Topic 1 Calorimetry

Objective Questions I (Only one correct option)

1. When M_1 gram of ice at -10° C (specific heat = 0.5 cal $g^{-1} \circ C^{-1}$) is added to M_2 gram of water at 50°C, finally no ice is left and the water is at 0°C. The value of latent heat of ice, in cal g^{-1} is (2019 Main, 12 April I)

(a)
$$\frac{50M_2}{M_1} - 5$$
 (b) $\frac{50M_1}{M_2} - 50$ (c) $\frac{50M_2}{M_1}$ (d) $\frac{5M_2}{M_1} - 5$

2. A thermally insulated vessel contains 150 g of water at 0°C. Then, the air from the vessel is pumped out adiabatically. A fraction of water turns into ice and the rest evaporates at 0°C itself. The mass of evaporated water will be closest to

(Latent heat of vaporisation of water = $210 \times 10^6 J kg^{-1}$ and latent heat of fusion of water = $3.36 \times 10^5 J kg^{-1}$)

(2019 Main, 8 April I) (a) 150 g (b) 20 g (c) 130 g (d) 35 g

3. When 100 g of a liquid A at 100°C is added to 50 g of a liquid B at temperature 75°C, the temperature of the mixture becomes 90°C. The temperature of the mixture, if 100 g of liquid A at 100°C is added to 50 g of liquid B at 50°C will be (2019 Main, 11 Jan II)

(a) 60°C (b) 80°C (c) 70°C (d) 85°C

- 4. In a process, temperature and volume of one mole of an ideal monoatomic gas are varied according to the relation VT = k, where k is a constant. In this process, the temperature of the gas is increased by ΔT . The amount of heat absorbed by gas is (where, R is gas constant) (2019 Main, 11 Jan II) (a) $\frac{1}{2} kR\Delta T$ (b) $\frac{2k}{3} \Delta T$ (c) $\frac{1}{2} R\Delta T$ (d) $\frac{3}{2} R\Delta T$
- 5. A metal ball of mass 0.1 kg is heated upto 500°C and dropped into a vessel of heat capacity 800 JK⁻¹ and containing 0.5 kg water. The initial temperature of water and vessel is 30°C. What is the approximate percentage increment in the temperature of the water? [Take, specific heat capacities of water and metal are respectively 4200 Jkg⁻¹K⁻¹ and 400 Jkg⁻¹K⁻¹] (2019 Main, 11 Jan II) (a) 25% (b) 15% (c) 30% (d) 20%

- **6.** Ice at -20° C is added to 50 g of water at 40°C. When the temperature of the mixture reaches 0°C, it is found that 20 g of ice is still unmelted. The amount of ice added to the water was close to (Take, specific heat of water = 4.2 J/g/°C specific heat of ice = 2.1 J/g/°C and heat of fusion of water at 0°C = 334 J/g)(2019 Main, 11 Jan I) (a) 40 g (b) 50 g (c) 60 g (d) 100 g
- 7. An unknown metal of mass 192 g heated to a temperature of 100°C was immersed into a brass calorimeter of mass 128 g containing 240 g of water at a temperature of 8.4°C. Calculate the specific heat of the unknown metal, if water temperature stabilises at 21.5°C. (Take, specific heat of brass is 394 J kg⁻¹K⁻¹) (2019 Main, 10 Jan II) (a) 916 J kg⁻¹ K⁻¹ (b) 654 J kg⁻¹ K⁻¹ (c) 1232 J kg⁻¹ K⁻¹ (d) 458 J kg⁻¹ K⁻¹
- **8.** C_p and C_V are specific heats at constant pressure and constant volume, respectively. It is observed that $C_p C_V = a$ for hydrogen gas $C_p C_V = b$ for nitrogen gas. The correct relation between *a* and *b* is (2017 Main) (a) a = b (b) a = 14b (c) a = 28b (d) $a = \frac{1}{14}b$
- **9.** A copper ball of mass 100 g is at a temperature *T*. It is dropped in a copper calorimeter of mass 100 g, filled with 170 g of water at room temperature. Subsequently, the temperature of the system is found to be 75°C. *T* is (Given, room temperature = 30°C, specific heat of copper = $0.1 \text{ cal/g}^{\circ}\text{C}$) (2017 Main) (a) 885°C (b) 1250°C (c) 825°C (d) 800°C
- **10.** A water cooler of storage capacity 120 litres can cool water at a constant rate of P watts. In a closed circulation system (as shown schematically in the figure), the water from the cooler is used to cool an external device that generates constantly 3 kW of heat (thermal load). The temperature of water fed into the device cannot exceed 30°C and the entire stored 120 litres of water is initially cooled to 10°C. The entire system is thermally insulated. The minimum value of P (in watts) for which the device can be operated for 3 hours is (2016 Adv.)



(Specific heat of water is 4.2 kJ kg⁻¹K⁻¹ and the density of water is 1000kg m⁻³)

(a) 1600 (b) 2067 (c) 2533 (d) 3933

- 11. Calorie is defined as the amount of heat required to raise temperature of 1 g of water by 1°C and it is defined under which of the following conditions ? (2005, 2M)
 (a) From 14.5°C to 15.5°C at 760 mm of Hg
 - (b) From 98.5°C to 99.5°C at 760 mm of Hg $\,$
 - (c) From 13.5°C to 14.5°C at 76 mm of Hg $\,$
 - (d) From 3.5°C to 4.5°C at 76 mm of Hg
- 12. Water of volume 2 L in a container is heated with a coil of 1 kW at 27°C. The lid of the container is open and energy dissipates at rate of 160 J/s. In how much time temperature will rise from 27°C to 77°C? (2005, 2M) [Specific heat of water is 4.2 kJ/kg]

 (a) 8 min 20 s
 (b) 6 min 2 s
 (c) 7 min
 (d) 14 min
- 13. Liquid oxygen at 50 K is heated to 300 K at constant pressure of 1 atm. The rate of heating is constant. Which of the following graphs represent the variation of temperature with time? (2004, 2M)



14. 2 kg of ice at -20°C is mixed with 5 kg of water at 20°C in an insulating vessel having a negligible heat capacity. Calculate the final mass of water remaining in the container. It is given that the specific heats of water and ice are 1 kcal/kg /°C and 0.5 kcal/kg /°C while the latent heat of fusion of ice is 80 kcal/kg (2003, 2M) (a) 7 kg (b) 6 kg

· ·			·	\sim
(c) 4 kg	(d)) 2	kg

15. A block of ice at -10°C is slowly heated and converted to steam at 100°C. Which of the following curves represents the phenomena qualitatively? (2000, 2M)



- 16. Steam at 100°C is passed into 1.1 kg of water contained in a calorimeter of water equivalent 0.02 kg at 15°C till the temperature of the calorimeter and its contents rises to 80°C. The mass of the steam condensed in kg is (1986, 2M) (a) 0.130 (b) 0.065 (c) 0.260 (d) 0.135
- 17. 70 cal of heat are required to raise the temperature of 2 moles of an ideal diatomic gas at constant pressure from 30°C to 35°C. The amount of heat required (in calorie) to raise the temperature of the same gas through the same range (30°C to 35°C) at constant volume is (1985, 2M) (a) 30 (b) 50

Integer Answer Type Question

18. A piece of ice (heat capacity = $2100 \text{ Jkg}^{-1} \circ \text{C}^{-1}$ and latent heat = $3.36 \times 10^5 \text{ J kg}^{-1}$) of mass *m* gram is at -5° C at atmospheric pressure. It is given 420 J of heat so that the ice starts melting. Finally when the ice-water mixture is in equilibrium, it is found that 1 g of ice has melted. Assuming there is no other heat exchange in the process, the value of *m* is (2010)

Fill in the Blanks

- **19.** Earth receives 1400 W/m^2 of solar power. If all the solar energy falling on a lens of area 0.2 m² is focussed onto a block of ice of mass 280 g, the time taken to melt the ice will be ...minutes. (Latent heat of fusion of ice = $3.3 \times 10^5 \text{ J/kg}$) (1997, 2M)
- 20. A substance of mass *M* kg requires a power input of *P* watts to remain in the molten state at its melting point. When the power source is turned off, the sample completely solidifies in time *t* seconds. The latent heat of fusion of the substance is (1992, 1M)
- 21. 300 g of water at 25°C is added to 100 g of ice at 0°C. The final temperature of the mixture is ... °C. (1989, 2M)



Analytical & Descriptive Questions

23. In an insulated vessel, 0.05 kg steam at 373 K and 0.45 kg of ice at 253 K are mixed. Find the final temperature of the mixture (in kelvin). (2006, 6M)

Given,
$$L_{\text{fusion}} = 80 \text{ cal/g} = 336 \text{ J/g},$$

 $L_{\text{vaporisation}} = 540 \text{ cal/g} = 2268 \text{ J/g},$
 $S_{\text{ice}} = 2100 \text{ J/kg}, K = 0.5 \text{ cal/g-K}$
and $S_{\text{water}} = 4200 \text{ J/kg}, K = 1 \text{ cal/g-K}.$

Topic 2 Thermal Expansion

Objective Questions I (Only one correct option)

1 A uniform cylindrical rod of length L and radius r, is made from a material whose Young's modulus of elasticity equals Y. When this rod is heated by temperature T and simultaneously subjected to a net longitudinal compressional force F, its length remains unchanged. The coefficient of volume expansion of the material of the rod, is (nearly) equal to (2019 Main, 12 April II)

(a)	$9F/(\pi r^2 YT)$	(b) $6F / (\pi r^2 YT)$
(c)	$3F/(\pi r^2 YT)$	(d) $F / (3\pi r^2 YT)$

2 At 40°C, a brass wire of 1 mm radius is hung from the ceiling. A small mass *M* is hung from the free end of the wire. When the wire is cooled down from 40°C to 20°C, it regains its original length of 0.2 m. The value of *M* is close to [Coefficient of linear expansion and Young's modulus of brass are 10^{-5} /°C and 10^{11} N/m² respectively, g = 10 ms⁻²] (2019 Main, 12 April I)

3 Two rods A and B of identical dimensions are at temperature 30°C. If A is heated upto 180°C and B upto T°C, then new lengths are the same. If the ratio of the coefficients of linear expansion of A and B is 4 : 3, then the value of T is (2019 Main, 11 Jan II)

4. An external pressure *p* is applied on a cube at 0°C so that it is equally compressed from all sides. *K* is the bulk modulus of the material of the cube and α is its coefficient of linear expansion. Suppose we want to bring the cube to its original size by heating. The temperature should be raised by

(2017 Main)

(

- **24.** An ice cube of mass 0.1 kg at 0°C is placed in an isolated container which is at 227°C. The specific heat *S* of the container varies with temperature *T* according to the empirical relation S = A + BT, where A = 100 cal/kg-K and $B = 2 \times 10^{-2} \text{ cal/kg-K}^2$. If the final temperature of the container is 27°C, determine the mass of the container. (Latent heat of fusion for water = $8 \times 10^4 \text{ cal/kg}$, specific heat of water = 10^3 cal/kg-K). (2001, 5M)
- 25. The temperature of 100 g of water is to be raised from 24°C to 90°C by adding steam to it. Calculate the mass of the steam required for this purpose. (1996, 2M)
- 26. A lead bullet just melts when stopped by an obstacle. Assuming that 25 per cent of the heat is absorbed by the obstacle, find the velocity of the bullet if its initial temperature is 27°C. (1981, 3M)

(Melting point of lead = 327°C, specific heat of lead

= 0.03 cal/g °C, latent heat of fusion of lead = 6 cal/g, J = 4.2 J/cal.)

(a)
$$\frac{p}{\alpha K}$$
 (b) $\frac{3\alpha}{pK}$ (c) $3pK\alpha$ (d) $\frac{p}{3\alpha K}$

- **5.** A pendulum clock loses 12 s a day if the temperature is 40°C and gains 4 s a day if the temperature is 20°C. The temperature at which the clock will show correct time, and the coefficient of linear expansion α of the metal of the pendulum shaft are, respectively. (2016 Main) (a) 25°C, $\alpha = 1.85 \times 10^{-5}$ /°C (b) 60°C, $\alpha = 1.85 \times 10^{-4}$ /°C (c) 30°C, $\alpha = 1.85 \times 10^{-3}$ /°C (d) 55°C, $\alpha = 1.85 \times 10^{-2}$ /°C
- **6.** The ends Q and R of two thin wires, PQ and RS, are soldered (joined) together. Initially, each of the wire has a length of 1 m 10°C. Now, the end P is maintained at 10°C, while the end S is heated and maintained at 400°C. The system is thermally insulated from its surroundings. If the thermal conductivity of wire PQ is twice that of the wire RS and the coefficient of linear thermal expansion of PQ is 1.2×10^{-5} K ⁻¹, the change in length of the wire PQ is (2016 Adv.) (a) 0.78 mm (b) 0.90 mm (c) 1.56 mm (d) 2.34 mm
- 7. Two rods, one of aluminium and the other made of steel, having initial length l_1 and l_2 are connected together to form a single rod of length $l_1 + l_2$. The coefficients of linear expansion for aluminium and steel are α_a and α_s respectively. If the length of each rod increases by the same amount when their temperature are raised by $t^{\circ}C$, then find the ratio l_1

a)
$$\frac{\alpha_s}{\alpha_a}$$
 (b) $\frac{\alpha_a}{\alpha_s}$ (c) $\frac{\alpha_s}{(\alpha_a + \alpha_s)}$ (d) $\frac{\alpha_a}{(\alpha_a + \alpha_s)}$

Objective Question II (One or more correct option)

- **8.** A bimetallic strip is formed out of two identical strips- one of copper and the other of brass. The coefficients of linear expansion of the two metals are α_C and α_B . On heating, the temperature of the strip goes up by ΔT and the strip bends to form an arc of radius of curvature *R*. Then, *R* is (1999, 3M) (a) proportional to ΔT
 - (b) inversely proportional to ΔT
 - (c) proportional to $|\alpha_B \alpha_C|$
 - (d) inversely proportional to $|\alpha_B \alpha_C|$

Integer Answer Type Question

9. Steel wire of length *L* at 40°C is suspended from the ceiling and then a mass *m* is hung from its free end. The wire is cooled down from 40°C to 30°C to regain its original length *L*. The coefficient of linear thermal expansion of the steel is 10^{-5} /°C, Young's modulus of steel is 10^{11} N/m² and radius of the wire is 1 mm. Assume that *L* >> diameter of the wire. Then the value of *m* in kg is nearly. (2011)

Analytical & Descriptive Questions

- **10.** A cube of coefficient of linear expansion α_s is floating in a bath containing a liquid of coefficient of volume expansion γ_l . When the temperature is raised by ΔT , the depth upto which the cube is submerged in the liquid remains the same. Find the relation between α_s and γ_l showing all the steps. (2004, 2M)
- **11.** The apparatus shown in figure consists of four glass columns connected by horizontal sections. The height of two central columns *B* and *C* are 49 cm each. The two outer columns *A* and *D* are open to the atmosphere.

Topic 3 Heat Transfer

Objective Questions I (Only one correct option)

1. Two materials having coefficients of thermal conductivity '3K' and 'K' and thickness 'd' and '3d' respectively, are joined to form a slab as shown in the figure. The temperatures of the outer surfaces are ' θ_2 ' and ' θ_1 ' respectively, ($\theta_2 > \theta_1$). The temperature at the interface is

(a)
$$\frac{\theta_2 \quad \frac{d}{3\kappa} \quad \frac{3d}{\kappa}}{2}$$
 (b) $\frac{\theta_1}{3} + \frac{2\theta_2}{3}$ (c) $\frac{\theta_1}{6} + \frac{5\theta_2}{6}$ (d) $\frac{\theta_1}{10} + \frac{9\theta_2}{10}$

2. Two identical beakers *A* and *B* contain equal volumes of two different liquids at 60°C each and left to cool down. Liquid in *A* has density of 8×10^2 kg/m³ and specific heat of 2000 J kg⁻¹K⁻¹ while liquid in *B* has density of 10^3 kg m⁻³ and specific heat of 4000 J kg⁻¹K⁻¹. Which of the following best describes their temperature *versus* time graph

A and C are maintained at a temperature of 95° C while the columns B and D are maintained at 5° C. The height of the liquid in A and D measured from the base line are 52.8 cm and 51 cm respectively. Determine the linear coefficient of thermal expansion of the liquid. (1997, 5M)



12. A composite rod is made by joining a copper rod, end to end, with a second rod of different material but of the same cross-section. At 25°C, the composite rod is 1 m in length, of which the length of the copper rod is 30 cm. At 125°C the length of the composite rod increases by 1.91 mm.

When the composite rod is not allowed to expand by holding it between two rigid walls, it is found that the length of the two constituents do not change with the rise of temperature. Find the Young's modulus and the coefficient of linear expansion of the second rod. (Given, Coefficient of linear expansion of copper = 1.7×10^{-5} per °C, Young's modulus of copper = 1.3×10^{11} N/m²) (1979)

13. A sinker of weight w_0 has an apparent weight w_1 when placed in a liquid at a temperature T_1 and w_2 when weighed in the same liquid at a temperature T_2 . The coefficient of cubical expansion of the material of the sinker is β . What is the coefficient of volume expansion of the liquid ? (1978)

schematically? (Assume the emissivity of both the beakers to be the same) (2019 Main, 8 April I)



3. A cylinder of radius *R* is surrounded by a cylindrical shell of inner radius *R* and outer radius 2R. The thermal conductivity of the material of the inner cylinder is K_1 and that of the outer cylinder is K_2 . Assuming no loss of heat, the effective

thermal conductivity of the system for heat flowing along the length of the cylinder is (2019 Main, 12 Jan I)

(a)
$$\frac{K_1 + K_2}{2}$$

(b) $\frac{K_1 + 3K_2}{4}$
(c) $\frac{2K_1 + 3K_2}{5}$
(d) $K_1 + K_2$

4. Temperature difference of 120°C is maintained between two ends of a uniform rod *AB* of length 2*L*. Another bent rod *PQ*, of same cross-section as *AB* and length $\frac{3L}{2}$ is connected across *AB* (see figure). In steady state, temperature difference between *P* and *Q* will be close to (2019 Main, 9 Jan I)

(a)
$$45^{\circ}$$
 (b) 35° C
(c) 75° C (d) 60° C

- **5.** Parallel rays of light of intensity $I = 912 \text{ Wm}^{-2}$ are incident on a spherical black body kept in surroundings of temperature 300 K. Take Stefan constant $\sigma = 5.7 \times 10^{-8} \ Wm^{-2} \ K^{-4}$ and assume that the energy exchange with the surroundings is only through radiation. The final steady state temperature of the black body is close to (2014 Main) (a) 330 K (b) 660 K (c) 990 K (d) 1550
- **6.** Three rods of copper, brass and steel are welded together to form a Y-shaped structure. Area of cross-section of each rod is 4 cm². End of copper rod is maintained at 100°C whereas ends of brass and steel are kept at 0°C. Lengths of the copper, brass and steel rods are 46, 13 and 12 cm respectively.

The rods are thermally insulated from surroundings except at ends. Thermal conductivities of copper, brass and steel are 0.92, 0.26 and 0.12 in CGS units, respectively. Rate of heat flow through copper rod is (2014 Main) (a) 1.2 cal/s (b) 2.4 cal/s (c) 4.8 cal/s (d) 6.0 cal/s

7. If a piece of metal is heated to temperature θ and then allowed to cool in a room which is at temperature θ_0 . The graph between the temperature *T* of the metal and time *t* will be closed to (2013 Main)



8. Two rectangular blocks, having indentical dimensions, can be arranged either in configuration I or in configuration II as

shown in the figure. One of the blocks has thermal conductivity K and the other 2K.

The temperature difference between the ends along the *X*-axis is the same in both the configurations. It takes 9s to transport a certain amount of heat from the hot end to the cold end in the configuration I. The time to transport the same amount of heat in the configuration II is (2013 Adv.)



9. Three very large plates of same area are kept parallel and close to each other. They are considered as ideal black surfaces and have very high thermal conductivity. The first and third plates are maintained at temperatures 2*T* and 3*T* respectively. The temperature of the middle (i.e. second) plate under steady state condition is (2012)

(a)
$$\left(\frac{65}{2}\right)^{\frac{1}{4}}T$$
 (b) $\left(\frac{97}{4}\right)^{\frac{1}{4}}T$
(c) $\left(\frac{97}{2}\right)^{\frac{1}{4}}T$ (d) $(97)^{\frac{1}{4}}T$

Variation of radiant energy emitted by sun, filament of tungsten lamp and welding arc as a function of its wavelength is shown in figure. Which of the following option is the correct match? (2005, 2M)



(a) Sun- T_1 , tungsten filament- T_2 , welding arc- T_3

(b) Sun- T_2 , tungsten filament- T_1 , welding arc- T_3

- (c) Sun- T_3 , tungsten filament- T_2 , welding arc- T_1
- (d) Sun- $T_{1,}$ tungsten filament- T_3 , welding arc- T_2
- **11.** In which of the following process, convection does not take place primarily? (2005, 2M)
 - (a) Sea and land breeze
 - (b) Boiling of water
 - (c) Warming of glass of bulb due to filament
 - (d) Heating air around a furnace
- **12.** Three discs, *A*, *B* and *C* having radii 2 m, 4 m and 6 m respectively are coated with carbon black on their outer surfaces. The wavelengths corresponding to maximum intensity are 300 nm, 400 nm and 500 nm, respectively. The power radiated by them are Q_A , Q_B and Q_C respectively (2004. 2M)

(a) Q_A is maximum	(b) Q_B is maximum
(c) Q_C is maximum	$(\mathbf{d}) Q_A = Q_B = Q_C$

13. Two identical conducting rods are first connected independently to two vessels, one containing water at 100°C and the other containing ice at 0°C. In the second case, the rods are joined end to end and connected to the same vessels. Let q_1 and q_2 gram per second be the rate of melting of ice in the two

cases respectively. The ratio $\frac{q_1}{q_2}$ is (2004, 2M) (a) $\frac{1}{2}$ (b) $\frac{2}{1}$ (c) $\frac{4}{1}$ (d) $\frac{1}{4}$

14. The graph, shown in the diagram, represents the variation of temperature (*T*) of the bodies, *x* and *y* having same surface area, with time (*t*) due to the emission of radiation. Find the correct relation between the emissivity and absorptivity power of the two bodies (2003, 2M)



- (a) $E_x > E_y$ and $a_x < a_y$ (b) $E_x < E_y$ and $a_x > a_y$ (c) $E_x > E_y$ and $a_x > a_y$
- (d) $E_x < E_y$ and $a_x < a_y$
- **15.** An ideal black body at room temperature is thrown into a furnace. It is observed that (2002, 2M)
 - (a) initially it is the darkest body and at later times the brightest
 - (b) it is the darkest body at all times
 - (c) it cannot be distinguished at all times
 - (d) initially it is the darkest body and at later times it cannot be distinguished
- 16. Three rods made of the same material and having the same cross-section have been joined as shown in the figure. Each rod is of the same length. The left and right ends are kept at 0°C and 90°C respectively. The temperature of junction of the three rods will be (2001, 2M)



17. The plots of intensity *versus* wavelength for three black bodies at temperatures T_1 , T_2 and T_3 respectively are as shown. Their temperatures are such that (2000, 2M)



- **18.** A black body is at a temperature of 2880 K. The energy of radiation emitted by this body with wavelength between 499 nm and 500 nm is U_1 , between 999 nm and 1000 nm is U_2 and between 1499 nm and 1500 nm is U_3 . The Wien constant, $b = 2.88 \times 10^6$ nm-K. Then, (1998, 2M) (a) $U_1 = 0$ (b) $U_3 = 0$ (c) $U_1 > U_2$ (d) $U_2 > U_1$
- **19.** A spherical black body with a radius of 12 cm radiates 450 W power at 500 K. If the radius were halved and the temperature doubled, the power radiated in watt would be (1997, 1M)

- 20. The intensity of radiation emitted by the sun has its maximum value at a wavelength of 510 nm and that emitted by the north star has the maximum value at 350 nm. If these stars behave like black bodies, then the ratio of the surface temperature of the sun and the north star is (1997, 1M) (a) 1.46 (b) 0.69 (c) 1.21 (d) 0.83
- **21.** Two metallic spheres S_1 and S_2 are made of the same material and have got identical surface finish. The mass of S_1 is thrice that of S_2 . Both the spheres are heated to the same high temperature and placed in the same room having lower temperature but are thermally insulated from each other. The ratio of the initial rate of cooling of S_1 to that of S_2 is

(1995, 2M)

(a)
$$\frac{1}{3}$$
 (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{\sqrt{3}}{1}$ (d) $\left(\frac{1}{3}\right)^{1/3}$

22. Three rods of identical cross-sectional area and made from the same metal form the sides of an isosceles triangle *ABC*, right angled at *B*. The points *A* and *B* are maintained at temperatures *T* and $(\sqrt{2})T$ respectively. In the steady state, the temperature of the point *C* is T_c . Assuming that only heat conduction takes place, T_c/T is (1995, 2M)

(a)
$$\frac{1}{2(\sqrt{2}-1)}$$
 (b) $\frac{3}{\sqrt{2}+1}$
(c) $\frac{1}{\sqrt{3}(\sqrt{2}-1)}$ (d) $\frac{1}{(\sqrt{2}+1)}$

23. A cylinder of radius R made of a material of thermal conductivity K_1 is surrounded by a cylindrical shell of inner radius R and outer radius 2R made of a material of thermal

conductivity K_2 . The two ends of the combined system are maintained at two different temperatures. There is no loss of heat across the cylindrical surface and the system is in steady state. The effective thermal conductivity of the system is (1988, 2M)

(a) $K_1 + K_2$	(b) $K_1 K_2 / (K_1 + K_2)$
(c) $(K_1 + 3K_2)/4$	(d) $(3K_1 + K_2)/4$

Objective Questions II (One or more correct option)

24. A composite block is made of slabs *A*,*B*,*C*, *D* and *E* of different thermal conductivities (given in terms of a constant *K*) and sizes (given in terms of length, *L*) as shown in the figure. All slabs are of same width. Heat *Q* flows only from left to right through the blocks. Then, in steady state (2011)



- (a) heat flow through A and E slabs are same
- (b) heat flow through slab *E* is maximum
- (c) temperature difference across slab E is smallest
- (d) heat flow through C = heat flow through B + heat flow through D
- **25.** A black body of temperature *T* is inside a chamber of temperature T_0 . Now the closed chamber is slightly opened to sun such that temperature of black body (*T*) and chamber (T_0) remains constant (2006, 3M)



- (a) black body will absorb more radiation
- (b) black body will absorb less radiation
- (c) black body emit more energy
- (d) black body emit energy equal to energy absorbed by it
- **26.** Two bodies *A* and *B* have thermal emissivities of 0.01 and 0.81 respectively. The outer surface areas of the two bodies are the same. The two bodies emit total radiant power at the same rate. The wavelength λ_B corresponding to maximum spectral radiancy in the radiation from *B* shifted from the wavelength corresponding to maximum spectral radiancy in the radiation from *A*, by 1.00 µm. If the temperature of *A* is 5802 K (1994, 2M)

(a) the temperature of *B* is 1934 K

(b)
$$\lambda_B = 1.5 \,\mu\text{m}$$

(c) the temperature of B is 11604 K

(d) the temperature of *B* is 2901 K

Numerical Value

27. Two conducting cylinders of equal length but different radii are connected in series between two heat baths kept at temperatures $T_1 = 300$ K and $T_2 = 100$ K, as shown in the figure. The radius of the bigger cylinder is twice that of the smaller one and the thermal conductivities of the materials of the smaller and the larger cylinders are K_1 and K_2 , respectively. If the temperature at the junction of the cylinders in the steady state is 200K, then $K_1/K_2 =$ (2018 Adv.)



Integer Answer Type Questions

- **28.** Two spherical stars *A* and *B* emit black body radiation. The radius of *A* is 400 times that of *B* and *A* emits 10⁴ times the power emitted from *B*. The ratio $\left(\frac{\lambda_A}{\lambda_B}\right)$ of their wavelengths λ_A and λ_B at which the peaks occur in their respective radiation curves is (2015 Adv.)
- **29.** A metal is heated in a furnace where a sensor is kept above the metal surface to read the power radiated (P) by the metal. The sensor has a scale that displays $\log_2(P/P_0)$, where P_0 is a constant. When the metal surface is at a temperature of 487°C, the sensor shows a value 1. Assume that the emissivity of the metallic surface remains constant. What is the value displayed by the sensor when the temperature of the metal surface is raised to 2767°C? (2016 Adv.)
- **30.** Two spherical bodies A (radius 6 cm) and B (radius 18 cm) are at temperatures T_1 and T_2 , respectively. The maximum intensity in the emission spectrum of A is at 500 nm and in that of B is at 1500 nm. Considering them to be black bodies, what will be the ratio of the rate of total energy radiated by A to that of B? (2010)

Fill in the Blanks

31. Two metal cubes A and B of same size are arranged as shown in figure. The extreme ends of the combination are maintained at the indicated temperatures. The arrangement is thermally insulated. The coefficients of thermal conductivity of A and B are 300 W/m °C and 200 W/m °C, respectively. After steady state is reached the temperature T of the interface will be (1996, 2M)



- 32. A point source of heat of power P is placed at the centre of a spherical shell of mean radius R. The material of the shell has thermal conductivity K. If the temperature difference between the outer and inner surface of the shell is not to exceed T, the thickness of the shell should not be less than (1991, 1M)
- **33.** A solid copper sphere (density ρ and specific heat *c*) of radius *r* at an initial temperature 200 K is suspended inside a chamber whose walls are at almost 0 K. The time required for the temperature of the sphere to drop to 100 K is (1991, 2M)
- **34.** The earth receives at its surface radiation from the sun at the rate of 1400 Wm^{-2} . The distance of the centre of the sun from the surface of the earth is 1.5×10^{11} m and the radius of the sun is 7×10^8 m. Treating the sun as a black body, it follows from the above data that its surface temperature is K. (1989, 2M)

True / False

35. Two spheres of the same material have radii 1 m and 4 m, temperature 4000 K and 2000 K respectively. The energy radiated per second by the first sphere is greater than that by the second. (1988, 2M)

Analytical & Descriptive Questions

36. A double-pane window used for insulating a room thermally from outside consists of two glass sheets each of area 1 m^2 and thickness 0.01 m separated by a 0.05 m thick stagnant air space. In the steady state, the room glass interface and the glass-outdoor interface are at constant temperatures of 27°C and 0°C respectively. Calculate the rate of heat flow through the window pane. Also, find the temperatures of other interfaces. Given, thermal conductivities of glass and air as 0.8 and 0.08 W m⁻¹K⁻¹ respectively. (1997C,5M)

Topic 4 Kinetic Theory of Gases and Gas Equations

Objective Questions I (Only one correct option)

 A diatomic gas with rigid molecules does 10 J of work when expanded at constant pressure. What would be the heat energy absorbed by the gas, in this process?
 (2019 Main, 12 April II)

2. Two moles of helium gas is mixed with three moles of hydrogen molecules (taken to be rigid). What is the molar specific heat of mixture at constant volume?

[Take, R = 8.3 J/mol-K]	(2019 Main, 12 April I)
(a) 19.7 J/mol-K	(b) 15.7 J/mol-K
(c) 17.4 J/mol-K	(d) 21.6 J/mol-K

3. One mole of an ideal gas passes through a process, where $\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$

pressure and volume obey the relation $p = p_0 \left[1 - \frac{1}{2} \left(\frac{V_0}{V} \right)^2 \right].$

- **37.** A cylindrical block of length 0.4 m and area of cross-section 0.04 m^2 is placed coaxially on a thin metal disc of mass 0.4 kg and of the same cross-section. The upper face of the cylinder is maintained at a constant temperature of 400 K and the initial temperature of the disc is 300 K. If the thermal conductivity of the material of the cylinder is 10 W/mK and the specific heat capacity of the material of the disc is 600 J/kg-K, how long will it take for the temperature of the disc to increase to 350 K? Assume, for purposes of calculation, the thermal conductivity of the disc to be very high and the system to be thermally insulated except for the upper face of the cylinder. (1992, 8M)
- **38.** An electric heater is used in a room of total wall area $137m^2$ to maintain a temperature of $+20^{\circ}$ C inside it, when the outside temperature is -10° C. The walls have three different layers. The innermost layer is of wood of thickness 2.5 cm, the middle layer is of cement of thickness 1.0 cm and the outermost layer is of brick of thickness 25.0 cm. Find the power of the electric heater. Assume that there is no heat loss through the floor and the ceiling. The thermal conductivities of wood, cement and brick are 0.125, 1.5 and 1.0 W/m/°C respectively. (1986, 8M)
- **39.** A solid sphere of copper of radius *R* and a hollow sphere of the same material of inner radius *r* and outer radius *R* are heated to the same temperature and allowed to cool in the same environment. Which of them starts cooling faster? (1982, 2M)
- **40.** A room is maintained at 20°C by a heater of resistance 20 Ω connected to 200 V mains. The temperature is uniform throughout the room and the heat is transmitted through a glass window of area 1 m² and thickness 0.2 cm. Calculate the temperature outside. Thermal conductivity of glass is 0.2 cal m⁻¹s⁻¹ (°C)⁻¹ and mechanical equivalent of heat is 4.2 Jcal⁻¹. (1978)

Here, p_0 and V_0 are constants. Calculate the change in the temperature of the gas, if its volume changes from V_0 to $2V_0$. (2019 Main. 10 April II)

(a)
$$\frac{1}{2} \frac{p_0 V_0}{R}$$
 (b) $\frac{1}{4} \frac{p_0 V_0}{R}$ (c) $\frac{3}{4} \frac{p_0 V_0}{R}$ (d) $\frac{5}{4} \frac{p_0 V_0}{R}$

- 4. A 25×10^{-3} m³ volume cylinder is filled with 1 mole of O₂ gas at room temperature (300 K). The molecular diameter of O₂ and its root mean square speed are found to be 0.3 nm and 200 m/s, respectively. What is the average collision rate (per second) for an O₂ molecule? (2019 Main, 10 April I) (a) ~ 10¹⁰ (b) ~ 10¹² (c) ~ 10¹¹ (d) ~ 10¹³
- **5.** A cylinder with fixed capacity of 67.2 L contains helium gas at STP. The amount of heat needed to raise the temperature of the gas by 20°C is

[Take,
$$R = 8.31 \text{ J mol}^{-1}\text{K}^{-1}$$
] (2019 Main, 10 April I)
(a) 700 J (b) 748 J (c) 374 J (d) 350 J

6. The specific heats, C_p and C_V of a gas of diatomic molecules,

A are given (in units of $J \mod^{-1} K^{-1}$) by 29 and 22, respectively. Another gas of diatomic molecules B, has the corresponding values 30 and 21. If they are treated as ideal (2019 Main, 9 April II) gases, then

- (a) A has a vibrational mode but B has none
- (b) Both A and B have a vibrational mode each
- (c) A has one vibrational mode and B has two
- (d) A is rigid but B has a vibrational mode
- 7. An HCl molecule has rotational, translational and vibrational motions. If the rms velocity of HCl molecules in its gaseous phase is \overline{v} , *m* is its mass and k_B is Boltzmann constant, then its temperature will be (2019 Main, 9 April I)

(a)
$$\frac{m\overline{v}^2}{3k_B}$$
 (b) $\frac{m\overline{v}^2}{7k_B}$ (c) $\frac{m\overline{v}^2}{5k_B}$ (d) $\frac{m\overline{v}^2}{6k_B}$

- 8. For a given gas at 1 atm pressure, rms speed of the molecules is 200 m/s at 127°C. At 2 atm pressure and at 227°C, the rms speed of the molecules will be (2019 Main, 9 April I) (a) $100\sqrt{5}$ m/s (b) $80 \,\text{m/s}$ (c) 100 m/s(d) $80\sqrt{5}$ m/s
- 9. The temperature, at which the root mean square velocity of hydrogen molecules equals their escape velocity from the earth, is closest to :

[Boltzmann constant $k_B = 1.38 \times 10^{-23}$ J/K, Avogadro number $N_A = 6.02 \times 10^{26}$ /kg, Radius of earth = 6.4×10^6 m, Gravitational acceleration on earth = 10 ms^{-2}]

(2019 Main, 8 Apri, II) (a) 10^4 K (c) 3×10^5 K (d) 800K (b) 650 K

- **10.** An ideal gas is enclosed in a cylinder at pressure of 2 atm and temperature, 300 K. The mean time between two successive collisions is 6×10^{-8} s. If the pressure is doubled and temperature is increased to 500 K, the mean time between two successive collisions will be close to (2019 Main, 12 Jan II) (b) 3×10^{-6} s (a) 4×10^{-8} s (d) 0.5×10^{-8} s (c) 2×10^{-7} s
- **11.** A vertical closed cylinder is separated into two parts by a frictionless piston of mass *m* and of negligible thickness. The piston is free to move along the length of the cylinder. The length of the cylinder above the piston is l_1 and that below the piston is l_2 , such that $l_1 > l_2$. Each part of the cylinder contains n moles of an ideal gas at equal temperature T. If the piston is stationary, its mass m, will be given by

(where, R is universal gas constant and g is the acceleration (2019 Main019, 12 Jan II) due to gravity)

(a) $\frac{nRT}{g} \left[\frac{l_1 - l_2}{l_1 l_2} \right]$	(b) $\frac{nRT}{g}\left[\frac{1}{l_2} + \frac{1}{l_1}\right]$
(c) $\frac{RT}{g} \left[\frac{2l_1 + l_2}{l_1 l_2} \right]$	(d) $\frac{RT}{ng} \left[\frac{l_1 - 3l_2}{l_1 l_2} \right]$

12. An ideal gas occupies a volume of 2 m^3 at a pressure of 3×10^6 Pa. The energy of the gas is (2019 Main, 12 Jan I) (a) 6×10^4 J (b) 10^8 J (c) 9×10^6 J (d) 3×10^2 J

13. A gas mixture consists of 3 moles of oxygen and 5 moles of argon at temperature T. Considering only translational and rotational modes, the total internal energy of the system is (2019 Main, 11 Jan I)

(b) 15 *RT* (c) 20 RT(a) 12 *RT* (d) 4 *RT*

- **14.** 2 kg of a monoatomic gas is at a pressure of 4×10^4 N/m². The density of the gas is 8 kg/m^3 . What is the order of energy of the gas due to its thermal motion ? (2019 Main, 10 Jan II) (b) 10^3 J (c) 10^4 J (d) 10^5 J (a) 10^6 J
- 15. A 15 g mass of nitrogen gas is enclosed in a vessel at a temperature 27°C. Amount of heat transferred to the gas, so that rms velocity of molecules is doubled is about

(Take, R = 8.3 J/K - mol)(2019 Main, 9 Jan II) (a) 10 kJ (b) 0.9 kJ (c) 14 kJ (d) 6 kJ

16. A mixture of 2 moles of helium gas (atomic mass = 4u) and 1 mole of argon gas (atomic mass = 40u) is kept at 300 K in a container. The ratio of their rms speeds

$$\begin{bmatrix} \frac{v_{\rm rms(helium)}}{v_{\rm rms(argon)}} \end{bmatrix}$$
 is close to
(2019 Main, 9 Jan I)
(a) 0.32 (b) 2.24 (c) 3.16 (d) 0.45

17. The mass of a hydrogen molecule is 3.32×10^{-27} kg. If 10^{23} hydrogen molecules strike per second, a fixed wall of area 2 cm^2 at an angle of 45° to the normal and rebound elastically with a speed of 10^3 m/s, then the pressure on the wall is nearly (2018 Main)

(a)
$$2.35 \times 10^4$$
 N/m²
(b) 2.35×10^3 N/m²
(c) 4.70×10^2 N/m²
(d) 2.35×10^3 N/m²

- **18.** The temperature of an open room of volume 30 m^3 increases from 17°C to 27°C due to the sunshine. The atmospheric pressure in the room remains 1×10^5 Pa. If n_i and n_f are the number of molecules in the room before and after heating, then $n_f - n_i$ will be (2017 Main) (b) 2.5×10^{25} (a) 1.38×10^{23} (c) -2.5×10^{25} (d) -1.61×10^{23}
- **19.** An ideal gas undergoes a quasi static, reversible process in which its molar heat capacity C remains constant. If during this process the relation of pressure p and volume V is given by pV^n = constant, then *n* is given by (Here, C_p and C_V are molar specific heat at constant pressure and constant volume, respectively) (2016 Main)

(a)
$$n = \frac{C_p}{C_V}$$

(b) $n = \frac{C - C_p}{C - C_V}$
(c) $n = \frac{C_p - C}{C - C_V}$
(d) $n = \frac{C - C_V}{C - C_p}$

20. Two non-reactive monoatomic ideal gases have their atomic masses in the ratio 2:3. The ratio of their partial pressures, when enclosed in a vessel kept at a constant temperature, is 4:3. The ratio of their densities is (2013 Adv.) (a) 1:4 (b) 1:2 (c)

21. A mixture of 2 moles of helium gas (atomic mass = 4 amu) and 1 mole of argon gas (atomic mass = 40 amu) is kept at 300 K in a container. The ratio of the rms speeds $\left(\frac{v_{\rm rms} \ (\rm helium)}{v_{\rm rms} \ (\rm argon)}\right)$ is (2012)

(a) 0.32 (b) 0.45 (c) 2.24 (d) 3.16

- **22.** A real gas behaves like an ideal gas if its (2010) (a) pressure and temperature are both high (b) pressure and temperature are both low (c) pressure is high and temperature is low (d) pressure is low and temperature is high
- **23.** A gas mixture consists of 2 moles of oxygen and 4 moles of argon at temperature T. Neglecting all vibrational modes, the total internal energy of the system is (1999, 2M) (a) 4 *RT* (b) 15 *RT* (c) 9 *RT* (d) 11 RT
- **24.** Two identical containers A and B with frictionless pistons contain the same ideal gas at the same temperature and the same volume V. The mass of the gas in A is m_A and that in B is m_{R} . The gas in each cylinder is now allowed to expand isothermally to the same final volume 2V. The changes in the pressure in A and B are found to be Δp and 1.5 Δp respectively. Then (1998, 2M) (a) $4 m_A = 9 m_B$
 - (b) $2 m_A = 3 m_B$ (d) $9 m_A = 4 m_B$ (c) $3 m_A = 2 m_B$
- **25.** A vessel contains a mixture of one mole of oxygen and two moles of nitrogen at 300 K. The ratio of the average rotational kinetic energy per O2 molecule to per N2 molecule (1998, 2M) is (a) 1 : 1 (b) 1 : 2 (c) 2 : 1

(d) depends on the moment of inertia of the two molecules

- **26.** A vessel contains 1 mole of O_2 gas (molar mass 32) at a temperature T. The pressure of the gas is p. An identical vessel containing one mole of the gas (molar mass 4) at a temperature 2T has a pressure of (1997, 1M) (d) 8 p (a) p/8(b) *p* (c) 2 p
- **27.** The average translational kinetic energy of O_2 (molar mass 32) molecules at a particular temperature is 0.048 eV. The translational kinetic energy of N₂ (molar mass 28) molecules in eV at the same temperature is (1997, 1M)

(a) 0.0015 (b) 0.003 (c) 0.048 (d) 0.768

28. The average translational energy and the rms speed of molecules in a sample of oxygen gas at 300 K are 6.21×10^{-21} J and 484 m/s respectively. The corresponding values at 600 K are nearly (assuming ideal gas behaviour)

(1997, 1M) (a) 12.42×10^{-21} J, 968 m/s (b) 8.78×10^{-21} J, 684 m/s (c) 6.21×10^{-21} J, 968 m/s (d) 12.42×10^{-21} J, 684 m/s

29. The temperature of an ideal gas is increased from 120 K to 480 K. If at 120 K the root mean square velocity of the gas molecules is v, at 480 K it becomes (1996, 2M) (a) 4 v (b) 2 v (c) v/2(d) v/4

- **30.** Three closed vessels A, B and C at the same temperature T and contain gases which obey the Maxwellian distribution of velocities. Vessel A contains only O_2 , B only N_2 and C a mixture of equal quantities of O2 and N2. If the average speed of the O_2 molecules in vessel A is v_1 , that of the N_2 molecules in vessel B is v_2 , the average speed of the O_2 molecules in vessel C is (1992, 2M)
 - (a) $(v_1 + v_2)/2$ (c) $(v_1 v_2)^{1/2}$ (b) v_1 (d) $\sqrt{3kT/M}$

where, M is the mass of an oxygen molecule.

- **31.** If one mole of a monoatomic gas $(\gamma = 5/3)$ is mixed with one mole of a diatomic gas ($\gamma = 7/5$), the value of γ for the mixture is (1988, 2M) (a) 1.40 (b) 1.50 (d) 3.07 (c) 1.53
- **32.** At room temperature, the rms speed of the molecules of a certain diatomic gas is found to be 1930 m/s. The gas is (a) H₂ (b) F₂ (c) O_2 (d) Cl_2

Assertion and Reason

Mark vour answer as

- (a) If Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
- (b) If Statement I is true, Statement II is true; Statement II is not a correct explanation for Statement I
- (c) If Statement I is true; Statement II is false
- (d) If Statement I is false; Statement II is true
- **33.** Statement I The total translational kinetic energy of all the molecules of a given mass of an ideal gas is 1.5 times the product of its pressure and its volume.

Statement II The molecules of a gas collide with each other and the velocities of the molecules change due to the collision. (2007, 3M)

Objective Questions II (One or more correct option)

- **34.** A container of fixed volume has a mixture of one mole of hydrogen and one mole of helium in equilibrium at temperature T. Assuming the gases are ideal, the correct statements is/are (2015 Adv.)
 - (a) The average energy per mole of the gas mixture is 2RT
 - (b) The ratio of speed of sound in the gas mixture to that in helium gas is $\sqrt{\frac{6}{5}}$
 - (c) The ratio of the rms speed of helium atoms to that of hydrogen molecules is $\frac{1}{2}$
 - (d) The ratio of the rms speed of helium atoms to that of hydrogen molecules is $\frac{1}{\sqrt{2}}$
- **35.** Let \overline{v} , $v_{\rm rms}$ and v_p respectively denote the mean speed, root mean square speed and most probable speed of the molecules in an ideal monoatomic gas at absolute temperature T. The mass of a molecule is m. Then, (1998, 2M)

(a) no molecule can have a speed greater than $\sqrt{2}v_{\rm rms}$

(b) no molecule can have speed less than $v_p/\sqrt{2}$

(c) $v_p < \overline{v} < v_{rms}$ (d) the average kinetic energy of a molecule is $\frac{3}{4} m v_p^2$

- **36.** From the following statements concerning ideal gas at any given temperature T, select the correct one (s). (1995, 2M) (a) The coefficient of volume expansion at constant pressure is the same for all ideal gases
 - (b) The average translational kinetic energy per molecule of oxygen gas is 3 kT, k being Boltzmann constant
 - (c) The mean-free path of molecules increases with decrease in the pressure
 - (d) In a gaseous mixture, the average translational kinetic energy of the molecules of each component is different

Fill in the Blanks

- **37.** A container of volume 1 m^3 is divided into two equal parts by a partition. One part has an ideal gas at 300 K and the other part is vacuum. The whole system is thermally isolated from the surroundings. When the partition is removed, the gas expands to occupy the whole volume. Its temperature will now be (1993, 1M)
- **38.** During an experiment, an ideal gas is found to obey an additional law $p^2 V = \text{constant}$. The gas is initially at a temperature T and volume V. When it expands to a volume 2V, the temperature becomes (1987, 2M)
- **39.** One mole of a monoatomic ideal gas is mixed with one mole of a diatomic ideal gas. The molar specific heat of the mixture at constant volume is (1984, 2M)

Topic 5 Thermodynamics

Objective Questions I (Only one correct option)

- 1. A Carnot engine has an efficiency of 1/6. When the temperature of the sink is reduced by 62°C, its efficiency is doubled. The temperatures of the source and the sink are respectively, (2019 Main, 12 April II)
 - (a) 62°C, 124°C
 - (b) 99°C, 37°C
 - (c) 124°C, 62°C
 - (d) 37°C, 99°C
- 2. A sample of an ideal gas is taken through the cyclic process abca as shown in the figure. The change in the internal energy of the gas along the path ca is -180 J. The gas absorbs 250 J of heat along the path ab and 60 J along the path bc. The work done by the gas along the path *abc* is

(2019 Main, 12 April I)

True / False

- **40.** The root mean square (rms) speed of oxygen molecules (O_2) at a certain temperature T (degree absolute) is V. If the temperature is doubled and oxygen gas dissociates into atomic oxygen, the rms speed remains unchanged. (1987, 2M)
- **41.** Two different gases at the same temperature have equal root mean square velocities. (1982.2M)
- **42.** The volume *V* versus temperature *T* graphs for a certain amount of a perfect gas at two pressure p_1 and p_2 are as shown in figure. It follows from the graphs that p_1 is greater than p_2 .(1982, 2M)



43. The root mean square speeds of the molecules of different ideal gases, maintained at the same temperature are the same. (1981, 2M)

Analytical & Descriptive Questions

44. A cubical box of side 1 m contains helium gas (atomic weight 4) at a pressure of 100 N/m^2 . During an observation time of 1 s, an atom travelling with the root mean square speed parallel to one of the edges of the cube, was found to make 500 hits with a particular wall, without any collision with other atoms. Take,

$$R = \frac{25}{3}$$
 J/mol-K and $k = 1.38 \times 10^{-23}$ J/K. (2002, 5M)

- (a) Evaluate the temperature of the gas.
- (b) Evaluate the average kinetic energy per atom.
- (c) Evaluate the total mass of helium gas in the box.



(a) 120 J (b) 130 J (c) 100 J (d) 140 J

3. When heat *Q* is supplied to a diatomic gas of rigid molecules, at constant volume, its temperature increases by ΔT . The heat required to produce the same change in temperature, at a constant pressure is (2019 Main, 10 April II)

(a)
$$\frac{2}{3}Q$$
 (b) $\frac{5}{3}Q$ (c) $\frac{3}{2}Q$ (d) $\frac{7}{5}Q$

4. *n* moles of an ideal gas with constant volume heat capacity C_V undergo an isobaric expansion by certain volume. The ratio of the work done in the process, to the heat supplied is (2019 Main, 10 April I)

(a)
$$\frac{4nR}{C_V + nR}$$
 (b) $\frac{4nR}{C_V - nR}$
(c) $\frac{nR}{C_V - nR}$ (d) $\frac{nR}{C_V + nR}$

5. Following figure shows two processes *A* and *B* for a gas. If ΔQ_A and ΔQ_B are the amount of heat absorbed by the system in two cases, and ΔU_A and ΔU_B are changes in internal energies respectively, then (2019 Main, 9 April I)



(a)
$$\Delta Q_A > \Delta Q_B$$
, $\Delta U_A > \Delta U_B$
(b) $\Delta Q_A < \Delta Q_B$, $\Delta U_A < \Delta U_B$
(c) $\Delta Q_A > \Delta Q_B$, $\Delta U_A = \Delta U_B$

- (d) $\Delta Q_A = \Delta Q_B$; $\Delta U_A = \Delta U_B$
- **6.** The given diagram shows four processes, i.e. isochoric, isobaric, isothermal and adiabatic. The correct assignment of the processes, in the same order is given by





For the given cyclic process CAB as shown for a gas, the work done is (2019 Main, 12 Jan I)



8. In a process, temperature and volume of one mole of an ideal monoatomic gas are varied according to the relation VT = k, where *k* is a constant. In this process, the temperature of the

gas is increased by ΔT . The amount of heat absorbed by gas is (where, *R* is gas constant) (2019 Main, 11 Jan II) (2019 Main, 11 Jan II)

(a)
$$\frac{1}{2} kR\Delta T$$
 (b) $\frac{2k}{3}\Delta T$ (c) $\frac{1}{2} R\Delta T$ (d) $\frac{5}{2} R\Delta T$

9. A rigid diatomic ideal gas undergoes an adiabatic process at room temperature. The relation between temperature and volume for this process is TV^x = constant, then x is

(b)
$$\frac{2}{3}$$
 (c) $\frac{5}{3}$ (d) $\frac{3}{5}$

10. Half-mole of an ideal monoatomic gas is heated at constant pressure of 1 atm from 20° C to 90° C. Work done by gas is close to (Take, gas constant, R = 8.31 J/mol-K) (Main 2019, 10 Jan II)

(a) $\frac{2}{5}$

11. Three Carnot engines operate in series between a heat source at a temperature T_1 and a heat sink at temperature T_4 (see figure). There are two other reservoirs at temperatures T_2 and T_3 , as shown with $T_1 > T_2 > T_3 > T_4$. The three engines are equally efficient if (Main 2019, 10 Jan I)



- **12.** Two Carnot engines *A* and *B* are operated in series. The first one, *A* receives heat at $T_1 (= 600 \text{ K})$ and rejects to a reservoir at temperature T_2 . The second engine *B* receives heat rejected by the first engine and in turn rejects to a heat reservoir at T_3 (= 400 K). Calculate the temperature T_2 if the work outputs of the two engines are equal. (2019 Main, 9 Jan II) (a) 600 K (b) 500 K (c) 400 K (d) 300 K
- 13. A gas can be taken from *A* to *B via* two different processes *ACB* and *ADB*. (2019 Main, 9 Jan I)



When path ACB is used 60 J of heat flows into the system and 30 J of work is done by the system. If path ADB is used work done by the system is 10 J the heat flow into the system in path ADB is

- (a) 80 J (b) 40 J (c) 100 J (d) 20 J
- 14. Two moles of an ideal monoatomic gas occupies a volume V at 27°C. The gas expands adiabatically to a volume 2V. Calculate (i) the final temperature of the gas and (ii) change in its internal energy. (2018 Main)
 (a) (i) 195 K (ii) 2.7 kJ
 (b) (i) 189 K (ii) 2.7 kJ
 (c) (i) 195 K (ii) -2.7 kJ
 (d) (i) 189 K (ii) -2.7 kJ
- **15.** *n* moles of an ideal gas undergoes a process *A* and *B* as shown in the figure. The maximum temperature of the gas during the process will be (2016 Main)



- **16.** A gas is enclosed in a cylinder with a movable frictionless piston. Its initial thermodynamic state at pressure $p_i = 10^5$ Pa and volume $V_1 = 10^{-3}$ m³ changes to a final state at $p_f = (1/32) \times 10^5$ Pa and $V_f = 8 \times 10^{-3}$ m³ in an adiabatic quasi-static process, such that $p^3 V^5$ = constant. Consider another thermodynamic process that brings the system from the same initial state to the same final state in two steps : an isobaric expansion at p_i followed by an isochoric (isovolumetric) process at volume V_f . The amount of heat supplied to the system in the two-step process is approximately (2016 Adv.) (a) 112 J (b) 294 J (c) 588 J (d) 813 J
- 17. One mole of diatomic ideal gas undergoes a cyclic process *ABC* as shown in figure. The process *BC* is adiabatic. The temperatures at *A*, *B* and *C* are 400 K, 800 K and 600 K, respectively. Choose the correct statement. (2014 Main)



- (a) The change in internal energy in whole cyclic process is 250 R
- (b) The change in internal energy in the process CA is 700 R
- (c) The change in internal energy in the process AB is -350R
- (d) The change in internal energy in the process BC is -500 R
- **18.** The shown *p*-*V* diagram represents the thermodynamic cycle of an engine, operating with an ideal monoatomic gas. The amount of heat, extracted from the source in a single cycle is (2013 Main)



19. One mole of a monatomic ideal gas is taken along two cyclic processes $E \to F \to G \to E$ and $E \to F \to H \to E$ as shown in the *p*-*V* diagram. (2013 Adv.)



The processes involved are purely isochoric, isobaric, isothermal or adiabatic.

Match the paths in List I with the magnitudes of the work done in List II and select the correct answer using the codes given below the lists.

-									
	List I				List II				
	Р.		G	$\rightarrow E$	1.	160 p	$_{0}V_{0}\ln$	2	
	(Q.	G	$\rightarrow H$	2.	$36 p_0 l$	0		
]	R.	F	$\rightarrow H$	3.	24 $p_0 l$	0		
	1	S.	F	$\rightarrow G$	4.	$31 p_0 V$	0		
Co	des								
	Р	Q	R	S		Р	Q	R	S
(a)	4	3	2	1		(b) 4	3	1	2
(c)	3	1	2	4		(d) 1	3	2	4

20. 5.6 L of helium gas at STP is adiabatically compressed to 0.7 L. Taking the initial temperature to be T_1 , the work done in the process is (2011)

(a)
$$\frac{9}{8}RT_1$$
 (b) $\frac{3}{2}RT_1$ (c) $\frac{15}{8}RT_1$ (d) $\frac{9}{2}RT_1$

- **21.** An ideal gas expands isothermally from a volume V_1 to V_2 and then compressed to original volume V_1 adiabatically. Initial pressure is p_1 and final pressure is p_3 . The total work done is *W*. Then, (2004, 2M)
 - (a) $p_3 > p_1, W > 0$ (b) $p_3 < p_1, W < 0$

(c)
$$p_3 > p_1, W < 0$$
 (d) $p_3 = p_1, W = 0$

22. An ideal gas is taken through the cycle $A \rightarrow B \rightarrow C \rightarrow A$, as shown in the figure. If the net heat supplied to the gas in the cycle is 5 J, the work done by the gas in the process $C \rightarrow A$ is (2002, 2M)



23. *p-V* plots for two gases during adiabatic processes are shown in the figure. Plots 1 and 2 should correspond respectively to (2001, 2M)



24. Starting with the same initial conditions, an ideal gas expands from volume V_1 to V_2 in three different ways, the work done by the gas is W_1 if the process is purely isothermal, W_2 if purely isobaric and W_3 if purely adiabatic, then (2000, 2M)

(a) $W_2 > W_1 > W_3$	(b) $W_2 > W_3 > W_1$
(c) $W_1 > W_2 > W_3$	(d) $W_1 > W_3 > W_2$

25. A monoatomic ideal gas, initially at temperature T_1 , is enclosed in a cylinder fitted with a frictionless piston. The gas is allowed to expand adiabatically to a temperature T_2 by releasing the piston suddenly. If L_1 and L_2 are the lengths of the gas column before and after expansion respectively, then T_1/T_2 is given by (2000, 2M) (a) $(L_1/L_2)^{2/3}$ (b) (L_1/L_2)

(c)
$$L_2/L_1$$
 (d) $(L_2/L_1)^{2/3}$

26. Two cylinders A and B fitted with pistons contain equal amounts of an ideal diatomic gas at 300 K. The piston of A is free to move, while that of B is held fixed. The same amount of heat is given to the gas in each cylinder. If the rise in temperature of the gas in A is 30 K, then the rise in temperature of the gas in B is (1998, 2M) (a) 30 K (b) 18 K (c) 50 K (d) 42 K

- 27. When an ideal diatomic gas is heated at constant pressure the fraction of the heat energy supplied which increases the internal energy of the gas is (1990, 2M) (a) $\frac{2}{5}$ (b) $\frac{3}{5}$ (c) $\frac{3}{7}$ (d) $\frac{5}{7}$
- 28. An ideal monoatomic gas is taken round the cycle ABCDA as shown in the p-V diagram (see figure). The work done during the cycle is (1983, 1M)

(a)
$$pV$$
 (b) 2 pV (c) $\frac{1}{2}pV$ (d) zero

Match the Columns

Directions (Q.Nos. 15-18) Matching the information given in the three columns of the following table.

An ideal gas is undergoing a cyclic thermodynamic process in different ways as shown in the corresponding *p-V* diagrams in column 3 of the table. Consider only the path from state 1 to state 2. *W* denotes the corresponding work done on the system. The equations and plots in the table have standards notations and used in thermodynamic processes. Here γ is the ratio of heat capacities at constant pressure and constant volume. The number of moles in the gas is *n*.

29. One mole of a monoatomic ideal gas undergoes four thermodynamic processes as shown schematically in the *pV*-diagram below. Among these four processes, one is isobaric, one is isochoric, one is isothermal and one is adiabatic. Match the processes mentioned in List-l with the corresponding statements in List-II. (2018 Adv.)

List-I			List-II		
	Р.	In process I	1.	Work done by the gas is zero	
	Q.	In process II	2.	Temperature of the gas remains unchanged	
	R.	In process III	3.	No heat is exchanged between the gas and its surroundings	
	S.	In process IV	4.	Work done by the gas is $6p_0V_0$	
((a) P	\rightarrow 4; Q \rightarrow 3; R	$\rightarrow 1$; $S \rightarrow 2$	
((b) $P \rightarrow 1$; $Q \rightarrow 3$; $R \rightarrow 2$; $S \rightarrow 4$				
((c) $P \rightarrow 3$; $Q \rightarrow 4$; $R \rightarrow 1$; $S \rightarrow 2$				
((d) P	\rightarrow 3; Q \rightarrow 4; R	$\rightarrow 2$; $S \rightarrow 1$	

	Column 1	C	Column 2		Column 3
(I)	$W_{1 \to 2} = \frac{1}{\gamma - 1} (p_2 V_2 - p_1 V_1)$	(i)	Isothermal	(P)	

(II) $W_{1\rightarrow2} = -pV_2 + pV_1$ (ii) Isochoric (Q)



30. Which one of the following options correctly represents a thermodynamic process that is used as a correction in the determination of the speed of sound in an ideal gas?

(2017 Adv.)

(a) (IV) (ii) (R)	(b) (I) (ii) (Q)
(c) (I), (iv) (Q)	(d) (III) (iv) (R)

- **31.** Which of the following options is the only correct representation of a process in which $\Delta U = \Delta Q p\Delta V$? (a) (II) (iii) (S) (b) (II) (iii) (P) (2017 Adv.) (c) (III) (iii) (P) (d) (II) (iv) (R)
- 32. Which one of the following options is the correct combination? (2017 Adv.)
 (a) (II) (iv) (P) (b) (III) (ii) (S)
 (c) (II) (iv) (R) (d) (IV) (ii) (S)
- **33.** One mole of a monatomic ideal gas is taken through a cycle *ABCDA* as shown in the *p-V* diagram. **Column II** gives the characteristics involved in the cycle. Match them with each of the processes given in **Column I**. (2011)



	Column I		Column II
(A)	Process $A \rightarrow B$	(p)	Internal energy decreases
(B)	Process $B \to C$	(q)	Internal energy increases
(C)	Process $C \rightarrow D$	(r)	Heat is lost
(D)	Process $D \to A$	(s)	Heat is gained
		(t)	Work is done on the gas

34. Match the following for the given process (2006, 6M)



	Column I	Column II		
(A)	Process $J \to K$	(p)	Q > 0	
(B)	Process $K \to L$	(q)	W < 0	
(C)	Process $L \to M$	(r)	W > 0	
(D)	Process $M \to J$	(s)	<i>Q</i> < 0	

Objective Questions II (One or more correct option)

35. One mole of a monoatomic ideal gas undergoes a cyclic process as shown in the figure (where, *V* is the volume and *T* is the temperature). Which of the statements below is (are) true ? (2018 Adv.)



- (a) Process I is an isochoric process
- (b) In process II, gas absorbs heat
- (c) In process IV, gas releases heat
- (d) Processes I and III are not isobaric
- **36.** An ideal monoatomic gas is confined in a horizontal cylinder by a spring loaded piston (as shown in the figure). Initially the gas is at temperature T_1 ,



pressure p_1 and volume V_1 and the spring is in its relaxed state. The gas is then heated very slowly to temperature T_2 , pressure p_2 and volume V_2 . During this process the piston moves out by a distance x. (2015 Adv.)

Ignoring the friction between the piston and the cylinder, the correct statements is/are

(a) If $V_2 = 2V_1$ and $T_2 = 3T_1$,

then the energy stored in the spring is $\frac{1}{4} p_1 V_1$

(b) If V₂ = 2V₁ and T₂ = 3T₁, then the change in internal energy is 3 p₁V₁
(c) If V₂ = 3V₁ and T₂ = 4T₁,

then the work done by the gas is $\frac{7}{3}p_1V_1$

- (d) If $V_2 = 3V_1$ and $T_2 = 4T_1$, then the heat supplied to the gas is
- **37.** One mole of an ideal gas in initial state *A* undergoes a cyclic process *ABCA*, as shown in the figure. Its pressure at *A* is p_0 . Choose the correct option(s) from the following. (2010)



- (a) Internal energies at A and B are the same
- (b) Work done by the gas in process AB is $p_0V_0 \ln 4$

(c) Pressure at *C* is
$$\frac{p_0}{4}$$

- (d) Temperature at C is $\frac{T_0}{4}$
- **38.** The figure shows the p-V plot an ideal gas taken through a cycle *ABCDA*. The part *ABC* is a semi-circle and *CDA* is half of an ellipse. Then, (2009)



- (a) the process during the path $A \rightarrow B$ is isothermal
- (b) heat flows out of the gas during the path $B \to C \to D$
- (c) work done during the path $A \rightarrow B \rightarrow C$ is zero (d) positive work is done by the gas in the cycle *ABCDA*
- 39. During the melting of a slab of ice at 273 K at atmospheric pressure (1998, 2M)
 (a) positive work is done by the ice-water system on the
 - atmosphere
 - (b) positive work is done on the ice-water system by the atmosphere
 - (c) the internal energy of the ice-water increases
 - (d) the internal energy of the ice-water system decreases
- **40.** For an ideal gas

(1989, 2M)

- (a) the change in internal energy in a constant pressure process from temperature T_1 to T_2 is equal to $nC_V (T_2 T_1)$, where C_V is the molar heat capacity at constant volume and n the number of moles of the gas
- (b) the change in internal energy of the gas and the work done by the gas are equal in magnitude in an adiabatic process
- (c) the internal energy does not change in an isothermal process
- (d) no heat is added or removed in an adiabatic process

Numerical Value

41. One mole of a monoatomic ideal gas undergoes an adiabatic expansion in which its volume becomes eight times its initial value. If the initial temperature of the gas is 100 K and the universal gas constant R = 8.0 j mol⁻¹ K⁻¹, the decrease in its internal energy in joule, is (2018 Adv.)

Integer Answer Type Questions

42. A thermodynamic system is taken from an initial state *i* with internal energy $U_i = 100$ J to the final state *f* along two different paths *iaf* and *ibf*, as schematically shown in the figure. The work done by the



system along the paths *af*, *ib* and *bf* are $W_{af} = 200 \text{ J}$, $W_{ib} = 50 \text{ J}$ and $W_{bf} = 100 \text{ J}$ respectively. The heat supplied to the system along the path *iaf*, *ib* and *bf* are Q_{iaf} , Q_{ib} and Q_{bf} respectively. If the internal energy of the system in the state *b* is $U_b = 200 \text{ J}$ and $Q_{iaf} = 500 \text{ J}$, the ratio Q_{bf} / Q_{ib} **ig2014 Adv.**)

43. A diatomic ideal gas is compressed adiabatically to $\frac{1}{32}$ of its

initial volume. If the initial temperature of the gas is T_i (in kelvin) and the final temperature is aT_i , the value of *a* is (2010)

Fill in the Blank

44. An ideal gas with pressure p, volume V and temperature T is expanded isothermally to a volume 2V and a final pressure p_i . If the same gas is expanded adiabatically to a volume 2V, the final pressure is p_a . The ratio of the specific heats of the gas is 1.67. The ratio p_a / p_i is (1994, 2M)

True / False

45. The curves *A* and *B* in the figure show *p*-*V* graphs for an isothermal and an adiabatic process for an ideal gas. The isothermal process is represented by the curve *A*. (1985, 3M)



Analytical & Descriptive Questions

- **46.** A metal of mass 1 kg at constant atmospheric pressure and at initial temperature 20°C is given a heat of 20000 J. Find the following : (2005, 6M)
 - (a) change in temperature,
 - (b) work done and

(c) change in internal energy.

(Given, Specific heat = 400 J/kg/°C, coefficient of cubical expansion, $\gamma = 9 \times 10^{-5}$ /°C, density $\rho = 9000 \text{ kg/m}^3$, atmospheric pressure = 10^5 N/m^2)

47. A monoatomic ideal gas of two moles is taken through a cyclic process starting from A as shown in the figure. The volume ratio are $\frac{V_B}{V_A} = 2$ and $\frac{V_D}{V_A} = 4$. If the temperature

 $T_{A} \text{ at } A \text{ is } 27^{\circ}\text{C}.$ $V_{D} \xrightarrow{\uparrow} V_{V_{B}} \xrightarrow{\downarrow} A \xrightarrow{\downarrow} A \xrightarrow{\downarrow} A \xrightarrow{\downarrow} T \xrightarrow{\downarrow} B$

Calculate

(2001, 10M)

(a) the temperature of the gas at point *B*,

(b) heat absorbed or released by the gas in each process,(c) the total work done by the gas during the complete cycle.Express your answer in terms of the gas constant *R*.

48. Two moles of an ideal monoatomic gas is taken through a cycle *ABCA* as shown in the *p*-*T* diagram. During the process *AB*, pressure and temperature of the gas vary such that $pT = \text{constant. If } T_1 = 300 \text{ K}$, calculate (2000, 10M)



- (a) the work done on the gas in the process AB and
- (b) the heat absorbed or released by the gas in each of the processes.

Give answers in terms of the gas constant R.

- **49.** Two moles of an ideal monoatomic gas initially at pressure p_1 and volume V_1 undergo an adiabatic compression until its volume is V_2 . Then the gas is given heat Q at constant volume V_2 . (1999, 10M)
 - (a) Sketch the complete process on a p-V diagram.
 - (b) Find the total work done by the gas, the total change in internal energy and the final temperature of the gas.

(Give your answer in terms of p_1, V_1, V_2, Q and R)

50. One mole of an ideal monoatomic gas is taken round the cyclic process *ABCA* as shown in figure. Calculate (**1998**, **8M**)



- (a) the work done by the gas.
- (b) the heat rejected by the gas in the path *CA* and the heat absorbed by the gas in the path *AB*.
- (c) the net heat absorbed by the gas in the path BC.
- (d) the maximum temperature attained by the gas during the cycle.

51. A sample of 2 kg monoatomic helium (assumed ideal) is taken through the process ABC and another sample of 2 kg of the same gas is taken through the process ADC (see fig). Given molecular mass of helium = 4. (1997C, 5M)



- (a) What is the temperature of helium in each of the states *A*, *B*, *C* and *D*?
- (b) Is there any way of telling afterwards which sample of helium went through the process *ABC* and which went through the process *ADC*? Write Yes or No.
- (c) How much is the heat involved in the process *ABC* and *ADC*?
- 52. One mole of a diatomic ideal gas (γ = 1.4) is taken through a cyclic process starting from point *A*. The process *A* → *B* is an adiabatic compression. *B* → *C* is isobaric expansion, *C* → *D* an adiabatic expansion and *D* → *A* is isochoric. The volume ratio are V_A/V_B = 16 and V_C/V_B = 2 and the temperature at *A* is *T_A* = 300 K. Calculate the temperature of the gas at the points *B* and *D* and find the efficiency of the cycle. (1997, 5M)
- **53.** At 27°C two moles of an ideal monoatomic gas occupy a volume *V*. The gas expands adiabatically to a volume 2*V*.

Calculate

(1996, 5M)

(a) the final temperature of the gas,(b) change in its internal energy,

(c) the work done by the gas during this process.

54. An ideal gas is taken through a cyclic thermodynamic process through four steps. The amounts of heat involved in these steps are $Q_1 = 5960J$, $Q_2 = -5585J$, $Q_3 = -2980J$ and $Q_4 = 3645J$ respectively. The corresponding quantities of work involved are $W_1 = 2200J$, $W_2 = -825J$, $W_3 = -1100J$ and W_4 respectively. (1994, 6M)

(a) Find the value of W_4 .

(b) What is the efficiency of the cycle?

- **55.** One mole of a monoatomic ideal P gas is taken through the cycle shown in figure (1993, 4+4+2M) $A \rightarrow B$: adiabatic expansion
 - $A \rightarrow B$: adiabatic expansion $B \rightarrow C$: cooling at constant



 $C \rightarrow D$: adiabatic compression

 $D \rightarrow A$: heating at

constant volume.

The pressure and temperature at *A*, *B*, etc., are denoted by p_A , T_A , p_B , T_B etc., respectively. Given that, $T_A = 1000 \text{ K}$, $p_B = (2/3)p_A$ and $p_C = (1/3)p_A$, calculate the following quantities

Г

- (a) The work done by the gas in the process $A \rightarrow B$.
- (b) The heat lost by the gas in the process $B \to C$.
- (c) The temperature T_D . (Given : $(2/3)^{2/5} = 0.85$)
- 56. Two moles of helium gas undergo a cyclic process as shown in figure. Assuming the gas to be ideal, calculate the following quantities in this process. (1992, 8M)



- (a) The net change in the heat energy.
- (b) The net work done.
- (c) The net change in internal energy.
- **57.** Three moles of an ideal gas $(C_p = \frac{7}{2}R)$ at pressure, p_A and

temperature T_A is isothermally expanded to twice its initial volume. It is then compressed at constant pressure to its original volume. Finally gas is compressed at constant volume to its original pressure p_A . (1991, 4+4 M) (a) Sketch *p*-*V* and *p*-*T* diagrams for the complete process.

- (b) Calculate the net work done by the gas, and net heat supplied to the gas during the complete process.
- **58.** An ideal gas having initial pressure p, volume V and temperature T is allowed to expand adiabatically until its volume becomes 5.66 V while its temperature falls to T/2.

Topic 6 Miscellaneous Problems

Objective Questions I (Only one correct option)

1 The number density of molecules of a gas depends on their distance *r* from the origin as, $n(r) = n_0 e^{-ar^4}$. Then, the total number of molecules is proportional to

(a)
$$n_0 \alpha^{-3/4}$$
 (b) $\sqrt{n_0} \alpha^{1/2}$ (c) $n_0 \alpha^{1/4}$ (d) $n_0 \alpha^{-3}$

II)

- 2 If 10^{22} gas molecules each of mass 10^{-26} kg collide with a surface (perpendicular to it) elastically per second over an area 1 m² with a speed 10^4 m/s, the pressure exerted by the gas molecules will be of the order of (2019 Main, 8 April I) (a) 10^4 N/m² (b) 10^8 N/m² (c) 10^3 N/m² (d) 10^{16} N/m²
- **3** A thermometer graduated according to a linear scale reads a value x_0 , when in contact with boiling water and $x_0 / 3$, when in contact with ice. What is the temperature of an object in °C, if this thermometer in the contact with the object reads $x_0 / 2$? (2019 Main, 11 Jan II) (a) 35 (b) 60 (c) 40 (d) 25

(1990, 7M)

- (a) How many degrees of freedom do gas molecules have?
- (b) Obtain the work done by the gas during the expansion as a function of the initial pressure *p* and volume *V*.
- **59.** An ideal gas has a specific heat at constant pressure $C_p = \frac{5R}{2}$. The gas is kept in a closed vessel of volume 0.0083 m³, at a

temperature of 300 K and a pressure of $1.6 \times 10^6 \text{ N/m}^2$. An amount of 2.49×10^4 J of heat energy is supplied to the gas. Calculate the final temperature and pressure of the gas. (1987, 7M)

- **60.** Calculate the work done when one mole of a perfect gas is compressed adiabatically. The initial pressure and volume of the gas are 10^5 N/m² and 6 L respectively. The final volume of the gas is 2 L molar specific heat of the gas at constant volume is 3R/2. (1982, 8M)
- **61.** A cyclic process *ABCA* shown in the *V-T* diagram is performed with a constant mass of an ideal gas. Show the same process on a p-V diagram. (1981, 4M)



(In the figure, line AB passes through origin).

4 A heat source at $T = 10^3$ K is connected to another heat reservoir at $T = 10^2$ K by a copper slab which is 1 m thick. Given that the thermal conductivity of copper is

 $0.1 \text{ WK}^{-1}\text{m}^{-1}$, the energy flux through it in the steady state is (2019 Main, 10 Jan I)

- (a) 90 Wm^{-2} (b) 65 Wm^{-2} (c) 120 Wm^{-2} (d) 200 Wm^{-2}
- **5.** Consider a spherical shell of radius *R* at temperature *T*. The black body radiation inside it can be considered as an ideal gas of photons with internal energy per unit volume $u = \frac{U}{V} \propto T^4$ and pressure $p = \frac{1}{3} \left(\frac{U}{V} \right)$. If the shell now undergoes an adiabatic expansion, the relation between *T* and *R* is (2015 Main)

(a)
$$T \propto e^{-R}$$
 (b) $T \propto \frac{1}{R}$

(c)
$$T \propto e^{-3R}$$
 (d) $T \propto \frac{1}{R^3}$

6. Consider an ideal gas confined in an isolated closed chamber. As the gas undergoes an adiabatic expansion, the average time of collision between molecules increases as V^q , where V

is the volume of the gas. The value of q is $\left(\gamma = \frac{C_p}{T}\right)$

(2015 Main)

(a)
$$\frac{3\gamma + 5}{6}$$

(b) $\frac{\gamma + 1}{2}$
(c) $\frac{3\gamma - 5}{6}$
(d) $\frac{\gamma - 1}{2}$

- 7. A solid body of constant heat capacity 1 J/°C is being heated by keeping it in contact with reservoirs in two ways
 - (i) Sequentially keeping in contact with 2 reservoirs such that each reservoir supplies same amount of heat.
 - (ii) Sequentially keeping in contact with 8 reservoirs such that each reservoir supplies same amount of heat.

In both the cases, body is brought from initial temperature 100°C to final temperature 200°C. Entropy change of the body in the two cases respectively, is (2015 Main) (a) $\ln 2$. $\ln 2$ (b) ln 2, 2 ln 2 (c) 2 ln 2, 8 ln 2 (d) ln 2, 4 ln 2

8. An ideal gas enclosed in a vertical cylindrical container supports a freely moving piston of mass M. The piston and the cylinder have equal cross-sectional area A. When the piston is in equilibrium, the volume of the gas is V_0 and its pressure is p_0 . The piston is slightly displaced from the equilibrium position and released. Assuming that the system is completely isolated from its surrounding, the piston executes a simple harmonic motion with frequency (2013 Main)

(a)
$$\frac{1}{2\pi} \frac{A_{\gamma} p_0}{V_0 M}$$
 (b) $\frac{1}{2\pi} \frac{V_0 M p_0}{A^2 \gamma}$
(c) $\frac{1}{2\pi} \sqrt{\frac{A^2 \gamma p_0}{M V_0}}$ (d) $\frac{1}{2\pi} \sqrt{\frac{M V_0}{A_{\gamma} p_0}}$

- 9. Two moles of ideal helium gas are in a rubber balloon at 30°C. The balloon is fully expandable and can be assumed to require no energy in its expansion. The temperature of the gas in the balloon is slowly changed to 35°C. The amount of heat required in raising the temperature is nearly (take R = 8.31 J/mol-K) (2012) (a) 62 J (b) 104 J (c) 124 J (d) 208 J
- **10.** An ideal gas is expanding such that $pT^2 = \text{constant}$. The coefficient of volume expansion of the gas is (2008, 3M)

(a) $\frac{1}{T}$	(b) $\frac{2}{T}$
(c) $\frac{3}{T}$	(d) $\frac{4}{T}$

11. A body with area A and temperature T and emissivity e = 0.6is kept inside a spherical black body. What will be the maximum energy radiated? (2005, 2M) (a) $0.60 \ eAT^4$ (b) $0.80 \ eAT^4$ (c) $1.00 \ eAT^4$ (d) $0.40 \ eAT^4$

12. The p-T diagram for an ideal gas is shown in the figure, where AC is an adiabatic process, find the corresponding p-Vdiagram (2003, 2M)



13. Which of the following graphs correctly represent the $\frac{dV/dp}{V}$ with p for an ideal gas at constant variation of $\beta =$ temperature? (2002, 2M)



- **14.** In a given process of an ideal gas, dW = 0 and dQ < 0. Then for the gas (2001, S)
 - (a) the temperature will decrease
 - (b) the volume will increase
 - (c) the pressure will remain constant
 - (d) the temperature will increase
- **15.** An ideal gas is initially at temperature T and volume V. Its volume is increased by ΔV due to an increase in temperature ΔT , pressure remaining constant. The quantity $\delta = \Delta V / V \Delta T$ varies with temperature as (2000, 2M)



 $\frac{p_0}{2}$



Passage Based Questions

Passage 1

In the figure a container is shown to have a movable (without friction) piston on top. The container and the piston are all made of perfectly insulating material allowing no heat transfer between outside and inside the container. The container is divided into two compartments by a rigid partition made of a thermally conducting material that allows slow transfer of heat.

The lower compartment of the container is filled with 2 moles of an ideal monoatomic gas at 700 K and the upper compartment is filled with 2 moles of an ideal diatomic gas at

400 K. The heat capacities per mole of an ideal monoatomic gas are
$$C_V = \frac{3}{2}R$$
, $C_p = \frac{5}{2}R$, and those for an ideal diatomic $\frac{5}{2}R$

gas are
$$C_V = \frac{5}{2}R, C_p = \frac{7}{2}R.$$
 (2014 Adv.)

- 16. Consider the partition to be rigidly fixed so that it does not move. When equilibrium is achieved, the final temperature of the gases will be (2014 Adv.)
 (a) 550 K
 (b) 525 K
 (c) 513 K
 (d) 490 K
- 17. Now consider the partition to be free to move without friction so that the pressure of gases in both compartments is the same. Then total work done by the gases till the time they achieve equilibrium will be (2014 Adv.) (a) 250R (b) 200R (c) 100R (d) -100R

Passage 2

A fixed thermally conducting cylinder has a radius R and height L_0 . The cylinder is open at its bottom and has a small hole at its top. A piston of mass M is held at a distance L from the top surface, as shown in the figure. The atmospheric pressure is p_0 .



18. The piston is now pulled out slowly and held at a distance 2L from the top. The pressure in the cylinder between its top and the piston will then be (2007, 4M)

(a)
$$p_0$$
 (b)

(c)
$$\frac{p_0}{2} + \frac{Mg}{\pi R^2}$$
 (d) $\frac{p_0}{2} - \frac{Mg}{\pi R^2}$

19. While the piston is at a distance 2L from the top, the hole at the top is sealed. The piston is then released, to a position where it can stay in equilibrium. In this condition, the distance of the piston from the top is (2007, 4M)

(a)
$$\left(\frac{2p_0\pi R^2}{\pi R^2 p_0 + Mg}\right)$$
 (2L) (b) $\left(\frac{p_0\pi R^2 - Mg}{\pi R^2 p_0}\right)$ (2L)
(c) $\left(\frac{p_0\pi R^2 + Mg}{\pi R^2 p_0}\right)$ (2L) (d) $\left(\frac{p_0\pi R^2}{\pi R^2 p_0 - Mg}\right)$ (2L)

20. The piston is taken completely out of the cylinder. The hole at the top is sealed. A water tank is brought below the cylinder and put in a position so that the water surface in the tank is at the same level as the top of the cylinder as shown in the figure. The density of the water is ρ . In



equilibrium, the height *H* of the water column in the cylinder satisfies (2007, 4M)

(a)
$$\rho g (L_0 - H)^2 + p_0 (L_0 - H) + L_0 p_0 = 0$$

(b) $\rho g (L_0 - H)^2 - p_0 (L_0 - H) - L_0 p_0 = 0$
(c) $\rho g (L_0 - H)^2 + p_0 (L_0 - H) - L_0 p_0 = 0$
(d) $\rho g (L_0 - H)^2 - p_0 (L_0 - H) + L_0 p_0 = 0$

Match the Columns

Column I contains a list of processes involving expansion of an ideal gas. Match this with Column II describing the thermodynamic change during this process. Indicate your answer by darkening the appropriate bubbles of the 4 × 4 matrix given in the ORS. (2008, 7M)

	Column I	С	olumn II
(A)	An insulated container has two chambers separated by a valve. Chamber I contains an ideal gas and the Chamber II has vacuum. The valve is opened.	(p)	The temperature of the gas decreases
(B)	An ideal monoatomic gas expands to twice its original volume such that its pressure $p \propto \frac{1}{V^2}$, where V is the volume of the gas.	(q)	The temperature of the gas increases or remains constant

	Column I	С	olumn II
(C)	An ideal monoatomic gas expands to twice its original volume such that its pressure $p \propto \frac{1}{V^{4/3}}$, where V is its volume.	(r)	The gas loses heat
(D) An ideal monoatomic gas expands such that its pressure p and volume V follows the behaviour shown in the graph.			The gas gains heat
	(P) = (P)		

22. Column I gives some devices and Column II gives some processes on which the functioning of these devices depend. Match the devices in Column I with the processes in Column II. (2007, 6M)

	Column I		Column II
(A)	Bimetallic strip	(p)	Radiation from a hot body
(B)	Steam engine	(q)	Energy conversion
(C)	Incandescent lamp	(r)	Melting
(D)	Electric fuse	(s)	Thermal expansion of solids

Objective Questions II (One or more correct option)

- **23.** A human body has a surface area of approximately 1 m². The normal body temperature is 10K above the surrounding room temperature T_0 . Take the room temperature to be $T_0 = 300$ K. For $T_0 = 300$ K, the value of $\sigma T_0^4 = 460$ Wm⁻² (where σ is the Stefan Boltzmann constant). Which of the following options is/are correct? (2017 Adv.)
 - (a) If the body temperature rises significantly, then the peak in the spectrum of electromagnetic radiation emitted by the body would shift to longer wavelengths
 - (b) If the surrounding temperature reduces by a small amount $\Delta T_0 \ll T_0$, then to maintain the same body temperature the same (living) human being needs to radiate $\Delta W = 4\sigma T_0^3 \Delta T_0$ more energy per unit time
 - (c) The amount of energy radiated by the body in 1s is close to 60 J
 - (d) Reducing the exposed surface area of the body (e.g. by curling up) allows humans to maintain the same body temperature while reducing the energy lost by radiation
- **24.** The figure below shows the variation of specific heat capacity (C) of a solid as a function of temperature (T). The temperature is increased continuously from 0 to 500 K at a constant rate. Ignoring any volume change, the following statement(s) is (are) correct to reasonable approximation.

- (a) the rate at which heat is absorbed in the range 0-100 K varies linearly with temperature T (2013 Adv.)
- (b) heat absorbed in increasing the temperature from 0-100 K is less than the heat required for increasing the temperature from 400-500 K
- (c) there is no change in the rate of heat absorbtion in the range 400-500 K
- (d) the rate of heat absorption increases in the range 200-300 K



- **25.** C_V and C_p denote the molar specific heat capacities of a gas at constant volume and constant pressure, respectively. Then,
 - (2009) (a) $C_p - C_V$ is larger for a diatomic ideal gas than for a monoatomic ideal gas
 - (b) $C_p + C_V$ is larger for a diatomic ideal gas than for a monoatomic ideal gas
 - (c) $\frac{C_p}{C_V}$ is larger for a diatomic ideal gas than for a monoatomic ideal gas
 - (d) C_p . C_V is larger for a diatomic ideal gas than for a monoatomic ideal gas
- **26.** An ideal gas is taken from the state *A* (pressure *p*, volume *V*) to the state *B* (pressure p/2, volume 2*V*) along a straight line path in the *p*-*V* diagram. Select the correct statements from the following (1993, 2M)
 - (a) The work done by the gas in the process *A* to *B* exceeds the work that would be done by it if the system were taken from *A* to *B* along an isotherm
 - (b) In the *T-V* diagram, the path *AB* becomes a part of a parabola
 - (c) In the *p*-*T* diagram, the path *AB* becomes a part of a hyperbola
 - (d) In going from *A* to *B*, the temperature *T* of the gas first increases to a maximum value and then decreases

Integer Answer Type Question

27. A metal rod *AB* of length 10*x* has its one end *A* in ice at 0°C and the other end *B* in water at 100°C. If a point *P* on the rod is maintained at 400°C, then it is found that equal amounts of water and ice evaporate and melt per unit time. The latent heat of evaporation of water is 540 calg⁻¹ and latent heat of melting of ice is 80 calg⁻¹. If the point *P* is at a distance of λx from the ice end *A*, find the value of λ . (Neglect any heat loss to the surrounding.) (2009)

Fill in the Blanks

28. A ring shaped tube contains two ideal gases with equal masses and relative molar masses $M_1 = 32$ and $M_2 = 28$. The gases are separated by one fixed partition and another movable stopper *S* which can move freely without friction inside the ring. The angle α as shown in the figure is degrees.





29. A gas thermometer is used as a standard thermometer for measurement of temperature. When the gas container of the thermometer is immersed in water at its triple point 273.16 K, the pressure in the gas thermometer reads $3.0 \times 10^4 \text{ N/m}^2$. When the gas container of the same thermometer is immersed in another system, the gas pressure reads $3.5 \times 10^4 \text{ N/m}^2$. The temperature of this system is therefore °C. (1997, 1M)

True / False

30. At a given temperature, the specific heat of a gas at a constant pressure is always greater than its specific heat at constant volume. (1987, 2M)

Analytical & Descriptive Questions

31. One end of a rod of length L and cross-sectional area A is kept in a furnace of temperature T_1 . The other end of the rod is kept at a temperature T_2 . The thermal conductivity of the material of the rod is K and emissivity of the rod is e.



It is given that $T_2 = T_s + \Delta T$, where $\Delta T < T_s$, T_s being the temperature of the surroundings. If $\Delta T \propto (T_1 - T_s)$, find the proportionality constant. Consider that heat is lost only by radiation at the end where the temperature of the rod is T_2 . (2004, 4M)

32. The piston cylinder arrangement shown contains a diatomic gas at temperature 300 K. The cross-sectional area of the cylinder is 1 m^2 . Initially the height of the piston above the base of the cylinder is 1 m. The temperature is now raised to 400 K at constant pressure. Find the new height of the piston above the base of the cylinder.



If the piston is now brought back to its original height without any heat loss, find the new equilibrium temperature of the gas. You can leave the answer in fraction. (2004, 2M)

33. The top of an insulated cylindrical container is covered by a disc having emissivity 0.6 and conductivity 0.167 W/Km and thickness 1 cm. The temperature is maintained by circulating oil as shown. (2003, 4M)



- (a) Find the radiation loss to the surroundings in W/m² if temperature of the upper surface of disc is 127°C and temperature of surroundings is 27°C.
- (b) Also find the temperature of the circulating oil. Neglect the heat loss due to convection.

(Given,
$$\sigma = \frac{17}{3} \times 10^{-8} \text{ Wm}^{-2} \text{K}^{-4}$$
)

- **34.** An insulated box containing a monoatomic gas of molar mass M moving with a speed v_0 is suddenly stopped. Find the increment in gas temperature as a result of stopping the box. (2003, 2M)
- **35.** A 5 m long cylindrical steel wire with radius 2×10^{-3} m is suspended vertically from a rigid support and carries a bob of mass 100 kg at the other end. If the bob gets snapped, calculate the change in temperature of the wire ignoring losses. (For the steel wire : Young's modulus = 2.1×10^{11} Pa; Density = 7860 kg/m³; Specific heat = 420 J/kg-K). (2001, 5M)
- **36.** A solid body X of heat capacity C is kept in an atmosphere whose temperature is $T_A = 300$ K. At time t = 0, the temperature of X is $T_0 = 400$ K. It cools according to Newton's law of cooling. At time t_1 its temperature is found to be 350 K.

At this time (t_1) the body X is connected to a large body Y at atmospheric temperature T_A through a conducting rod of length L, cross-sectional area A and thermal conductivity K. The heat capacity of Y is so large that any variation in its temperature may be neglected. The cross-sectional area A of the connecting rod is small compared to the surface area of X. Find the temperature of X at time $t = 3t_1$. (1998, 8M)

37. A gaseous mixture enclosed in a vessel of volume *V* consists of one gram mole of gas *A* with $\gamma = C_p/C_V = 5/3$ and another gas *B* with $\gamma = 7/5$ at a certain temperature *T*. The gram molecular weights of the gases *A* and *B* are 4 and 32 respectively. The gases *A* and *B* do not react with each other and are assumed to be ideal. The gaseous mixture follows the equation $pV^{19/13} = c$ onstant, in adiabatic process.

(1995, 10M)

- (a) Find the number of gram moles of the gas *B* in the gaseous mixture.
- (b) Compute the speed of sound in the gaseous mixture at 300 K.
- (c) If *T* is raised by 1 K from 300 K, find the percentage change in the speed of sound in the gaseous mixture.
- (d) The mixture is compressed adiabatically to 1/5 of its initial volume V. Find the change in its adiabatic compressibility in terms of the given quantities.
- **38.** A closed container of volume 0.02 m^3 contains a mixture of neon and argon gases, at a temperature of 27°C and pressure of $1 \times 10^5 \text{ Nm}^{-2}$. The total mass of the mixture is 28 g. If the molar masses of neon and argon are 20 and 40 g mol⁻¹ respectively, find the masses of the individual gases in the container assuming them to be ideal.

(Universal gas constant R = 8.314 J/mol-K). (1994, 6M)

39. An ideal monoatomic gas is confined in a cylinder by a spring-loaded piston of cross-section 8.0×10^{-3} m². Initially the gas is at 300 K and occupies a volume of 2.4×10^{-3} m³ and the spring is in its relaxed (unstretched, uncompressed) state. The gas is heated by a small electric heater until the piston moves out slowly by 0.1 m.



Calculate the final temperature of the gas and the heat supplied (in joules) by the heater. The force constant of the spring is 8000 N/m, and the atmospheric pressure 2.0×10^5 Nm⁻². The cylinder and the piston are thermally insulated. The piston is massless and there is no friction between the piston and the cylinder. Neglect heat loss through the lead wires of the heater. The heat capacity of the heater coil is negligible. Assume the spring to the massless. (1989, 8M)

40. Two moles of helium gas ($\gamma = 5/3$) are initially at temperature 27°C and occupy a volume of 20 L. The gas is first expanded at constant pressure until the volume is doubled. Then it undergoes an adiabatic change until the temperature returns to its initial value. (1988, 6M)

- (a) Sketch the process on a p-V diagram.
- (b) What are the final volume and pressure of the gas?
- (c) What is the work done by the gas?
- **41.** A thin tube of uniform cross-section is sealed at both ends. It lies horizontally, the middle 5 cm containing mercury and the two equal ends containing air at the same pressure p. When the tube is held at an angle of 60° with the vertical direction, the length of the air column above and below the mercury column are 46 cm and 44.5 cm respectively. Calculate the pressure p in centimetre of mercury. (The temperature of the system is kept at 30° C). (1986, 6M)
- **42.** Two glass bulbs of equal volume are connected by a narrow tube and are filled with a gas at 0°C and a pressure of 76 cm of mercury. One of the bulbs is then placed in melting ice and the other is placed in a water bath maintained at 62°C. What is the new value of the pressure inside the bulbs? The volume of the connecting tube is negligible. (1985, 6M)
- **43.** The rectangular box shown in figure has a partition which can slide without friction along the length of the box.



Initially each of the two chambers of the box has one mole of a monoatomic ideal gas ($\gamma = 5/3$) at a pressure p_0 , volume V_0 and temperature T_0 . The chamber on the left is slowly heated by an electric heater. The walls of the box and the partition are thermally insulated. Heat loss through the lead wires of the heater is negligible. The gas in the left chamber expands pushing the partition until the final pressure in both chambers becomes $243 p_0/32$. Determine (a) the final temperature of the gas in each chamber and (b) the work done by the gas in the right chamber. (1984, 8M)

44. One gram mole of oxygen at 27°C and one atmospheric pressure is enclosed in a vessel. (a) Assuming the molecules to be moving with $v_{\rm rms}$, find the number of collisions per second which the molecules make with one square metre area of the vessel wall. (b) The vessel is next thermally insulated and moved with a constant speed v_0 . It is then suddenly stopped. The process results in a rise of the temperature of the gas by 1°C. Calculate the speed v_0 . (1983, 8M)

Answers

Topic 1				17. (b)	18. (c)	19. (b)	20. (d)
1. (a)	2. (b)	3. (b)	4. (c)	21. (d)	22. (d)	23. (d)	24. (c)
5. (d)	6. (a)	7. (a)		25. (a)	26. (c)	27. (c)	28. (d)
8. (b)	9. (a)	10. (b)	11. (a)	29. (b)	30. (b)	31. (b)	32. (a)
12. (a)	13. (c)	14. (b)	15. (a)	33. (b)			
16. (a)	17. (b)	18. 8	19. 5.5	34. (a, b, d)	35. (c, d)	36. (a, c)	37. 300 K
20 $\frac{Pt}{T}$	21 0° C			38. $\sqrt{2}T$	39. 2 <i>R</i>	40. F	41. F
20. M	21.0 C			42. F	43. F 4	4. (a) 160 K (b) 3.3	312×10^{-21} J (c) 0.3 g
22. partly sol	id and partly liq	uid.	23. 273K				
24. 0.495 kg	25. 12 g	26. 409.8 m/s		Topic 5			
Topic 2				1. (b)	2. (b)	3. (d)	
1. (c)	2. (a)	3. (a)		4. (d)	5. (c)	6. (a)	7. (b)
4. (d)	5. (a)	6. (a)	7. (c)	8. (c)	9. (a)	10. (a)	11. (b)
8. (b, d)	9. 3	10. $\gamma_l = 2\alpha_s$		12. (b)	13. (b)	14. (d)	
11. 6.7×10^{-5}	/ °C			15. (a)	16. (c)	17. (d)	18. (b)
19 1 105 × 10	$11 \text{ N}/m^2 2 \times 10^{11}$	-5		19. (a)	20. (a)	21. (c)	22. (a)
12. 1.105 × 10	N/m , 2×10	per °C		23. (b)	24. (a)	25. (d)	26. (d)
13. $\beta_{i} = \beta \left(\frac{N}{2} \right)$	$\left(\frac{w_0 - w_1}{w_0}\right) + \frac{(1)}{w_0}$	$w_2 - w_1$)		27. (d)	28. (a)	29. (c)	30. (c)
П Г Г (и	$(w_0 - w_2) (w_0 - w_2) = (w_0 - w_2)$	$w_2(T_2 - T_1)$		31. (b)	32. (b)		
Topic 3				33. (A) \rightarrow p, t	$\mathbf{r},\mathbf{t};(\mathbf{B})\to\mathbf{p},\mathbf{r};(\mathbf{B})\to\mathbf{p},(\mathbf$	$(C) \rightarrow q, s; (D) \rightarrow t$:, t
	2 (b)	3 (b)	1 (2)	34. (A) \rightarrow s;	$(B) \rightarrow p, r; (C$	C) p; (D) \rightarrow q,s	
1. (u)	2. (0) 6. (c)	5. (6)	4. (a)	35. (b,c,d)			
9 (c)	0. (c) 10 (c)	11 (c)	6. (a) 12. (b)	36. (a, b, c)	37. (a, b)	38. (b, d)	39. (b, c)
13. (c)	10. (c) 14. (c)	11. (c) 15. (a)	12. (b)	40. (a, b, c, d) 41. –900 J	42. (2)	43. 4
17. (b)	18. (d)	19. (d)	20. (b)	44. 0.628	45. T	46. (a) 50°C (b)	0.05 J (c) 19999.95 J
21. (d)	22. (b)	23. (c)	24. (a, c, d)	47. (a) 600 K	(b) 1500 <i>R</i> , 831.	6 R, -900 R, -83	1.6 R (c) 600 R
25. (d)	26. (a, b)	27. 4	28. 2	48. (a) -1200	R (b) $Q_{AB} = -2$	100 <i>R</i> , $Q_{BC} = 1500$	$Q_{CA} = 831.6R$
29. 9	30. 9	31. 60°C	$32. \frac{4\pi KTR^2}{P}$	49. (b) (i) <i>W</i> _{Te}	$p_{\text{tal}} = -\frac{3}{2} p_1 V_1 \left[\left(\frac{1}{2} \sum_{i=1}^{n} p_i V_1 \right) \right] \left(\frac{1}{2} \sum_{i=1}^{n} p_i V_1 \right] \left(\frac{1}{2} \sum_{i=1}^{n} p_i V_1 \right) \right]$	$\left(\frac{V_1}{V_2}\right)^{2/3} - 1$	
33. 1.71 ρ <i>rc</i>	34. 5803	35. F			Ľ		
36. 41.6 W, 2	26.48 °C, 0.52 °C	C		(ii) A	$U_{\rm Tatal} = \frac{3}{2} p_1 V_1$	$\left(\frac{V_1}{V_1}\right)^{2/3} - 1 + 0$	
37. 166.32 s		38. 9091 W		()	$2^{P_1 + 1}$	$\begin{pmatrix} V_2 \end{pmatrix}$	
39. Hollow sp	ohere	40. 15.24°C		(iii) T _{Fir}	$\frac{Q}{3R} = \frac{Q}{3R} + \frac{p_1 V_1}{2R}$	$\left(\frac{V_1}{V_2}\right)^{2/3}$	
1 (b)	9 (c)	3 (d)	A (*)			p_0V_0 25 p_0	V ₀
I. (U)	6 (c)	J. (u)	T • ()	50. (a) $p_0 V_0$ (b) $\frac{-p_0V_0}{2}$, $3p_0V_0$	(c) $\frac{r_0}{2}$ (d) $\frac{r_0}{8} \frac{r_0}{R}$	<u>~</u>
ə. (u)	0. (a)	(. (U)	o. (a)	51. (a) $T_A = 12$	20.34 K, $T_B = 24$	$0.68 \mathrm{K}, T_C = 481.30$	6K, $T_D = 240.68$ K
9. (a)	10. (a)	11. (a)	12. (c)	(b) No (c	$Q_{ABC} = 3.25 \times$	$10^6 \text{ J}, Q_{ADC} = 2.75$	$\times 10^6$ J
13. (b)	14. (c)	15. (a)	16. (c)		~ ADC	$, \mathcal{L}_{ADC}$	-

- **52.** $T_B = 909 \text{ K}, T_D = 791.4 \text{ K}, 61.4\%$ **53.** (a) 189 K (b) – 2767 J (c) 2767 J **54.** (a) 765 J (b) 10.82% **55.** (a) 1869.75 J (b) 5297.6 J (c) 500 K **56.** (a) 1152 J (b) 1152 J (c) zero **57.** (b) 0.58 RT_4 **58.** (a) f = 5 (b) $W = 1.23 \ pV$ **59.** 675K, 3.6×10^6 N/m² **60.** –972 J Topic 6 **1.** (a) 2. (*) 3. (d) **4.** (a) **5.** (b) **6.** (b) 7. (a) 8. (c)
- **9.** (d) **10.** (c) 11. Question is incomplete **12.** (none) **13.** (a) 14. (a) 15. (c) **16.** (d) 17. (d) 18. (a) 19. (d) **20.** (c) **21.** (A) \rightarrow q; (B) \rightarrow p, r; (C) \rightarrow p, s; (D) \rightarrow q, s **22.** (A) \rightarrow s; (B) \rightarrow q; (C) \rightarrow p, q; (D) \rightarrow q, r 23. (b, c, d) **24.** (c, d) **25.** (b. d) **26.** (a, b, d) 27.9
- **28.** 192° **29.** 45.68°C 30. T $\frac{K}{4e\sigma LT_s^3 + K}$ **31.** Proportionality constant = **32.** $\frac{4}{2}$ m, 448.8K **33.** (a) 595 W/m² (b) 162.6 °C $\mathbf{34.}\ \Delta T = \frac{Mv_0^2}{3R}$ **35.** 4.568×10^{-3} °C **36.** $[300 + 12.5 e^{\frac{-2KAt_1}{CL}}]$ K **37.** (a) 2 mol (b) 401 m/s (c) 0.167% (d) $-8.27 \times 10^{-5} V$ **38.** Mass of neon = 4.074 g, mass of argon = 23.926 g 39.800 K, 720 J **40.** (a) $\uparrow A \rightarrow B_{C}$ (b) 113 L, 0.44×10^5 N/m² (c) 12459 J 41. 75.4 cm of Hg 42. 83.83 cm of Hg **43.** (a) $T_1 = 12.94 T_0, T_2 = 2.25 T_0$ (b) $-1.875 RT_0$ **44.** (a) 1.96×10^{27} / s (b) 36 m/s

Hints & Solutions

Topic 1 Calorimetry

1. Key Idea In such kind of heat transfer problems, Heat given by water = Heat gained by ice and (heat) $_{\text{liquid}} = \text{mass} \times \text{specific heat} \times$ temperature = { $m_l s_l \Delta T_l$ } $(Heat)_{solid} = (mass \times latent heat)$ + (mass \times specific heat \times temperature) $= (m_s \times L) + m_s \times s_s \times \Delta T_s$

Heat given by water is (specific heat of water is 1 cal $g^{-1} \circ C^{-1}$) $(\Delta H)_{\text{water}} = M_2 \times 1 \times (50 - 0) = 50M_2$...(i) Heat taken by ice is

$$(\Delta H)_{ice} = M_1 \times 0.5 \times [0 - (-10)] + M_1 \times L_{ice}$$

$$\Rightarrow \qquad (\Delta H)_{ice} = 5M_1 + M_1 L_{ice} \qquad \dots (ii)$$

Comparing Eqs. (i) and (ii), we get

$$(\Delta H)_{\text{water}} = (\Delta H)_{\text{ice}}$$

... \Rightarrow

$$50M_2 = 5M_1 + M_1L_{ice}$$
$$L_{ice} = \frac{50M_2 - 5M_1}{1 - 1}$$

$$L_{\rm ice} = 50 \frac{M_2}{M_1} - 5$$

2. Let *x* grams of water is evaporated.

According to the principle of calorimetry,

Heat lost by freezing water (that turns into ice) = Heat gained by evaporated water

Given, mass of water = 150 g \Rightarrow (150-x)×10⁻³ × 3.36×10⁵ $= x \times 10^{-3} \times 2.10 \times 10^{6}$ \Rightarrow (150 - x) × 3.36 = 21x $x = \frac{150}{725} = 20.6$ $x \approx 20 \,\mathrm{g}$ *.*.. 3. In first case according to principle of calorimetry, heat lost by liquid A = heat gained by liquid B $m_A S_A \Delta T_A = m_B S_B \Delta T_B$ or where, S_A is specific heat capacity of A and S_B is specific heat capacity of B $100 \times S_A (100 - 90) = 50 \times S_B (90 - 75)$ \Rightarrow $1000S_A = 50 \times 15S_B$ \Rightarrow $4S_{A} = 3S_{B}$ or ...(i) Similarly, in second case, $100 \times S_{A}(100 - T) = 50 \times S_{B}(T - 50)$ where, T = Final temperature of the mixture. $4S_A(100-T) = 2S_B(T-50)$ \Rightarrow Using Eq. (i), $3S_{R}(100 - T) = 2S_{R}(T - 50)$

300 - 3T = 2T - 100or 5T = 400or $T = 80^{\circ} \text{C}$ or

4. Given, VT = k, (k is constant)

or
$$T \propto \frac{1}{V}$$
 ...(i)

Using ideal gas equation, pV = nRT

 \Rightarrow

or

i.e a polytropic process with x = 2.

(Polytropic process means, $pV^x = \text{constant}$)

 $pV \propto T$ $pV \propto \frac{1}{V}$

 $pV^2 = \text{constant}$

We know that, work done in a polytropic process is given by

$$\Delta W = \frac{p_2 V_2 - p_1 V_1}{1 - x} \text{ (for } x \neq 1\text{)} \qquad \dots \text{(iii)}$$

...(ii)

and,
$$\Delta W = pV \ln \left(\frac{V_2}{V_1}\right)$$
 (for $x = 1$)

Here, x = 2,

 \Rightarrow

$$\therefore \qquad \Delta W = \frac{p_2 V_2 - p_1 V_1}{1 - x} = \frac{nR(T_2 - T_1)}{1 - x}$$
$$\Rightarrow \qquad \Delta W = \frac{nR\Delta T}{1 - x^2} = -nR\Delta T \qquad \dots (iv)$$

Now, for monoatomic gas change in internal energy is given by

$$\Delta U = \frac{3}{2} R \Delta T \qquad \dots (v)$$

Using first law of thermodynamics, heat absorbed by one mole gas is

$$\Delta Q = \Delta W + \Delta U = \frac{3}{2}R\Delta T - R\Delta T$$
$$\Delta Q = \frac{1}{2}R\Delta T$$

5. Using heat lost or gained without change in state is $\Delta Q = ms\Delta T$, where *s* is specific heat capacity and T = change in temperature

Let final temperature of ball be *T*.

Then heat lost by ball is,

$$\Delta Q = 0.1 \times 400(500 - T) \qquad ...(i)$$

This lost heat by ball is gained by water and vessel and given as

Heat gained by water,

$$\Delta Q_1 = 0.5 \times 4200(T - 30) \qquad \dots (ii)$$

and heat gained by vessel is

$$\Delta Q_2 = \text{heat capacity} \times \Delta T$$

= 800 × (T - 30) ...(iii)

According to principle of calorimetry, total heat lost = total heat gained

$$\Rightarrow \qquad 0.1 \times 400(500 - T) \\ = 0.5 \times 4200(T - 30) + 800(T - 30)$$

$$\Rightarrow (500 - T) = \frac{(2100 + 800)(T - 30)}{40}$$

$$\Rightarrow 500 - T = 72.5(T - 30)$$

$$\Rightarrow 500 + 217.5 = 72.5T \text{ or } T = 36.39 \text{ K}$$

So, percentage increment in temperature of water

$$\% = \frac{36.39 - 30}{30} \times 100 \approx 20\%$$

6. Let amount of ice be '*x*' *gm*. According to the principle of calorimeter,

heat lost by water = heat gained by ice

Here, heat lost by water

$$\Delta Q = ms_{\text{water}} \Delta T$$

Substituting the given values, we get
$$\Delta Q = 50 \times 4.2 \times 40$$

Heat gained by ice,

$$\Delta Q = x \, s_{ice} \, \Delta T + (x - 20) \, L$$

= x × 2.1 × 20 + (x - 20) × 334
= 20 x × 2.1 + 334x - 6680
: 20 x × 2.1 + 334x - 6680 = 50 × 4.2 × 40
42x + 334x - 6680 = 8400
$$\Rightarrow \qquad 376 \, x = 15080$$

or
$$x = 40.10 \, g$$

$$x \simeq 40 \, g$$

7. **Key Idea** The principle of calorimetry states that total heat lost by the hotter body equals to the total heat gained by colder body, provided that there is no exchange of heat with the surroundings.

Let specific heat of unknown metal is *s* and heat lost by this metal is ΔQ .

Heat lost and specific heat of a certain material/substance are related as

$$\Delta Q = ms\Delta T \qquad \qquad \dots (i)$$

For unknown metal, m = 192 g and

...

 $\Delta T = (100 - 21.5) \,^{\circ}\mathrm{C}$

$$\Delta Q' = 192(100 - 21.5) \times s$$
 ...(ii)

Now, this heat is gained by the calorimeter and water inside it.

As, heat gained by calorimeter can be calculated by Eq. (i). So, for brass specific heat,

$$s = 394 \text{ J kg}^{-1} \text{ K}^{-1}$$
 (given)
= 0.394 J g⁻¹ K⁻¹

Mass of calorimeter, m = 128 g

Change in temperature, $\Delta T = (21.5 - 8.4)^{\circ}$ C

So, using Eq. (i) for calorimeter, heat gained by brass

$$\Delta Q_1 = 128 \times 0.394 \times (21.5 - 8.4) \qquad \dots (iii)$$

Heat gained by water can be calculated as follows mass of water, m = 240 g,

 $\frac{1}{2}$

specific heat of water, $s = 4.18 \text{ J g}^{-1}\text{K}^{-1}$,

change in temperature, $\Delta T = (21.5 - 8.4)^{\circ}$ C

heat gained by water,

$$\Delta Q_2 = 240 \times 4.18 \times (21.5 - 8.4) \qquad \dots (iv)$$

Now, according to the principle of calorimeter, the total heat gained by the calorimeter and water must be equal to heat lost by unknown metal

$$\Delta Q' = \Delta Q_1 + \Delta Q_2$$
Using Eqs. (ii), (iii) and (iv), we get

$$\Rightarrow = 192(100 - 21.5) \times s$$

$$= 128 \times 0.394 \times (21.5 - 8.4) + 240$$

$$\times 4.18 \times (21.5 - 8.4)$$

$$\Rightarrow 15072 \, s = 660.65 + 13142$$

$$\Rightarrow s = 0.916 \, \text{J g}^{-1} \, \text{K}^{-1}$$
or
$$s = 916 \, \text{J g}^{-1} \text{K}^{-1}.$$

8. By Mayor's relation, for 1 g mole of a gas, $C_p - C_V = R$

So, when *n* gram moles are given,

$$C_p - C_V = \frac{R}{n}$$

As per given question,

$$a = C_p - C_V = \frac{R}{2}; \text{ for H}_2$$
$$b = C_p - C_V = \frac{R}{28}; \text{ for N}_2$$
$$a = 14b$$

9. Heat gained (water + calorimeter) = Heat lost by copper ball $\Rightarrow m_w s_w \Delta T + m_c s_c \Delta T = m_B s_B \Delta T$ $\Rightarrow 170 \times 1 \times 45 + 100 \times 0.1 \times 45$

$$= 100 \times 0.1 \times (T - 75)$$

T = 885°C

10. Heat generated in device in 3 h

...

= time × power = $3 \times 3600 \times 3 \times 10^{3} = 324 \times 10^{5}$ J

Heat used to heat water

$$= ms\Delta\theta = 120 \times 1 \times 4.2 \times 10^3 \times 20J$$

Heat absorbed by coolant

$$= Pt = 324 \times 10^{5} - 120 \times 1 \times 4.2 \times 10^{3} \times 20J$$
$$Pt = (325 - 100.8) \times 10^{5} J$$
$$222.2 \times 10^{8}$$

$$P = \frac{223.2 \times 10}{3600} = 2067 \text{ W}$$

- 1 calorie is the heat required to raise the temperature of 1 g of water from 14.5°C to 15.5°C at 760 mm of Hg. Hence, correct option is (a).
- 12. Energy gained by water (in 1 s) = energy supplied – energy lost = 1000 J - 160 J = 840 JTotal heat required to raise the temperature of water from 27° C to 77° C is $ms\Delta\theta$.

Hence, the required time

$$t = \frac{ms\Delta\theta}{\text{rate by which energy is gained by water}}$$

$$= \frac{(2) (4.2 \times 10^3) (50)}{840}$$

$$= 500 \text{ s} = 8 \text{ min } 20 \text{ s}$$

- **13.** Temperature of liquid oxygen will first increase in the same phase. Then, phase change (liquid to gas) will take place. During which temperature will remain constant. After that temperature of oxygen in gaseous state will further increase.
- 14. Heat released by 5 kg of water when its temperature falls from 20° C to 0° C is,

$$Q_1 = mc\Delta\theta = (5)(10^3)(20 - 0) = 10^5$$
 cal

when 2 kg ice at -20° C comes to a temperature of 0° C, it takes an energy

$$Q_2 = mc\Delta\theta = (2)(500)(20) = 0.2 \times 10^5$$
 cal

The remaining heat $Q = Q_1 - Q_2 = 0.8 \times 10^5$ cal will melt a mass *m* of the ice, where,

$$m = \frac{Q}{L} = \frac{0.8 \times 10^5}{80 \times 10^3} = 1 \,\mathrm{kg}$$

So, the temperature of the mixture will be 0°C, mass of water in it is 5 + 1 = 6 kg and mass of ice is 2 - 1 = 1 kg.

15. The temperature of ice will first increase from -10° C to 0° C. Heat supplied in this process will be

$$Q_{1} = ms_{i}(10),$$
where, $m = \text{mass of ice}$

$$s_{i} = \text{specific heat of ice}$$

$$0^{\circ}C$$

$$-10^{\circ}C$$

$$Q_{1} \qquad (Q_{1}+Q_{2}) \qquad (Q_{1}+Q_{2}+Q_{3})$$
Heat supplied

Then, ice starts melting. Temperature during melting will remain constant (0°C).

Heat supplied in this process will be

 $Q_2 = mL, L =$ latent heat of melting.

Now, the temperature of water will increase from 0°C to 100°C. Heat supplied will be

$$Q_3 = ms_w(100)$$

where, $s_w =$ Specific heat of water.

Finally, water at 100°C will be converted into steam at 100°C and during this process temperature again remains constant. Temperature *versus* heat supplied graph will be as shown in above figure.

16. Heat required $Q_1 = (1.1 + 0.02) \times 10^3 \times 1 \times (80 - 15)$ = 72800 cal

Heat given it *m* (in kg) steam is condensed :

$$Q_2 = (m \times 540 \times 10^3) + m \times 1 \times 10^3 \times (100 - 80)$$

Equating $Q_1 = Q_2$ we get

$$m = 0.130 \text{ kg}$$

17.
$$Q_1 = nC_p \ \Delta T, Q_2 = nC_V \ \Delta T, \frac{Q_2}{Q_1} = \frac{C_V}{C_p} = \frac{1}{\gamma}$$

or $Q_2 = \frac{Q_1}{\gamma} = \frac{70}{1.4} = 50$ cal

18. Language of question is slightly wrong. As heat capacity and specific heat are two different physical quantities. Unit of heat capacity is $J - kg^{-1}$ not $J - kg^{-1} - {}^{\circ}C^{-1}$. The heat capacity given in the question is really the specific heat. Now applying the heat exchange equation.

$$420 = (m \times 10^{-3}) (2100) (5) + (1 \times 10^{-3}) (3.36 \times 10^{5})$$

Solving this equation, we get

- \therefore The correct answer is 8.
- **19.** Heat required to melt the block

$$= (0.28 \text{kg}) (3.3 \times 10^5 \text{ J/kg}) = 9.24 \times 10^4 \text{ J}$$

Heat received per second = $(1400 \text{J/m}^2)(0.2 \text{m}^2) = 280 \text{J}$

$$\therefore \text{ Time taken (in minutes)} = \frac{9.24 \times 10^4}{280 \times 60} = 5.5$$

- **20.** Heat lost in time $t = Pt = ML \implies \therefore L = \frac{Pt}{M}$
- **21.** Heat liberated when 300 g water at 25°C goes to water at 0° C : $Q = ms \Delta \theta = (300) (1) (25) = 7500$ cal

From Q = mL, this much heat can melt mass of ice given by

$$m = \frac{Q}{L} = \frac{7500}{80} = 93.75 \,\mathrm{g}$$

i.e. whole ice will not melt.

Hence, the mixture will be at 0°C.

Mass of water in mixture

$$= 300 + 93.75$$

$$= 393.75 \,\mathrm{g}$$
 and

Mass of ice in mixture

$$= 100 - 93.75 = 6.25 \text{ g}$$

- **22.** *O* to $A \longrightarrow$ material is in solid state (only temperature is increasing).
 - A to $B \longrightarrow$ material is partially solid and partially in liquid state.
 - *B* to $C \longrightarrow$ material is in liquid state.
 - C to $D \longrightarrow$ material is partially liquid and partially in vapour state.

23. 0.05 kg steam at 373 K
$$\xrightarrow{Q_1}$$
 0.05 kg water at 373 K
0.05 kg water at 373 K $\xrightarrow{Q_2}$ 0.05 kg water at 273 K
0.45 kg ice at 253 K $\xrightarrow{Q_3}$ 0.45 kg ice at 273 K
0.45 kg ice at 273 K $\xrightarrow{Q_4}$ 0.45 kg water at 273 K
 $Q_1 = (50) (540) = 27000 \text{ cal} = 27 \text{ kcal}$
 $Q_2 = (50) (1) (100) = 5000 \text{ cal} = 5 \text{ kcal}$
 $Q_3 = (450) (0.5) (20) = 4500 \text{ cal} = 4.5 \text{ kcal}$
 $Q_4 = (450) (80) = 36000 \text{ cal} = 36 \text{ kcal}$
Now, since $Q_1 + Q_2 > Q_2$ but $Q_1 + Q_2 < Q_2 + Q_3$ ice wi

Now, since $Q_1 + Q_2 > Q_3$ but $Q_1 + Q_2 < Q_3 + Q_4$ ice will come to 273 K from 253 K, but whole ice will not melt. Therefore, temperature of the mixture is 273 K.

24. Let *m* be the mass of the container. Initial temperature of container,

$$T_i = (227 + 273) = 500 \,\mathrm{K}$$

and final temperature of container,

$$T_f = (27 + 273) = 300 \,\mathrm{K}$$

Now, heat gained by the ice cube = heat lost by the container $\therefore (0.1)(8 \times 10^4) + (0.1)(10^3)(27) = -m \int_{500}^{300} (A + BT) dT$

or
$$10700 = -m \left[AT + \frac{BT^2}{2} \right]_{500}^{300}$$

After substituting the values of *A* and *B* and the proper limits, we get

$$m = 0.495 \, \text{kg}$$

25. Let *m* be the mass of the steam required to raise the temperature of 100 g of water from 24° C to 90° C. Heat lost by steam = Heat gained by water

$$\therefore m(L + s\Delta\theta_1) = 100s\Delta\theta_2 \text{ or } m = \frac{(100)(s)(\Delta\theta_2)}{L + s(\Delta\theta_1)}$$

Here, $s = \text{specific heat of water} = 1 \text{ cal/g-}^{\circ}\text{C}$,

L =latent heat of vaporisation = 540 cal/g.

$$\Delta \theta_1 = (100 - 90) = 10^{\circ} 0$$

$$\Delta \theta_2 = (90 - 24) = 66^{\circ} \text{ C}$$

Substituting the values, we have

$$m = \frac{(100)(1)(66)}{(540) + (1)(10)} = 12 \text{ g} \implies \therefore m = 12 \text{ g}$$

26. 75% heat is retained by bullet

and

$$\frac{3}{4} \left[\frac{1}{2} mv^2 \right] = ms \,\Delta\theta + mL \text{ or } v = \sqrt{\frac{(8s\Delta\theta + 8L)}{3}}$$

Substituting the values, we have

$$v = \sqrt{\frac{(8 \times 0.03 \times 4.2 \times 300) + (8 \times 6 \times 4.2)}{3 \times 10^{-3}}}$$

=409.8 m/s

Topic 2 Thermal Expansion

1. As length of rod remains unchanged,

$$F \longrightarrow \overbrace{\underbrace{\leftarrow}}^{\underbrace{\leftarrow}} \qquad \xrightarrow{\frown} \\ \Delta \qquad \Delta \qquad \Delta \qquad \Delta$$

Strain caused by compressive forces is equal and opposite to the thermal strain.

Now, compressive strain is obtained by using formula for Young's modulus,

$$Y = \frac{\frac{F}{A}}{\frac{\Delta l}{l}}$$

Compressive strain,

$$\Rightarrow \qquad \frac{\Delta l}{l} = \frac{F}{AY} = \frac{F}{\pi Y r^2} \qquad \dots (i)$$

Also, thermal strain in rod is obtained by using formula for expansion in rod,

$$\Delta l = l \alpha \ \Delta T$$

l

 \Rightarrow Thermal strain,

$$\frac{\Delta l}{l} = \alpha \ \Delta T \qquad \dots (ii)$$

From Eqs. (i) and (ii), we get

$$\frac{F}{\pi r^2 Y} = \alpha T \qquad [\because \Delta T = T]$$
$$\alpha = \frac{F}{\pi r^2 Y T}$$

 \Rightarrow

$$\gamma = 3\alpha = \frac{3F}{\pi r^2 YT}$$

2. Given,
$$T_1 = 40^\circ$$
 C and $T_2 = 20^\circ$ C
 $\Rightarrow \Delta T = T_1 - T_2 = 40 - 20 = 20^\circ$ C
Also, Young's modulus,

 $Y = 10^{11}$ N/m²

Coefficient of linear expansion, $\alpha = 10^{-5}/^{\circ}C$

Area of the brass wire, $A = \pi \times (10^{-3})^2 \text{ m}^2$

Now, expansion in the wire due to rise in temperature is

$$\Delta l = l \, \alpha \Delta T \implies \quad \frac{\Delta l}{l} = \alpha \Delta T \qquad \qquad \dots (i)$$

We know that, Young's modulus is defined as

$$Y = \frac{Mgl}{A\Delta l} \implies M = \frac{YA\Delta l}{gl} \qquad \dots (ii)$$

Using Eq. (i), we get

$$M = \frac{YA}{g} \times \alpha \Delta T = \frac{10^{11} \times 22 \times 10^{-6} \times 10^{-5} \times 20}{7 \times 10}$$
$$\Rightarrow \qquad M = \frac{22 \times 20}{7 \times 10} = \frac{44}{7} = 6.28 \text{ kg}$$

Which is closest to 9, so option (a) is nearly correct.

3. Let initial length of identical rods is l_0 Thermal expansion in length of rod due to heating is given by the relation

 $\Delta l = l_0 \alpha (\Delta T) = l_0 \alpha (T_2 - T_1)$

Here, α is coefficient of linear expansion.

So, change in length of rods are

$$\Delta l_1 = l_0 \,\alpha_1 (180 - 30)$$

$$\Delta l_2 = l_0 \alpha_2 (T - 30)$$

Because new lengths are same, so change in lengths of both rods are equal.

i.e.
$$\Delta l_1 = \Delta l_2$$

$$\Rightarrow \qquad l_0 \alpha_1 (180 - 30) = l_0 \alpha_2 (T - 30)$$

or
$$\frac{\alpha_1}{\alpha_2} = \frac{(T - 30)}{150}$$

Given,
$$\alpha_1 : \alpha_2 = 4 : 3$$

$$\therefore \quad \frac{T-30}{150} = \frac{4}{3} \implies T-30 = \frac{4}{3} \times 150 = 200$$

or $T = 200 + 30 = 230^{\circ}\text{C}$

4.
$$K = \frac{p}{(-\Delta V/V)} \Rightarrow \frac{\Delta V}{V} = \frac{p}{K}$$

 $\Rightarrow -\Delta V = \frac{pV}{K} \Rightarrow \frac{pV}{K} = V(3\alpha) \Delta T$
 $\therefore \qquad \Delta T = \frac{p}{3\alpha K}$
5. $T_0 = 2\pi \sqrt{\frac{L}{g}}$
 $T' = T_0 + \Delta T = 2\pi \sqrt{\frac{L+\Delta L}{g}}$
 $\therefore \qquad T' = T_0 + \Delta T = 2\pi \sqrt{\frac{L(1+\alpha \Delta \theta)}{g}}$

$$T' = T_0 + \Delta T = 2\pi \sqrt{\frac{L(1+\alpha \Delta \theta)}{g}}$$
$$= \left\{ 2\pi \sqrt{\frac{L}{g}} \right\} (1+\alpha \Delta \theta)^{\frac{1}{2}} \approx T_0 \left(1 + \frac{\alpha \Delta \theta}{2} \right)$$

$$\Delta T = T' - T_0 = \frac{\alpha \,\Delta \Theta T_0}{2} \qquad \dots (i)$$

or
$$\frac{\Delta T_1}{\Delta T_2} = \frac{\alpha \Delta \theta_1 T_0}{\alpha \Delta \theta_2 T_0}$$
$$\Rightarrow \quad \frac{12}{4} = \frac{40 - \theta}{\theta - 20}$$
$$\Rightarrow \quad 3(\theta - 20) = 40 - \theta$$
$$\Rightarrow \quad 4\theta = 100$$

:..

$$\Rightarrow \qquad \theta = 25^{\circ} C$$

Time gained or lost is given by

$$\Delta T = \left(\frac{\Delta T}{T_0 + \Delta T}\right) t \approx \frac{\Delta t}{T_0} t$$

From Eq. (i), $\frac{\Delta T}{T_0} = \frac{\alpha \Delta \theta}{2}$

$$\Delta t = \frac{\alpha (\Delta \theta) t}{2}$$

$$12 = \frac{\alpha (40 - 25)(24 \times 3600)}{2}$$

$$\alpha = 1.85 \times 10^{-5} / ^{\circ} \text{ C}$$

6.

Rate of heat flow from P to Q

$$\frac{dQ}{dt} = \frac{2KA(T-10)}{1}$$
Rate of heat flow from Q to S

$$\frac{dQ}{dt} = \frac{KA(4000-T)}{1}$$
At steady state rate of heat flow is same

$$\therefore \frac{2KA(T-10)}{1} = KA(400-T)$$
or $2T - 20 = 400 - T$ or $3T = 420$

$$\therefore T = 140^{\circ}$$



Temperature of junction is 140°C Temperature at a distance x from end P is $T_x = (130x + 10^\circ)$ Change in length dx is suppose dy Then, $dy = \alpha dx (T_x - 10)$ $\int_0^{\Delta y} dy = \int_0^1 \alpha dx (130x + 10 - 10)$ $\Delta y = \left[\frac{\alpha x^2}{2} \times 130\right]^1$

$$\Delta y = 1.2 \times 10^{-5} \times 65$$

 $\Delta y = 78.0 \times 10^{-5} \text{ m} = 0.78 \text{ mm}$

- 7. Given $\Delta l_1 = \Delta l_2$ or $l_1 \alpha_a t = l_2 \alpha_s t$ $\therefore \qquad \frac{l_1}{l_2} = \frac{\alpha_s}{\alpha_a}$ or $\frac{l_1}{l_1 + l_2} = \frac{\alpha_s}{\alpha_a + \alpha_s}$
- Let *l*₀ be the initial length of each strip before heating. Length after heating will be



$$l_{B} = l_{0}(1 + \alpha_{B}\Delta T) = (R + d)\theta$$

and
$$l_{C} = l_{0}(1 + \alpha_{C}\Delta T) = R \theta$$

$$\therefore \qquad \frac{R + d}{R} = \left(\frac{1 + \alpha_{B}\Delta T}{1 + \alpha_{C}\Delta T}\right)$$

$$\therefore \qquad 1 + \frac{d}{R} = 1 + (\alpha_{B} - \alpha_{C})\Delta T$$

[From binomial expansion]

$$R = \frac{a}{(\alpha_B - \alpha_C)\Delta T} \quad \text{or} \quad R \propto \frac{1}{\Delta T} \propto \frac{1}{|\alpha_B - \alpha_C|}$$

9.
$$\Delta l_1 = \frac{FL}{AY} = \frac{mgL}{\pi r^2 Y}$$
 = Increase in length

$$\Delta l_2 = L \alpha \ \Delta \theta$$
 = Decrease in length.

To regain its original length, $\Delta l_1 = \Delta l_2$

$$\therefore \quad \frac{mgL}{\pi r^2 Y} = L \alpha \ \Delta \theta \quad \Rightarrow \quad \therefore \quad m = \left(\frac{r^2 Y \ \alpha \ \Delta \theta}{g}\right)$$

Substituting the values we get, $m \approx 3 \text{ kg}$ \therefore Answer is 3.

10. When the temperature is increased, volume of the cube will increase while density of liquid will decrease. The depth upto which the cube is submerged in the liquid remains the same.

Upthrust = Weight. Therefore, upthrust should not change F = F'

$$\therefore \quad V_i \rho_L g = V_i' \rho'_L g \qquad (V_i = \text{volume immersed})$$

$$\therefore \quad (Ah_i)(\rho_L)(g) = A(1 + 2\alpha_s \Delta T)(h_i) \left(\frac{\rho_L}{1 + \gamma_L \Delta T}\right) g$$

Solving this equation, we get $\gamma_l = 2 \alpha_s$

11. Density of a liquid varies with temperature as

$$\rho_{t^{\circ}C} = \left(\frac{\rho_{0^{\circ}C}}{1 + \gamma t}\right)$$

Here, γ is the coefficient of volume expansion of temperature.



In the figure

 $h_1 = 52.8 \text{ cm}, h_2 = 51 \text{ cm} \text{ and } h = 49 \text{ cm}$ Now, pressure at B = pressure at C

 $p_0 + h_1 \rho_{95^{\circ}}g - h\rho_{5^{\circ}}g = p_0 + h_2 \rho_{5^{\circ}}g - h\rho_{95^{\circ}}g$ $\Rightarrow \qquad \rho_{95^{\circ}}(h_1 + h) = \rho_{5^{\circ}}(h_2 + h)$

$$\Rightarrow \qquad \frac{\rho_{95^{\circ}}}{\rho_{5^{\circ}}} = \frac{h_2 + h}{h_1 + h} \quad \Rightarrow \quad \frac{\frac{\rho_{0^{\circ}}}{1 + 95\gamma}}{\frac{\rho_{0^{\circ}}}{1 + 5\gamma}} = \frac{h_2 + h}{h_1 + h}$$

$$\Rightarrow \qquad \frac{1+5\gamma}{1+95\gamma} = \frac{51+49}{52.8+49} = \frac{100}{101.8}$$

Solving this equation, we get

 $\gamma = 2 \times 10^{-4} / ^{\circ} C$

: Coefficient of linear expansion of temperature,

$$\alpha = \frac{\gamma}{3} = 6.7 \times 10^{-5/\circ} \text{C}$$

12. $\Delta l = l \alpha \Delta \theta$



$$(1.91 \times 10^{-3}) = 0.3 \times 1.7 \times 10^{-5} \times 100 + \alpha \times 0.7 \times 100$$

Solving this equation, we get, $\alpha = 2 \times 10^{-5}$ per °C

Length of two rods will not change if force on joint due to thermal stress from both sides is equal.

i.e.
$$F_1 = F_2$$

or
$$Y_1 \times \left(\frac{\Delta l_1}{l_1}\right) \times A = Y_2 \times \left(\frac{\Delta l_2}{l_2}\right) \times A$$

or
$$Y_1 \times \alpha_1 \times \Delta \theta = Y_2 \times \alpha_2 \times \Delta \theta$$

$$\therefore \quad Y_2 = \frac{\alpha_1 Y_1}{\alpha_2} = \frac{\alpha_{Cu} Y_{Cu}}{\alpha_2}$$

$$= \frac{(1.7 \times 10^{-5})(1.3 \times 10^{11})}{(2.0 \times 10^{-5})} = 1.105 \times 10^{11} \text{ N/m}^2$$

13. Apparent weight = actual weight – upthrust

 $w_1 = w_0 - F_1$ Here, F_1 = upthrust at temperature T_1

= (volume of sinker at temperature T_1) ×

(density of liquid at temperature
$$T_1$$
) × g

$$= \frac{w_0}{g} \times \frac{1}{(\rho_s)_{T_1}} \times g \times (\rho_l)_{T_1} = w_0 \left\lfloor \frac{(\rho_l)_{T_1}}{(\rho_s)_{T_1}} \right\rfloor$$

$$\therefore \quad w_1 = w_0 - w_0 \left\lfloor \frac{(\rho_l)_{T_1}}{(\rho_s)_{T_1}} \right\rfloor \qquad \dots (i)$$

Similarly,
$$w_2 = w_0 - w_0 \left[\frac{(\rho_1)_{T_2}}{(\rho_s)_{T_2}} \right]$$
...(ii)

From Eqs. (i) and (ii), we have

$$\frac{(\rho_I)_{T_1}}{(\rho_s)_{T_2}} = \frac{w_0 - w_1}{w_0} \text{ and } \frac{(\rho_I)_{T_2}}{(\rho_s)_{T_2}} = \frac{w_0 - w_2}{w_0}$$

From these two equations we have,

$$\frac{(\rho_I)_{T_1}}{(\rho_I)_{T_2}} \times \frac{(\rho_s)_{T_2}}{(\rho_s)_{T_1}} = \frac{w_0 - w_1}{w_0 - w_2}$$

Using,
$$\frac{\rho'}{\rho} = \frac{1}{1 + \beta \Delta T}$$

Here, β = thermal coefficient of volume expansion

$$\therefore \qquad \frac{1+\beta_{l} (T_{2}-T_{1})}{1+\beta_{s} (T_{2}-T_{1})} = \frac{w_{0}-w_{1}}{w_{0}-w_{2}}$$

Given, $\beta_{s} = \beta$
$$\therefore \qquad \left(\frac{w_{0}-w_{2}}{w_{0}-w_{1}}\right) [1+\beta_{l} (T_{2}-T_{1})] = 1+\beta (T_{2}-T_{1})$$

Solving this equation, we get

.

$$\beta_{I} = \frac{\{1 + \beta (T_{2} - T_{1})\} \left\{ \frac{w_{0} - w_{1}}{w_{0} - w_{2}} \right\} - 1}{(T_{2} - T_{1})}$$
$$= \beta \left(\frac{w_{0} - w_{1}}{w_{0} - w_{2}} \right) + \frac{w_{2} - w_{1}}{(w_{0} - w_{2})(T_{2} - T_{1})}$$

Topic 3 Heat Transfer

1. Let interface temperature in steady state conduction is θ , then assuming no heat loss through sides;

$$\begin{pmatrix} \text{Rate of heat} \\ \text{flow through} \\ \text{first slab} \end{pmatrix} = \begin{pmatrix} \text{Rate of heat} \\ \text{flow through} \\ \text{second slab} \end{pmatrix}$$
$$\Rightarrow \qquad \frac{(3K) A(\theta_2 - \theta)}{d} = \frac{KA(\theta - \theta_1)}{3d}$$
$$\Rightarrow \qquad 9(\theta_2 - \theta) = \theta - \theta_1$$
$$\Rightarrow \qquad 9\theta_2 + \theta_1 = 10\theta$$
$$\Rightarrow \qquad \theta = \frac{9}{10}\theta_2 + \frac{1}{10}\theta_1$$

2. Key Idea From Newton's law of cooling, we have rate of cooling, $\frac{dQ}{dt} = \frac{h}{ms}(T - T_0)$

where, h = heat transfer coefficient,

T = temperature of body,

- T_0 = temperature of surrounding,
- m = mass and s = specific heat.

We know, $m = V \cdot \rho$

where, V = volume and $\rho =$ density.

÷.

$$\frac{dQ}{dt} = \frac{h}{ms} \left(T - T_0 \right) = \frac{h(T - T_0)}{V \cdot \rho s}$$

Since, h, $(T - T_0)$ and V are constant for both beaker.

$$\frac{dQ}{dt} \propto \frac{1}{\rho s}$$

We have given that $\rho_A = 8 \times 10^2 \text{ kgm}^{-3}$,

$$\rho_B = 10^3 \text{ kgm}^{-3},$$

 $s_A = 2000 \text{ J kg}^{-1} \text{ K}^{-1} \text{ and}$

 $s_B = 4000 \text{ J kg}^{-1} \text{ K}^{-1},$

$$\rho_A s_A = 16 \times 10^5$$

$$\rho_B s_B = 4 \times 10^6$$

So, $\rho_A < \rho_B$, $s_A < s_B$ and $\rho_A s_A < \rho_B s_B$

$$\Rightarrow \frac{1}{\rho_A s_A} > \frac{1}{\rho_B s_B} \Rightarrow \frac{dQ_A}{dt} > \frac{dQ_B}{dt}$$

So, for container *B*, rate of cooling is smaller than the container A. Hence, graph of B lies above the graph of A and it is not a straight line (slope of A is greater than B).

3. Both the given cylinders are in parallel as heat flow is given along length. In parallel, equivalent thermal conductivity of system is

$$K_{\rm eq} = \frac{K_1 A_1 + K_2 A_2}{A_1 + A_2}$$

So, in given system

$$K_{\rm eq} = \frac{K_1(\pi R^2) + K_2[\pi(2R)^2 - \pi R^2]}{(\pi R^2) + (4\pi R^2 - \pi R^2)} \text{ or } K_{\rm eq} = \frac{K_1 + 3K_2}{4}$$

4. According to the given question, the given figure with its length for each section is given as below

$$A \xrightarrow{L/4} \overbrace{L/2}^{L/4} B$$

The above figure considering that every section has the same thermal conductivity, then in terms of thermal resistance is shown in the figure below,

Net resistance of the section *PQRS* is $=\frac{R \times \frac{3R}{2}}{\frac{5R}{5}} = \frac{3R}{5}$...(i)

Total resistance of the net network, $R_{\rm net}$

$$= \frac{R}{2} + \frac{R}{2} + \frac{3R}{5} = \frac{8R}{5}$$

: Thermal current, $I = \frac{\Delta T_{AB}}{R_{\text{not}}}$

$$I = \frac{120 - 0}{\left(\frac{8R}{5}\right)} = \frac{120 \times 5}{8R}$$

Thus, the net temperature difference between point P and Qis

$$T_P - T_Q = I \times \frac{3R}{5} \qquad \text{[using Eq. (i)]}$$
$$= \frac{120 \times 5}{8R} \times \frac{3R}{5} = 45^{\circ}\text{C}$$

5. In steady state



Energy incident per second = Energy radiated per second $\therefore I\pi R^2 = \sigma \left(T^4 - T_0^4\right) 4\pi R^2 \implies I = \sigma \left(T^4 - T_0^4\right) 4$ $T^4 - T_0^4 = 40 \times 10^8 \implies T^4 - 81 \times 10^8 = 40 \times 10^8$ ⇒ $T^4 = 121 \times 10^8 \implies T \approx 330 \,\mathrm{K}$ ⇒

6. In thermal conduction, it is found that in steady state the heat current is directly proportional to the area of cross-section A which is proportional to the change in temperature $(T_1 - T_2)$.

Then,
$$\frac{\Delta Q}{\Delta t} = \frac{KA(T_1 - T_2)}{x}$$

According to thermal conductivity, we get 100°C



7. According to Newton's cooling law, option (c) is correct answer.

8.
$$R_{\rm I} = R_1 + R_2 = \left(\frac{l}{KA}\right) + \left(\frac{l}{2KA}\right) = \frac{3}{2}\left(\frac{l}{KA}\right)$$

 $\frac{1}{R_{\rm II}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{KA}{l} + \frac{2KA}{l}$
or $R_{\rm II} = \frac{l}{3KA} = \frac{R_{\rm I}}{4.5}$

Since thermal resistance $R_{\rm II}$ is 4.5 times less than thermal resistance R_{I} .

$$t_{\rm II} = \frac{t_{\rm I}}{4.5} = \frac{9}{4.5} \, {\rm s} = 2 \, {\rm s}$$

...

and

9. Let temperature of middle plate in steady state is T_0

$$Q_1 = Q_2$$

$$Q = \text{ net rate of heat flow}$$

$$\therefore \quad \sigma A (3T)^4 - \sigma A T_0^4 = \sigma A T_0^4 - \sigma A (2T)^4$$

Solving this equation, we get

$$T_0 = \left(\frac{97}{2}\right)^{1/4} T$$

10. $\lambda_m T = \text{constant}$

From the graph $T_3 > T_2 > T_1$ Temperature of Sun will be maximum. Therefore, (c) is the correct option. NOTE Although graphs are not very clear.

11. Glass of bulb heats due to filament by radiation.

 $Q \propto \frac{A}{\left(\lambda_m\right)^4}$

12. $Q \propto AT^4$ and $\lambda_m T = \text{constant}$ Hence,

С

or
$$Q \propto \frac{r^2}{(\lambda_m)^4}$$

 $Q_A : Q_B : Q_C = \frac{(2)^2}{(3)^4} : \frac{(4)^2}{(4)^4} : \frac{(6)^2}{(5)^4}$
 $= \frac{4}{81} : \frac{1}{16} : \frac{36}{625} = 0.05 : 0.0625 : 0.0576$

i.e. Q_B is maximum.

13.
$$\frac{dQ}{dt} = L\left(\frac{dm}{dt}\right)$$

or $\frac{\text{Temperature difference}}{\text{Thermal resistance}} = L\left(\frac{dm}{dt}\right)$
or $\frac{dm}{dt} \propto \frac{1}{\text{Thermal resistance}}$

 $q \propto \frac{1}{R}$ \Rightarrow

In the first case rods are in parallel and thermal resistance is $\frac{R}{2}$ while in second case rods are in series and thermal resistance is 2R.

$$\frac{q_1}{q_2} = \frac{2R}{R/2} = \frac{4}{1}$$
14. Rate of cooling $\left(-\frac{dT}{dt}\right) \propto$ emissivity (e)
From the graph,

$$\left(-\frac{dT}{dt}\right)_{x} > \left(-\frac{dT}{dt}\right)_{y}$$

:.. $E_x > E_v$ Further emissivity (E) = absorptive power (a)(good absorbers are good emitters also) *:*.. $a_x > a_y$

- 15. Black body radiates maximum number of wavelength and maximum energy if all other conditions (e.g. temperature surface area etc.) are same. So, when the temperature of black body becomes equal to the temperature of the furnace, the black body will radiate maximum energy and it will be brightest of all. Initially, it will absorb all the radiant energy incident on it, so it is the darkest one.
- **16.** Let θ be the temperature of the junction (say *B*). Thermal resistance of all the three rods is equal. Rate of heat flow through AB + Rate of heat flow through CB = Rate of heat flow through BD



Here, R = Thermal resistance

:..

 $3\theta = 180^{\circ}$ or $\theta = 60^{\circ}$ C *:*.. NOTE Rate of heat flow $(H) = \frac{\text{Temperature difference (TD)}}{\text{Thermal resistance (R)}}$ $R = \frac{I}{KA}$ where, K = Thermal conductivity of the rod. This is similar to the current flow through a resistance (R) where current (i) = Rate of flow of charge = Potential difference (PD) Electrical resistance (R)

Here,
$$R = \frac{l}{\sigma A}$$
 where σ = Electrical conductivity.

17. Wien's displacement law for a perfectly black body is

 $\lambda_m T = \text{constant} = \text{Wien's constant } b$

Here, λ_m is the minimum wavelength corresponding to maximum intensity I.

$$\lambda_m \propto \frac{1}{T}$$

From the figure $(\lambda_m)_1 < (\lambda_m)_3 < (\lambda_m)_2$ Therefore. $T_1 > T_3 > T_2$

18. Wien's displacement law is

or

...

...

$$\lambda_m T = b$$
 (b = Wien's constant)

$$\lambda_m = \frac{b}{T} = \frac{2.88 \times 10^6 \text{ nm-K}}{2880 \text{ K}}$$

$$\lambda = 1000 \text{ nm}$$

Energy distribution with wavelength will be as follows :



- **19.** Power radiated \propto (surface area) $(T)^4$. The radius is halved,
 - hence, surface area will become $\frac{1}{4}$ times. Temperature is doubled, therefore, T^4 becomes 16 times. New power = $(450)\left(\frac{1}{4}\right)(16) = 1800$ W.
- 20. From Wien's displacement law

 $\lambda_m T = \text{constant}$ $T = \frac{1}{\lambda}$

or

...

$$\frac{T_{\text{sun}}}{T_{\text{north star}}} = \frac{(\lambda_m)_{\text{north star}}}{(\lambda_m)_{\text{sun}}}$$
$$= \frac{350}{510} \approx 0.69$$

21. The rate at which energy radiates from the object is $\frac{\Delta Q}{\Delta t} = e \sigma A T^4$

Since,

$$\Delta Q = mc\Delta T, \text{ we get}$$
$$\frac{\Delta T}{\Delta t} = \frac{e\sigma A T^4}{mc}$$

Also, since
$$m = \frac{4}{3} \pi r^3 \rho$$
 for a sphere, we get

 $A = 4\pi r^2 = 4\pi \left(\frac{3m}{4\pi\rho}\right)^{2/3}$ Hence, $\frac{\Delta T}{\Delta t} = \frac{e\sigma T^4}{mc} \left[4\pi \left(\frac{3m}{4\pi\rho}\right)^{2/3}\right]$ $= K \left(\frac{1}{m}\right)^{1/3}$

For the given two bodies

$$\frac{\left(\Delta T/\Delta t\right)_1}{\left(\Delta T/\Delta t\right)_2} = \left(\frac{m_2}{m_1}\right)^{1/3} = \left(\frac{1}{3}\right)^{1/3}$$

22. The diagramatic representation of the given problem is shown in figure. Since, $T_B > T_A$ the heat will flow from *B* to *A*. Similarly, heat will also flow from *B* to *C* and *C* to *A*.



Applying the conduction formula

$$\frac{\Delta Q}{\Delta t} = \frac{KA}{l} (\Delta T)$$

$$\left(\frac{\Delta T}{\sqrt{2}a}\right)_{CA} = \left(\frac{\Delta T}{a}\right)_{BC} \implies \frac{T_C - T}{\sqrt{2}a} = \frac{\sqrt{2}T - T_C}{a}$$

$$3T = T_C (\sqrt{2} + 1) \implies \frac{T_C}{T} = \frac{3}{(\sqrt{2} + 1)}$$

23. Let R_1 and R_2 be the thermal resistances of inner and outer portions. Since, temperature difference at both ends is same, the resistances are in parallel. Hence,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{K(4\pi R^2)}{l} = \frac{K_1(\pi R^2)}{l} + \frac{K_2(3\pi R^2)}{l}$$

$$K = \frac{3K_2 + K_1}{4}$$
Thermal resistance $R = \frac{l}{KA}$

$$\therefore \qquad R_A = \frac{L}{(2K)(4Lw)} \qquad \text{(Here } w = \text{width)}$$

$$= \frac{1}{8Kw},$$

$$R_B = \frac{4L}{3K(Lw)} = \frac{4}{3Kw}$$

$$R_C = \frac{4L}{(4K)(2Lw)} = \frac{1}{2Kw}$$

$$R_D = \frac{4L}{(5K)(Lw)} = \frac{4}{5Kw}$$

24.

$$R_{E} = \frac{L}{(6K)(Lw)} = \frac{1}{6Kw}$$
$$R_{A} : R_{B} : R_{C} : R_{D} : R_{E}$$
$$= 15:160:60:96:12$$

So, let us write, $R_A = 15 R$, $R_B = 160 R$ etc and draw a simple electrical circuit as shown in figure



H = Heat current = Rate of heat flow.

$$H_A = H_E = H \tag{let}$$

In parallel current distributes in inverse ratio of resistance.

$$\therefore \quad H_B : H_C : H_D = \frac{1}{R_B} : \frac{1}{R_C} : \frac{1}{R_D}$$

$$= \frac{1}{160} : \frac{1}{60} : \frac{1}{96}$$

$$= 9 : 24 : 15$$

$$\therefore \quad H_B = \left(\frac{9}{0 + 24 + 15}\right) H = \frac{3}{16} H$$

...

$$(9+24+15) = 16$$
$$H_C = \left(\frac{24}{9+24+15}\right)H = \frac{1}{2}H$$
$$H_D = \left(\frac{15}{9+24+15}\right)H = \frac{5}{16}H$$

and

$$H_C = H_B + H_D$$

Temperature difference (let us call it *T*) = (Heat current) \times (Thermal resistance)

$$T_{A} = H_{A}R_{A} = (H)(15R) = 15 HR$$

$$T_{B} = H_{B}R_{B} = \left(\frac{3}{16}H\right)(160 R) = 30 HR$$

$$T_{C} = H_{C}R_{C} = \left(\frac{1}{2}H\right)(60 R) = 30 HR$$

$$T_{D} = H_{D}R_{D} = \left(\frac{5}{16}H\right)(96 R) = 30 HR$$

$$T_{E} = H_{E}R_{E} = (H)(12 R) = 12 HR$$

Here, T_E is minimum. Therefore option (c) is also correct.

: Correct options are (a), (c) and (d).

25. Since, the temperature of black body is constant, total heat absorbed = total heat radiated.

26. Power radiated and surface area is same for both A and B. $e_A \sigma T_A^4 A = e_B \sigma T_B^4 A$ Therefore, $= \left(\frac{0.81}{0.01}\right)^{1/4} = 3$

$$\therefore \qquad \frac{T_A}{T_B} = \left(\frac{e_B}{e_A}\right)^{1/4} = \left($$
$$\therefore \qquad T_B = \frac{T_A}{3} = \frac{5802}{3}$$
$$= 1934 \text{ K}$$
$$T_B = 1934 \text{ K}$$

According to Wien's displacement law, T

$$\lambda_m T = \text{constant}$$

$$\lambda_A T_A = \lambda_B T_B$$
or
$$\lambda_A = \lambda_B \left(\frac{T_B}{T_A}\right) = \frac{\lambda_B}{3}$$
Given,
$$\lambda_B - \lambda_A = 1 \mu m$$

$$\Rightarrow \qquad \lambda_B - \frac{\lambda_B}{3} = 1 \mu m$$
or
$$\frac{2}{3} \lambda_B = 1 \mu m$$

$$\Rightarrow \qquad \lambda_B = 1.5 \mu m$$

NOTE $\lambda_m T = b$ = Wien's constant value of this constant for perfectly black body is 2.89×10^{-3} m-K. For other bodies this constant will have some different value. In the opinion of author option (b) has been framed by assuming b to be constant for all bodies. If we take b different for different bodies. Option (b) is incorrect.

27. (*d*) Rate of heat flow will be same,

$$\therefore \quad \frac{300 - 200}{R_1} = \frac{200 - 100}{R_2} \qquad \left(\text{as } H = \frac{dQ}{dt} = \frac{T \cdot D}{R} \right)$$

$$\therefore \qquad R_1 = R_2 \quad \Rightarrow \quad \frac{L_1}{K_1 A_1} = \frac{L_2}{K_2 A_2} \quad \Rightarrow \quad \frac{K_1}{K_2} = \frac{A_2}{A_1} = 4$$

. . .

...(i)

28. Power,
$$P = (\sigma T^4 A) = \sigma T^4 (4\pi R^2)$$

or, $P \propto T^4 R^2$

According to Wien's law,

$$\lambda \propto \frac{1}{T}$$

(λ is the wavelength at which peak occurs) \therefore Eq. (i) will become,

$$P \propto \frac{R^2}{\lambda^4} \text{ or } \lambda \propto \left[\frac{R^2}{P}\right]^{1/4}$$
$$\Rightarrow \quad \frac{\lambda_A}{\lambda_B} = \left[\frac{R_A}{R_B}\right]^{1/2} \left[\frac{P_B}{P_A}\right]^{1/4} = [400]^{1/2} \left[\frac{1}{10^4}\right]^{1/4} = 2$$

29.
$$\log_2 \frac{P_1}{P_0} = 1$$
Therefore, $\frac{P_1}{P_0} = 2$

According to Stefan's law,

$$P \propto T^{2}$$

$$\Rightarrow \qquad \frac{P_{2}}{P_{1}} = \left(\frac{T_{2}}{T_{1}}\right)^{4} = \left(\frac{2767 + 273}{487 + 273}\right)^{4} = 4^{4}$$

$$\Rightarrow \qquad \frac{P_{2}}{P_{1}} = \frac{P_{2}}{2P_{0}} = 4^{4} \implies \frac{P_{2}}{P_{0}} = 2 \times 4^{4}$$

$$\log_{2} \frac{P_{2}}{P_{0}} = \log_{2} [2 \times 4^{4}] = \log_{2} 2 + \log_{2} 4^{4}$$

$$= 1 + \log_{2} 2^{8} = 9$$

30.
$$\lambda_m \propto \frac{1}{T}$$

 $\therefore \frac{\lambda_A}{\lambda_B} = \frac{T_B}{T_A} = \frac{500}{1500} = \frac{1}{3}$
 $E \propto T^4 A$ (where, $A = \text{surface area} = 4\pi R^2$)
 $\therefore E \propto T^4 R^2$
 $\frac{E_A}{E_B} = \left(\frac{T_A}{T_B}\right)^4 \left(\frac{R_A}{R_B}\right)^2 = (3)^4 \left(\frac{6}{18}\right)^2 = 9$
 $\therefore \text{ Answer is } 0$

 \therefore Answer is 9.

 \Rightarrow

31. Thermal resistance, $R = \frac{l}{KA}$. i.e. $R \propto \frac{1}{K}$, *l* and *A* being the same for both the blocks $\therefore \qquad \frac{R_A}{R_B} = \frac{K_B}{K_A} = \frac{200}{300} = \frac{2}{3}$

Rate of flow of heat (thermal current) along both the cubes will be equal (in series). Therefore, temperature difference across both the cubes will be in the ratio of their thermal resistances. Hence,

$$\frac{(\text{TD})_A}{(\text{TD})_B} = \frac{R_A}{R_B} = \frac{2}{3} \text{ or } \frac{100^\circ \text{C} - T}{T - 0^\circ \text{C}} = \frac{2}{3}$$

$$300^\circ \text{C} = 3T = 2T \implies T = 60^\circ \text{C}$$

32. Thermal resistance $= \frac{l}{KA} = \frac{t}{K (4\pi R^2)} (t = \text{thickness})$

Now, rate of heat transfer = $\frac{\text{Temperature difference}}{\text{Thermal resistance}}$

Thermal resistance = $\frac{T}{t / 4\pi KR^2} = \frac{4\pi KTR^2}{t}$

Ρ.

Equating this rate with the power of the source.

$$\therefore P = \frac{4\pi KTR^2}{t} \quad \text{or} \quad t = \frac{4\pi KTR^2}{P}$$

or thickness *t* should not exceed $\frac{4\pi KTR^2}{P}$.

33. Area of sphere, $A = 4\pi r^2$

Mass of sphere,
$$m = \left(\frac{4}{3}\pi\rho r^3\right)$$

Now, energy radiated per second = (σAT^4)

$$\therefore \qquad mc\left(-\frac{dT}{dt}\right) = \sigma A T^4$$

or
$$\int_0^t dt = \left(\frac{-mc}{\sigma A}\right) \int_{200}^{100} T^{-4} dT$$

or
$$t = \frac{mc}{3\sigma A} \left[\frac{1}{(100)^3} - \frac{1}{(200)^3}\right]$$

 $l = \frac{1}{3\sigma A} \left[\frac{1}{(100)^3} - \frac{1}{(200)^3} \right]$ $= \frac{7mc}{7mc} \times 10^{-6} - \frac{7mc}{10^{-6}} \times 10^{-6}$

$$= \frac{1}{3 \times 8 \, \sigma \, A} \times 10^{\circ} = \frac{1}{24 \, \sigma A}$$

Substituting the values,

$$t = \frac{\left(7 \times \frac{4}{3}\pi\rho r^{3}\right)(c) \times 10^{-6}}{3 \times 8 \times 5.67 \times 10^{-8} \times 4\pi r^{2}} = 1.71\rho rc$$

34. Given, $\frac{(\sigma T^{4})(4\pi R^{2})}{(4\pi r^{2})} = 1400$
 $\therefore T = \left(\frac{1400 \times r^{2}}{\sigma R^{2}}\right)^{1/4}$
 $= \left[\frac{1400 \times (1.5 \times 10^{11})^{2}}{(5.67 \times 10^{-8}) \times (7 \times 10^{8})^{2}}\right]^{1/4}$
 $= 5803 \text{ K}$

35. Energy radiated per second $\propto AT^4 \propto (R^2)T^4$

$$\frac{E_1}{E_2} = \left(\frac{1}{4}\right)^2 \left(\frac{4000}{2000}\right)^4 = 1 \text{ or } E_1 = E_2$$

36. Let θ_1 and θ_2 be the temperatures of the two interfaces as shown in figure.



Thermal resistance,
$$R = \frac{l}{KA}$$

:.
$$R_1 = R_3 = \frac{(0.01)}{(0.8)(1)} = 0.0125 \text{ K/W or °C/W}$$

and
$$R_2 = \frac{(0.05)}{(0.08)(1)} = 0.625^{\circ} \text{ C/W}$$

Now, the rate of heat flow $\left(\frac{dQ}{dt}\right)$ will be equal from all the

three sections and since rate of heat flow is given by

$$\frac{dQ}{dt} = \frac{\text{Temperature difference}}{\text{Thermal resistance}}$$

and
$$\left(\frac{dQ}{dt}\right)_1 = \left(\frac{dQ}{dt}\right)_2 = \left(\frac{dQ}{dt}\right)_3$$

Therefore,

$$\frac{27 - \theta_1}{0.0125} = \frac{\theta_1 - \theta_2}{0.625} = \frac{\theta_2 - 0}{0.0125}$$

Solving this equation, we get

and
$$\theta_1 = 26.48^{\circ} \text{ C}$$
 and $\theta_2 = 0.52^{\circ} \text{ C}$
 $\frac{dQ}{dt} = \frac{27 - \theta_1}{0.0125} \implies \frac{dQ}{dt} = \frac{(27 - 26.48)}{0.0125}$
 $= 41.6 \text{ W}$

37. Let at any time temperature of the disc be θ . At this moment rate of heat flow,

$$\frac{dQ}{dt} = \frac{KA(\Delta\theta)}{l} = \frac{KA}{l}(\theta_0 - \theta) \qquad \dots (i)$$

This heat is utilised in increasing the temperature of the disc. Hence,

 $\frac{d\theta}{\theta_0 - \theta} = \frac{KA}{msl}dt$

$$\frac{dQ}{dt} = ms\frac{d\theta}{dt} \qquad \dots (ii)$$

Equating Eqs. (i) and (ii), we have $ms\frac{d\theta}{dt} = \frac{KA}{l}(\theta_0 - \theta)$

Therefore,

or

or
$$\int_{300K}^{350K} \frac{d\theta}{\theta_0 - \theta} = \frac{KA}{msl} \int_0^t dt$$

or
$$[-\ln(\theta_0 - \theta)]_{300K}^{350K} = \frac{KA}{msl} t$$
$$\int_{l=0.4m}^{\infty} \frac{dQ}{dt}$$
$$\frac{dQ}{dt}$$
$$\vdots$$
$$t = \frac{msl}{KA} \ln\left(\frac{\theta_0 - 300}{\theta_0 - 350}\right)$$

Substituting the values, we have

$$t = \frac{(0.4)(600)(0.4)}{(10)(0.04)} \ln\left(\frac{400 - 300}{400 - 350}\right)$$

$$t = 166.32 \text{ s}$$

38. Let R_1 , R_2 and R_3 be the thermal resistances of wood, cement and brick. All the resistances are in series. Hence,

$$R = R_{1} + R_{2} + R_{3}$$

$$= \frac{2.5 \times 10^{-2}}{0.125 \times 137} + \frac{1.0 \times 10^{-2}}{1.5 \times 137} + \frac{25 \times 10^{-2}}{1.0 \times 137} \left(\text{as } R = \frac{l}{KA} \right)$$

$$= 0.33 \times 10^{-2} \text{ °C/W}$$

$$20^{\circ}\text{C} \boxed{R_{1} \quad R_{2} \qquad R_{3}} -10^{\circ}\text{C}$$

:. Rate of heat transfer,

$$\frac{dQ}{dt} = \frac{\text{temperature difference}}{\text{thermal resistance}} = \frac{30}{0.33 \times 10^{-2}}$$
$$\approx 9091 \,\text{W}$$

... Power of heater should be 9091 W.

39. Net rate of heat radiation $\left(\frac{dQ}{dt}\right)$ will be same in both the

cases, as temperature and area are same. Therefore, from equation

$$ms\left(-\frac{d\,\theta}{dt}\right) = \frac{dQ}{dt}$$
 or $-\frac{d\theta}{dt} \propto \frac{1}{m}$

The hollow sphere will cool faster as its mass is less.

40. Power produced by heater

$$\Rightarrow \text{ rate of heat flow through window}$$
$$\therefore \qquad \frac{V^2}{R} = \frac{\text{Temperature difference}}{\text{Thermal resistance}} = \frac{20 - \theta}{(l/KA)}$$
$$\therefore \qquad \theta = 20 - \frac{V^2 l}{KAR}$$

Substituting the values we have

$$\theta = 20 - \frac{(200)^2 (0.2 \times 10^{-2})}{(0.2 \times 4.2)(1)(20)} = 15.24^{\circ} \text{ C}$$

Topic 4 Kinetic Theory of Gases and Gas Equation

1. Work done by a gas is

or

 $W = p\Delta V = nR\Delta T = 10$ J (given) ...(i)

As the gas is diatomic,

$$C_p$$
 = specific heat of gas at constant pressure = $\frac{7}{2}R$

So, heat absorbed by gas at constant pressure is

$$\Delta Q = nC_p \ \Delta T = n\frac{7}{2}R \ \Delta T$$
$$= \frac{7}{2}(nR \ \Delta T)$$
$$= \frac{7}{2} \times 10 \qquad [From Eq. (i)]$$
$$= 35 \text{J}$$

7

2. Let molar specific heat of the mixture is C_V .

Total number of molecules in the mixture
=
$$3 + 2 = 5$$

$$\therefore C_V \text{ can be determined using} n C_V dT = n_1 C_{V_1} dT + n_2 C_{V_2} dT \text{ or } (n_1 + n_2) (C_V)_{\text{mix}} = n_1 C_{V_1} + n_2 C_{V_2} [here, n = n_1 + n_2] 3R$$

Here,
$$C_{V_1} = \frac{5R}{2}$$
 (for helium); $n_1 = 2$
 $C_{V_2} = \frac{5R}{2}$ (for hydrogen); $n_2 = 3$

[For monoatomic gases, $C_V = \frac{3}{2}R$ and for diatomic gases, $C_V = \frac{5}{2}R$]

$$\therefore \qquad 5 \times C_V = \left(2 \times \frac{3R}{2}\right) + \left(3 \times \frac{5R}{2}\right)$$
21R

 $5C_V = \frac{21R}{2}$ \Rightarrow

or

...

$$C_V = \frac{21R}{10} = \frac{21 \times 8.3}{10} = \frac{174.3}{10}$$

or $C_V = 17.4 \text{ J/mol-K}$

3. Given process equation for 1 mole of an ideal gas is

$$p = p_0 \left(1 - \frac{1}{2} \left(\frac{V_0}{V} \right)^2 \right)$$
 ...(i)

Also, for 1 mole of ideal gas,

$$pV = RT$$
$$p = \frac{RT}{V} \qquad \dots (ii)$$

So, from Eqs. (i) and (ii), we have

$$\frac{RT}{V} = p_0 \left(1 - \frac{1}{2} \left(\frac{V_0}{V} \right)^2 \right)$$

$$\therefore \qquad T = \frac{p_0 V}{R} \left(1 - \frac{1}{2} \left(\frac{V_0}{V} \right)^2 \right) \qquad \dots (iii)$$

When volume of gas is V_0 , then by substituting $V = V_0$ in Eq. (iii), we get

Temperature of gas is

$$T_1 = \frac{p_0 V_0}{R} \left(1 - \frac{1}{2} \left(\frac{V_0}{V_0} \right)^2 \right) = \frac{p_0 V_0}{2R}$$

Similarly, at volume, $V = 2V_0$ Temperature of gas is

$$T_2 = \frac{p_0(2V_0)}{R} \left(1 - \frac{1}{2} \left(\frac{V_0}{2V_0} \right)^2 \right) = \frac{7}{4} \frac{p_0 V_0}{R}$$

So, change in temperature as volume changes from V_0 to $2V_0$ is

$$\Delta T = T_2 - T_1 = \left(\frac{7}{4} - \frac{1}{2}\right) \frac{p_0 V_0}{R} = \frac{5}{4} \frac{p_0 V_0}{R}$$

4. Given,

Volume, $V = 25 \times 10^{-3} \text{ m}^{-3}$

$$N = 1 \text{ mole of } O_2 = 6.023 \times 10^{23} \text{ atoms of } O_2$$

$$T = 300 \,\mathrm{K}$$

Root mean square velocity of a gas molecule of O_2 , $v_{rms} = 200 \text{ m/s}$

Radius,
$$r = \frac{0.3}{2}$$
 nm $= \frac{0.3}{2} \times 10^{-9}$ m
Now, average time, $\frac{1}{\tau} = \frac{v_{av}}{\lambda}$

As,

$$p = \frac{RT}{V} \implies \lambda = \frac{V}{\sqrt{2} N\pi r^2}$$

 $\lambda = \frac{RT}{\sqrt{2} N\pi r^2 p}$

$$\frac{1}{\tau} = \frac{v_{av}}{\lambda} = \frac{\sqrt{\frac{8}{3\pi}} \times v_{rms}}{\lambda}$$
$$\frac{1}{\tau} = \sqrt{\frac{8}{3\pi}} \times \frac{200 \times \sqrt{2} \times 6.023 \times 0^{23} \times \pi \times \frac{0.09}{4} \times 10^{-18}}{25 \times 10^{-3}}$$
$$\Rightarrow \frac{1}{\tau} = 4.4 \times 10^8 \text{ per second} \approx 10^8$$

: No option given is correct.

5. Given, capacity of cylinder is = 67.2 L and $\Delta T = 20^{\circ} \text{ C}$ At STP volume = 22.4 L

: Number of moles =
$$\frac{67.2}{22.4} = 3$$

Now, change in heat is given as

$$\Delta Q = nC_V \ \Delta T$$

Substituting the given values, we get

$$\Delta Q = 3 \times \frac{3R}{2} \times 20 \qquad (\because \text{ for He gas}, C_V = \frac{3}{2}R)$$
$$= 90 R \text{ J}$$
Given, $R = 8.31 \text{ J} \text{ mol}^{-1}\text{K}^{-1}$
$$\therefore \quad \Delta Q = 90 \times 8.31 = 747.9 \text{ J} = 748 \text{ J}$$

6. Key Idea A diatomic gas molecule has 5 degrees of freedom, i.e. 3 translational and 2 rotational, at low temperature ranges (~ 250 K to 750 K). At temperatures above 750 K, molecular vibrations occurs and this causes two extra degrees of freedom.

Now, in given case, For gas $A, C_p = 29, C_V = 22$ For gas $B, C_p = 30, C_V = 21$ By using

$$\gamma = \frac{C_p}{C_V} = 1 + \frac{2}{f}$$

We have,

...

For gas A,
$$1 + \frac{2}{f} = \frac{29}{22} \approx 1.3 \Longrightarrow f = 6.67 \approx 7$$

So, gas A has vibrational mode of degree of freedom. For gas B,

$$1 + \frac{2}{f} = \frac{30}{21} \approx 1.4 \Longrightarrow f = 5$$

Hence, gas B does not have any vibrational mode of degree of freedom.

7. Key Idea According to the law of equipartition of energy, $\frac{1}{2}mv_{\rm rms}^2 = \frac{n}{2}k_BT$ where, n is the degree of freedom.

Since, HCl is a diatomic molecule that has rotational, translational and vibrational motion.

So,

Here,

$$n = 7 \Longrightarrow \frac{1}{2} m v_{\rm rms}^2 = \frac{7}{2} k_B T$$
$$v_{\rm rms}^2 = \bar{v} \implies T = \frac{m \bar{v}^2}{7 k_B}$$

8. Key Idea For a gas molecule, $v_{\rm rms} = \sqrt{\frac{3RT}{M}}$ $v_{\rm rms} \propto \sqrt{T}$ *.*..

Let unknown rms speed be $v_{\rm rms, 2}$

at
$$T_2 = 227^{\circ}$$
C (or 500 K)

and at $T_1 = 127^{\circ}$ C (or 400 K)

$$v_{\rm rms,1} = 200 \,{\rm m/s}$$

: Using the relation $v_{\rm rms} \propto \sqrt{T}$, we can write

$$\frac{V_{\rm rms,2}}{V_{\rm rms,1}} = \sqrt{\frac{T_2}{T_1}}$$
 ... (i)

Substituting these given values in Eq. (i), we get

:.
$$v_{\text{rms, 2}} = \sqrt{\frac{500}{400}} \times 200 \text{ m/s}$$

= $\frac{1}{2}\sqrt{5} \times 200 \text{ m/s} = 100\sqrt{5} \text{ m/s}$

9. Root mean square velocity of hydrogen molecule is given as

$$v_{\rm rms} = \sqrt{\frac{3k_BT}{m}}$$

Escape velocity of hydrogen molecule from the earth is given as

$$v_e = \sqrt{2gR_e}$$

Given,

or

$$\sqrt{\frac{3k_BT}{m}} = \sqrt{2gR_e} \implies T = \frac{2gR_em}{3 \times k_B}$$

 $v_{\rm rms} = v_e$

Substituting the given values, we get

$$T = \frac{2 \times 10 \times 6.4 \times 10^{6} \times 2}{3 \times 1.38 \times 10^{-23} \times 6.02 \times 10^{26}} \approx 10^{4} \text{ K}$$

Alternate Solution

At rms speed, average thermal kinetic energy of a hydrogen gas molecule is $=\frac{3}{2}k_BT$

and if v_e = escape velocity of the gas molecule from the earth, then its kinetic energy is $KE = \frac{1}{2}mv_e^2$, where *m* is the mass of the gas molecule.

Equating the above thermal and kinetic energies, we have

$$\frac{3}{2}k_BT = \frac{1}{2}mv_e^2$$
 or $T = \frac{mv_e^2}{3k_B}$...(i)

Here,
$$k_B = 1.38 \times 10^{-23} \text{ JK}^{-1}$$
,
 $v_e = 11.2 \times 10^3 \text{ ms}^{-1}$,
 $m = \frac{2}{6.02 \times 10^{26}} = 0.332 \times 10^{-26} \text{ kg}$

Substituting these value in Eq. (i), we get $\frac{26}{26}$

Т

.

ν

$$= \frac{0.332 \times 10^{-20} \times (11.2 \times 10^{3})^{2}}{3 \times 1.38 \times 10^{-23}} \approx 10^{4} \text{ K}$$

10. Mean time elapsed between two successive, collisions is $t = \frac{\lambda}{\lambda}$

where,
$$\lambda$$
 = mean free path length and

$$v = \text{mean speed of gas molecule}$$

$$\therefore \qquad t = \frac{\left(\frac{k_B t}{\sqrt{2\pi d^2 p}}\right)}{\sqrt{\frac{8}{\pi} \cdot \frac{k_B T}{M}}} \Rightarrow t = \frac{C\sqrt{T}}{p}$$

where, $C = \frac{1}{4d^2} \sqrt{\frac{k_B M}{\pi}} = \text{a constant for a gas.}$
So,

$$\frac{t_2}{t_1} = \frac{\sqrt{T_2}}{\sqrt{T_1}} \cdot \left(\frac{p_1}{p_2}\right) \qquad \dots (i)$$

Here given, $\frac{p_1}{p_2} = \frac{1}{2}, \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{500}{300}} = \sqrt{\frac{5}{3}}$
and

$$t_1 = 6 \times 10^{-8} \text{ s}$$

Substituting there values in (i), we get

$$t_2 = 6 \times 10^{-8} \times \sqrt{\frac{5}{3}} \times \frac{1}{2} = 3.86 \times 10^{-8} \text{ s} \approx 4 \times 10^{-8} \text{ s}$$

11. Key Idea As piston is in equilibrium, so net force on piston is zero.

When the piston is stationary, i.e. on equilibrium as shown in the figure below,



 $p_1A + mg = p_2A$ then $mg = p_2A - p_1A$ \Rightarrow $mg = \left(\frac{nRTA}{V_2} - \frac{nRTA}{V_1}\right)$ or

or

{:: pV = nRT (ideal gas equation)}

$$= nRT\left(\frac{A}{Al_2} - \frac{A}{Al_1}\right) = nRT\left(\frac{l_1 - l_2}{l_1 l_2}\right)$$
$$m = \frac{nRT}{g}\left(\frac{l_1 - l_2}{l_1 l_2}\right)$$

Key Idea Internal energy of 'n' moles of a gas with degree of freedom f (= 3 for an ideal gas), at temperature T is $E = \frac{f}{2} \cdot nRT = \frac{3}{2} nRT$

For an ideal gas, internal energy, $E = \frac{3}{2}nRT$

=

$$\frac{3}{2}p \cdot V \qquad (\because \text{ from } pV = nRT)$$

Substituting the given values, we get

$$=\frac{3}{2}\times3\times10^6\times2=9\times10^6\mathrm{J}$$

13. Internal energy of a gas with f degree of freedom is

 $U = \frac{n f R T}{2}$, where *n* is the number of moles.

Internal energy due to O_2 gas which is a diatomic gas is $U_1 = \frac{n_1 f_1 RT}{2} = 3 \times \frac{5}{2} RT$

> (:: $n_1 = 3$ moles, degree of freedom for a diatomic gas $f_1 = 5$)

Internal energy due to Ar gas which is a monoatomic gas is

$$U_2 = \frac{n_2 f_2 RT}{2} = 5 \times \frac{3}{2} RT$$

(:: $n_2 = 5$ moles, degree of freedom for a monoatomic gas $f_2 = 3$)

 \therefore Total internal energy = $U_1 + U_2$ U = 15 RT \Rightarrow

14. Given, mass of gas, m = 2 kgPressure on gas, $p = 4 \times 10^4 \text{ N/m}^2$

Density of gas,
$$\rho = 8 \text{ kg/m}^3$$

 \Rightarrow Volume of gas = $\frac{\text{Mass}}{\text{Density}}$

$$\Rightarrow \qquad V = \frac{2}{8} = \frac{1}{4} \text{ m}^3 \qquad \dots \text{ (i)}$$

nV - nRT

Using ideal gas equation,

$$\Rightarrow \qquad nRT = 4 \times 10^4 \times \frac{1}{4}$$

 $nRT = 10^4$... (ii) Internal energy of n moles of a monoatomic gas is given by,

$$U = \frac{3}{2} nRT$$

$$\Rightarrow \qquad U = \frac{3}{2} \times 10^4 \text{ J} = 1.5 \times 10^4 \text{ J}$$

i.e. in order of 10^4 J.

15. We know that from kinetic theory of gases,

$$V_{\rm rms} = \sqrt{\frac{3RT}{M}} \text{ or } V_{\rm rms} \propto \sqrt{T}$$

where, R is gas constant, T is temperature and M is molecular mass of a gas.

Here, to double the $v_{\rm rms}$, temperature must be 4 times of the initial temperature.

: New temperature,

$$T_2 = 4 \times T_1$$
$$T_1 = 27^{\circ} \text{ C} = 300 \text{ K}$$

...

 $T_2 = 4 \times 300 = 1200 \,\mathrm{K}$

As, gas is kept inside a closed vessel.

$$\therefore \qquad Q = nC_V \cdot dT$$

$$= \frac{15}{28} \times \frac{5R}{2} (1200 - 300)$$

$$\begin{bmatrix} \text{As, } n = \frac{m}{M} = \frac{15}{28} \text{ for nitrogen} \end{bmatrix}$$

$$= \frac{15}{28} \times \frac{5}{2} \times 8.3 \times 900$$

$$[\text{Given, } R = 8.3 \text{ J/K-mole}]$$
or
$$Q = 10004.4 \text{ J} \simeq 10 \text{ kJ}$$

 T_1

$$Q$$
 :

16. Root mean square (rms) velocity of the molecules of a gas is given as

$$v_{\rm rms} = \sqrt{\frac{3RT}{M}}$$

where, M is the atomic mass of the gas.

$$\Rightarrow \qquad \nu_{\rm rms} \propto \sqrt{\frac{1}{M}}$$

$$\therefore \qquad \frac{\nu_{\rm rms \ (helium)}}{\nu_{\rm rms \ (argon)}} = \sqrt{\frac{M_{\rm argon}}{M_{\rm helium}}} \qquad \dots (i)$$

Given, $M_{\text{argon}} = 40 u$ and $M_{\text{helium}} = 4 u$ Substituting these values in Eq. (i), we get

$$\frac{v_{\rm rms (helium)}}{v_{\rm rms (argon)}} = \sqrt{\frac{40}{4}} = \sqrt{10} = 3.16$$

17. Pressure =
$$\frac{\text{Force}}{\text{Area}}$$

=

 $\frac{\text{Number of Collisions}}{\text{collision}} \times \frac{\text{Change in momentum}}{\text{collision}}$ collision

$$\frac{10^{23} \times 2mv \cos 45^{\circ}}{2 \times 10^{-4}} = 2.35 \times 10^{3} \text{ N/m}^{2}$$

18. From
$$pV = nRT = \frac{N}{N_A}RT$$

We have,
$$n_f - n_i = \frac{pVN_A}{RT_c} - \frac{pVN_A}{RT_i}$$

$$\Rightarrow n_f - n_i = \frac{10^5 \times 30}{8.3} \times 6.02 \times 10^{23} \cdot \left(\frac{1}{300} - \frac{1}{290}\right)$$
$$= -2.5 \times 10^{25}$$
$$\therefore \qquad \Delta n = -2.5 \times 10^{25}$$

19. $\Delta Q = \Delta U + \Delta W$

In the process pV^n = constant, molar heat capacity is given by

$$C = \frac{R}{\gamma - 1} + \frac{R}{1 - n} = C_V + \frac{R}{1 - n}$$
$$C - C_V = \frac{R}{1 - n} \implies 1 - n = \frac{C_p - C_V}{C - C_V}$$
$$\implies n = 1 - \left(\frac{C_p - C_V}{C - C_V}\right)$$
$$= \frac{(C - C_V) - (C_p - C_V)}{C - C_V} = \frac{C - C_p}{C - C_V}$$

20.
$$\rho = \frac{pM}{RT}$$
 \therefore $\rho \propto pM$
or $\frac{\rho_1}{\rho_2} = \left(\frac{p_1}{p_2}\right) \left(\frac{M_1}{M_2}\right) = \left(\frac{2}{3}\right) \left(\frac{4}{3}\right) = \frac{8}{9}$

 $v_{\rm rms} = \sqrt{\frac{3KI}{M}}$ or $v_{\rm rms} \propto \frac{1}{\sqrt{M}}$ 21.

:.
$$\frac{(v_{\rm rms})_{\rm He}}{(v_{\rm rms})_{\rm Ar}} = \sqrt{\frac{M_{\rm Ar}}{M_{\rm He}}} = \sqrt{\frac{40}{4}} = \sqrt{10} \approx 3.16$$

- 22. A real gas behaves like an ideal gas at low pressure and high temperature.
- **23.** Internal energy of *n* moles of an ideal gas at temperature *T* is given by

$$U = n \left(\frac{f}{2} RT\right)$$

where,
$$f =$$
 degrees of freedom.
= 5 for Ω_2 and 3 for Ar

= 5 for
$$O_2$$
 and 3 for Ar

Hence,
$$U = U_{O_2} + U_{Ar} = 2\left(\frac{5}{2}RT\right) + 4\left(\frac{3}{2}RT\right) = 11RT$$

24. Process is isothermal. Therefore, $T = \text{constant.} \left(p \propto \frac{1}{V} \right)$

volume is increasing, therefore, pressure will decrease. In chamber $A \rightarrow$

In chamber $B \rightarrow$

$$-1.5\Delta p = (p_B)_i - (p_B)_f = \frac{n_B RT}{V} - \frac{n_B RT}{2V}$$
$$= \frac{n_B RT}{2V} \qquad \dots (ii)$$

From Eqs. (i) and (ii)

or

$$\frac{n_A}{n_B} = \frac{1}{1.5} = \frac{2}{3} \quad \text{or} \quad \frac{m_A/M}{m_B/M} = \frac{2}{3}$$
$$\frac{m_A}{m_B} = \frac{2}{3} \quad \text{or} \quad 3m_A = 2m_B$$

25. Average kinetic energy per molecule per degree of freedom $=\frac{1}{2}kT$. Since, both the gases are diatomic and at same temperature (300 K), both will have the same number of rotational degree of freedom i.e. two. Therefore, both the gases will have the same average rotational kinetic energy per molecule

$$= 2 \times \frac{1}{2} kT$$
 or kT

Thus, ratio will be 1 : 1. \Rightarrow

26.

$$pV = nRT$$

or $p = \frac{nRT}{V}$ or $p \propto T$

if V and n are same. Therefore, if T is doubled, pressure also becomes two times i.e. 2p.

- 27. Average translational kinetic energy of an ideal gas molecule is $\frac{3}{2}kT$ which depends on temperature only. Therefore, if temperature is same, translational kinetic energy of O₂ and N₂ both will be equal.
- **28.** The average translational KE = $\frac{3}{2}kT$ which is directly proportional to T, while rms speed of molecules is given by

$$v_{\rm rms} = \sqrt{\frac{3RT}{M}}$$
 i.e. $v_{\rm rms} \propto \sqrt{T}$

When temperature of gas is increased from 300 K to 600 K (i.e. 2 times), the average translational KE will increase to 2 times and rms speed to $\sqrt{2}$ or 1.414 times.

: Average translational KE =
$$2 \times 6.21 \times 10^{-21}$$
 J

 $= 12.42 \times 10^{-21} \text{ J}$ $v_{\rm rms} = (1.414) (484) \, {\rm m/s}$

and

29.

i.e.

When temperature is increased from 120 K to 480 K (i.e. four times), the root mean square speed will become $\sqrt{4}$ or 2 times i.e. 2v.

 $v_{\rm rms} = \sqrt{\frac{3RT}{M}}$

 $v_{\rm rms} \propto \sqrt{T}$

≈ 684 m/s

30. The average speed of molecules of an ideal gas is given by

$$\langle v \rangle = \sqrt{\frac{8RT}{\pi M}}$$

i.e. $\langle v \rangle \propto \sqrt{T}$ for same gas.

Since, temperatures of A and C are same, average speed of O_2 molecules will be equal in A and C i.e. v_1 .

31. $\gamma_1 = \frac{5}{3}$ means gas is monoatomic or $C_{V_1} = \frac{3}{2}R$

 $\gamma_2 = \frac{7}{5}$ means gas is diatomic or $C_{V_2} = \frac{5}{2}R$ C_V (of the mixture)

$$= \frac{n_1 C_{V_1} + n_2 C_{V_2}}{n_1 + n_2} = \frac{(1) \left(\frac{3}{2}R\right) + (1) \left(\frac{5}{2}\right)R}{1 + 1} = 2R$$

C_n (of the mixture) = C_V + R = 3R

$$\therefore \quad \gamma_{\text{mixture}} = \frac{C_p}{C_V} = \frac{3R}{2R} = 1.5$$

32. $v_{\rm rms} = \sqrt{\frac{3RT}{M}}$

Room temperature $T \approx 300$ K

$$\therefore \qquad 1930 = \sqrt{\frac{3 \times 8.31 \times 10^3 \times 300}{M}}$$
$$\therefore \qquad M = 2.0 \text{ g/mol or the gas is}$$

$$M = 2.0 \text{ g/mol or the gas is H}_2.$$

33. Total translational kinetic energy
$$=\frac{3}{2}nRT = \frac{3}{2}pV = 1.5pV$$

34. (a) Total internal energy
$$U = \frac{f_1}{2}nRT + \frac{f_2}{2}nRT$$

$$(U_{\text{ave}})_{\text{per mole}} = \frac{U}{2n} = \frac{1}{4} [5RT + 3RT] = 2RT$$

$$(b)\gamma_{\text{mix}} = \frac{n_1 C_{p_1} + n_2 C_{p_2}}{n_2 C_{V_1} + n_2 C_{V_2}} = \frac{(1)\frac{7R}{2} + (1)\frac{5R}{2}}{(1)\frac{5R}{2} + (1)\frac{3R}{2}} = \frac{3}{2}$$

$$M_{\text{mix}} = \frac{n_1 M_1 + n_2 M_2}{n_1 + n_2} = \frac{M_1 + M_2}{2} = \frac{2 + 4}{2} = 3$$
Speed of sound $v = \sqrt{\frac{\gamma RT}{M}} \implies v \propto \sqrt{\frac{\gamma}{M}}$

$$\frac{V_{\text{mix}}}{V_{\text{He}}} = \sqrt{\frac{\gamma \min}{\gamma_{\text{He}}} \times \frac{M_{\text{He}}}{M_{\text{mix}}}} = \sqrt{\frac{3/2}{5/3} \times \frac{4}{3}} = \sqrt{\frac{6}{5}}$$

$$(d) \ V_{\text{rms}} = \sqrt{\frac{3RT}{M}} \implies V_{\text{rms}} \propto \frac{1}{\sqrt{M}},$$

$$\frac{V_{\text{He}}}{V_{\text{H}}} = \sqrt{\frac{M_{\text{H}}}{M_{\text{He}}}} = \sqrt{\frac{2}{4}} = \frac{1}{\sqrt{2}}$$
35.
$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}}, \ \bar{v} = \sqrt{\frac{8}{\pi} \cdot \frac{RT}{M}} \approx \sqrt{\frac{2.5RT}{M}}$$
and
$$v_p = \sqrt{\frac{2RT}{M}}$$

From these expressions we can see that,

 $v_p < \overline{v} < v_{\rm rms}$ $v_{\rm rms} = \sqrt{\frac{3}{2}} v_p$ Secondly,

and average kinetic energy of a gas molecule

$$= \frac{1}{2}mv_{\rm rms}^2 = \frac{1}{2}m\left(\sqrt{\frac{3}{2}}v_p\right)^2 = \frac{3}{4}mv_p^2$$

36. For one mole of an ideal gas

$$pV = RT$$

The coefficient of volume expansion at constant pressure is given by,

$$\left(\frac{\Delta V}{\Delta T}\right)_p = \frac{R}{p} = \text{constant}$$
 [option (a)]

The average translational kinetic energy per molecule is (3/2)kT and not 3 kT. With decrease of pressure, volume of the gas increases so its mean free path. [option (c)] The average translational kinetic energy of the molecules is independent of their nature, so each component of the gaseous mixture will have the same value of average translational kinetic energy.

37. Since, the system is insulated, Q=0. Other part is vacuum, therefore, work done by the gas W is also zero. Hence, from first law of thermodynamics, $\Delta U = 0$ i.e. temperature remains constant.

38.
$$Vp^2 = \text{constant}$$

Putting,
$$p = \frac{nRT}{V}$$
, we have $\frac{T^2}{V}$ = constant
or $T \propto \sqrt{V}$

So, if V is doubled, T becomes $\sqrt{2}$ times.

39.
$$C_V = \frac{n_1 C_{V_1} + n_2 C_{V_2}}{n_1 + n_2} = \frac{(1)\left(\frac{3}{2}R\right) + (1)\left(\frac{5}{2}R\right)}{1+1} = 2R$$

40. $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$ or $v_{\text{rms}} \propto \sqrt{\frac{T}{M}}$...(i)

When oxygen gas dissociates into atomic oxygen its atomic mass M will remain half. Temperature is doubled. So, from Eq. (i) $v_{\rm rms}$ will become two times.

41.
$$v_{\rm rms} = \sqrt{\frac{3RT}{M}}$$

So, $v_{\rm rms}$ not only depends on T but also on M.

DT

T 7

$$pV = nRT$$
$$\therefore \qquad V = \left(\frac{nR}{p}\right)T$$

Comparing this with y = mx, V-T graph is a straight line passing through origin with slope = $\frac{nR}{n}$.

or
$$slope \propto \frac{1}{p} (slope)_1 > (slope)_2$$

 $\therefore \qquad p_1 < p_2$

$$43. \quad v_{\rm rms} = \sqrt{\frac{3RT}{M}}$$

42.

It not only depends on T but also on M.

44. Volume of the box = 1 m^3

Pressure of the gas = 100N/m^2



(a) Time between two consecutive collisions with one wall $=\frac{1}{500}$ s. This time should be equal to $\frac{2l}{v_{max}}$, where *l* is the

side of the cube.

$$\frac{2l}{v_{\rm rms}} = \frac{1}{500}$$

or $v_{\rm rms} = 1000 \,\mathrm{m/s} \,(\mathrm{as} \, l = 1 \,\mathrm{m})$
or $\sqrt{\frac{3RT}{M}} = 1000$

$$\therefore \qquad T = \frac{(1000)^2 M}{3R} = \frac{(10)^6 (4 \times 10^{-3})}{3(25/3)} = 160 \,\mathrm{K}$$

(b) Average kinetic energy per atom

$$= \frac{3}{2}kT = \frac{3}{2}(1.38 \times 10^{-23})(160) \text{ J}$$
$$= 3.312 \times 10^{-21} \text{ J}$$
(c) From $pV = nRT = \frac{m}{M}RT$

We get mass of helium gas in the box, $m = \frac{pVM}{RT}$

Substituting the values, we get $m = \frac{(100)(1)(4)}{(25/3)(160)} = 0.3 \text{ g}$

Topic 5 Thermodynamics

1. Efficiency of a Carnot engine working between source of temperature T_1 and sink of temperature T_2 is given by

$$\eta = 1 - \frac{T_2}{T_1}$$

Here, T_2 and T_1 are absolute temperatures. Initially,

 $\eta = \frac{1}{6}$

 $\frac{1}{6} = 1 - \frac{T_2}{T_1} \Longrightarrow \frac{T_2}{T_1} = \frac{5}{6}$ *:*..

Finally, efficiency is doubled on reducing sink temperature by 62°C.

:.
$$\eta = \frac{2}{6}, T_{\text{sink}} = T_2' = T_2 - 62$$

So, $\eta = 1 - \frac{T'_2}{T'_1}$

$$\Rightarrow \quad \frac{2}{6} = 1 - \frac{T_2 - 62}{T_1} \Rightarrow \frac{T_2 - 62}{T_1} = \frac{4}{6} \Rightarrow \quad \frac{T_2}{T_1} - \frac{62}{T_1} = \frac{4}{6} \Rightarrow \quad \frac{5}{6} - \frac{62}{T_1} = \frac{4}{6} \Rightarrow T_1 = 6 \times 62 = 372 \text{ K} = 372 - 273 = 99^{\circ} \text{ C} \Rightarrow \quad T_2 = \frac{5}{6} \times T_1 \approx 310 \text{ K} = 310 - 273 = 37^{\circ} \text{ C}$$
Key Idea In *p-V* curve, work done *dW*, change in internal

energy ΔU and heat absorbed ΔQ are connected with first law of thermodynamics, i.e. $\Delta Q = \Delta U + dW$...(i) and total change in internal energy in complete cycle is always zero. Using this equation in different part of the curve, we can solve the

given problem.

2.

In Process $a \rightarrow b$ $\Delta Q_{ab} = 250 \,\mathrm{J}$ Given, $250 \text{ J} = \Delta U_{ab} + dW_{ab}$ *.*.. ...(ii) In Process $b \rightarrow c$ Given, $\Delta Q_{bc} = 60 \,\mathrm{J}$ Also, V is constant, so dV = 0 $dW_{bc} = p(dV)_{bc} = 0$ \Rightarrow ...(iii) $60 \,\mathrm{J} = \Delta U_{bc} + 0$ *.*.. $\Delta U_{bc} = 60 \,\mathrm{J}$ \Rightarrow In Process $c \rightarrow a$ $\Delta U_{ca} = -180 \,\mathrm{J}$ Given, ...(iv) Now, for complete cycle, $\Delta U_{abca} = \Delta U_{ab} + \Delta U_{bc} + \Delta U_{ca} = 0$...(v) From Eqs. (iii), (iv) and (v), we get $\Delta U_{ab} = -\Delta U_{bc} - \Delta U_{ca}$ $\Delta U_{ab} = -60 + 180 = 120 \text{ J}$...(vi) From Eq. (ii), we get $250 J = 120 J + dW_{ab}$ $dW_{ab} = 130 \text{ J}$...(vii) \Rightarrow From Eqs. (i) and (vii), we get Work done by the gas along the path *abc*, $dW_{abc} = dW_{ab} + dW_{bc}$ = 130 J + 0 K $dW_{abc} = 130 \text{ J}$ \Rightarrow

3. Given, heat supplied at constant volume is *Q*.

...

So, we have

$$Q = nC_V \Delta T$$

For same change in temperature, if heat supplied at constant pressure is *O*, then

$$Q' = nC_p\Delta T$$

$$\frac{Q'}{Q} = \frac{nC_p\Delta T}{nC_V\Delta T} \Rightarrow \frac{Q'}{Q} = \frac{C_p}{C_V}$$

As given gas is diatomic, so $\gamma = \frac{7}{5}$

 \Rightarrow

 $Q' = \frac{7}{5}Q$ *.*..

4. For isobaric process, work done is given as $W = nR\Delta T$ Heat supplied, $\Delta Q = nC_n \Delta T$ and

$$C_{p} - \frac{C_{V}}{n} = R$$

$$\Rightarrow \qquad C_{p} = R + C_{V} / n$$

$$\therefore \qquad \frac{W}{\Delta Q} = \frac{nR\Delta T}{nC_{p}\Delta T} = \frac{R}{C_{p}} = \frac{R}{R + \frac{C_{V}}{n}}$$

$$\Rightarrow \qquad \frac{W}{\Delta Q} = \frac{nR}{nR + C_{V}}$$

5. According to the first law of thermodynamics, Heat supplied (ΔQ) = Work done (W) + Change in internal energy of the system (ΔU)

$$\Delta Q_A = \Delta U_A + W_A$$

Similarly, for proces

$$\Delta Q_B = \Delta U_B + W_B$$

Now, we know that, work done for a process = area under it's *p*-*V* curve Here,



Thus, it is clear from the above graphs,

$$W_A > W_B$$
 ...(i)

Also, since the initial and final state are same in both process, so

$$\Delta U_A = \Delta U_B \qquad \qquad \dots (ii)$$
 So, from Eqs. (i) and (ii), we can conclude that

 $\Delta Q_A > \Delta Q_B$

6. Given processes are



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For process *a*, pressure is constant.

$$\therefore$$
 a is isobaric.

For process d, volume is constant.

 \therefore d is isochoric.

Also, as we know that, slope of adiabatic curve in p-Vdiagram is more than that of isothermal curve.

- \therefore *b* is isothermal and *c* is adiabatic.
- 7. Key Idea In a cyclic thermodynamic process work done = area under p - V diagram. Also in clockwise cycle, work done is positive.

In the given cyclic process, work done = $\oint pdV$ = area enclosed by the cycle

 $=\frac{1}{2} \times \text{base} \times \text{height of triangle } (CAB)$ made by cycle

$$=\frac{1}{2}(V_2 - V_1)(p_2 - p_1)$$

From graph, given

$$V_2 = 5 \text{ m}^3, V_1 = 1 \text{ m}^3,$$

$$p_2 = 6 \text{ Pa}, \quad p_1 = 1 \text{ Pa}$$

$$= \frac{1}{2} (5-1) (6-1) = \frac{1}{2} \times 4 \times 5 = 10 \text{ J}$$

 $T \propto \frac{1}{T}$

8. Given, VT = k, (k is constant)

or

...(i)

...(ii)

Using ideal gas equation,

$$pV = nKT$$

 $pV \propto T \implies pV \propto \frac{1}{V}$

or

i.e a polytropic process with x = 2.

(Polytropic process means, $pV^x = \text{constant}$)

We know that, work done in a polytropic process is given by

 $pV^2 = \text{constant}$

$$\Delta W = \frac{p_2 V_2 - p_1 V_1}{1 - x} (\text{ for } x \neq 1) \qquad \dots (\text{iii})$$

 $\Delta W = pV \ln\left(\frac{V_2}{V_1}\right) \text{(for } x = 1\text{)}$ and,

Here, x = 2,

=

$$\therefore \qquad \Delta W = \frac{p_2 V_2 - p_1 V_1}{1 - x} = \frac{nR(T_2 - T_1)}{1 - x}$$
$$\Rightarrow \qquad \Delta W = \frac{nR\Delta T}{1 - 2} = -nR\Delta T \qquad \dots (iv)$$

Now, for monoatomic gas change in internal energy is given by

$$\Delta U = \frac{3}{2} R \Delta T \qquad \dots (v)$$

Using first law of thermodynamics, heat absorbed by one mole gas is

$$\Delta Q = \Delta W + \Delta U = \frac{3}{2}R\Delta T - R\Delta T \Longrightarrow \Delta Q = \frac{1}{2}R\Delta T$$

9.	Key Idea For an ideal gas undergoing an adiabatic process at room temperature,					
	$pV^{\gamma} = \text{constant or } TV^{\gamma-1} = \text{constant}$					
	For a diatomic gas, degree of freedom, $f = 5$					
	:. $\gamma = 1 + 2/f = 1 + \frac{2}{5} = \frac{7}{5}$					
	As for adiabatic process, $TV^{\gamma - 1} = \text{constant}$ (i)					
	and it is given that, here $TV^x = \text{constant}$ (ii)					
	Comparing Eqs. (i) and (ii), we get					
	$\gamma - 1 = x \implies \frac{7}{5} - 1 = x$					
	or $x = 2/5$					
0.	Work done by gas during heat process at constant pressure is given by					
	$\Delta W = p \Delta V$					
	Using ideal gas equation, rV = rPT					
	$\Rightarrow \qquad p_V - nRT$					
	So, $\Delta W = nR\Delta T$ (i)					
	Now, it is given that, $n = \frac{1}{2}$					
	and $\Delta T = 90^{\circ} \mathrm{C} - 20^{\circ} \mathrm{C}$					
	= 363 K - 293 K = 70 K					
	and R (gas constant) = 8.31 J/mol-K					
	Substituting these values in Eq. (1), we get $\frac{1}{1}$					
	$\Delta W = \frac{1}{2} \times 8.31 \times 70 = 290.85 \text{ J}$					
	$\Delta W \simeq 291 \mathrm{J}$					

11. Given, Carnot engines operates as,



As, efficiency of a Carnot's engine is given by

$$\eta = 1 - \frac{T_{\text{sink}}}{T_{\text{source}}}$$

We have,

$$\eta_1$$
 = efficiency of engine $\varepsilon_1 = 1 - \frac{T_2}{T_1}$
 η_2 = efficiency of engine $\varepsilon_2 = 1 - \frac{T_3}{T_2}$

$$\begin{aligned} \eta_{3} &= \text{efficiency of engine } \varepsilon_{3} = 1 - \frac{T_{4}}{T_{3}} \\ \text{For equal efficiencies,} \\ \eta_{1} &= \eta_{2} = \eta_{3} \\ \Rightarrow & 1 - \frac{T_{2}}{T_{1}} = 1 - \frac{T_{3}}{T_{2}} = 1 - \frac{T_{4}}{T_{3}} \\ \Rightarrow & \frac{T_{2}}{T_{1}} = \frac{T_{3}}{T_{2}} = \frac{T_{4}}{T_{3}} \\ \Rightarrow & T_{2}^{2} = T_{1}T_{3} \text{ and } T_{3}^{2} = T_{2}T_{4} \\ \Rightarrow & T_{2}^{4} = T_{1}^{2}T_{2}^{2} \\ & T_{2}^{4} = T_{1}^{2}T_{2}T_{4} \\ & T_{2}^{3} = T_{4}T_{1}^{2} \\ \text{or} & T_{2} = (T_{1}^{2} T_{4})^{\frac{1}{3}} \\ \text{Also,} & T_{3}^{4} = T_{2}^{2} T_{4}^{2} = T_{1}T_{3}T_{4}^{2} \\ & T_{3}^{3} = T_{1}T_{4}^{2} \text{ or } T_{3} = (T_{1}T_{4}^{2})^{\frac{1}{3}} \end{aligned}$$

12. Key idea In a Carnot engine the heat flow from higher temperature source (at T_1) to lower temperature sink (at T_2) and give the work done equal to the $W = Q_1 - Q_2$.



For the given condition, Carnot engine A and B are operated in series as shown below

$$T_{1} = 600K \xrightarrow{A} Q_{1} \xrightarrow{Q_{2}} \xrightarrow{B} Q_{2} \xrightarrow{Q_{2}} \xrightarrow{Q_{3}} T_{3} = 400K \xrightarrow{Q_{3}} C$$

where,

 \Rightarrow

 Q_1 = heat rejected by engine A at T_1K , Q_2 = heat received by engine B at T_2K and Q_3 = heat rejected by engine B to source C at T_3K . According to Carnot engine principle,

 $W_1 = Q_1 - Q_2$ (work-output from source A and B)

 $W_2 = Q_2 - Q_3$ (work output from source *B* and *C*) As per the given condition, if the work outputs of the two

engines are equal, then ~ ~

$$Q_1 - Q_2 = Q_2 - Q_3$$

 $Q_1 + Q_3 = 2Q_2$

$$\frac{Q_1}{Q_2} + \frac{Q_3}{Q_2} = 2 \qquad \dots (i)$$
$$\frac{T_1}{T_2} + \frac{T_3}{T_2} = 2$$

Therefore

$$\frac{T_2 - T_2}{T_1 + T_3} = 2$$

So,

$$\Rightarrow \qquad T_2 = \frac{T_2}{2} = \frac{600 + 400}{2}$$
$$T_2 = 500 \,\mathrm{K}$$

13. For the *ABC* as shown in the figure below,



According to the first law of thermodynamics, heat supplied, ΔQ = work done (ΔW) + internal energy (ΔU)

$$\Rightarrow \quad \Delta Q_{CB} = \Delta W_{ACB} + (U_B - U_A)$$

[where, $\Delta U = U_B - U_A$]

Substituting the given values,

 $U_B - U_A = 60 - 30 = 30 \text{ J} \qquad \dots(i)$ Similarly for the *ADB* as shown in the figure below,



$$\Delta Q_{ADB} = \Delta W_{ADB} + (U_B - U_A)$$

$$\Rightarrow \Delta Q_{ADB} = 10 + 30 \qquad [using Eq. (i)]$$

$$= 40 \text{ J}$$

14. For adiabatic process,

=

$$T_1 V_1^{\gamma - 1} = T_2 V_2^{\gamma - 1} \qquad \left(\because \gamma = \frac{5}{3} \right)$$

$$\Delta U = \frac{f}{2} nR\Delta T = \left(\frac{3}{2}\right)(2)\left(\frac{25}{3}\right) = \frac{300}{2^{\frac{2}{3}}} \approx 189K$$
$$\Delta U = \frac{f}{2} nR\Delta T = \left(\frac{3}{2}\right)(2)\left(\frac{25}{3}\right)(189 - 300)$$
$$= -2.7 \text{ kJ}$$

15. p-V equation for path AB

$$p = -\left(\frac{p_0}{V_0}\right)V + 3p_0$$

$$\Rightarrow \qquad pV = 3p_0V - \frac{p_0}{V_0}V^2$$

or
$$T = \frac{pV}{nR} = \frac{1}{nR} \left(3p_0V - \frac{p_0}{V_0}V^2\right)$$

For maximum temperature,

$$\frac{dT}{dV} = 0 \implies 3p_0 - \frac{2p_0V}{V_0} = 0$$
$$V = \frac{3}{2}V_0 \text{ and } p = 3p_0 - \frac{p_0}{V_0} = \frac{3p_0}{2}$$

Therefore, at these values :

 \Rightarrow

:.

 \Rightarrow

$$T_{\max} = \frac{\left(\frac{3p_0}{2}\right)\left(\frac{3V_0}{2}\right)}{nR} = \frac{9p_0V_0}{4nR}$$

16. In the first process : $p_i V_i^{\gamma} = p_f V_f^{\gamma}$



For the two step process

$$W = p_i (V_f - V_i) = 10^5 (7 \times 10^{-3}) = 7 \times 10^2 \text{ J}$$
$$\Delta U = \frac{f}{2} (p_f V_f - p_i V_i) = \frac{1}{\gamma - 1} \left(\frac{1}{4} \times 10^2 - 10^2\right)$$
$$\Delta U = -\frac{3}{2} \cdot \frac{3}{4} \times 10^2 = -\frac{9}{8} \times 10^2 \text{ J}$$
$$Q - W = \Delta U$$
$$Q = 7 \times 10^2 - \frac{9}{8} \times 10^2 = \frac{47}{8} \times 10^2 \text{ J} = 588 \text{ J}$$

17. According to first law of thermodynamics, we get (i) Change in internal energy from A to B i.e. ΔU_{AB}

$$\Delta U_{AB} = nC_V (T_B - T_A) = 1 \times \frac{5R}{2} (800 - 400) = 1000 R$$

(ii) Change in internal energy from B to C

$$\Delta U_{BC} = nC_V (T_C - T_B) = 1 \times \frac{5R}{2} (600 - 800)$$

= -500 R

(iii) $\Delta U_{\text{isothermal}} = 0$ (iv) Change in internal energy from C to A i.e. ΔU_{CA} $\Delta U_{CA} = nC_V (T_A - T_C)$ $= 1 \times \frac{5R}{2} (400 - 600) = -500 R$

18. Heat is extracted from the source means heat is given to the system (or gas) or Q is positive. This is positive only along the path ABC.Heat supplied

$$\therefore \quad Q_{ABC} = \Delta U_{ABC} + W_{ABC}$$

= $nC_V (T_f - T_i)$ + Area under p - V graph
= $n\left(\frac{3}{2}R\right)(T_C - T_A) + 2p_0V_0$
= $\frac{3}{2}(nRT_C - nRT_A) + 2p_0V_0$
= $\frac{3}{2}(p_CV_C - p_AV_A) + 2p_0V_0$
= $\frac{3}{2}(4p_0V_0 - p_0V_0) + 2p_0V_0 = \frac{13}{2}p_0V_0$

19. $p_{1} = F$ $32p_{0} = 7$ $p_{0} = 7$ $p_{0} =$

In $F \to G$ work done in isothermal process is

$$nRT \ln\left(\frac{V_f}{V_i}\right) = 32 \ p_0 V_0 \ \ln\left(\frac{32 V_0}{V_0}\right)$$
$$= 32 \ p_0 V_0 \ \ln 2^5 = 160 \ p_0 \ V_0 \ \ln 2$$

In $G \to E$, $\Delta W = p_0 \Delta V = p_0 (31V_0) = 31 p_0 V_0$ In $G \to H$ work done is less than 31 $p_0 V_0$ i.e. 24 $p_0 V_0$ In $F \to H$ work done is 36 $p_0 V_0$

20. At STP, 22.4 L of any gas is 1 mole.

:. 5.6 L =
$$\frac{5.6}{22.4} = \frac{1}{4}$$
 moles = n

In adiabatic process, $TV^{\gamma-1} = \text{constant}$

$$\therefore \qquad T_2 V_2^{\gamma - 1} = T_1 V_1^{\gamma - 1} \text{ or } T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma - 1}$$
$$\gamma = \frac{C_p}{C_V} = \frac{5}{3} \text{ for monoatomic He gas.}$$
$$\therefore \qquad T_2 = T_1 \left(\frac{5.6}{0.7}\right)^{\frac{5}{3} - 1} = 4 T_1$$

Further in adiabatic process, Q = 0

$$\therefore \quad W + \Delta U = 0$$

or
$$W = -\Delta U = -nC_V \Delta T = -n\left(\frac{R}{\gamma - 1}\right)(T_2 - T_1)$$
$$= -\frac{1}{4}\left(\frac{R}{\frac{5}{3} - 1}\right)(4T_1 - T_1) = -\frac{9}{8}RT_1$$

21. Slope of adiabatic process at a given state (p, V, T) is more than the slope of isothermal process. The corresponding p-V graph for the two processes is as shown in figure.



In the graph, AB is isothermal and BC is adiabatic.

 W_{AB} = positive (as volume is increasing)

and W_{BC} = negative (as volume is decreasing) plus,

 $|W_{BC}| > |W_{AB}|$, as area under *p*-*V* graph gives the work done.

Hence,
$$W_{AB} + W_{BC} = W < 0$$

From the graph itself, it is clear that $p_3 > p_1$. **NOTE** At point *B*, slope of adiabatic (process *BC*) is greater than the slope of isothermal (process *AB*).

22.
$$\Delta W_{AB} = p\Delta V = (10)(2-1) = 10 \text{ J}$$

$$\Delta W_{BC} = 0 \qquad (as V = constant)$$

From first law of thermodynamics

$$\Delta Q = \Delta W + \Delta U$$

$$\Delta U = 0$$
 (process *ABCA* is cyclic)

$$\Delta Q = \Delta W_{AB} + \Delta W_{BC} + \Delta W_{CA}$$
$$\Delta W_{CA} = \Delta Q - \Delta W_{AB} - \Delta W_{BC}$$
$$= 5 - 10 - 0 = -5 \text{ J}$$

23. In adiabatic process

...

...

...

slope of
$$p$$
- V graph, $\frac{dp}{dV} = -\gamma \frac{p}{V}$

(with negative sign)

From the given graph,

$$(slope)_2 > (slope)_1$$

slope $\propto \gamma$

$$\gamma_2 > \gamma_1$$

Therefore, 1 should correspond to O_2 ($\gamma = 1.4$) and 2 should correspond to He ($\gamma = 1.67$).

24. The corresponding p-V graphs (also called indicator diagram) in three different processes will be as shown :



Area under the graph gives the work done by the gas.

25. During adiabatic expansion, we know

$$TV^{\gamma - 1} = \text{constant}$$
 or $T_1 V_1^{\gamma - 1} = T_2 V_2^{\gamma - 1}$

For a monoatomic gas, $\gamma = \frac{5}{3}$

$$\therefore \qquad \frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma - 1} = \left(\frac{AL_2}{AL_1}\right)^{(5/3) - 1}$$

$$(A = \text{Area of cross-section of piston})$$

$$= \left(\frac{L_2}{L_1}\right)^{2/3}$$

26. *A* is free to move, therefore, heat will be supplied at constant pressure

$$\therefore \qquad dQ_A = nC_p dT_A \qquad \dots (i)$$

B is held fixed, therefore, heat will be supplied at constant volume.

$$\therefore \qquad dQ_B = nC_V dT_B \qquad \dots (ii)$$

But
$$dQ_A = dQ_B \qquad (given)$$

$$\therefore \qquad nC_p dT_A = nC_V dT_B$$

$$\therefore \qquad dT_B = \left(\frac{C_p}{C_V}\right) dT_A$$

$$= \gamma (dT_A) \qquad [\gamma = 1.4 \text{ (diatomic)}]$$

$$(dT_A = 30 \text{ K})$$

$$= (1.4)(30K)$$

 $\therefore \qquad dT_B = 42 \text{ K}$

27. The desired fraction is

$$f = \frac{\Delta U}{\Delta Q} = \frac{nC_V \Delta T}{nC_p \Delta T} = \frac{C_V}{C_p} = \frac{1}{\gamma} \text{ or } f = \frac{5}{7} \qquad \left(as \gamma = \frac{7}{5} \right)$$

28. Work done in a cyclic process = area between the cycle = $AB \times BC = (2p - p) \times (2V - V) = pV$

NOTE If cycle is clockwise (*p* on *Y*-axis and *V* on *X*-axis) work done is positive and if it is anti-clockwise work done is negative.

29.
$$\left(\frac{dp}{dV}\right)_{\text{adiabatic}} = \gamma \left(\frac{dp}{dV}\right)_{\text{isothermal}}$$

List-I

- (P) Process I \Rightarrow Adiabatic $\Rightarrow Q = 0$
- (Q) Process II \Rightarrow Isobaric

 $\therefore \qquad W = p\Delta V = 3p_0[3V_0 - V_0] = 6p_0V_0$

(R) Process III \Rightarrow Isochoric $\Rightarrow W = 0$

(S) Process (IV) \Rightarrow Isothermal \Rightarrow Temperature = Constant

30. $v_{\text{sound}} = \sqrt{\frac{\gamma RT}{M}}$. As the sound wave propagates, the air in a

chamber undergoes compression and rarefaction very fastly, hence undergo a adiabatic process. So, curves are steeper than isothermal.

$$\left(\frac{dp}{dV}\right)_{\rm Adi} = -\gamma\left(\frac{p}{V}\right) \qquad \dots (i)$$

$$\left(\frac{dp}{dV}\right)_{\rm Iso} = -\left(\frac{p}{V}\right) \qquad \dots (ii)$$

Graph 'Q' satisfies Eq. (i)

31.
$$\Delta U = \Delta Q - p \Delta V$$

 $\Delta U + p\Delta V = \Delta Q$

As $\Delta U \neq 0, W \neq 0, \Delta Q \neq 0$. The process represents, isobaric process

$$W_{\text{gas}} = -p(\Delta V) = -p(V_2 - V_1) = -pV_2 + pV_1$$

Graph 'P' satisfies isobaric process.

32. Work done in isochoric process is zero.

 $W_{12} = 0$ as $\Delta V = 0$ Graph 'S' represents isochoric process.

33. Internal energy $\propto T \propto pV$

This is because
$$U = \frac{nf}{2}RT = \frac{f}{2}(pV)$$

Here, n = number of moles, f = degree of freedom.

 \therefore If the product pV increases then internal energy will increase and if product decreases the internal energy will decrease.

Further, work is done on the gas, if volume of gas decreases. For heat exchange,

$$Q = W + \Delta U$$

Work done is area under p - V graph. If volume increases work done by gas is positive and if volume decreases work done by gas is negative. Further ΔU is positive if product of pV is increasing and ΔU is negative if product of pV is decreasing.

If heat is taken by the gas Q is positive and if heat is lost by the gas Q is negative. Keeping the above points in mind the answer to this question is as under.

$$(A) \rightarrow (p, r, t); (B) \rightarrow (p, r); (C) \rightarrow (q, s); (D) \rightarrow (r, t)$$

34. In process $J \rightarrow K V$ is constant whereas p is decreasing. Therefore, T should also decrease.

$$\therefore \qquad W = 0, \Delta U = -\text{ ve and } Q < 0$$

In process $K \to L p$ is constant while V is increasing. Therefore, temperature should also increase.

$$W > 0, \Delta U > 0$$
 and $Q > 0$

In process $L \to M$ This is inverse of process $J \to K$.

$$W = 0, \Delta U > 0 \text{ and } Q > 0$$

In process $M \rightarrow J$

...

...

V is decreasing. Therefore, W < 0 $(pV)_J < (pV)_M$

	$(pV)_J < (pV)_J$
<i>.</i>	$T_J < T_M$
or	$\Delta U < 0$
Therefore,	Q < 0.

35. Process-II is isothermal expansion,

$$\Delta U = 0, W > 0$$

$$\Delta Q = W > 0$$

Process-IV is isothermal compression,

$$\Delta U = 0, W < 0$$
$$\Delta Q = W < 0$$

Process-I and III are not isobaric because in isobaric process $T \propto V$ hence, T-V graph should be a straight line passing through origin.

Option b, c and d are true.

36. Note This question can be solved if right hand side hand side chamber is assumed open, so that its pressure remains constant even if the piston shifts towards right.



(a)

$$pV = nRT$$
$$p \propto \frac{T}{V}$$

Temperature is made three times and volume is doubled

$$\Rightarrow \qquad p_2 = \frac{3}{2} p_1$$

Further
$$x = \frac{\Delta V}{A} = \frac{V_2 - V_1}{A} = \frac{2V_1 - V_1}{A} = \frac{V_1}{A}$$
$$p_2 = \frac{3p_1}{2} = p_1 + \frac{kx}{A} \Rightarrow kx = \frac{p_1 A}{2}$$

Energy of spring

$$\frac{1}{2}kx^{2} = \frac{p_{1}A}{4}x = \frac{p_{1}V_{1}}{4}$$
(b) $\Delta U = nc_{v}\Delta T = n\left(\frac{3}{2}R\right)\Delta T$
 $= \frac{3}{2}(p_{2}V_{2} - p_{1}V_{1})$
 $= \frac{3}{2}\left[\left(\frac{3}{2}p_{1}\right)(2V_{1}) - p_{1}V_{1}\right] = 3p_{1}V_{1}$
(c) $p_{2} = \frac{4p_{1}}{3}$
 $\Rightarrow p_{2} = \frac{4}{3}p_{1} = p_{1} + \frac{kx}{A}$

$$\Rightarrow \qquad kx = \frac{p_1 A}{3}$$
$$\Rightarrow \qquad x = \frac{\Delta V}{A} = \frac{2V_1}{A}$$

$$W_{\text{gas}} = (p_0 \ \Delta V + W_{\text{spring}}) = (p_1 A x + \frac{1}{2} k x \cdot x)$$
$$= + \left(p_1 A \cdot \frac{2V_1}{A} + \frac{1}{2} \cdot \frac{p_1 A}{3} \cdot \frac{2V_1}{A} \right)$$
$$= 2 p_1 V_1 + \frac{p_1 V_1}{3} = \frac{7 p_1 V_1}{3}$$

(d)
$$\Delta Q = W + \Delta U$$

 $= \frac{7p_1V_1}{3} + \frac{3}{2}(p_2V_2 - p_1V_1)$
 $= \frac{7p_1V_1}{3} + \frac{3}{2}\left(\frac{4}{3}p_1 \cdot 3V_1 - p_1V_1\right)$
 $= \frac{7p_1V_1}{3} + \frac{9}{2}p_1V_1 = \frac{41p_1V_1}{6}$

NOTE $\Delta U = \frac{3}{2} (p_2 V_2 - p_1 V_1)$ has been obtained in part (b).

37.
$$T_A = T_B \implies \therefore \quad U_A = U_B$$

 $W_{AB} = (1) (R) T_0 \ln \left(\frac{V_f}{V_i}\right)$
 $= RT_0 \ln \left(\frac{4V_0}{V_0}\right) = p_0 V_0 \ln (4)$

Information regarding p and T at C can not be obtained from the given graph. Unless it is mentioned that line BC passes through origin or not.

Hence, the correct options are (a) and (b).

- **38.** (A) *p-V* graph is not rectangular hyperbola. Therefore, process *A-B* is not isothermal.
 - (B) In process *BCD*, product of *pV* (therefore temperature and internal energy) is decreasing. Further, volume is decreasing. Hence, work done is also negative. Hence, *Q* will be negative or heat will flow out of the gas.

(C) W_{ABC} = positive

- (D) For clockwise cycle on *p-V* diagram with *P* on *Y*-axis, net work done is positive.
- **39.** There is a decrease in volume during melting on an ice slab at 273 K. Therefore, negative work is done by ice-water system on the atmosphere or positive work is done on the ice-water system by the atmosphere. Hence, option (b) is correct. Secondly heat is absorbed during melting (*i.e.*, dQ is positive) and as we have seen, work done by ice-water system is negative (dW is negative). Therefore, from first law of thermodynamics dU = dQ dW

change in internal energy of ice-water system, dU will be positive or internal energy will increase.

40. (a) $\Delta U = nC_V \Delta T = nC_V (T_2 - T_1)$ in all processes

(b) In adiabatic process $\Delta Q = 0$

- $\therefore \qquad \Delta U = -\Delta W \quad \text{or} \quad |\Delta U| = |\Delta W|$
- (c) In isothermal process $\Delta T = 0$

$$\Delta U = 0 \qquad (as \ \Delta U = nC_V \Delta T)$$

(d) In adiabatic process $\Delta Q = 0$

: All the options are correct.

...

41. Given,
$$n = 1, \gamma = \frac{5}{3}$$

 $T \cdot V$ equation in adiabatic process is
 $TV^{\gamma - 1} = \text{constant}$
 \therefore $T_1V_1^{\gamma - 1} = T_2V_2^{\gamma - 1}$
 \Rightarrow $T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma - 1}$
 $= 100 \times \left(\frac{1}{8}\right)^{\frac{2}{3}}$
 \Rightarrow $T_2 = 25 \text{ K}$
 $C_V = \frac{3}{2} R$ for monoatomic gas
 \therefore $\Delta U = nC_V \Delta T = n \times \left(\frac{3R}{2}\right)(T_2 - T_1)$
 $= 1 \times \frac{3}{2} \times 8 \times (25 - 100)$
 $= -900 \text{ J}$
 \therefore Decrease in internal energy = 900 J
42. $W_{ibf} = W_{ib} + W_{bf} = 50 \text{ J} + 100 \text{ J} = 150 \text{ J}$
 $W_{iaf} = W_{ia} + W_{af} = 0 + 200 \text{ J} = 200$
 $Q_{iaf} = 500 \text{ J}$
So $\Delta U_{iaf} = Q_{iaf} - W_{iaf}$
 $= 500 \text{ J} - 200 \text{ J}$
 $= 300 \text{ J} + 100 \text{ J} = 400 \text{ J}$
 $\Delta U_{ib} = U_b - U_i$
 $= 200 \text{ J} - 100 \text{ J} = 100 \text{ J}$
 $Q_{ib} = \Delta U_{ib} + W_{ib}$
 $= 100 \text{ J} + 50 \text{ J} = 150 \text{ J}$
 $Q_{ibf} = \Delta U_{ib} + W_{ib}$
 $= 100 \text{ J} + 50 \text{ J} = 150 \text{ J}$
So, the required ratio
 $\frac{Q_{bf}}{Q_{ib}} = \frac{Q_{ibf} - Q_{ib}}{Q_{ib}}$
 $= \frac{450 - 150}{150} = 2$

43. In adiabatic process,

 $TV^{\gamma - 1} = \text{constant}$

 $T_i V_i^{0.4} = T_f V_f^{0.4}$ (as $\gamma = 1.4$ for diatomic gas) *:*..

or

 $T_i V_i^{0.4} = (q T_i) \left(\frac{V_i}{32}\right)^{0.4}$ $q(32)^{0.4} = 4$ or

 \therefore The correct answer is 4.

44. Under isothermal conditions, pV = constant $(p_i)(2V) = pV$ *.*..

 $p_i = p/2$ \Rightarrow Under adiabatic conditions, $pV^{\gamma} = \text{constant}$ where $\gamma = 1.67$ $\therefore (p_a)(2V)^{1.67} = pV^{1.67}$ $p_0 = p/(2)^{1.67}$ \Rightarrow Therefore, the ratio $p_a/p_i = (2/2^{1.67}) = 0.628$

45. In *p*-*V* graph slope of adiabatic = y (slope of isothermal) or slope of adiabatic > slope of isothermal.

46. (a) From
$$\Delta Q = ms \Delta T$$

$$\Delta T = \frac{\Delta Q}{ms} = \frac{20000}{1 \times 400} = 50^{\circ}\text{C}$$

(b) $\Delta V = V\gamma \Delta T = \left(\frac{1}{9000}\right) (9 \times 10^{-5}) (50)$
 $= 5 \times 10^{-7} \text{ m}^3$
 $\therefore W = p \cdot \Delta V = (10^5) (5 \times 10^{-7}) = 0.05 \text{J}$
(c) $\Delta U = \Delta Q - W = (20000 - 0.05) \text{J}$
 $= 19999.95 \text{J}$

47. Given,

200 J

Number of moles, n = 2



(monoatomic)

$$T_A = 27^{\circ} \text{C} = 300 \text{ K}$$

Let
$$V_A = V_0$$
 then $V_B = 2V_0$
and $V_D = V_C = 4V_0$

(a) Process
$$A \to B$$

$$\Rightarrow \qquad \begin{array}{c} V \propto T \\ \frac{T_B}{T_A} = \frac{V_B}{V_A} \\ \therefore \qquad T_B = T_A \left(\frac{V_B}{V_A}\right) = (300)(2) = 600 \text{ K} \\ \therefore \qquad T_B = 600 \text{ K} \\ \text{(b) Process } A \to B \end{array}$$

$$V \sim T$$

$$\Rightarrow \qquad p = \text{constant}$$

$$\therefore \qquad Q_{AB} = nC_p dT = nC_p (T_B - T_A)$$

$$= (2) \left(\frac{5}{2}R\right) (600 - 300)$$

(absorbed)

 $Q_{AB} = 1500 R$ **Process** $B \rightarrow C$

$$T = \text{constant}$$

$$dU = 0$$

$$\therefore \qquad Q_{BC} = W_{BC} = nRT_B \ln\left(\frac{V_C}{V_B}\right)$$

$$= (2)(R)(600) \ln\left(\frac{4V_0}{2V_0}\right)$$

$$= (1200R) \ln (2) = (1200R)(0.693)$$
or
$$Q_{BC} \approx 831.6R \text{ (absorbed)}$$

Process $C \rightarrow D$ V = constant

$$\therefore \qquad Q_{CD} = nC_V \ dT = nC_V (T_D - T_C)$$
$$= n \left(\frac{3}{2}R\right) (T_A - T_B)$$
$$(T_D = T_A \text{ and } T_C = T_B)$$
$$= (2) \left(\frac{3}{2}R\right) (300 - 600)$$
$$Q_A = -000 R \text{ (released)}$$

 $Q_{CD} = -900 R$ (released) **Process** $D \rightarrow A T$ = constant

$$\Rightarrow \qquad \Delta U = 0$$

$$\therefore \qquad Q_{DA} = W_{DA} = nRT_D \ln\left(\frac{V_A}{V_D}\right)$$

$$= (2)(R)(300) \ln\left(\frac{V_0}{4V_0}\right)$$

$$= 600R \ln\left(\frac{1}{4}\right)$$

 $Q_{DA} \approx -831.6 R$ (released)

(c) In the complete cycle $\Delta U = 0$

Therefore, from conservation of energy

$$W_{\text{net}} = Q_{AB} + Q_{BC} + Q_{CD} + Q_{DA}$$

$$W_{\text{net}} = 1500 R + 831.6 R - 900 R - 831.6 R$$

or $W_{\text{net}} = W_{\text{total}} = 600 R$

48. (a) Number of moles, n = 2, $T_1 = 300$ K During the process $A \rightarrow B$

$$pT = \text{constant or } p^2 V = \text{constant} = K \text{ (say)}$$

$$\therefore \qquad p = \frac{\sqrt{K}}{\sqrt{V}}$$

$$\therefore \qquad W_{A \to B} = \int_{V_A}^{V_B} p \cdot dV = \int_{V_A}^{V_B} \frac{\sqrt{K}}{\sqrt{V}} dV$$

$$= 2\sqrt{K} [\sqrt{V_B} - \sqrt{V_A}]$$

$$= 2[\sqrt{KV_B} - \sqrt{KV_A}]$$

$$= 2[\sqrt{(p_B^2 V_B)V_B} - \sqrt{(p_A^2 V_A)V_A}]$$

= 2 [$p_B V_B - p_A V_A$]
= 2 [$nRT_B - nRT_A$] = 2 nR [$T_1 - 2T_1$]
= (2)(2)(R)[300 - 600] = -1200 R

 \therefore Work done on the gas in the process *AB* is 1200 *R*. Alternate solution



From the given condition

From

 $\therefore W_A$

$$pT = \text{constant}$$

$$pdT + Tdp = 0 \qquad \dots \text{(ii)}$$
Eqs. (i) and (ii), we get
$$pdV = 2nRdT$$

$$\rightarrow B = \int pdV = 2nR \int_{T_A}^{T_B} dT = 2nR(T_B - T_A)$$

$$= 2nR(T_1 - 2T_1) = (2)(2)(R)(300 - 600)$$

or $W_{A \rightarrow B} = -1200 R$

(b) **Heat absorbed/released in different processes.** Since, the gas is monoatomic.

Therefore,
$$C_V = \frac{3}{2}R$$
 and $C_p = \frac{5}{2}R$ and $\gamma = \frac{5}{3}$
Process $A \rightarrow B \ \Delta U = nC_V \Delta T$
 $= (2)\left(\frac{3}{2}R\right)(T_B - T_A)$
 $= (2)\left(\frac{3}{2}R\right)(300 - 600) = -900R$
 $\therefore \qquad Q_{A \rightarrow B} = W_{A \rightarrow B} + \Delta U$
 $= (-1200R) - (900R)$
 $Q_{A \rightarrow B} = -2100R$ (released)
Alternate solution
In the process pV^x = constant
Molar heat capacity, $C = \frac{R}{\gamma - 1} + \frac{R}{1 - x}$
Here the process is p^2V = constant
or $pV^{1/2}$ = constant
i.e. $x = \frac{1}{2}$

$C = \frac{R}{\frac{5}{3} - 1} + \frac{R}{1 - \frac{1}{2}}$ C = 3.5 R $Q_{A \to B} = nC\Delta T$ = (2)(3.5 R)(300 - 600)or $Q_{A \to B} = -2100 R$ Process $B \to C$ Process is isobaric. $Q_{B \to C} = nC_p\Delta T = (2)\left(\frac{5}{2}R\right)(T_C - T_B)$

$$= 2\left(\frac{5}{2}R\right)(2T_1 - T_1)$$

= (5R)(600 - 300)

 $Q_{B \to C} = 1500 R \text{ (absorbed)}$

Process $C \rightarrow A$ Process is isothermal.

$$\therefore \qquad \Delta U = 0$$
and $Q_{C \to A} = W_{C \to A} = nRT_C \ln\left(\frac{p_C}{p_A}\right)$

$$= nR(2T_1)\ln\left(\frac{2p_1}{p_1}\right) = (2)(R)(600)\ln(2)$$

$$Q_{C \to A} = 831.6R \text{ (absorbed)}$$

NOTE

• In first law of thermodynamics, (*dQ* = *dU* + *dW*) we come across three terms *dQ*, *dU* and *dW*.

 $dU = nC_V dT$ for all the processes whether it is isobaric, isochoric or else and dQ = nCdT where

$$C = \frac{R}{g-1} + \frac{R}{1-X}$$

in the process $pV^r = \text{constant}$.

- In both the terms we require $dT = (T_f T_i)$ only. The third term dW is obviously dQ dU. Therefore if in any process change in temperature (dT) and p-V relation is known, then the above method is the simplest one. Note that even if we have V-T or T-p relation, it can be converted into p-V relation by the equation pV = nRT
- **49.** (a) The p-V diagram for the complete process will be as shown below :



Process $A \rightarrow B$ is adiabatic compression and Process $B \rightarrow C$ is isochoric.

(b) (i) Total work done by the gas

Process
$$A \rightarrow B$$

$$W_{AB} = \frac{p_A V_A - p_B V_B}{\gamma - 1}$$

$$W_{\text{adiabatic}} = \frac{p_i V_i - p_f V_f}{\gamma - 1} = \frac{p_1 V_1 - p_2 V_2}{\frac{5}{3} - 1}$$

 $\gamma = 5/3$ for monoatomic gas

$$= \frac{p_1 V_1 - p_1 \left(\frac{V_1}{V_2}\right)^{\gamma} V_2}{2/3} \qquad \begin{bmatrix} p_1 V_1^{\gamma} = p_2 V_2^{\gamma} \\ \therefore p_2 = p_1 \left(\frac{V_1}{V_2}\right)^{\gamma} \end{bmatrix}$$
$$= \frac{3}{2} p_1 V_1 \left[1 - \left(\frac{V_1}{V_2}\right)^{\gamma-1} \right] = -\frac{3}{2} p_1 V_1 \left[\left(\frac{V_1}{V_2}\right)^{5/3-1} - 1 \right]$$
$$= -\frac{3}{2} p_1 V_1 \left[\left(\frac{V_1}{V_2}\right)^{2/3} - 1 \right]$$

Process $B \rightarrow C W_{BC} = 0 (V = \text{constant})$

$$W_{\text{Total}} = W_{AB} + W_{BC} = -\frac{3}{2} p_1 V_1 \left[\left(\frac{V_1}{V_2} \right)^{2/3} - 1 \right]$$

(ii) Total change in internal energy

...

Process $A \rightarrow B Q_{AB} = 0$ (Process is adiabatic)

:.
$$\Delta U_{AB} = -W_{AB} = \frac{3}{2} p_1 V_1 \left[\left(\frac{V_1}{V_2} \right)^{2/3} - 1 \right]$$

Process $\boldsymbol{B} \to \boldsymbol{C} \ W_{BC} = 0$

$$\therefore \quad \Delta U_{BC} = Q_{BC} = Q \quad \text{(Given)}$$

$$\therefore \quad \Delta U_{\text{Total}} = \Delta U_{AB} + \Delta U_{BC}$$

$$= \frac{3}{2} p_1 V_1 \left[\left(\frac{V_1}{V_2} \right)^{2/3} - 1 \right] + Q$$

(iii) Final temperature of the gas

$$\Delta U_{\text{Total}} = nC_V \Delta T = 2\left(\frac{R}{\gamma - 1}\right)(T_C - T_A)$$

$$\therefore \frac{3}{2} p_1 V_1 \left[\left(\frac{V_1}{V_2}\right)^{2/3} - 1 \right] + Q = \frac{2R}{5/3 - 1} \left(T_C - \frac{p_A V_A}{2R}\right)$$

or $\frac{3}{2} p_1 V_1 \left[\left(\frac{V_1}{V_2}\right)^{2/3} - 1 \right] + Q = 3R \left(T_C - \frac{p_1 V_1}{2R}\right)$

$$\therefore T_C = \frac{Q}{3R} + \frac{p_1 V_1}{2R} \left(\frac{V_1}{V_2}\right)^{2/3} = T_{\text{final}}$$

50. (a) *ABCA* is a clockwise cyclic process.



.:. Work done by the gas

$$W = +$$
 Area of triangle ABC

$$= \frac{1}{2} (\text{base}) (\text{height}) = \frac{1}{2} (2V_0 - V_0) (3p_0 - p_0)$$

$$W = p_0 V_0$$

=

(b) Number of moles n = 1 and gas is monoatomic, therefore

$$C_V = \frac{3}{2}R \quad \text{and} \quad C_p = \frac{5}{2}R$$

$$\frac{C_V}{R} = \frac{3}{2} \quad \text{and} \quad \frac{C_p}{R} = \frac{5}{2}$$

(i) Heat rejected in path CA (Process is isobaric)

$$\therefore \quad dQ_{CA} = C_p dT = C_p (T_f - T_i)$$
$$= C_p \left(\frac{p_f V_f}{R} - \frac{p_i V_i}{R}\right) = \frac{C_p}{R} (p_f V_f - p_i V_i)$$

Substituting the values

$$dQ_{CA} = \frac{5}{2}(p_0V_0 - 2p_0V_0) = -\frac{5}{2}p_0V_0$$

Therefore, heat rejected in the process CA is $\frac{5}{2}p_0V_0$.

(ii) Heat absorbed in path AB (Process is isochoric) $dQ_{uv} = C_{uv} dT = C_{uv} (T_u)$ T

$$C_{V} dQ_{AB} = C_{V} dI = C_{V} (I_{f} - I_{i})$$

$$= C_{V} \left(\frac{p_{f}V_{f}}{R} - \frac{p_{i}V_{i}}{R} \right)$$

$$= \frac{C_{V}}{R} (p_{f}V_{f} - p_{i}V_{i})$$

$$= \frac{3}{2} (p_{f}V_{f} - p_{i}V_{i})$$

$$= \frac{3}{2} (3p_{0}V_{0} - p_{0}V_{0})$$

$$dQ_{AB} = 3p_{0}V_{0}$$

 \therefore Heat absorbed in the process *AB* is 3 p_0V_0 .

(c) Let dQ_{BC} be the heat absorbed in the process BC:

Total heat absorbed,

$$dQ = dQ_{CA} + dQ_{AB} + dQ_{BC}$$

$$dQ = \left(-\frac{5}{2}p_0V_0\right) + (3p_0V_0) + dQ_{BC}$$

$$dQ = dQ_{BC} + \frac{p_0V_0}{2}$$

Change in internal energy, dU = 0

.

$$\therefore \qquad dQ = dW$$

$$\therefore \qquad dQ_{BC} + \frac{p_0 V_0}{2} = p_0 V_0 \therefore dQ_{BC} = \frac{p_0 V_0}{2}$$

 \therefore Heat absorbed in the process *BC* is $\frac{p_0V_0}{2}$.

(d) Maximum temperature of the gas will be somewhere between B and C. Line BC is a straight line. Therefore, *p*-*V* equation for the process *BC* can be written as

$$p = -mV + c \qquad (y = mx + c)$$

Here, $m = \frac{2p_0}{V_0}$ and $c = 5p_0$
 $\therefore \qquad p = -\left(\frac{2p_0}{V_0}\right)V + 5p_0$

Multiplying the equation by V,

$$pV = -\left(\frac{2p_0}{V_0}\right)V^2 + 5p_0V \qquad (pV = RT \text{ for } n = 1)$$
$$RT = -\left(\frac{2p_0}{V_0}\right)V^2 + 5p_0V$$
or $T = \frac{1}{R}\left[5p_0V - \frac{2p_0}{V_0}V^2\right] \qquad \dots(i)$ For T to be maximum $\frac{dT}{dt} = 0$

For *T* to be maximum, $\frac{dT}{dV}$

$$\Rightarrow \qquad 5p_0 - \frac{4p_0}{V_0}V = 0 \Rightarrow V = \frac{5V_0}{4}$$

i.e. at $V = \frac{\Im V_0}{4}$, (on line *BC*), temperature of the gas is maximum. From Eq. (i) this maximum temperature will be

$$T_{\max} = \frac{1}{R} \left[5p_0 \left(\frac{5V_0}{4} \right) - \frac{2p_0}{V_0} \left(\frac{5V_0}{4} \right)^2 \right]$$
$$T_{\max} = \frac{25}{8} \frac{p_0 V_0}{R}$$

51. Number of gram moles of He,

$$n = \frac{m}{M} = \frac{2 \times 10^{3}}{4} = 500$$
(a) $V_{A} = 10 \text{ m}^{3}$, $p_{A} = 5 \times 10^{4} \text{ N/m}^{2}$
 \therefore $T_{A} = \frac{p_{A}V_{A}}{nR} = \frac{(10)(5 \times 10^{4})}{(500)(8.31)} \text{ K}$
or $T_{A} = 120.34 \text{ K}$
Similarly, $V_{B} = 10 \text{ m}^{3}$, $p_{B} = 10 \times 10^{4} \text{ N/m}^{2}$
 \therefore $T_{B} = \frac{(10)(10 \times 10^{4})}{(500)(8.31)} \text{ K}$
 \therefore $T_{B} = 240.68 \text{ K}$
 $V_{C} = 20 \text{ m}^{3}$, $p_{C} = 10 \times 10^{4} \text{ N/m}^{2}$
 \therefore $T_{C} = \frac{(20)(10 \times 10^{4})}{(500)(8.31)} \text{ K}$

=

$T_C = 481.36$ K and $V_D = 20 \text{ m}^3$, $p_D = 5 \times 10^4 \text{ N/m}^2$ $V_D = \frac{(20)(5 \times 10^4)}{(500)(8.31)} \,\mathrm{K}$ *:*.

 $T_D = 240.68 \,\mathrm{K}$

(b) No, it is not possible to tell afterwards which sample went through the process ABC or ADC. But during the process if we note down the work done in both the processes, then the process which require more work goes through process ABC.

(c) In the process *ABC*

$$\Delta U = nC_V \ \Delta T = n\left(\frac{3}{2}R\right)(T_C - T_A)$$

= (500) $\left(\frac{3}{2}\right)$ (8.31) (481.36 - 120.34) J
 $\Delta U = 2.25 \times 10^6$ J

and ΔW = Area under *BC* $= (20 - 10) (10) \times 10^4 \text{ J} = 10^6 \text{ J}$

$$\therefore \Delta Q_{ABC} = \Delta U + \Delta W = (2.25 \times 10^6 + 10^6) \,\mathrm{J}$$

$$\Delta Q_{ABC} = 3.25 \times 10^6 \text{ J}$$

In the process ADC ΔU will be same (because it depends on initial and final temperatures only)

 ΔW = Area under AD $= (20 - 10) (5 \times 10^4) J = 0.5 \times 10^6 J$ $\Delta Q_{ADC} = \Delta U + \Delta W = (2.25 \times 10^6 + 0.5 \times 10^6) \text{ J}$ $\Delta Q_{ADC} = 2.75 \times 10^6 \text{ J}$

- **52.** The corresponding p-V diagram is as shown Given $T_A = 300 \text{ K}, n = 1, \gamma = 1.4, V_A / V_B = 16$ and $V_C/V_B = 2$
 - Let $V_B = V_0$ and $p_B = p_0$
 - Then, $V_{C} = 2V_{0}$ and $V_{A} = 16V_{0}$

Temperature at **B**

Process A-B is adiabatic. Hence,



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Temperature at **D**

:..

...

 $B \rightarrow C$ is an isobaric process (p = constant)

$$T \propto V, V_C = 2V_B$$
$$T_C = 2T_B = (2)(909) \text{ K}$$
$$T_C = 1818 \text{ K}$$

Now, the process $C \rightarrow D$ is adiabatic.

Therefore,
$$T_D = T_C \left(\frac{V_C}{V_D}\right)^{\gamma - 1} = (1818) \left(\frac{2}{16}\right)^{1.4 - 1}$$

 $T_D = 791.4 \text{ K}$

Efficiency of cycle

Efficiency of cycle (in percentage) is defined as

$$\eta = \frac{\text{Net work done in the cycle}}{\text{Heat absorbed in the cycle}} \times 100$$

or $\eta = \frac{W_{\text{Total}}}{2} \times 100$

$$= \frac{Q_{+ve}}{Q_{+ve}} \times 100 = \left(1 - \frac{Q_1}{Q_2}\right) \times 100 \qquad \dots (i)$$

where, Q_1 = negative heat in the cycle (heat released) and Q_2 = positive heat in the cycle (heat absorbed) In the cycle

$$Q_{AB} = Q_{CD} = 0 \qquad \text{(Adiabatic process)}$$
$$Q_{DA} = nC_V \ \Delta T = (1) \left(\frac{5}{2}R\right) (T_A - T_D)$$
$$(C_V = \frac{5}{2}R \text{ for a diatomic gas)}$$

$$= \frac{5}{2} \times 8.31(300 - 791.4) \,\mathrm{J}$$

and
$$Q_{BC} = nC_p \Delta T = (1) \left(\frac{7}{2}R\right) (T_C - T_B)$$

 $(C_p = \frac{7}{2}R \text{ for a diatomic gas})$

$$= \left(\frac{7}{2}\right)(8.31)(1818 - 909).$$

 $Q_{BC} = 26438.3 \,\mathrm{J}$ or

Therefore, substituting $Q_1 = 10208.8 \text{ J}$ and $Q_2 = 26438.3$ J in Eq. (i), we get

$$\therefore \quad \eta = \left\{ 1 - \frac{10208.8}{26438.3} \right\} \times 100 \quad \text{or} \quad \eta = 61.4\%$$

53. Given, $T_1 = 27^\circ \text{C} = 300 \text{ K}, V_1 = V, V_2 = 2V$ (a) Final temperature

In adiabatic process, $TV^{\gamma - 1} = \text{constant}$

$$T_1 V_1^{\gamma - 1} = T_2 V_2^{\gamma - 1}$$

or
$$T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma - 1} = 300 \left(\frac{V}{2V}\right)^{5/3 - 1}$$

or

$$\gamma = \frac{5}{3} \text{ for a monoatomic gas } T_2 \approx 189 \text{ K}$$
(b) Change in internal energy,

$$\Delta U = nC_F \Delta T$$

$$\Delta U = (2) \left(\frac{3}{2}R\right) (T_2 - T_1)$$

$$\Delta U = 2 \left(\frac{3}{2}\right) (8.31) (189 - 300) \text{ J}$$

$$\Delta U = -2767 \text{ J}$$
(c) Work done
Process is adiabatic, therefore $\Delta Q = 0$
and from first law of thermodynamics,

$$\Delta Q = \Delta W + \Delta U$$

$$\Delta W = -\Delta U = -(-2767 \text{ J})$$

$$\Delta W = 2767 \text{ J}$$
54. (a) In a cyclic process $\Delta U = 0$
Therefore, $Q_{\text{net}} = W_{\text{net}}$
or $Q_1 + Q_2 + Q_3 + Q_4 = W_1 + W_2 + W_3 + W_4$
Hence, $W_4 = (Q_1 + Q_2 + Q_3 + Q_4) - (W_1 + W_2 + W_3)$
 $= \{(5960 - 5585 - 2980 + 3645)$
 $-(2200 - 825 - 1100)\}$
or $W_4 = 765 \text{ J}$
(b) Efficiency,
 $\eta = \frac{\text{Total work done in the cycle}}{\text{Heat absorbed (positive heat)}} \times 100$
 $g = \left\{\frac{(2200 - 825 - 1100 + 765)}{5960 + 3645}\right\} \times 100$
 $g = \left\{\frac{(2200 - 825 - 1100 + 765)}{5960 + 3645}\right\} \times 100$
 $g = \left\{\frac{(2200 - 825 - 1100 + 765)}{5960 + 3645}\right\} \times 100$
 $\eta = 10.82\%$
NOTE
* From energy conservation
 $W_{\text{net}} = Q_{\text{net}} < 0 = (Q_{\text{net}} - Q_{\text{net}}) \times 100 = \left(1 - \frac{Q_{\text{net}}}{Q_{\text{net}}}\right) \times 100$
In the above question
 $Q_{\text{net}} = Q_{\text{let}} + |Q_{\text{l}}| = (5585 + 2980) \text{ J} = 8565 \text{ J}$
and $Q_{\text{net}} = Q_{\text{l}} + Q_{\text{l}} = (5685 + 2980) \text{ J} = 8565 \text{ J}$
and $Q_{\text{net}} = Q_{\text{l}} + Q_{\text{l}} = (5685 + 2980) \text{ J} = 8565 \text{ J}$
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(a) $A \rightarrow B$ is an adiabatic process, therefore

$$p_{A}^{1-\gamma}T_{A}^{\gamma} = p_{B}^{1-\gamma}T_{B}^{\gamma}$$

$$\therefore \quad T_{B} = T_{A} \left(\frac{p_{A}}{p_{B}}\right)^{\frac{1-\gamma}{\gamma}}$$

$$= (1000) \left(\frac{3}{2}\right)^{\frac{1-5/3}{5/3}} = (1000) \left(\frac{3}{2}\right)^{-2/5}$$

$$= (1000) \left(\frac{2}{3}\right)^{2/5}$$

$$T_{B} = (1000)(0.85)$$

$$\therefore \quad T_{B} = 850 \text{ K}$$

$$p_{A} = \int_{D} \int_{D} \int_{C} \int$$

Now, work done in the process $A \rightarrow B$ will be

$$W_{AB} = \frac{R}{1 - \gamma} (T_B - T_A)$$

= $\frac{8.31}{1 - 5/3} (850 - 1000)$ or $W_{AB} = 1869.75 \text{ J}$

(b) $B \rightarrow C$ is an isochoric process (V = constant)

$$\therefore \quad \frac{T_B}{T_C} = \frac{p_B}{p_C}$$

$$\therefore \quad T_C = \left(\frac{p_C}{p_B}\right) T_B$$

$$= \left(\frac{(1/3)p_A}{(2/3)p_A}\right) 850 \text{ K}$$

$$T_C = 425 \text{ K}$$
Therefore, $Q_{BC} = nC_V \Delta T = (1) \left(\frac{3}{2}R\right) (T_C - T_B)$

$$= \left(\frac{3}{2}\right)(8.31)(425 - 850)$$

$$Q_{BC} = -5297.6 \,\mathrm{J}$$

Therefore, heat lost in the process *BC* is 5297.6 J. (c) $C \rightarrow D$ and $A \rightarrow B$ are adiabatic processes. Therefore,

$$p_{C}^{1-\gamma}T_{C}^{\gamma} = p_{D}^{1-\gamma}T_{D}^{\gamma} \Rightarrow \frac{p_{C}}{p_{D}} = \left(\frac{T_{D}}{T_{C}}\right)^{\frac{\gamma}{1-\gamma}} \qquad \dots(i)$$

$$p_{A}^{1-\gamma}T_{A}^{\gamma} = p_{B}^{1-\gamma}T_{B}^{\gamma}$$

$$\Rightarrow \qquad \frac{p_{A}}{p_{B}} = \left(\frac{T_{B}}{T_{A}}\right)^{\frac{\gamma}{1-\gamma}} \qquad \dots(ii)$$

Multiplying Eqs. (i) and (ii), we get

$$\frac{p_C p_A}{p_D p_B} = \left(\frac{T_D T_B}{T_C T_A}\right)^{\frac{\gamma}{1-\gamma}} \dots \dots (\text{iii})$$

Processes $B \rightarrow C$ and $D \rightarrow A$ are isochoric. (V = constant)

Therefore,
$$\frac{p_C}{p_B} = \frac{T_C}{T_B}$$
 and $\frac{p_A}{p_D} = \frac{T_A}{T_D}$

Multiplying these two equations, we get

$$\frac{p_C p_A}{p_D p_B} = \frac{T_C T_A}{T_B T_D} \qquad \dots \text{(iv)}$$

From Eqs. (iii) and (iv), we have

$$\left(\frac{T_D T_B}{T_C T_A}\right)^{\frac{1}{1-\gamma}} = \left(\frac{T_C T_A}{T_B T_D}\right)$$

or
$$\left(\frac{T_C T_A}{T_D T_B}\right)^{\frac{1-\gamma}{\gamma}} = \left(\frac{T_C T_A}{T_B T_D}\right)$$
$$\Rightarrow \qquad \frac{T_C T_A}{T_D T_B} = 1$$

or
$$T_D = \frac{T_C T_A}{T_B} = \frac{(425)(1000)}{850}$$

or
$$T_D = 500 \text{ K}$$

56. Given number of moles n = 2

Process *AB* and *CD* are isobaric. Hence, $Q_{AB} = -Q_{CD}$ [Because $(\Delta T)_{AB} = +100$ K whereas $(\Delta T)_{CD} = -100$ K and $Q_{\text{isobaric}} = nC_p \Delta T$] or $Q_{AB} + Q_{CD} = 0$ Process *BC* is isothermal ($\Delta U = 0$)

 $Q_{BC} = W_{BC} = nRT_B \ln\left(\frac{p_B}{p_C}\right)$ *.*.. $= (2)(8.31)(400)\ln\left(\frac{2}{1}\right)$

$$Q_{BC} = 4608 \, \mathrm{J}$$

Similarly, process DA is also isothermal hence,

$$Q_{DA} = W_{DA} = nRT_D \ln\left(\frac{p_D}{p_A}\right)$$
$$= (2)(8.31)(300) \ln\left(\frac{1}{2}\right)$$

 $Q_{BC} = -3456 \,\mathrm{J}$ (a) Net heat exchange in the process

$$Q = Q_{AB} + Q_{BC} + Q_{CD} + Q_{DA} = (4608 - 3456) \text{ J}$$

 $Q = 1152 \text{ J}$

- (b) From first law of thermodynamics, $\Delta U = 0$ (in complete cycle) $Q_{\text{net}} = W_{\text{net}}$ $W = Q = 1152 \,\mathrm{J}$ Hence,
- (c) Since, $T_i = T_f$, therefore net change in internal energy, dU = 0.

57. (a) The p-V and p-T diagrams are shown below



(b) **Process** A - B T = constant

 $\therefore p \propto \frac{1}{V}, V$ is doubled. Therefore, p will become half.

Further,
$$V_A = \frac{nRT_A}{p_A} = \frac{3RT_A}{p_A}$$

 $\Delta U_{AB} = 0$
 $\therefore Q_{AB} = W_{AB} = nRT_A \ln\left(\frac{2V_A}{V_A}\right)$
 $= 3RT_A \ln(2) = 2.08RT_A$

Process B-C $Q_{BC} = nC_{n} (T_{C})$

$$= (3)\left(\frac{7}{2}R\right)\left(\frac{T_A}{2} - T_A\right) = -\frac{21}{4}RT_A$$

= -5.25*R*T_4

Process C-A V = constant

$$\therefore \qquad W_{CA} = 0$$

or
$$Q_{CA} = \Delta U_{CA} = nC_V (T_A - T_C)$$
$$= (3) \left(\frac{5}{2}R\right) \left(T_A - \frac{T_A}{2}\right) = 3.75 RT_A$$

In a cyclic process,

$$\Delta U = 0$$

$$Q_{\text{net}} = W_{\text{net}} = 0.58 \, RT_A$$

58. (a) In adiabatic process $TV^{\gamma - 1} = \text{constant}$

$$TV^{\gamma-1} = \left(\frac{T}{2}\right) (5.66V)^{\gamma-1}$$

or $(5.66)^{\gamma-1} = 2$
$$(\gamma - 1) \ln (5.66) = \ln (2)$$

$$\gamma - 1 = 0.4$$

or $\gamma = 1.4$
i.e. degree of freedom, $f = 5$ as $\gamma = 1 + \frac{2}{f}$

(b) Using, $pV^{\gamma} = \text{constant}$

...

$$pV^{1.4} = p_f (5.66V)^{1.4}$$

 $p_f = 0.09 \ p$ Now, work done in adiabatic process

$$W = \frac{p_i V_i - p_f V_f}{\gamma - 1}$$

= $\frac{(pV) - (0.09p)(5.66V)}{1.4 - 1}$
= 1.23 pV

59. Vessel is closed. Therefore, $\Delta W = 0$

or

$$\Delta Q = \Delta U = nC_V \ \Delta T$$
$$\Delta T = \frac{\Delta Q}{nC_V} = \frac{\Delta Q}{\left(\frac{pV}{RT}\right)(C_p - R)}$$
$$= \frac{(\Delta Q)(RT)}{pV(C_p - R)}$$

Substituting the values, we have

$$\Delta T = \frac{(2.49 \times 10^4) (300)}{(1.6 \times 10^6) (0.0083) (3/2)} = 375 \,\mathrm{K}$$

$$\therefore$$
 $T_f = \Delta T + T = 675 \text{ K}$

Further at constant volume

$$\frac{p_2}{p_1} = \frac{T_2}{T_1}$$

$$\therefore \qquad p_2 = \frac{T_2}{T_1} p_1$$

$$= \left(\frac{675}{300}\right) (1.6 \times 10^6)$$

$$= 3.6 \times 10^6 \text{ N/m}^2$$

60. In adiabatic process

$$p_{i} V_{i}^{\gamma} = p_{f} V_{f}^{\gamma}$$

$$p_{f} = \left(\frac{V_{i}}{V_{f}}\right)^{\gamma} p_{i} \qquad \dots (i)$$

$$C_{V} = \frac{3R}{2}$$

Further,

or

:.
$$p_f = \left(\frac{6}{2}\right)^{5/3} (10)^5 = 6.24 \times 10^5 \text{ N/m}^2$$

Now, work done in adiabatic process is given by V = V

 $C_p = C_V + R = \frac{5R}{2}$

 $\gamma = \frac{C_p}{C_V} = \frac{5}{3}$

$$W = \frac{p_i v_i - p_f v_f}{\gamma - 1}$$

= $\frac{10^5 \times 6 \times 10^{-3} - 6.24 \times 10^5 \times 2 \times 10^{-3}}{\left(\frac{5}{3}\right) - 1}$
= - 972 J

NOTE Work done is negative because volume of the gas is decreasing.

61.	Name of the process	Information obtained from the graph	Pressure	Volume	Nature of <i>p-V</i> graph
	$A \rightarrow B$	$V \propto T$ $\therefore p = \text{constant}$ V and T both are increasing	Constant	Increasing	Straight line parallel to V-axis as p = constant
	$B \rightarrow C$	V = constant $\therefore p \propto T$ <i>T</i> is decreasing. $\therefore p \text{ will}$ decrease	Decreasing	Constant	Straight line parallel to p-axis, as V = constant
	$C \rightarrow A$	T = constant $\therefore p \propto \frac{1}{V}$ V is decreasing $\therefore p \text{ should}$ increase	Increasing	Decreasing	Rectangular hyperbola as $p \propto \frac{1}{V}$





Topic 6 Miscellaneous Problems

1. Number density of gas molecules, $n = \frac{\text{Number of molecules}}{\text{Number of molecules}}$

$$\Rightarrow$$
 Number of molecules

$$N = n \times \text{volume of gas}$$



Now, consider a shell of radius r and thickness dr with centre at r = 0.

Volume of shell of differentiable thickness (*dr*), dV =surface area × thickness = $4\pi r^2 dr$

Now, number of molecules in this shell is

$$dN = n(r) \cdot dV = n_0 e^{-\alpha r^4} \cdot 4\pi r^2 dr$$

So, total number of molecules present in given volume (extending from r = 0 to $r = \infty$) is

$$N = \int_0^\infty n(r) \cdot dV = \int_0^\infty n_0 \ e^{-\alpha r^4} \cdot 4\pi r^2 \ dr$$

$$= \int_0^\infty 4\pi n_0 \ e^{-\alpha r^4} \cdot r^2 \ dr \qquad \dots (i)$$

Here, we take

$$\alpha r^4 = t \Longrightarrow r = t^{\frac{1}{4}} \cdot \alpha^{\frac{-1}{4}}$$
$$4\alpha r^3 dr = dt$$

 \Rightarrow \Rightarrow

$$r^{2} dr = \frac{dt}{4\alpha r} = \frac{dt}{4\alpha t^{\frac{1}{4}} \cdot \alpha^{-\frac{1}{4}}} = \frac{dt}{4\alpha^{3/4} t^{\frac{1}{4}}}$$

Also, when r = 0, t = 0 and when $r = \infty$, $t = \infty$ substituting in Eq. (i), we get

$$N = \int_0^\infty 4\pi n_0 e^{-t} \cdot \frac{dt}{4\alpha^{\frac{3}{4}} t^{\frac{1}{4}}}$$
$$N = \pi \alpha^{-\frac{3}{4}} \cdot n_0 \cdot \int_0^\infty e^{-t} \cdot t^{-\frac{1}{4}} dt$$

As value of definite integral $\int_0^\infty e^{-t} \cdot t^{-\frac{1}{4}} dt$ is a constant (= k

let), we have

$$N = \pi k n_0 \alpha^{-\frac{3}{4}} \Longrightarrow N \propto n_0 \alpha^{-\frac{3}{4}}$$

2. Momentum imparted to the surface in one collision,

$$\Delta p = (p_i - p_f) = mv - (-mv) = 2mv \qquad \dots (i)$$

Force on the surface due to *n* collision per second,

$$F = \frac{n}{t}(\Delta p) = n\Delta p \qquad (\because t = 1s)$$

[from Eq. (i)]

$$= 2 mnv$$

So, pressure on the surface,

$$p = \frac{F}{A} = \frac{2mnv}{A}$$
Here, $m = 10^{-26}$ kg, $n = 10^{22}$ s⁻¹,
 $v = 10^4$ ms⁻¹, $A = 1$ m²
 \therefore Pressure, $p = \frac{2 \times 10^{-26} \times 10^{22} \times 10^4}{10^{22} \times 10^4} = 2$ N/m²

So, pressure exerted is order of 10^0 .

3. (d) By principle of thermometry for any liner temperature scale,

1

$$\frac{T - T_{\rm LFP}}{T_{\rm UFP} - T_{\rm LFP}} = a \text{ (constant)}$$

where,

T = temperature measured

 $T_{\rm LFP}$ = temperature of melting ice or lower fixed point. $T_{\rm UFP}$ = temperature of boling water or upper fixed point. If, T = temperature of given object.

r

$$\frac{T - 0^{\circ} \text{C}}{100^{\circ} \text{C} - 0^{\circ} \text{C}} = \frac{\frac{x_0}{2} - \frac{x_o}{3}}{x_0 - \frac{x_0}{3}} \text{ or } \frac{T}{100} = \frac{1}{4} \text{ or } T = 25^{\circ} \text{C}$$

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4. Energy flux is the rate of heat flow per unit area through the rod.

Also, rate of flow of heat per unit time through a material of area of cross-section A and thermal conductivity k between the temperatures T_1 and T_2 ($T_1 > T_2$) is given as,

$$\frac{\Delta Q}{\Delta t} = \frac{kA(T_1 - T_2)}{l} \qquad \dots (i)$$

Energy flux using Eq. (i), we get

$$= \frac{1}{A} \cdot \frac{\Delta Q}{\Delta t} = \frac{k(T_1 - T_2)}{l}$$
Here, $k = 0.1 \text{ W K}^{-1}\text{m}^{-1}$, $l = 1 \text{ m}$
 $T_1 = 1000 \text{ K}$ and $T_2 = 100 \text{ K}$
 \therefore Energy flux = $\frac{0.1(1000 - 100)}{1}$
 $= 90 \text{ Wm}^{-2}$.

5. Given,
$$\frac{U}{V} \propto T^4$$

 $\frac{U}{V} = \alpha T^4$...(i)

It is also given that, $P = \frac{1}{3} \left(\frac{U}{V} \right)$

or

...

:..

and

$$\Rightarrow \qquad \frac{nR_0T}{V} = \frac{1}{3}(\alpha T^4) \qquad (R_0 = \text{Gas constant})$$

$$VT^3 = \frac{3nR_0}{\alpha} = \text{constant}$$

$$\left(\frac{4}{3}\pi R^3\right)T^3 = \text{constant or } RT = \text{constant}$$

 $T \propto \frac{1}{R}$

6. Average time between two collisions is given by

$$\tau = \frac{1}{\sqrt{2\pi}nv_{\rm rms}d^2} \qquad \dots(i)$$

Here, n = number of molecules per unit volume = $\frac{N}{V}$

$$v_{\rm rms} = \sqrt{\frac{3RT}{M}}$$

Substituting these values in Eq.(i) we have,

$$\tau \propto \frac{V}{\sqrt{T}}$$
 ...(ii)

For adiabatic process, $TV^{\gamma - 1} = \text{constant}$

substituting in Eq. (ii), we have
$$\tau \propto \frac{V}{\sqrt{\left(\frac{1}{V^{\gamma-1}}\right)}}$$

or $\tau \propto V^{1+\left(\frac{\gamma-1}{2}\right)}$ or $\tau \propto V^{\left(\frac{1+\gamma}{2}\right)}$

7. Entropy is a state functions. Therefore in both cases answer should be same.

8. In equilibrium,

$$p_0 A = Mg$$
 . when slightly displaced downwards,

..(i)



$$dp = -\gamma \left(\frac{p_0}{V_0}\right) dV \left(\text{As in adiabatic process}, \frac{dp}{dV} = -\gamma \frac{p}{V}\right)$$

$$\therefore \text{ Restoring force,}$$

$$F = (dp)A = -\left(\frac{\gamma p_0}{V_0}\right)(A)(Ax)$$

 $F \propto -x$

Therefore, motion is simple harmonic comparing with F = -kx we have

$$k = \frac{\gamma p_0 A^2}{V_0}$$

$$\therefore \qquad f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = \frac{1}{2\pi} \sqrt{\frac{\gamma p_0 A^2}{MV_0}}$$

9. The process may be assumed to be isobaric.

$$\therefore \qquad Q = n C_p \Delta T = (2) \left(\frac{5}{2}R\right) (5)$$
$$= 5 \times 8.31 \times 5 = 207.75 \text{ J} \approx 208 \text{ J}$$

10. $pT^2 = \text{constant}$

$$\therefore \quad \left(\frac{nRT}{V}\right)T^2 = \text{ constant} \quad \text{or} \quad T^3V^{-1} = \text{constant}$$

Differentiating the equation, we get

$$\frac{3T^2}{V}.dT - \frac{T^3}{V^2}dV = 0 \quad \text{or} \quad 3dT = \frac{T}{V}.dV \qquad \dots (i)$$

From the equation

$$\gamma = \text{coefficient of volume expansion of gas} = \frac{dV}{V.dT}$$

 $dV = V\gamma dT$

From Eq. (i) $\gamma = \frac{dV}{V dT} = \frac{3}{T}$

- **11.** This question is incomplete.
- 12. Out of the alternatives provided, none appears completely correct.

AB is an isothermal process.

$$(T = \text{constant}, p \propto \frac{1}{V})$$
. So, *p*-*V* graph should be rectangular

hyperbola with p decreasing and V increasing. BC is an isobaric process.

 $(p = \text{constant}, V \propto T)$. Temperature is increasing. Hence, volume should also increase.

CA is an adiabatic process (pV^{γ} = constant). Pressure is increasing. So, volume should decrease. At point A, an isotherm AB and an adiabatic curve AC are meeting. We know that

(slope of an adiabatic graph in *p*-*V* diagram) = γ (slope of an isothermal graph in the same diagram) with $\gamma > 1$ or $(Slope)_{adiabatic} > (Slope)_{isothermal}.$

None of the given examples fulfill all the above requirements.

13.
$$\beta = -\frac{dV / dp}{V} = \text{compressibility of gas}$$

= $\frac{1}{\text{Bulk modulus of elasticity}}$

and $\beta = \frac{1}{p}$ under isothermal conditions.

Thus, β versus p graph will be a rectangular hyperbola.

14. From first law of thermodynamics,

$$\begin{aligned} dQ &= dU + dW \\ dQ &= dU, \text{ if } dW = 0 \\ \text{Since,} & dQ < 0 \\ \text{Therefore,} & dU < 0 \\ \text{or} & U_{\text{final}} < U_{\text{initial}} \end{aligned}$$

or temperature will decrease.

or

NOTE Internal energy *U* of an ideal gas depends only on the temperature of the gas. Internal energy of *n* moles of an ideal gas is given by

$$U = n(f / 2)RT$$
$$U \propto T$$

Here, f is the degree of freedom of the gas.

15. For an ideal gas, pV = nRT

For
$$p = \text{constant}$$

 $p\Delta V = nR\Delta T$
 $\therefore \qquad \frac{\Delta V}{\Delta T} = \frac{nR}{p} = \frac{nR}{\frac{nRT}{V}} = \frac{V}{T}$
 $\therefore \qquad \frac{\Delta V}{V\Delta T} = \frac{1}{T} \text{ or } \delta = \frac{1}{T}$

Therefore, δ is inversely proportional to temperature *T*. i.e. when T increases, δ decreases and vice-versa.



Hence, δ -*T* graph will be a rectangular hyperbola as shown in the above figure.

16. Let final equilibrium temperature of gases is T Heat rejected by gas by lower compartment

$$= nC_V \Delta T = 2 \times \frac{3}{2} R (700 - T)$$

Heat received by the gas in above compartment

$$= nC_p \Delta T = 2 \times \frac{7}{2} R (T - 400)$$

Equating the two, we get 2100 - 3T = 7T - 2800 $T = 490 \, \text{K}$ \Rightarrow

$$17. \quad \Delta W_1 + \Delta U_1 = \Delta Q_1$$

 $\Delta W_2 + \Delta U_2 = \Delta Q_2$ $\Delta Q_1 + \Delta Q_2 = 0$ $\therefore (nC_p \Delta T)_1 + (nC_p \Delta T)_2 = 0$ $n_1 = n_2 = 2$ But $5_{R(T-700)} + \frac{7}{-R(T-400)} = 0$

$$\frac{-R}{2}(T - 700) + \frac{-R}{2}(T - 400) =$$

Solving, we get $T = 525$ K

Now, from Eqs. (i) and (ii), we get

as

$$\Delta Q_{1} + \Delta Q_{2} = 0$$

$$\therefore \quad \Delta W_{1} + \Delta W_{2} = -[(nC_{V}\Delta T)_{1} + (nC_{V}\Delta T)_{2}]$$

$$= -\left[2 \times \frac{3}{2}R \times (525 - 700) + 2 \times \frac{5}{2}R \times (525 - 400)\right]$$

$$= -100R$$

 $\Delta W_1 + \Delta W_2 = -\Delta U_1 - \Delta U_2$

- **18.** Since it is open from top, pressure will be p_0 .
- **19.** Let p be the pressure in equilibrium.

$$\begin{array}{c} & & & \\ & \downarrow \\ & & \downarrow \\ & & \downarrow \\ & & Mg & p_0 A \end{array}$$

Then,

.

$$pA = p_0 A - Mg$$
$$p = p_0 - \frac{Mg}{A} = p_0 - \frac{Mg}{\pi R^2}$$

Applying,
$$p_1V_1 = p_2V_2$$

 $\therefore \quad p_0(2AL) = (p)(AL')$

$$\therefore L' = \frac{2p_0L}{p} = \left(\frac{p_0}{p_0 - \frac{Mg}{\pi R^2}}\right) (2L)$$
$$= \left(\frac{p_0\pi R^2}{\pi R^2 p_0 - Mg}\right) (2L)$$

 \therefore Correct option is (d).

$$p_1 = p_2$$

 $p_0 + \rho g (L_0 - H) = p$...(i)



Now, applying $p_1V_1 = p_2V_2$ for the air inside the cylinder, we have $p_0(L_0) = p(L_0)$ H)

$$\therefore \qquad p = \frac{p_0 L_0}{L_0 - H}$$

...(i)

...(ii)

Substituting in Eq. (i), we have

$$p_0 + \rho g (L_0 - H) = \frac{p_0 L_0}{L_0 - H}$$
$$\rho g (L_0 - H)^2 + p_0 (L_0 - H) - p_0 L_0 = 0$$

 \therefore Correct option is (c).

or

21. (A) In case of free expansion under adiabatic conditions, change in internal energy $\Delta U = 0$.

: Internal energy and temperature will remain constant.

(B)
$$p \propto \frac{1}{V^2}$$

 $\Rightarrow \therefore pV^2 = \text{constant} \qquad \dots (i)$

or
$$\left(\frac{nRT}{V}\right) V^2 = \text{constant} \implies \therefore T \propto \frac{1}{V} \qquad \dots \text{(ii)}$$

If volume is doubled, temperature will decrease as per Eq. (ii).

Further, molar heat capacity in process pV^x = constant is

$$C = C_V + \frac{R}{1 - x}$$

From Eq. (i), x = 2

:..

$$C = \frac{3}{2}R + \frac{R}{1-2} = +\frac{R}{2}$$

Since, molar heat capacity is positive, according to $Q = nC\Delta T$, Q will be negative if ΔT is negative. Or gas loses heat if temperature is decreasing.

(C)
$$p \propto \frac{1}{V^{4/3}}$$
, $pV^{4/3} = \text{constant}$
 $\therefore \qquad \left(\frac{nRT}{V}\right)V^{4/3} = \text{constant}$
 $\therefore \qquad T \propto \frac{1}{V^{1/3}}$

Further, with increase in volume temperature will decrease.

Here,
$$x = \frac{4}{3} \implies C = \frac{3}{2}R + \frac{R}{1 - 4/3} = -1.5R$$

As molar heat capacity is negative, Q will be positive if ΔT is negative. Or gas gains heat with decrease in temperature.

(D) $T \propto pV$

In expansion from V_1 to $2V_1$, product of pV is increasing. Therefore, temperature will increase. Or $\Delta U = +$ ve. Further, in expansion work done is also positive.

Hence, $Q = W + \Delta U = +$ ve or, gas gains heat.

[black body radiation] **23.** Assumption e = 1

$$P = \sigma A (T^4 - T_0^4)$$
(c) $P_{\text{rad}} = \sigma A T^4 = \sigma \cdot 1 \cdot (T_0 + 10)^4$

$$= \sigma \cdot T_0^4 \left(1 + \frac{10}{T_0} \right)^4 \qquad [T_0 = 300 \text{ K given}]$$

$$= \sigma \cdot (300)^4 \cdot \left(1 + \frac{40}{300} \right) \approx 460 \times \frac{17}{15} \approx 520 \text{ J}$$

$$P_{\text{net}} = 520 - 460 \approx 60 \text{ W}$$

$$\Rightarrow \text{ Energy radiated in 1 s} = 60 \text{ J}$$
(b) $P = \sigma A (T^4 - T_0^4)$
 $dP = \sigma A (0 - 4T_0^3 \cdot dT)$
and $dT = -\Delta T$

and

- $dP = 4\sigma A T_0^3 \Delta T$ \Rightarrow
- (d) If surface area decreases, then energy radiation also decreases.

NOTE

While giving answer (b) and (c) it is assumed that energy radiated refers the net radiation. If energy radiated is taken as only emission, then (b) and (c) will not be included in answer.

24.
$$Q = mCT \implies \frac{dQ}{dt} = mc\frac{dT}{dt}$$

R = rate of absortion of heat = $\frac{dQ}{dt} \propto C$

- (i) in 0 100 K C increase, so R increases but not linearly
- (ii) $\Delta Q = mC\Delta T$ as C is more in (400 K-500 K) then (0 - 100 K) so heat is increasing
- (iii) C remains constant so there no change in R from (400 K - 500 K)
- (iv) C is increases so R increases in range(200 K 300 K)

 $C_V = \frac{3}{2}R$

 $C_V = \frac{5}{2}R$

25. For monoatomic gas,
$$C_p = \frac{5}{2}R$$

and

For diatomic gas,
$$C_p = \frac{7}{2}$$

and

26. (a) Work done = Area under p-V graph

$$A_1 > A_2$$

$$W : \qquad > W : \quad .$$

$$\therefore \qquad W_{\text{given process}} > W_{\text{isothermal process}}$$

(b) In the given process p-V equation will be of a straight line with negative slope and positive intercept i.e., $p = -\alpha V + \beta$ (Here α and β are positive constants)



This is an equation of parabola in T and V.

(d

$$\frac{dT}{dV} = 0 = \beta - 2\alpha V$$

$$\Rightarrow \qquad V = \frac{B}{2\alpha}$$
Now,
$$\frac{d^2T}{dV^2} = -2\alpha = -ve$$
ie, *T* has some maximum value.
Now,
$$T \propto pV$$
and
$$(pA)_A = (pV)_B$$

$$\Rightarrow \qquad T_A = T_B$$
We conclude that temperatures are same at *A* and *B*

B and in between temperature has a maximum value. Therefore, in going from A to B, T will first increase to a maximum value and then decrease.

Heat will flow both sides from point P.

$$L_{1} \frac{dm_{1}}{dt} = \left(\frac{\text{Temperature difference}}{\text{Thermal resistance}}\right)_{1}$$
$$= \frac{400}{(\lambda x)/kA} \qquad \dots (i)$$

$$L_1 \frac{dm_2}{dt} = \frac{400 - 100}{(100 - \lambda)x/kA} \qquad \dots (ii)$$

In about two equations, $\frac{dm_1}{dt} = \frac{dm_2}{dt}$ (given)

$$L_1 = 80 \text{ calg}^{-1} \text{ and } L_2 = 540 \text{ calg}^{-1}$$

Solving these two equations, we get $\lambda = 9$.

28. Pressure on both sides will be equal

i.e.

$$p_1 = p_2$$

$$\frac{n_1 RT}{V_1} = \frac{n_2 RT}{V_2}$$

$$\left(n = \frac{m}{M}\right)$$

or

or

$$M_1V_1$$
 M

$$\frac{V_2}{V_1} = \frac{M_1}{M_2} = \frac{32}{28} = \frac{8}{7}$$

$$\alpha = \left(\frac{360^\circ}{8+7}\right) \times 8 = 192^\circ$$

29. The gas thermometer works at constant volume. Therefore, $T \propto p$

or
$$T_2/T_1 = p_2/p_1$$

or $T_2 = T_1(p_2/p_1)$
or $T_2 = (273.16) \left(\frac{3.5 \times 10^4}{3 \times 10^4}\right) K$
 $= 318.68 K$

or temperature at given pressure will be

or **30.**

...

 $C_p = C_V + R$ $\implies \therefore \quad C_p > C_V$

31. Rate of heat conduction through rod = rate of the heat lost from right end of the rod.

$$\therefore \frac{KA(T_1 - T_2)}{L} = eA\sigma (T_2^4 - T_s^4) \qquad \dots (i)$$

Given that $T_2 = T_s + \Delta T$

$$T_2^4 = (T_s + \Delta T)^4$$

$$=T_s^4 \left(1 + \frac{\Delta I}{T_s}\right)$$

Using binomial expansion, we have

$$T_2^4 = T_s^4 \left(1 + 4\frac{\Delta T}{T_s} \right) \qquad (\text{as } \Delta T \ll T_s)$$
$$T_2^4 - T_s^4 = 4(\Delta T)(T_s^3)$$

Substituting in Eq. (i), we have

$$\frac{K(T_1 - T_s - \Delta T)}{L} = 4e\sigma T_s^3 \cdot \Delta T$$
$$\frac{K(T_1 - T_s)}{L} = \left(4e\sigma T_s^3 + \frac{K}{L}\right)\Delta T$$

or

...

$$\Delta T = \frac{K(T_1 - T_s)}{(4e\sigma LT_s^3 + K)}$$

Comparing with the given relation, proportionality constant

$$=\frac{K}{4e\sigma LT_s^3+K}$$

32. At constant pressure, $V \propto T$

or
$$\frac{V_2}{V_1} = \frac{T_2}{T_1}$$
 or $\frac{Ah_2}{Ah_1} = \frac{T_2}{T_1}$
 $\therefore \qquad h_2 = h_1 \left(\frac{T_2}{T_1}\right) = (1.0) \left(\frac{400}{300}\right) m = \frac{4}{3} m$

As there is no heat loss, process is adiabatic. For adiabatic process,

$$T_{f}V_{f}^{\gamma-1} = T_{i}V_{i}^{\gamma-1}$$

$$\therefore \qquad T_{f} = T_{i}\left(\frac{V_{i}}{V_{f}}\right)^{\gamma-1} = (400)\left(\frac{h_{i}}{h_{f}}\right)^{1.4-1}$$
$$= 400\left(\frac{4}{3}\right)^{0.4} = 448.8 \,\mathrm{K}$$

33. (a) Rate of heat loss per unit area due to radiation $I = e\sigma (T^4 - T_0^4)$

Here,
$$T = 127 + 273 = 400 \text{ K}$$

and $T_0 = 27 + 273 = 300 \text{ K}$
 \therefore $I = 0.6 \times \frac{17}{3} \times 10^{-8} [(400)^4 - (300)^4]$
 $= 595 \text{ W/m}^2$

(b) Let θ be the temperature of the oil.

Then, rate of heat flow through conduction = rate of heat loss due to radiation

$$\frac{\text{Temperature difference}}{\text{Thermal resistance}} = (595)A$$
$$\frac{(\theta - 127)}{\left(\frac{l}{KA}\right)} = (595)A$$

Here, A = area of disc ; K = thermal conductivity and l = thickness (or length) of disc : $(0, -127)^{K} = 505$

$$\therefore \theta = 595 \left(\frac{l}{K}\right) + 127 = \frac{595 \times 10^{-2}}{0.167} + 127 = 162.6^{\circ} \text{C}$$

34. Decrease in kinetic energy = increase in internal energy of the gas

$$\therefore \qquad \frac{1}{2}mv_0^2 = nC_V \Delta T = \left(\frac{m}{M}\right) \left(\frac{3}{2}R\right) \Delta T$$
$$\therefore \qquad \Delta T = \frac{Mv_0^2}{3R}$$

35. Given,

÷.

Length of the wire, l = 5 mRadius of the wire, $r = 2 \times 10^{-3} \text{ m}$ Density of wire, $\rho = 7860 \text{kg/m}^3$

$$M = 100 \text{ kg}$$

Young's modulus,

$$Y = 2.1 \times 10^{11} \,\mathrm{N/m^2}$$

and specific heat, s = 420 J/kg-KMass of wire, m = (density) (volume)

=
$$(\rho) (\pi r^2 l)$$

= $(7860) (\pi) (2 \times 10^{-3})^2 (5) \text{ kg} = 0.494 \text{ kg}$

Elastic potential energy stored in the wire,

$$U = \frac{1}{2} \text{ (stress) (strain) (volume)}$$
$$\left[\because \frac{\text{Energy}}{\text{Volume}} = \frac{1}{2} \times \text{stress} \times \text{strain} \right]$$
$$U = \frac{1}{2} \left(\frac{Mg}{\pi r^2} \right) \left(\frac{\Delta l}{l} \right) (\pi r^2 l)$$
$$= \frac{1}{2} (Mg) \cdot \Delta l \qquad \left(\Delta l = \frac{Fl}{AY} \right)$$
$$= \frac{1}{2} (Mg) \frac{(Mgl)}{(\pi r^2) Y} = \frac{1}{2} \frac{M^2 g^2 l}{\pi r^2 Y}$$

Substituting the values, we have

$$U = \frac{1}{2} \frac{(100)^2 (10)^2 (5)}{(3.14) (2 \times 10^{-3})^2 (2.1 \times 10^{11})} J$$

= 0.9478 J

When the bob gets snapped, this energy is utilised in raising the temperature of the wire.

So,

or

So,

$$U = ms \Delta \theta$$

$$\Delta \theta = \frac{U}{ms} = \frac{0.9478}{0.494 (420)} \circ \text{C or K}$$

$$\Delta \theta = 4.568 \times 10^{-3} \circ \text{C}$$

36. In the first part of the question $(t \le t_1)$



At $t = 0, T_X = T_0 = 400$ K and at $t = t_1, T_X = T_1 = 350$ K Temperature of atmosphere, $T_A = 300$ K (constant) This cools down according to Newton's law of cooling. Therefore, rate of cooling ∞ temperature difference.

 $\left(-\frac{dT}{dt}\right) = k(T - T_A)$ $\frac{dT}{T - T_A} = -k dt$ *:*..

 \Rightarrow

$$\Rightarrow \qquad \int_{T_0}^{T_1} \frac{dT}{T - T_A} = -k \int_0^{t_1} dt$$

$$\Rightarrow \qquad \ln\left(\frac{T_1 - T_A}{T_0 - T_A}\right) = -kt_1$$

$$\Rightarrow kt_1 = -\ln\left(\frac{350 - 300}{400 - 300}\right)$$
$$\Rightarrow kt_1 = \ln(2) \dots (i)$$

In the second part $(t > t_1)$, body X cools by radiation (according to Newton's law) as well as by conduction.



Therefore, rate of cooling

...

= (cooling by radiation) + (cooling by conduction)

$$\left(-\frac{dT}{dt}\right) = k(T - T_A) + \frac{KA}{CL}(T - T_A) \qquad \dots \text{(ii)}$$

In conduction,
$$\frac{dQ}{dt} = \frac{KA(T - T_A)}{L} = C\left(-\frac{dT}{dt}\right)$$

 $\therefore \qquad \left(-\frac{dT}{dt}\right) = \frac{KA}{LC}(T - T_A)$

where, C = heat capacity of body X

$$\left(-\frac{dT}{dt}\right) = \left(k + \frac{KA}{CL}\right)(T - T_A) \qquad \dots \text{(iii)}$$

Let at $t = 3t_1$, temperature of X becomes T_2 Then from Eq. (iii)

$$\int_{T_1}^{T_2} \frac{dT}{T - T_A} = -\left(k + \frac{KA}{LC}\right) \int_{t_1}^{3t_1} dt$$
$$\ln\left(\frac{T_2 - T_A}{T_1 - T_A}\right) = -\left(k + \frac{KA}{LC}\right)(2t_1)$$
$$= -\left(2kt_1 + \frac{2KA}{LC}t_1\right)$$
or
$$\ln\left(\frac{T_2 - 300}{350 - 300}\right) = -2\ln(2) - \frac{2KAt_1}{LC};$$

 $kt_1 = \ln(2)$ from Eq. (i). This equation gives

$$T_2 = \left(300 + 12.5e^{\frac{-2KAt_1}{CL}}\right) \mathrm{K}$$

37. (a) Number of moles of gas A are $n_A = 1$ (given) Let the number of moles of gas *B* be $n_B = n$ For a mixture

$$\frac{n_1 + n_2}{\gamma - 1} = \frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1} \qquad \dots (i)$$

Substituting the values in Eq. (i), we have

$$\frac{1+n}{(19/13)-1} = \frac{1}{(5/3)-1} + \frac{n}{(7/5)-1}$$

Solving this, we get n = 2.

(b) Molecular weight of the mixture will be given by,

$$M = \frac{n_A M_A + n_B M_B}{n_A + n_B} = \frac{(1)(4) + 2(32)}{1+2}$$
$$M = 22.67$$

Speed of sound in a gas is given by $v = \sqrt{\frac{\gamma RT}{r}}$

$$v = \sqrt{\frac{1}{M}}$$

Therefore, in the mixture of the gas

$$v = \sqrt{\frac{(19/13)(8.31)(300)}{22.67 \times 10^{-3}}} \text{ m/s}$$

$$v \approx 401 \text{ m/s}$$

(c)

$$v \propto \sqrt{T}$$

or $v = KT^{1/2}$... (ii)
$$\Rightarrow \frac{dv}{dT} = \frac{1}{2}KT^{-1/2} \Rightarrow dv = K\left(\frac{dT}{2\sqrt{T}}\right)$$
$$\Rightarrow \frac{dv}{v} = \frac{K}{v} \cdot \left(\frac{dT}{2\sqrt{T}}\right) \Rightarrow \frac{dv}{v} = \frac{1}{\sqrt{T}}\left(\frac{dT}{2\sqrt{T}}\right) = \frac{1}{2}\left(\frac{dT}{T}\right)$$
$$\Rightarrow \frac{dv}{v} \times 100 = \frac{1}{2}\left(\frac{dT}{T}\right) \times 100 = \frac{1}{2}\left(\frac{1}{300}\right) \times 100$$
$$= 0.167$$

Therefore, percentage change in speed is 0.167%.

(d) Compressibility =
$$\frac{1}{\text{Bulk modulus}} = \beta$$
 (say)

Adiabatic bulk modulus is given by

$$B_{\rm adi} = \gamma \ p \qquad \left(B = -\frac{dp}{dV/V} \right)$$

/

>

 \therefore Adiabatic compressibility will be given by 1

$$\beta_{adi} = \frac{1}{\gamma \ p} \text{ and } \beta'_{adi} = \frac{1}{\gamma p'} = \frac{1}{\gamma p(5)^{\gamma}} (pV^{\gamma} = \text{constant})$$

$$[pV^{\gamma} = p'(V/5)^{\gamma} \implies p' = p(5)^{\gamma}]$$

$$\therefore \ \Delta\beta = \beta'_{adi} - \beta_{adi} = \frac{-1}{\gamma \ p} \left[1 - \left(\frac{1}{5}\right)^{\gamma} \right]$$

$$= \frac{-V}{\gamma \ (n_A + n_B)RT} \left[1 - \left(\frac{1}{5}\right)^{\gamma} \right] \qquad \left(\because p = \frac{nRT}{V}\right)$$

$$= \frac{-V}{\left(\frac{19}{13}\right)(1+2)(8.31)(300)} \left[1 - \left(\frac{1}{5}\right)^{\frac{19}{13}} \right]$$

$$\left(\gamma = \gamma_{\text{mixture}} = \frac{19}{13}\right)$$

$$\Delta\beta = -8.27 \times 10^{-5} \text{ V}$$

38. Given, temperature of the mixture,
$$T = 27^{\circ}$$
 C = 300 K
Let *m* be the mass of the neon gas in the mixture. Then, mass
of argon would be $(28 - m)$

Number of gram moles of neon, $n_1 = \frac{m}{20}$

Number of gram moles of argon,
$$n_2 = \frac{(28 - m)}{40}$$

From Dalton's law of partial pressures.

Total pressure of the mixture (p) = Pressure due to neon (p_1) + Pressure due to argon (p_2)

or
$$p = p_1 + p_2 = \frac{n_1 RT}{V} + \frac{n_2 RT}{V} = (n_1 + n_2)\frac{RT}{V}$$

Substituting the values

$$1.0 \times 10^5 = \left(\frac{m}{20} + \frac{28 - m}{40}\right) \frac{(8.314)(300)}{0.02}$$

Solving this equation, we get

$$28 - m = 23.926 \text{ g} \Longrightarrow m = 4.074 \text{ g}$$

Therefore, in the mixture, 4.074 g neon is present and the rest i.e. 23.926 g argon is present.

39. Final pressure =
$$p_0 + \frac{kx}{A}$$

= $1.0 \times 10^5 + \frac{(8000)(0.1)}{8 \times 10^{-3}} = 2 \times 10^5 \text{ N/m}^2$

Final volume =
$$2.4 \times 10^{-3} + (0.1) (8 \times 10^{-3})$$

= 3.2×10^{-3} m³

$$= 3.2 \times 10^{-5} \text{ r}$$

Applying,
$$\frac{p_i V_i}{T_i} = \frac{p_f V_f}{T_f}$$

we have,
$$T_f = \left(\frac{p_f V_f}{p_i V_i}\right) T_i$$

= $\frac{(2 \times 10^5) (3.2 \times 10^{-3})}{(1 \times 10^5) (2.4 \times 10^{-3})} \times 300 = 800 \text{ K}$

Heat supplied $Q = W_{gas} + \Delta U$

$$= p_0 (\Delta V) + \frac{1}{2} kx^2 + nC_V \Delta T \qquad \left(\text{as } n = \frac{p_i V_i}{RT_i} \right)$$
$$= (10)^5 (3.2 - 2.4) \times 10^{-3} + \frac{1}{2} \times 8000 \times (0.1)^2$$
$$+ \frac{10^5 \times 2.4 \times 10^{-3}}{8.31 \times 300} \times \frac{3}{2} \times 8.31 \times (800 - 300)$$

= 80 + 40 + 600 = 720 J

40. (a) and (b) Process AB p = constant

:. $V \propto T$ i.e. if V is doubled, T also becomes two times.

$$T_A = 300 \text{ K} \implies \therefore \quad T_B = 600 \text{ K}$$
$$V_A = 20 \text{ L} \implies \therefore \quad V_B = 40 \text{ L}$$
$$p_A = \frac{n R T_A}{V_A} = \frac{(2) (8.31) (300)}{20 \times 10^{-3}}$$

$$= 2.49 \times 10^{5} \text{ N/m}^{2}$$

Process 2 Process is adiabatic. So, applying $T^{\gamma} p^{1-\gamma} = \text{constant}$



Similarly, using pV^{γ} = constant we can find that V_c = 113 L The corresponding p-V graph is shown above

(c)
$$W_{AB} = \text{Area under } p - V \text{ graph}$$

= $(2.49 \times 10^5) (40 - 20) \times 10^{-3} = 4980 \text{ J}$
 $W_{BC} = -\Delta U = nC_V (T_B - T_C)$
= $(2) \left(\frac{3}{2} \times 8.31\right) (600 - 300) = 7479 \text{ J}$
 $\therefore W_{\text{net}} = (4980 + 7479) \text{ J} = 12459 \text{ J}$

41. From the two figures we can see that



 $2x + 5 = 44.5 + 5 + 46 \implies x = 45.25 \text{ cm}$

Let A be the area of cross-section of the tube. Process is given isothermal. Hence, apply pV = constant in two sides of mercury column.

 $pAx = p_2A$ (44.5) or p (45.25) = p_2 (44.5) ...(i)

or
$$p(45.25) = p_1(46)$$
 ...(ii)

 $pAx = p_1 A(46)$

From figure (b),
$$p_2 = p_1 + 5 \sin 30^\circ$$
 ...(iii)

Solving these three equations, we get

p = 75.4 cm of Hg

42. Let x moles transfer from bulb of higher temperature to lower temperature.

Applying pV = nRT for both the bulbs initially and finally

$$76 \times V = nR \times 273$$
 ...(i)
 $p' \times V = (n + x) R \times 273$...(ii)

$$'p_{X} \not \sqsubseteq (-n) x \not \And 335 \dots$$
(iii)

Solving these equations, we get $n = \frac{602}{62} x$

Now, dividing Eq. (ii) with Eq. (i), we get

$$\frac{p'}{76} = \frac{n+x}{n} = 1 + \frac{x}{n} = 1 + \frac{62}{602} = 1.103$$

∴ $p' = 83.83 \text{ cm of Hg}$

.

43. (a) Applying $\frac{\mu\nu}{T}$ = constant for both the chambers.

$$\frac{p_0 V_0}{T_0} = \frac{p V_1}{T_1} = \frac{p V_2}{T_2} \qquad \left(\text{Here, } p = \frac{243 p_0}{32}\right)$$

$$\therefore \quad V_1 = \frac{32}{243} \left(\frac{T_1}{T_0}\right) V_0 \quad \text{and} \quad V_2 = \left(\frac{32}{243}\right) \left(\frac{T_2}{T_0}\right) V_0$$

Further,
$$V_1 + V_2 = 2 V_0$$

or $\left(\frac{16}{243}\right) (T_1 + T_2) = T_0 \text{ or } T_1 + T_2 = \frac{243}{16} T_0 \qquad \dots(i)$

Applying the p-T equation of adiabatic process to the right chamber,

$$\left(\frac{T_0}{T_2}\right)^{5/3} = \left(\frac{243\,p_0}{32\,p_0}\right)^{1-5/3}$$

Solving this equation, we get

$$T_2 = 2.25 T_0$$

From Eq. (i), $T_1 = 12.94 T_0$

(b) Work done by the gas in right chamber $(\Delta Q = 0, \text{ adiabatic process})$

$$\Delta W = -\Delta U = nC_V (T_i - T_f)$$

= (1) $\left(\frac{3}{2}R\right) (T_0 - 2.25T_0) = -1.875 RT_0$

44. (a)
$$v_{\rm rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3 \times 8.31 \times 300}{32 \times 10^{-3}}} = 483.4 \,\mathrm{m/s}$$

Given, $p_0 = 1.01 \times 10^5$ N/m² = Force per unit area. Let *n* molecules of oxygen strike the wall per second per m² and recoil with same speed. Change in momentum is $(2nmv_{rms})$. The change in momentum per unit time is the force.

Hence,
$$p_0 = 2nmv_{\text{rms}}$$

 \therefore $n = \frac{p_0}{2mv_{\text{rms}}} = \frac{1.01 \times 10^5}{2\left[\frac{32}{6.02 \times 10^{26}}\right](483.4)}$
 $= 1.96 \times 10^{27}/\text{s}$

(b)
$$\frac{1}{2}(m_{\text{gas}})v_0^2 = nC_V \Delta T$$

 $\therefore v_0 = \sqrt{\frac{2nC_V \Delta t}{m_{\text{gas}}}} = \sqrt{\frac{(2)(n)(\frac{5}{2} \times 8.31)(1)}{(n)(32 \times 10^{-3})}} = 36 \text{ m/s}$