}} = \mathbf{F}_e + \mathbf{F}_B = q\mathbf{E} + q\mathbf{v} \times \mathbf{B}$

For particle to move in straight line with constant velocity, $\mathbf{F}_{\text{net}} = 0$

$\therefore$ $q\mathbf{E} + q\mathbf{v} \times \mathbf{B} = 0$

27. For path to be helix with axis along positive z-direction, particle should experience a centripetal acceleration in xy-plane.

For the given set of options, only option (c) satisfy the condition. Path is helical with increasing pitch.

28. ZPath C is undeviated. Therefore, it is of neutron's path. From Fleming's left hand rule magnetic force on positive charge will be leftwards and on negative charge is rightwards. Therefore, track D is of electron. Among A and B one is of proton and other of α-particle.

Further, $r=\frac{mv}{Bq}$ or $r\propto\frac{m}{q}$

Since, $\left(\frac{m}{q}\right)_\alpha > \left(\frac{m}{q}\right)_P$

$\therefore$ $r_\alpha > r_P$

or track B is of α-particle.

29. $r=\frac{\sqrt{2km}}{Bq}$ or $r\propto\sqrt{m}$ (k, q and B are same)

$m_p > m_e \Rightarrow \therefore r_p > r_e$

30. The path will be a helix. Path is circle when it enters normal to the magnetic field.

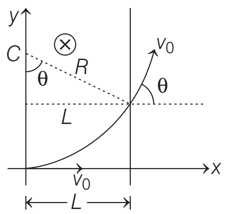
31. Magnetic force acting on a charged particle is always perpendicular to its velocity or work done by a magnetic force is always zero. Hence, a magnetic force cannot change the energy of charged particle.

32. $r=\frac{\sqrt{2qVm}}{Bq}$ or $r\propto\sqrt{\frac{m}{q}}$

$$\frac{r_P}{r_\alpha}=\sqrt{\frac{m_P}{m_\alpha}}\sqrt{\frac{q_\alpha}{q_P}}=\sqrt{\frac{1}{4}}\sqrt{\frac{2}{1}}=\frac{1}{\sqrt{2}}$$

33. (a) $\theta = 30°$

$$\sin\theta = \frac{L}{R}$$

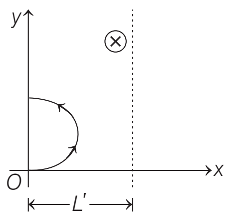


Here, $R=\frac{mv_0}{B_0q}$

$\therefore$ $\sin 30° = \frac{L}{\frac{mv_0}{B_0q}}$

or $\frac{1}{2}=\frac{B_0qL}{mv_0} \Rightarrow L=\frac{mv_0}{2B_0q}$

(b) In part (a)



$\sin 30° = \frac{L}{R}$ or $\frac{1}{2}=\frac{L}{R}$

or $L=R/2$

Now when $L' = 2.1\,L$

or $\frac{2.1}{2}R \Rightarrow L' > R$

Therefore, deviation of the particle is $\theta = 180°$ as shown.

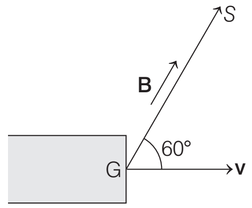
$\therefore$ $\mathbf{v}_f = -v_0\hat{\mathbf{i}}$

and $t_{AB} = T/2 = \frac{\pi m}{B_0 q}$

34. Kinetic energy of electron, $K=\frac{1}{2}mv^2 = 2$ keV

$\therefore$ Speed of electron, $v=\sqrt{\frac{2K}{m}}$

$$v=\sqrt{\frac{2\times 2\times 1.6\times 10^{-16}}{9.1\times 10^{-31}}}\text{ m/s} = 2.65\times 10^7\text{ m/s}$$



Since, the velocity ($\mathbf{v}$) of the electron makes an angle of $\theta = 60°$ with the magnetic field $\mathbf{B}$, the path will be a helix. So, the particle will hit S if GS = np

Here, $n = 1, 2, 3$ ....

$$p = \text{pitch of helix} = \frac{2\pi m}{qB}v\cos\theta$$

But for B to be minimum, $n = 1$

Hence, $GS = p = \frac{2\pi m}{qB}v\cos\theta$

$$B = B_{\min} = \frac{2\pi mv\cos\theta}{q(GS)}$$

Substituting the values, we have

$$B_{\min} = \frac{(2\pi)(9.1\times 10^{-31})(2.65\times 10^7)\left(\frac{1}{2}\right)}{(1.6\times 10^{-19})(0.1)}$$

or $B_{\min} = 4.73\times 10^{-3}$ T

35. (i) $r=\frac{mv\sin\theta}{Bq}=\frac{(1.67\times 10^{-27})(4\times 10^5)(\sin 60°)}{(0.3)(1.6\times 10^{-19})}$

$= 1.2\times 10^{-2}$ m

(ii) $p=\left(\frac{2\pi m}{Bq}\right)(v\cos\theta)$

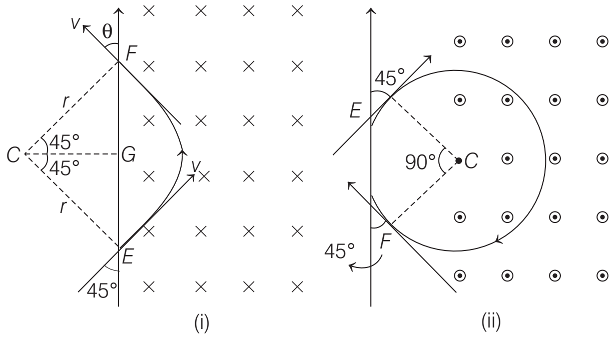
$=\frac{(2\pi)(1.67\times 10^{-27})(4\times 10^5)(\cos 60°)}{(0.3)(1.6\times 10^{-19})}$

$= 4.37\times 10^{-2}$ m

36. Inside a magnetic field speed of charged particle does not change. Further, velocity is perpendicular to magnetic field in both the cases hence path of the particle in the magnetic field will be circular. Centre of circle can be obtained by drawing perpendiculars to velocity (or tangent to the circular path) at E and F. Radius and angular speed of circular path would be

$$r=\frac{mv}{Bq}$$

and $$\omega=\frac{Bq}{m}$$



(a) Refer figure (i)

$\angle CFG = 90° - \theta$ and $\angle CEG = 90° - 45° = 45°$

Since, $\quad CF = CE$

$\therefore \quad \angle CFG = \angle CEG$

or $\quad 90° - \theta = 45° \quad$ or $\quad \theta = 45°$

Further, $\quad FG = GE = r\cos 45°$

$$\therefore \; EF = 2FG = 2r\cos 45° = \frac{2mv\cos 45°}{Bq}$$

$$= \frac{2\,(1.6\times 10^{-27})\,(10^{7})\left(\frac{1}{\sqrt{2}}\right)}{(1)\,(1.6\times 10^{-19})} = 0.14 \text{ m}$$

NOTE That in this case particle completes 1/4th of circle in the magnetic field.

(b) Refer figure (ii) In this case particle will complete $\frac{3}{4}$th of circle in the magnetic field. Hence, the time spent in the magnetic field

$$t = \frac{3}{4} \qquad \text{(time period of circular motion)}$$

$$= \frac{3}{4}\left(\frac{2\pi m}{Bq}\right) = \frac{3\pi m}{2Bq}$$

$$= \frac{(3\pi)\,(1.6\times 10^{-27})}{(2)\,(1)(1.6\times 10^{-19})}$$

$$= 4.712\times 10^{-8} \text{ s}$$

37. $\mathbf{F}_e = q\,\mathbf{E} = (1.6\times 10^{-19})(-\,102.4\times 10^{3})\hat{\mathbf{k}}$

$$= -\,(1.6384\times 10^{-16})\hat{\mathbf{k}} \text{ N}$$

$\mathbf{F}_m = q\,(\mathbf{v}\times\mathbf{B}) = 1.6\times 10^{-19}\;(1.28\times 10^{6}\;\hat{\mathbf{i}}\times 8\times 10^{-2}\;\hat{\mathbf{j}})$

$$= 1.6384\times 10^{-16}\;\hat{\mathbf{k}} \text{ N}$$

Since, $\mathbf{F}_e + \mathbf{F}_m = 0$

$\therefore$ Net force on the charged particle is zero. Particle will move undeviated.

In time $t = 5\times 10^{-6}$ s, the x-coordinate of particle will become,

$$x = v_x t \;= (1.28\times 10^{6})\,(5\times 10^{-6}) = 6.4 \text{ m}$$

while y and z-coordinates will be zero.

At $x = 5\times 10^{-6}$ s, electric field is switched-off. Only magnetic field is left which is perpendicular to its velocity. Hence, path of the particle will now become circular.

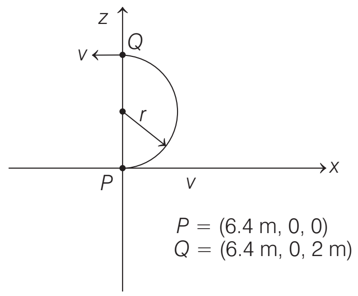
Plane of circle will be perpendicular to magnetic field i.e. x-z. Radius and angular velocity of circular path will become

$$r = \frac{mv}{Bq} = \frac{(10^{-26})\,(1.28\times 10^{6})}{(8\times 10^{-2})\,(1.6\times 10^{-19})} = 1 \text{m}$$

$$\omega = \frac{Bq}{m} = \frac{(8\times 10^{-2})\,(1.6\times 10^{-19})}{(10^{-26})}$$

$$= 1.28\times 10^{6} \text{ rad/s}$$

In the remaining time i.e. $(7.45 - 5)\times 10^{-6} = 2.45\times 10^{-6}$s



Angle rotated by particle,

$$\theta = \omega t = (1.28\times 10^{6})\,(2.45\times 10^{-6}) = 3.14 \text{ rad} \approx 180°$$

$\therefore$ z-coordinate of particle will become

$$z = 2r = 2m$$

while y-coordinate will be zero.

$\therefore$ Position of particle at $t = 5\times 10^{-6}$ s is $P \equiv (6.4 \text{ m}, 0, 0)$

and at $t = 7.45\times 10^{-6}$ s is $Q \equiv (6.4 \text{m}, 0, 2 \text{m})$

38. Electron pass undeviated. Therefore,

$|\mathbf{F}_e| = |\mathbf{F}_m|$ or $eE = eBv$

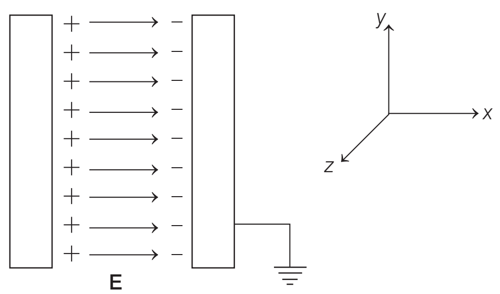
or $\quad B = \frac{E}{v} = \frac{V/d}{v}$

(V = potential difference between the plates)

or $\quad B = \frac{V}{dv}$

Substituting the values, we have

$$B = \frac{600}{3\times 10^{-3}\times 2\times 10^{6}} = 0.1\text{T}$$



Further, direction of $\mathbf{F}_e$ should be opposite of $\mathbf{F}_m$.

or $\qquad e\mathbf{E} \uparrow\downarrow e(\mathbf{v}\times\mathbf{B})$

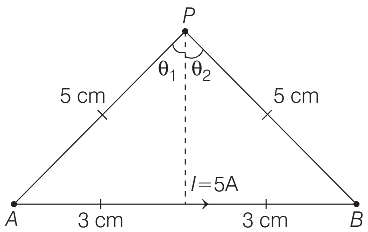
$\therefore \qquad \mathbf{E} \uparrow\downarrow \mathbf{v}\times\mathbf{B}$

Here, $\mathbf{E}$ is in positive x-direction.

Therefore, $\mathbf{v}\times\mathbf{B}$ should be in negative x-direction or $\mathbf{B}$ should be in negative z-direction or perpendicular to paper inwards, because velocity of electron is in positive y-direction.

Topic 2 Biot-Savart Law and Ampere's Circuital Law

1. The given figure can be drawn as shown below



By Biot-Savart's law, magnetic field due to a wire segment at point P is

$$B = \frac{\mu_0 I}{4\pi d}(\sin\theta_1 + \sin\theta_2)$$

Here, $\qquad \theta_1 = \theta_2 = \theta \qquad$ (say)

Then, $\qquad B = \frac{\mu_0 I}{4\pi d}\times 2\sin\theta \qquad$ ...(i)

From given data,

$$I = 5\text{A},$$
$$\mu = 4\pi\times10^{-7}\ \text{NA}^{-2}$$
$$d = \sqrt{5^2 - 3^2} = \sqrt{16} = 4\ \text{cm}$$
$$\sin\theta = \frac{3}{5}$$

On substituting these values in Eq. (i), we get

$$B = \frac{\mu_0 I}{4\pi d}\times 2\sin\theta = \frac{4\pi\times10^{-7}\times5}{4\pi\times4\times10^{-2}}\times2\times\left(\frac{3}{5}\right)$$
$$= \frac{5\times2\times3}{4\times5}\times10^{-5} = 1.5\times10^{-5}\ \text{T}$$

2. Given, $\mu_0 = 4\pi\times10^{-7}\ \text{N/A}^2$,

$$\omega = 40\pi\ \text{rad/s}$$
$$B_{\text{at centre}} = 3.8\times10^{-9}\ \text{T}$$

and $\qquad R = 10\ \text{cm} = 0.1\ \text{m}$

Now, we know that, magnetic field at the centre of a current carrying ring is given by

$$B = \frac{\mu_0 I}{2r} \qquad \text{...(i)}$$

Here, I can be determined by flow of charge per rotation, i.e.

$$I = \frac{Q}{T} \qquad \text{...(ii)}$$

Here, $\qquad T = \frac{2\pi}{\omega}$

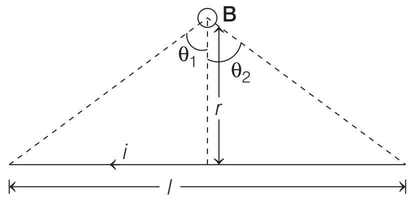
$\Rightarrow \qquad I = \frac{Q\omega}{2\pi} \qquad$ ...(iii)

By putting value of I from Eq. (iii) to Eq. (i), we get

$$B = \frac{\mu_0 Q\omega}{2r\times2\pi} \text{ or } Q = \frac{2Br\times2\pi}{\mu_0\omega}$$
$$= \frac{2\times3.8\times10^{-9}\times0.1\times2\pi}{4\pi\times10^{-7}\times40\pi}$$
$$= \frac{2\times3.8\times0.1}{2\times40\pi}\times10^{-2}$$
$$= 0.003022\times10^{-2}\ \text{C}$$
$$= 3.022\times10^{-5}\ \text{C}$$

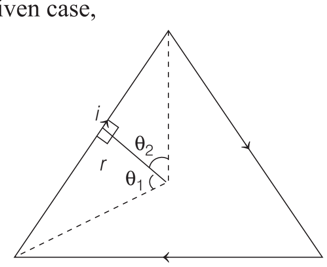
or $\qquad Q = 3\times10^{-5}\ \text{C}$

3. For a current carrying wire, from result obtained by Biot-Savart's law, magnetic field at a distance r is given by



$$B = \frac{\mu_0 i}{4\pi r}(\sin\theta_1 + \sin\theta_2)$$

Now, in given case,



Due to symmetry of arrangement, net field at centre of triangle is

$$B_{\text{net}} = \text{Sum of fields of all wires (sides)}$$
$$= 3\times\frac{\mu_0 i}{4\pi r}(\sin\theta_1 + \sin\theta_2)$$

Here, $\theta_1 = \theta_2 = 60^\circ$

$\therefore \quad \sin\theta_1 = \sin\theta_2 = \frac{\sqrt{3}}{2},\ i = 10\text{A},\ \frac{\mu_0}{4\pi} = 10^{-7}\text{NA}^{-2}$

and $\qquad r = \frac{1}{3}\times\text{altitude}$

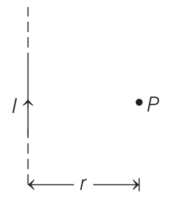
$$= \frac{1}{3} \times \frac{\sqrt{3}}{2} \times \text{sides length} = \frac{1}{2\sqrt{3}} \times 1\,\text{m} = \frac{1}{2\sqrt{3}}\,\text{m}$$

So,

$$B_{\text{net}} = \frac{3 \times 10^{-7} \times 10 \times 2\left(\frac{\sqrt{3}}{2}\right)}{\left(\frac{1}{2\sqrt{3}}\right)} = 18 \times 10^{-6}\ \text{T}$$

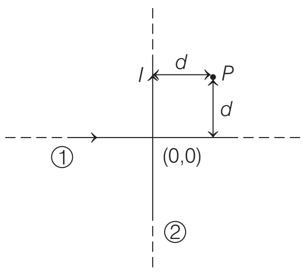
$\Rightarrow B_{\text{net}} = 18\,\mu\text{T}$

4. Magnetic field due to an infinitely long straight wire at point P is given as



$$|\mathbf{B}| = \frac{\mu_0 I}{2\pi r}$$

Thus, in the given situation, magnetic field due to wire 1 at point P is



$$\mathbf{B}_1 = \frac{\mu_0}{2\pi}\frac{I}{d}, \otimes$$

Similarly, magnetic field due to wire 2 at point P is

$$\mathbf{B}_2 = \frac{\mu_0}{2\pi}\frac{I}{d},$$

Resultant field at point P is

$$\mathbf{B}_{\text{net}} = \mathbf{B}_1 + \mathbf{B}_2$$

Since, $|\mathbf{B}_1| = |\mathbf{B}_2|$, but they are opposite in direction.

Thus, $\mathbf{B}_{\text{net}} = 0$

$\therefore$ Net magnetic field at point P will be zero.

5. There is no magnetic field along axis of a current-carrying wire.

Also, magnetic field near one of end of an infinitely long wire is $\frac{\mu_0 I}{4\pi r}$ tesla.

Hence, magnetic field due to segments LP and MQ at 'O' is zero.

Using right hand rule, we can check that magnetic field due to segments PS and QN at 'O' is in same direction perpendicularly into the plane of paper.

Hence, $B_O = B_{PS} + B_{QN}$

$$= \frac{\mu_0 i}{4\pi r} + \frac{\mu_0 i}{4\pi r} = \frac{\mu_0 i}{2\pi r}$$

So, $\quad i = \dfrac{2\pi r B_0}{\mu_0}$

Here, $r = OP = OQ = 4$ cm

and $B_O = 10^{-4}$ T.

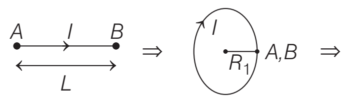
Substituting values, we get

$$\Rightarrow i = \frac{2\pi \times 4 \times 10^{-2} \times 10^{-4}}{4\pi \times 10^{-7}}$$

$\Rightarrow i = 20\,\text{A}$, Also, magnetic field points perpendicular into the plane of paper.

6. Let consider the length of first wire is L, then according to question, if radius of loop formed is R_1, then,

For wire 1,

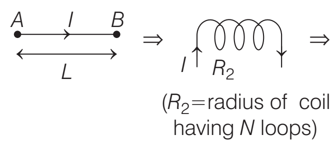


$$L = 2\pi R_1 \Rightarrow R_1 = \frac{L}{2\pi}$$

The magnetic field due to this loop at its centre is

$$B_L = \frac{\mu_0 I}{2R_1} = \frac{\mu_0 I}{2L} \times 2\pi \qquad \ldots\text{(i)}$$

Now, several wire is made into a coil of N- turns.



Thin, $\quad L = N(2\pi R_2) \Rightarrow R_2 = \dfrac{L}{2\pi N}$

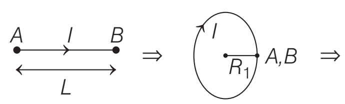
The magnetic field due to this circular coil of N-turns is

$$B_C = \left(\frac{\mu_0 I}{2R_2}\right)N = N \cdot \frac{\mu_0 I \cdot (2\pi N)}{2L} \qquad \ldots\text{(ii)}$$

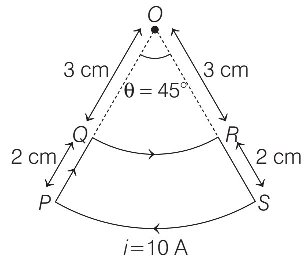
Using Eqs. (i) and (ii), the ratio of $\frac{B_L}{B_C}$ is :

$$\Rightarrow \quad \frac{B_L}{B_C} = \frac{\frac{\mu_0 I}{2R_1}}{N\frac{\mu_0 I}{2R_2}} = \frac{\frac{\mu_0 I}{2L}\cdot(2\pi)}{\frac{\mu_0 I}{2L}\cdot(2\pi)N^2} = \frac{1}{N^2}$$

7. **Key Idea** When a point 'P' lies on the axial position of current-carrying conductor, then magnetic field at P is always zero.



From the given figure as shown below,



The magnetic field at point 'O' due to wires PQ and RS will be zero.

Magnetic field due to arc QR at point 'O' will be

$$B_1 = \frac{\theta}{2\pi}\left(\frac{\mu_0 i}{2a}\right)$$

Here, $\theta = 45° = \frac{\pi}{4}$ rad, $i = 10$ A

and $a = 3 \text{ cm} = 3 \times 10^{-2}$ m

$$\Rightarrow \quad B_1 = \frac{\pi}{2\pi \times 4}\left(\frac{\mu_0 \times 10}{2 \times 3 \times 10^{-2}}\right)$$

$$= \frac{\mu_0 \times 5}{2 \times 12 \times 10^{-2}} = \frac{5 \times \mu_0 \times 10^2}{24}$$

Direction of field B_1 will be coming out of the plane of figure.

Similarly, field at point 'O' due to arc SP will be

$$B_2 = \frac{\pi}{4}\left(\frac{1}{2\pi}\right)\left[\frac{\mu_0 \times 10}{2 \times (2+3) \times 10^{-2}}\right] = \frac{\mu_0 \times 5}{2 \times 20 \times 10^{-2}}$$

$$= \frac{\mu_0}{2 \times 4 \times 10^{-2}} = \frac{\mu_0 \times 10^2}{8}$$

Direction of B_2 is going into the plane of the figure.

$\therefore$ The resultant field at O is

$$B = B_1 - B_2 = \frac{1}{2}\left(\frac{5 \times \mu_0}{12 \times 10^{-2}} - \frac{\mu_0}{4 \times 10^{-2}}\right)$$

$$= \frac{1}{2}\left(\frac{5\mu_0 - 3\mu_0}{12 \times 10^{-2}}\right) = \frac{1}{2}\left(\frac{2\mu_0}{12 \times 10^{-2}}\right)$$

$$= \frac{4\pi \times 10^{-7}}{12 \times 10^{-2}} \cong 1 \times 10^{-5} \text{ T}$$

8. B at centre of a circle $= \frac{\mu_0 I}{2R}$

B at centre of a square

$$= 4 \times \frac{\mu I}{4\pi \cdot \frac{l}{2}}[\sin 45° + \sin 45°] = 4\sqrt{2}\,\frac{\mu_0 I}{2\pi l}$$

Now, $\quad R = \frac{L}{2\pi}$ and $l = \frac{L}{4}$ $\qquad$ (as $L = 2\pi R = 4l$)

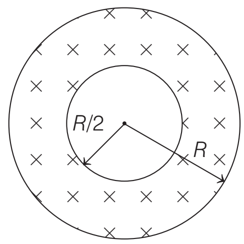
where, $\quad L$ = length of wire.

$$\therefore \quad B_A = \frac{\mu_0 I}{2 \cdot \frac{L}{2\pi}} = \frac{\pi \mu_0 I}{L} = \pi\left[\frac{\mu_0 I}{L}\right]$$

$$B_B = 4\sqrt{2}\,\frac{\mu_0 I}{2\pi\left(\frac{L}{4}\right)} = \frac{8\sqrt{2}\mu_0 I}{\pi L} = \frac{8\sqrt{2}}{\pi}\left[\frac{\mu_0 I}{L}\right]$$

$$\therefore \quad \frac{B_A}{B_B} = \frac{\pi^2}{8\sqrt{2}}$$

9.



r = distance of a point from centre.

For $r \le R/2$ Using Ampere's circuital law,

$$\oint \mathbf{B} \cdot d\mathbf{l} \quad \text{or} \quad Bl = \mu_0 (I_{in})$$

or $\quad B(2\pi r) = \mu_0 (I_{in})$ or $\quad B = \frac{\mu_0}{2\pi}\frac{I_{in}}{r} \qquad \ldots\text{(i)}$

Since, $I_{in} = 0 \Rightarrow \therefore B = 0$

For $\frac{R}{2} \le r \le R$ $\quad I_{in} = \left[\pi r^2 - \pi\left(\frac{R}{2}\right)^2\right]\sigma$

Here, σ = current per unit area

Substituting in Eq. (i), we have

$$B = \frac{\mu_0}{2\pi}\frac{\left[\pi r^2 - \pi\frac{R^2}{4}\right]\sigma}{r} = \frac{\mu_0 \sigma}{2r}\left(r^2 - \frac{R^2}{4}\right)$$

At $\quad r = \frac{R}{2}, B = 0$

At $\quad r = R, B = \frac{3\mu_0 \sigma R}{8}$

For $r \ge R$ $\quad I_{in} = I_{Total} = I$ (say)

Therefore, substituting in Eq. (i), we have

$$B = \frac{\mu_0}{2\pi} \cdot \frac{I}{r} \quad \text{or} \quad B \propto \frac{1}{r}$$

10. If we take a small strip of dr at distance r from centre, then number of turns in this strip would be,

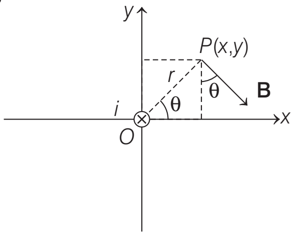
$$dN = \left(\frac{N}{b-a}\right) dr$$

Magnetic field due to this element at the centre of the coil will be

$$dB = \frac{\mu_0 (dN) I}{2r} = \frac{\mu_0 NI}{(b-a)}\frac{dr}{r}$$

$$\therefore \quad B = \int_{r=a}^{r=b} dB = \frac{\mu_0 NI}{2(b-a)} \ln\left(\frac{b}{a}\right)$$

11. Magnetic field at P is **B**, perpendicular to OP in the direction shown in figure.



So, $\mathbf{B} = B\sin\theta\hat{\mathbf{i}} - B\cos\theta\hat{\mathbf{j}}$

Here, $B = \dfrac{\mu_0 I}{2\pi r}$

$$\sin\theta = \frac{y}{r} \quad \text{and} \quad \cos\theta = \frac{x}{r}$$

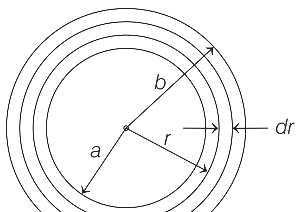
$$\therefore \quad \mathbf{B} = \frac{\mu_0 I}{2\pi}\cdot\frac{1}{r^2}(y\hat{\mathbf{i}} - x\hat{\mathbf{i}})$$

$$= \frac{\mu_0 I(y\hat{\mathbf{i}} - x\hat{\mathbf{j}})}{2\pi(x^2 + y^2)} \qquad (\text{as } r^2 = x^2 + y^2)$$

12. Consider an element of thickness dr at a distance r from the centre. The number of turns in this element,

$$dN = \left(\frac{N}{b-a}\right)dr$$

Magnetic field due to this element at the centre of the coil will be

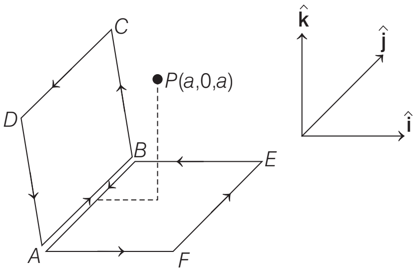


$$dB = \frac{\mu_0 (dN) I}{2r} = \frac{\mu_0 I}{2}\frac{N}{b-a}\cdot\frac{dr}{r}$$

$$\therefore \quad B = \int_{r=a}^{r=b} dB = \frac{\mu_0 NI}{2(b-a)}\ln\left(\frac{b}{a}\right)$$

NOTE The idea of this question is taken from question number 3.245 of IE Irodov.

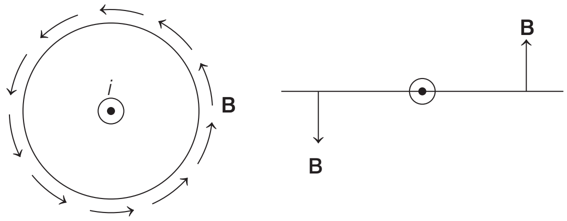
13. The magnetic field at $P(a, 0, a)$ due to the loop is equal to the vector sum of the magnetic fields produced by loops $ABCDA$ and $AFEBA$ as shown in the figure.



Magnetic field due to loop $ABCDA$ will be along $\hat{\mathbf{i}}$ and due to loop $AFEBA$, along $\hat{\mathbf{k}}$. Magnitude of magnetic field due to both the loops will be equal. Therefore, direction of resultant magnetic field at P will be $\dfrac{1}{\sqrt{2}}(\hat{\mathbf{i}} + \hat{\mathbf{k}})$.

NOTE This is a common practice, when by assuming equal currents in opposite directions in an imaginary wire (here AB) loops are completed and solution becomes easy.

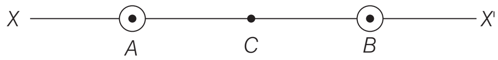
14. If the current flows out of the paper, the magnetic field at points to the right of the wire will be upwards and to the left will be downwards as shown in figure.



Now, let us come to the problem.

Magnetic field at $C = 0$

Magnetic field in region BX' will be upwards (+ve) because all points lying in this region are to the right of both the wires.



Magnetic field in region AC will be upwards (+ve), because points are closer to A, compared to B. Similarly, magnetic field in region BC will be downwards (–ve).

Graph (b) satisfies all these conditions. Therefore, correct answer is (b).

15. H_1 = Magnetic field at M due to PQ + Magnetic field at M due to QR. But magnetic field at M due to $QR = 0$

$\therefore$ Magnetic field at M due to PQ (or due to current I in PQ) $= H_1$

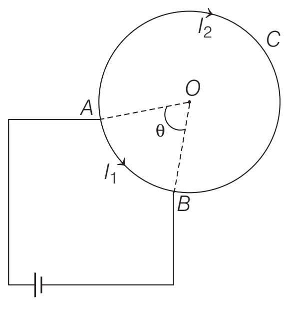
Now H_2 = Magnetic field at M due to PQ (current I)
+ magnetic field at M due to QS (current $I/2$)
+ magnetic field at M due to QR

$$= H_1 + \frac{H_1}{2} + 0 = \frac{3}{2}H_1$$

$$\frac{H_1}{H_2} = \frac{2}{3}$$

NOTE Magnetic field at any point lying on the current carrying straight conductor is zero.

16. For a current flowing into a circular arc, the magnetic induction at the centre is



$$B = \left(\frac{\mu_0 i}{4\pi r}\right)\theta \text{ or } B \propto i\theta$$

In the given problem, the total current is divided into two arcs

$$i \propto \frac{1}{\text{resistance of arc}} \propto \frac{1}{\text{length of arc}}$$

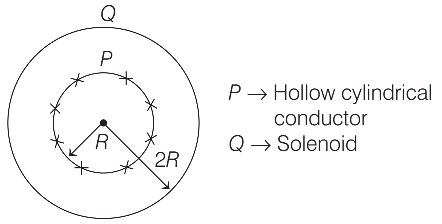
$$\propto \frac{1}{\text{angle subtended at centre } (\theta)}$$

or $\quad i\theta = \text{constant}$

i.e. magnetic field at centre due to arc AB is equal and opposite to the magnetic field at centre due to arc ACB. Or the net magnetic field at centre is zero.

17. Using Ampere's circuital law over a circular loop of any radius less than the radius of the pipe, we can see that net current inside the loop is zero. Hence, magnetic field at every point inside the loop will be zero.

18.



In the region, $0 < r < R$

$$B_P = 0,$$

$B_Q \neq 0$, along the axis

$\therefore \quad B_{\text{net}} \neq 0$

In the region, $\quad R < r < 2R$

$B_P \neq 0$, tangential to the circle of radius r, centred on the axis.

$B_Q \neq 0$, along the axis.

$\therefore$ $B_{\text{net}} \neq 0$ neither in the directions mentioned in options (b) or (c). In region, $r > 2R$

$$B_P \neq 0$$

$$B_Q \neq 0 \Rightarrow B_{\text{net}} \neq 0$$

19. $B_R = B_T - B_C$

R = Remaining portion

T = Total portion and

C = Cavity

$$B_R = \frac{\mu_0 I_T}{2a\pi} - \frac{\mu_0 I_C}{2\,(3a/2)\pi} \qquad \ldots\text{(i)}$$

$$I_T = J\,(\pi a^2)$$

$$I_C = J\left(\frac{\pi a^2}{4}\right)$$

Substituting the values in Eq. (i), we have

$$B_R = \frac{\mu_0}{a\pi}\left[\frac{I_T}{2} - \frac{I_C}{3}\right]$$

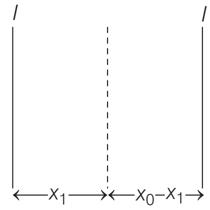
$$= \frac{\mu_0}{a\pi}\left[\frac{\pi a^2 J}{2} - \frac{\pi a^2 J}{12}\right] = \frac{5\mu_0 a J}{12}$$

$\therefore \quad N = 5$

20. $B_2 = \dfrac{\mu_0 I}{2\pi x_1} + \dfrac{\mu_0 I}{2\pi\,(x_0 - x_1)}$

(when currents are in opposite directions)

$$B_1 = \frac{\mu_0 I}{2\pi x_1} - \frac{\mu_0 I}{2\pi\,(x_0 - x_1)}$$



(when currents are in same direction)

Substituting $x_1 = \dfrac{x_0}{3}$ $\quad$ (as $\dfrac{x_0}{x_1} = 3$)

$$B_1 = \frac{3\mu_0 I}{2\pi x_0} - \frac{3\mu_0 I}{4\pi x_0} = \frac{3\mu_0 I}{4\pi x_0}$$

$$R_1 = \frac{mv}{qB_1} \text{ and } B_2 = \frac{9\mu_0 I}{4\pi x_0}$$

$$R_2 = \frac{mv}{qB_2}$$

$$\Rightarrow \qquad \frac{R_1}{R_2} = \frac{B_2}{B_1} = \frac{9}{3} = 3$$

21. At C magnetic field due to wires PQ and RS will be zero. Due to wire QR,

$$B_1 = \frac{1}{2}\left(\frac{\mu_0 I}{2R_1}\right) = \frac{\mu_0 I}{4R_1} \quad \text{(perpendicular to paper outwards)}$$

And due to wire SP,

$$B_2 = \frac{1}{2}\left(\frac{\mu_0 I}{2R_2}\right) = \frac{\mu_0 I}{4R_2} \quad \text{(perpendicular to paper inwards)}$$

$\therefore$ Net magnetic field would be,

$$B = \frac{\mu_0 I}{4}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{(perpendicular to paper outwards)}$$

22. Magnetic field at O due to L and M is zero. Due to P magnetic field at O is

$$B_1 = \frac{1}{2}\left(\frac{\mu_0}{2\pi}\frac{i}{OR}\right) = \frac{(10^{-7})(10)}{0.02} = 5.0 \times 10^{-5}\,\text{T}$$

(perpendicular to paper outwards)

Similarly, field at O due to Q would be,

$$B_2 = \frac{1}{2}\left(\frac{\mu_0}{2\pi}\frac{i}{OS}\right)$$

$$= \frac{(10^{-7})(10)}{0.02} = 5.0 \times 10^{-5}\,\text{T}$$

(perpendicular to paper outwards)

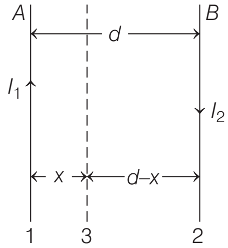
Since, both the fields are in same direction, net field will be sum of these two.

$\therefore \qquad B_{\text{net}} = B_1 + B_2 = 10^{-4}\,\text{T}$

Direction of field is perpendicular to the paper outwards.

Topic 3 Magnetic Force on Current Carrying Wires

1. Net force on the third wire, carrying current I in the following first case is



$$F_{13} + F_{23} = 0$$

Using thumb rule, direction of **B** at inside region of wires A and B will be same.

$$\therefore \quad \frac{\mu_0 I_1 I}{2\pi x} + \frac{\mu_0 I_2 I}{2\pi (d - x)} = 0$$

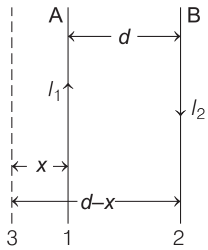
$$\Rightarrow \quad \frac{I_1}{x} + \frac{I_2}{d - x} = 0$$

$$\Rightarrow \quad \frac{I_1}{x} = \frac{I_2}{x - d} \text{ or } (x - d) I_1 = x I_2$$

$$\Rightarrow \quad x (I_1 - I_2) = d I_1$$

$$\Rightarrow \quad x = \frac{I_1}{(I_1 - I_2)} \cdot d \qquad \ldots \text{(i)}$$

Second case of balanced force can be as shown



Using thumb rule, directions of **B** at any point on wires A and B will be opposite, so net force,

$$\frac{\mu_0 I_1 I}{2\pi x} - \frac{\mu_0 I_2 I}{2\pi (d + x)} = 0 \text{ or } \frac{I_1}{x} - \frac{I_2}{(d + x)} = 0$$

$$\Rightarrow \quad \frac{I_1}{x} = \frac{I_2}{d + x}$$

$$\Rightarrow \quad (d + x) I_1 = x I_2$$

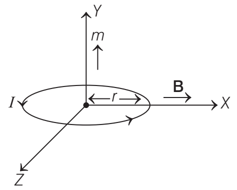
$$\Rightarrow \quad (I_2 - I_1) x = d I_1$$

$$\Rightarrow \quad x = -\frac{I_1}{(I_1 - I_2)} \cdot d \qquad \ldots \text{(ii)}$$

From Eqs. (i) and (ii), it is clear that

$$x = \pm \frac{I_1}{(I_1 - I_2)} d$$

2. According to the question, the situation can be drawn as



Let the current I is flowing in anti-clockwise direction, then the magnetic moment of the coil is

$$m = NIA$$

where, N = number of turns in coil

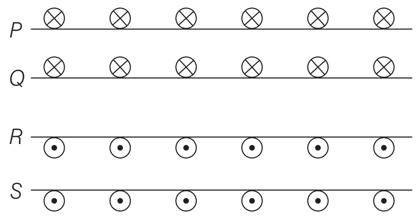
and A = area of each coil = πr^2.

Its direction is perpendicular to the area of coil and is along Y-axis.

Then, torque on the current coil is

$$\tau = \mathbf{m} \times \mathbf{B} = mB \sin 90^\circ = NIAB = NI\pi r^2 B \text{(N-m)}$$

3.

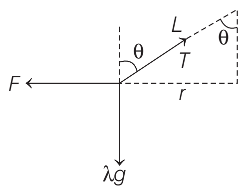


If we calculate the force on inner solenoid. Force on Q due to P is outwards (attraction between currents in same direction. Similarly, force on R due to S is also outwards. Hence, net force $\mathbf{F}_1$ is zero)

Force on P due to Q and force on S due to R is inwards. Hence, net force $\mathbf{F}_2$ is also zero.

Alternate Thought Field of one solenoid is uniform and other solenoid may be assumed a combination of circular closed loops. In uniform magnetic field, net force on a closed current carrying loop is zero.

4.



$$r = L \sin \theta$$

F = Magnetic force (repulsion) per unit length

$$= \frac{\mu_0}{2\pi} \frac{I^2}{2r} = \frac{\mu_0}{4\pi} \frac{I^2}{L \sin \theta}$$

λg = weight per unit length

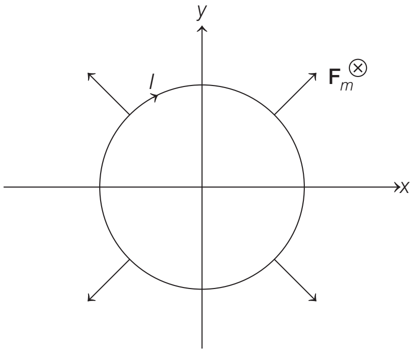
Each wire is in equilibrium under three concurrent forces as shown in figure. Therefore applying Lami's theorem.

$$\frac{F}{\sin(180^\circ - \theta)} = \frac{\lambda g}{\sin (90^\circ + \theta)}$$

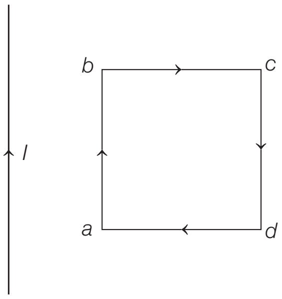
or $$\frac{\frac{\mu_0}{4\pi}\frac{I^2}{L\sin\theta}}{\sin\theta}=\frac{\lambda g}{\cos\theta}$$

$\therefore$ $$I=2\sin\theta\sqrt{\frac{\pi\lambda g L}{\mu_0\cos\theta}}$$

5. Net force on a current carrying loop in uniform magnetic field is zero. Hence, the loop cannot translate. So, options (c) and (d) are wrong. From Fleming's left hand rule we can see that if magnetic field is perpendicular to paper inwards and current in the loop is clockwise (as shown) the magnetic force $\mathbf{F}_m$ on each element of the loop is radially outwards, or the loops will have a tendency to expand.



6. Force per unit length between two wires carrying currents i_1 and i_2 at distance r is given by



$$\frac{F}{l}=\frac{\mu_0}{2\pi}\frac{i_1 i_2}{r}$$

Here, $i_1=i_2=i$

and $r=b$

$\therefore$ $$\frac{F}{l}=\frac{\mu_0 i^2}{2\pi b}$$

7. Straight wire will produce a non-uniform field to the right of it. $\mathbf{F}_{bc}$ and $\mathbf{F}_{da}$ will be calculated by integration but these two forces will cancel each other. Further force on wire ab will be towards the long wire and on wire cd will be away from the long wire. But since the wire ab is nearer to the long wire, force of attraction towards the long wire will be more. Hence, the loop will move towards the wire.

8. Net force is on an imaginary wire FA having current I from F to A.

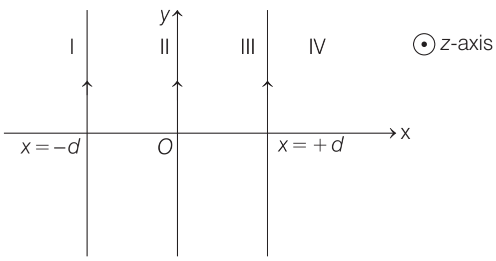
$$\mathbf{F}=I[(L\hat{\mathbf{i}})\times(B\hat{\mathbf{j}})]=ILB\hat{\mathbf{k}}$$

$\because$ Magnitude of force is ILB and direction of force is positive z.

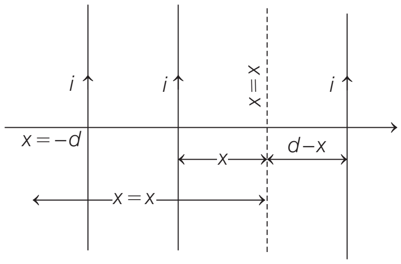
9. A current carrying coil is a magnetic dipole. Net magnetic force on a magnetic dipole in uniform field is zero.

10. Magnetic field will be zero on the y-axis i.e.

$$x=0=z$$



Magnetic field cannot be zero in region I and region IV because in region I magnetic field will be along positive z-direction due to all the three wires, while in region IV magnetic field will be along negative z-axis due to all the three wires. It can zero only in region II and III.



Let magnetic field is zero on line ($z=0$) and $x=x$. Then magnetic field on this line due to wires 1 and 2 will be along negative z-axis and due to wire 3 along positive z-axis. Thus

$$B_1+B_2=B_3$$

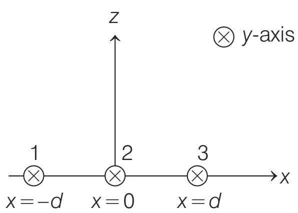
or $$\frac{\mu_0}{2\pi}\frac{i}{d+x}+\frac{\mu_0}{2\pi}\frac{i}{x}=\frac{\mu_0}{2\pi}\frac{i}{d-x}$$

or $$\frac{1}{d+x}+\frac{1}{x}=\frac{1}{d-x}$$

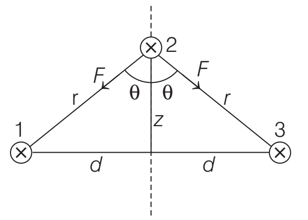
This equation givesz $x=\pm\frac{d}{\sqrt{3}}$

where magnetic field is zero.

(b) In this part, we change our coordinate axes system, just for better understanding.



There are three wires 1, 2 and 3 as shown in figure. If we displace the wire 2 towards the z-axis, then force of attraction per unit length between wires (1 and 2) and (2 and 3) will be given as



$$F = \frac{\mu_0}{2\pi}\frac{i^2}{r}$$

The components of F along x-axis will be cancelled out. Net resultant force will be towards negative z-axis (or mean position) and will be given by

$$F_{\text{net}} = \frac{\mu_0}{2\pi}\frac{i^2}{r}(2\cos\theta) = 2\left\{\frac{\mu_0}{2\pi}\frac{i^2}{r}\right\}\frac{z}{r}$$

$$F_{\text{net}} = \frac{\mu_0}{\pi}\frac{i^2}{(z^2+d^2)}z$$

If $z << d$, then

$$z^2 + d^2 = d^2 \text{ and } F_{\text{net}} = -\left(\frac{\mu_0}{\pi}\frac{i^2}{d^2}\right).z$$

Negative sign implies that F_{net} is restoring in nature

Therefore, $F_{\text{net}} \propto -z$

i.e. the wire will oscillate simple harmonically.

Let a be the acceleration of wire in this position and λ is the mass per unit length of this wire then

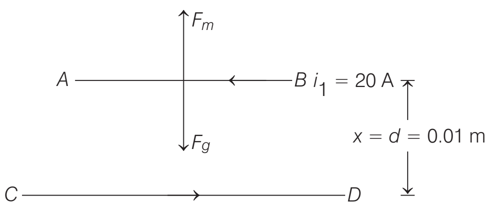
$$F_{\text{net}} = \lambda\,.\,a = -\left(\frac{\mu_0}{\pi}\frac{i^2}{d^2}\right)z \quad\text{or}\quad a = -\left(\frac{\mu_0 i^2}{\pi\lambda d^2}\right).z$$

$\therefore$ Frequency of oscillation

$$f = \frac{1}{2\pi}\sqrt{\frac{\text{acceleration}}{\text{displacement}}} = \frac{1}{2\pi}\sqrt{\frac{a}{z}}$$

$$= \frac{1}{2\pi}\frac{i}{d}\sqrt{\frac{\mu_0}{\pi\lambda}} \quad\text{or}\quad f = \frac{i}{2\pi d}\sqrt{\frac{\mu_0}{\pi\lambda}}$$

11. Let m be the mass per unit length of wire AB. At a height x above the wire CD, magnetic force per unit length on wire AB will be given by



$$F_m = \frac{\mu_0}{2\pi}\frac{i_1 i_2}{x} \quad \text{(upwards)} \qquad \ldots\text{(i)}$$

Weight per unit length of wire AB is

$$F_g = mg \qquad \text{(downwards)}$$

Here, m = mass per unit length of wire AB

At $x = d$, wire is in equilibrium i.e.,

$$F_m = F_g$$

or
$$\frac{\mu_0}{2\pi}\frac{i_1 i_2}{d} = mg$$

or
$$\frac{\mu_0}{2\pi}\frac{i_1 i_2}{d^2} = \frac{mg}{d} \qquad \ldots\text{(ii)}$$

When AB is depressed, x decreases therefore, F_m will increase, while F_g remains the same. Let AB is displaced by dx downwards.

Differentiating Eq. (i) w.r.t. x, we get

$$dF_m = -\frac{\mu_0}{2\pi}\frac{i_1 i_2}{x^2}.dx \qquad \ldots\text{(iii)}$$

i.e. restoring force, $F = dF_m \propto -dx$

Hence, the motion of wire is simple harmonic.

From Eqs. (ii) and (iii), we can write

$$dF_m = -\left(\frac{mg}{d}\right).dx \qquad (\because x = d)$$

$\therefore$ Acceleration of wire $a = -\left(\dfrac{g}{d}\right).dx$

Hence, period of oscillation

$$T = 2\pi\sqrt{\left|\frac{dx}{a}\right|}$$

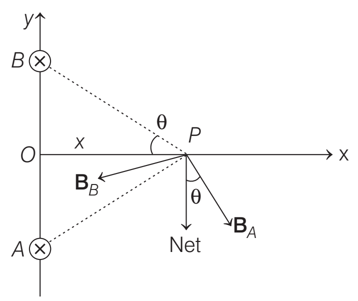
$$= 2\pi\sqrt{\frac{|\text{displacement}|}{|\text{acceleration}|}}$$

or
$$T = 2\pi\sqrt{\frac{d}{g}}$$

$$= 2\pi\sqrt{\frac{0.01}{9.8}}$$

or
$$T = 0.2 \text{ s}$$

12. (a) Let us assume a segment of wire OC at a point P, a distance x from the centre of length dx as shown in figure.



Magnetic field at P due to current in wires A and B will be in the directions perpendicular to AP and BP respectively as shown.

$$|\mathbf{B}| = \frac{\mu_0}{2\pi}\frac{I}{AP}$$

Therefore, net magnetic force at P will be along negative y-axis as shown

$$B_{\text{net}} = 2|\mathbf{B}|\cos\theta = 2\left(\frac{\mu_0}{2\pi}\right)\frac{I}{AP}\left(\frac{x}{AP}\right)$$

$$B_{\text{net}} = \left(\frac{\mu_0}{\pi}\right)\frac{I.x}{(AP)^2}$$

$$B_{\text{net}} = \frac{\mu_0}{\pi}\cdot\frac{Ix}{(a^2+x^2)}$$

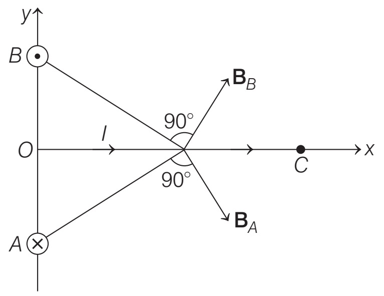
Therefore, force on this element will be

$$dF = I\left\{\frac{\mu_0}{\pi}\frac{Ix}{a^2+x^2}\right\}dx \qquad \text{(in negative } z\text{-direction)}$$

$\therefore$ Total force on the wire will be

$$F = \int_{x=0}^{x=L} dF = \frac{\mu_0 I^2}{\pi}\int_0^L \frac{xdx}{x^2+a^2}$$

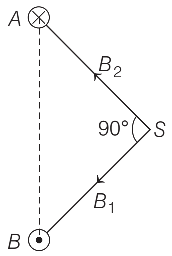
$$= \frac{\mu_0 I^2}{2\pi}\ln\left(\frac{L^2+a^2}{a^2}\right) \qquad \text{(in negative } z\text{-axis)}$$



Hence, $\mathbf{F} = -\frac{\mu_0 I^2}{2\pi}\ln\left(\frac{L^2+a^2}{a^2}\right)\hat{\mathbf{k}}$

(b) When direction of current in B is reversed, net magnetic field is along the current. Hence, force is zero.

13. (a) Direction of current at B should be perpendicular to paper outwards. Let current in this wire be i_B. Then,



$$\frac{\mu_0}{2\pi}\frac{i_A}{\left(2+\frac{10}{11}\right)} = \frac{\mu_0}{2\pi}\frac{i_B}{(10/11)}$$

or $$\frac{i_B}{i_A} = \frac{10}{32}$$

or $$i_B = \frac{10}{32}\times i_A = \frac{10}{32}\times 9.6 = 3\text{A}$$

(b) Since, $AS^2 + BS^2 = AB^2$

$\therefore \quad \angle ASB = 90°$

At S : B_1 = Magnetic field due to i_A

$$= \frac{\mu_0}{2\pi}\frac{i_A}{1.6}$$

$$= \frac{(2\times10^{-7})(9.6)}{1.6} = 12\times10^{-7}\ \text{T}$$

B_2 = Magnetic field due to i_B

$$= \frac{\mu_0}{2\pi}\frac{i_B}{1.2} = \frac{(2\times10^{-7})(3)}{1.2} = 5\times10^{-7}\ \text{T}$$

Since, B_1 and B_2 are mutually perpendicular. Net magnetic field at S would be

$$B = \sqrt{B_1^2 + B_2^2}$$

$$= \sqrt{(12\times10^{-7})^2 + (5\times10^{-7})^2}$$

$$= 13\times10^{-7}\ \text{T}$$

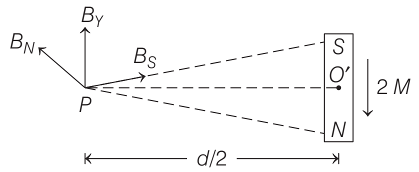
(c) Force per unit length on wire B

$$\frac{F}{l} = \frac{\mu_0}{2\pi}\frac{i_A i_B}{r} \qquad (r = AB = 2\text{ m})$$

$$= \frac{(2\times10^{-7})(9.6\times3)}{2} = 2.88\times10^{-6}\ \text{N/m}$$

Topic 4 Magnetic Dipole

1. Let $2l_1$ and $2l_2$ be the length of dipole X and Y, respectively. For dipole X, point P lies on its axial line. So, magnetic field strength at P due to X is



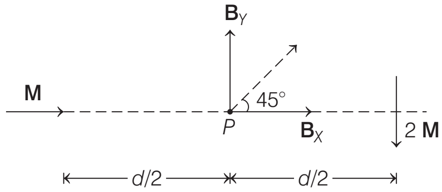
$$\mathbf{B}_X = \frac{\mu_0}{4\pi}\cdot\frac{2Mr}{(r^2-l_1^2)^2}, \text{ along } OP$$

Here, $r = \frac{d}{2}$

Also, $d >> l_1$

$$\Rightarrow \quad |\mathbf{B}_X| = \frac{\mu_0}{4\pi}\cdot\frac{2M(d/2)}{(d/2)^4} = \frac{\mu_0}{4\pi}\frac{2M}{(d/2)^3}$$

Similarly, for dipole Y, point P lies on its equatorial line. So, magnetic field strength at P due to Y is



$$\mathbf{B}_Y = \frac{\mu_0}{4\pi}\cdot\frac{2M}{(r^2+l_2^2)^{3/2}}, \text{ (along a line perpendicular to } O'P)$$

Here, $r = \frac{d}{2}$

Also, $d >> l_2$

$$\Rightarrow \quad |\mathbf{B}_Y| = \frac{\mu_0}{4\pi}\frac{2M}{(d/2)^3}$$

Thus, the resultant magnetic field due to X and Y at P is

$$\mathbf{B}_{\text{net}} = \mathbf{B}_X + \mathbf{B}_Y$$

Since, $|\mathbf{B}_Y| = |\mathbf{B}_X|$

Thus, the resultant magnetic field ($\mathbf{B}_{net}$) at P will be at 45° with the horizontal.

This means, direction of $\mathbf{B}_{net}$ and velocity of the charged particle is same.

$\therefore$ Force on the charged particle moving with velocity v in the presence of magnetic field which is

$$\mathbf{B} = q(\mathbf{v} \times \mathbf{B}) = q\,|\mathbf{v}||\mathbf{B}|\sin\theta$$

where, θ is the angle between $\mathbf{B}$ and $\mathbf{v}$.

According to the above analysis, we get

$$\theta = 0$$

$$\therefore \qquad \mathbf{F} = 0$$

Thus, magnitude of force on the particle at that instant is zero.

2. **Key Idea** Magnetic dipole moment of a current carrying loop is

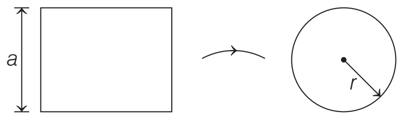
$m = IA(\text{A - m}^2)$

where, I = current in loop and A = area of loop.

Let the given square loop has side a, then its magnetic dipole moment will be

$$m = Ia^2$$

When square is converted into a circular loop of radius r,



Then, wire length will be same in both areas,

$$\Rightarrow \qquad 4a = 2\pi r \quad \Rightarrow \quad r = \frac{4a}{2\pi} = \frac{2a}{\pi}$$

Hence, area of circular loop formed is, $A' = \pi r^2$

$$= \pi\left(\frac{2a}{\pi}\right)^2 = \frac{4a^2}{\pi}$$

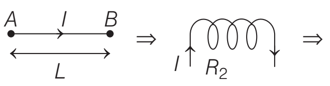
Magnitude of magnetic dipole moment of circular loop will be

$$m' = IA' = I\frac{4a^2}{\pi}$$

Ratio of magnetic dipole moments of both shapes is,

$$\frac{m'}{m} = \frac{I \cdot \frac{4a^2}{\pi}}{Ia^2} = \frac{4}{\pi} \quad \Rightarrow \quad m' = \frac{4m}{\pi}(\text{A-m})$$

3.

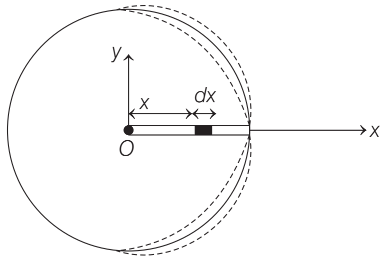


(R_2=radius of coil having N loops)

Key Idea A rotating charge constitutes a current. Hence, a rotating charged rod behaves like a current carrying coil.
If charge q rotates with a frequency n, then equivalent current is $I = qn$ and magnetic moment associated with this current is $M = IA$

where, A = area of coil or area swept by rotating rod.

Let dq be the charge on dx length of rod at a distance x from origin as shown in the figure below.



The magnetic moment dm of this portion dx is given as

$$dm = (dI)\,A$$

$$dm = ndqA \qquad [\because I = qn, \ \therefore dI = n.dq]$$

$$= n\,\rho\,dx\,A$$

where, ρ = charge density of rod $= \rho_0 \dfrac{x}{l}$.

$$\text{So,} \qquad dm = \frac{n\rho_0 x\,dx\,\pi x^2}{l} = \frac{\pi\,n\,\rho_0}{l}\cdot x^3 \cdot dx$$

Total magnetic moment associated with rotating rod is sum of all the magnetic moments of such differentiable elements of rod.

So, magnetic moment associated with complete rod is

$$M = \int_{x=0}^{x=l} dm$$

$$= \int_0^l \frac{\pi\,n\,\rho_0}{l}\cdot x^3\,dx = \frac{\pi\,n\,\rho_0}{l}\cdot\int_0^l x^3\,dx$$

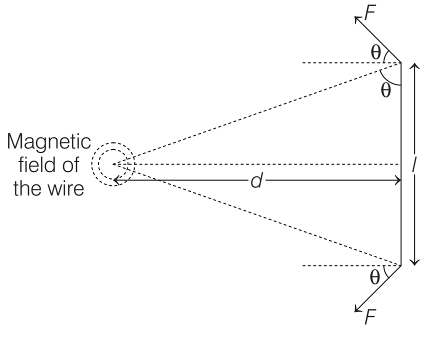
$$= \frac{\pi\,n\,\rho_0}{l}\left[\frac{x^4}{4}\right]_0^l = \frac{\pi\,n\,\rho_0\,l^3}{4}$$

at $\qquad x = l$

$$\rho = \rho_0$$

$$\therefore \qquad M = \frac{\pi}{4}n\rho l^3$$

4. In the given condition, the current-carrying loop is at a very large distance from the long current-carrying conducting wire. Thus, it can be considered as a dipole (a magnet with north pole facing in upward direction and south in the downward direction). Suppose the effective length of this dipole be 'l'. Thus, the top view of the condition can be shown in the figure given below.



Now, the net force on the loop (i.e. at the two poles) due to the wire is given as,

$$F_{net} = 2F\cos\theta = 2mB\cos\theta$$

where, m is the pole strength.

From the figure, we have

$$\cos\theta = \frac{\frac{l}{2}}{\sqrt{d^2 + \frac{l^2}{4}}}$$

$$\Rightarrow \quad F_{net} = \frac{2mB\,l}{2\sqrt{d^2 + \frac{l^2}{4}}} \quad \ldots(i)$$

Since, the magnetic moment of a loop of radius r is

$$M = IA = I\pi r^2 = ml \quad \ldots(ii)$$

and magnetic field due to a straight infinitely long current-carrying conductor at a distance x is

$$B = \frac{\mu_0 I'}{2\pi x} \quad \ldots(iii)$$

$\therefore$ Using Eqs. (ii) and (iii), rewriting Eq. (i), we get

$$F_{net} = \frac{I\pi a^2 \mu_0 I'}{2\pi\left(\sqrt{d^2 + \frac{l^2}{4}}\right)^2}$$

$$\Rightarrow \quad F_{net} \propto \frac{a^2}{\left(d^2 + \frac{l^2}{4}\right)}$$

l can be neglected as the loop is kept at very large distance.

or $\quad F_{net} \propto \left(\frac{a}{d}\right)^2$

5. $m = I \times \pi R^2$

$$2m = I \times \pi (R')^2 \quad \Rightarrow \quad R' = \sqrt{2}R$$

$$B = \frac{\mu_0 I}{2\pi R} \Rightarrow B \propto \frac{1}{R} \Rightarrow \frac{B_1}{B_2} = \frac{R'}{R} = \sqrt{2}$$

6. Direction of magnetic dipole moment **M** is given by screw law and this is perpendicular to plane of loop.

In stable equilibrium position, angle between **M** and **B** is 0° and in unstable equilibrium this angle is 180°.

7. Area of the given loop is

A = (area of two circles of radius $\frac{a}{2}$ and area of a square of side a)

$$= 2\pi\left(\frac{a}{2}\right)^2 + a^2 = \left(\frac{\pi}{2} + 1\right)a^2$$

$$|\mathbf{M}| = IA = \left(\frac{\pi}{2} + 1\right)a^2 I$$

From screw law direction of **M** is outwards or in positive z-direction.

$$\therefore \quad \mathbf{M} = \left(\frac{\pi}{2} + 1\right)a^2\, I\,\hat{\mathbf{k}}$$

8. $U = -\mathbf{M}\mathbf{B} = -MB\cos\theta$

Here, **M** = magnetic moment of the loop

θ = angle between **M** and **B**

U is maximum when $\theta = 180°$ and minimum when $\theta = 0°$. So, as θ decreases from 180° to 0°, its PE also decreases.

9. Ratio of magnetic moment and angular momentum is given by

$$\frac{M}{L} = \frac{q}{2m}$$

which is a function of q and m only. This can be derived as follows,

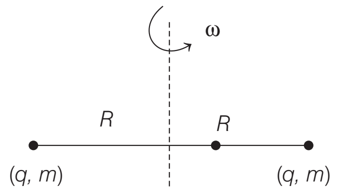
$$M = i\,A = (q\,f).(\pi r^2)$$

$$= (q)\left(\frac{\omega}{2\pi}\right)(\pi r^2) = \frac{q\omega r^2}{2}$$

and $\quad L = I\omega = (mr^2\omega)$

$$\therefore \quad \frac{M}{L} = \frac{q\frac{\omega r^2}{2}}{mr^2\omega} = \frac{q}{2m}$$

10. Current, i = (frequency) (charge) $= \left(\frac{\omega}{2\pi}\right)(2q) = \frac{q\omega}{\pi}$



Magnetic moment, $M = (i)(A) = \left(\frac{q\omega}{\pi}\right)(\pi R^2) = (q\omega R^2)$

Angular momentum, $L = 2I\omega = 2(mR^2)\,\omega$

$$\therefore \quad \frac{M}{L} = \frac{q\omega R^2}{2(mR^2)\omega} = \frac{q}{2m}$$

11. Magnetic force on a current carrying loop in uniform magnetic field is zero.

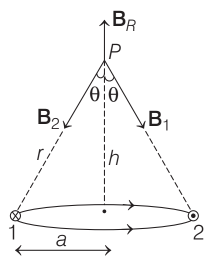
12. $\mathbf{B}_R$ = **B** due to ring

$\mathbf{B}_1$ = **B** due to wire-1

$\Rightarrow$ $\mathbf{B}_2$ = **B** due to wire-2

In magnitudes,

$$\mathbf{B}_1 = \mathbf{B}_2 = \frac{\mu_0 I}{2\pi r}$$



Resultant of $\mathbf{B}_1$ and $\mathbf{B}_2$

$$= 2\mathbf{B}_1\cos\theta = 2\left(\frac{\mu_0 I}{2\pi r}\right)\left(\frac{h}{r}\right) = \frac{\mu_0 I h}{\pi r^2}$$

$$\mathbf{B}_R = \frac{\mu_0 I R^2}{2(R^2+x^2)^{3/2}} = \frac{2\mu_0 I\pi a^2}{4\pi r^3}$$

As, $R = a, x = h$ and $a^2 + h^2 = r^2$

For zero magnetic field at P, $\frac{\mu_0 I h}{\pi r^2} = \frac{2\mu_0 I\pi a^2}{4\pi r^3}$

$\Rightarrow \quad \pi a^2 = 2rh \quad \Rightarrow h \approx 1.2a$

13. Magnetic field at mid-point of two wires

$= 2$ (magnetic field due to one wire)

$$= 2\left[\frac{\mu_0}{2\pi}\frac{I}{d}\right] = \frac{\mu_0 I}{\pi d} \otimes$$

Magnetic moment of loop $M = IA = I\pi a^2$

Torque on loop $= MB \sin 150°$

$$= \frac{\mu_0 I^2 a^2}{2d}$$

14. $\mathbf{F}_{BA} = 0$, because magnetic lines are parallel to this wire.

$\mathbf{F}_{CD} = 0$, because magnetic lines are anti-parallel to this wire.

$\mathbf{F}_{CB}$ is perpendicular to paper outwards and $\mathbf{F}_{AD}$ is perpendicular to paper inwards. These two forces (although calculated by integration) cancel each other but produce a torque which tend to rotate the loop in clockwise direction about an axis OO'.

15. Equivalent current $i = qf$

and magnetic moment $m = (i\pi r^2) = \pi q f r^2$

Substituting the values, we have

$$M = (\pi)(0.5\times 10^{-10})^2 (10^{16})(1.6\times 10^{-19})$$

$$= 1.26\times 10^{-23} \text{ A-m}^2$$

16. Let R be the radius of circle. Then,

$$2\pi R = L \quad \text{or} \quad R = \frac{L}{2\pi}$$

$$M = iA = i\pi R^2 = \frac{L^2 i}{4\pi}$$

17. Magnetic moment of the loop, $\mathbf{M} = (iA)\hat{\mathbf{k}} = (I_0 L^2)\hat{\mathbf{k}}$

Magnetic field, $\mathbf{B} = (B\cos 45°)\hat{\mathbf{i}} + (B\sin 45°)\hat{\mathbf{j}}$

$$= \frac{B}{\sqrt{2}}(\hat{\mathbf{i}} + \hat{\mathbf{j}})$$

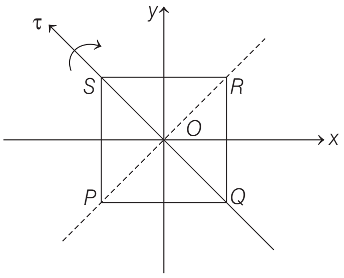
(a) Torque acting on the loop, $\tau = \mathbf{M}\times\mathbf{B}$

$$= (I_0 L^2 \hat{\mathbf{k}}) \times \left[\frac{B}{\sqrt{2}}(\hat{\mathbf{i}} + \hat{\mathbf{j}})\right]$$

$$\therefore \quad \tau = \frac{I_0 L^2 B}{\sqrt{2}}(\hat{\mathbf{j}} - \hat{\mathbf{i}})$$

or $\quad |\tau| = I_0 L^2 B$

(b) Axis of rotation coincides with the torque and since torque is in $\hat{\mathbf{j}} - \hat{\mathbf{i}}$ direction or parallel to QS. Therefore, the loop will rotate about an axis passing through Q and S as shown in the figure.



Angular acceleration, $\alpha = \frac{|\tau|}{I}$

where, I = moment of inertia of loop about QS.

$$I_{QS} + I_{PR} = I_{ZZ}$$

(From theorem of perpendicular axis)

But $\quad I_{QS} = I_{PR}$

$$\therefore \quad 2I_{QS} = I_{ZZ} = \frac{4}{3}ML^2$$

$$I_{QS} = \frac{2}{3}ML^2$$

$$\therefore \quad \alpha = \frac{|\tau|}{I} = \frac{I_0 L^2 B}{2/3 ML^2} = \frac{3}{2}\frac{I_0 B}{M}$$

$\therefore$ Angle by which the frame rotates in time Δt is

$$\theta = \frac{1}{2}\alpha(\Delta t)^2 \text{ or } \theta = \frac{3}{4}\frac{I_0 B}{M}.(\Delta t)^2$$

18. In ground state ($n = 1$) according to Bohr's theory

$$mvR = \frac{h}{2\pi} \text{ or } v = \frac{h}{2\pi mR}$$

Now, time period,

$$T = \frac{2\pi R}{v} = \frac{2\pi R}{h/2\pi mR} = \frac{4\pi^2 mR^2}{h}$$

Magnetic moment, $M = iA$

where, $\quad i = \frac{\text{charge}}{\text{time period}} = \frac{e}{\frac{4\pi^2 mR^2}{h}}$

$$= \frac{eh}{4\pi^2 mR^2} \quad \text{and} \quad A = \pi R^2$$

$$\therefore \quad M = (\pi R^2)\left(\frac{eh}{4\pi^2 mR^2}\right) \text{ or } M = \frac{eh}{4\pi m}$$

Direction of magnetic moment $\mathbf{M}$ is perpendicular to the plane of orbit.

(b) $\quad \tau = \mathbf{M}\times\mathbf{B} \Rightarrow |\tau| = MB\sin\theta$

where, θ is the angle between $\mathbf{M}$ and $\mathbf{B}$

$\theta = 30°$

$$\therefore \quad \tau = \left(\frac{eh}{4\pi m}\right)(B)\sin 30°$$

$$\therefore \quad \tau = \frac{ehB}{8\pi m}$$

The direction of τ is perpendicular to both $\mathbf{M}$ and $\mathbf{B}$.

Topic 5 Magnetism

1. Given, at first place, angle of dip, $\theta_1 = 45^\circ$

Time period, $T_1 = \frac{60}{30} = 2\text{s}$

At second place, angle of dip, $\theta_2 = 30^\circ$

Time period, $T_2 = \frac{60}{40} = \frac{3}{2}\text{ s}$

Now, at first place,

$$B_{H_1} = B_1 \cos\theta_1 = B_1 \cos 45^\circ = \frac{B_1}{\sqrt{2}} \quad \ldots\text{(i)}$$

and at second place,

$$B_{H_2} = B_2 \cos\theta_2 = B_2 \cos 30^\circ = \frac{\sqrt{3}}{2} B_2 \quad \ldots\text{(ii)}$$

Also, time period of a magnetic needle is given by

$$T = 2\pi\sqrt{\frac{I}{MB_H}} \quad \ldots\text{(iii)}$$

$$\therefore \quad T \propto \sqrt{\frac{1}{B_H}} \text{ or } \frac{T_1}{T_2} = \sqrt{\frac{B_{H_2}}{B_{H_1}}} \quad \ldots\text{(iv)}$$

By putting the values from Eqs. (i) and (ii) into Eq. (iv), we get

$$\frac{2}{\frac{3}{2}} = \sqrt{\frac{\frac{\sqrt{3}\,B_2}{2}}{\frac{B_1}{\sqrt{2}}}} \text{ or } \left(\frac{4}{3}\right)^2 = \frac{\sqrt{3}\times\sqrt{2}\,B_2}{2B_1}$$

$$\Rightarrow \quad \frac{B_1}{B_2} = \frac{\sqrt{3}\times\sqrt{2}}{2} \times \frac{9}{16}$$

$$\Rightarrow \quad \frac{B_1}{B_2} = \frac{9\sqrt{3}}{16\sqrt{2}}$$

$$\Rightarrow \quad \frac{B_1}{B_2} = \frac{9\times 1.732}{16\times 1.414} = \frac{15.588}{22.624}$$

$$\Rightarrow \quad \frac{B_1}{B_2} = 0.689 \approx 0.7\text{ T}$$

2. In a moving coil galvanometer in equilibrium, torque on coil due to current is balanced by torque of torsion band.

As, torque on coil,

$$\tau = \mathbf{M}\times\mathbf{B} = NIAB\sin\alpha$$

where, B = magnetic field strength,

I = current,

N = number of turns of coil

Since, plane of the coil is parallel to the field.

$$\therefore \quad \alpha = 90^\circ \Rightarrow \tau = NIBA$$

Torque of torsion band, $T = k\theta$

where, k = torsion constant of torsion band

and θ = deflection of coil in radians or angle of twist of restoring torque.

$$\therefore \quad BINA = k\theta \text{ or } B = \frac{k\theta}{INA} \quad \ldots\text{(i)}$$

Here, $k = 10^{-6}\text{ N-m/rad}$,

$I = 1\times 10^{-3}\text{ A}$,

$N = 175$,

$A = 1\text{ cm}^2 = 1\times 10^{-4}\text{ m}^2$

$\theta = 1^\circ = \frac{\pi}{180}\text{ rad}$

Substituting values in Eq. (i), we get

$$B = \frac{10^{-6}\times 22}{1\times 10^{-3}\times 175\times 7\times 180\times 10^{-4}}$$

$$= 0.998\times 10^{-3} \simeq 10^{-3}\text{ T}$$

3. From Curie's law for paramagnetic substance, we have

Magnetic susceptiblity $\chi \propto \frac{1}{T}$

$$\therefore \quad \frac{\chi_2}{\chi_1} = \frac{T_1}{T_2} \Rightarrow \chi_2 = \frac{\chi_1\cdot T_1}{T_2}$$

$$\Rightarrow \quad \chi_2 = \frac{2.8\times 10^{-4}\times 350}{300} = 3.267\times 10^{-4}$$

4. Given, side of cube = 1cm = 10^{-2} m

$\therefore$ Volume, $V = 10^{-6}\text{m}^3$

Dipole moment, $M = 20\times 10^{-6}$ J/T

Applied magnetic intensity, $H = 60\times 10^3$ A/m

Intensity of magnetisation

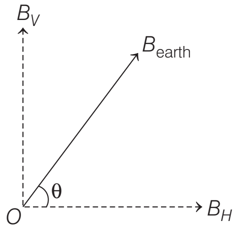
$$I = \frac{M}{V} = \frac{20\times 10^{-6}}{10^{-6}} = 20\text{ A/m}$$

Now, magnetic susceptibility χ is

$$\chi = \frac{\text{Intensity of magnetisation}}{\text{Applied magnetic intensity}} = \frac{I}{H} = \frac{20}{60\times 10^3}$$

$$\Rightarrow \quad \chi = \frac{1}{3}\times 10^{-3} = 3.33\times 10^{-4}$$

5. Without applied forces, (in equilibrium position) the needle will stay in the resultant magnetic field of earth. Hence, the dip 'θ' at this place is 45° (given).



We know that, horizontal and vertical components of earth's magnetic field (B_H and B_V) are related as

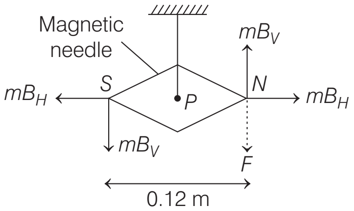
$$\frac{B_V}{B_H} = \tan\theta$$

Here, $\theta = 45^\circ$ and $B_H = 18\times 10^{-6}$T

$$\Rightarrow \quad B_V = B_H \tan 45^\circ$$

$$\Rightarrow \quad B_V = B_H = 18\times 10^{-6}\text{T} \quad (\because \tan 45^\circ = 1)$$

Now, when the external force F is applied, so as to keep the needle stays in horizontal position is shown below,



Taking torque at point P, we get

$$mB_V \times 2l = Fl$$

$$\therefore \quad F = 2 \times mB_V$$

Substituting the given values, we get

$$= 2 \times 1.8 \times 18 \times 10^{-6}$$

$$= 6.48 \times 10^{-5} = 6.5 \times 10^{-5} \text{ N}$$

6. Coercivity of a bar magnet is the value of magnetic field intensity (H) that is needed to reduce magnetisation to zero.

Since, for a solenoid magnetic induction is given as,

$$B = \mu_0 nI \quad \text{...(i)}$$

where, n is the number of turns (N) per unit, length (l) and I is the current.

Also, $$B = \mu_0 H \quad \text{...(ii)}$$

$\therefore$ From Eqs. (i) and (ii), we get

$$\mu_0 nI = \mu_0 H \text{ or } H = nI = \frac{N}{l} I$$

Substituting the given values, we get

$$H = \frac{100}{0.2} \times 5.2 = 2600 \text{ A/m}$$

Thus, the value of coercivity of the bar magnet is 2600 A/m.

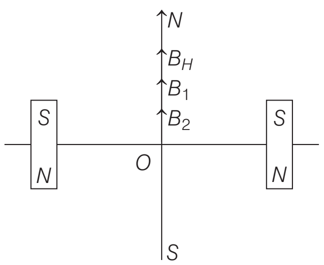
7. We need high retentivity and high coercivity for electromagnets and small area of hysteresis loop for transformers.

8. For solenoid, the magnetic field needed to be magnetised the magnet. $B = \mu_0 nI$

$$\text{where, } n = 100, l = 10\text{cm} = \frac{10}{100}\text{m} = 0.1 \text{ m}$$

$$\Rightarrow \quad 3 \times 10^3 = \frac{100}{0.1} \times I \Rightarrow I = 3\text{A}$$

9. $B_{\text{net}} = B_1 + B_2 + B_H$



$$B_{\text{net}} = \frac{\mu_0}{4\pi} \frac{(M_1 + M_2)}{r^3} + B_H$$

$$= \frac{10^{-7} (1.2 + 1)}{(0.1)^3} + 3.6 \times 10^{-5}$$

$$= 2.56 \times 10^{-4} \text{ Wb/m}^2$$

10. Magnetic lines form closed loop. Inside magnet, these are directed from South to North-pole.

11. In non-uniform magnetic field, the needle will experience both a force and a torque.

12. $c\theta = BINA \Rightarrow \theta = \left(\frac{BNA}{c}\right) I$

Using iron core, value of magnetic field increases. So, deflection increases for same current. Hence, sensitivity increases.

Soft iron can be easily magnetised or demagnetised.

13. $\tau = MB = ki \Rightarrow k = \frac{MB}{i} = \frac{(NiA)B}{i} = NBA$

(b) $\tau = k \cdot \theta = BiNA$

$$\therefore \quad k = \frac{2\,BiNA}{\pi} \quad (\text{as } \theta = \pi/2)$$

(c) $\tau = BiNA$

$$\text{or} \quad \int_0^t \tau\, dt = BNA \int_0^t i\, dt$$

$$I\omega = BNAQ$$

$$\text{or} \quad \omega = \frac{BNAQ}{I} \quad \text{...(i)}$$

At maximum deflection, whole kinetic energy (rotational) will be converted into potential energy of spring.

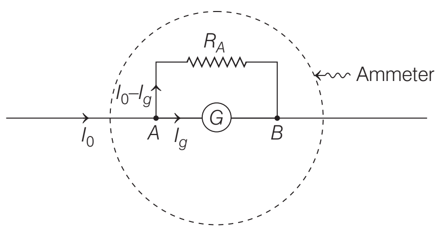
$$\text{Hence,} \quad \frac{1}{2} I\omega^2 = \frac{1}{2} k\theta^2_{\max}$$

Substituting the values, we get

$$\theta_{\max} = Q\sqrt{\frac{BN\pi A}{2I}}$$

Topic 6 Miscellaneous Problems

1. To use galvanometer as an ammeter, a low resistance in parallel is used.



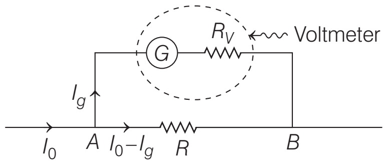
In ammeter, if I_g = full scale deflection current, then equating potential drops across points marked AB, we have

$$V_{AB} = I_g G = (I_0 - I_g) R_A$$

$$\Rightarrow \qquad R_A = \frac{I_g G}{I_0 - I_g} \qquad \text{...(i)}$$

Here, G = resistance of galvanometer coil.

When a galvanometer is used as a voltmeter, a high resistance (R_V) in series is used.



Equating potential across point AB,

$$V_{AB} = (G + R_V) I_g$$

But $\qquad V_{AB} = I_0 G \qquad$ (given)

So, $\qquad I_0 G = (G + R_V) I_g$

$$\Rightarrow \qquad R_V = \frac{(I_0 - I_g) G}{I_g} \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we have

$$\frac{R_A}{R_V} = \frac{\left(\dfrac{I_g G}{I_0 - I_g}\right)}{\dfrac{(I_0 - I_g) G}{I_g}} = \frac{I_g^2}{(I_0 - I_g)^2}$$

and $\qquad R_A \times R_V = \dfrac{I_g G}{(I_0 - I_g)} \times \dfrac{(I_0 - I_g) G}{I_g} = G^2$

2. Given, divisions in scale of galvanometer, $n = 50$

Sensitivity of galvanometer,

$$\frac{I_g}{n} = 20\,\mu\text{A / division}$$

$\therefore$ Current in galvanometer,

$$I_g = \frac{I_g}{n} \times n = 20\mu\text{A} \times 50$$

$\Rightarrow \qquad I_g = 1000\,\mu\text{A} = 1 \text{ mA}$

Now, for R, it should be converted into 2V voltmeter.

$\therefore \qquad V_1 = I_g (R_1 + G)$

$$2 = 10^{-3}(R_1 + 100)$$

$\Rightarrow \qquad 2000 = R_1 + 100$

$\therefore \qquad R_1 = 1900\,\Omega \qquad$ ...(i)

For R_2, it should be converted into 10V voltmeter.

$\therefore \qquad V_2 = I_g [(R_1 + R_2) + G]$

$\Rightarrow \qquad 10 = 10^{-3} [(R_1 + R_2) + 100]$

$\Rightarrow \qquad 10000 = R_1 + R_2 + 100 = 2000 + R_2$

$\therefore \qquad R_2 = 8000\,\Omega \qquad$ ...(ii)

For R_3, it should be converted into 20V voltmeter.

$\therefore \qquad V_3 = I_g[(R_1 + R_2 + R_3 + G]$

$\Rightarrow \qquad 20 = 10^{-3}[1900 + 8000 + R_3 + 100]$

$\Rightarrow \qquad 20000 = R_3 + 10000$

$\therefore \qquad R_3 = 10000\,\Omega \qquad$ ...(iii)

From Eqs. (i), (ii) and (iii), it is clear that option (c) is correct.

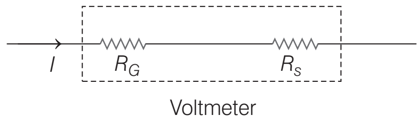
3. Given data,

$$I = 10^{-4} \text{ A},$$

$$R_S = 2 \text{ m}\Omega = 2 \times 10^6\,\Omega,$$

$$V_{\max} = 5 \text{ V}$$

Let internal resistance of galvanometer is R_G.



Then, $\qquad I \times R_S + I \times R_G = V_{\max}$

$\Rightarrow \qquad 2 \times 10^6 \times 10^{-4} + 10^{-4} \times R_G = 5$

$\Rightarrow \qquad 10^{-4} R_G = 5 - 200 = -195$

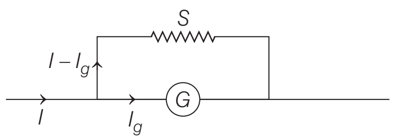
or $\qquad R_G = -195 \times 10^4\,\Omega$

Resistance cannot be negative.

$\therefore$ No option is correct

4. **Key Idea** An ammeter is a type of galvanometer with a shunt connected in parallel to the galvanometer.

Ammeter circuit is shown in the figure below,



So, $\qquad I_g G = (I - I_g) S$

Here, $\qquad I_g = 0.002$ A,

$\qquad I = 0.5$A,

$\qquad G = 50\,\Omega$

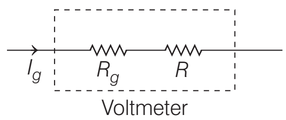
So, shunt resistance required is

$$S = \frac{I_g G}{I - I_g} = \frac{0.002 \times 50}{(0.5 - 0.002)} \approx 0.2\,\Omega$$

5. Given, resistance of galvanometer, $R_g = 50\Omega$

Current, $I_g = 4 \text{ mA} = 4 \times 10^{-3}$ A

Resistance used in converting a galvanometer in voltmeter, $R = 5 \text{ k}\Omega = 5 \times 10^3\,\Omega$



$\therefore$ Maximum current in galvanometer is

$$I_g = \frac{E}{R + R_g}$$

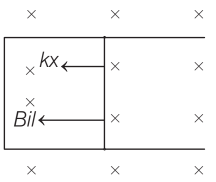
$\therefore \quad E = I_g(R + R_g)$

$= 4 \times 10^{-3} \times (5 \times 10^3 + 50)$

$= 5050 \times 4 \times 10^{-3}$

$= 20.2\,V \simeq 20\,V$

6. There are two forces on slider



Spring force $= kx$

where, $\quad k =$ spring constant.

As the slider is kept in a uniform magnetic field $B = 0.1\,T$, hence it will experience a force, i.e.

Magnetic force $= Bil$

where, $\quad l =$ length of the strip.

Now, using $\quad F_{net} = ma$

We have, $\quad (-kx) + (-Bil) = ma$

$\Rightarrow \quad -kx - Bil - ma = 0$

$\Rightarrow \quad -kx - \frac{B^2 l^2}{R} \cdot v - ma = 0 \quad \left[\because i = \frac{Blv}{R}\right]$

and acceleration, $a = \frac{d^2x}{dt^2}$

Hence, the modified equation becomes

$$\Rightarrow \quad \frac{md^2x}{dt^2} + \frac{B^2l^2}{R}\left(\frac{dx}{dt}\right) + kx = 0$$

This is the equation of damped simple harmonic motion.

So, amplitude of oscillation varies with time as

$$A = A_0 e^{-\frac{B^2l^2}{2Rm} \cdot t}$$

Now, when amplitude is $\frac{A_0}{e}$, then

$$\frac{A_0}{e} = \frac{A_0}{e^{\frac{B^2l^2}{2Rm} \cdot t}} \quad \text{(as given)}$$

$$\Rightarrow \quad \left(\frac{B^2l^2}{2Rm}\right)t = 1 \quad \text{or} \quad t = \frac{2Rm}{B^2l^2}$$

According to the question, magnetic field $B = 0.1\,T$, mass of strip $m = 50 \times 10^{-3}$ kg,

resistance $R = 10\,\Omega$, $l = 10\,cm = 10 \times 10^{-2}$ m

$$\therefore \quad t = \frac{2Rm}{B^2l^2} = \frac{2 \times 10 \times 50 \times 10^{-3}}{(0.1)^2 \times (10 \times 10^{-2})^2}$$

$$= \frac{1}{10^{-4}} = 10000\,s$$

Given, spring constant, $k = 0.5\,Nm^{-1}$

Also, time period of oscillation is

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{50 \times 10^{-3}}{0.5}} = \frac{2\pi}{\sqrt{10}} \approx 2\,s$$

So, number of oscillations is $N = \frac{t}{T} = \frac{10000}{2} = 5000$

7. Mean radiation intensity is

$$I = \varepsilon_0 c E_{rms}^2$$

$$= \varepsilon_0 c (cB_{rms})^2 \quad \left[\because \frac{E_{rms}}{B_{rms}} = c\right]$$

$$= \varepsilon_0 c^3 B_{rms}^2$$

$$\Rightarrow \quad B_{rms} = \sqrt{\frac{I}{\varepsilon_0 c^3}}$$

Substituting the given values, we get

$$= \sqrt{\left(\frac{10^8}{8.85 \times 10^{-12} \times (3 \times 10^8)^3}\right)}$$

$$= \sqrt{\left(\frac{10^8}{8.85 \times 27 \times 10^{12}}\right)} \approx \sqrt{(10^{-8})} \approx 10^{-4}\,T$$

8. The time-period of oscillations made by a magnet of magnetic moment M, moment of inertia I, placed in a magnetic field is given by

$$T = 2\pi\sqrt{\frac{I}{MB}} \quad \ldots (i)$$

For the hoop, let us assume its moment of inertia I_h and magnetic moment M_h, then its time period will be

$$T_h = 2\pi\sqrt{\frac{I_h}{M_h B}} \quad \ldots (ii)$$

Similarly, for solid cylinder, time period is,

$$T_c = 2\pi\sqrt{\frac{I_c}{M_c B}} \quad \ldots (iii)$$

Dividing Eq. (ii) by Eq. (iii), we get

$$\frac{T_h}{T_c} = \sqrt{\frac{I_h M_c}{M_h I_c}} \quad \ldots (iv)$$

Now, it is given that,

$$M_h = 2M_c$$

and we know that, moment of inertia of hoop $I_h = mR^2$ and moment of inertia of solid

cylinder $I_c = \frac{1}{2}mR^2$

Substituting these values in Eq. (iv), we get

$$\frac{T_h}{T_c} = \sqrt{\frac{mR^2 \times M_c}{\frac{1}{2}mR^2 \times 2M_c}} = 1$$

$$\Rightarrow \quad T_h = T_c$$

9. Work done in reversing dipole is

$$W = 2\,MB$$

where, M = magnetic dipole moment

$$= 10^{-2}\ \text{A-m}^2$$

and B = external field

$$= B\cos\omega t = 1\times\cos(0.125\times 1)$$

$$= \cos(7^\circ) = 0.992$$

Substituting these values, we get,

$$W = 2\times 10^{-2}\times 0.992$$

$$= 0.0198\ \text{J}$$

which is nearest to 0.014 J

10. Since, the magnetic field is dependent on time, so the net charge flowing through the loop will be given as

$$Q = \frac{\text{change in magnetic flux, } \Delta\phi_B}{\text{resistance, } R}$$

As, $\Delta\phi_B = \mathbf{B}\,\mathbf{A} = BA\cos\theta$

where, A is the surface area of the loop and 'θ' is an angle between B and A.

Here, $\theta = 0 \Rightarrow \Delta\phi_B = BA$

$\therefore$ For the time interval, $t = 0$ ms to $t = 10$ ms,

$$Q = \frac{\Delta\phi_B}{R}$$

$$= \frac{A}{R}(B_{f\ \text{at}\ 0.01\ \text{s}} - B_{i\ \text{at}\ 0\ \text{s}})$$

Substituting the given values, we get

$$= \frac{3.5\times 10^{-3}}{10}[0.4\sin(0.5\pi) - 0.4\ \sin 0]$$

$$= 3.5\times 10^{-4}(0.4\sin\pi/2)$$

$$= 1.4\times 10^{-4}\,\text{C} = 14\ \text{mC}$$

11. Given,

Area of the rectangular coil, $A = 5\,\text{cm}\times 2.5\,\text{cm}$

$$\Rightarrow \quad A = 12.5\ \text{cm}^2 = 12.5\times 10^{-4}\ \text{m}^2$$

Number of turns, $N = 100$ turns

Current through the coil, $I = 3$ A

Magnetic field applied, $B = 1$ T

Angle between the magnetic field and area vector of the coil, $\theta = 45^\circ$

As we know that, when a coil is tilted by an angle θ in the presence of some external magnetic field, then the net torque experienced by the coil is,

$$\tau = \mathbf{M}\times\mathbf{B} = NI(\mathbf{A}\times\mathbf{B}) = NIAB\sin\theta$$

Substituting the given values, we get

$$\tau = 100\times 3\times 12.5\times 10^{-4}\times 1\times\sin 45^\circ$$

$$\tau = 0.707\times 100\times 3\times 12.5\times 10^{-4}\ \text{N-m}$$

$$= 2.651\times 10^{-1}\ \text{N-m}$$

$$\approx 0.27\ \text{N-m}$$

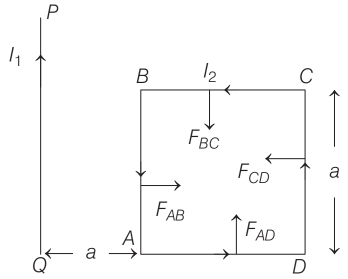
12. **Key Idea** Net force experienced by two wires separated by same distance is attractive, if current flow in them in same direction. However, this force is repulsive in nature, if current in them flows in opposite direction.

Force on a wire 1 in which current I_1 is flowing due to another wire 2 which are separated by a distance r is given as

$$\mathbf{F} = I_1(l\times\mathbf{B}_2)$$

or $$F = \frac{\mu_0 I_1 I_2}{2\pi r}\cdot l\sin\theta \qquad \left[\because \mathbf{B}_2 = \frac{\mu_0 I_2}{2\pi r}\right]$$

Thus, the given square loop can be drawn as shown below



$$F_{AB} = \frac{\mu_0 I_1}{2\pi a}\cdot I_2 a \text{ (away from wire } PQ\text{)}$$

$$F_{BC} = F_{AD} = 0 \qquad [\because \theta = 0^\circ]$$

$$F_{CD} = \frac{\mu_0 I_1}{2\pi(2a)}\cdot I_2 a$$

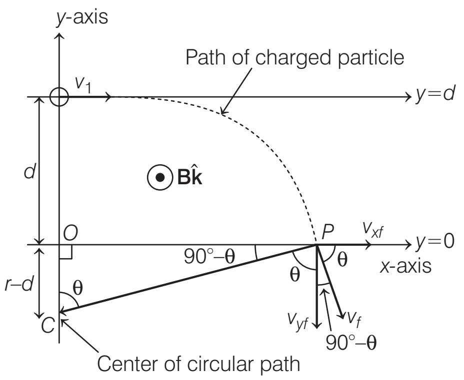
$$= \frac{\mu_0 I_1}{4\pi a}\cdot I_2 a \text{(towards the wire } PQ\text{)}$$

$$\therefore \quad F_{\text{net}} = F_{AB} - F_{CD}$$

$$= \frac{\mu_0 I_1 I_2}{2\pi} - \frac{\mu_0 I_1 I_2}{4\pi} \qquad \text{(away from wire)}$$

$$= \frac{\mu_0 I_1 I_2}{4\pi} \qquad \text{(repulsive in nature)}$$

13. Situation given in question is shown below;



Path taken by particle of charge 'q' and mass 'm' is a circle of radius r where,

$$r = \frac{mv}{Bq}$$

Here final velocity

$$\mathbf{v}_f = \mathbf{v}_x\hat{\mathbf{i}} + \mathbf{v}_{yf}(-\hat{\mathbf{j}}) = v\cos 60\,\hat{\mathbf{i}} - v\sin 60^\circ\,\hat{\mathbf{j}}$$

$$= v\left(\frac{1}{2}\hat{\mathbf{i}} - \frac{\sqrt{3}}{2}\hat{\mathbf{j}}\right)$$

So, change of velocity of charged particle is

$$\Delta\mathbf{v} = \mathbf{v}_f - \mathbf{v}_i = v\left(\frac{1}{2}\hat{\mathbf{i}} - \frac{\sqrt{3}}{2}\hat{\mathbf{j}}\right) - v\hat{\mathbf{i}}$$

$$= -v\left(\frac{1}{2}\hat{\mathbf{i}} + \frac{\sqrt{3}}{2}\hat{\mathbf{j}}\right)$$

It t = time taken by charged particle to cross region of magnetic field then,

$$t = \frac{\text{distance } OP}{\text{speed in direction } OP}$$

$$= \frac{r\times\frac{\sqrt{3}}{2}}{v} = \frac{\frac{mv}{Bq}\times\frac{\sqrt{3}}{2}}{v} = \frac{\sqrt{3}m}{2Bq}$$

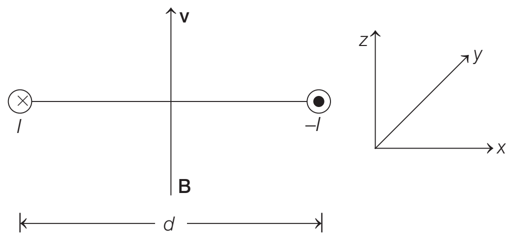
So, acceleration of charged particle at the point its emergence is;

$$\text{Acceleration, } \mathbf{a} = \frac{\Delta\mathbf{v}}{\Delta t} = \frac{-v\left(\frac{1}{2}\hat{\mathbf{i}} + \frac{\sqrt{3}}{2}\hat{\mathbf{j}}\right)}{\frac{\sqrt{3}}{2}\frac{m}{Bq}}$$

$$= \frac{-Bqv}{m}\left(\frac{\hat{\mathbf{i}}}{\sqrt{3}} + \hat{\mathbf{j}}\right)\text{ms}^{-2}$$

14. Correct answer is (c), because induced electric field lines (produced by change in magnetic field) and magnetic field lines form closed loops.

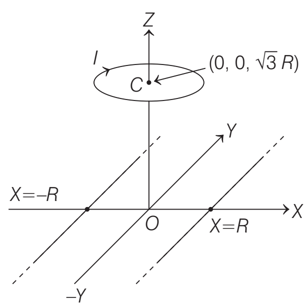
15. Net magnetic field due to both the wires will be downward as shown in the figure.



Since, angle between $\mathbf{v}$ and $\mathbf{B}$ is 180°.
Therefore, magnetic force

$$\mathbf{F}_m = q(\mathbf{v}\times\mathbf{B}) = 0$$

16.

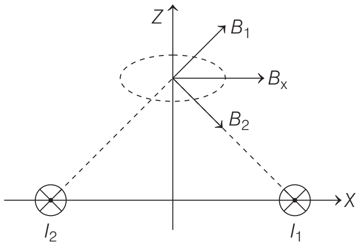


(a) At origin, $\mathbf{B}=0$ due to two wires if $I_1 = I_2$, hence ($\mathbf{B}_{\text{net}}$) at origin is equal to $\mathbf{B}$ due to ring. which is non-zero.

(b) If $I_1 > 0$ and $I_2 < 0$, B at origin due to wires will be along $+\hat{k}$. Direction of B due to ring is along $-\hat{k}$ direction and hence B can be zero at origin.

(c) If $I_1 < 0$ and $I_2 > 0$, B at origin due to wires is along $-\hat{k}$ and also along $-\hat{k}$ due to ring, hence B cannot be zero.

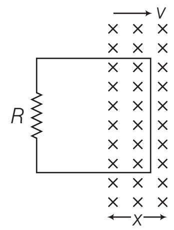
(d)



At centre of ring, B due to wires is along x-axis.

Hence, z-component is only because of ring which $B = \frac{\mu_0 i}{2R}(-\hat{k})$.

17.



When loop was entering (x < L)

$$\phi = BLx$$

$$e = -\frac{d\phi}{dt} = -BL\frac{dx}{dt}$$

$$|e| = BLv$$

$$i = \frac{e}{R} = \frac{BLv}{R} \qquad \text{(anti-clockwise)}$$

$$F = ilB \text{ (Left direction)} = \frac{B^2L^2v}{R} \text{ (in left direction)}$$

$$\Rightarrow \qquad a = \frac{F}{m} = -\frac{B^2L^2v}{mR}$$

$$a = v\frac{dv}{dx}$$

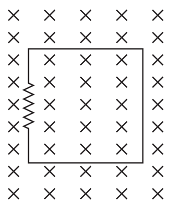
$$v\frac{dv}{dx} = -\frac{B^2L^2v}{mR}$$

$$\Rightarrow \qquad \int_{v_0}^{v} dv = -\frac{B^2L^2}{mR}\int_0^x dx$$

$$\Rightarrow v = v_0 - \frac{B^2L^2v}{mR}x \text{ (straight line of negative slope for } x < L)$$

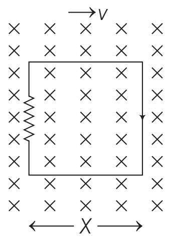
$I = \dfrac{BL}{R} v \Rightarrow$ (I *vs* x will also be straight line of negative slope for $x < L$)

$L \le x \le 3L$



$\dfrac{d\phi}{dt} = 0, \;\; e = 0, i = 0$

$F = 0, \quad x > 4L$



$e = Blv$

Force also will be in left direction.

$$i = \frac{BLv}{R} \text{ (clockwise)}$$

$$a = \frac{B^2L^2v}{mR} = v\frac{dv}{dx}$$

$$F = \frac{B^2L^2v}{R}$$

$$\int_L^x -\frac{B^2L^2}{mR}dx = \int_{v_i}^{v_f} dv$$

$$\Rightarrow -\frac{B^2L^2}{mR}(x - L) = v_f - v_i$$

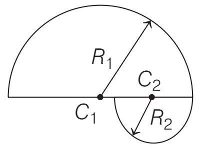
$$v_f - v_i - \frac{B^2L^2}{mR}(x - L) \text{ (straight line of negative slope)}$$

$$I = \frac{BLv}{R} \rightarrow \text{(Clockwise) (straight line of negative slope)}$$

18. If average speed is considered along x-axis,

$$R_1 = \frac{mv_0}{qB_1}, R_2 = \frac{mv_0}{qB_2} = \frac{mv_0}{4qB_1}$$

$$R_1 > R_2$$



Distance travelled along x-axis, $\Delta x = 2(R_1 + R_2) = \dfrac{5mv_0}{2qB_1}$

$$\text{Total time} = \frac{T_1}{2} + \frac{T_2}{2} = \frac{\pi m}{qB_1} + \frac{\pi m}{qB_2}$$

$$= \frac{\pi m}{qB_1} + \frac{\pi m}{4qB_1} = \frac{5\pi m}{4qB_1}$$

$$\text{Magnitude of average speed} = \frac{\dfrac{5mv_0}{2qB_1}}{\dfrac{5\pi m}{4qB_1}} = 2$$

19. $F_B = Bev = Be\dfrac{I}{nAe} = \dfrac{BI}{nA}$

$$F_e = eE$$

$$F_e = F_B$$

$$eE = \frac{BI}{nA} \Rightarrow \quad E = \frac{B}{nAe}$$

$$V = Ed = \frac{BI}{nAe}\cdot w = \frac{BIw}{n(wd)e} = \frac{BI}{ned}$$

$$\frac{V_1}{V_2} = \frac{d_2}{d_1}$$

$\Rightarrow$ if $w_1 = 2w_2$

and $d_1 = d_2$

$V_1 = V_2$

20. $V = \dfrac{BI}{ned} \Rightarrow \dfrac{V_1}{V_2} = \dfrac{B_1}{B_2} \times \dfrac{n_2}{n_1}$

If $B_1 = B_2$ and $n_1 = 2n_2$, then $V_2 = 2V_1$

If $B_1 = 2B_2$ and $n_1 = n_2$, then $V_2 = 0.5V_1$

21. If $B_2 > B_1$, critical temperature, (at which resistance of semiconductors abruptly becomes zero) in case 2 will be less than compared to case 1.

22. With increase in temperature, T_C is decreasing.

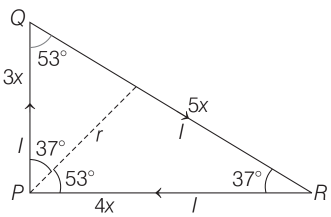
$$T_C(0) = 100 \text{ K}$$

$$T_C = 75 \text{ K at } B = 7.5 \text{ T}$$

Hence, at $B = 5$ T, T_C should lie between 75K and 100K. Hence, the correct option should be (b).

26. Magnetic field at point P due to wires RP and RQ is zero. Only wire QR will produce magnetic field at P.

$$r = 3x\cos 37° = (3x)\left(\frac{4}{5}\right) = \frac{12x}{5}$$



Now, $\qquad B = \dfrac{\mu_0}{4\pi}\dfrac{I}{12x/5}\quad[\sin 37° + \sin 53°]$

$$= 7\left(\frac{\mu_0 I}{48\pi x}\right)$$

27. Radius of circular path is given by

$$R = \frac{mv}{Bq} = \frac{p}{Bq} = \frac{\sqrt{2Km}}{Bq}$$

Radius in both the cases are equal. Therefore,

$$\frac{\sqrt{2K_\alpha m_\alpha}}{Bq_\alpha} = \frac{\sqrt{2K_d m_d}}{2.3Bq_d},$$

$$\frac{q_d}{q_\alpha} = \frac{e}{2e} = \frac{1}{2}$$

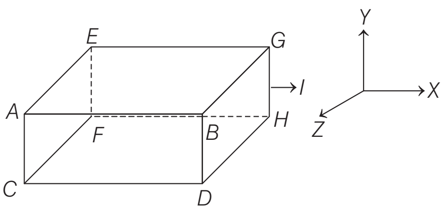
and $$\frac{m_\alpha}{m_d} = \frac{4}{2} = 2$$

$\therefore$ $$K_d = \left(\frac{2.3q_d}{q_\alpha}\right)^2 \left(\frac{m_\alpha}{m_d}\right) \cdot K_\alpha$$

$$K_d = \left(2.3 \times \frac{1}{2}\right)^2 (2)(5.3)\,\text{MeV},$$

$$K_d = 14.0185\ \text{MeV}$$

28.



In metals, charge carries are free electrons. Current is in positive direction of x-axis. Therefore, charge carries will be moving in negative direction of x-axis.

$$\mathbf{F}_m = q(\mathbf{v} \times \mathbf{B}) = (-e)(-v\,\hat{\mathbf{i}}) \times (B\hat{\mathbf{j}})$$

$$\mathbf{F}_m = evB\hat{\mathbf{k}}$$

Due to this magnetic force, electrons will be collecting at face $ABCD$, therefore lowering its potential.

29. In equilibrium, $2T_0 = mg$ or $T_0 = \dfrac{mg}{2}$ ...(i)

Magnetic moment, $M = iA = \left(\dfrac{\omega}{2\pi}Q\right)(\pi R^2)$

$$\tau = MB \sin 90° = \frac{\omega BQR^2}{2}$$

Let T_1 and T_2 be the tensions in the two strings when magnetic field is switched on ($T_1 > T_2$).
For translational equilibrium,

$$T_1 + T_2 = mg \qquad \text{...(ii)}$$

For rotational equilibrium,

$$(T_1 - T_2)\frac{D}{2} = \tau = \frac{\omega BQR^2}{2} \text{ or } T_1 - T_2 = \frac{\omega BQR^2}{2} \qquad \text{... (iii)}$$

Solving Eqs. (ii) and (iii), we have

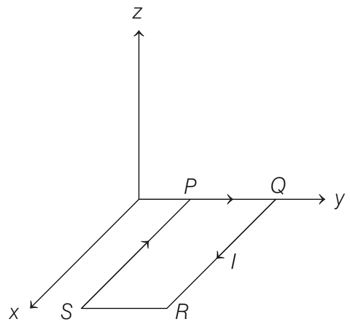
$$T_1 = \frac{mg}{2} + \frac{\omega BQR^2}{2D}$$

As $T_1 > T_2$ and maximum values of T_1 can be $\dfrac{3T_0}{2}$, we have

$$\frac{3T_0}{2} = T_0 + \frac{\omega_{\max} BQR^2}{2D} \qquad \left(\frac{mg}{2} = T_0\right)$$

$\therefore$ $$\omega_{\max} = \frac{DT_0}{BQR^2}$$

30. Let the direction of current in wire PQ is from P to Q and its magnitude be I.



The magnetic moment of the given loop is

$$\mathbf{M} = -Iab\hat{\mathbf{k}}$$

Torque on the loop due to magnetic forces is

$$\tau_1 = \mathbf{M} \times \mathbf{B} = (-Iab\hat{\mathbf{k}}) \times (3\hat{\mathbf{i}} + 4\hat{\mathbf{k}})B_0\hat{\mathbf{i}} = -3IabB_0\hat{\mathbf{j}}$$

Torque of weight of the loop about axis PQ is

$$\tau_2 = \mathbf{r} \times \mathbf{F} = \left(\frac{a}{2}\hat{\mathbf{i}}\right) \times (-mg\hat{\mathbf{k}}) = \frac{mga}{2}\hat{\mathbf{j}}$$

We see that when the current in the wire PQ is from P to Q, τ_1 and τ_2 are in opposite directions, so they can cancel each other and the loop may remain in equilibrium. So, the direction of current I in wire PQ is from P to Q. Further for equilibrium of the loop

$$|\tau_1| = |\tau_2|$$

or $$3IabB_0 = \frac{mga}{2}$$

$$I = \frac{mg}{6bB_0}$$

(b) Magnetic force on wire RS is

$$\mathbf{F} = I(\mathbf{I} \times \mathbf{B}) = I[(-b\hat{\mathbf{j}}) \times \{(3\hat{\mathbf{i}} + 4\hat{\mathbf{k}})B_0\}]$$

$$\mathbf{F} = IbB_0(3\hat{\mathbf{k}} - 4\hat{\mathbf{i}})$$

31. Given, $i = 10$ A, $r_1 = 0.08$ m and $r_2 = 0.12$ m. Straight portions i.e. CD etc, will produce zero magnetic field at the centre. Rest eight arcs will produce the magnetic field at the centre in the same direction i.e. perpendicular to the paper outwards or vertically upwards and its magnitude is

$$B = B_{\text{inner arcs}} + B_{\text{outer arcs}}$$

$$= \frac{1}{2}\left\{\frac{\mu_0 i}{2r_1}\right\} + \frac{1}{2}\left\{\frac{\mu_0 i}{2r_2}\right\} = \left(\frac{\mu_0}{4\pi}\right)(\pi i)\left(\frac{r_1 + r_2}{r_1 r_2}\right)$$

Substituting the values, we have

$$B = \frac{(10^{-7})(3.14)(10)(0.08 + 0.12)}{(0.08 \times 0.12)} \Rightarrow B = 6.54 \times 10^{-5}\ \text{T}$$

(vertically upward or outward normal to the paper)

(b) Force on *AC*

Force on circular portions of the circuit i.e. *AC* etc, due to the wire at the centre will be zero because magnetic field due to the central wire at these arcs will be tangential ($\theta = 180°$).

Force on *CD*

Current in central wire is also $i = 10$ A. Magnetic field at distance x due to central wire

$$B = \frac{\mu_0}{2\pi} \cdot \frac{i}{x}$$

$\therefore$ Magnetic force on element dx due to this magnetic field

$$dF = (i)\left(\frac{\mu_0}{2\pi} \cdot \frac{i}{x}\right) \cdot dx = \left(\frac{\mu_0}{2\pi}\right) i^2 \frac{dx}{x}$$

$$(F = ilB \sin 90°)$$

Therefore, net force on *CD* is

$$F = \int_{x = r_1}^{x = r_2} dF = \frac{\mu_0 i^2}{2\pi} \int_{0.08}^{0.12} \frac{dx}{x} = \frac{\mu_0}{2\pi} i^2 \ln\left(\frac{3}{2}\right)$$

Substituting the values

$$F = (2 \times 10^{-7})(10)^2 \ln(1.5)$$

or $\quad F = 8.1 \times 10^{-6}$ N (inwards)

Force on wire at the centre

Net magnetic field at the centre due to the circuit is in vertical direction and current in the wire in centre is also in vertical direction. Therefore, net force on the wire at the centre will be zero. ($\theta = 180°$). Hence,

(i) force acting on the wire at the centre is zero.
(ii) force on arc $AC = 0$.
(iii) force on segment *CD* is 8.1×10^{-6} N (inwards).

32. Magnetic field (**B**) at the origin = magnetic field due to semicircle KLM + Magnetic field due to other semicircle KNM

$$\therefore \quad \mathbf{B} = -\frac{\mu_0 I}{4R}(-\hat{\mathbf{i}}) + \frac{\mu_0 I}{4R}(\hat{\mathbf{j}})$$

$$\mathbf{B} = -\frac{\mu_0 I}{4R}\hat{\mathbf{i}} + \frac{\mu_0 I}{4R}\hat{\mathbf{j}} = \frac{\mu_0 I}{4R}(-\hat{\mathbf{i}} + \hat{\mathbf{j}})$$

$\therefore$ Magnetic force acting on the particle

$$\mathbf{F} = q(\mathbf{v} \times \mathbf{B}) = q\{(-v_0\hat{\mathbf{i}}) \times (-\hat{\mathbf{i}} + \hat{\mathbf{j}})\}\frac{\mu_0 I}{4R}$$

$$\mathbf{F} = -\frac{\mu_0 q v_0 I}{4R}\hat{\mathbf{k}}$$

(b) $\mathbf{F}_{KLM} = \mathbf{F}_{KMN} = \mathbf{F}_{KM}$ and $\mathbf{F}_{KM} = BI(2R)\hat{\mathbf{i}} = 2BIR\hat{\mathbf{i}}$

$$\mathbf{F}_1 = \mathbf{F}_2 = 2BIR\,\hat{\mathbf{i}}$$

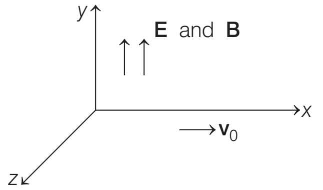
Total force on the loop, $\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2$ or $\mathbf{F} = 4BIR\hat{\mathbf{i}}$

NOTE If a current carrying wire ADC (of any shape) is placed in a uniform magnetic field **B**.

Then, $\quad F_{ADC} = F_{AC}$ or $|F_{ADC}| = \hat{i}(AC)B$

From this we can conclude that net force on a current carrying loop in uniform magnetic field is zero. In the question, segments *KLM* and *KNM* also form a loop and they are also placed in a uniform magnetic field but in this case net force on the loop will not be zero. It would had been zero if the current in any of the segments was in opposite direction.

33. $\hat{\mathbf{j}} = \dfrac{\mathbf{E}}{E}$ or $\dfrac{\mathbf{B}}{B}$; $\hat{\mathbf{i}} = \dfrac{\mathbf{v}_0}{v_0} \Rightarrow \hat{\mathbf{k}} = \dfrac{\mathbf{v}_0 \times \mathbf{B}_0}{v_0 B}$



Force due to electric field will be along y-axis. Magnetic force will not affect the motion of charged particle in the direction of electric field (or y-axis). So,

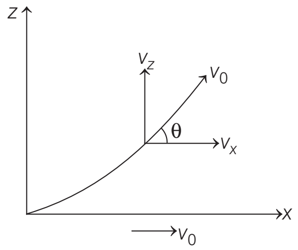
$$a_y = \frac{F_e}{m} = \frac{qE}{m} = \text{constant.}$$

Therefore, $\quad v_y = a_y t = \dfrac{qE}{m} \cdot t \quad$ ...(i)

The charged particle under the action of magnetic field describes a circle in x-z plane (perpendicular to **B**) with

$$T = \frac{2\pi m}{Bq} \text{ or } \omega = \frac{2\pi}{T} = \frac{qB}{m}$$

Initially ($t = 0$), velocity was along x-axis. Therefore, magnetic force ($\mathbf{F}_m$) will be along positive z-axis [$\mathbf{F}_m = q(\mathbf{v}_0 \times \mathbf{B})$]. Let it makes an angle θ with x-axis at time t, then



$$\theta = \omega t$$

$\therefore \quad v_x = v_0 \cos \omega t = v_0 \cos\left(\dfrac{qB}{m}t\right) \quad$ ...(ii)

$\quad v_z = v_0 \sin \omega t = v_0 \sin\left(\dfrac{qB}{m}t\right) \quad$ ...(iii)

From Eqs. (i), (ii) and (iii),

$$\mathbf{v} = v_x\,\hat{\mathbf{i}} + v_y\,\hat{\mathbf{j}} + v_z\hat{\mathbf{k}}$$

$$\therefore \quad \mathbf{v} = v_0 \cos\left(\frac{qB}{m}t\right)\left(\frac{\mathbf{v}_0}{v_0}\right) + \frac{qE}{m}t\left(\frac{\mathbf{E}}{E}\right) + v_0 \sin\left(\frac{qB}{m}t\right)\left(\frac{\mathbf{v}_0 \times \mathbf{B}}{v_0 B}\right)$$

or $\mathbf{v} = \cos\left(\frac{qB}{m}t\right)(\mathbf{v}_0) + \left(\frac{q}{m}t\right)(\mathbf{E}) + \sin\left(\frac{qB}{m}t\right)\left(\frac{\mathbf{v}_0 \times \mathbf{B}}{B}\right)$

NOTE The path of the particle will be a helix of increasing pitch. The axis of the helix will be along y-axis.

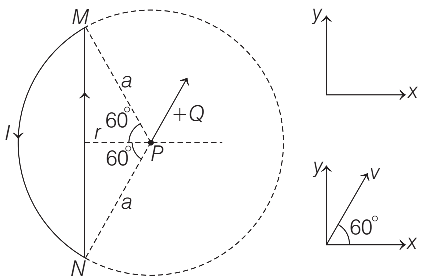
34. Magnetic field at P due to arc of circle,

Subtending an angle of 120° at centre would be

$$B_1 = \frac{1}{3}(\text{field due to circle}) = \frac{1}{3}\left(\frac{\mu_0 I}{2a}\right)$$

$$= \frac{\mu_0 I}{6a} \qquad \text{(outwards)}$$

$$= \frac{0.16\mu_0 I}{a} \qquad \text{(outwards)}$$



or $\qquad \mathbf{B}_1 = \frac{0.16\mu_0 I}{a}\hat{\mathbf{k}}$

Magnetic field due to straight wire NM at P

$$B_2 = \frac{\mu_0}{4\pi}\frac{I}{r}(\sin 60° + \sin 60°)$$

Here, $\qquad r = a\cos 60°$

$\therefore \qquad B_2 = \frac{\mu_0}{4\pi}\frac{I}{a\cos 60°}(2\sin 60°)$

or $\qquad B_2 = \frac{\mu_0}{2\pi}\frac{I}{a}\tan 60° = \frac{0.27\mu_0 I}{a} \qquad$ (inwards)

or $\qquad \mathbf{B}_2 = -\frac{0.27\mu_0 I}{a}\hat{\mathbf{k}}$

$\therefore \qquad \mathbf{B}_{\text{net}} = \mathbf{B}_1 + \mathbf{B}_2 = -\frac{0.11\mu_0 I}{a}\hat{\mathbf{k}}$

Now, velocity of particle can be written as,

$$\mathbf{v} = v\cos 60°\,\hat{\mathbf{i}} + v\sin 60°\,\hat{\mathbf{j}} = \frac{v}{2}\hat{\mathbf{i}} + \frac{\sqrt{3}v}{2}\hat{\mathbf{j}}$$

Magnetic force

$$\mathbf{F}_m = Q(\mathbf{v} \times \mathbf{B}) = \frac{0.11\mu_0 IQv}{2a}\hat{\mathbf{j}} - \frac{0.11\sqrt{3}\,\mu_0 IQv}{2a}\hat{\mathbf{i}}$$

$\therefore$ Instantaneous acceleration

$$\mathbf{a} = \frac{\mathbf{F}_m}{m} = \frac{0.11\mu_0 IQv}{2am}(\hat{\mathbf{j}} - \sqrt{3}\hat{\mathbf{i}})$$

(b) In uniform magnetic field, force on a current loop is zero. Further, magnetic dipole moment of the loop will be,

$$\mathbf{M} = (IA)\,\hat{\mathbf{k}}$$

Here, A is the area of the loop.

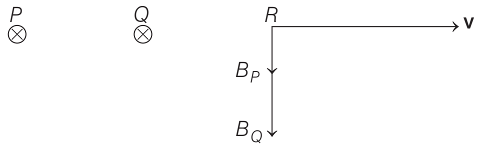
$$A = \frac{1}{3}(\pi a^2) - \frac{1}{2}[2 \times a\sin 60°][a\cos 60°]$$

$$= \frac{\pi a^2}{3} - \frac{a^2}{2}\sin 120° = 0.61\,a^2$$

$\therefore \qquad \mathbf{M} = (0.61\,Ia^2)\hat{\mathbf{k}}$

Given, $\qquad \mathbf{B} = B\hat{\mathbf{i}} \Rightarrow \tau = \mathbf{M} \times \mathbf{B} = (0.61\,Ia^2 B)\,\hat{\mathbf{j}}$

35. Magnetic field at R due to both the wires P and Q will be downwards as shown in figure.



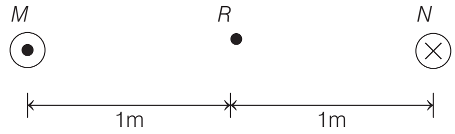
Therefore, net field at R will be sum of these two

$$B = B_P + B_Q = \frac{\mu_0}{2\pi}\frac{I_P}{5} + \frac{\mu_0}{2\pi}\frac{I_Q}{2}$$

$$= \frac{\mu_0}{2\pi}\left(\frac{2.5}{5} + \frac{I}{2}\right) = \frac{\mu_0}{4\pi}(I + 1)$$

$$= 10^{-7}(I + 1)$$

(a) Net force on the electron will be,



$$F_m = Bqv\sin 90°$$

or $(3.2 \times 10^{-20}) = (10^{-7})(I + 1)(1.6 \times 10^{-19})(4 \times 10^5)$

or $\qquad I + 1 = 5 \Rightarrow I = 4\text{A}$

(b) Net field at R due to wires P and Q is

$$B = 10^{-7}(I + 1)\text{ T} = 5 \times 10^{-7}\text{ T}$$

Magnetic field due to third wire carrying a current of 2.5 A should be 5×10^{-7} T in upward direction so, that net field at R becomes zero. Let distance of this wire from R be r. Then,

$$\frac{\mu_0}{2\pi}\frac{2.5}{r} = 5 \times 10^{-7} \text{ or } \frac{(2 \times 10^{-7})(2.5)}{r} = 5 \times 10^{-7}\text{ m}$$

or $\qquad r = 1\text{ m}$

So, the third wire can be put at M or N as shown in figure.

If it is placed at M, then current in it should be outwards and if placed at N, then current be inwards.