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Electrostatics

Topic 1 Electrostatic Force and Field Strength

Objective Questions I (Only one correct option)

1. Let a total charge $2Q$ be distributed in a sphere of radius R, with the charge density given by $\rho(r) = kr$, where r is the distance from the centre. Two charges A and B, of $-Q$ each, are placed on diametrically opposite points, at equal distance a, from the centre. If A and B do not experience any force, then **(Main 2019, 12 April II)**

(a) $a = 8^{-1/4}R$ (b) $a = \frac{3R}{2^{1/4}}$
(c) $a = 2^{-1/4}R$ (d) $a = R/\sqrt{3}$

2. Four point charges $-q, +q, +q$ and $-q$ are placed on Y-axis at $y = -2d$, $y = -d$, $y = +d$ and $y = +2d$, respectively. The magnitude of the electric field E at a point on the X-axis at $x = D$, with $D >> d$, will behave as **(Main 2019, 9 April II)**

(a) $E \propto \frac{1}{D}$ (b) $E \propto \frac{1}{D^3}$ (c) $E \propto \frac{1}{D^2}$ (d) $E \propto \frac{1}{D^4}$

3. Two point charges q_1 $(\sqrt{10}\,\mu C)$ and q_2 $(-25\,\mu C)$ are placed on the x-axis at $x = 1$ m and $x = 4$ m, respectively. The electric field (in V/m) at a point $y = 3$ m on Y-axis is **(Main 2019, 9 Jan II)**

$\left(\text{Take, } \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ N-m}^2\text{C}^{-2}\right)$

(a) $(63\,\hat{\mathbf{i}} - 27\hat{\mathbf{j}}) \times 10^2$ (b) $(81\,\hat{\mathbf{i}} - 81\hat{\mathbf{j}}) \times 10^2$
(c) $(-81\,\hat{\mathbf{i}} + 81\hat{\mathbf{j}}) \times 10^2$ (d) $(-63\,\hat{\mathbf{i}} + 27\hat{\mathbf{j}}) \times 10^2$

4. Charge is distributed within a sphere of radius R with a volume charge density $\rho(r) = \frac{A}{r^2}e^{\frac{-2r}{a}}$, where A and a are constants. If Q is the total charge of this charge distribution, the radius R is **(Main 2019, 9 Jan Shift II)**

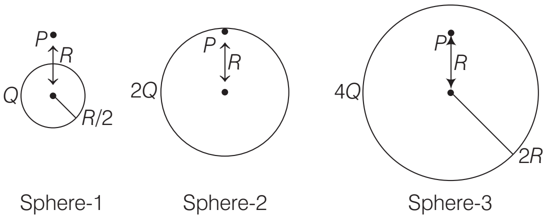
(a) $a \log\left(\frac{1}{1 - \frac{Q}{2\pi aA}}\right)$ (b) $a \log\left(1 - \frac{Q}{2\pi aA}\right)$

(c) $\frac{a}{2}\log\left(1 - \frac{Q}{2\pi aA}\right)$ (d) $\frac{a}{2}\log\left(\frac{1}{1 - \frac{Q}{2\pi aA}}\right)$

5. Three charges $+Q, q, +Q$ are placed respectively at distance $0, \frac{d}{2}$ and d from the origin on the X-axis. If the net force experienced by $+Q$ placed at $x = 0$ is zero, then value of q is **(Main 2019, 9 Jan Shift I)**

(a) $\frac{+Q}{2}$ (b) $\frac{+Q}{4}$ (c) $\frac{-Q}{2}$ (d) $\frac{-Q}{4}$

6. Charges $Q, 2Q$ and $4Q$ are uniformly distributed in three dielectric solid spheres 1, 2 and 3 of radii $R/2, R$ and $2R$ respectively, as shown in figure. If magnitudes of the electric fields at point P at a distance R from the centre of spheres 1, 2 and 3 are E_1, E_2 and E_3 respectively, then **(2014 Adv.)**

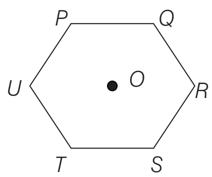


(a) $E_1 > E_2 > E_3$ (b) $E_3 > E_1 > E_2$
(c) $E_2 > E_1 > E_3$ (d) $E_3 > E_2 > E_1$

7. Two charges, each equal to q, are kept at $x = -a$ and $x = a$ on the x-axis. A particle of mass m and charge $q_0 = \frac{q}{2}$ is placed at the origin. If charge q_0 is given a small displacement $y(y << a)$ along the y-axis, the net force acting on the particle is proportional to **(2013 Main)**

(a) y (b) $-y$ (c) $\frac{1}{y}$ (d) $-\frac{1}{y}$

8. Six charges, three positive and three negative of equal magnitude are to be placed at the vertices of a regular hexagon such that the electric field at O is double the electric field when only one positive charge of same magnitude is placed at R. Which of the following arrangements of charge is possible for, P, Q, R, S, T and U respectively? **(2004, 1M)**



(a) +, −, +, −, −, + (b) +, −, +, −, +, −
(c) +, +, −, +, −, − (d) −, +, +, −, +, −

9. An electron of mass m_e, initially at rest, moves through a certain distance in a uniform electric field in time t_1. A proton of mass m_p, also, initially at rest, takes time t_2 to move through an equal distance in this uniform electric field. Neglecting the effect of gravity, the ratio t_2/t_1 is nearly equal to **(1997, 1M)**

(a) 1 (b) $(m_p/m_e)^{1/2}$ (c) $(m_e/m_p)^{1/2}$ (d) 1836

10. A charge q is placed at the centre of the line joining two equal charges Q. The system of the three charges will be in equilibrium if q is equal to **(1987, 2M)**

(a) $-\frac{Q}{2}$ (b) $-\frac{Q}{4}$ (c) $+\frac{Q}{4}$ (d) $+\frac{Q}{2}$

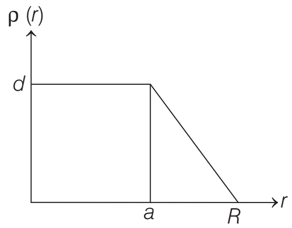
11. Two equal negative charges $-q$ are fixed at points $(0, -a)$ and $(0, a)$ on y-axis. A positive charge Q is released from rest at the point $(2a, 0)$ on the x-axis. The charge Q will **(1984, 2M)**

(a) execute simple harmonic motion about the origin
(b) move to the origin and remain at rest
(c) move to infinity
(d) execute oscillatory but not simple harmonic motion

Passage Based Questions

Passage

The nuclear charge (Ze) is non-uniformly distributed within a nucleus of radius R. The charge density $\rho(r)$ (charge per unit volume) is dependent only on the radial distance r from the centre of the nucleus as shown in figure. The electric field is only along the radial direction.



12. The electric field at $r = R$ is **(2008, 4M)**

(a) independent of a (b) directly proportional to a
(c) directly proportional to a^2 (d) inversely proportional to a

13. For $a = 0$, the value of d (maximum value of ρ as shown in the figure) is **(2008, 4M)**

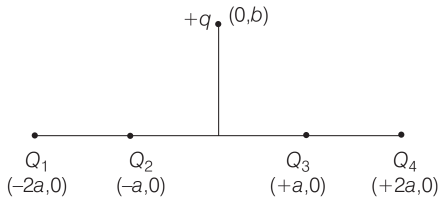
(a) $\frac{3Ze}{4\pi R^3}$ (b) $\frac{3Ze}{\pi R^3}$ (c) $\frac{4Ze}{3\pi R^3}$ (d) $\frac{Ze}{3\pi R^3}$

14. The electric field within the nucleus is generally observed to be linearly dependent on r. This implies **(2008, 4M)**

(a) $a = 0$ (b) $a = \frac{R}{2}$ (c) $a = R$ (d) $a = \frac{2R}{3}$

Match the Column

15. Four charges Q_1, Q_2, Q_3 and Q_4 of same magnitude are fixed along the x-axis at $x = -2a, -a, +a$ and $+2a$ respectively. A positive charge q is placed on the positive y-axis at a distance $b > 0$. Four options of the signs of these charges are given in List I. The direction of the forces on the charge q is given in List II. Match List I with List II and select the correct answer using the code given below the lists. **(2014 Adv.)**



	List I		List II
P.	Q_1, Q_2, Q_3, Q_4 all positive	1.	$+x$
Q.	Q_1, Q_2 positive; Q_3, Q_4 negative	2.	$-x$
R.	Q_1, Q_4 positive; Q_2, Q_3 negative	3.	$+y$
S.	Q_1, Q_3 positive; Q_2, Q_4 negative	4.	$-y$

Codes

(a) P-3, Q-1, R-4, S-2 (b) P-4, Q-2, R-3, S-1
(c) P-3, Q-1, R-2, S-4 (d) P-4, Q-2, R-1, S-3

Objective Questions II (One or more correct option)

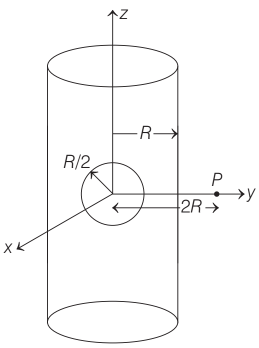
16. Let $E_1(r), E_2(r)$ and $E_3(r)$ be the respective electric fields at a distance r from a point charge Q, an infinitely long wire with constant linear charge density λ, and an infinite plane with uniform surface charge density σ. If $E_1(r_0) = E_2(r_0) = E_3(r_0)$ at a given distance r_0, then **(2014 Adv.)**

(a) $Q = 4\sigma\pi r_0^2$
(b) $r_0 = \frac{\lambda}{2\pi\sigma}$
(c) $E_1(r_0/2) = 2E_2(r_0/2)$
(d) $E_2(r_0/2) = 4E_3(r_0/2)$

17. A non-conducting solid sphere of radius R is uniformly charged. The magnitude of the electric field due to the sphere at a distance r from its centre **(1998, 2M)**

(a) increases as r increases for $r < R$
(b) decreases as r increases for $0 < r < \infty$
(c) decreases as r increases for $R < r < \infty$
(d) is discontinuous at $r = R$

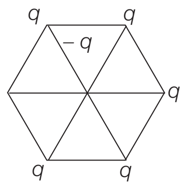
18. An infinitely long solid cylinder of radius R has a uniform volume charge density ρ. It has a spherical cavity of radius $R/2$ with its centre on the axis of the cylinder, as shown in the figure. The magnitude of the electric field at the point P, which is at a distance $2R$ from the axis of the cylinder, is given by the expression $\frac{23\rho R}{16k\varepsilon_0}$. The value of k is **(2012)**



19. A solid sphere of radius R has a charge Q distributed in its volume with a charge density $\rho = kr^a$, where k and a are constants and r is the distance from its centre. If the electric field at $r = \frac{R}{2}$ is $\frac{1}{8}$ times that at $r = R$, find the value of a **(2009)**

Fill in the Blanks

20. Five point charges, each of value $+q$ coulomb, are placed on five vertices of a regular hexagon of side L metre. The magnitude of the force on the point charge of value $-q$ coulomb placed at the centre of the hexagon is ....newton. **(1992, 1 M)**



21. Two small balls having equal positive charges Q (coulomb) on each are suspended by two insulating strings of equal length L (metre) from a hook fixed to a stand. The whole setup is taken in a satellite into space where there is no gravity (state of weightlessness). The angle between the strings is ...... and the tension in each string is ......newton. **(1986, 2M)**

True/False

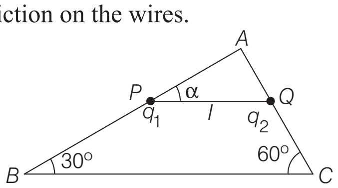
22. A ring of radius R carries a uniformly distributed charge $+Q$. A point charge $-q$ is placed on the axis of the ring at a distance $2R$ from the centre of the ring and released from rest. The particle executes a simple harmonic motion along the axis of the ring. **(1988, 2M)**

Analytical & Descriptive Questions

23. Three particles, each of mass 1 g and carrying a charge q, are suspended from a common point by insulated massless strings, each 100 cm long. If the particles are in equilibrium and are located at the corners of an equilateral triangle of side length 3 cm, calculate the charge q on each particle. (Take $g = 10\ \text{m/s}^2$). **(1988, 5M)**

24. A pendulum bob of mass 80 mg and carrying a charge of 2×10^{-8} C is at rest in a horizontal uniform electric field of 20,000 V/m. Find the tension in the thread of the pendulum and the angle it makes with the vertical. (Take $g = 9.8\ \text{ms}^{-2}$) **(1979)**

25. A rigid insulated wire frame in the form of a right angled triangle ABC, is set in a vertical plane as shown in figure. Two beads of equal masses m each and carrying charges q_1 and q_2 are connected by a cord of length l and can slide without friction on the wires.



Considering the case when the beads are stationary determine

(a) (i) The angle α **(1978)**

(ii) The tension in the cord

(iii) The normal reaction on the beads

(b) If the cord is now cut what are the value of the charges for which the beads continue to remain stationary?

Topic 2 Electrostatics Potential, Potential Energy, Work Done and Energy Conservation

Objective Questions I (Only one correct option)

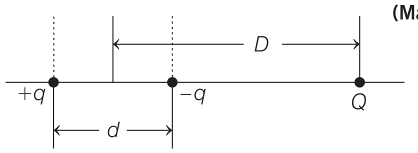
1. In free space, a particle A of charge $1\,\mu$C is held fixed at a point P. Another particle B of the same charge and mass $4\,\mu$g is kept at a distance of 1 mm from P. If B is released, then its velocity at a distance of 9 mm from P is

$$\left[\text{Take, } \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ \text{N-m}^2\text{C}^{-2}\right]$$ **(Main 2019, 10 April II)**

(a) 1.5×10^2 m/s (b) 3.0×10^4 m/s

(c) 1.0 m/s (d) 2.0×10^3 m/s

2. A system of three charges are placed as shown in the figure **(Main 2019, 9 April I)**

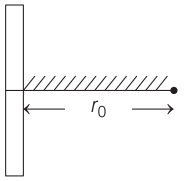


If $D >> d$, the potential energy of the system is best given by

(a) $\frac{1}{4\pi\varepsilon_0}\left[-\frac{q^2}{d} + \frac{2qQd}{D^2}\right]$ (b) $\frac{1}{4\pi\varepsilon_0}\left[+\frac{q^2}{d} + \frac{qQd}{D^2}\right]$

(c) $\frac{1}{4\pi\varepsilon_0}\left[-\frac{q^2}{d} - \frac{qQd}{2D^2}\right]$ (d) $\frac{1}{4\pi\varepsilon_0}\left[-\frac{q^2}{d} - \frac{qQd}{D^2}\right]$

3. A positive point charge is released from rest at a distance r_0 from a positive line charge with uniform density. The speed (v) of the point charge, as a function of instantaneous distance r from line charge, is proportional to **(Main 2019, 8 April II)**



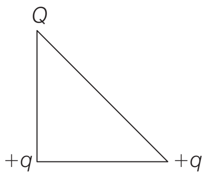
(a) $v \propto \left(\frac{r}{r_0}\right)$ (b) $v \propto e^{+\frac{r}{r_0}}$

(c) $v \propto \ln\left(\frac{r}{r_0}\right)$ (d) $v \propto \sqrt{\ln\left(\frac{r}{r_0}\right)}$

4. The electric field in a region is given by $\mathbf{E} = (Ax + B)\hat{\mathbf{i}}$, where E is in NC^{-1} and x is in metres. The values of constants are $A = 20$ SI unit and $B = 10$ SI unit. If the potential at $x = 1$ is V_1 and that at $x = -5$ is V_2, then $V_1 - V_2$ is **(Main 2019, 8 April II)**

(a) -48 V (b) -520 V
(c) 180 V (d) 320 V

5. Three charges Q, $+q$ and $+q$ are placed at the vertices of a right angle isosceles triangle as shown below. The net electrostatic energy of the configuration is zero, if the value of Q is **(Main 2019, 11 Jan I)**



(a) $-2q$ (b) $\frac{-q}{1+\sqrt{2}}$

(c) $+q$ (d) $\frac{-\sqrt{2}q}{\sqrt{2}+1}$

6. Four equal point charges Q each are placed in the xy-plane at (0, 2), (4, 2), (4, –2) and (0, – 2). The work required to put a fifth charge Q at the origin of the coordinate system will be **(Main 2019, 10 Jan II)**

(a) $\frac{Q^2}{4\pi\varepsilon_0}$ (b) $\frac{Q^2}{4\pi\varepsilon_0}\left(1+\frac{1}{\sqrt{3}}\right)$

(c) $\frac{Q^2}{2\sqrt{2}\pi\varepsilon_0}$ (d) $\frac{Q^2}{4\pi\varepsilon_0}\left(1+\frac{1}{\sqrt{5}}\right)$

7. A charge Q is distributed over three concentric spherical shells of radii $a, b, c\ (a < b < c)$ such that their surface charge densities are equal to one another.

The total potential at a point at distance r from their common centre, where $r < a$ would be **(Main 2019, 10 Jan I)**

(a) $\frac{Q(a^2+b^2+c^2)}{4\pi\varepsilon_0(a^3+b^3+c^3)}$ (b) $\frac{Q(a+b+c)}{4\pi\varepsilon_0(a^2+b^2+c^2)}$

(c) $\frac{Q}{4\pi\varepsilon_0(a+b+c)}$ (d) $\frac{Q}{12\pi\varepsilon_0}\cdot\frac{ab+bc+ca}{abc}$

8. A uniformly charged ring of radius $3a$ and total charge q is placed in xy-plane centred at origin. A point charge q is moving towards the ring along the Z-axis and has speed v at $z = 4a$. The minimum value of v such that it crosses the origin is **(Main 2019, 10 April I)**

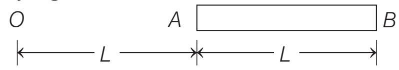
(a) $\sqrt{\frac{2}{m}}\left(\frac{1}{5}\frac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$ (b) $\sqrt{\frac{2}{m}}\left(\frac{4}{15}\frac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$

(c) $\sqrt{\frac{2}{m}}\left(\frac{1}{15}\frac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$ (d) $\sqrt{\frac{2}{m}}\left(\frac{2}{15}\frac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$

9. Assume that an electric field $\mathbf{E} = 30x^2\ \hat{\mathbf{i}}$ exists in space. Then, the potential difference $V_A - V_O$, where V_O is the potential at the origin and V_A the potential at $x = 2$ m is **(2014 Main)**

(a) 120 J (b) -120 J (c) -80 J (d) 80 J

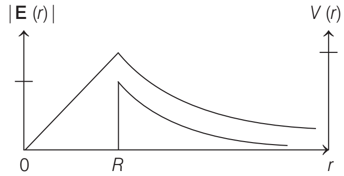
10. A charge Q is uniformly distributed over a long rod AB of length L as shown in the figure. The electric potential at the point O lying at distance L from the end A is **(2013 Main)**

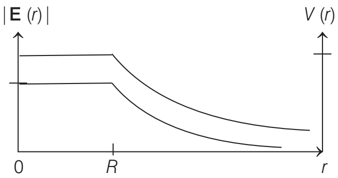


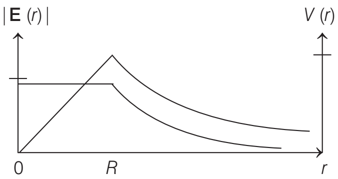
(a) $\frac{Q}{8\pi\varepsilon_0 L}$ (b) $\frac{3Q}{4\pi\varepsilon_0 L}$

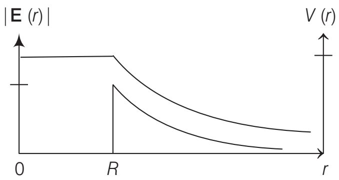
(c) $\frac{Q}{4\pi\varepsilon_0 L\ln 2}$ (d) $\frac{Q\ln 2}{4\pi\varepsilon_0 L}$

11. Consider a thin spherical shell of radius R with its centre at the origin, carrying uniform positive surface charge density. The variation of the magnitude of the electric field $|\mathbf{E}(r)|$ and the electric potential $V(r)$ with the distance r from the centre, is best represented by which graph? **(2012)**

(a)


(b)


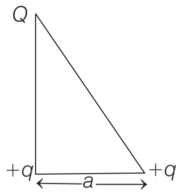
(c)


(d)


12. Positive and negative point charges of equal magnitude are kept at $(0, 0, a/2)$ and $(0, 0, -a/2)$, respectively. The work done by the electric field when another positive point charge in moved from $(-a, 0, 0)$ to $(0, a, 0)$ is **(2007, 3M)**
(a) positive
(b) negative
(c) zero
(d) depends on the path connecting the initial and final positions

13. A uniform electric field pointing in positive x-direction exists in a region. Let A be the origin, B be the point on the x-axis at $x = +1$cm and C be the point on the y-axis at $y = +1$cm. Then the potentials at the points A, B and C satisfy **(2001, 1M)**
(a) $V_A < V_B$ (b) $V_A > V_B$ (c) $V_A < V_C$ (d) $V_A > V_C$

14. Three charges Q, $+q$ and $+q$ are placed at the vertices of a right angle triangle (isosceles triangle) as shown. The net electrostatic energy of the configuration is zero, if Q is equal to **(2000, 2M)**



(a) $\frac{-q}{1+\sqrt{2}}$ (b) $\frac{-2q}{2+\sqrt{2}}$
(c) $-2q$ (d) $+q$

15. A charge $+q$ is fixed at each of the points $x = x_0, x = 3x_0, x = 5x_0 \ldots \infty$ on the x-axis and a charge $-q$ is fixed at each of the points $x = 2x_0, x = 4x_0, x = 6x_0 \ldots, \infty$. Here, x_0 is a positive constant. Take the electric potential at a point due to a charge Q at a distance r from it to be $Q/4\pi\varepsilon_0 r$. Then the potential at the origin due to the above system of charges is **(1998, 2M)**
(a) zero (b) $\frac{q}{8\pi\varepsilon_0 x_0 \ln 2}$
(c) infinite (d) $\frac{q \ln(2)}{4\pi\varepsilon_0 x_0}$

16. A non-conducting ring of radius 0.5 m carries a total charge of 1.11×10^{-10} C distributed non-uniformly on its circumference producing an electric field E everywhere in space. The value of the integral $\int_{l=\infty}^{l=0} -\mathbf{E}\cdot d\mathbf{l}$ ($l = 0$ being centre of the ring) in volt is **(1997, 2M)**
(a) +2 (b) − 1
(c) − 2 (d) zero

17. An alpha particle of energy 5 MeV is scattered through 180° by a fixed uranium nucleus. The distance of closest approach is of the order of **(1981, 2M)**
(a) 1 Å (b) 10^{-10}cm
(c) 10^{-12}cm (d) 10^{-15}cm

Assertion and Reason

Mark your answer as
(a) If Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I.
(b) If Statement I is true, Statement II is true; Statement II is not a correct explanation for Statement I.
(c) If Statement I is true; Statement II is false.
(d) If Statement I is false; Statement II is true.

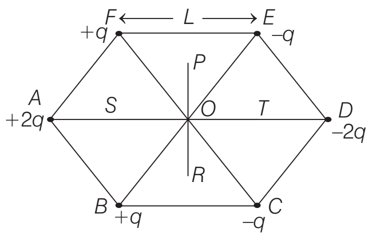
18. **Statement I** For practical purposes, the earth is used as a reference at zero potential in electrical circuits.

Statement II The electrical potential of a sphere of radius R with charge Q uniformly distributed on the surface is given by $\frac{Q}{4\pi\varepsilon_0 R}$. **(2008, 3M)**

Objective Questions II (One or more correct option)

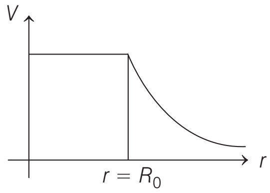
19. A uniformly charged solid sphere of radius R has potential V_0 (measured with respect to ∞) on its surface. For this sphere, the equipotential surfaces with potentials $\frac{3V_0}{2}, \frac{5V_0}{4}, \frac{3V_0}{4}$ and $\frac{V_0}{4}$ have radius R_1, R_2, R_3, and R_4 respectively. Then,
(a) $R_1 \neq 0$ and $(R_2 - R_1) > (R_4 - R_3)$ **(2015 Main)**
(b) $R_1 = 0$ and $R_2 > (R_4 - R_3)$
(c) $2R < R_4$
(d) $R_1 = 0$ and $R_2 < (R_4 - R_3)$

20. Six point charges are kept at the vertices of a regular hexagon of side L and centre O as shown in the figure. Given that $K = \frac{1}{4\pi\varepsilon_0}\frac{q}{L^2}$, which of the following statements(s) is(are) correct. **(2012)**



(a) The electric field at O is $6K$ along OD
(b) The potential at O is zero
(c) The potential at all points on the line PR is same
(d) The potential at all points on the line ST is same

21. For spherical symmetrical charge distribution, variation of electric potential with distance from centre is given in diagram. Given that **(2006, 5M)**



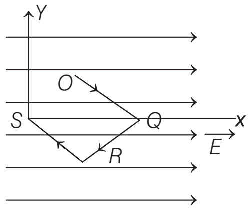
$$V = \frac{q}{4\pi\varepsilon_0 R_0} \text{ for } r \leq R_0 \quad \text{and} \quad V = \frac{q}{4\pi\varepsilon_0 r} \text{ for } r \geq R_0$$

Then, which option(s) are correct
(a) Total charge within $2R_0$ is q
(b) Total electrostatic energy for $r \le R_0$ is zero
(c) At $r = R_0$ electric field is discontinuous
(d) There will be no charge anywhere except at $r = R_0$

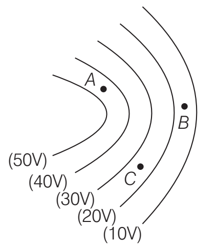
Fill in the Blanks

22. The electric potential V at any point x, y, z (all in metre) in space is given by $V = 4x^2$ volt. The electric field at the point (1m, 0, 2 m) is ……… V/m. **(1992, 1M)**

23. A point charge q moves from point P to point S along the path $PQRS$ (Fig.) in a uniform electric field E pointing parallel to the positive direction of the X-axis. The coordinates of points P, Q, R and S are $(a, b, 0)$, $(2a, 0, 0)$ $(a, -b, 0)$ $(0,0,0)$ respectively. The work done by the field in the above process is given by the expression …… **(1989, 2M)**



24. Figure shows lines of constant potential in a region in which an electric field is present. The values of the potential are written in brackets. Of the points A, B and C, the magnitude of the electric field is greatest at the point…… . **(1984, 2M)**

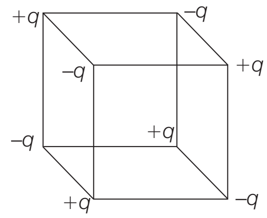


True/False

25. The work done in carrying a point charge from one point to another in an electrostatic field depends on the path along which the point charge is carried. **(1981, 2M)**

Analytical & Descriptive Questions

26. Eight point charges are placed at the corners of a cube of edge a as shown in figure. Find the work done in disassembling this system of charges. **(2003, 2M)**



27. Four point charges $+8\,\mu C, -1\,\mu C, -1\,\mu C$ and $+8\,\mu C$ are fixed at the points $-\sqrt{27/2}$ m, $-\sqrt{3/2}$ m, $+\sqrt{3/2}$ m and $+\sqrt{27/2}$ m respectively on the y-axis. A particle of mass 6×10^{-4} kg and charge $+0.1\,\mu C$ moves along the x-direction. Its speed at $x = +\infty$ is v_0. Find the least value of v_0 for which the particle will cross the origin. Find also the kinetic energy of the particle at the origin. Assume that space is gravity free. $(1/4\pi\varepsilon_0 = 9 \times 10^9\ \text{Nm}^2/\text{C}^2)$. **(2000, 10 M)**

28. Two fixed charges $-2Q$ and Q are located at the points with coordinates $(-3a, 0)$ and $(+3a, 0)$ respectively in the x-y plane. **(1991, 8M)**

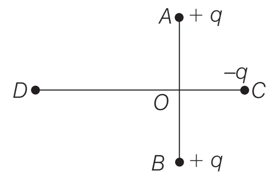
(a) Show that all points in the x-y plane where the electric potential due to the two charges is zero, lie on a circle. Find its radius and the location of its centre.

(b) Give the expression $V(x)$ at a general point on the x-axis and sketch the function $V(x)$ on the whole x-axis.

(c) If a particle of charge $+q$ starts form rest at the centre of the circle, show by a short quantitative argument that the particle eventually crosses the circle. Find its speed when it does so.

29. Three point charges q, $2q$ and $8q$ are to be placed on a 9 cm long straight line. Find the positions where the charges should be placed such that the potential energy of this system is minimum. In this situation, what is the electric field at the position of the charge q due to the other two charges ? **(1987, 7M)**

30. Two fixed, equal, positive charges, each of magnitude $q = 5 \times 10^{-5}$ C are located at points A and B separated by a distance of 6 m. An equal and opposite charge moves towards them along the line COD, the perpendicular bisector of the line AB. The moving charge, when reaches the point C at a distance of 4 m from O, has a kinetic energy of 4 J. Calculate the distance of the farthest point D which the negative charge will reach before returning towards C. **(1985, 6M)**

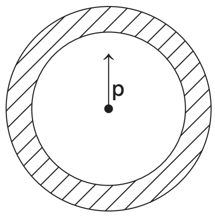


Topic 3 Gauss Theorem and Spherical Shells

Objective Questions I (Only one correct option)

1. Shown in the figure is a shell made of a conductor. It has inner radius a and outer radius b and carries charge Q. At its centre is a dipole $\mathbf{p}$ as shown.

In this case, **(Main 2019, 12 April I)**



(a) surface charge density on the inner surface is uniform and equal to $\dfrac{\left(\dfrac{Q}{2}\right)}{4\pi a^2}$

(b) electric field outside the shell is the same as that of a point charge at the centre of the shell

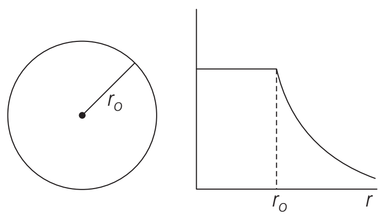
(c) surface charge density on the outer surface depends on $|\mathbf{p}|$

(d) surface charge density on the inner surface of the shell is zero everywhere

2. A solid conducting sphere, having a charge Q, is surrounded by an uncharged conducting hollow spherical shell. Let the potential difference between the surface of the solid sphere and that of the outer surface of the hollow shell be V. If the shell is now given a charge of $-4\,Q$, the new potential difference between the same two surfaces is **(Main 2019, 8 April I)**

(a) $-2V$ (b) $2V$ (c) $4V$ (d) V

3. The given graph shows variation (with distance r from centre) of **(Main 2019, 11 Jan I)**



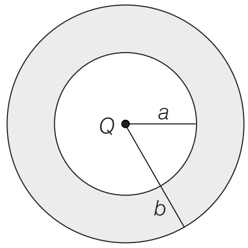
(a) electric field of a uniformly charged spherical shell
(b) potential of a uniformly charged spherical shell
(c) electric field of a uniformly charged sphere
(d) potential of a uniformly charged sphere

4. Three concentric metal shells A, B and C of respective radii a, b and c $(a < b < c)$ have surface charge densities $+\sigma$, $-\sigma$ and $+\sigma$, respectively. The potential of shell B is **(2018 Main)**

(a) $\dfrac{\sigma}{\varepsilon_0}\left(\dfrac{b^2-c^2}{c}+a\right)$ (b) $\dfrac{\sigma}{\varepsilon_0}\left(\dfrac{a^2-b^2}{a}+c\right)$

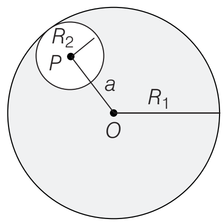
(c) $\dfrac{\sigma}{\varepsilon_0}\left(\dfrac{a^2-b^2}{b}+c\right)$ (d) $\dfrac{\sigma}{\varepsilon_0}\left(\dfrac{b^2-c^2}{b}+a\right)$

5. The region between two concentric spheres of radii a and b, respectively (see the figure), has volume charge density $\rho = \dfrac{A}{r}$, where, A is a constant and r is the distance from the centre. At the centre of the spheres is a point charge Q. The value of A, such that the electric field in the region between the spheres will be constant, is **(2016 Main)**



(a) $\dfrac{Q}{2\pi a^2}$ (b) $\dfrac{Q}{2\pi(b^2-a^2)}$ (c) $\dfrac{2Q}{\pi(a^2-b^2)}$ (d) $\dfrac{2Q}{\pi a^2}$

6. Consider a uniform spherical charge distribution of radius R_1 centred at the origin O. In this distribution, a spherical cavity of radius R_2, centred at P with distance $OP = a = R_1 - R_2$ (see figure) is made. If the electric field inside the cavity at position $\mathbf{r}$ is $\mathbf{E}(\mathbf{r})$, then the correct statements is/are



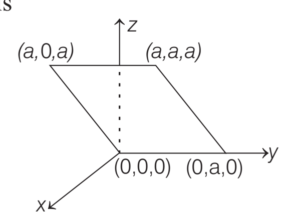
(a) $\mathbf{E}$ is uniform, its magnitude is independent of R_2 but its direction depends on $\mathbf{r}$

(b) $\mathbf{E}$ is uniform, its magnitude depends on R_2 and its direction depends on $\mathbf{r}$

(c) $\mathbf{E}$ is uniform, its magnitude is independent of 'a' but its direction depends on $\mathbf{a}$

(d) $\mathbf{E}$ is uniform and both its magnitude and direction depend on $\mathbf{a}$

7. Consider an electric field $\mathbf{E} = E_0\,\hat{\mathbf{x}}$, where E_0 is a constant. The flux through the shaded area (as shown in the figure) due to this field is **(2011)**



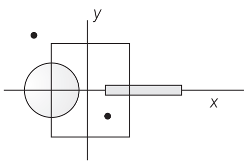
(a) $2E_0a^2$ (b) $\sqrt{2}\,E_0a^2$

(c) E_0a^2 (d) $\dfrac{E_0a^2}{\sqrt{2}}$

8. Three concentric metallic spherical shells of radii $R, 2R$ and $3R$ are given charges Q_1, Q_2 and Q_3, respectively. It is found that the surface charge densities on the outer surfaces of the shells are equal. Then, the ratio of the charges given to the shells, $Q_1 : Q_2 : Q_3$, is **(2009)**

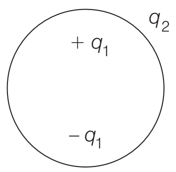
(a) 1 : 2 : 3 (b) 1 : 3 : 5 (c) 1 : 4 : 9 (d) 1 : 8 : 18

9. A disc of radius $\frac{a}{4}$ having a uniformly distributed charge 6C is placed in the x-y plane with its centre at $\left(\frac{-a}{2}, 0, 0\right)$. A rod of length a carrying a uniformly distributed charge $8C$ is placed on the x-axis from $x = \frac{a}{4}$ to $x = \frac{5a}{4}$. Two point charges −7C and 3C are placed at $\left(\frac{a}{4}, \frac{-a}{4}, 0\right)$ and $\left(\frac{-3a}{4}, \frac{3a}{4}, 0\right)$, respectively. Consider a cubical surface formed by six surfaces $x = \pm\frac{a}{2}$, $y = \pm\frac{a}{2}$, $z = \pm\frac{a}{2}$. The electric flux through this cubical surface is **(2009)**



(a) $\frac{-2\,C}{\varepsilon_0}$ (b) $\frac{2C}{\varepsilon_0}$ (c) $\frac{10C}{\varepsilon_0}$ (d) $\frac{12C}{\varepsilon_0}$

10. Consider the charge configuration and a spherical Gaussian surface as shown in the figure. When calculating the flux of the electric field over the spherical surface, the electric field will be due to **(2004, 1M)**



(a) q_2 (b) only the positive charges
(c) all the charges (d) $+q_1$ and $-q_1$

11. A solid conducting sphere having a charge Q is surrounded by an uncharged concentric conducting hollow spherical shell. Let the potential difference between the surface of the solid sphere and that of the outer surface of the hollow shell be V. If the shell is now given a change of $-3Q$, the new potential difference between the same two surfaces is **(1989, 2M)**

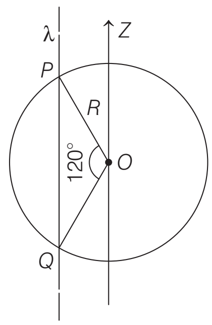
(a) V (b) 2V
(c) 4V (d) – 2V

12. A hollow metal sphere of radius 5 cm is charged such that the potential on its surface is 10 V. The potential at the centre of the sphere is **(1983, 1M)**

(a) zero
(b) 10 V
(c) same as at a point 5 cm away from the surface
(d) same as at a point 25 cm away from the surface

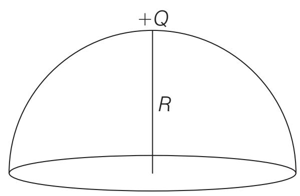
Objective Questions II (One or more correct option)

13. An infinitely long thin non-conducting wire is parallel to the Z-axis and carries a uniform line charge density λ. It pierces a thin non-conducting spherical shell of radius R in such a way that the arc PQ subtends an angle 120° at the centre O of the spherical shell, as shown in the figure. The permittivity of free space is ε_0. Which of the following statements is (are) true? **(2018 Adv.)**



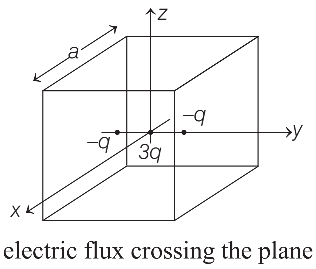
(a) The electric flux through the shell is $\sqrt{3}\, R\lambda / \varepsilon_0$
(b) The z-component of the electric field is zero at all the points on the surface of the shell
(c) The electric flux through the shell is $\sqrt{2}\, R\lambda / \varepsilon_0$
(d) The electric field is normal to the surface of the shell at all points

14. A point charge $+Q$ is placed just outside an imaginary hemispherical surface of radius R as shown in the figure. Which of the following statements is/are correct? **(2017 Adv.)**



(a) The electric flux passing through the curved surface of the hemisphere is $-\frac{Q}{2\varepsilon_0}\left(1 - \frac{1}{\sqrt{2}}\right)$.
(b) The component of the electric field normal to the flat surface is constant over the surface
(c) Total flux through the curved and the flat surfaces is $\frac{Q}{\varepsilon_0}$
(d) The circumference of the flat surface is an equipotential

15. A cubical region of side a has its centre at the origin. It encloses three fixed point charges, $-q$ at $(0, -a/4, 0)$, $+3q$ at $(0, 0, 0)$ and $-q$ at $(0, +a/4, 0)$. Choose the correct option(s). **(2012)**



(a) The net electric flux crossing the plane $x = +a/2$ is equal to the net electric flux crossing the plane $x = -a/2$
(b) The net electric flux crossing the plane $y = +a/2$ is more than the net electric flux crossing the plane $y = -a/2$
(c) The net electric flux crossing the entire region is q/ε_0
(d) The net electric flux crossing the plane $z = +a/2$ is equal to the net electric flux crossing the plane $x = +a/2$

16. Which of the following statement(s) is/are correct? **(2011)**

(a) If the electric field due to a point charge varies as $r^{-2.5}$ instead of r^{-2}, then the Gauss's law will still be valid

(b) The Gauss's law can be used to calculate the field distribution around an electric dipole

(c) If the electric field between two point charges is zero somewhere, then the sign of the two charges is the same

(d) The work done by the external force in moving a unit positive charge from point A at potential V_A to point B at potential V_B is $(V_B - V_A)$

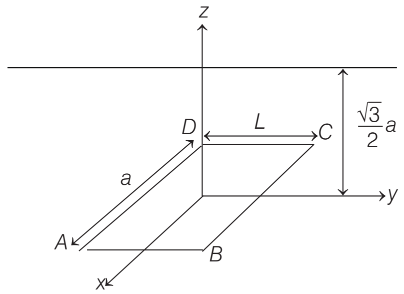
17. A spherical metal shell A of radius R_A and a solid metal sphere B of radius $R_B (< R_A)$ are kept far apart and each is given charge $+Q$. Now they are connected by a thin metal wire. Then **(2011)**

(a) $E_A^{\text{inside}} = 0$ (b) $Q_A > Q_B$

(c) $\frac{\sigma_A}{\sigma_B} = \frac{R_B}{R_A}$ (d) $E_A^{\text{on surface}} < E_B^{\text{on surface}}$

Integer Answer Type Question

18. An infinitely long uniform line charge distribution of charge per unit length λ lies parallel to the y-axis in the y-z plane at $z = \frac{\sqrt{3}}{2}a$ (see figure). If the magnitude of the flux of the electric field through the rectangular surface $ABCD$ lying in the x-y plane with its centre at the origin is $\frac{\lambda L}{n\varepsilon_0}$ (ε_0 = permittivity of free space), then the value of n is **(2015 Adv.)**



Analytical & Descriptive Questions

19. Three concentric spherical metallic shells, A, B and C of radii a, b and c $(a < b < c)$ have surface charge densities $\sigma, -\sigma$ and σ respectively. **(1990, 7M)**

(a) Find the potential of the three shells A, B and C.

(b) If the shells A and C are at the same potential, obtain the relation between the radii a, b and c.

20. A charge Q is distributed over two concentric hollow spheres of radii r and R $(> r)$ such that the surface densities are equal. Find the potential at the common centre. **(1981, 3M)**

Topic 4 Electric Field Lines, Behaviour of Conductor and Electric Dipole

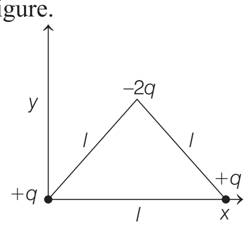
Objective Questions I (Only one correct option)

1. A point dipole $\mathbf{p} = -p_0\hat{\mathbf{x}}$ is kept at the origin. The potential and electric field due to this dipole on the Y-axis at a distance d are, respectively [Take, $V = 0$ at infinity] **(Main 2019, 12 Apr I)**

(a) $\frac{|\mathbf{p}|}{4\pi\varepsilon_0 d^2}, \frac{\mathbf{p}}{4\pi\varepsilon_0 d^3}$ (b) $0, \frac{-\mathbf{p}}{4\pi\varepsilon_0 d^3}$

(c) $0, \frac{\mathbf{p}}{4\pi\varepsilon_0 d^3}$ (d) $\frac{|\mathbf{p}|}{4\pi\varepsilon_0 d^2}, \frac{-\mathbf{p}}{4\pi\varepsilon_0 d^3}$

2. Determine the electric dipole moment of the system of three charges, placed on the vertices of an equilateral triangle as shown in the figure. **(Main 2019, 12 Jan I)**



(a) $\sqrt{3}\, ql \frac{\hat{\mathbf{j}} - \hat{\mathbf{i}}}{\sqrt{2}}$ (b) $2ql\,\hat{\mathbf{j}}$

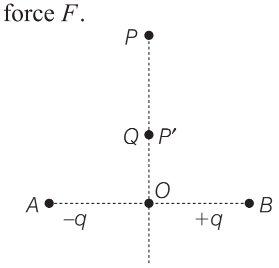
(c) $-\sqrt{3}\, ql\,\hat{\mathbf{j}}$ (d) $(ql)\frac{\hat{\mathbf{i}} + \hat{\mathbf{j}}}{\sqrt{2}}$

3. An electric field of 1000 V/m is applied to an electric dipole at angle of 45°. The value of electric dipole moment is 10^{-29} C-m. What is the potential energy of the electric dipole? **(Main 2019, 11 Jan II)**

(a) -9×10^{-20} J (b) -10×10^{-29} J

(c) -20×10^{-18} J (d) -7×10^{-27} J

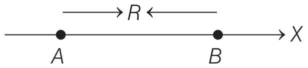
4. Charges $-q$ and $+q$ located at A and B, respectively, constitute an electric dipole. Distance $AB = 2a$, O is the mid point of the dipole and OP is perpendicular to AB. A charge Q is placed at P, where $OP = y$ and $y >> 2a$. The charge Q experiences an electrostatic force F. **(Main 2019, 10 Jan II)**



If Q is now moved along the equatorial line to P' such that $OP' = \left(\frac{y}{3}\right)$, the force on Q will be close to $\left(\frac{y}{3} >> 2a\right)$

(a) $\frac{F}{3}$ (b) $3F$ (c) $9F$ (d) $27F$

5. Two electric dipoles, A, B with respective dipole moments $\mathbf{d}_A = -4\,qa\,\hat{\mathbf{i}}$ and $\mathbf{d}_B = -2\,qa\,\hat{\mathbf{i}}$ are placed on the X-axis with a separation R, as shown in the figure



The distance from A at which both of them produce the same potential is **(Main 2019, 10 Jan I)**

(a) $\frac{\sqrt{2}\,R}{\sqrt{2}+1}$ (b) $\frac{\sqrt{2}\,R}{\sqrt{2}-1}$ (c) $\frac{R}{\sqrt{2}+1}$ (d) $\frac{R}{\sqrt{2}-1}$

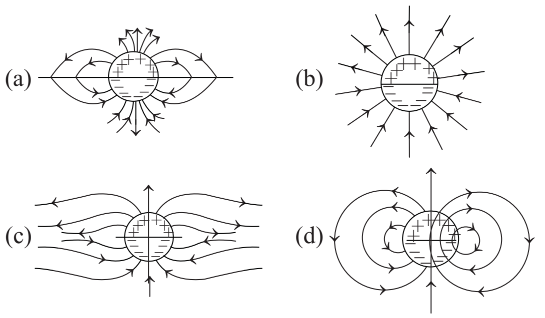
6. For a uniformly charged ring of radius R, the electric field on its axis has the largest magnitude at a distance h from its centre. Then, value of h is **(Main 2019, 9 Jan I)**

(a) $\frac{R}{\sqrt{2}}$ (b) $R\sqrt{2}$ (c) R (d) $\frac{R}{\sqrt{5}}$

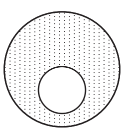
7. An electric dipole has a fixed dipole moment $\mathbf{P}$, which makes angle θ with respect to X-axis. When subjected to an electric field $\mathbf{E}_1 = E\hat{\mathbf{i}}$, it experiences a torque $\mathbf{T}_1 = \tau\hat{\mathbf{k}}$. When subjected to another electric field $\mathbf{E}_2 = \sqrt{3}E_1\,\hat{\mathbf{j}}$, it experiences a torque $\mathbf{T}_2 = -\mathbf{T}_1$. The angle θ is **(2017 Main)**

(a) 45° (b) 60° (c) 90° (d) 30°

8. A long cylindrical shell carries positive surface charge σ in the upper half and negative surface charge $-\sigma$ in the lower half. The electric field lines around the cylinder will look like figure given in (figures are schematic and not drawn to scale) **(2015 Main)**

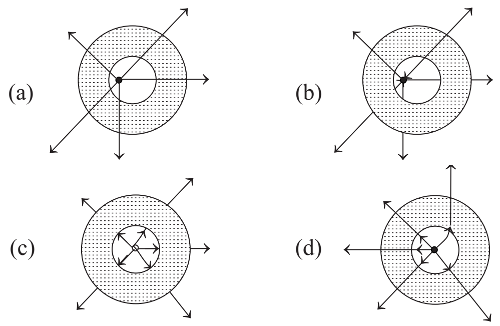


9. Consider a neutral conducting sphere. A positive point charge is placed outside the sphere. The net charge on the sphere is then **(2007, 3M)**

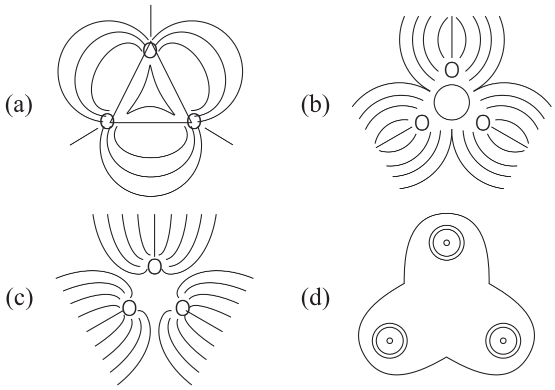


(a) negative and distributed uniformly over the surface of the sphere
(b) negative and appears only at the point on the sphere closest to the point charge
(c) negative and distributed non-uniformly over the entire surface of the sphere
(d) zero

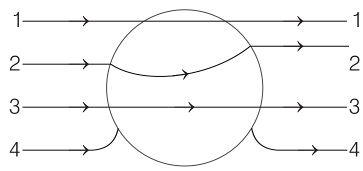
10. A metallic shell has a point charge q kept inside its cavity. Which one of the following diagrams correctly represents the electric lines of forces ? **(2003, 1M)**



11. Three positive charges of equal value q are placed at the vertices of an equilateral triangle. The resulting lines of force should be sketched as in **(2001, 1M)**



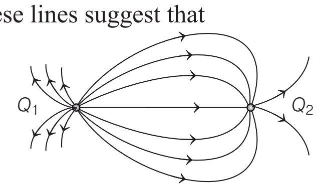
12. A metallic solid sphere is placed in a uniform electric field. The lines of force follow the path(s) shown in figure as **(1996, 2M)**



(a) 1 (b) 2 (c) 3 (d) 4

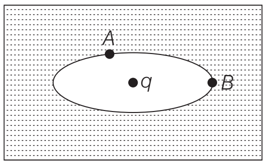
Objective Questions II (One or more correct option)

13. A few electric field lines for a system of two charges Q_1 and Q_2 fixed at two different points on the x-axis are shown in the figure. These lines suggest that **(2010)**



(a) $|Q_1| > |Q_2|$
(b) $|Q_1| < |Q_2|$
(c) at a finite distance to the left of Q_1 the electric field is zero
(d) at a finite distance to the right of Q_2 the electric field is zero

14. An elliptical cavity is carved within a perfect conductor. A positive charge q is placed at the centre of the cavity. The points A and B are on the cavity surface as shown in the figure. Then **(1999, 3M)**



(a) electric field near A in the cavity = electric field near B in the cavity.
(b) charge density at A = charge density at B
(c) potential at A = potential at B
(d) total electric field flux through the surface of the cavity is q/ε_0.

True/False

15. An electric line of force in the x-y plane is given by the equation $x^2 + y^2 = 1$. A particle with unit positive charge, initially at rest at the point $x = 1$, $y = 0$ in the x-y plane, will move along the circular line of force. **(1988, 2M)**

Analytical & Descriptive Questions

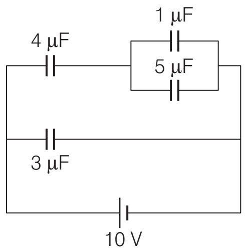
16. A positive point charge q is fixed at origin. A dipole with a dipole moment **p** is placed along the x-axis far away from the origin with **p** pointing along positive x-axis. Find : (a) the kinetic energy of the dipole when it reaches a distance d from the origin, and (b) the force experienced by the charge q at this moment. **(2003, 4M)**

17. A charged particle is free to move in an electric field. Will it always move along an electric line of force? **(1979)**

Topic 5 Capacitors

Objective Questions I (Only one correct option)

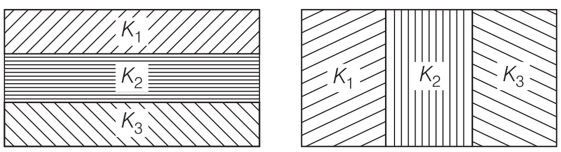
1. In the given circuit, the charge on 4 μF capacitor will be **(Main 2019, 12 April II)**



(a) 5.4 μC (b) 9.6 μC (c) 13.4 μC (d) 24 μC

2. Two identical parallel plate capacitors of capacitance C each, have plates of area A, separated by a distance d. The space between the plates of the two capacitors, is filled with three dielectrics of equal thickness and dielectric constants K_1, K_2 and K_3.

The first capacitor is filled as shown in Fig. I, and the second one is filled as shown in Fig. II. If these two modified capacitors are charged by the same potential V, the ratio of the energy stored in the two, would be (E_1 refers to capacitor (I) and E_2 to capacitor (II)) : **(Main 2019, 12 April I)**



(I) (II)

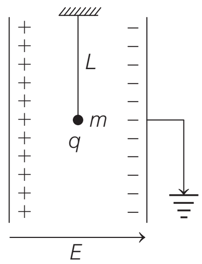
(a) $\frac{E_1}{E_2} = \frac{K_1K_2K_3}{(K_1 + K_2 + K_3)(K_2K_3 + K_3K_1 + K_1K_2)}$

(b) $\frac{E_1}{E_2} = \frac{(K_1 + K_2 + K_3)(K_2K_3 + K_3K_1 + K_1K_2)}{K_1K_2K_3}$

(c) $\frac{E_1}{E_2} = \frac{9K_1K_2K_3}{(K_1 + K_2 + K_3)(K_2K_3 + K_3K_1 + K_1K_2)}$

(d) $\frac{E_1}{E_2} = \frac{(K_1 + K_2 + K_3)(K_2K_3 + K_3K_1 + K_1K_2)}{9K_1K_2K_3}$

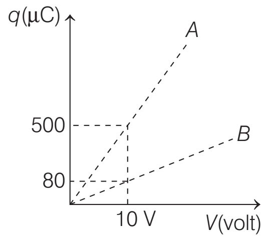
3. A simple pendulum of length L is placed between the plates of a parallel plate capacitor having electric field E, as shown in figure. Its bob has mass m and charge q. The time period of the pendulum is given by **(Main 2019, 10 April II)**



(a) $2\pi\sqrt{\frac{L}{\sqrt{g^2 + \left(\frac{qE}{m}\right)^2}}}$ (b) $2\pi\sqrt{\frac{L}{\sqrt{g^2 - \frac{q^2E^2}{m^2}}}}$

(c) $2\pi\sqrt{\frac{L}{\left(g + \frac{qE}{m}\right)}}$ (d) $2\pi\sqrt{\frac{L}{\left(g - \frac{qE}{m}\right)}}$

4. Figure shows charge (q) *versus* voltage (V) graph for series and parallel combination of two given capacitors. The capacitances are **(Main 2019, 10 April I)**



(a) 60 μF and 40 μF (b) 50 μF and 30 μF
(c) 20 μF and 30 μF (d) 40 μF and 10μF

5 The parallel combination of two air filled parallel plate capacitors of capacitance C and nC is connected to a battery of voltage, V. When the capacitors are fully charged, the battery is removed and after that a dielectric material of dielectric constant K is placed between the two plates of the first capacitor. The new potential difference of the combined system is **(Main 2019, 9 April II)**

(a) $\frac{(n+1)V}{(K+n)}$ (b) $\frac{nV}{K+n}$

(c) V (d) $\frac{V}{K+n}$

6. A capacitor with capacitance $5\,\mu F$ is charged to $5\,\mu C$. If the plates are pulled apart to reduce the capacitance to $2\,\mu F$, how much work is done? **(Main 2019, 9 April I)**

(a) 6.25×10^{-6} J (b) 2.16×10^{-6} J

(c) 2.55×10^{-6} J (d) 3.75×10^{-6} J

7. A parallel plate capacitor has $1\,\mu F$ capacitance. One of its two plates is given $+2\mu C$ charge and the other plate $+4\mu C$ charge. The potential difference developed across the capacitor is **(Main 2019, 8 April II)**

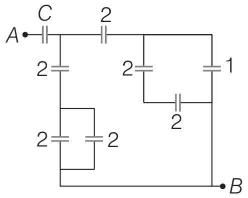
(a) 1 V (b) 5 V (c) 2 V (d) 3 V

8. Voltage rating of a parallel plate capacitor is 500 V. Its dielectric can withstand a maximum electric field of 10^6 V/m. The plate area is $10^{-4}\,m^2$. What is the dielectric constant, if the capacitance is 15 pF?

(Take, $\varepsilon_0 = 8.86 \times 10^{-12} C^2/N\text{-}m^2$) **(Main 2019, 8 April I)**

(a) 3.8 (b) 8.5 (c) 4.5 (d) 6.2

9. In the circuit shown, find C if the effective capacitance of the whole circuit is to be $0.5\,\mu F$. All values in the circuit are in μF. **(Main 2019, 12 Jan II)**



(a) $\frac{6}{5}\,\mu F$ (b) $4\,\mu F$

(c) $\frac{7}{10}\,\mu F$ (d) $\frac{7}{11}\,\mu F$

10. A parallel plate capacitor with plates of area $1\,m^2$ each, are at a separation of 0.1 m. If the electric field between the plates is 100 N/C, the magnitude of charge on each plate is $\left(\text{Take, } \varepsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{N - m^2}\right)$ **(Main 2019, 12 Jan II)**

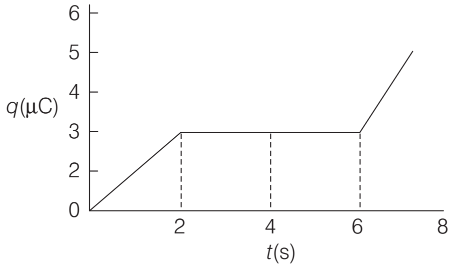
(a) 9.85×10^{-10} C

(b) 8.85×10^{-10} C

(c) 7.85×10^{-10} C

(d) 6.85×10^{-10} C

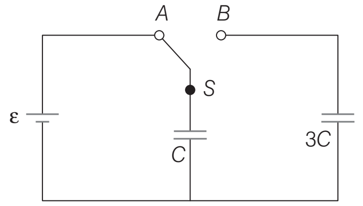
11. The charge on a capacitor plate in a circuit as a function of time is shown in the figure. **(Main 2019, 12 Jan II)**



What is the value of current at $t = 4$ s?

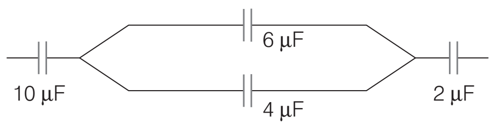
(a) $2\,\mu A$ (b) $1.5\,\mu A$ (c) Zero (d) $3\,\mu A$

12. In the figure shown, after the switch 'S' is turned from position 'A' to position 'B', the energy dissipated in the circuit in terms of capacitance 'C' and total charge 'Q' is **(Main 2019, 12 Jan I)**



(a) $\frac{3}{4} \cdot \frac{Q^2}{C}$ (b) $\frac{5}{8} \cdot \frac{Q^2}{C}$ (c) $\frac{1}{8} \cdot \frac{Q^2}{C}$ (d) $\frac{3}{8} \cdot \frac{Q^2}{C}$

13. In the figure shown below, the charge on the left plate of the $10\,\mu F$ capacitor is $-30\,\mu C$. The charge on the right plate of the $6\,\mu F$ capacitor is **(Main 2019, 11 Jan I)**

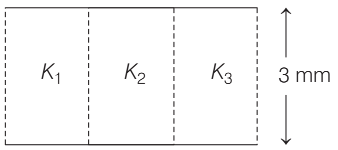


(a) $+12\,\mu C$ (b) $+18\,\mu C$ (c) $-12\,\mu C$ (d) $-18\,\mu C$

14. A parallel plate capacitor having capacitance 12 pF is charged by a battery to a potential difference of 10 V between its plates. The charging battery is now disconnected and a porcelain slab of dielectric constant 6.5 is slipped between the plates. The work done by the capacitor on the slab is **(Main 2019, 10 Jan II)**

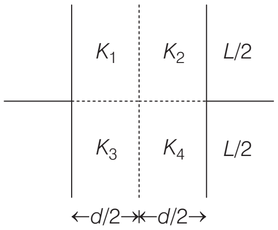
(a) 560 pJ (b) 508 pJ (c) 692 pJ (d) 600 pJ

15. A parallel plate capacitor is of area 6 cm^2 and a separation 3 mm. The gap is filled with three dielectric materials of equal thickness (see figure) with dielectric constants $K_1 = 10$, $K_2 = 12$ and $K_3 = 14$. The dielectric constant of a material which give same capacitance when fully inserted in above capacitor, would be **(Main 2019, 10 Jan I)**



(a) 4 (b) 36 (c) 12 (d) 14

16. A parallel plate capacitor with square plates is filled with four dielectrics of dielectric constants K_1, K_2, K_3, K_4 arranged as shown in the figure. The effective dielectric constant K will be **(Main 2019, 9 Jan II)**



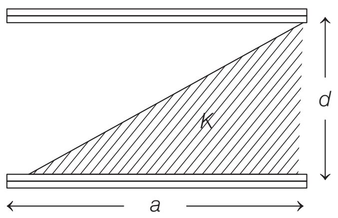
(a) $K = \frac{(K_1+K_2)(K_3+K_4)}{2(K_1+K_2+K_3+K_4)}$

(b) $K = \frac{(K_1+K_2)(K_3+K_4)}{K_1+K_2+K_3+K_4}$

(c) $K = \frac{(K_1+K_3)(K_2+K_4)}{K_1+K_2+K_3+K_4}$

(d) $K = \frac{(K_1+K_4)(K_2+K_3)}{2(K_1+K_2+K_3+K_4)}$

17. A parallel plate capacitor is made of two square plates of side 'a' separated by a distance d (d<<a). The lower triangular portions is filled with a dielectric of dielectric constant k, as shown in the figure. Capacitance of this capacitor is **(Main 2019, 9 Jan I)**



(a) $\frac{K\varepsilon_0 a^2}{d}\ln K$ (b) $\frac{K\varepsilon_0 a^2}{d(K-1)}\ln K$

(c) $\frac{K\varepsilon_0 a^2}{2d(K+1)}$ (d) $\frac{1}{2}\cdot\frac{K\varepsilon_0 a^2}{d}$

18. A parallel plate capacitor of capacitance 90 pF is connected to a battery of emf 20 V. If a dielectric material of dielectric constant $K = 5/3$ is inserted between the plates, the magnitude of the induced charge will be **(2018 Main)**

(a) 0.9 μC (b) 1.2 μC

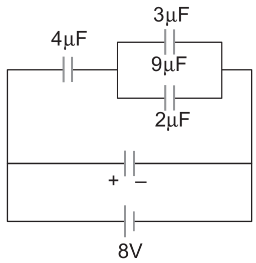
(c) 0.3 μC (d) 2.4 μC

19. A capacitance of 2 μ**F** is required in an electrical circuit across a potential difference of 1kV. A large number of 1 μ**F** capacitors are available which can withstand a potential difference of not more than 300 V. The minimum number of capacitors required to achieve this is **(2017 Main)**

(a) 16 (b) 24

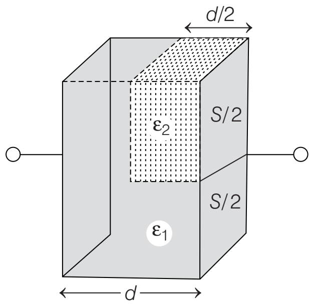
(c) 32 (d) 2

20. A combination of capacitors is set-up as shown in the figure. The magnitude of the electric field, due to a point charge Q (having a charge equal to the sum of the charges on the 4 μF and 9 μF capacitors), at a point distant 30 m from it, would equal to **(2016 Main)**



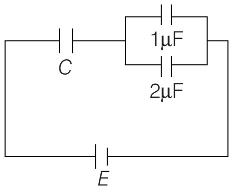
(a) 240 N/C (b) 360 N/C (c) 420 N/C (d) 480 N/C

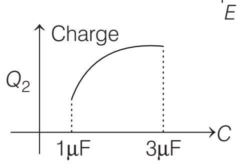
21. A parallel plate capacitor having plates of area S and plate separation d, has capacitance C_1 in air. When two dielectrics of different relative permittivities ($\varepsilon_1 = 2$ and $\varepsilon_2 = 4$) are introduced between the two plates as shown in the figure, the capacitance becomes C_2. The ratio $\frac{C_2}{C_1}$ is **(2015 Adv.)**

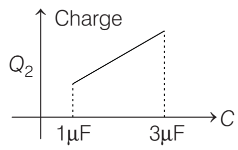


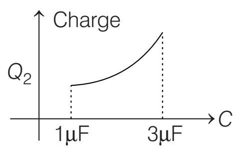
(a) $\frac{6}{5}$ (b) $\frac{5}{3}$ (c) $\frac{7}{5}$ (d) $\frac{7}{3}$

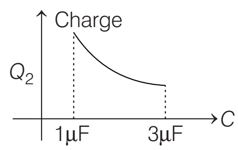
22. In the given circuit, charge Q_2 on the 2μF capacitor changes as C is varied from 1μF to 3μF. Q_2 as a function of C is given properly by (figures are drawn schematically and are not to scale) **(2015 Main)**



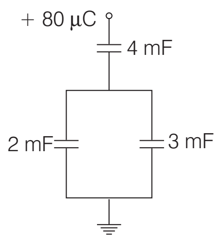
(a)


(b)


(c)


(d)


23. In the given circuit, a charge of +80 μC is given to the upper plate of the 4 μF capacitor. Then in the steady state, the charge on the upper plate of the 3 μF capacitor is **(2012)**



(a) + 32 μC

(b) + 40 μC

(c) + 48 μC

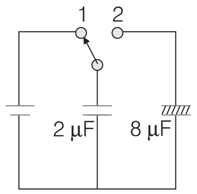
(d) + 80 μC

24. A parallel plate capacitor is made of two circular plates separated by a distance of 5 mm and with a dielectric of dielectric constant 2.2 between them. When the electric field in the dielectric is 3×10^4 V/m, the charge density of the positive plate will be close to **(2014 Main)**

(a) 6×10^{-7} C/m^2 (b) 3×10^{-7} C/m^2

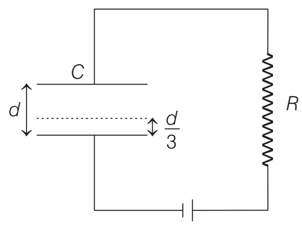
(c) 3×10^{4} C/m^2 (d) 6×10^{4} C/m^2

25. A 2 μF capacitor is charged as shown in the figure. The percentage of its stored energy dissipated after the switch S is turned to position 2 is **(2011)**



(a) 0% (b) 20% (c) 75% (d) 80%

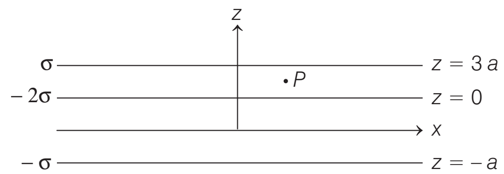
26. A parallel plate capacitor C with plates of unit area and separation d is filled with a liquid of dielectric constant $K = 2$. The level of liquid is $d/3$ initially. Suppose the liquid level decreases at a constant speed v, the time constant as a function of time t is **(2008, 3M)**



(a) $\dfrac{6\varepsilon_0 R}{5d + 3vt}$ (b) $\dfrac{(15d + 9vt)\varepsilon_0 R}{2d^2 - 3dvt - 9v^2t^2}$

(c) $\dfrac{6\varepsilon_0 R}{5d - 3vt}$ (d) $\dfrac{(15d - 9vt)\varepsilon_0 R}{2d^2 + 3dvt - 9v^2t^2}$

27. Three infinitely long charge sheets are placed as shown in figure. The electric field at point P is **(2005, 1M)**



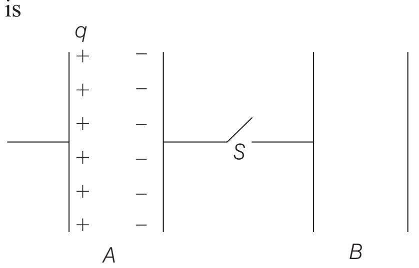
(a) $\dfrac{2\sigma}{\varepsilon_0}\hat{\mathbf{k}}$ (b) $-\dfrac{2\sigma}{\varepsilon_0}\hat{\mathbf{k}}$ (c) $\dfrac{4\sigma}{\varepsilon_0}\hat{\mathbf{k}}$ (d) $-\dfrac{4\sigma}{\varepsilon_0}\hat{\mathbf{k}}$

28. Two identical capacitors, have the same capacitance C. One of them is charged to potential V_1 and the other to V_2. Likely charged plates are then connected. Then, the decrease in energy of the combined system is **(2002, 1M)**

(a) $\dfrac{1}{4}C(V_1^2 - V_2^2)$ (b) $\dfrac{1}{4}C(V_1^2 + V_2^2)$

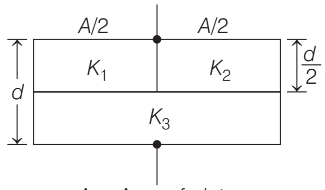
(c) $\dfrac{1}{4}C(V_1 - V_2)^2$ (d) $\dfrac{1}{4}C(V_1 + V_2)^2$

29. Consider the situation shown in the figure. The capacitor A has a charge q on it whereas B is uncharged. The charge appearing on the capacitor B a long time after the switch is closed is **(2001, 1M)**



(a) zero (b) $q/2$ (c) q (d) $2q$

30. A parallel plate capacitor of area A, plate separation d and capacitance C is filled with three different dielectric materials having dielectric constants K_1, K_2 and K_3 as shown. If a single dielectric material is to be used to have the same capacitance C in this capacitor then its dielectric constant K is given by **(2000, 2M)**

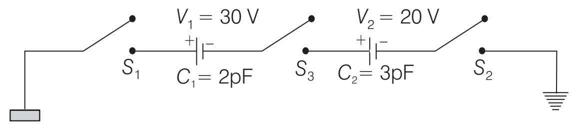


A = Area of plates

(a) $\dfrac{1}{K} = \dfrac{1}{K_1} + \dfrac{1}{K_2} + \dfrac{1}{2K_3}$ (b) $\dfrac{1}{K} = \dfrac{1}{K_1 + K_2} + \dfrac{1}{2K_3}$

(c) $\dfrac{1}{K} = \dfrac{K_1K_2}{K_1 + K_2} + 2K_3$ (d) $K = \dfrac{K_1K_3}{K_1 + K_3} + \dfrac{K_2K_3}{K_2 + K_3}$

31. For the circuit shown, which of the following statements is true ? **(1999, 2M)**



(a) With S_1 closed, $V_1 = 15\text{V}, V_2 = 20\text{V}$
(b) With S_3 closed, $V_1 = V_2 = 25\text{V}$
(c) With S_1 and S_2 closed, $V_1 = V_2 = 0$
(d) With S_3 closed, $V_1 = 30\text{V}, V_2 = 20\text{V}$

32. Two identical metal plates are given positive charges Q_1 and Q_2 $(< Q_1)$ respectively. If they are now brought close together to form a parallel plate capacitor with capacitance C, the potential difference between them is **(1999, 2M)**

(a) $(Q_1 + Q_2)/2C$ (b) $(Q_1 + Q_2)/C$

(c) $(Q_1 - Q_2)/C$ (d) $(Q_1 - Q_2)/2C$

33. A parallel combination of 0.1 MΩ resistor and a 10 μF capacitor is connected across a 1.5 V source of negligible resistance. The time required for the capacitor to get charged upto 0.75 V is approximately (in second) **(1997, 2M)**

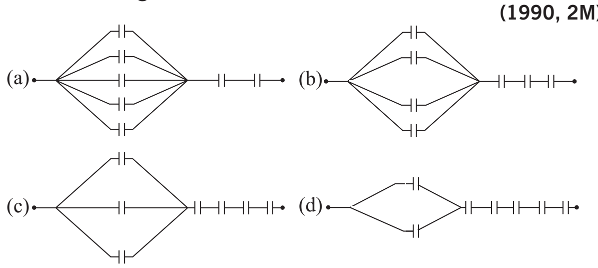
(a) infinite (b) $\log_e 2$

(c) $\log_{10} 2$ (d) zero

34. The magnitude of electric field **E** in the annular region of a charged cylindrical capacitor **(1996, 2M)**
(a) is same throughout.
(b) is higher near the outer cylinder than near the inner cylinder.
(c) varies as $1/r$ where r is the distance from the axis.
(d) varies as $1/r^2$ where r is the distance from the axis.

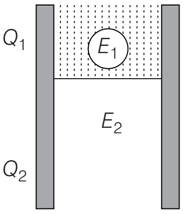
35. A parallel plate capacitor of capacitance C is connected to a battery and is charged to a potential difference V. Another capacitor of capacitance $2C$ is similarly charged to a potential difference $2V$. The charging battery is now disconnected and the capacitors are connected in parallel to each other in such a way that the positive terminal of one is connected to the negative terminal of the other. The final energy of the configuration is **(1995, 2M)**
(a) zero (b) $\frac{3}{2}CV^2$ (c) $\frac{25}{6}CV^2$ (d) $\frac{9}{2}CV^2$

36. Seven capacitors each of capacitance $2\mu F$ are connected in a configuration to obtain an effective capacitance $\frac{10}{11}\mu F$. Which of the following combination will achieve the desired result ? **(1990, 2M)**



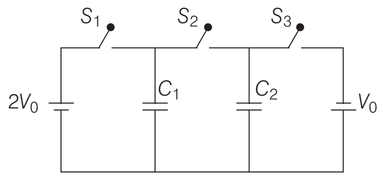
Objective Questions II (One or more correct option)

37. A parallel plate capacitor has a dielectric slab of dielectric constant K between its plates that covers 1/3 of the area of its plates, as shown in the figure. The total capacitance of the capacitor is C while that of the portion with dielectric in between is C_1. When the capacitor is charged, the plate area covered by the dielectric gets charge Q_1 and the rest of the area gets charge Q_2. The electric field in the dielectric is E_1 and that in the other portion is E_2. Choose the correct option/options, ignoring edge effects. **(2014 Adv.)**



(a) $\frac{E_1}{E_2} = 1$ (b) $\frac{E_1}{E_2} = \frac{1}{K}$
(c) $\frac{Q_1}{Q_2} = \frac{3}{K}$ (d) $\frac{C}{C_1} = \frac{2+K}{K}$

38. In the circuit shown in the figure, there are two parallel plate capacitors each of capacitance C. The switch S_1 is pressed first to fully charge the capacitor C_1 and then released. The switch S_2 is then pressed to charge the capacitor C_2. After some time, S_2 is released and then S_3 is pressed. After some time **(2013 Adv.)**



(a) the charge on the upper plate of C_1 is $2CV_0$
(b) the charge on the upper plate of C_1 is CV_0
(c) the charge on the upper plate of C_2 is 0
(d) the charge on the upper plate of C_2 is $-CV_0$

39. Two capacitors C_1 and C_2 are charged to 120 V and 200 V respectively. It is found that by connecting them together the potential on each one can be made zero. Then **(2013 Main)**
(a) $5C_1 = 3C_2$ (b) $3C_1 = 5C_2$
(c) $3C_1 + 5C_2 = 0$ (d) $9C_1 = 4C_2$

40. A dielectric slab of thickness d is inserted in a parallel plate capacitor whose negative plate is at $x = 0$ and positive plate is at $x = 3d$. The slab is equidistant from the plates. The capacitor is given some charge. As x goes from 0 to $3d$ **(1998, 2M)**
(a) the magnitude of the electric field remains the same.
(b) the direction of the electric field remains the same.
(c) the electric potential increases continuously.
(d) the electric potential increases at first, then decreases and again increases.

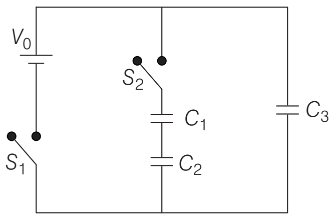
41. A parallel plate capacitor of plate area A and plate separation d is charged to potential difference V and then the battery is disconnected. A slab of dielecric constant K is then inserted between the plates of the capacitor so as to fill the space between the plates. If Q, E and W denote respectively, the magnitude of charge on each plate, the electric field between the plates (after the slab is inserted), and work done on the system, in question, in the process of inserting the slab, then **(1991, 2M)**
(a) $Q = \frac{\varepsilon_0 AV}{d}$ (b) $Q = \frac{\varepsilon_0 KAV}{d}$
(c) $E = V/Kd$ (d) $W = \frac{\varepsilon_0 AV^2}{2d}\left[1 - 1/K\right]$

42. A parallel plate capacitor is charged and the charging battery is then disconnected. If the plates of the capacitor are moved farther apart by means of insulating handles **(1987, 2M)**
(a) the charge on the capacitor increases
(b) the voltage across the plates increases
(c) the capacitance increases
(d) the electrostatic energy stored in the capacitor increases

43. A parallel plate air capacitor is connected to a battery. The quantities charge, voltage, electric field and energy associated with this capacitor are given by Q_0, V_0, E_0 and U_0 respectively. A dielectric slab is now introduced to fill the space between the plates with the battery still in connection. The corresponding quantities now given by Q, V, E and U are related to the previous one as **(1985, 2M)**
(a) $Q > Q_0$ (b) $V > V_0$
(c) $E > E_0$ (d) $U > U_0$

Numerical Value Based Question

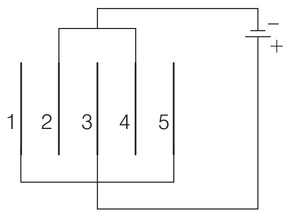
44. Three identical capacitors C_1, C_2 and C_3 have a capacitance of 1.0μF each and they are uncharged initially. They are connected in a circuit as shown in the figure and C_1 is then filled completely with a dielectric material of relative permittivity ε_r. The cell electromotive force (emf) $V_0 = 8V$. First the switch S_1 is closed while the switch S_2 is kept open. When the capacitor C_3 is fully charged, S_1 is opened and S_2 is closed simultaneously. When all the capacitors reach equilibrium, the charge on C_3 is found to be 5μC. The value of ε_r =... **(2018 Adv.)**



Fill in the Blanks

45. Two parallel plate capacitors of capacitances C and $2C$ are connected in parallel and charged to a potential difference V. The battery is then disconnected and the region between the plates of capacitor C is completely filled with a material of dielectric constant K. The potential difference across the capacitors now becomes…. **(1988, 2M)**

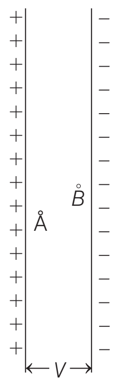
46. Five identical capacitor plates, each of area A, are arranged such that adjacent plates are at a distance d apart, the plates are connected to a source of emf V as shown in the figure. **(1984, 2M)**



The charge on plate 1 is …… and on plate 4 is …… .

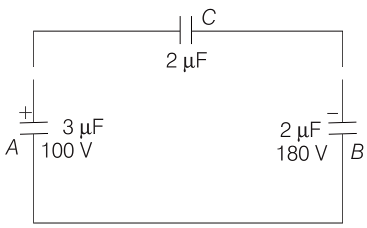
True / False

47. Two protons A and B are placed in between the two plates of a parallel plate capacitor charged to a potential difference V as shown in the figure. The forces on the two protons are identical. **(1986, 3M)**



Analytical & Descriptive Questions

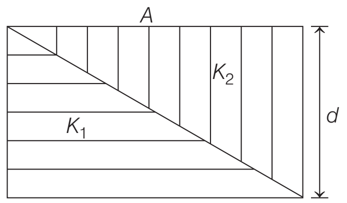
48. Two capacitors A and B with capacities 3μF and 2μF are charged to a potential difference of 100 V and 180 V respectively. The plates of the capacitors are connected as shown in the figure with one wire of each capacitor free. The upper plate of A is positive and that of B is negative. An uncharged 2 μF capacitor C with lead wires falls on the free ends to complete the circuit. Calculate **(1997, 5M)**



(a) the final charge on the three capacitors and

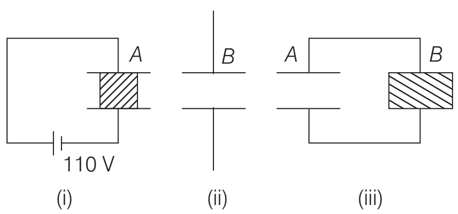
(b) the amount of electrostatic energy stored in the system before and after completion of the circuit.

49. The capacitance of a parallel plate capacitor with plate area A and separation d, is C. The space between the plates is filled with two wedges of dielectric constants K_1 and K_2 respectively (figure). Find the capacitance of the resulting capacitor. **(1996, 2M)**



50. Two square metal plates of side 1 m are kept 0.01 m apart like a parallel plate capacitor in air in such a way that one of their edges is perpendicular to an oil surface in a tank filled with an insulating oil. The plates are connected to a battery of emf 500 V. The plates are then lowered vertically into the oil at a speed of 0.001 ms^{-1}. Calculate the current drawn from the battery during the process. (Dielectric constant of oil = 11, $\varepsilon_0 = 8.85 \times 10^{-12}\ C^2N^{-1}m^{-2}$). **(1994, 6M)**

51. Two parallel plate capacitors A and B have the same separation $d = 8.85 \times 10^{-4}$m between the plates. The plate areas of A and B are 0.04 m^2 and 0.02 m^2 respectively. A slab of dielectric constant (relative permittivity) $K = 9$ has dimensions such that it can exactly fill the space between the plates of capacitor B. **(1993, 7M)**

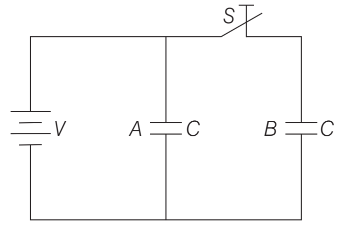


(a) The dielectric slab is placed inside A as shown in figure (i) A is then charged to a potential difference of 110 V. Calculate the capacitance of A and the energy stored in it.

(b) The battery is disconnected and then the dielectric slab is removed from A. Find the work done by the external agency in removing the slab from A.

(c) The same dielectric slab is now placed inside B, filling it completely. The two capacitors A and B are then connected as shown in figure (iii). Calculate the energy stored in the system.

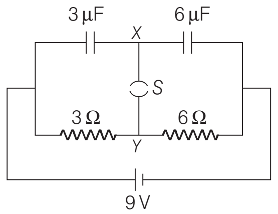
52. The figure shows two identical parallel plate capacitors connected to a battery with the switch S closed. The switch is now opened and the free space between the plates of the capacitors is filled with a dielectric of dielectric constant (or relative permittivity) 3. Find the ratio of the total electrostatic energy stored in both capacitors before and after the introduction of the dielectric. **(1983, 6M)**



Topic 6 *C-R* Circuits

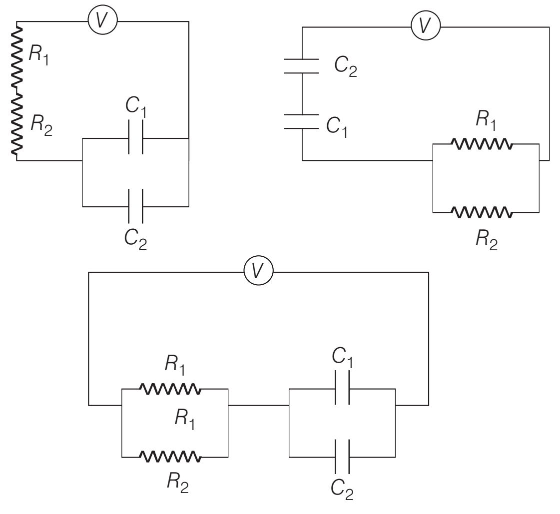
Objective Questions I (Only one correct option)

1. A circuit is connected as shown in the figure with the switch S open. When the switch is closed, the total amount of charge that flows from Y to X is **(2007, 3M)**



(a) zero (b) 54 μC (c) 27 μC (d) 81 μC

2. Find the time constant for the given RC circuits in correct order (in μs). **(2006, 3M)**



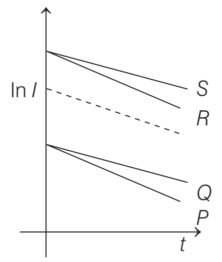
$R_1 = 1\,\Omega, R_2 = 2\,\Omega, C_1 = 4\,\mu F, C_2 = 2\mu F.$

(a) 18, 4, 8/9 (b) 18, 8/9, 4 (c) 4, 18, 8/9 (d) 4, 8/9, 18

3. A 4μF capacitor and a resistance of 2.5 MΩ are in series with 12 V battery. Find the time after which the potential difference across the capacitor is 3 times the potential difference across the resistor. [Given, ln (2) = 0.693] **(2005, 2M)**

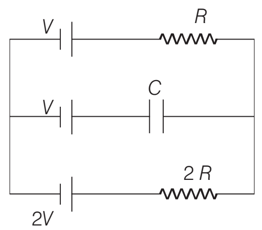
(a) 13.86 s (b) 6.93 s (c) 7 s (d) 14 s

4. A capacitor is charged using an external battery with a resistance x in series. The dashed line shows the variation of ln I with respect to time. If the resistance is changed to $2x$, the new graph will be **(2004, 2M)**



(a) P (b) Q (c) R (d) S

5. In the given circuit, with steady current, the potential difference across the capacitor must be **(2001, 2M)**



(a) V (b) $V/2$ (c) $V/3$ (d) $2V/3$

Objective Questions II (One or more correct option)

6. Capacitor C_1 of capacitance 1 μF and capacitor C_2 of capacitance 2 μF are separately charged fully by a common battery. The two capacitors are then separately allowed to discharge through equal resistors at time $t = 0$. **(1989, 2M)**

(a) The current in each of the two discharging circuits is zero at $t = 0$

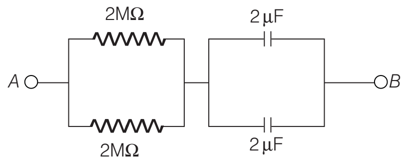
(b) The currents in the two discharging circuits at $t = 0$ are equal but not zero

(c) The currents in the two discharging circuits at $t = 0$ are unequal

(d) Capacitor C_1, loses 50% of its initial charge sooner than C_2 loses 50% of its initial charge

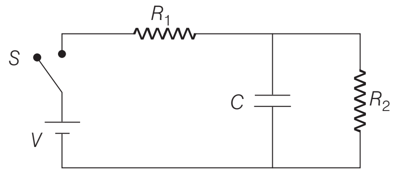
Integer Answer Type Questions

7. At time $t = 0$, a battery of 10 V is connected across points A and B in the given circuit. If the capacitors have no charge initially, at what time (in second) does the voltage across them become 4 V? [Take : $\ln 5 = 1.6$, $\ln 3 = 1.1$] **(2010)**

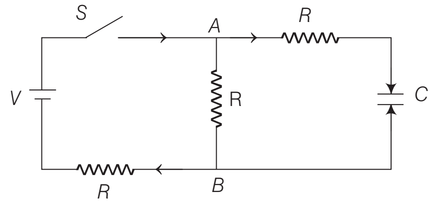


Analytical & Descriptive Questions

8. At $t = 0$, switch S is closed. The charge on the capacitor is varying with time as $Q = Q_0(1 - e^{-\alpha t})$. Obtain the value of Q_0 and α in the given circuit parameters. **(2005, 4M)**



9. In the circuit shown in figure, the battery is an ideal one, with emf V. The capacitor is initially uncharged. The switch S is closed at time $t = 0$. **(1998, 8M)**

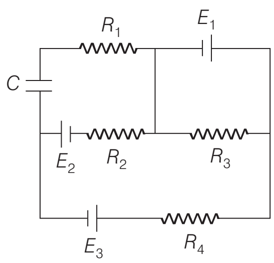


(a) Find the charge Q on the capacitor at time t.

(b) Find the current in AB at time t. What is its limiting value as $t \to \infty$?

10. A leaky parallel plate capacitor is filled completely with a material having dielectric constant $K = 5$ and electrical conductivity $\sigma = 7.4 \times 10^{-12}\ \Omega^{-1}\ m^{-1}$. If the charge on the capacitor at instant $t = 0$ is $q = 8.85\ \mu C$, then calculate the leakage current at the instant $t = 12$ s. **(1997 C, 5M)**

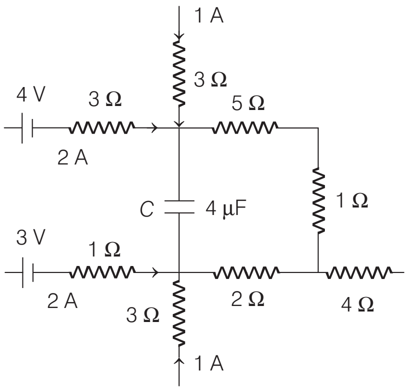
11. In the given circuit, **(1988, 5M)**



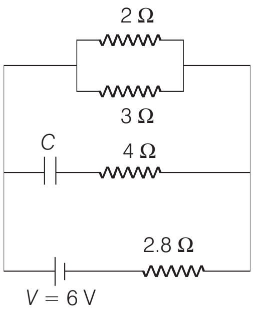
$E_1 = 3E_2 = 2E_3 = 6$ V and $R_1 = 2R_4 = 6\Omega$, $R_3 = 2R_2 = 4\Omega$, $C = 5\mu F$.

Find the current in R_3 and the energy stored in the capacitor.

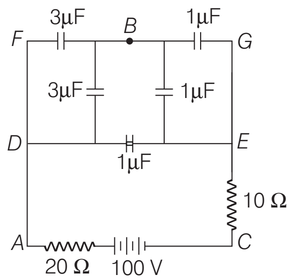
12. A part of circuit in a steady state along with the currents flowing in the branches, the values of resistances etc, is shown in the figure. Calculate the energy stored in the capacitor $C\ (4\mu F)$. **(1986, 4M)**



13. Calculate the steady state current in the 2 Ω resistor shown in the circuit (see figure). The internal resistance of the battery is negligible and the capacitance of the condenser C is 0.2 μF. **(1982, 5M)**



14. Find the potential difference between the points A and B and between the points B and C in the steady state. **(1980)**



Topic 7 Miscellaneous Problems

Objective Questions I (Only one correct option)

1. The magnetic field of a plane electromagnetic wave is given by

$\mathbf{B} = B_0[\cos(kz - \omega t)]\hat{\mathbf{i}} + B_1 \cos(kz + \omega t)\hat{\mathbf{j}}$

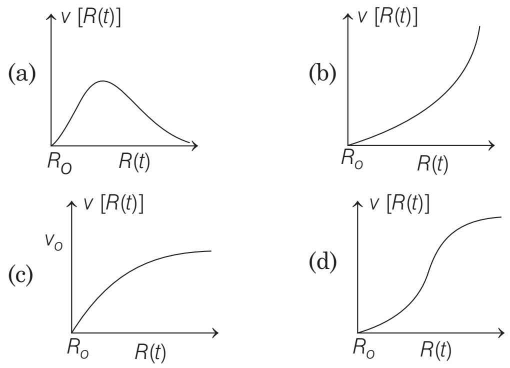
where, $B_0 = 3 \times 10^{-5}$ T and $B_1 = 2 \times 10^{-6}$ T. The rms value of the force experienced by a stationary charge $Q = 10^{-4}$ C at $z = 0$ is closest to **(Main 2019, 9 April I)**

(a) 0.1 N (b) 3×10^{-2} N (c) 0.6 N (d) 0.9 N

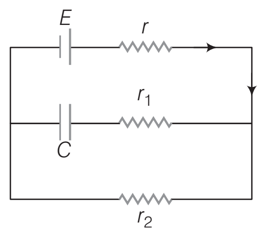
2. An electric dipole is formed by two equal and opposite charges q with separation d. The charges have same mass m. It is kept in a uniform electric field E. If it is slightly rotated from its equilibrium orientation, then its angular frequency ω is **(Main 2019, 8 April II)**

(a) $\sqrt{\frac{2qE}{md}}$ (b) $2\sqrt{\frac{qE}{md}}$ (c) $\sqrt{\frac{qE}{md}}$ (d) $\sqrt{\frac{qE}{2md}}$

3. There is uniform spherically symmetric surface charge density at a distance R_0 from the origin. The charge distribution is initially at rest and starts expanding because of mutual repulsion. The figure that represents best the speed $v[R(t)]$ of the distribution as a function of its instantaneous radius $R(t)$ is **(Main 2019, 12 Jan I)**



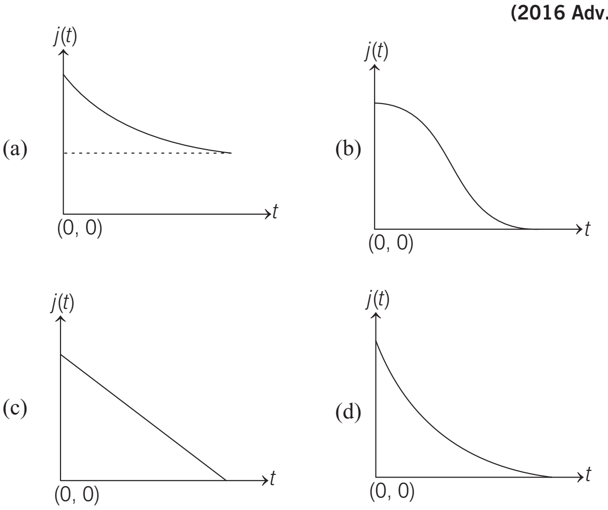
4. In the given circuit diagram, when the current reaches steady state in the circuit, the charge on the capacitor of capacitance C will be **(2017 Main)**



(a) $CE\frac{r_1}{(r_2 + r)}$ (b) $CE\frac{r_2}{(r + r_2)}$

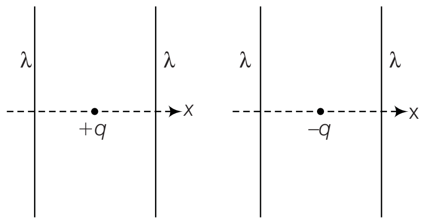
(c) $CE\frac{r_1}{(r_1 + r)}$ (d) CE

5. An infinite line charge of uniform electric charge density λ lies along the axis of an electrically conducting infinite cylindrical shell of radius R. At time $t = 0$, the space inside the cylinder is filled with a material of permittivity ε and electrical conductivity σ. The electrical conduction in the material follows Ohm's law. Which one of the following graphs best describes the subsequent variation of the magnitude of current density $j(t)$ at any point in the material? **(2016 Adv.)**



6. The figures below depict two situations in which two infinitely long static line charges of constant positive line charge density λ are kept parallel to each other

In their resulting electric field, point charges q and $-q$ are kept in equilibrium between them. The point charges are confined to move in the x direction only. If they are given a small displacement about their equilibrium positions, then the correct statements is/are



(2015 Adv.)

(a) both charges execute simple harmonic motion.

(b) both charges will continue moving in the direction of their displacement.

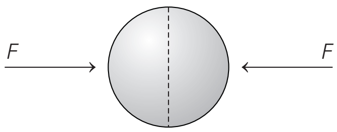
(c) charge $+q$ executes simple harmonic motion while charge $-q$ continues moving in the direction of its displacement.

(d) charge $-q$ executes simple harmonic motion while charge $+q$ continues moving in the direction of its displacement.

7. Two large vertical and parallel metal plates having a separation of 1 cm are connected to a DC voltage source of potential difference X. A proton is released at rest midway between the two plates. It is found to move at 45° to the vertical just after release. Then X is nearly **(2012)**

(a) 1×10^{-5} V (b) 1×10^{-7} V
(c) 1×10^{-9} V (d) 1×10^{-10} V

8. A uniformly charged thin spherical shell of radius R carries uniform surface charge density of σ per unit area. It is made of two hemispherical shells, held together by pressing them with force F (see figure). F is proportional to **(2010)**

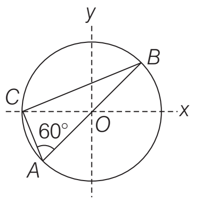


(a) $\frac{1}{\varepsilon_0}\sigma^2R^2$ (b) $\frac{1}{\varepsilon_0}\sigma^2R$
(c) $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R}$ (d) $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R^2}$

9. A tiny spherical oil drop carrying a net charge q is balanced in still air with a vertical uniform electric field of strength $\frac{81\pi}{7}\times10^5\ \text{Vm}^{-1}$. When the field is switched off, the drop is observed to fall with terminal velocity $2\times10^{-3}\ \text{ms}^{-1}$. Given $g = 9.8\,\text{ms}^{-2}$, viscosity of the air $= 1.8\times10^{-5}\ \text{Ns m}^{-2}$ and the density of oil $= 900\,\text{kg m}^{-3}$, the magnitude of q is **(2010)**

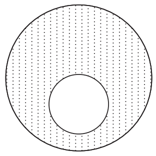
(a) 1.6×10^{-19} C (b) 3.2×10^{-19} C
(c) 4.8×10^{-19} C (d) 8.0×10^{-19} C

10. Consider a system of three charges $q/3, q/3$ and $-2q/3$ placed at points A, B and C respectively, as shown in the figure. Take O to be the centre of the circle of radius R and angle $CAB = 60°$ **(2008, 3M)**



(a) The electric field at point O is $\frac{q}{8\pi\varepsilon_0R^2}$ directed along the negative x-axis.
(b) The potential energy of the system is zero
(c) The magnitude of the force between the charges at C and B is $\frac{q^2}{54\pi\varepsilon_0R^2}$.
(d) The potential at point O is $\frac{q}{12\pi\varepsilon_0R}$.

11. A spherical portion has been removed from a solid sphere having a charge distributed uniformly in its volume as shown in the figure. The electric field inside the emptied space is **(2007, 3M)**



(a) zero everywhere (b) non-zero and uniform
(c) non-uniform (d) zero only at its centre

12. A long, hollow conducting cylinder is kept co-axially inside another long, hollow conducting cylinder of larger radius. Both the cylinders are initially electrically neutral. **(2007, 3M)**

(a) A potential difference appears between the two cylinders when a charge density is given to the inner cylinder.
(b) A potential difference appears between the two cylinders when a charge density is given to the outer cylinder.
(c) No potential difference appears between the two cylinders when a uniform line charge is kept along the axis of the cylinders.
(d) No potential difference appears between the two cylinders when same charge density is given to both the cylinders.

13. Two equal point charges are fixed at $x = -a$ and $x = +a$ on the x-axis. Another point charge Q is placed at the origin. The change in the electrical potential energy of Q, when it is displaced by a small distance x along the x-axis, is approximately proportional to **(2002, 1M)**

(a) x (b) x^2 (c) x^3 (d) $1/x$

14. Two point charges $+q$ and $-q$ are held fixed at $(-d, 0)$ and $(d, 0)$ respectively of a x-y co-ordinate system. Then **(1995, 2M)**

(a) the electric field E at all points on the x-axis has the same direction.
(b) work has to be done in bringing a test charge from ∞ to the origin.
(c) electric field at all point on y-axis is along x-axis.
(d) the dipole moment is $2qd$ along the x-axis.

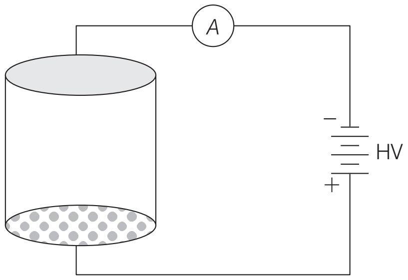
15. Two identical thin rings, each of radius R, are coaxially placed a distance R apart. If Q_1 and Q_2 are respectively the charges uniformly spread on the two rings, the work done in moving a charge q from the centre of one ring to that of the other is **(1992, 2M)**

(a) zero
(b) $\frac{q(Q_1-Q_2)(\sqrt{2}-1)}{\sqrt{2}\,(4\pi\varepsilon_0 R)}$
(c) $\frac{q\sqrt{2}(Q_1+Q_2)}{(4\pi\varepsilon_0 R)}$
(d) $q(Q_1/Q_2)(\sqrt{2}+1)\sqrt{2}\,(4\pi\varepsilon_0 R)$

Passage Based Questions

Passage

Consider an evacuated cylindrical chamber of height h having rigid conducting plates at the ends and an insulating curved surface as shown in the figure. A number of spherical balls made of a light weight and soft material and coated with a conducting material are placed on the bottom plate. The balls have a radius $r << h$. Now, a high voltage source (HV) connected across the conducting plates such that the bottom plate is at $+V_0$ and the top plate at $-V_0$. Due to their conducting surface, the balls will get charge, will become equipotential with the plate and are repelled by it. The balls will eventually collide with the top plate, where the coefficient of restitution can be taken to be zero due to te soft nature of the material of the balls. The electric field in the chamber can be considered to be that of a parallel plate capacitor. Assume that there are no collisions between the balls and the interaction between them is negligible. (Ignore gravity)



16. Which one of the following statement is correct? **(2016 Adv.)**
(a) The balls will execute simple harmonic motion between the two plates.
(b) The balls will bounce back to the bottom plate carrying the same charge they went up with.
(c) The balls will stick to the top plate and remain there.
(d) The balls will bounce back to the bottom plate carrying the opposite charge they went up with.

17. The average current in the steady state registered by the ammeter in the circuit will be **(2016 Adv.)**
(a) proportional to V_0^2.
(b) proportional to the potential V_0.
(c) zero
(d) proportions to $V_0^{1/2}$

Match the Column

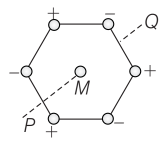
18. The electric field E is measured at a point $P\,(0, 0, d)$ generated due to various charge distributions and the dependence of E on d is found to be different for different charge distributions. List-I contains different relations between E and d. List-II describes different electric charge distributions, along with their locations. Match the functions in List-I with the related charge distributions in List-II.

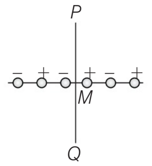
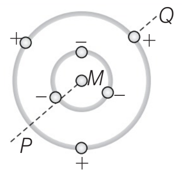
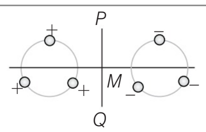
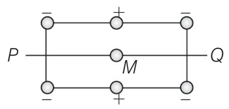
	List-I		List-II
P.	E is independent of d	1.	A point charge Q at the origin
Q.	$E \propto \frac{1}{d}$	2.	A small dipole with point charges Q at $(0, 0, l)$ and $-Q$ at $(0, 0, -1)$. (Take, $2l << d$)
R.	$E \propto \frac{1}{d^2}$	3.	An infinite line charge coincident with the X-axis, with uniform linear charge density λ.
S.	$E \propto \frac{1}{d^3}$	4.	Two infinite wires carrying a uniform linear charge density parallel to the X- axis. The one along $(y = 0, z = l)$ has a charge density $+\lambda$ and the one along $(y = 0, z = -l)$ has a charge density $-\lambda$. (Take, $2l << d$).
		5.	Infinite plane charge coincident with the xy-plane with uniform surface charge density.

(2018 Adv.)

(a) P → 5; Q → 3,4; R → 1; S → 2
(b) P → 5; Q → 3; R → 1, 4; S → 2
(c) P → 5; Q → 3; R → 1, 2; S → 4
(d) P → 4; Q → 2, 3; R → 1; S → 5

19. Six point charges, each of the same magnitude q, are arranged in different manners as shown in **Column II**. In each case, a point M and a line PQ passing through M are shown. Let E be the electric field and V be the electric potential at M (potential at infinity is zero) due to the given charge distribution when it is at rest. Now, the whole system is set into rotation with a constant angular velocity about the line PQ. Let B be the magnetic field at M and μ be the magnetic moment of the system in this condition. Assume each rotating charge to be equivalent to a steady current. **(2009)**

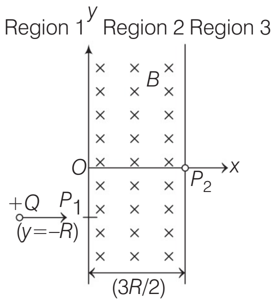
Column I		Column II
(A) $E = 0$	(p)	 Charges are at the corners of a regular hexagon. M is at the centre of the hexagon. PQ is perpendicular to the plane of the hexagon.

Column I	Column II
(B) $V \neq 0$	(q)  Charges are on a line perpendicular to PQ at equal intervals. M is the mid-point between the two innermost charges.
(C) $B = 0$	(r)  Charges are placed on two coplanar insulating rings at equal intervals. M is the common centre of the rings. PQ is perpendicular to the plane of the rings.
(D) $\mu \neq 0$	(s)  Charges are placed at the corners of a rectangle of sides a and $2a$ and at the mid points of the longer sides. M is at the centre of the rectangle. PQ is parallel to the longer sides.
	(t)  Charges are placed on two coplanar, identical insulating rings at equal intervals. M is the mid points between the centres of the rings. PQ is perpendicular to the line joining the centres and coplanar to the rings.

Objective Questions II (One or more correct option)

20. A uniform magnetic field B exists in the region between $x = 0$ and $x = \frac{3R}{2}$ (region 2 in the figure) pointing normally into the plane of the paper. A particle with charge $+Q$ and momentum p directed along X-axis enters region 2 from region 1 at point P_1 $(y = -R)$. **(2017 Main)**

Which of the following option(s) is/are correct?



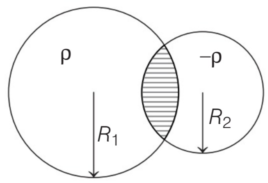
(a) When the particle re-enters region 1 through the longest possible path in region 2, the magnitude of the change in its linear momentum between point P_1 and the farthest point from Y-axis is $\frac{p}{\sqrt{2}}$.

(b) For $B = \frac{8}{13}\frac{p}{QR}$, the particle will enter region 3 through the point P_2 on X-axis.

(c) For $B > \frac{2}{3}\frac{p}{QR}$, the particle will re-enter region 1.

(d) For a fixed B, particles of same charge Q and same velocity v, the distance between the point P_1 and the point of re-entry into region 1 is inversely proportional to the mass of the particle.

21. Two non-conducting spheres of radii R_1 and R_2 and carrying uniform volume charge densities $+\rho$ and $-\rho$, respectively, are placed such that they partially overlap, as shown in the figure. At all points in the overlapping region **(2013 Adv.)**



(a) the electrostatic field is zero.
(b) the electrostatic potential is constant.
(c) the electrostatic field is constant in magnitude.
(d) the electrostatic field has same direction.

22. Two non-conducting solid spheres of radii R and $2R$, having uniform volume charge densities ρ_1 and ρ_2 respectively, touch each other. The net electric field at a distance $2R$ from the centre of the smaller sphere, along the line joining the centre of the spheres, is zero. The ratio ρ_1 / ρ_2 can be **(2013 Adv.)**

(a) -4 (b) $\frac{-32}{25}$ (c) $\frac{32}{25}$ (d) 4

23. Under the influence of the coulomb field of charge $+Q$, a charge $-q$ is moving around it in an elliptical orbit. Find out the correct statement(s). **(2009)**

(a) The angular momentum of the charge $-q$ is constant
(b) The linear momentum of the charge $-q$ is constant
(c) The angular velocity of the charge $-q$ is constant
(d) The linear speed of the charge $-q$ is constant

24. A positively charged thin metal ring of radius R is fixed in the x-y plane with its centre at the origin O. A negatively charged particle P is released from rest at the point $(0,0,z_0)$ where $z_0 > 0$. Then the motion of P is **(1998, 2M)**

(a) periodic for all values of z_0 satisfying $0 < z_0 < \infty$

(b) simple harmonic for all values of z_0 satisfying $0 < z_0 \le R$

(c) approximately simple harmonic provided $z_0 << R$

(d) such that P crosses O and continues to move along the negative z-axis towards $z = -\infty$

Integer Answer Type Questions

25. Four point charges, each of $+q$, are rigidly fixed at the four corners of a square planar soap film of side a. The surface tension of the soap film is γ. The system of charges and planar film are in equilibrium, and $a = k\left[\dfrac{q^2}{\gamma}\right]^{1/N}$, where k is a constant. Then N is **(2011)**

True/False

26. A small metal ball is suspended in a uniform electric field with the help of an insulated thread. If high energy X-ray beam falls on the ball, the ball will be deflected in the direction of the field. **(1983, 2M)**

27. Two identical metallic spheres of exactly equal masses are taken. One is given a positive charge Q coulomb and the other an equal negative charge. Their masses after charging are different. **(1983, 2M)**

Analytical & Descriptive Questions

28. A conducting bubble of radius a, thickness t $(t << a)$ has potential V. Now the bubble collapses into a droplet. Find the potential of the droplet. **(2005, 2M)**

29. There are two large parallel metallic plates S_1 and S_2 carrying surface charge densities σ_1 and σ_2 respectively $(\sigma_1 > \sigma_2)$ placed at a distance d apart in vacuum. Find the work done by the electric field in moving a point charge q a distance a $(a < d)$ from S_1 towards S_2 along a line making an angle $\pi/4$ with the normal to the plates. **(2004, 2M)**

30. A small ball of mass 2×10^{-3} kg having a charge of $1\,\mu$C is suspended by a string of length 0.8 m. Another identical ball having the same charge is kept at the point of suspension. Determine the minimum horizontal velocity which should be imparted to the lower ball, so that it can make complete revolution. **(2001, 5M)**

31. A non-conducting disc of radius a and uniform positive surface charge density σ is placed on the ground with its axis vertical. A particle of mass m and positive charge q is dropped, along the axis of the disc from a height H with zero initial velocity. The particle has $q/m = 4\varepsilon_0\, g/\sigma$. **(1999, 10M)**

(a) Find the value of H if the particle just reaches the disc.

(b) Sketch the potential energy of the particle as a function of its height and find its equilibrium position.

32. A conducting sphere S_1 of radius r is attached to an insulating handle. Another conducting sphere S_2 of radius R is mounted on an insulating stand. S_2 is initially uncharged.

S_1 is given a charge Q, brought into contact with S_2 and removed. S_1 is recharged such that the charge on it is again Q and it is again brought into contact with S_2 and removed. This procedure is repeated n times. **(1998, 8M)**

(a) Find the electrostatic energy of S_2 after n such contacts with S_1.

(b) What is the limiting value of this energy as $n \to \infty$?

33. Two isolated metallic solid spheres of radii R and $2R$ are charged such that both of these have same charge density σ. The spheres are located far away from each other and connected by a thin conducting wire. Find the new charge density on the bigger sphere. **(1996, 3M)**

34. A circular ring of radius R with uniform positive charge density λ per unit length is located in the y-z plane with its centre at the origin O. A particle of mass m and positive charge q is projected from the point P $(R\sqrt{3}, 0, 0)$ on the positive x-axis directly towards O, with an initial speed v. Find the smallest (non-zero) value of the speed v such that the particle does not return to P. **(1993, 4M)**

35. Assume the earth to be a sphere of uniform mass density. Calculate this energy, given the product of the mass and the radius of the earth to be 2.5×10^{31} kg-m.

(a) If the same charge of Q as in part (a) above is given to a spherical conductor of the same radius R, what will be the energy of the system? **(1992, 10M)**

(b) A charge of Q is uniformly distributed over a spherical volume of radius R. Obtain an expression for the energy of the system.

(c) What will be the corresponding expression for the energy needed to completely disassemble the planet earth against the gravitational pull amongst its constituent particles?

Answers

Topic 1

1. (a) **2.** (d) **3.** (a)
4. (d) **5.** (d) **6.** (c) **7.** (a)
8. (d) **9.** (b) **10.** (b) **11.** (d)
12. (a) **13.** (b) **14.** (c) **15.** (a)
16. (c) **17.** (a, c) **18.** 6 **19.** 2
20. $9 \times 10^9 \dfrac{q^2}{L^2}$ **21.** $180°$, $\dfrac{Q^2}{16\pi\varepsilon_0 L^2}$ **22.** F
23. 3.17×10^{-9} C **24.** $T = 8.79 \times 10^{-4}$ N and $\theta = 27°$

25. (a) (i) $\alpha = 60°$ (ii) $T = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{q_1q_2}{l^2} + mg$

(iii) $N_P = \sqrt{3}mg, N_Q = mg$ (b) $q_1q_2 = -(-4\pi\varepsilon_0)mgl^2$

Topic 2

1. (d) **2.** (d) **3.** (d) **4.** (c)
5. (d) **6.** (d) **7.** (b) **8.** (d) **9.** (c)
10. (d) **11.** (d) **12.** (c) **13.** (b)
14. (b) **15.** (d) **16.** (a) **17.** (c)
18. (b) **19.** (b, c) **20.** (a, b, c)
21. (a, b, c, d) **22.** $-8\hat{\mathbf{i}}$ **23.** $-qEa$ **24.** B
25. F **26.** $W = 5.824\left(\frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}\right)$

27. $(v_0)_{\min} = 3$ m/s, $K = 3 \times 10^{-4}$ J

28. (a) Radius $= 4a$, Centre $= (5a, 0)$

(b) $V_x = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{3a - x} - \frac{2}{3a + x}\right)$ for $x \le 3a$,

$V_x = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{x - 3a} - \frac{2}{3a + x}\right)$ for $x > 3a$

(c) $v = \sqrt{\frac{Qq}{8\pi\varepsilon_0 ma}}$

29. (a) Charge q should be at a distance of 3 cm from $2q$ (b) Electric field = 0

30. Maximum distance from $O = 8.48$ m

Topic 3

1. (b) **2.** (d) **3.** (b) **4.** (c)
5. (a) **6.** (d) **7.** (c) **8.** (b)
9. (a) **10.** (c) **11.** (a) **12.** (b)
13. (a,b) **14.** (a, d) **15.** (a, c) **16.** (c, d)
17. (a, b, c, d) **18.** (6)

19. (a) $V_A = \frac{\sigma}{\varepsilon_0}(a - b + c), V_B = \frac{\sigma}{\varepsilon_0}\left(\frac{a^2}{b} - b + c\right)$,

$V_C = \frac{\sigma}{\varepsilon_0}\left(\frac{a^2}{c} - \frac{b^2}{c} + c\right)$ (b) $a + b = c$ **20.** $\frac{Q(R + r)}{4\pi\varepsilon_0(R^2 + r^2)}$

Topic 4

1. (b) **2.** (c) **3.** (d) **4.** (d)
5. (a) **6.** (a) **7.** (b) **8.** (d)
9. (d) **10.** (c) **11.** (c) **12.** (d)
13. (a, d) **14.** (c, d) **15.** F
16. (a) $\text{KE} = \frac{qp}{4\pi\varepsilon_0 d^2}$ (b) $\mathbf{F} = \frac{pq}{2\pi\varepsilon_0 d^3}\hat{\mathbf{i}}$ **17.** No

Topic 5

1. (d) **2.** (d) **3.** (a) **4.** (d)
5. (a) **6.** (d) **7.** (a) **8.** (b)
9. (d) **10.** (b) **11.** (c) **12.** (d)
13. (b) **14.** (b) **15.** (c) **16.** (*)
17. (b) **18.** (b)
19. (c) **20.** (c) **21.** (d) **22.** (a)
23. (c) **24.** (a) **25.** (d) **26.** (a)
27. (b) **28.** (c) **29.** (a) **30.** (d)
31. (d) **32.** (d) **33.** (d) **34.** (c)
35. (b) **36.** (a) **37.** (a, d) **38.** (b, d)
39. (b, c) **40.** (b, c) **41.** (a, c, d) **42.** (b, d)
43. (a, d) **44.** (1.50) **45.** $\left(\frac{3}{K + 2}\right)V$

46. $\frac{\varepsilon_0 AV}{d}, -\frac{2\varepsilon_0 AV}{d}$ **47.** T

48. (a) $q_1 = 90\,\mu\text{C}, q_2 = 210\,\mu\text{C}, q_3 = 150\,\mu\text{C}$ (b) (i) $U_i = 47.4$ mJ (ii) $U_f = 18$ mJ

49. $C_R = \frac{CK_1K_2}{K_2 - K_1}\ln\frac{K_2}{K_1}$ where, $C = \frac{\varepsilon_0 A}{d}$

50. $i = 4.43 \times 10^{-9}$ A

51. (a) $C_A = 2 \times 10^{-9}\,\text{F}, U_A = 1.21 \times 10^{-5}\,\text{J}$ (b) $W = 4.84 \times 10^{-5}$ J (c) $U = 1.1 \times 10^{-5}$ J **52.** 3/5

Topic 6

1. (c) **2.** (b) **3.** (a) **4.** (b)
5. (c) **6.** (b, d) **7.** 2

8. $Q_0 = \frac{CVR_2}{R_1 + R_2}, \alpha = \frac{R_1 + R_2}{CR_1R_2}$ **9.** (a) $Q = \frac{CV}{2}(1 - e^{-2t/3RC})$

(b) $i_2 = \frac{V}{2R} - \frac{V}{6R}e^{-2t/3RC}, \frac{V}{2R}$ **10.** 0.198 μA

11. 1.5 A from right to left, 1.44×10^{-5} J

12. 0.288 mJ **13.** 0.9 A **14.** $V_{AB} = 25$ V, $V_{BC} = 75$ V

Topic 7

1. (c) **2.** (a) **3.** (c)
4. (b) **5.** (d) **6.** (c) **7.** (c)
8. (a) **9.** (d) **10.** (c) **11.** (b)
12. (a) **13.** (b) **14.** (c) **15.** (b)
16. (d) **17.** (a) **18.** (b)
19. A→ p, r, s; B→ r, s ; C→ p, q, t; D→ r, s **20.** (b, c)
21. (c, d) **22.** (b, d) **23.** (a) **24.** (a, c)
25. 3 **26.** T **27.** T **28.** $V' = V\left(\frac{a}{3t}\right)^{1/3}$

29. $W = \frac{(\sigma_1 - \sigma_2)qa}{\sqrt{2}\varepsilon_0}$ **30.** 5.86 m/s

31. (a) $H = \frac{4}{3}a$ (b) $H = \frac{a}{\sqrt{3}}$

32. (a) $U_n = \frac{q_n^2}{8\pi\varepsilon_0 R}$

(b) $U_\infty = \frac{Q^2R}{8\pi\varepsilon_0 r^2}$ Here, $q_n = \frac{QR}{r}\left[1 - \left(\frac{R}{R + r}\right)^n\right]$

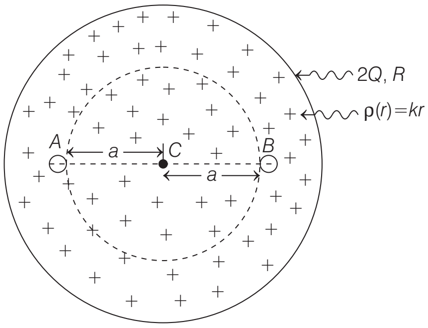
33. $\frac{5}{6}\sigma$ **34.** $v_{\min} = \sqrt{\frac{q\lambda}{2\varepsilon_0 m}}$ **35.** (a) $U = \frac{3}{20}\frac{Q^2}{\pi\varepsilon_0 R}$

(b) $U = -\frac{3}{5}\frac{GM^2}{R}, E = 1.5 \times 10^{32}$ J (c) $U = \frac{Q^2}{8\pi\varepsilon_0 R}$

Hints & Solutions

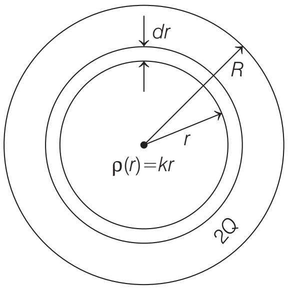
Topic 1 Electrostatics Force and Field Strength

1.



Key Idea Force on A is zero only when repulsion of A and B = attraction of positive charge distribution of radius a and charge A.

In given charge distribution, let r is radius of a shell of thickness dr.



Charge dQ present in shell of thickness dr = Volume of shell × Volumetric charge density

$$\Rightarrow \quad dQ = (4\pi r^2 \times dr) \times (kr)$$
$$= 4\pi k r^3 \, dr$$

Total charge in sphere is

$$2Q = \int_0^R dQ = \int_0^R 4\pi k r^3 \, dr$$

$$\Rightarrow \quad 2Q = 4\pi k \left[\frac{r^4}{4}\right]_0^R \Rightarrow k = \frac{2Q}{\pi R^4}$$

Now, using Gauss' law, electric field on the surface of sphere of radius a is

$$E \int_0^a dA = \frac{1}{\varepsilon_0} \cdot \int_0^a (kr \, 4\pi r^2 \, dr)$$

$$\Rightarrow \quad E \cdot 4\pi a^2 = \frac{1}{\varepsilon_0} \cdot 4\pi k \left(\frac{a^4}{4}\right)$$

$$\Rightarrow \quad E = \frac{ka^2}{4\varepsilon_0} = \frac{2Q\,a^2}{4\pi\varepsilon_0 R^4}$$

Force of attraction on charge A (or B) due to this field is

$$F_1 = QE = \frac{2Q^2 a^2}{4\pi\varepsilon_0 R^4}$$

Force of repulsion on charge A due to B is

$$F_2 = \frac{1}{4\pi\varepsilon_0} \frac{Q^2}{(2a)^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q^2}{4a^2}$$

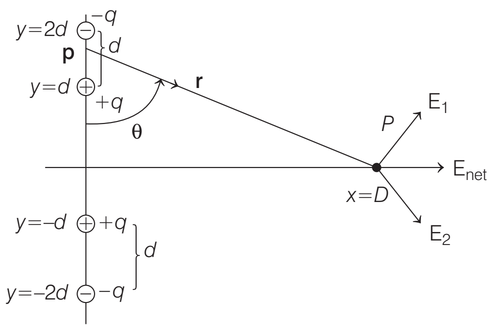
If charge A (or B) does not feel any force, then

$$F_1 = F_2$$

$$\Rightarrow \quad \frac{2Q^2 a^2}{4\pi\varepsilon_0 R^4} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q^2}{4a^2}$$

$$\Rightarrow \quad 8a^4 = R^4 \quad \Rightarrow a = 8^{-1/4} R$$

2. Given charge distribution is as shown below



So, we can view above point charges as combination of pair of dipoles or a quadrupole.

By symmetry, the field components parallel to quadrupole cancels and the resultant perpendicular field is

$$E = \frac{2q}{4\pi\varepsilon_0}\left(\frac{1}{D^2} - \frac{D}{(D^2 + d^2)^{3/2}}\right)$$

$$= \frac{2q}{4\pi\varepsilon_0 D^2}\left(1 - \left(1 + \frac{d^2}{D^2}\right)^{-3/2}\right)$$

As, $\left(1 + \frac{d^2}{D^2}\right)^{-\frac{3}{2}} \approx \left(1 - \frac{3}{2}\frac{d^2}{D^2}\right)$

(using binomial expansion)

We have, $\quad E = \dfrac{3qd^2}{4\pi\varepsilon_0 D^4}$

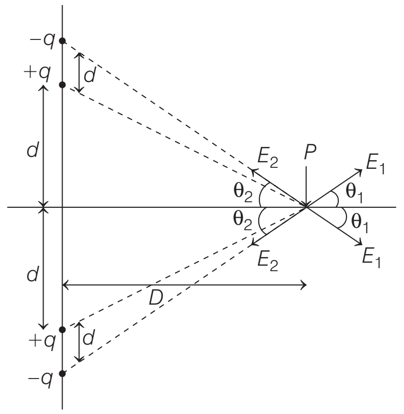
$$\therefore \quad E \propto \frac{1}{D^4}$$

NOTE Dependence of field for a point charge is

$$E \propto \frac{1}{r^2}$$

For a dipole, it is $E \propto \dfrac{1}{r^3}$

For a quadrupole, it is $E \propto \dfrac{1}{r^4}$ ..... etc.



Alternate Solution The given distribution of charges can be shown as the figure below

Electric field at point P,

$$E = E_1 \cos\theta_1 + E_1 \cos\theta_1 - E_2 \cos\theta_2 - E_2 \cos\theta_2$$

$$= 2E_1 \cos\theta_1 - 2E_2 \cos\theta_2$$

$$= \frac{2kq}{(d^2 + D^2)} \cos\theta_1 - \frac{2kq}{(2d)^2 + D^2} \cos\theta_2$$

As, $\cos\theta_1 = \dfrac{D}{(d^2 + D^2)^{1/2}}$

Similarly, $\cos\theta_2 = \dfrac{D}{[(2d)^2 + D^2]^{1/2}}$

$$= 2kqD[(d^2 + D^2]^{-3/2} - (4d^2 + D^2)^{-3/2}]$$

$$= \frac{2kqD}{D^3}\left[\left(1 + \frac{d^2}{D^2}\right)^{-3/2} - \left(1 + \frac{4d^2}{D^2}\right)^{-3/2}\right]$$

As $D >> d$, then by applying binomial approximation, we get

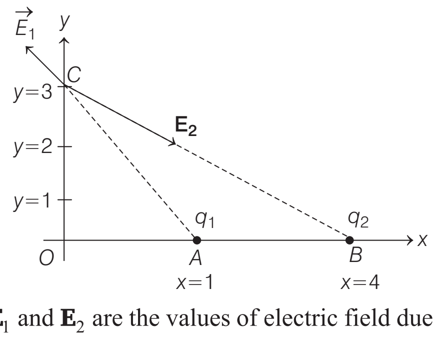
$$= \frac{2kq}{D^2}\left[1 - \frac{3}{2}\frac{d^2}{D^2} - \left(1 - \frac{3}{2}\left(\frac{4d^2}{D^2}\right)\right)\right]$$

$$= \frac{2kq}{D^2}\left[\frac{9d^2}{2D^2}\right] = \frac{9kqd^2}{D^4}$$

$$\Rightarrow \qquad E \propto \frac{1}{D^4}$$

3. Here, $q_1 = \sqrt{10}\,\mu\text{C} = \sqrt{10} \times 10^{-6}$ C

$q_2 = -25\,\mu\text{C} = -25 \times 10^{-6}$ C



Let $\mathbf{E}_1$ and $\mathbf{E}_2$ are the values of electric field due to q_1 and q_2 respectively.

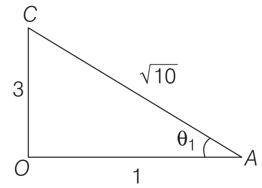
Here, $E_1 = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{q_1}{AC^2} = \dfrac{1}{4\pi\varepsilon_0} \times \dfrac{\sqrt{10} \times 10^{-6}}{(1^2 + 3^2)}$

$$= 9 \times 10^9 \times \sqrt{10} \times 10^{-7}$$

$$= 9\sqrt{10} \times 10^2$$

$\therefore \quad \mathbf{E}_1 = 9\sqrt{10} \times 10^2[\cos\theta_1(-\hat{\mathbf{i}}) + \sin\theta_1 \hat{\mathbf{j}}]$

From ΔOAC,



$\sin\theta_1 = \dfrac{3}{\sqrt{10}}$ and $\cos\theta_1 = \dfrac{1}{\sqrt{10}}$

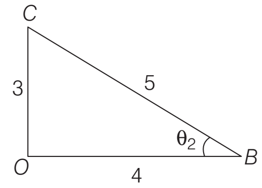
$\therefore \quad E_1 = 9\sqrt{10} \times 10^2\left[\dfrac{1}{\sqrt{10}}(-\hat{\mathbf{i}}) + \dfrac{3}{\sqrt{10}}\hat{\mathbf{j}}\right]$

$$= 9 \times 10^2[-\hat{\mathbf{i}} + 3\hat{\mathbf{j}}]$$

$$= (-9\hat{\mathbf{i}} + 27\hat{\mathbf{j}}) \times 10^2 \text{ V/m}$$

and $E_2 = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{-25 \times 10^{-6}}{(4^2 + 3^2)} = 9 \times 10^3$ V/m

From ΔOBC,



$\sin\theta_2 = \dfrac{3}{5}$

$\cos\theta_2 = \dfrac{4}{5}$

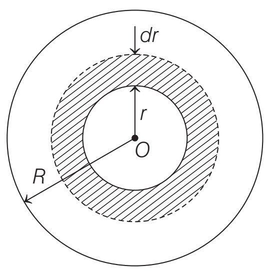
$\therefore \quad \mathbf{E}_2 = 9 \times 10^3[\cos\theta_2 \hat{\mathbf{i}} - \sin\theta_2 \hat{\mathbf{j}}]$

$\mathbf{E}_2 = 9 \times 10^3\left[\dfrac{4}{5}\hat{\mathbf{i}} - \dfrac{3}{5}\hat{\mathbf{j}}\right] = (72\hat{\mathbf{i}} - 54\hat{\mathbf{j}}) \times 10^2$

$\therefore \quad \mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 = (63\hat{\mathbf{i}} - 27\hat{\mathbf{j}}) \times 10^2$ V/m

4. Here, volume charge density,

$$\rho(r) = \frac{A}{r^2} \cdot e^{-\frac{2r}{a}}$$



where, a and A are constant.

Let a spherical region of small element of radius r.
If Q is total charge distribution upto radius R, then

$$Q = \int_0^R \rho \cdot dV = \int_0^R \frac{A}{r^2} e^{-2r/a} (4\pi r^2 \, dr)$$

(From figure, we observe $dV = A \cdot dr = 4\pi r^2 \cdot dr$)

$$= 4\pi A \int_0^R e^{-2r/a} dr = 4\pi A \left(\frac{e^{-2r/a}}{-2/a} \right)_0^R$$

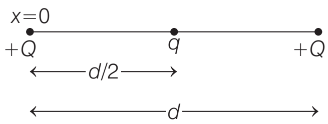
$$= 4\pi A \times \left(\frac{-a}{2} \right) (e^{-2R/a} - e^0)$$

$$= 2\pi A(-a)[e^{-2R/a} - 1]$$

or $\quad Q = 2\pi a \, A \, (1 - e^{-2R/a})$

or $\quad R = \frac{a}{2} \log \left(\frac{1}{1 - \frac{Q}{2\pi a A}} \right)$

5. The given condition is shown in the figure given below,



Then, according to the Coulomb's law, the electrostatic force between two charges q_1 and q_2 such that the distance between them is (r) given as,

$$F = \frac{1 \cdot q_1 q_2}{4\pi\varepsilon_0 \cdot r^2}$$

∴ Net force on charge 'Q' placed at origin i.e. at $x = 0$ in accordance with the principle of superposition can be given as

$$F_{\text{net}} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q \times q}{\left(\frac{d}{2}\right)^2} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q \times Q}{(d)^2}$$

Since, it has been given that, $F_{\text{net}} = 0$.

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q \times q}{\left(\frac{d}{2}\right)^2} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q \times Q}{(d)^2} = 0$$

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q \times q}{\left(\frac{d}{2}\right)^2} = -\frac{1}{4\pi\varepsilon_0} \cdot \frac{Q \times Q}{(d)^2} \text{ or } q = -\frac{Q}{4}$$

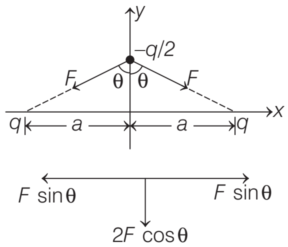
6. $E_1 = \frac{kQ}{R^2}$, where $k = \frac{1}{4\pi\varepsilon_0}$

$$\Rightarrow \quad mE_2 = \frac{k(2Q)}{R^2}$$

$$\Rightarrow \quad E_2 = \frac{2kQ}{R^2} \quad \Rightarrow E_3 = \frac{k(4Q)R}{(2R)^3}$$

$$\Rightarrow \quad E_3 = \frac{kQ}{2R^2}$$

7.



$$F_{\text{net}} = 2F\cos\theta$$

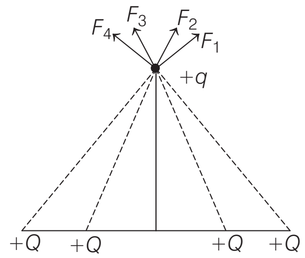
$$\Rightarrow \quad F_{\text{net}} = \frac{2kq\left(\frac{q}{2}\right)}{(\sqrt{y^2 + a^2})^2} \cdot \frac{y}{\sqrt{y^2 + a^2}}$$

$$F_{\text{net}} = \frac{2kq\left(\frac{q}{2}\right) y}{(y^2 + a^2)^{3/2}} \Rightarrow \frac{kq^2 y}{a^3} \propto y$$

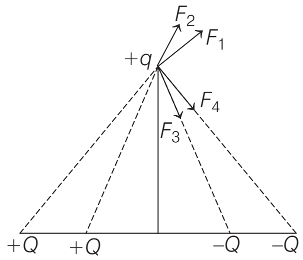
8. According to option (d) the electric field due to P and S and due to Q and T add to zero. While due to U and R will be added up. Hence, the correct option is (d).

(P) Component of forces along x-axis will vanish. Net force along positive y-axis

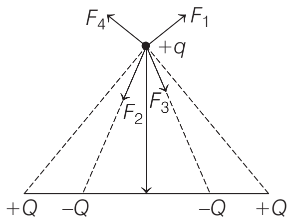
$$E_3 < E_1 < E_2$$



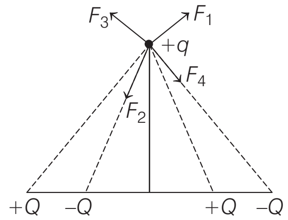
(Q) Component of forces along y-axis will vanish. Net force along positive x-axis



(R) Component of forces along x-axis will vanish. Net force along negative y-axis



(S) Component of forces along y-axis will vanish. Net force along negative x-axis



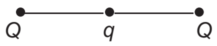
9. Electrostatic force, $F_e = eE$ (for both the particles)

But acceleration of electron, $a_e = F_e / m_e$ and

acceleration of proton, $a_p = F_e / m_p$

$$S = \frac{1}{2} a_e t_1^2 = \frac{1}{2} a_p t_2^2 \quad \Rightarrow \quad \therefore \quad \frac{t_2}{t_1} = \sqrt{\frac{a_e}{a_p}} = \sqrt{\frac{m_p}{m_e}}$$

10. Since, q is at the centre of two charges Q and Q, net force on it is zero, whatever the magnitude and sign of charge



on it. For the equilibrium of Q, q should be negative because other charge Q will repel it, so q should attract it. Simultaneously these attractions and repulsions should be equal.

$$\frac{1}{4\pi\varepsilon_0} \frac{QQ}{r^2} = \frac{1}{4\pi\varepsilon_0} \frac{Qq}{(r/2)^2}$$

or $\quad q = \frac{Q}{4}$

or with sign $\quad q = -\frac{Q}{4}$

11. Motion is simple harmonic only if Q is released from a point not very far from the origin on x-axis. Otherwise motion is periodic but not simple harmonic.

12. At $r = R$, from Gauss's law

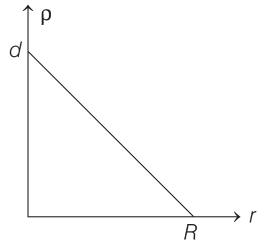
$$E\,(4\pi R^2) = \frac{q_{\text{net}}}{\varepsilon_0} = \frac{Ze}{\varepsilon_0} \quad \text{or} \quad E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Ze}{R^2}$$

E is independent of a.

$\therefore$ Correct option is (a).

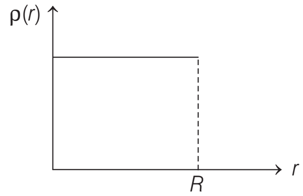
13. For $a = 0 \Rightarrow \rho(r) = \left(-\frac{d}{R} \cdot r + d\right)$

Now $\quad \int_0^R (4\pi r^2)\left(d - \frac{d}{R} r\right) dr =$ net charge $= Ze$.



Solving this equation, we get $d = \frac{3Ze}{\pi R^3}$

14. In case of solid sphere of charge of uniform volume density



$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R^3} \cdot r \qquad \text{or} \qquad E \propto r$$

Thus, for E to be linearly dependent on r, volume charge density should be constant.

or $\quad a = R$.

$\therefore$ Correct option is (c).

16. $\frac{Q}{4\pi\varepsilon_0 r_0^2} = \frac{\lambda}{2\pi\varepsilon_0 r_0} = \frac{\sigma}{2\varepsilon_0} \quad \Rightarrow \quad Q = 2\pi\sigma r_0^2$

(a) is incorrect, $\; r_0 = \frac{\lambda}{\pi\sigma}$

(b) is incorrect, $\; E_1\left(\frac{r_0}{2}\right) = 4E_1(r_0)$

As $\quad E_1 \propto \frac{1}{r^2}$

$$E_2\left(\frac{r_0}{2}\right) = 2E_2(r_0) \;\text{ as }\; E_2 \propto \frac{1}{r}$$

(c) is correct

$$E_3\left(\frac{r_0}{2}\right) = E_3(r_0) = E_2(r_0)$$

as $E_3 \propto r^0 \Rightarrow$ (d) option is incorrect

17. Inside the sphere $E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{R^3} r$

$\Rightarrow \quad E \propto r \;$ for $\; r \le R$

i.e. E at centre $= 0$ as $r = 0$ and E at surface $= \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{R^2}$

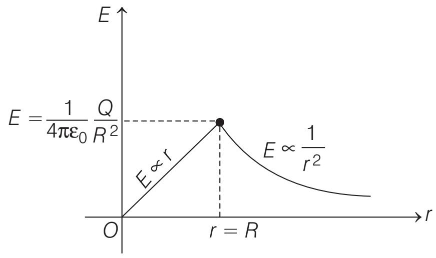
as $\quad r = R$

Outside the sphere

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2} \;\text{ for }\; r \ge R$$

or $\quad E \propto \frac{1}{r^2}$

Thus, variation of electric field (E) with distance (r) from the centre will be as shown



18. Volume of cylinder per unit length ($l = 1$) is

$$V = \pi R^2 l = (\pi R^2)$$

$\therefore$ Charge per unit length,

λ = (Volume per unit length) $\times$ (Volume charge density)

$= (\pi R^2 \rho)$

Now at *P*

$$E_R = E_T - E_C$$

R = Remaining portion

T = Total portion and

C = cavity

$$\therefore \quad E_R = \frac{\lambda}{2\pi\varepsilon_0 (2R)} - \frac{1}{4\pi\varepsilon_0}\frac{Q}{(2R)^2}$$

Q = charge on sphere

$$= \frac{4}{3}\pi\left(\frac{R}{2}\right)^3 \rho = \frac{\pi R^3 \rho}{6}$$

Substituting the values, we have

$$E_R = \frac{(\pi R^2 \rho)}{4\pi\varepsilon_0 R} - \frac{1}{4\pi\varepsilon_0} \cdot \frac{(\pi R^3\,\rho/6)}{4R^2}$$

$$= \frac{23\rho R}{96\varepsilon_0} = \frac{23\rho R}{(16)\,(6)\,\varepsilon_0}$$

$$\therefore \quad k = 6$$

19. From Gauss theorem,

$$E \propto \frac{q}{r^2} \qquad (q = \text{charge enclosed})$$

$$\therefore \quad \frac{E_2}{E_1} = \frac{q_2}{q_1} = \frac{r_1^2}{r_2^2}$$

$$\text{or} \quad 8 = \frac{\int_0^R (4\pi r^2) k r^a \, dr}{\int_0^{R/2} (4\pi r^2) k r^a \, dr} \times \frac{(R/2)^2}{(R)^2}$$

Solving this equation we get,

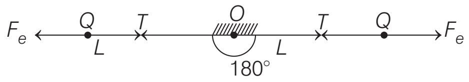
$$a = 2.$$

20. Force on $-q$ due to charges at 1 and 4 are equal and opposite. Similarly, forces on $-q$ due to charges at 2 and 5 are also equal and opposite. Therefore, net force on $-q$ due to charges at 1, 2, 4 and 5 is zero. Only unbalanced force is between $-q$ and $+q$ at 3 which is equal to

$$\frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2}{L^2}$$

$$\text{or} \quad 9.0 \times 10^9 \, \frac{q^2}{L^2} \text{ (attraction)}$$

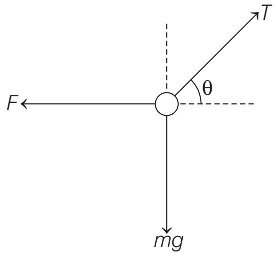
21. Due to electrostatic repulsion the charges will move as farthest as possible and the angle between the two strings will be 180° as shown in figure. Tension in each string will be equal to the electrostatic repulsion between the two charges. Thus,



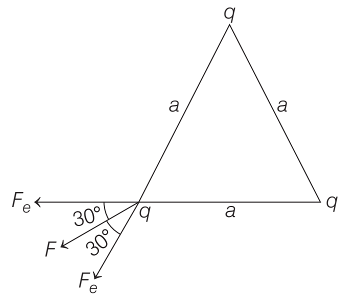
$$T = F_e = \frac{1}{4\pi\varepsilon_0}\frac{Q \times Q}{(2L)^2} = \frac{Q^2}{16\pi\varepsilon_0 L^2}$$

22. Motion is simple harmonic only when charge $-q$ is not very far from the centre of ring on its axis. Otherwise motion is periodic but not simple harmonic in nature.

23.



F is the resultant of electrostatic forces between two charges.



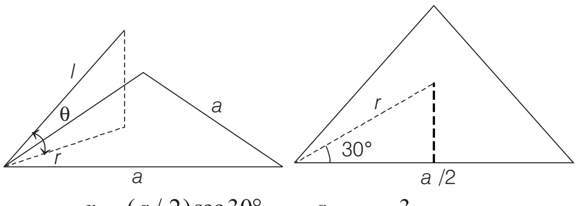
$$F = 2F_e \cos 30°$$

$$= 2\left[\frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2}{a^2}\right]\frac{\sqrt{3}}{2}$$

$$= \frac{2 \times 10^9 \times 9 \times q^2 \times \sqrt{3}}{(3 \times 10^{-2})^2 \times 2}$$

$$= \sqrt{3} \times 10^{13} q^2 \qquad \text{...(i)}$$

θ is the angle of string with horizontal in equilibrium,



$$\cos\theta = \frac{r}{l} = \frac{(a/2)\sec 30°}{l} = \frac{a}{\sqrt{3}l} = \frac{3}{100\sqrt{3}}$$

$$\therefore \quad \theta \approx 89°$$

Now, the particle is in equilibrium under three concurrent forces, F, T and mg. Therefore, applying Lami's theorem

$$\frac{F}{\sin\,(90° + \theta)} = \frac{mg}{\sin\,(180° - \theta)}$$

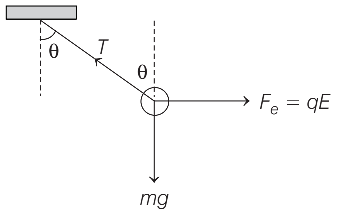
$$\text{or} \quad \sqrt{3} \times 10^{13} q^2 = (1 \times 10^{-3})(10) \cot\, 89°$$

Solving this equation, we get

$$q = 0.317 \times 10^{-8}\,C \quad \text{or} \quad q = 3.17 \times 10^{-9}\,C$$

24. For equilibrium of bob

$$T\cos\theta = mg \quad \text{and} \quad T\sin\theta = qE$$



From these two equations we find

$$T = \sqrt{(mg)^2 + (qE)^2} \text{ and } \theta = \tan^{-1}\left(\frac{qE}{mg}\right)$$

Substituting the values we have

$$T = \sqrt{(80 \times 10^{-6} \times 9.8)^2 + (2 \times 10^{-8} \times 20000)^2}$$

$$= 8.79 \times 10^{-4}\text{ N}$$

$$\theta = \tan^{-1}\left(\frac{2 \times 10^{-8} \times 20000}{80 \times 10^{-6} \times 9.8}\right) = 27^\circ$$

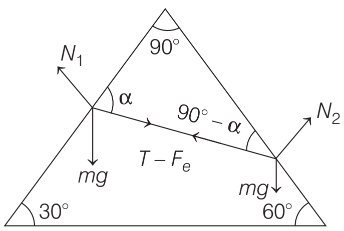
25. Tension and electrostatic force are in opposite direction and along the string. Now each bead is in equilibrium under three concurrent forces

(i) Normal reaction (N)

(ii) Weight (mg) and

(iii) $T - F_e$, where $F_e = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{l^2}$

Applying Lami's theorem for both beads.



$$\frac{N_1}{\sin(90^\circ - \alpha)} = \frac{mg}{\cos\alpha} = \frac{T - F_e}{\cos 60^\circ} \quad \text{...(i)}$$

$$\frac{N_2}{\sin(60^\circ + \alpha)} = \frac{mg}{\sin\alpha} = \frac{T - F_e}{\cos 30^\circ} \quad \text{...(ii)}$$

Dividing Eq. (i) by Eq. (ii), we have

$$\tan\alpha = \frac{\cos 30^\circ}{\cos 60^\circ} = \sqrt{3} \Rightarrow \alpha = 60^\circ$$

$$T = F_e + mg = \left(\frac{1}{4\pi\varepsilon_0}\right) \cdot \frac{q_1 q_2}{l^2} + mg \quad \text{...(iii)}$$

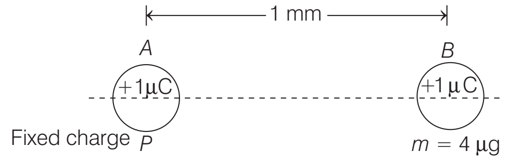
$$N_1 = \sqrt{3}\,mg \quad \text{and} \quad N_2 = mg$$

From Eq. (iii) $T = 0$ when string is cut.

or $\quad q_1 q_2 = -(4\pi\varepsilon_0)\,mgl^2$

Topic 2 Electrostatic Potential, Potential Energy, Work Done and Energy Conservation

1. Given situation is shown in the figure below,



When charged particle B is released due to mutual repulsion, it moves away from A. In this process, potential energy of system of charges reduces and this change of potential energy appears as kinetic energy of B.

Now, potential energy of system of charges at separation of 1 mm is

$$U_1 = \frac{Kq_1q_2}{r}$$

Here, $\quad q_1 = q_2 = 1 \times 10^{-6}$ C

$$r = 1\text{mm} = 1 \times 10^{-3}\text{ m}$$

$\therefore$ $$U_1 = \frac{9 \times 10^9 \times 1 \times 10^{-6} \times 1 \times 10^{-6}}{1 \times 10^{-3}} = 9\text{ J}$$

Potential energy of given system of charges at separation of 9 mm is

$$U_2 = \frac{Kq_1q_2}{r} = \frac{9 \times 10^9 \times (1 \times 10^{-6})^2}{9 \times 10^{-3}} = 1\text{ J}$$

By energy conservation,
Change in potential energy of system of A and B
= Kinetic energy of charged particle B

$$\Rightarrow \quad U_1 - U_2 = \frac{1}{2} m_B v_B^2$$

where, m_B = mass of particle B = 4 μg
$= 4 \times 10^{-6} \times 10^{-3}$ kg $= 4 \times 10^{-9}$ kg

and v_B = velocity of particle B at separation of 9 mm

$$\Rightarrow \quad 9 - 1 = \frac{1}{2} \times 4 \times 10^{-9} \times v_B^2$$

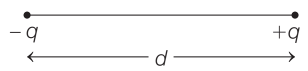
$$\Rightarrow \quad v_B^2 = 4 \times 10^9 \Rightarrow v_B = 2 \times 10^3\text{ ms}^{-1}$$

2. The system of two charges, i.e. $+q$ and $-q$ that are separated by distance d can be considered as a dipole. Thus, the charge Q would be at D distance from the centre of an electric dipole on its axial line.

So, the total potential energy of the system will be due to two components.

(1) Potential energy of dipole's own system

$$(\text{PE})_1 = \frac{Kq_1q_2}{d} = -\frac{Kq^2}{d} \quad \text{...(i)}$$



(2) Potential energy of charge Q and dipole system

$$(\text{PE})_2 = -\frac{KQq}{D^2}\cdot d \qquad \text{...(ii)}$$

Hence, total potential energy of the system

$$(\text{PE})_{\text{total}} = (\text{PE})_1 + (\text{PE})_2 = -\frac{Kq^2}{d} - \frac{KQq}{D^2}\cdot d$$

$$\Rightarrow \quad (\text{PE})_{\text{total}} = -\frac{1}{4\pi\varepsilon_0}\left[\frac{q^2}{d} + \frac{Qqd}{D^2}\right]$$

3. For a positive line charge or charged wire with uniform density λ, electric field at distance x is

$$E = \frac{2k\lambda}{x} = \frac{\lambda}{2\pi\varepsilon_0 x} \qquad \text{...(i)}$$

So, force on charge q which is at a distance r_0 due to this line charge is $F = qE = \dfrac{2kq\lambda}{x}$ ...(ii) [using Eq. (i)]

Now, work done when charge is pushed by field by a small displacement dx is

$$dW = F\cdot dx = \frac{2kq\lambda}{x}\cdot dx \qquad \text{[using Eq. (ii)]}$$

$\therefore$ Total work done by field of wire in taking charge q from distance r_0 to distance r will be

$$W = \int_{r_0}^{r} dW = \int_{r_0}^{r} \frac{2kq\lambda}{x}\cdot dx$$

$$= 2kq\lambda[\log x]_{r_0}^{r} = 2kq\lambda(\log r - \log r_0)$$

$$= 2kq\lambda \log\left|\frac{r}{r_0}\right| \qquad \text{...(iii)}$$

As we know, from work-kinetic energy theorem,

$$K_{\text{final}} - K_{\text{initial}} = W$$

$$\Rightarrow \quad \frac{1}{2}mv^2 - 0 = 2kq\times\log\left|\frac{r}{r_0}\right| \qquad \text{[using Eq. (iii)]}$$

$$\Rightarrow \quad v = \left(\frac{4kq\lambda}{m}\log\left|\frac{r}{r_0}\right|\right)^{1/2}$$

$$\therefore \quad v \propto \left(\log\left|\frac{r}{r_0}\right|\right)^{1/2}$$

4. Given, $\mathbf{E} = (Ax + B)\hat{\mathbf{i}}$ N-C^{-1}

The relation between electric field and potential is given as

$$dV = -\mathbf{E}\cdot d\mathbf{x}$$

Integrating on both sides within the specified limits, we get

$$\therefore \quad \int_1^2 dV = V_2 - V_1 = -\int_{x_1}^{x_2}\mathbf{E}\cdot d\mathbf{x}$$

$$\Rightarrow \quad V_1 - V_2 = \int_{x_1}^{x_2}\mathbf{E}\cdot d\mathbf{x}$$

$$= \int_{x_1}^{x_2}(Ax + B)\hat{\mathbf{i}}\cdot(dx\hat{\mathbf{i}}) = \int_{x_1}^{x_2}(Ax + B)\cdot dx$$

Here, $A = 20$ SI unit, $B = 10$ SI unit, $x_1 = 1$ and $x_2 = -5$

$$\Rightarrow \quad V_1 - V_2 = \int_1^{-5}(20x + 10)\cdot dx$$

$$= \left[\frac{20x^2}{2} + 10x\right]_1^{-5} = 10[x^2 + x]_1^{-5}$$

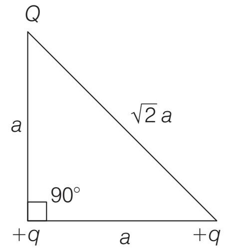
$$= 10[(-5)^2 + (-5) - (1)^2 - (1)]$$

$$= 10(25 - 5 - 2) = 180\ \text{V}$$

5. Electrostatic energy between two charges q_1 and q_2 such that the distance between them r is given as

$$U = \frac{K\ q_1 q_2}{r}$$

In accordance to the principle of superposition, total energy of the charge system as shown in the figure below is



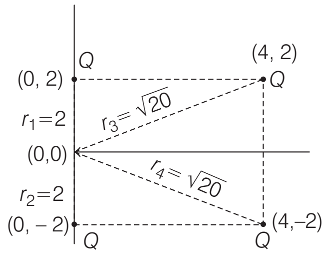
$$U = \frac{Kq^2}{a} + \frac{KQq}{a} + \frac{KQq}{\sqrt{2}a}$$

It is given that, $\quad U = 0$

$$\therefore \quad \frac{Kq}{a}\left[q + Q + \frac{Q}{\sqrt{2}}\right] = 0$$

$$\Rightarrow \quad Q = \frac{-\sqrt{2}\times q}{(\sqrt{2} + 1)}$$

6. The four charges are shown in the figure below,



Electric potential at origin (0, 0) due to these charges can be found by scalar addition of electric potentials due to each charge.

$$\therefore \quad V = \frac{KQ}{r_1} + \frac{KQ}{r_2} + \frac{KQ}{r_3} + \frac{KQ}{r_4} \qquad \text{... (i)}$$

$$\Rightarrow V = KQ\left[\frac{1}{2} + \frac{1}{2} + \frac{1}{\sqrt{20}} + \frac{1}{\sqrt{20}}\right] = KQ\left[1 + \frac{1}{\sqrt{5}}\right]$$

$$\Rightarrow V = KQ\frac{(\sqrt{5} + 1)}{\sqrt{5}}\ \text{V} \qquad \text{...(ii)}$$

Now, if another charge Q is placed at origin, then work done to get the charge at origin

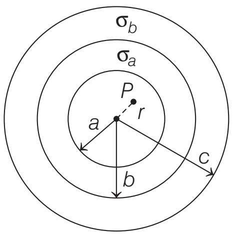
$$W = QV \quad \text{...(iii)}$$

By putting the value of V from Eq. (ii) in Eq. (iii), we get

$$W = KQ^2 \frac{(\sqrt{5}+1)}{\sqrt{5}} \text{ J}$$

or
$$W = \frac{Q^2}{4\pi\varepsilon_0}\left(1+\frac{1}{\sqrt{5}}\right) \text{ J}$$

7. Given charge distribution is shown in the figure below,



Given surface charge densities of each shell are same.

$\therefore$
$$\sigma_a = \sigma_b = \sigma_c \quad \text{...(i)}$$

As, surface charge density of shell of radius 'r' and having charge 'Q' is given as $\sigma = \dfrac{Q}{4\pi r^2}$

So, relation (i) can be rewritten as

$$\frac{Q_a}{4\pi a^2} = \frac{Q_b}{4\pi b^2} = \frac{Q_c}{4\pi c^2}$$

$$\Rightarrow \quad Q_a : Q_b : Q_c = a^2 : b^2 : c^2$$

where Q_a, Q_b and Q_c are charges on shell of radius a, b and c, respectively.

Also,
$$Q_a + Q_b + Q_c = Q$$

Hence,
$$Q_a = \frac{a^2}{a^2+b^2+c^2}\cdot Q$$

$$Q_b = \frac{b^2}{a^2+b^2+c^2}\cdot Q$$

$$Q_c = \frac{c^2}{a^2+b^2+c^2}\cdot Q$$

As we know for charged spherical shell with charge Q of radius 'R', the potential at a point 'P' at distance r such that $r < R$ is

$$V_P = \frac{kQ}{R}.$$

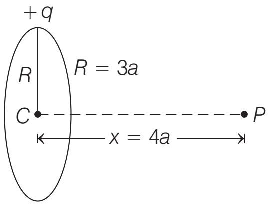
$\therefore$ potential at point P at a distance 'r' = Potential due to Q_a + Potential due to Q_b + Potential due to Q_c

$$= \frac{kQ_a}{a} + \frac{kQ_b}{b} + \frac{kQ_c}{c}$$

Substituting the values of Q_a, Q_b and Q_c, we get

$$V = \frac{Q\,(a+b+c)}{4\pi\varepsilon_0\,(a^2+b^2+c^2)}.$$

8. Potential at any point at distance x from the centre of the ring is given by



$$V_p = \frac{Kq}{\sqrt{R^2+x^2}}$$

Given, $R = 3a$ and $x = 4a$

$$\therefore \quad V_P = \frac{Kq}{\sqrt{9a^2+16a^2}} = \frac{Kq}{5a} \quad \text{... (i)}$$

At centre, $x = 0$

So, potential at centre is

$$V_C = \frac{Kq}{R} = \frac{Kq}{3a} \quad \text{... (ii)}$$

Now, energy required to get this charge from $x = 4a$ to the centre is

$$\Delta U = q\,\Delta V = q\,[V_C - V_P] = q\left[\frac{Kq}{3a} - \frac{Kq}{5a}\right]$$

$$= \frac{Kq^2}{a}\left[\frac{1}{3} - \frac{1}{5}\right]$$

$$\Delta U = \frac{2}{15}\frac{Kq^2}{a} \quad \text{... (iii)}$$

This energy must be equal to (or less than) the kinetic energy of the charge, i.e.

$$\frac{1}{2}mv^2 \geq \frac{2}{15}\frac{Kq^2}{a}$$

So, minimum energy required is

$$\frac{1}{2}mv^2 = \frac{2}{15}\times\frac{1}{4\pi\varepsilon_0}\times\frac{q^2}{a} \quad (\text{put } K = 1/4\pi\varepsilon_0)$$

$\therefore$ Minimum velocity,

$$v^2 = \frac{2}{m}\times\frac{2}{4\pi\varepsilon_0}\times\frac{q^2}{15a}$$

or
$$v = \sqrt{\frac{2}{m}}\times\sqrt{\frac{2q^2}{4\pi\varepsilon_0 a\times 15}}$$

9. As we know, potential difference $V_A - V_O$ is

$$dV = -E dx$$

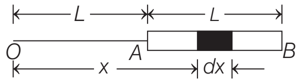
$$\Rightarrow \quad \int_{V_o}^{V_A} dV = -\int_0^2 30x^2 dx$$

$$V_A - V_O = -30\times\left[\frac{x^3}{3}\right]_0^2$$

$$= -10\times[2^3 - (0)^3]$$

$$= -10\times 8 = -80 \text{ J}$$

10.



$$V=\int_L^{2L}\frac{kdQ}{x}=\int_L^{2L}\frac{k\left(\frac{Q}{L}\right)dx}{x}=\frac{Q}{4\pi\varepsilon_0 L}\int_L^{2L}\left(\frac{1}{x}\right)dx$$

$$=\frac{Q}{4\pi\varepsilon_0 L}[\log_e x]_L^{2L}$$

$$=\frac{Q}{4\pi\varepsilon_0 L}[\log_e 2L-\log_e L]=\frac{Q}{4\pi\varepsilon_0 L}\ln(2)$$

11. **For inside points** $(r \le R)$

$$E=0 \Rightarrow V=\text{constant}=\frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

For outside points $(r \ge R)$

$$E=\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2} \quad \text{or} \quad E\propto\frac{1}{r^2}$$

and $$V=\frac{1}{4\pi\varepsilon_0}\frac{q}{r} \quad \text{or} \quad V\propto\frac{1}{r}$$

On the surface $(r=R)$

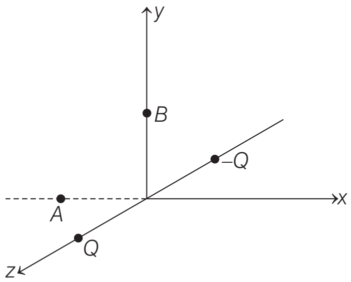
$$V=\frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

$$\Rightarrow \quad E=\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R^2}=\frac{\sigma}{\varepsilon_0}$$

where, $\sigma=\frac{q}{4\pi R^2}$ = surface charge density

corresponding to above equations the correct graphs are shown in option (d).

12. $A\equiv(-a,0,0)$, $B\equiv(0,a,0)$

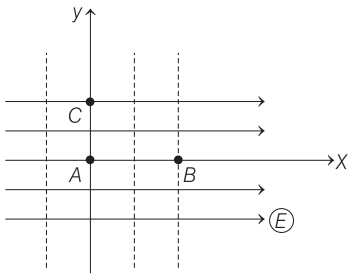


Point charge is moved from A to B

$$V_A=V_B=0 \Rightarrow W=0$$

or the correct option is (c).

13. Potential decreases in the direction of electric field. Dotted lines are equipotential lines.



$$\therefore \quad V_A=V_C \quad \text{and} \quad V_A>V_B$$

14. Net electrostatic energy of the configuration will be

$$U=K\left[\frac{q.q}{a}+\frac{Q.q}{\sqrt{2}a}+\frac{Q.q}{a}\right].$$

Here, $K=\frac{1}{4\pi\varepsilon_0}$

Putting $U=0$ we get, $Q=\frac{-2q}{2+\sqrt{2}}$

15. Potential at origin will be given by

$$V=\frac{q}{4\pi\,\varepsilon_0}\left[\frac{1}{x_0}-\frac{1}{2x_0}+\frac{1}{3x_0}-\frac{1}{4x_0}+....\right]$$

$$=\frac{q}{4\pi\varepsilon_0}\cdot\frac{1}{x_0}\left[1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\ldots\right]=\frac{q}{4\pi\varepsilon_0 x_0}\ln(2)$$

16. $-\int_{l=\infty}^{l=0}\mathbf{E}\cdot\mathbf{dl}=\int_{l=\infty}^{l=0}dV=V\text{ (centre)}-V\text{ (infinity)}$

but V (infinity) = 0

$\therefore -\int_{l=\infty}^{l=0}\mathbf{E}\cdot\mathbf{dl}$ corresponds to potential at centre of ring.

and $V\text{ (centre)}=\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R}$

$$=\frac{(9\times10^9)(1.11\times10^{-10})}{0.5}\approx 2\text{V}$$

17. From conservation of mechanical energy

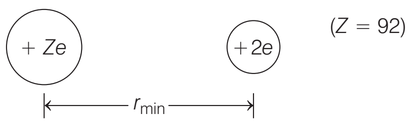
decrease in kinetic energy = increase in potential energy

or $$\frac{1}{4\pi\varepsilon_0}\frac{(Ze)(2e)}{r_{\min}}=5\text{ MeV}$$

$$=5\times1.6\times10^{-13}\text{ J}$$

$$\therefore \quad r_{\min}=\frac{1}{4\pi\varepsilon_0}\frac{2Ze^2}{5\times1.6\times10^{-13}}$$

$$=\frac{(9\times10^9)(2)(92)(1.6\times10^{-19})^2}{5\times1.6\times10^{-13}}$$



$$=5.3\times10^{-14}\text{ m}$$

$$=5.3\times10^{-12}\text{ cm}$$

i.e. $r_{\min}$ is of the order of 10^{-12} cm.

$\therefore$ Correct option is (c).

18. Option (b) is correct.

19. V_0 = potential on the surface $=\frac{Kq}{R}$

where, $K=\frac{1}{4\pi\varepsilon_0}$ and q is total charge on sphere.

Potential at centre $=\frac{3}{2}\frac{Kq}{R}=\frac{3}{2}V_0$

Hence, $R_1 = 0$

From centre to surface potential varies between $\frac{3}{2}V_0$ and V_0

From surface to infinity, it varies between V_0 and 0, $\frac{5V_0}{4}$ will be potential at a point between centre and surface. At any point, at a distance $r(r \le R)$ from centre potential is given by

$$V = \frac{Kq}{R^3}\left(\frac{3}{2}R^2 - \frac{1}{2}r^2\right)$$

$$= \frac{V_0}{R^2}\left(\frac{3}{2}R^2 - \frac{1}{2}r^2\right) \quad (\text{as } V_0 = \frac{Kq}{R})$$

Putting $V = \frac{5}{4}V_0$ and $r = R_2$ in this equation, we get

$$R_2 = \frac{R}{\sqrt{2}}$$

$\frac{3V_0}{4}$ and $\frac{V_0}{4}$ are the potentials lying between V_0 and zero hence these potentials lie outside the sphere. At a distance $r(\ge R)$ from centre potential is given by $V = \frac{Kq}{r} = \frac{V_0 R}{r}$

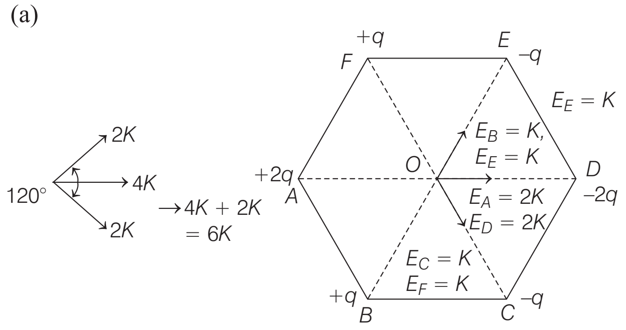
Putting $V = \frac{3}{4}V_0$ and $r = R_3$ in this equation we get, $R_3 = \frac{4}{3}R$

Further putting $V = \frac{V_0}{4}$ and $r = R_4$ in above equation,

we get $\quad R_4 = 4R$

Thus, $R_1 = 0$, $R_2 = \frac{R}{\sqrt{2}}$, $R_3 = \frac{4R}{3}$ and $R_4 = 4R$ with these values, option (b) and (c) are correct.

20. (a)



Resultant of $2K$ and $2K$ (at 120°) is also $2K$ towards $4K$. Therefore, net electric field is $6K$.

(b) $$V_0 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_A}{L} + \frac{q_B}{L} + \frac{q_C}{L} + \frac{q_D}{L} + \frac{q_E}{L} + \frac{q_F}{L}\right]$$

$$= \frac{1}{4\pi\varepsilon_0 L}(q_A + \dots + q_F) = 0$$

Because $q_A + q_B + q_C + q_D + q_E + q_F = 0$

(c) Only line PR, potential is same (= 0).

21. The given graph is of charged conducting sphere of radius R_0. The whole charge q distributes on the surface of the sphere.

22. $\mathbf{E} = -\left[\frac{\partial V}{\partial x}\hat{\mathbf{i}} + \frac{\partial V}{\partial y}\hat{\mathbf{j}} + \frac{\partial V}{\partial z}\hat{\mathbf{k}}\right] \Rightarrow V = 4x^2$

Therefore, $\frac{\partial V}{\partial x} = 8x$ and $\frac{\partial V}{\partial y} = 0 = \frac{\partial V}{\partial z}$

$$\mathbf{E} = -8x\hat{\mathbf{i}}$$

or $\mathbf{E}$ at (1 m, 0, 2 m) is $-8\hat{\mathbf{i}}$ V/m

23. $W_{Fe} = \mathbf{F}\cdot\mathbf{d}$ ($\mathbf{d}$ = displacement)

$= (qE\hat{\mathbf{i}})\cdot[\mathbf{r}_s - \mathbf{r}_p] = qE\hat{\mathbf{i}}\cdot[(-a\hat{\mathbf{i}} - b\hat{\mathbf{j}})] = -qEa$

24. Magnitude of electric field is greatest at a point where electric lines of force are most close to each other.

24. Electrostatic force is conservative in nature and in conservative force field work done is path independent.

26. For potential energy of the system of charges, total number of charge pairs will be ${}^8C_2$ or 28 of these 28 pairs 12 unlike charges are at a separation a, 12 like charges are at separation $\sqrt{2}\,a$ and 4 unlike charges are at separation $\sqrt{3}\,a$. Therefore, the potential energy of the system is given as

$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{(12)(q)(-q)}{a} + \frac{(12)(q)(q)}{\sqrt{2}a} + \frac{(4)(q)(-q)}{\sqrt{3}a}\right]$$

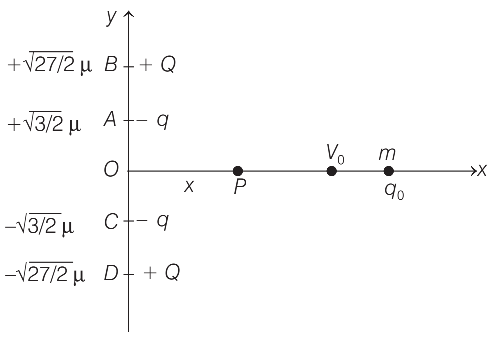
$$= -5.824\left(\frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{a}\right)$$

The binding energy of this system is therefore,

$$|U| = 5.824\left(\frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}\right).$$

So, work done by external forces in disassembling, this system of charges is $W = 5.824\left(\frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{a}\right)$

27.



In the figure, $q = 1\,\mu\text{C} = 10^{-6}\,\text{C}$, $q_0 = +0.1\,\mu\text{C} = 10^{-7}\,\text{C}$ and $m = 6\times10^{-4}$ kg and $Q = 8\,\mu\text{C} = 8\times10^{-6}$ C

Let P be any point at a distance x from origin O. Then

$$AP = CP = \sqrt{\frac{3}{2} + x^2}$$

$$BP = DP = \sqrt{\frac{27}{2} + x^2}$$

Electric potential at point P will be, $V = \frac{2KQ}{BP} - \frac{2Kq}{AP}$

where, $K = \frac{1}{4\pi\varepsilon_0} = 9\times10^9\,\text{Nm}^2/\text{C}^2$

$$\therefore \quad V = 2\times9\times10^9\left[\frac{8\times10^{-6}}{\sqrt{\frac{27}{2}+x^2}} - \frac{10^{-6}}{\sqrt{\frac{3}{2}+x^2}}\right]$$

$$V = 1.8\times10^4\left[\frac{8}{\sqrt{\frac{27}{2}+x^2}} - \frac{1}{\sqrt{\frac{3}{2}+x^2}}\right] \quad \ldots\text{(i)}$$

$\therefore$ Electric field at point P is

$$E = -\frac{dV}{dX} = 1.8\times10^4\left[(8)\left(\frac{-1}{2}\right)\left(\frac{27}{2}+x^2\right)^{-3/2} \left(-\frac{1}{2}\right)\left(\frac{3}{2}+x^2\right)^{-3/2}\right](2x)$$

$E = 0$ on x-axis where $x = 0$,

or $$\frac{8}{\left(\frac{27}{2}+x^2\right)^{3/2}} = \frac{1}{\left(\frac{3}{2}+x^2\right)^{3/2}}$$

$$\Rightarrow \quad \frac{(4)^{3/2}}{\left(\frac{27}{2}+x^2\right)^{3/2}} = \frac{1}{\left(\frac{3}{2}+x^2\right)^{3/2}}$$

$$\Rightarrow \quad \left(\frac{27}{2}+x^2\right) = 4\left(\frac{3}{2}+x^2\right)$$

This equation gives, $x = \pm\sqrt{\frac{5}{2}}$ m

The least value of kinetic energy of the particle at infinity should be enough to take the particle upto $x = +\sqrt{\frac{5}{2}}$ m

because at $x = +\sqrt{\frac{5}{2}}$ m, $E = 0$.

$\Rightarrow$ Electrostatic force on charge q is zero or $F_e = 0$.

For at $x > \sqrt{\frac{5}{2}}$ m, E is repulsive (towards positive x-axis)

and for $x < \sqrt{\frac{5}{2}}$ m, E is attractive (towards negative x-axis)

Now, from Eq. (i), potential at $x = \sqrt{\frac{5}{2}}$ m

$$V = 1.8\times10^4\left[\frac{8}{\sqrt{\frac{27}{2}+\frac{5}{2}}} - \frac{1}{\sqrt{\frac{3}{2}+\frac{5}{2}}}\right]$$

$$V = 2.7\times10^4\ \text{V}$$

Applying energy conservation at $x = \infty$ and $x = \sqrt{\frac{5}{2}}$ m

$$\frac{1}{2}mv_0^2 = q_0V \quad \ldots\text{(ii)}$$

$$\therefore \quad v_0 = \sqrt{\frac{2q_0V}{m}}$$

Substituting these values,

$$v_0 = \sqrt{\frac{2\times10^{-7}\times2.7\times10^4}{6\times10^{-4}}}. \Rightarrow v_0 = 3\ \text{m/s}$$

$\therefore$ Minimum value of v_0 is 3 m/s

From Eq. (i), potential at origin $(x = 0)$ is

$$V_0 = 1.8\times10^4\left[\frac{8}{\sqrt{\frac{27}{2}}} - \frac{1}{\sqrt{\frac{3}{2}}}\right] \approx 2.4\times10^4\ \text{V}$$

Let K be the kinetic energy of the particle at origin.

Applying energy conservation at $x =$`0 and at $x = \infty$

$$K + q_0V_0 = \frac{1}{2}mv_0^2$$

But $\frac{1}{2}mv_0^2 = q_0V$ [from Eq. (ii)]

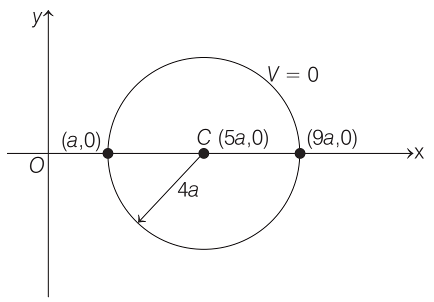
$$K = q_0\,(V - V_0)$$

$$K = (10^{-7})\,(2.7\times10^4 - 2.4\times10^4)$$

$$K = 3\times10^{-4}\ \text{J}$$

NOTE $E = 0$ or F_e on q_0 is zero at $x = 0$ and $x = \pm\sqrt{\frac{5}{2}}$ m of these $x = 0$ is stable equilibrium position and $x = \pm\sqrt{\frac{5}{2}}$ is unstable equilibrium position.

28. (a) Let $P(x, y)$ be a general point on x-y plane. Electric potential at point P would be,



V = (potential due to Q) + (potential due to $-2Q$)

or $$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{\sqrt{(3a-x)^2+y^2}}\right] + \frac{1}{4\pi\varepsilon_0}\left[\frac{-2Q}{\sqrt{(3a+x)^2+y^2}}\right] \quad \ldots\text{(i)}$$

Given $V = 0$

$\therefore \quad 4[(3a - x)^2 + y^2] = (3a + x)^2 + y^2$

On simplifying, we get $(x - 5a)^2 + y^2 = (4a)^2$

This is the equation of a circle of radius $4a$ and centre at $(5a, 0)$.

(b) On x-axis, potential will be undefined (or say $\pm\infty$) at $x = 3a$ and $x = -3a$, because charge Q and $-2Q$ are placed at these two points.

So, between $-3a < x < 3a$ we can find potential by putting $y = 0$ in Eq. (i). Therefore,

$$V = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{3a - x} - \frac{2}{3a + x}\right] \text{ for } -3a < x < 3a$$

$$V = 0 \text{ at } x = a$$

$$V \to -\infty \text{ at } x \to -3a$$

and $\quad V \to +\infty$ at $x \to 3a$

For $x > 3a$, there is again a point where potential will become zero so for $x > 3a$, we can write

$$V = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{x - 3a} - \frac{2}{3a + x}\right] \text{ for } x > 3a$$

$V = 0$ at $x = 9a$

For $x < -3a$, we can write

$$V = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{3a - x} - \frac{2}{3a - x}\right] \text{ for } x < -3a$$

In this region potential will be zero only at $x \to -\infty$

Thus, we can summarise it as under.

(i) At $x = 3a, V = +\infty$

(ii) At $x = -3a, V = -\infty$

(iii) For $x < -3a$, $V = \dfrac{Q}{4\pi\varepsilon_0}\left[\dfrac{1}{3a - x} - \dfrac{2}{3a + x}\right]$

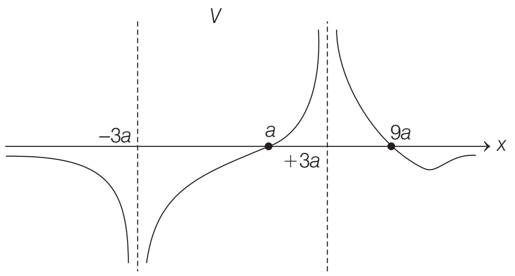
(iv) For $-3a < x < 3a$, expression of V is same i.e.

$$V = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{3a - x} - \frac{2}{3a + x}\right]$$

(v) For $x > 3a$, $V = \dfrac{Q}{4\pi\varepsilon_0}\left[\dfrac{1}{x - 3a} - \dfrac{2}{3a + x}\right]$

Potential on x-axis is zero at two places at $x = a$ and $x = 9a$.

The V- x graph is shown below,



Exercise (i) Find potential at $x = 0$.

(ii) In the graph for $x > 9a$, find where $|V|$ will be maximum and what will be its value ?

(c) Potential at centre i.e. at $x = 5a$ will be,

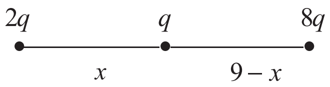
$$V = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{2a} - \frac{2}{8a}\right] = \frac{Q}{16\pi\varepsilon_0 a} = \text{positive}$$

Potential on the circle will be zero.

Since, potential at centre > potential on circumference on it, the particle will cross the circle because positive charge moves from higher potential to lower potential. Speed of particle, while crossing the circle would be,

$$v = \sqrt{\frac{2q(\Delta V)}{m}} = \sqrt{\frac{Qq}{8\pi\varepsilon_0 ma}}$$

Here, ΔV is the potential difference between the centre and circumference of the circle.

29. 

For potential energy to be minimum the bigger charges should be farthest. Let x be the distance of q from $2q$. Then potential energy of the system shown in figure would be

$$U = K\left[\frac{(2q)(q)}{x} + \frac{(8q)(q)}{(9 - x)} + \frac{(2q)(8q)}{9}\right]$$

Here, $\quad K = \dfrac{1}{4\pi\varepsilon_0}$

For U to be minimum $\dfrac{2}{x} + \dfrac{8}{9 - x}$ should be minimum.

$$\frac{d}{dx}\left[\frac{2}{x} + \frac{8}{9 - x}\right] = 0$$

$\therefore \quad \dfrac{-2}{x^2} + \dfrac{8}{(9 - x)^2} = 0$

$\therefore \quad \dfrac{x}{9 - x} = \dfrac{1}{2}$ or $x = 3$ cm

i.e. distance of charge q from $2q$ should be 3 cm.

Electric field at q

$$E = \frac{K(2q)}{(3 \times 10^{-2})^2} - \frac{K(8q)}{(6 \times 10^{-2})^2} = 0$$

30. Equating the energy of $(-q)$ at C and D

$$K_C + U_C = K_D + U_D$$

Here, $\quad K_C = 4$ J

$$U_C = 2\left[\frac{1}{4\pi\varepsilon_0}\frac{(q)(-q)}{AC}\right]$$

$$= \frac{-2 \times 9 \times 10^9 \times (5 \times 10^{-5})^2}{5} = -9\text{J}$$

$$K_D = 0$$

and $$U_D = 2\left[\frac{1}{4\pi\varepsilon_0}\frac{(q)(-q)}{AD}\right]$$

$$= \frac{-2\times 9\times 10^9 \times (5\times 10^{-5})^2}{AD}$$

$$= -\frac{45}{AD}$$

Substituting these values in Eq. (i)

$$4 - 9 = 0 - \frac{45}{AD}$$

$\therefore$ $$AD = 9\,\text{m}$$

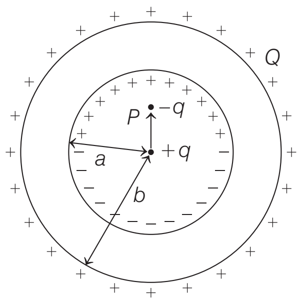
$\therefore$ $$OD = \sqrt{AD^2 - OA^2}$$

$$= \sqrt{(9)^2 - (3)^2}$$

$$= \sqrt{81 - 9} = 8.48\text{ m}$$

Topic 3 Gauss Theorem and Spherical Shells

1. Electric charge distribution at inner and outer surface of spherical shell due to the electric dipole can be shown as below



Here, we need to consider two different factors

(i) charge on the spherical shell is $+Q$ which will be distributed on its outer surface as shown in figure.

(ii) Electric dipole will create non-uniform electric field inside the shell which will distribute the charges on inner surface as shown in figure. But its net contribution to the outer side of the shell will be zero as net charge of a dipole is zero.

$\therefore$ Net charge on outer surface of shell will be $+Q$.

Hence, using (ii), option (a) is incorrect as field inside shell is not uniform. Option (b) is correct, as net charge on outer surface is $+Q$ even in the presence of dipole.

Option (c) is incorrect, as surface charge density at outer surface is uniform $\left(= \frac{Q}{A} = \frac{Q}{4\pi b^2}\right)$.

Option (d) is incorrect, as surface charge density at inner surface is non-zero.

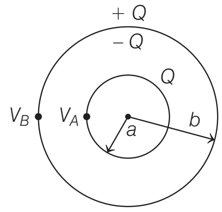
So, option (b) is correct.

Alternate Solution Using Gauss' law at outer surface, let charge on dipole is q,

$$\phi = \frac{\Sigma q}{\varepsilon_0} = E\cdot A \text{ or } E = \frac{1}{A\varepsilon_0}\Sigma q$$

$$= \frac{(+Q+q-q)}{A\varepsilon_0} = \frac{Q}{A\varepsilon_0} = \frac{\sigma}{\varepsilon_0} = \text{constant}$$

2. Initially when uncharged shell encloses charge Q, charge distribution due to induction will be as shown,



The potential on surface of inner shell is

$$V_A = \frac{kQ}{a} + \frac{k(-Q)}{b} + \frac{kQ}{b} \quad \ldots\text{(i)}$$

where, k = proportionality constant.

Potential on surface of outer shell is

$$V_B = \frac{kQ}{b} + \frac{k(-Q)}{b} + \frac{kQ}{b} \quad \ldots\text{(ii)}$$

Then, potential difference is

$$\Delta V_{AB} = V_A - V_B = kQ\left(\frac{1}{a} - \frac{1}{b}\right)$$

Given, $$\Delta V_{AB} = V$$

So, $$kQ\left(\frac{1}{a} - \frac{1}{b}\right) = V \quad \ldots\text{(iii)}$$

Finally after giving charge $-4Q$ to outer shell, potential difference will be

$$\Delta V_{AB} = V_A - V_B = \left(\frac{kQ}{a} + \frac{k(-4Q)}{b}\right) - \left(\frac{kQ}{b} + \frac{k(-4Q)}{b}\right)$$

$$= kQ\left(\frac{1}{a} - \frac{1}{b}\right) = V \quad \text{[from Eq. (iii)]}$$

Hence, we obtain that potential difference does not depend on the charge of outer sphere, hence potential difference remains same.

3. For a uniformly charged spherical shell, electric potential inside it is given by

$V_{\text{inside}} = V_{\text{surface}} = kq/r_0$ = constant,

(where r_0 = radius of the shell).

and electric potential outside the shell at a distance r is

$$V_{\text{outside}} = \frac{kq}{r} \Rightarrow V \propto 1/r$$

$\therefore$ The given graph represents the variation of r and potential of a uniformly charged spherical shell.

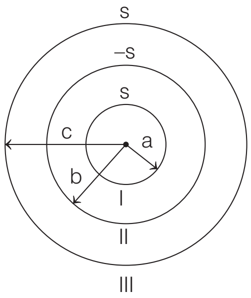
4. (c) $q_I = (4\pi a^2)(\sigma), q_{II} = 4\pi b^2(-\sigma), q_{III} = 4\pi c^2(\sigma)$

V_B = (Potential due to I) + (Potential due to II) + (Potential due to III)

$$V_B = \frac{K\,4\pi a^2\sigma}{b} + \frac{K\,4\pi b^2}{b}(-\sigma) + \frac{K\,4\pi c^2(\sigma)}{c}$$

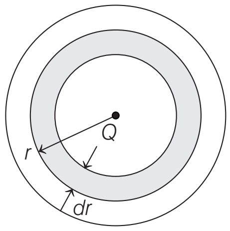
Substituting $K = \frac{1}{4\pi\varepsilon_0}$, we get

$$V_B = \frac{\sigma}{\varepsilon_0}\left(\frac{a^2 - b^2}{b} + c\right)$$



5. As E is constant,

Hence, $E_a = E_b$



As per Guass theorem, only Q_{in} contributes in electric field.

$$\therefore \qquad \frac{kQ}{a^2} = \frac{k\left[Q + \int_a^b 4\pi r^2 dr \cdot \frac{A}{r}\right]}{b^2}$$

Here, $\qquad k = \frac{1}{4\pi\varepsilon_0}$

$$\Rightarrow \qquad Q\frac{b^2}{a^2} = Q + 4\pi A\left[\frac{r^2}{2}\Bigg|_a^b\right] = Q + 4\pi A \cdot \left(\frac{b^2 - a^2}{2}\right)$$

$$\Rightarrow \qquad Q\left(\frac{b^2}{a^2}\right) = Q + 2\pi A\,(b^2 - a^2)$$

$$\Rightarrow \qquad Q\left(\frac{b^2 - a^2}{a^2}\right) = 2\pi A\,(b^2 - a^2) \;\Rightarrow\; A = \frac{Q}{2\pi a^2}$$

6. The sphere with cavity can be assumed as a complete sphere with positive charge of radius R_1 + another complete sphere with negative charge and radius R_2.

$E_+ \rightarrow$ **E** due to total positive charge

$E_- \rightarrow$ **E** due to total negative charge.

$$\mathbf{E} = \mathbf{E}_+ + \mathbf{E}_-$$

If we calculate it at P, then $\mathbf{E}_-$ comes out to be zero.

$\therefore \qquad \mathbf{E} = \mathbf{E}_+$

and $\mathbf{E}_+ = \frac{1}{4\pi\varepsilon_0}\frac{q}{R_1^3}(OP)$, in the direction of OP.

Here, q is total positive charge on whole sphere.

It is in the direction of OP or **a**.

Now, inside the cavity electric field comes out to be uniform at any point. This is a standard result.

7. Electric flux, $\phi = \mathbf{E} \cdot \mathbf{S}$ or $\phi = ES\cos\theta$

Here, θ is the angle between **E** and **S**.

In this question $\theta = 45°$, because **S** is perpendicular to surface.

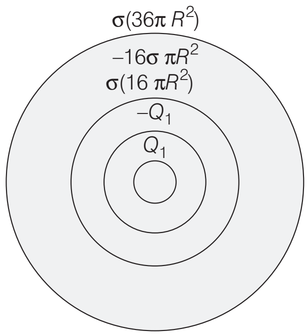
$$E = E_0$$

$$S = (\sqrt{2}a)\,(a) = \sqrt{2}a^2$$

$$\therefore \qquad \phi = (E_0)\,(\sqrt{2}a^2)\cos 45° = E_0\, a^2$$

$\therefore$ Correct option is (c).

8. $Q_1 = \sigma(4\pi R^2) = 4\pi\sigma R^2$



$$Q_2 = 16\pi\sigma R^2 - Q_1 = 12\pi\sigma R^2$$

$$Q_3 = 36\pi\sigma R^2 - 16\pi\sigma R^2 = 20\pi\sigma R^2$$

$$Q_1 : Q_2 : Q_3 = 1:3:5$$

9. Total enclosed charge as already shown is

$$q_{\text{net}} = \frac{6C}{2} + \frac{8C}{4} - 7C = -\,2C$$

From Gauss-theorem, net flux, $\phi_{\text{net}} = \frac{q_{\text{net}}}{\varepsilon_0} = \frac{-2C}{\varepsilon_0}$

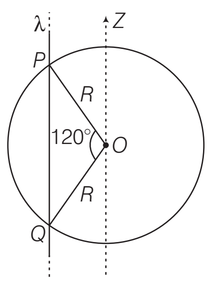
10. At any point over the spherical Gaussian surface, net electric field is the vector sum of electric fields due to $+q_1, -q_1$ and q_2. Don't confuse with the electric flux which is zero (net) passing over the Gaussian surface as the net charge enclosing the surface is zero.

Hence, the correct option is (c).

11. In such situation potential difference depends only on the charge on inner sphere. Since, charge on inner sphere is unchanged. Therefore, potential difference V will remain unchanged.

12. Electric potential at any point inside a hollow metallic sphere is constant. Therefore, if potential at surface is 10 V, potential at centre will also be 10 V.

13. $PQ = (2)\,R\sin 60°$



$$= (2R)\frac{\sqrt{3}}{2} = (\sqrt{3}R)$$

$$q_{\text{enclosed}} = \lambda\,(\sqrt{3}R)$$

We have, $\phi = \dfrac{q_{\text{enclosed}}}{\varepsilon_0}$

$$\Rightarrow \quad \phi = \left(\frac{\sqrt{3}\lambda R}{\varepsilon_0}\right)$$

Also, electric field is perpendicular to wire, so Z-component will be zero.

14. (a) $\Omega = 2\pi(1-\cos\theta)$; $\theta = 45°$

$$\phi = -\frac{\Omega}{4\pi}\times\frac{Q}{\varepsilon_0} = -\frac{2\pi(1-\cos\theta)}{4\pi}\frac{Q}{\varepsilon_0}$$

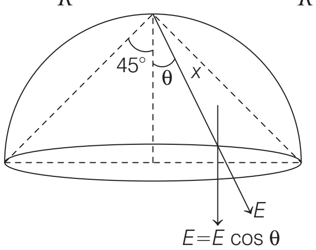
$$= -\frac{Q}{2\varepsilon_0}\left(1-\frac{1}{\sqrt{2}}\right)$$

(b) The component of the electric field perpendicular to the flat surface will decrease so we move away from the centre as the distance increases (magnitude of electric field decreases) as well as the angle between the normal and electric field will increase. Hence, the component of the electric field normal to the flat surface is not constant.

Alternate solution

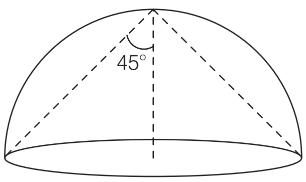
$$x = \frac{R}{\cos\theta}$$

$$E = \frac{KQ}{x^2} = \frac{KQ\cos^2\theta}{R^2} \Rightarrow E_{\perp} = \frac{KQ\cos^3\theta}{R^2}$$



As we move away from centre $\theta\uparrow$ $\cos\theta\downarrow$ so $E_{\perp}\downarrow$

(c) Total flux ϕ due to charge Q is $\dfrac{Q}{\varepsilon_0}$.



So, ϕ through the curved and flat surface will be less than $\dfrac{Q}{\varepsilon_0}$.

(d) Since, the circumference is equidistant from Q it will be equipotential $V = \dfrac{KQ}{\sqrt{2}R}$.

15. Option (a) is correct due to symmetry.

Option (b) is wrong again due to symmetry.

Option (c) is correct because as per Gauss's theorem, net electric flux passing through any closed surface $= \dfrac{q_{\text{in}}}{\varepsilon_0}$

Here, $q_{\text{in}} = 3q - q - q = q$

$\therefore$ Net electric flux $= \dfrac{q}{\varepsilon_0}$

Option (d) is wrong because there is no symmetry in two given planes.

16. If charges are of opposite signs then the two fields are along the same direction. So, they cannot be zero. Hence, the charges should be of same sign.

Therefore, option (c) is correct.

Further, Work done by external force = change in potential energy

$\therefore$ $W_{A\to B} = q\,(\Delta V) = (+1)\,(V_B - V_A)$

or $W_{A\to B} = V_B - V_A$

Therefore, option (d) is also correct.

$\therefore$ Correct options are (c) and (d).

17. Inside a conducting shell electric field is always zero. Therefore, option (a) is correct. When the two are connected, their potentials become the same.

$\therefore$ $V_A = V_B$ or $\dfrac{Q_A}{R_A} = \dfrac{Q_B}{R_B}$ $\left(V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{R}\right)$

Since, $R_A > R_B$ $\therefore$ $Q_A > Q_B$

$\therefore$ Option (b) is correct.

Potential is also equal to, $V = \dfrac{\sigma R}{\varepsilon_0}$, $V_A = V_B$

$\therefore$ $\sigma_A R_A = \sigma_B R_B$ or $\dfrac{\sigma_A}{\sigma_B} = \dfrac{R_B}{R_A}$ or $\sigma_A < \sigma_B$

$\therefore$ Option (c) is correct.

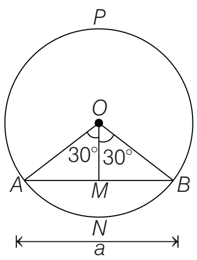
Electric field on surface, $E = \dfrac{\sigma}{\varepsilon_0}$ or $E \propto \sigma$

Since, $\sigma_A < \sigma_B$ $\therefore$ $E_A < E_B$

$\therefore$ Option (d) is also correct.

$\therefore$ Correct options are (a), (b), (c) and (d).

18. $ANBP$ is cross-section of a cylinder of length L. The line charge passes through the centre O and perpendicular to paper.



$$AM = \frac{a}{2},\ MO = \frac{\sqrt{3}a}{2}$$

$$\therefore \angle AOM = \tan^{-1}\left(\frac{AM}{OM}\right)$$

$$= \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30°$$

Electric flux passing from the whole cylinder

$$\phi_1 = \frac{q_{in}}{\varepsilon_0} = \frac{\lambda L}{\varepsilon_0}$$

$\therefore$ Electric flux passing through $ABCD$ plane surface (shown only AB) = Electric flux passing through cylindrical surface ANB

$$= \left(\frac{60°}{360°}\right)(\phi_1)$$

$$= \frac{\lambda L}{6\varepsilon_0}$$

$\therefore \quad n = 6$

19. (a) Potential at any shell will be due to all three charges.

$$V_A = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_A}{a} + \frac{q_B}{b} + \frac{q_C}{c}\right]$$

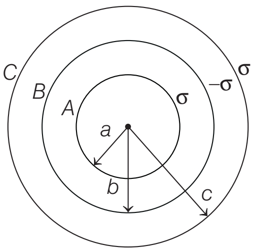
$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{(4\pi a^2)(\sigma)}{a} + \frac{(4\pi b^2)(-\sigma)}{b} + \frac{(4\pi c^2)(\sigma)}{c}\right]$$

$$= \frac{\sigma}{\varepsilon_0}(a - b + c)$$

$$V_B = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_A}{b} + \frac{q_B}{b} + \frac{q_C}{c}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{(4\pi a^2)(\sigma)}{b} + \frac{(4\pi b^2)(-\sigma)}{b} + \frac{(4\pi c^2)(\sigma)}{c}\right]$$

$$= \frac{\sigma}{\varepsilon_0}\left[\frac{a^2}{b} - b + c\right]$$



Similarly, $V_C = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_A}{c} + \frac{q_B}{c} + \frac{q_C}{c}\right]$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{(4\pi a^2)(\sigma)}{c} + \frac{(4\pi b^2)(-\sigma)}{c} + \frac{(4\pi c^2)(\sigma)}{c}\right]$$

$$= \frac{\sigma}{\varepsilon_0}\left[\frac{a^2}{c} - \frac{b^2}{c} + c\right]$$

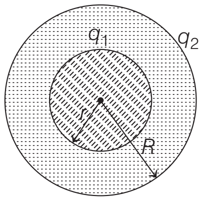
(b) Given $V_A = V_C$

$$\therefore \quad \frac{\sigma}{\varepsilon_0}(a - b + c) = \frac{\sigma}{\varepsilon_0}\left(\frac{a^2}{c} - \frac{b^2}{c} + c\right)$$

$$\therefore \quad a - b + c = \frac{a^2}{c} - \frac{b^2}{c} + c$$

or $\quad a + b = c$

20. Let q_1 and q_2 be the charges on them.



$$\sigma_1 = \sigma_2$$

$$\therefore \quad \frac{q_1}{4\pi r^2} = \frac{q_2}{4\pi R^2} \Rightarrow \frac{q_1}{q_2} = \frac{r^2}{R^2}$$

i.e., charge on them is distributed in above ratio.

or $\quad q_1 = \frac{r^2}{r^2 + R^2}Q$ and $q_2 = \frac{R^2}{r^2 + R^2}Q$

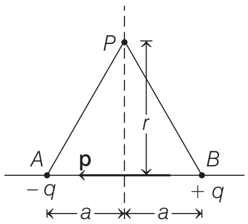
Potential at centre V = potential due to q_1 + potential due to q_2

or $$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1}{r} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_2}{R}$$

$$= \frac{Q(R + r)}{4\pi\varepsilon_0(r^2 + R^2)}$$

Topic 4 Electric Field Lines, Behaviour of Conductor and Electric Dipole

1. The given problem can be shown as clearly potential difference at point P due to dipole is



$$V = V_{AP} + V_{BP} \text{ (scalar addition)}$$

$$\Rightarrow \quad V = \frac{k(-q)}{AP} + \frac{k(q)}{BP} \quad \text{...(i)}$$

Here, $AP = BP = \sqrt{a^2 + r^2}$

$$\therefore \quad V = -\frac{kq}{\sqrt{a^2 + r^2}} + \frac{kq}{\sqrt{a^2 + r^2}} = 0 \quad \text{...(ii)}$$

Now, electric field at any point on Y-axis, i.e. equatorial line of the dipole can be given by

$$\mathbf{E} = -\frac{k\,\mathbf{p}}{r^3} \quad \text{(standard expression)}$$

$$\Rightarrow \quad \mathbf{E} = -\frac{1}{4\pi\varepsilon_0}\frac{\mathbf{p}}{r^3}$$

Given, $\quad r = d$

$$\therefore \quad \mathbf{E} = -\frac{1}{4\pi\varepsilon_0}\frac{\mathbf{p}}{d^3} \quad \text{...(iii)}$$

From Eqs. (ii) and (iii), correct option is (b).

Alternate Solution

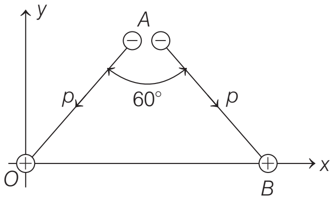
Electric field at any point at θ angle from axial line of dipole is given by

$$\mathbf{E} = -\frac{k\mathbf{p}}{r^3}\sqrt{3\cos^2\theta + 1}$$

Here, $\theta = 90° \Rightarrow \cos\theta = \cos 90° = 0$ and $r = d$

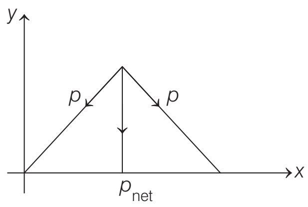
$$\therefore \quad \mathbf{E} = -\frac{k\mathbf{p}}{d^3} = -\frac{\mathbf{p}}{4\pi\varepsilon_0 d^3}$$

2. Given system is equivalent to two dipoles inclined at 60° to each other as shown in the figure below,



Now, magnitude of resultant of these dipole moments is

$$p_{\text{net}} = \sqrt{p^2 + p^2 + 2p \cdot p\cos 60°} = \sqrt{3}p = \sqrt{3}ql$$



As, resultant is directed along negative y-direction

$$p_{\text{net}} = -\sqrt{3}p\hat{\mathbf{j}} = -\sqrt{3}ql\,\hat{\mathbf{j}}$$

3. Given, $\mathbf{E} = 1000$ V/m

$\theta = 45°$

and $\mathbf{p} = 10^{-29}$ C-m

We know that, electric potential energy stored in an electric dipole kept in uniform electric field is given by the relation

$$U = -\,\mathbf{p}\cdot\mathbf{E} = -PE\cos\theta$$

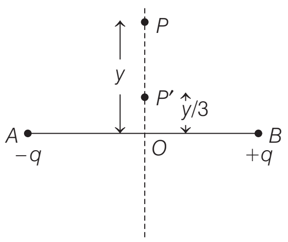
$$= -10^{-29} \times 1000 \times \cos 45°$$

$$\Rightarrow \quad U \approx -7 \times 10^{-27} \text{ J}$$

4. Electric field on the equatorial line of a dipole at any point, which is at distance r from the centre is given by

$$E = \frac{2kP}{(r^2 + a^2)^{3/2}} \quad \ldots \text{(i)}$$

where, P is the dipole moment of the charges.



In first case $\quad r = y$

$$\Rightarrow \quad E_1 = \frac{2kP}{(y^2 + a^2)^{3/2}}$$

Here, $\quad y^2 >> a^2$

$$\Rightarrow \quad y^2 + a^2 \approx y^2$$

$$\text{or} \quad E_1 = \frac{2kP}{y^3} \quad \ldots \text{(ii)}$$

So, force on the charge in its position at P will be

$$F = QE_1 = \frac{2kPQ}{y^3} \quad \ldots \text{(iii)}$$

In second case $r = y/3$

From Eq. (i), electric field at point P' will be

$$E_2 = \frac{2kP}{\left[\left(\frac{y}{3}\right)^2 + a^2\right]^{3/2}}$$

Again, $\frac{y}{3} >> a \Rightarrow \left(\frac{y}{3}\right)^2 + a^2 \approx \left(\frac{y}{3}\right)^2$

$$\Rightarrow \quad E_2 = \frac{2kP}{(y/3)^3} \Rightarrow E_2 = 27 \times \frac{2kP}{y^3}$$

Force on charge in this position,

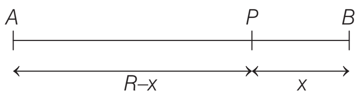
$$F' = QE_2 = 27 \times \frac{2kPQ}{y^3} \quad \ldots \text{(iv)}$$

From Eqs. (iii) and (iv), we get

$$F' = 27F$$

5. **Key Idea** As, dipole moments points in same direction

So, potential of both dipoles can be same at some point between A and B. Let potentials are same at P, distant x from B as shown below



$$\text{Then,} \quad \frac{4qa}{(R-x)^2} = \frac{2qa}{(x)^2}$$

$$2x^2 = (R-x)^2$$

$$\sqrt{2}x = R - x \quad \Rightarrow x = \frac{R}{\sqrt{2}+1}$$

Distance from A is

$$\Rightarrow \quad R - x = R - \frac{R}{\sqrt{2}+1} = \frac{\sqrt{2}R}{\sqrt{2}+1}$$

6. Electric field at a distance 'h' from the centre of uniformly charged ring of total charge q (say) on its axis is given as,

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qh}{(h^2 + R^2)^{3/2}}$$

For the magnitude to be maximum, then

$$\frac{dE}{dh} = 0$$

$$\Rightarrow \quad \frac{dE}{dh} = \frac{q}{4\pi\varepsilon_0}\left[\frac{(h^2+R^2)^{3/2} - h[3/2(h^2+R^2)^{1/2}\,2h]}{(h^2+R^2)^3}\right] = 0$$

$$\Rightarrow \quad 0 = \frac{(h^2+R^2)^{3/2} - 3h^2(h^2+R^2)^{1/2}}{(h^2+R^2)^3}$$

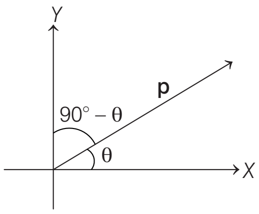
$$\Rightarrow (h^2+R^2)^{3/2} = 3h^2(h+R^2)^{1/2} \Rightarrow 3h^2 = (h^2+R^2)$$

$$3h^2 - h^2 = R^2$$

$$2h^2 = R^2 \Rightarrow h = \pm\frac{R}{\sqrt{2}}$$

$\therefore$ At $\frac{R}{\sqrt{2}}$, the value of electric field associated with a charged ring on its axis has the maximum value.

7.



Torque applied on a dipole, $\tau = pE\sin\theta$

where, θ = angle between axis of dipole and electric field.

For electric field $E_1 = E\,\hat{\mathbf{i}}$.

it means field is directed along positive X direction, so angle between dipole and field will remain θ, therefore torque in this direction

$$E_1 = pE_1\sin\theta$$

In electric field $E_2 = \sqrt{3}\,E\hat{\mathbf{j}}$, it means field is directed along positive Y-axis, so angle between dipole and field will be $90 - \theta$

Torque in this direction $T_2 = pE\sin(90° - \theta)$.

$$= p\sqrt{3}\,E_1\cos\theta$$

According to question $\tau_2 = -\tau_1 \Rightarrow |\tau_2| = |\tau_1|$

$$\therefore \quad pE_1\sin\theta = p\sqrt{3}\,E_1\cos\theta$$

$$\tan\theta = \sqrt{3} \Rightarrow \tan\theta = \tan 60°$$

$$\therefore \quad \theta = 60°$$

8. Electric field lines originate from position charge and termination negative charge. They cannot form closed loops and they are smooth curves. Hence, the most appropriate answer is (d).

9. Charge will be induced in the conducting sphere, but net charge on it will be zero.

$\therefore$ Option (d) is correct.

10. Electric field is zero everywhere inside a metal (conductor) i.e. field lines do not enter a metal. Simultaneously these are perpendicular to a metal surface (equipotential surface).

11. Electric lines of force never form a closed loop.

12. Electric field lines never enter a metallic conductor ($E = 0$, inside a conductor) and they fall normally on the surface of a metallic conductor (because whole surface is at same potential and lines are perpendicular to equipotential surface).

13. From the behaviour of electric lines, we can say that Q_1 is positive and Q_2 is negative. Further, $|Q_1| > |Q_2|$

At some finite distance to the right of Q_2, electric field will be zero. Because electric field due to Q_1 is towards right (away from Q_1) and due to Q_2 is towards left (towards Q_2). But since magnitude of Q_1 is more, the two fields may cancel each other because distance of that point from Q_1 will also be more.

$\therefore$ The correct options are (a) and (d).

14. Under electrostatic condition, all points lying on the conductor are at same potential. Therefore, potential at A = potential at B. Hence, option (c) is correct. From Gauss theorem, total flux through the surface of the cavity will be q/ε_0.

NOTE Instead of an elliptical cavity, if it would had been a spherical cavity then options (a) and (b) were also correct.

15. Electric field lines of force does not represent the path of the charged particle but tangent to the path at any point on the line shows the direction of electric force on it and it is not always necessary that motion of the particle is in the direction of force acting on it.

16. (a) Applying energy conservation principle, increase in kinetic energy of the dipole = decrease in electrostatic potential energy of the dipole.

$\therefore$ Kinetic energy of dipole at distance d from origin

$$= U_i - U_f$$

or $$\text{KE} = 0 - (-\,\mathbf{p}\cdot\mathbf{E}) = \mathbf{p}\cdot\mathbf{E}$$

$$= (p\hat{\mathbf{i}})\cdot\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{d^2}\hat{\mathbf{i}}\right) = \frac{qp}{4\pi\varepsilon_0 d^2}$$

(b) Electric field at origin due to the dipole,

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\frac{2p}{d^3}\hat{\mathbf{i}} \quad (\mathbf{E}_{\text{axis}} \uparrow\uparrow \mathbf{p})$$

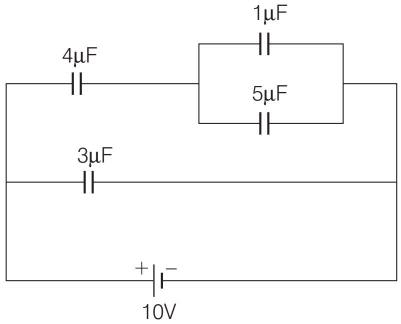
$\therefore$ Force on charge q

$$\mathbf{F} = q\mathbf{E} = \frac{pq}{2\pi\varepsilon_0 d^3}\,\hat{\mathbf{i}}$$

17. Force on the charged particle is along the tangent of electric line. A particle not always moves in the direction of force acting on it.

Topic 5 Capacitors

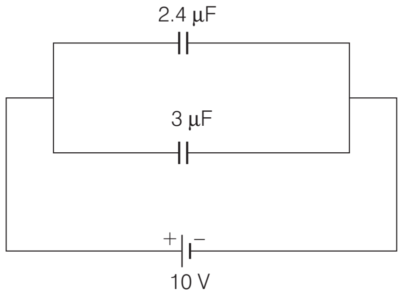
1. Given circuit is



In parallel, $C_{eq} = 5 + 1 = 6\,\mu F$

and in series, $C'_{eq} = \dfrac{6 \times 4}{6+4} = 2.4\,\mu F$

This is equivalent to



So, potential difference across upper branch $= 10\,V$

Using, $Q = C \times V$, charge delivered to upper branch is

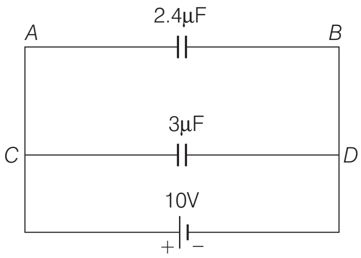
$$Q = C'_{eq} \cdot V = 2.4\,\mu F \times 10 V$$
$$= 24\,\mu C$$

As we know, in series connection, same charge is shared by capacitors, so charge on $4\,\mu F$ capacitor and $6\,\mu F$ capacitor would be same,

i.e., $$Q'_{4\mu F} = 24\,\mu C$$

Alternate Solution

The circuit obtained,



This can be further simplified as, $2.4\,\mu F$ and $3\,\mu F$ are in parallel.

So, net capacitance, $C_{net} = 2.4 + 3 = 5.4\,\mu F$

Net charge flow through circuit,

$$Q = C_{net} V = 5.4 \times 10 = 54\,\mu C$$

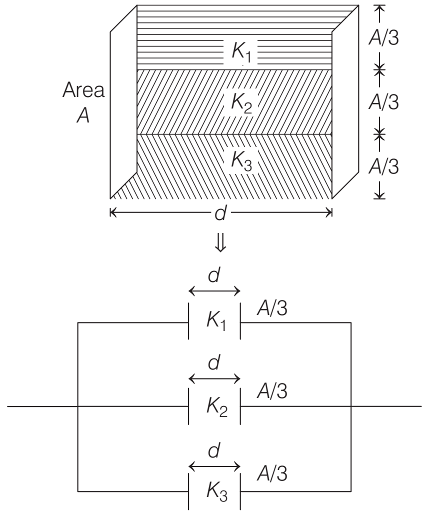
$\therefore$ This charge will be distributed in the ratio of capacitance in the two branches AB and CD as

$$\frac{Q_1}{Q_2} = \frac{2.4}{3} = \frac{4}{5} \Rightarrow 9x = 54\,\mu C \text{ or } x = 6\,\mu C$$

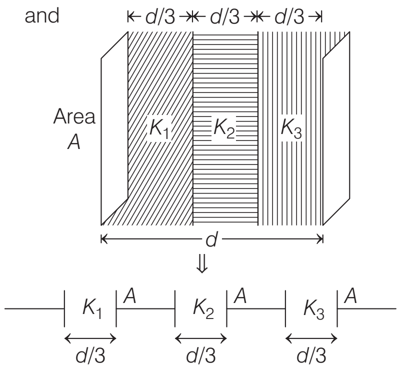
$\therefore$ Charge on $4\,\mu F$ capacitor is $= 4 \times 6\,\mu C = 24\,\mu C$

2. **Key Idea** A capacitor filled with dielectrics can be treated/compared as series/parallel combinations of capacitor having individual dielectric.

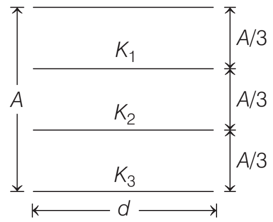
e.g.

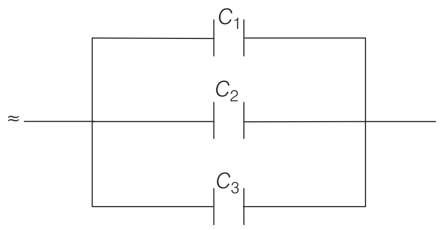


and



Case I





Capacitance in the equivalent circuit are

$$C_1 = \frac{\varepsilon_0\left(\frac{A}{3}\right)}{d} K_1 = \frac{\varepsilon_0 A}{3d} K_1,\ C_2 = \frac{\varepsilon_0\left(\frac{A}{3}\right)}{d} K_2 = \frac{\varepsilon_0 A}{3d} K_2$$

and $$C_3 = \frac{\varepsilon_0\left(\frac{A}{3}\right)}{d} K_3 = \frac{\varepsilon_0 A}{3d} K_3$$

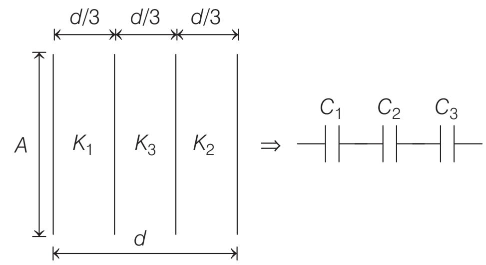
So, equivalent capacitance,

$$C_{\mathrm{I}} = C_1 + C_2 + C_3$$

$$= \frac{\varepsilon_0 A}{3d} K_1 + \frac{\varepsilon_0 A}{3d} K_2 + \frac{\varepsilon_0 A}{3d} K_3$$

$$C_{\mathrm{I}} = \frac{\varepsilon_0 A}{3d}(K_1 + K_2 + K_3) \quad \ldots\text{(i)}$$

Case II



Capacitance of equivalent circuit are

$$C_1 = \frac{\varepsilon_0 A}{\left(\frac{d}{3}\right)} \cdot K_1 = \frac{3\varepsilon_0 A}{d} K_1$$

$$C_2 = \frac{\varepsilon_0 A}{\left(\frac{d}{3}\right)} K_2 = \frac{3\varepsilon_0 A}{d} K_2$$

and $$C_3 = \frac{\varepsilon_0 A}{\left(\frac{d}{3}\right)} K_3 = \frac{3\varepsilon_0 A}{d} K_3$$

So, equivalent capacitance,

$$\frac{1}{C_{\mathrm{II}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$= \frac{d}{3\varepsilon_0 AK_1} + \frac{d}{3\varepsilon_0 AK_2} + \frac{d}{3\varepsilon_0 AK_3}$$

$$\Rightarrow \frac{1}{C_{\mathrm{II}}} = \frac{d}{3\varepsilon_0 A}\left[\frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3}\right]$$

$$= \frac{d}{3\varepsilon_0 A}\left[\frac{K_2K_3 + K_1K_3 + K_1K_2}{K_1K_2K_3}\right]$$

$$C_{\mathrm{II}} = \frac{3\varepsilon_0 A}{d}\left[\frac{K_1 K_2 K_3}{K_1 K_2 + K_2 K_3 + K_3 K_1}\right] \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{C_{\mathrm{I}}}{C_{\mathrm{II}}} = \frac{\varepsilon_0 A}{3d}(K_1 + K_2 + K_3) \times \frac{d(K_1K_2 + K_2K_3 + K_3K_1)}{3\varepsilon_0 A(K_1K_2K_3)}$$

$$= \frac{(K_1 + K_2 + K_3)(K_1K_2 + K_2K_3 + K_3K_1)}{9K_1K_2K_3}$$

Now, energy stored in capacitor, $E = \frac{1}{2}CV^2$

$$\Rightarrow \quad E \propto C$$

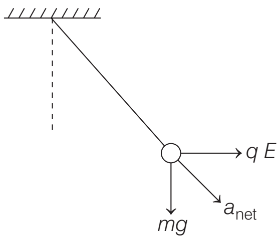
$$\therefore \quad \frac{E_{\mathrm{I}}}{E_{\mathrm{II}}} = \frac{C_{\mathrm{I}}}{C_{\mathrm{II}}}$$

3. When pendulum is oscillating between capacitor plates, it is subjected to two forces;

(i) Weight downwards = mg

(ii) Electrostatic force acting horizontally = qE

So, net acceleration of pendulum bob is resultant of accelerations produced by these two perpendicular forces.



Net acceleration is, $a_{\mathrm{net}} = \sqrt{a_1^2 + a_2^2} = \sqrt{g^2 + \left(\frac{qE}{m}\right)^2}$

So, time period of oscillations of pendulum is

$$T = 2\pi\sqrt{\frac{l}{a_{\mathrm{net}}}} = 2\pi\sqrt{\frac{L}{\sqrt{g^2 + \left(\frac{qE}{m}\right)^2}}}$$

4. In the given figure,

Slope of OA > Slope of OB

Since, we know that, net capacitance of parallel combination > net capacitance of series combination

$\therefore$ Parallel combination's capacitance,

$$C_P = C_1 + C_2 = \frac{500\,\mu\mathrm{C}}{10\ \mathrm{V}} = 50\,\mu\mathrm{F} \quad \ldots\text{(i)}$$

Series combination's capacitance,

$$C_S = \frac{C_1C_2}{C_1 + C_2} = \frac{80\,\mu\mathrm{C}}{10\mathrm{V}} = 8\,\mu\mathrm{F} \quad \ldots\text{(ii)}$$

or $$C_1C_2 = 8 \times (C_1 + C_2) = 8 \times 50\,\mu\mathrm{F}$$
$$= 400\,\mu\mathrm{F} \quad \text{[using Eq. (i)]} \quad \ldots\text{(iii)}$$

From Eqs. (i) and (iii), we get

$$C_1 = 50 - C_2$$

and $$C_1C_2 = 400$$

$$\Rightarrow \quad C_2(50 - C_2) = 400$$

$$\Rightarrow \quad 50C_2 - C_2^2 = 400$$

or $$C_2^2 - 50C_2 + 400 = 0$$

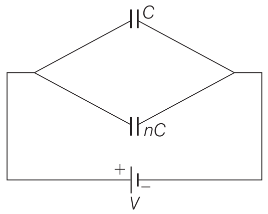
$$\Rightarrow C_2 = \frac{+50 \pm \sqrt{2500 - 1600}}{2} = \frac{+50 \pm 30}{2}$$

$\Rightarrow \quad C_2 = +40\,\mu F$ or $+10\,\mu F$

Also, $C_1 = 50 - C_2 \Rightarrow C_1 = +10\,\mu F$ or $+40\,\mu F$

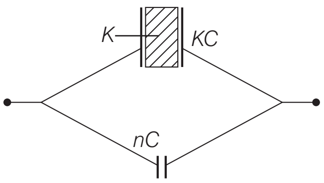
Hence, capacitance of two given capacitors is $10\,\mu F$ and $40\,\mu F$.

5. When parallel combination is fully charged, charge on the combination is



$$Q = C_{eq}V = C(1+n)V$$

When battery is removed and a dielectric slab is placed between two plates of first capacitor, then charge on the system remains same. Now, equivalent capacitance after insertion of dielectric is



$$C_{eq} = KC + nC = (n+K)C$$

If potential value after insertion of dielectric is V', then charge on system is

$$Q' = C_{eq}V' = (n+K)CV'$$

As $Q = Q'$, we have

$$C(1+n)V = (n+K)CV'$$

$$\therefore \quad V' = \frac{(1+n)V}{(n+K)}$$

6. Potential energy stored in a capacitor is

$$U = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C}$$

So, initial energy of the capacitor, $U_i = \frac{1}{2}Q^2/C_1$

Final energy of the capacitor, $U_f = \frac{1}{2}Q^2/C_2$

As we know, work done, $W = \Delta U = U_f - U_i$

$$= \frac{1}{2}Q^2\left[\frac{1}{C_2} - \frac{1}{C_1}\right]$$

Here, $Q = 5\,\mu C = 5\times 10^{-6}$ C,

$C_1 = 5\,\mu F = 5\times 10^{-6}$F,

$C_2 = 2\,\mu F = 2\times 10^{-6}$ F

$$\Rightarrow \quad \Delta U = \frac{1}{2}\times(5\times 10^{-6})^2\left[\frac{1}{2\times 10^{-6}} - \frac{1}{5\times 10^{-6}}\right]$$

$$= \frac{1}{2}\times\frac{5\times 5\times 10^{-12}}{10^{-6}}\times\frac{3}{10}$$

$$= \frac{25\times 3}{20}\times 10^{-6}\text{ J}$$

$$\Rightarrow \quad \Delta U = 3.75\times 10^{-6}\text{ J}$$

$\therefore$ Work done in reducing the capacitance from $5\,\mu F$ to $2\,\mu F$ by pulling plates of capacitor apart is 3.75×10^{-6} J.

7. Net value of charge on plates of capacitor after steady state is reached is

$$q_{net} = \frac{q_2 - q_1}{2}$$

where, q_2 and q_1 are the charges given to plates.

(Note that this formula is valid for any polarity of charge.)

Here, $q_2 = 4\mu C$, $q_1 = 2\mu C$

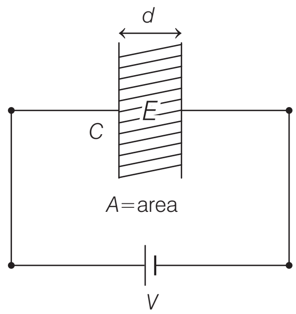
$\therefore$ Charge of capacitor is $q = \Delta q_{net} = \frac{4-2}{2} = 1\,\mu C$

Potential difference between capacitor plates is

$$V = \frac{Q}{C} = \frac{1\mu C}{1\mu F} = 1\text{ V}$$

8. As we know, capacitance of a capacitor filled with dielectric medium,

$$C = \frac{\varepsilon_0 KA}{d} \quad \ldots(i)$$



and potential difference between plates is

$$E = \frac{V}{d} \Rightarrow d = \frac{V}{E} \quad \ldots(ii)$$

So, by combining both Eqs. (i) and (ii), we get

$$K = \frac{CV}{\varepsilon_0 AE} \quad \ldots(iii)$$

Given, $C = 15\text{pF} = 15\times 10^{-12}$ F,

$V = 500$ V, $E = 10^6$ Vm^{-1},

$A = 10^{-4}$ m^2

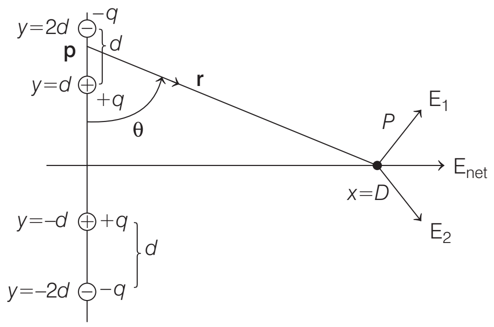
and $\varepsilon_0 = 8.85\times 10^{-12}$ C^{2}N^{-1}m^{-2}

Substituting the values in Eq. (iii), we get

$$K = \frac{15\times 10^{-12}\times 500}{8.85\times 10^{-12}\times 10^{-4}\times 10^6}$$

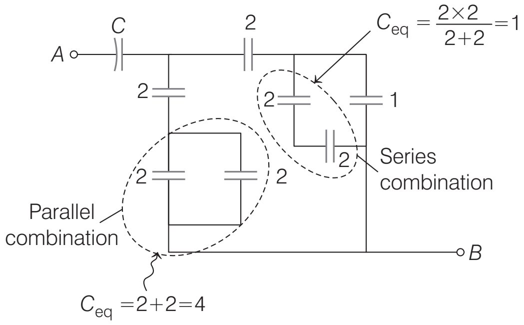
$$= 8.47 \approx 8.5$$

9. **Key Idea** Two capacitors C_1 and C_2, if connected in series, then their equivalent capacitance is

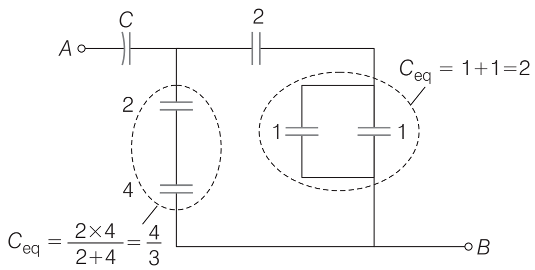


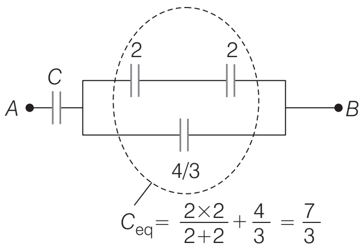
$$C_{eq} = \frac{C_1 \times C_2}{C_1 + C_2}$$

If they are connected in parallel, then their equivalent capacitance is, $C_{eq} = C_1 + C_2$.



We simplify given circuit as





So, $$C_{AB} = \frac{C \times 7/3}{C + 7/3} = \frac{1}{2} \quad \text{(given)}$$

$$\Rightarrow \quad \frac{7C}{3} = \frac{C}{2} + \frac{7}{6}$$

$$\Rightarrow \quad \left(\frac{14-3}{6}\right) C = \frac{7}{6} \Rightarrow C = \frac{7}{11}\,\mu\text{F}$$

10. If Q = charge on each plate, then

$$Q = CV = \frac{\varepsilon_0 A}{d} \cdot Ed = \varepsilon_0 AE$$

Here, $A = 1\text{ m}^2, E = 100\text{ N/C}$

and $$\varepsilon_0 = 8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N}-\text{m}^2}$$

So, by substituting given values, we get

$Q = 8.85 \times 10^{-12} \times 1 \times 100 = 8.85 \times 10^{-10}$ C

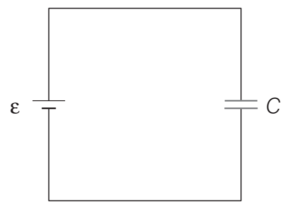
11. As we know, Current,

$$I = \frac{dq}{dt}$$

= Slope of q *versus* t graph

= Zero at t = 4s; (as graph is a line parallel to time axis at t = 4s)

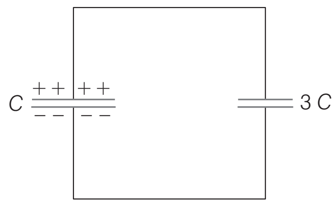
12. In position 'A' of switch, we have a capacitor joined with battery.



So, energy stored in position

$$U_1 = \frac{1}{2} C \varepsilon^2$$

When switch is turned to position B, we have a charged capacitor joined to an uncharged capacitor.



Common potential in steady state will be

$$V = \frac{\text{total charge}}{\text{total capacity}} = \frac{C\varepsilon}{4C} = \frac{\varepsilon}{4}$$

Now, energy stored will be

$$U_2 = \frac{1}{2} (C_{eq})(V_{common})^2$$

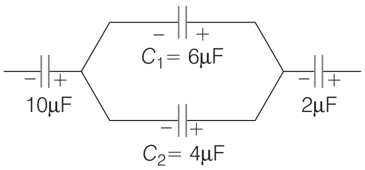
$$= \frac{1}{2} 4C \times \left(\frac{\varepsilon}{4}\right)^2 = \frac{1}{8} C\varepsilon^2$$

So, energy dissipated is

$$\Delta U = U_1 - U_2 = \frac{1}{2} C\varepsilon^2 - \frac{1}{8} C\varepsilon^2$$

$$= \frac{3}{8} C\varepsilon^2 = \frac{3}{8} C\left(\frac{Q}{C}\right)^2 = \frac{3}{8} \cdot \frac{Q^2}{C}$$

13. Applying the concept of charge conservation on isolated plates of 10 μF, 6 μF and 4 μF. Since, 6 μF and 4 μF are in parallel, so total charge on this combination will be 30 μC.



∴ Charge on 6 μF, capacitor

$$= \left(\frac{C_1}{C_1 + C_2}\right) q = \frac{6}{6+4} \times 30 = 18\,\mu\text{C}$$

Since, the charge has been asked on the right plate of the capacitor. Thus, it would be + 18 μC.

Alternative method

Let charge on 6 μF capacitor is q μC.

Now, V at 6 μF = V at 4 μF

$$\therefore \quad \frac{q}{6\,\mu\text{F}} = \frac{30 - q}{4\,\mu\text{F}} \qquad (\because V = q/C)$$

$$\Rightarrow \quad 4q = -6q + 180$$

$$\Rightarrow \quad q = 18\,\mu\text{C}.$$

14. Energy stored in a charged capacitor is given by

$$U = \frac{1}{2} CV^2 = \frac{1}{2} \cdot \frac{Q^2}{C} \qquad \ldots \text{(i)}$$

Here, $C = 12 \times 10^{-12}$ F and $V = 10$ V.

$$\Rightarrow \quad U = \frac{1}{2} \times 12 \times 10^{-12} \times 100$$

$$U = 6 \times 10^{-10}\ \text{J} \qquad \ldots \text{(ii)}$$

After insertion of slab, capacitance will be $C' = KC$ and final energy,

$$U' = \frac{1}{2} \cdot \frac{Q^2}{C'} = \frac{1}{2} \frac{Q^2}{KC}$$

$$\Rightarrow U' = \frac{1}{K} U = \frac{1}{6.5} \times 6 \times 10^{-10}\ \text{J} \quad \ldots \text{(iv)} \quad (\because \text{given}, K = 6.5)$$

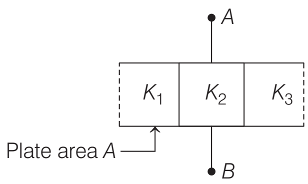
So, energy dissipated in the process will be equal to work done on the slab, i.e.

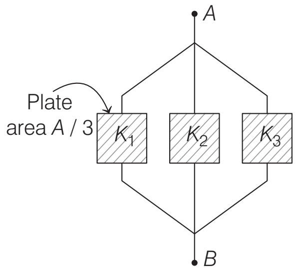
$$\Delta U = U - U' = \left(1 - \frac{1}{6.5}\right) \times 6 \times 10^{-10}\ \text{J}$$

$$\Rightarrow \Delta U = \frac{5.5}{6.5} \times 6 \times 10^{-10}\ \text{J}$$

$$\cong 5.08 \times 10^{-10}\ \text{J or 508 pJ}$$

15. In the given arrangement, capacitor can be viewed as three-different capacitors connected in parallel as shown below,





So, equivalent capacity of the system is

$$C_{\text{eq}} = C_1 + C_2 + C_3$$

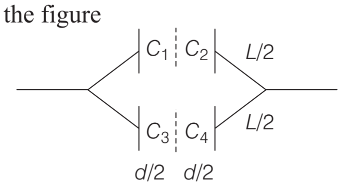
$$\Rightarrow \frac{K\varepsilon_0 A}{d} = \frac{K_1 \varepsilon_0 A/3}{d} + \frac{K_2 \varepsilon_0 A/3}{d} + \frac{K_3 \varepsilon_0 A/3}{d}$$

$$\Rightarrow \quad K = \frac{K_1}{3} + \frac{K_2}{3} + \frac{K_3}{3}$$

Here, $K_1 = 10$, $K_2 = 12$ and $K_3 = 14$

$$\text{So,} \quad K = \frac{10 + 12 + 14}{3} \Rightarrow K = 12$$

16 This capacitor system can be converted into two parts as shown in the figure



where C_1, C_2, C_3 and C_4 are capacitance of the capacitor having dielectric constants K_1, K_2, K_3 and K_4 respectively.

$$\text{Here,} \quad C_1 = \frac{K_1 \varepsilon_0 A/2}{d/2} = \frac{K_1 \varepsilon_0 A}{d}$$

Similarly, $C_2 = \dfrac{K_2 \varepsilon_0 A}{d}$, $C_3 = \dfrac{K_3 \varepsilon_0 A}{d}$ and $C_4 = \dfrac{K_4 \varepsilon_0 A}{d}$

Since, equivalent capacitance in series combination is

$$C_{\text{eq}} = \frac{C_1 \cdot C_2}{C_1 + C_2}$$

Here, C_1, C_2 and C_3, C_4 are in series combination.

$$\therefore \quad (C_{\text{eq}})_{12} = \frac{C_1 \cdot C_2}{C_1 + C_2} = \frac{\frac{K_1 \varepsilon_0 A}{d} \cdot \frac{K_2 \varepsilon_0 A}{d}}{\frac{K_1 \varepsilon_0 A}{d} + \frac{K_2 \varepsilon_0 A}{d}}$$

$$= \frac{K_1 \cdot K_2}{K_1 + K_2} \cdot \frac{\varepsilon_0 A}{d}$$

$$\text{Similarly,} \ (C_{\text{eq}})_{34} = \frac{K_3 \cdot K_4}{K_3 + K_4} \cdot \frac{\varepsilon_0 A}{d}$$

Now, $(C_{\text{eq}})_{12}$ and $(C_{\text{eq}})_{34}$ are in parallel combination.

$$\therefore \quad C_{\text{net}} = (C_{\text{eq}})_{12} + (C_{\text{eq}})_{34}$$

$$= \frac{K_1 \cdot K_2}{K_1 + K_2} \cdot \frac{\varepsilon_0 A}{d} + \frac{K_3 \cdot K_4}{K_3 + K_4} \cdot \frac{\varepsilon_0 A}{d}$$

$$\Rightarrow \quad C_{\text{net}} = \left(\frac{K_1 \cdot K_2}{K_1 + K_2} + \frac{K_3 \cdot K_4}{K_3 + K_4}\right) \frac{\varepsilon_0 A}{d} \qquad \ldots \text{(i)}$$

If K is effective dielectric constant, then

$$C_{net} = \frac{K\varepsilon_0 A}{d} \quad \text{...(ii)}$$

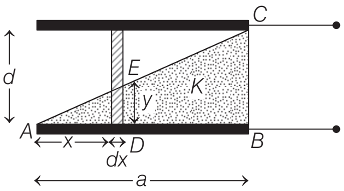
From Eqs. (i) and (ii),

$$\frac{K\varepsilon_0 A}{d} = \left(\frac{K_1 \cdot K_2}{K_1 + K_2} + \frac{K_3 \cdot K_4}{K_3 + K_4}\right)\frac{\varepsilon_0 A}{d}$$

or $$K = \left(\frac{K_1 \cdot K_2}{K_1 + K_2} + \frac{K_3 \cdot K_4}{K_3 + K_4}\right)$$

17. Let's consider a strip of thickness 'dx' at a distance of 'x' from the left end as shown in the figure. From the figure, ΔABC and ΔADE are similar triangles,

$$\Rightarrow \quad \frac{y}{x} = \frac{d}{a} \Rightarrow y = \left(\frac{d}{a}\right)x \quad \text{...(i)}$$



We know that, the capacitance of parallel plate capacitor,

$$C = \frac{\varepsilon_0 A}{d}$$

$$C_1 = \frac{\varepsilon_0 (adx)}{(d-y)} \text{ and } C_2 = \frac{K\varepsilon_0 (adx)}{y}$$

Here, two capacitor are placed in series with variable thickness, therefore

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} \Rightarrow C_{eq} = \frac{K\varepsilon_0 adx}{Kd + (1-K)y} \quad \text{...(ii)}$$

Now, integrate it from 0 to a

$$C = \int_0^a \frac{K\varepsilon_0 adx}{Kd + (1-K)y}$$

Using Eq. (i), $y = \left(\frac{d}{a}\right)x$, we get

$$C = \varepsilon_0 a \int_0^a \frac{dx}{d + \left(\frac{1}{K} - 1\right)\frac{d}{a}x}$$

$$\Rightarrow \quad C = \frac{\varepsilon_0 a}{\left(\frac{1-K}{K}\right)\frac{d}{a}} \ln\left[\frac{1}{K}\right]$$

$$\Rightarrow \quad C = \frac{\varepsilon_0 a^2 K \ln K}{(K-1)d}$$

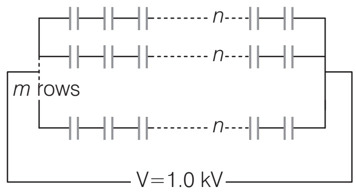
18. $$Q = KC_0 V$$

$$|Q_{in}| = \left|Q\left(1 - \frac{1}{K}\right)\right|$$

$$= (90 \times 10^{-12})(20)\left(\frac{5}{3}\right)\left(1 - \frac{3}{5}\right)\text{C}$$

$$= 1200 \times 10^{-12}\text{ C} = 1.2\,\mu\text{C}$$

19. Let there are n capacitors in a row with m such rows in parallel.



As voltage not to exceed 300 V

$$\therefore \quad n \times 300 > 1000$$

[a voltage greater than 1 kV to be withstand]

$$\Rightarrow \quad n > \frac{10}{3} \Rightarrow n = 4 \quad \text{(or 3.33)}$$

Also, $$C_{\mathbf{Eq}} = \frac{mC}{n} = 2\mu\mathbf{F}$$

$$\Rightarrow \quad \frac{m}{n} = 2 \Rightarrow m = 8 \quad [\because C = 1\mu\mathbf{F}]$$

So, total number of capacitors required

$$= m \times n = 8 \times 4 = 32$$

20. $3\,\mu$F and 9μF $= 12\mu$F

$$4\mu\text{ F and } 12\mu\text{F} = \frac{4 \times 12}{4 + 12} = 3\mu\text{F}$$

$$Q = CV = 3 \times 8 = 24\mu\text{C} \text{ (on } 4\mu\text{F and } 3\mu\text{F)}$$

Now, this 24μC distributes in direct ratio of capacity between 3μF and 9μF. Therefore,

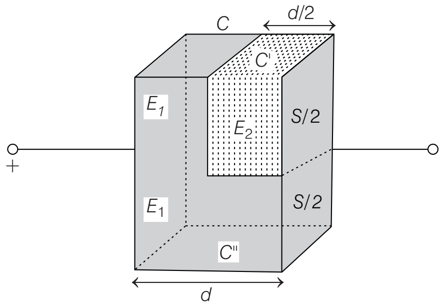
$$Q_{9\,\mu\text{F}} = 18\,\mu\text{C}$$

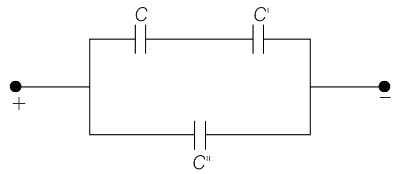
$$\therefore Q_{4\mu\text{F}} + Q_{9\,\mu\text{F}} = 24 + 18 = 42\mu\text{C} = Q \quad \text{(say)}$$

$$E = \frac{kQ}{R^2} = \frac{9 \times 10^9 \times 42 \times 10^{-6}}{30^2}$$

$$= 420 \text{ N/C}$$

21.





$$C_1 = \frac{\varepsilon_0 s}{d}, C = \frac{2\varepsilon_0 \frac{s}{2}}{\frac{d}{2}} = \frac{2\varepsilon_0 s}{d} \Rightarrow C' = \frac{4\varepsilon_0 \frac{s}{2}}{\frac{d}{2}} = \frac{4\varepsilon_0 s}{d}$$

and $$C'' = \frac{2\varepsilon_0 \frac{s}{2}}{d} = \frac{\varepsilon_0 s}{d}$$

$$C_2 = \frac{CC'}{C + C'} + C'' = \frac{4}{3}\frac{\varepsilon_0 s}{d} + \frac{\varepsilon_0 s}{d} = \frac{7}{3}\frac{\varepsilon_0 s}{d}\quad \frac{C_2}{C_1} = \frac{7}{3}$$

22. Resultant of 1 μF and 2 μF is 3 μF. Now in series, potential difference distributes in inverse ratio of capacity.

$$\therefore \quad \frac{V_{3\mu F}}{V_c} = \frac{c}{3} \quad \text{or} \quad V_{3\mu F} = \left(\frac{c}{c+3}\right)E$$

This is also the potential difference across 2 μF.

$$\therefore \quad Q_2 = (2\,\mu F)(V_{2\mu F})$$

$$\text{or} \quad Q_2 = \left(\frac{2cE}{c+3}\right) = \left(\frac{2}{1+\frac{3}{c}}\right)E$$

From this expression of Q_2, we can see that Q_2 will increase with increase in the value of c (but not linearly). Therefore, only options (a) and (b) may be correct.

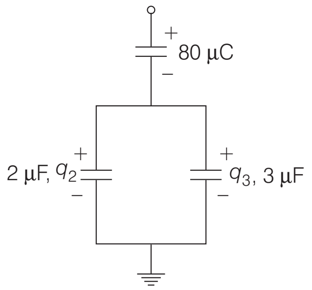
$$\text{Further,} \quad \frac{d}{dc}(Q_2) = 2E\left[\frac{(c+3)-c}{(c+3)^2}\right] = \frac{6E}{(c+3)^2}$$

= Slope of Q_2 verus c graph.

i.e. slope of Q_2 versus c graph decreases with increase in the value of c. Hence, the correct graph is (a).

23. Between 3 μF and 2 μF (in parallel), total charge of 80 μC will distribute in direct ratio of capacity.

$$\frac{q_3}{q_2} = \frac{3}{2}$$



$$q_3 = \left(\frac{3}{3+2}\right)(80) = 48\,\mu C$$

24. When free space between parallel plates of capacitor,

$$E = \frac{\sigma}{\varepsilon_0}$$

When dielectric is introduced between parallel plates of capacitor, $E' = \dfrac{\sigma}{K\varepsilon_0}$

Electric field inside dielectric, $\dfrac{\sigma}{K\varepsilon_0} = 3 \times 10^4$

where, K = dielectric constant of medium = 2.2

ε_0 = permitivity of free space = 8.85×10^{-12}

$$\Rightarrow \quad \sigma = 2.2 \times 8.85 \times 10^{-12} \times 3 \times 10^4$$
$$= 6.6 \times 8.85 \times 10^{-8} = 5.841 \times 10^{-7}$$
$$= 6 \times 10^{-7}\ \text{C/m}^2$$

25. $q_i = C_iV = 2V = q$ (say)

This charge will remain constant after switch is shifted from position 1 to position 2.

$$U_i = \frac{1}{2}\frac{q^2}{C_i} = \frac{q^2}{2\times 2} = \frac{q^2}{4}$$

$$U_f = \frac{1}{2}\frac{q^2}{C_f} = \frac{q^2}{2\times 10} = \frac{q^2}{20}$$

$$\therefore \text{Energy dissipated} = U_i - U_f = \frac{q^2}{5}$$

This energy dissipated $\left(= \dfrac{q^2}{5}\right)$ is 80% of the initial stored energy $\left(= \dfrac{q^2}{4}\right)$.

∴ Correct option is (d).

26. After time t, thickeness of liquid will remain $\left(\dfrac{d}{3} - vt\right)$.

Now, time constant as function of time

$$\tau_c = CR = \frac{\varepsilon_0(1)\cdot R}{(d - \frac{d}{3} + vt) + \frac{d/3 - vt}{2}}$$

$$\left(\text{Applying } C = \frac{\varepsilon_0 A}{d - t + \frac{t}{k}}\right)$$

$$= \frac{6\varepsilon_0 R}{5d + 3vt}$$

∴ Correct option is (a).

27. All the three plates will produce electric field at P along negative z-axis. Hence,

$$\mathbf{E}_p = \left[\frac{\sigma}{2\varepsilon_0} + \frac{2\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0}\right](-\hat{\mathbf{k}}) = -\frac{2\sigma}{\varepsilon_0}\hat{\mathbf{k}}$$

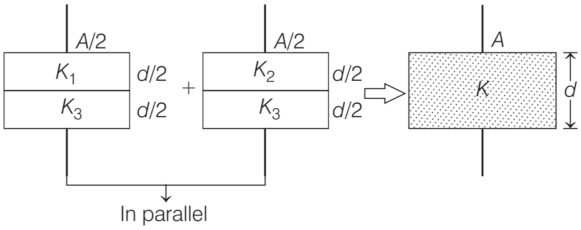
∴ Correct answer is (b).

28. ΔU = decrease in potential energy

$$= U_i - U_f$$
$$= \frac{1}{2}C(V_1^2 + V_2^2) - \frac{1}{2}(2C)\left(\frac{V_1 + V_2}{2}\right)^2$$
$$= \frac{1}{4}C(V_1 - V_2)^2$$

29. Due to attraction with positive charge, the negative charge on capacitor A will not flow through the switch S.

30. Applying $C = \dfrac{\varepsilon_0 A}{d - t_1 - t_2 + \frac{t_1}{K_1} + \frac{t_2}{K_2}}$, we have



$$\frac{\varepsilon_0(A/2)}{d - d/2 - d/2 + \frac{d/2}{K_1} + \frac{d/2}{K_3}} + \frac{\varepsilon_0(A/2)}{d - d/2 - d/2 + \frac{d/2}{K_2} + \frac{d/2}{K_3}} = \frac{K\varepsilon_0 A}{d}$$

Solving this equation, we get

$$K = \frac{K_1K_3}{K_1 + K_3} + \frac{K_2K_3}{K_2 + K_3}$$

31. When S_3 is closed, due to attraction with opposite charge, no flow of charge takes place through S_3. Therefore, potential difference across capacitor plates remains unchanged or $V_1 = 30$ V and $V_2 = 20$ V.

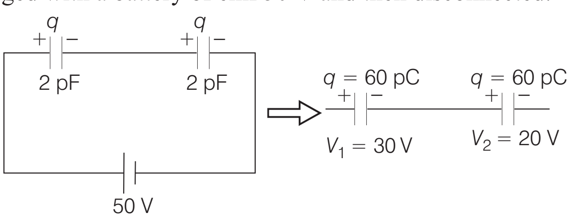
Alternate Solution

Charges on the capacitors are

$$q_1 = (30)(2) = 60 \text{ pC}$$

and $q_2 = (20)(3) = 60$ pC or $q_1 = q_2 = q$ (say)

The situation is similar as the two capacitors in series are first charged with a battery of emf 50 V and then disconnected.

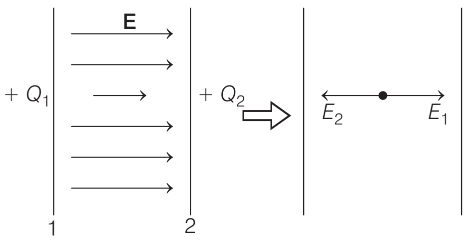


∴ When S_3 is closed,

$$V_1 = 30 \text{ V}$$

and $$V_2 = 20 \text{ V}.$$

32. Electric field within the plates $\mathbf{E} = \mathbf{E}_{Q_1} + \mathbf{E}_{Q_2}$

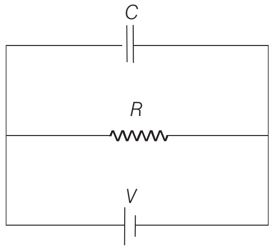


$$E = E_1 - E_2 = \frac{Q_1}{2A\varepsilon_0} - \frac{Q_2}{2A\varepsilon_0} \Rightarrow E = \frac{Q_1 - Q_2}{2A\varepsilon_0}$$

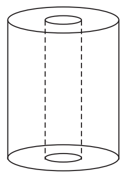
∴ Potential difference between the plates

$$V_A - V_B = Ed = \left(\frac{Q_1 - Q_2}{2A\varepsilon_0}\right)d = \frac{Q_1 - Q_2}{2\left(\frac{A\varepsilon_0}{d}\right)} = \frac{Q_1 - Q_2}{2C}$$

33. Since, the capacitor plates are directly connected to the battery, it will take no time in charging.



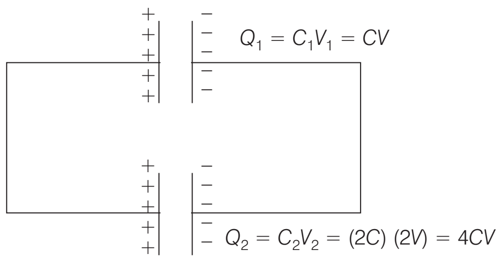
34. The magnitude of electric field at a distance r from the axis is given as :



$$E = \frac{\lambda}{2\pi\,\varepsilon_0 r} \text{ i.e. } E \propto \frac{1}{r}$$

Here, λ is the charge per unit length of the capacitor.

35. The diagramatic representation of given problem is shown in figure.



The net charge shared between the two capacitors is

$$Q' = Q_2 - Q_1 = 4CV - CV = 3CV$$

The two capacitors will have the same potential, say V'.

The net capacitance of the parallel combination of the two capacitors will be

$$C' = C_1 + C_2 = C + 2C = 3C$$

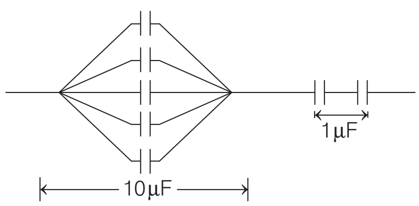
The potential difference across the capacitors will be

$$V' = \frac{Q'}{C'} = \frac{3CV}{3C} = V$$

The electrostatic energy of the capacitors will be

$$U' = \frac{1}{2}C'V'^2 = \frac{1}{2}(3C)V^2 = \frac{3}{2}CV^2$$

36. In series, $C = \frac{C_1C_2}{C_1 + C_2}$



∴ $$C_{\text{net}} = \frac{(10)(1)}{10 + 1} = \frac{10}{11}\ \mu\text{F}$$

37.
$$C = C_1 + C_2$$
$$C_1 = \frac{K\varepsilon_0 A/3}{d}$$
$$C_2 = \frac{\varepsilon_0 2A/3}{d}$$
$$\Rightarrow \quad C = \frac{(K+2)\varepsilon_0 A}{3d}$$
$$\Rightarrow \quad \frac{C}{C_1} = \frac{K+2}{K}$$

Also, $E_1 = E_2 = V/d$, where V is potential difference between the plates.

38. After pressing S_1 charge on upper plate of C_1 is $+2CV_0$.

After pressing S_2 this charge equally distributes in two capacitors. Therefore, charge an upper plates of both capacitors will be $+CV_0$.

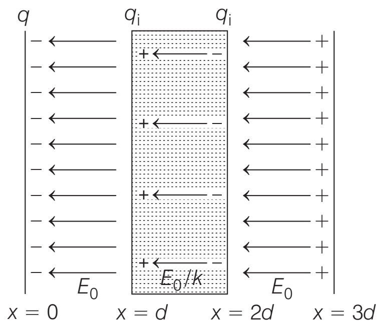
When S_2 is released and S_3 is pressed, charge on upper plate of C_1 remains unchanged $(= +CV_0)$ but charge on upper plate of C_2 is according to new battery $(= -CV_0)$.

39. Polarity should be mentioned in the question. Potential on each of them can be zero if, $q_{net} = 0$

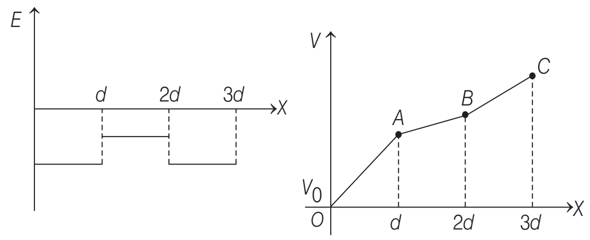
or $\quad q_1 \pm q_2 = 0$

or $\quad 120C_1 \pm 200C_2 = 0$ or $3C_1 \pm 5C_2 = 0$

40. The magnitude and direction of electric field at different points are shown in figure. The direction of the electric field remains the same. Hence, option (b) is correct. Similarly, electric lines always flow from higher to lower potential, therefore, electric potential increases continuously as we move from $x = 0$ to $x = 3d$.



Therefore, option (c) is also correct. The variation of electric field (E) and potential (V) with x will be as follows



$OA \parallel BC$ and $(\text{Slope})_{OA} > (\text{Slope})_{AB}$

Because $\quad E_{O-d} = E_{2d-3d}$

and $\quad E_{O-d} > E_{d-2d}$

41. Battery is removed. Therefore, charge stored in the plates will remain constant.
$$Q = CV = \frac{\varepsilon_0 A}{d}V \quad \text{or } Q = \text{constant.}$$

Now, dielectric slab is inserted. Therefore, C will increase. New capacity will be,
$$C' = KC = \frac{\varepsilon_0 KA}{d} \Rightarrow V' = \frac{Q}{C'} = \frac{V}{K}$$

and new electric field, $E = \frac{V'}{d} = \frac{V}{K \cdot d}$

Potential energy stored in the capacitor,

Initially, $\quad U_i = \frac{1}{2}CV^2 = \frac{\varepsilon_0 AV^2}{2d}$

Finally, $\quad U_f = \frac{1}{2}C'V'^2 = \frac{1}{2}\left(\frac{K\varepsilon_0 A}{d}\right)\left(\frac{V}{K}\right)^2 = \frac{\varepsilon_0 AV^2}{2Kd}$

Work done on the system will be
$$|\Delta U| = \frac{\varepsilon_0 AV^2}{2d}\left(1 - \frac{1}{K}\right)$$

$\therefore$ Correct options are (a), (c) and (d).

42. Charging battery is removed. Therefore, q = constant

Distance between the plates is increased. Therefore, C decreases.

Now, $V = q/C$, q is constant and C is decreasing. Therefore, V should increase.

$U = \frac{1}{2}\frac{q^2}{C}$ again q is constant and C is decreasing.

Therefore U should increase.

$\therefore$ Correct options are (b) and (d).

43. When dielectric slab is introduced capacity gets increased while potential difference remains unchanged.

$\therefore \quad V = V_0,\ C > C_0$

$Q = CV \quad \therefore \quad Q > Q_0$

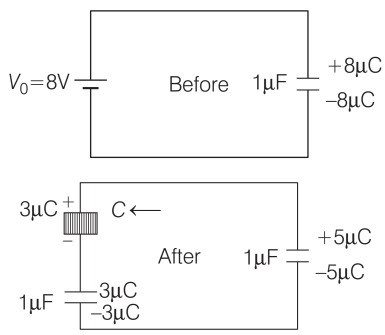
$U = \frac{1}{2}CV^2 \quad \therefore \quad U > U_0$

$E = \frac{V}{d}$ but V and d both are unchanged.

Therefore, $\quad E = E_0$

Therefore, correct options are (a) and (d).

44.



$C = \varepsilon_r C_1 = (\varepsilon_r)\,\mu F$

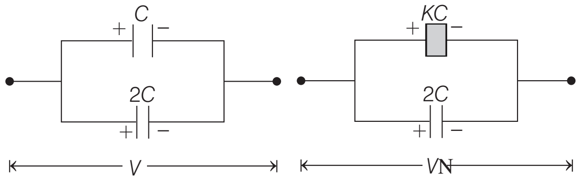
Applying loop rule,

$$\frac{5}{1}-\frac{3}{\varepsilon_r}-\frac{3}{1}=0 \Rightarrow \frac{3}{\varepsilon_r}=2$$

$$\varepsilon_r = 1.50$$

45. Total charge will remain unchanged.

Hence, $Q = Q'$ or $3CV = (KC + 2C)V'$

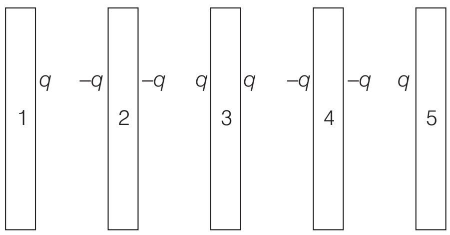


$$\therefore \qquad V' = \left(\frac{3}{K+2}\right)V$$

46. In the circuit shown in figure, there is a capacitor between plates 1 and 2 , the capacity of which is : $C_1 = \dfrac{\varepsilon_0 A}{d}$ and potential difference between its plates is V. Therefore, charge stored in it is, $q = C_1V = \dfrac{\varepsilon_0 AV}{d}$

Since, plate 1 is connected with positive terminal, hence this charge q will be positive.

Plate 4 is making two capacitors, one with 3 and other with 5. Hence, charge on it will be $-2q$ or $\dfrac{-2\varepsilon_0 AV}{d}$. Charge on it is negative because this is connected with negative plate. Charges on both sides of the plates are shown below.

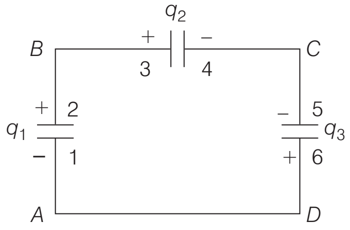


Here, $\qquad q = \dfrac{\varepsilon_0 AV}{d}$

47. Electric field between the plates of capacitor is almost uniform. Therefore, force on both the protons will be identical. It hardly matters whether they are placed near positive plate or negative plate.

48. (a) Charge on capacitor A, before joining with an uncharged capacitor

$$q_A = CV = (100)(3)\mu C = 300\,\mu C$$



Similarly, charge on capacitor B

$$q_B = (180)(2)\,\mu C = 360\,\mu C$$

Let q_1, q_2 and q_3 be the charges on the three capacitors after joining them as shown in figure.

(q_1, q_2 and q_3 are in microcoulombs)

From conservation of charge

Net charge on plates 2 and 3 before joining

= net charge after joining

$$\therefore \qquad 300 = q_1 + q_2 \qquad \ldots\text{(i)}$$

Similarly, net charge on plates 4 and 5 before joining

$$= \text{net charge after joining } -360 = -q_2 - q_3$$

$$\text{or} \qquad 360 = q_2 + q_3 \qquad \ldots\text{(ii)}$$

Applying Kirchhoff's second law in closed loop $ABCDA$

$$\frac{q_1}{3}-\frac{q_2}{2}+\frac{q_3}{2}=0$$

$$\text{or} \qquad 2q_1 - 3q_2 + 3q_3 = 0 \qquad \ldots\text{(iii)}$$

Solving Eqs. (i) , (ii) and (iii), we get

$$q_1 = 90\,\mu C,\ q_2 = 210\,\mu C$$

$$\text{and} \qquad q_3 = 150\ \mu C$$

(b) (i) Electrostatic energy stored before, completing the circuit

$$U_i = \frac{1}{2}(3\times10^{-6})(100)^2 + \frac{1}{2}(2\times10^{-6})(180)^2 \qquad \left(\because U = \frac{1}{2}CV^2\right)$$

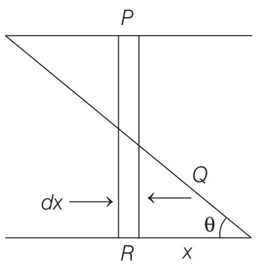
$$= 4.74\times10^{-2}\ \text{J or } U_i = 47.4\ \text{mJ}$$

(ii) Electrostatic energy stored after, completing the circuit

$$U_f = \frac{1}{2}\frac{(90\times10^{-6})^2}{(3\times10^{-6})} + \frac{1}{2}\frac{(210\times10^{-6})^2}{(2\times10^{-6})} + \frac{1}{2}\frac{(150\times10^{-6})^2}{(2\times10^{-6})} \qquad \left[U = \frac{1}{2}\frac{q^2}{C}\right]$$

$$= 1.8\times10^{-2}\ \text{J} \quad \text{or} \quad U_f = 18\ \text{mJ}$$

49. Let length and breadth of the capacitor be l and b respectively and d be the distance between the plates as shown in figure. Then, consider a strip at a distance x of width dx.



Now, $\qquad QR = x \tan\theta$

and $\qquad PQ = d - x\tan\theta$

where, $\tan\theta = d/l$

Capacitance of PQ

$$C_1 = \frac{K_1\varepsilon_0(bdx)}{d - x\tan\theta} = \frac{K_1\varepsilon_0(bdx)}{d - \frac{xd}{l}}$$

$$C_1 = \frac{K_1\varepsilon_0 bldx}{d(l-x)} = \frac{K_1\ \varepsilon_0 A(dx)}{d(l-x)}$$

and $\quad C_2 =$ capacitance of $QR = \dfrac{K_2\varepsilon_0\, b(dx)}{x\tan\theta}$

$$C_2 = \frac{K_2\varepsilon_0 A\ (dx)}{xd} \qquad \left(\tan\theta = \frac{d}{l}\right)$$

Now, C_1 and C_2 are in series. Therefore, their resultant capacity C_0 will be given by

$$\frac{1}{C_0} = \frac{1}{C_1} + \frac{1}{C_2}$$

Then, $\dfrac{1}{C_0} = \dfrac{1}{C_1} + \dfrac{1}{C_2} = \dfrac{d\ (l-x)}{K_1\ \varepsilon_0 A\ (dx)} + \dfrac{x.d}{K_2\varepsilon_0 A(dx)}$

$$\frac{1}{C_0} = \frac{d}{\varepsilon_0 A(dx)}\left(\frac{l-x}{K_1} + \frac{x}{K_2}\right)$$

$$= \frac{d\ \{K_2(l-x) + K_1 x\}}{\varepsilon_0 A K_1 K_2 (dx)}$$

$$\therefore \quad C_0 = \frac{\varepsilon_0 A K_1 K_2}{d\ \{K_2(l-x) + K_1 x\}}dx$$

$$C_0 = \frac{\varepsilon_0 A K_1 K_2}{d\{K_2 l + (K_1 - K_2)\,x\}}dx$$

Now, the net capacitance of the given parallel plate capacitor is obtained by adding such infinitesimal capacitors placed parallel from $x = 0$ to $x = l$

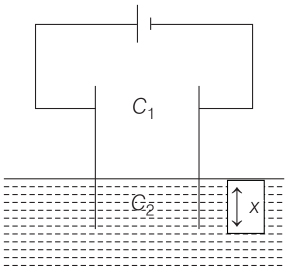
i.e. $\quad C_R = \displaystyle\int_{x=0}^{x=l} C_0 = \int_0^l \frac{\varepsilon_0 A K_1 K_2}{d\ \{K_2 l + (K_1 - K_2)x\}}\,dx$

Finally we get $C_R = \dfrac{K_1 K_2 \varepsilon_0 A}{(K_2 - K_1)d}\ln\dfrac{K_2}{K_1}$

$$= \frac{CK_1K_2}{K_2 - K_1}\ln\frac{K_2}{K_1} \qquad \text{where, } C = \frac{\varepsilon_0 A}{d}$$

50. Let a be the side of the square plate.

As shown in figure, C_1 and C_2 are in parallel. Therefore, total capacity of capacitors in the position shown is



$$C = C_1 + C_2$$

$$C = \frac{\varepsilon_0 a(a-x)}{d} + \frac{K\varepsilon_0 ax}{d}$$

$$\therefore \quad q = CV = \frac{\varepsilon_0 aV}{d}(a - x + Kx)$$

As plates are lowered in the oil, C increases or charge stored will increase.

Therefore, $\quad i = \dfrac{dq}{dt} = \dfrac{\varepsilon_0 aV}{d}(K-1)\cdot\dfrac{dx}{dt}$

Substituting the values

$$\varepsilon_0 = 8.85\times10^{-12}\ \text{C}^2/\text{N-m}^2$$

$$a = 1\,\text{m},\ V = 500\,\text{volt},\ d = 0.01\,\text{m},\ K = 11$$

and $\quad \dfrac{dx}{dt} =$ speed of plate $= 0.001\,\text{m/s}$

We get current $\ i = \dfrac{(8.85\times10^{-12})(1)(500)\,(11-1)\,(0.001)}{(0.01)}$

$$i = 4.43\times10^{-9}\ \text{A}$$

51. (a) Capacitor A is a combination of two capacitors C_K and C_O in parallel. Hence,

$$C_A = C_K + C_O = \frac{K\varepsilon_0 A}{d} + \frac{\varepsilon_0 A}{d} = (K+1)\frac{\varepsilon_0 A}{d}$$

Here, $A = 0.02\ \text{m}^2$. Substituting the values, we have

$$C_A = (9+1)\frac{8.85\times10^{-12}\ (0.02)}{(8.85\times10^{-4})}$$

$$C_A = 2.0\times10^{-9}\ \text{F}$$

Energy stored in capacitor A, when connected with a 110 V battery is

$$U_A = \frac{1}{2}C_A V^2 = \frac{1}{2}(2\times10^{-9})\,(110)^2$$

$$U_A = 1.21\times10^{-5}\ \text{J}$$

(b) Charge stored in the capacitor

$$q_A = C_A V = (2.0\times10^{-9})\,(110) \Rightarrow q_A = 2.2\times10^{-7}\text{C}$$

Now, this charge remains constant even after battery is disconnected. But when the slab is removed, capacitance of A will get reduced. Let it be C'_A

$$C'_A = \frac{\varepsilon_0(2A)}{d} = \frac{(8.85\times10^{-12})\,(0.04)}{8.85\times10^{-4}}$$

$$C'_A = 0.4\times10^{-9}\ \text{F}$$

Energy stored in this case would be

$$U'_A = \frac{1}{2}\frac{(q_A)^2}{C'_A} = \frac{1}{2}\frac{(2.2\times10^{-7})^2}{(0.4\times10^{-9})}$$

$$U'_A = 6.05\times10^{-5}\ \text{J} > U_A$$

Therefore, work done to remove the slab would be

$$W = U'_A - U_A = (6.05 - 1.21)\times10^{-5}\ \text{J}$$

or $\quad W = 4.84\times10^{-5}\ \text{J}$

(c) Capacity of B when filled with dielectric is

$$C_B = \frac{K\varepsilon_0 A}{d} = \frac{(9)\,(8.85\times10^{-12})\,(0.02)}{(8.85\times10^{-4})}$$

$$C_B = 1.8\times10^{-9}\ \text{F}$$

These two capacitors are in parallel. Therefore, net capacitance of the system is

$$C = C'_A + C_B = (0.4 + 1.8) \times 10^{-9}\ \text{F}$$

$$C = 2.2 \times 10^{-9}\ \text{F}$$

Charge stored in the system is $q = q_A = 2.2 \times 10^{-7}\text{C}$

Therefore, energy stored, $U = \dfrac{1}{2}\dfrac{q^2}{C}$

$$U = \frac{1}{2}\frac{(2.2 \times 10^{-7})^2}{(2.2 \times 10^{-9})} \quad \text{or} \quad U = 1.1 \times 10^{-5}\ \text{J}$$

52. Before opening the switch potential difference across both the capacitors is V, as they are in parallel. Hence, energy stored in them is,

$$U_A = U_B = \frac{1}{2}CV^2$$

$\therefore \quad U_{\text{Total}} = CV^2 = U_i \quad \ldots\text{(i)}$

After opening the switch, potential difference across it is V and its capacity is $3C$

$\therefore \quad U_A = \dfrac{1}{2}(3C)V^2 = \dfrac{3}{2}CV^2$

In case of capacitor B, charge stored in it is $q = CV$ and its capacity is also $3C$.

Therefore, $U_B = \dfrac{q^2}{2(3C)} = \dfrac{CV^2}{6}$

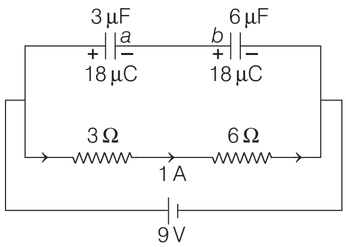
$\therefore \quad U_{\text{Total}} = \dfrac{3CV^2}{2} + \dfrac{CV^2}{6} = \dfrac{10}{6}CV^2 = \dfrac{5CV^2}{3} = U_f \quad \ldots\text{(ii)}$

From Eqs. (i) and (ii), we get $\dfrac{U_i}{U_f} = \dfrac{3}{5}$

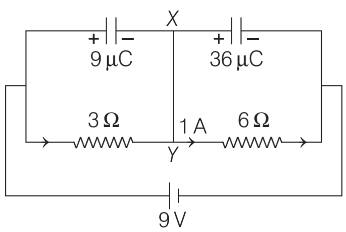
Topic 6 *C-R* Circuits

1. From Y to X charge flows to plates a and b.

$$(q_a + q_b)_i = 0,\ (q_a + q_b)_f = 27\,\mu\text{C}$$



Initial figure (when switch was open)



Final figure (when switch is closed)

$\therefore$ 27 μC charge flows from Y to X.

$\therefore$ Correct option is (c).

2. $\tau = CR$

$$\tau_1 = (C_1 + C_2)(R_1 + R_2) = 18\,\mu\text{s}$$

$$\tau_2 = \left(\frac{C_1C_2}{C_1 + C_2}\right)\left(\frac{R_1R_2}{R_1 + R_2}\right) = \frac{8}{6} \times \frac{2}{3} = \frac{8}{9}\,\mu\text{s}$$

$$\tau_3 = (C_1 + C_2)\left(\frac{R_1R_2}{R_1 + R_2}\right) = (6)\left(\frac{2}{3}\right) = 4\,\mu\text{s}$$

3. Given : $V_C = 3V_R = 3(V - V_C)$

Here, V is the applied potential.

$\therefore \quad V_C = \dfrac{3}{4}V \quad \text{or} \quad V(1 - e^{-t/\tau_c}) = \dfrac{3}{4}V$

$\therefore \quad e^{-t/\tau_c} = \dfrac{1}{4} \quad \ldots\text{(i)}$

Here, $\quad \tau_c = CR = 10\ \text{s}$

Substituting this value of τ_c in Eq. (i) and solving for t, we get

$$t = 13.86\ \text{s}$$

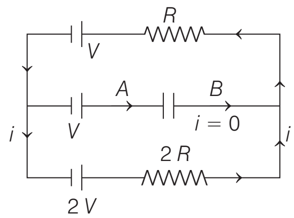
$\therefore$ Correct answer is (a).

4. Charging current, $I = \dfrac{E}{R}e^{-\frac{t}{RC}}$

Taking log on both sides, $\log I = \log\left(\dfrac{E}{R}\right) - \dfrac{t}{RC}$

When R is doubled, slope of curve decreases. Also at $t = 0$, the current will be less. Graph Q represents the best. Hence, the correct option is (b).

5. In steady state condition, no current will flow through the capacitor C. Current in the outer circuit,



$$i = \frac{2V - V}{2R + R} = \frac{V}{3R}$$

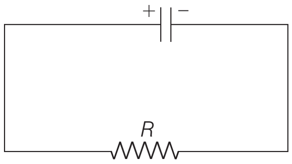
Potential difference between A and B

$$V_A - V + V + iR = V_B$$

$\therefore \quad V_B - V_A = iR = \left(\dfrac{V}{3R}\right)R = \dfrac{V}{3}$

NOTE In this problem charge stored in the capacitor can also be asked, which is equal to $q = C\dfrac{V}{3}$ with positive charge on B side and negative on A side because $V_B > V_A$.

6. The discharging current in the circuit is,



$$i = i_0 e^{-t/CR}$$

Here, $i_0 =$ initial current $= \dfrac{V}{R}$

Here, V is the potential with which capacitor was charged.

Since, V and R for both the capacitors are same, initial discharging current will be same but non-zero.

$\therefore$ Correct option is (b).

Further, $\tau_c = CR \Rightarrow C_1 < C_2$ or $\tau_{C_1} < \tau_{C_2}$

or C_1 loses its 50% of initial charge sooner than C_2.

$\therefore$ Option (d) is also correct .

7. Voltage across the capacitors will increase from 0 to 10 V exponentially. The voltage at time t will be given by

$$V = 10\,(1 - e^{-t/\tau_C})$$

Here, $\tau_c = C_{net}\, R_{net}$

$$= (1 \times 10^6)\,(4 \times 10^{-6}) = 4\,\text{s}$$

$\therefore$ $V = 10\,(1 - e^{-t/4})$

Substituting $V = 4$ volt, we have

$$4 = 10\,(1 - e^{-t/4})$$

or $e^{-t/4} = 0.6 = \dfrac{3}{5}$

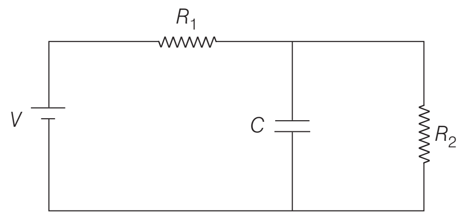
Taking log on both sides we have,

$$-\frac{t}{4} = \ln 3 - \ln 5$$

or $t = 4\,(\ln 5 - \ln 3) = 2\,\text{s}$

Hence, the answer is 2 .

8.



Q_0 is the steady state charge stored in the capacitor.

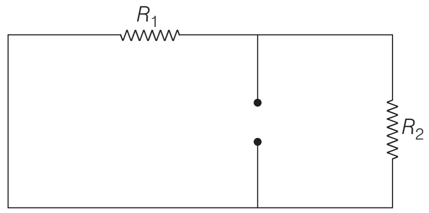
$Q_0 = C$ [PD across capacitor in steady state]

$= C$ [steady state current through R_2] (R_2)

$$= C\left(\frac{V}{R_1 + R_2}\right)\cdot R_2$$

$\therefore$ $Q_0 = \dfrac{C\,V\,R_2}{R_1 + R_2}$

α is $1/\tau_c$ or $\dfrac{1}{C\,R_{net}}$



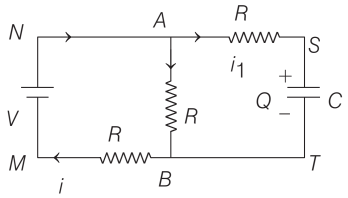
Here, R_{net} is equivalent resistance across capacitor after short circuiting the battery. Thus,

$$R_{net} = \frac{R_1 R_2}{R_1 + R_2} \qquad \text{(As } R_1 \text{ and } R_2 \text{ are in parallel)}$$

$$\alpha = \frac{1}{C\left(\dfrac{R_1 R_2}{R_1 + R_2}\right)} = \frac{R_1 + R_2}{C\,R_1 R_2}$$

9. Let at any time t charge on capacitor C be Q and currents are as shown. Since, charge Q will increase with time t.

Therefore,



$$i_1 = \frac{dQ}{dt}$$

(a) Applying Kirchhoff 's second law in loop $MNABM$

$$V = (i - i_1)\,R + iR$$

or $V = 2iR - i_1 R$ ...(i)

Similarly, applying Kirchhoff's second law in loop $MNSTM$, we have

$$V = i_1 R + \frac{Q}{C} + iR \qquad \text{(ii)}$$

Eliminating i from Eqs. (i) and (ii), we get

$$V = 3i_1 R + \frac{2Q}{C}$$

or $3i_1 R = V - \dfrac{2Q}{C}$ or $i_1 = \dfrac{1}{3R}\left(V - \dfrac{2Q}{C}\right)$

or $\dfrac{dQ}{dt} = \dfrac{1}{3R}\left(V - \dfrac{2Q}{C}\right)$ or $\dfrac{dQ}{V - \dfrac{2Q}{C}} = \dfrac{dt}{3R}$

or $\displaystyle\int_0^Q \frac{dQ}{V - \dfrac{2Q}{C}} = \int_0^t \frac{dt}{3R}$

This equation gives $Q = \dfrac{CV}{2}\,(1 - e^{-2t/3RC})$

(b) $i_1 = \dfrac{dQ}{dt} = \dfrac{V}{3R}\,e^{-2t/3RC}$

From Eq. (i) $i = \dfrac{V + i_1 R}{2R} = \dfrac{V + \dfrac{V}{3}e^{-2t/3RC}}{2R}$

$\therefore$ Current through AB

$$i_2 = i - i_1 = \frac{V + \frac{V}{3}e^{-2t/3RC}}{2R} - \frac{V}{3R}e^{-2t/3RC}$$

$$i_2 = \frac{V}{2R} - \frac{V}{6R}e^{-2t/3RC}$$

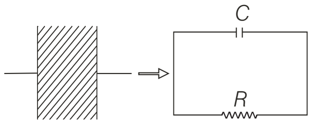
$$i_2 = \frac{V}{2R} \text{ as } t \to \infty$$

10. The problem is basically of discharging of CR circuit, because between the plates of the capacitor, there is capacitor as well as resistance.

$$R = \frac{d}{\sigma A} \qquad \left(R = \frac{l}{\sigma A}\right)$$

and $$C = \frac{K\varepsilon_0 A}{d}$$

$\therefore$ Time constant, $\tau_c = CR = \dfrac{K\varepsilon_0}{\sigma}$



Substituting the values, we have

$$\tau_c = \frac{5 \times 8.86 \times 10^{-12}}{7.4 \times 10^{-12}} = 5.98\text{ s}$$

Charge at any time decreases exponentially as

$$q = q_0 e^{-t/\tau_c}$$

Here, $q_0 = 8.85 \times 10^{-6}$ C (Charge at time $t = 0$)

Therefore, discharging (leakage) current at time t will be given by

$$i = \left(-\frac{dq}{dt}\right) = \frac{q_0}{\tau_c}e^{-t/\tau_c}$$

or current at $t = 12$ s is

$$i = \frac{(8.85 \times 10^{-6})}{5.98}e^{-12/5.98}$$

$$= 0.198 \times 10^{-6}\text{ A} = 0.198\,\mu\text{A}$$

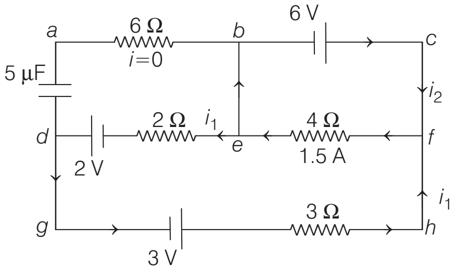
$$i = 0.198\,\mu\text{A}$$

11. In steady state no current will flow through $R_1 = 6\Omega$.

Potential difference across R_3 or 4 Ω is E_1 or 6 V

$\therefore$ Current through it will be $\dfrac{6}{4} = 1.5$A from right to left.

Because left hand side of this resistance is at higher potential.



Now, suppose this 1.5 A distributes in i_1 and i_2 as shown. Applying Kirchhoff's second law in loop $dghfed$

$$3 - 3i_1 - 4 \times 1.5 - 2i_1 + 2 = 0$$

$$\therefore \qquad i_1 = -\frac{1}{5}\text{A} = -0.2\text{A}$$

To find energy stored in capacitor we will have to find potential difference across it. Or V_{ad}.

Now, $\quad V_a - 2i_1 + 2 = V_d$

or $\quad V_a - V_d = 2i_1 - 2 = -2.4$ V

or $\quad V_d - V_a = 2.4\text{ V} = V_{da}$

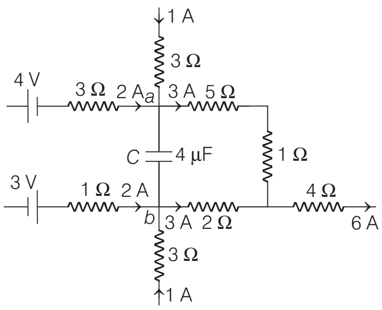
Energy stored in capacitor:

$$U = \frac{1}{2}CV_{da}^2$$

$$= \frac{1}{2}(5 \times 10^{-6})(2.4)^2$$

$$= 1.44 \times 10^{-5}\text{ J}$$

12. Using Kirchhoff's first law at junctions a and b, we have found the current in other wires of the circuit on which currents were not shown.



Now, to calculate the energy stored in the capacitor we will have to first find the potential difference V_{ab} across it.

$$V_a - 3 \times 5 - 3 \times 1 + 3 \times 2 = V_b$$

$$\therefore \qquad V_a - V_b = V_{ab} = 12\text{ V}$$

$$\therefore \qquad U = \frac{1}{2}CV_{ab}^2 = \frac{1}{2}(4 \times 10^{-6})(12)^2\text{ J} = 0.288\text{ mJ}$$

13. In steady state situation no current will flow through the capacitor. 2Ω and 3Ω are in parallel.

Therefore, their combined resistance will be

$$R = \frac{2 \times 3}{2 + 3} = 1.2\,\Omega$$

Net current through the battery

$$i = \frac{6}{1.2 + 2.8} = 1.5\text{ A}$$

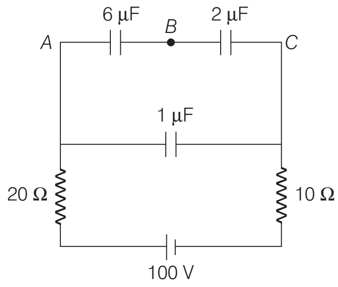
This current will distribute in inverse ratio of their resistances in 2Ω and 3Ω.

$$\therefore \qquad \frac{i_2}{i_3} = \frac{3}{2}$$

or $$i_2 = \left(\frac{3}{3+2}\right)(1.5) = 0.9\text{A}$$

14. Two capacitors of $3\,\mu F$ each and two capacitors of $1\,\mu F$ each are in parallel.

Therefore, simplified circuit can be drawn as below.



In steady state no current will flow in the circuit and capacitors are fully charged.

Points A, B and C in original circuit are shown in the simplified circuit.

Between points A and C, $6\,\mu F$ and $2\,\mu F$ are in series. 100 V is applied across this series combination. In series potential drops in inverse ratio of capacity.

$$\therefore \quad V_{AB} = V_{6\,\mu F} = \left(\frac{2}{6+2}\right) \times 100 = 25\text{ V}$$

$$V_{BC} = V_{2\,\mu F} = \left(\frac{6}{6+2}\right) \times 100 = 75\text{ V}$$

Topic 7 Miscellaneous Problems

1. Given, magnetic field of an electromagnetic wave is

$$\mathbf{B} = B_0[\cos(kz - \omega t)]\,\hat{\mathbf{i}} + B_1[\cos(kz + \omega t)]\,\hat{\mathbf{j}}$$

Here, $B_0 = 3 \times 10^{-5}$ T and $B_1 = 2 \times 10^{-6}$ T

Also, stationary charge, $Q = 10^{-4}$C at $z = 0$

As charge is released from the rest at $z = 0$, in this condition.

Maximum electric field, $E_0 = cB_0$ and $E_1 = cB_1$

So, $E_0 = c \times 3 \times 10^{-5}$ and $E_1 = c \times 2 \times 10^{-6}$

Now,the direction of electric field of an electromagnetic wave is perpendicular to $\mathbf{B}$ and to the direction of propagation of wave $(\mathbf{E} \times \mathbf{B})$ which is $\hat{\mathbf{k}}$.

So, for E_0, $\mathbf{E}_0 \times \mathbf{B}_0 = \hat{\mathbf{k}} \Rightarrow \mathbf{E}_0 \times \hat{\mathbf{i}} = \hat{\mathbf{k}} \Rightarrow \mathbf{E}_0 = -\hat{\mathbf{j}}$

Similarly,

for E_1 $\mathbf{E}_1 \times \mathbf{B}_1 = \mathbf{k} \Rightarrow \mathbf{E}_1 \times \hat{\mathbf{j}} = \hat{\mathbf{k}}$

$$\Rightarrow \quad \mathbf{E}_1 = \hat{\mathbf{i}}$$

$$\therefore \quad \mathbf{E}_0 = c \times 3 \times 10^{-5}(-\hat{\mathbf{j}})\text{NC}^{-1}$$

$$\mathbf{E} = c \times 2 \times 10^{-6}(+\hat{\mathbf{i}})\text{NC}^{-1}$$

$\therefore$ Maximum force experienced by stationary charge is

$$\mathbf{F}_{max} = Q\mathbf{E} = Q(\mathbf{E}_0 + \mathbf{E}_1)$$

$$= Q \times c\,[-3 \times 10^{-5}\,\hat{\mathbf{j}} + 2 \times 10^{-6}\,\hat{\mathbf{i}}]$$

$$\Rightarrow |\mathbf{F}_{max}| = 10^{-4} \times 3 \times 10^{8} \times \sqrt{(3 \times 10^{-5})^2 + (2 \times 10^{-6})^2}$$

$$= 3 \times 10^{4} \times 10^{-6}\sqrt{900 + 4}$$

$$= 3 \times 10^{-2} \times \sqrt{904} \approx 0.9\text{ N}$$

$\therefore$ rms value of experienced force is

$$F_{rms} = \frac{F_{max}}{\sqrt{2}} = \frac{0.9}{\sqrt{2}} = 0.707 \times 0.9$$

$$= 0.6363\text{ N} \approx 0.6\text{ N}$$

2. **Key Idea** When an electric dipole is placed in an electric field $\mathbf{E}$ at some angle θ, then two forces equal in magnitude but opposite in direction acts on the +ve and –ve charges, respectively. These forces forms a couple which exert a torque, which is given as

$$\tau = \mathbf{p} \times \mathbf{E}$$

where, $\mathbf{p}$ is dipole moment.

Torque on the dipole is given as

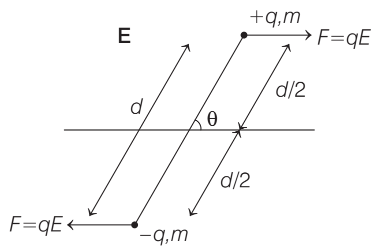
$$\tau = I\alpha = -\,pE\sin\theta$$

where, I is the moment of inertia and α is angular acceleration.

For small angles, $\sin\theta \approx \theta$

$$\therefore \quad \alpha = -\left(\frac{pE}{I}\right)\theta \quad \ldots\text{(i)}$$

Moment of inertia of the given system is



$$I = m\left(\frac{d}{2}\right)^2 + m\left(\frac{d}{2}\right)^2 = \frac{2md^2}{4} = \frac{md^2}{2}$$

Substituting the value of I in Eq. (i), we get

$$\Rightarrow \quad \alpha = -\left(\frac{2pE}{md^2}\right)\cdot\theta \quad \text{...(ii)}$$

The above equation is similar to the equation for a system executing angular SHM.

Comparing Eq. (ii), with the general equation of angular SHM, i.e.

$$\alpha = -\,\omega^2\theta$$

where, ω is the angular frequency,

we get

$$\omega^2 = \frac{2pE}{md^2} \quad \text{or} \quad \omega = \sqrt{\frac{2pE}{md^2}}$$

As, $\quad p = qd$

$$\therefore \quad \omega = \sqrt{\frac{2qdE}{md^2}} = \sqrt{\frac{2qE}{md}}$$

3. **Key Idea** As, electrostatic force is conserved in nature so, total energy of charge distribution remains constant in absence of any external interaction.

Let radius of distribution at some instant t is R. At $t = 0$, radius is given R_0

Now by conservation of energy, we have

$$0 + \frac{kQ^2}{2R_0} = \frac{1}{2}mv^2 + \frac{kQ^2}{2R}$$

($\because$ The distribution starts from rest, so, initial kinetic energy is zero.)

Differentiating this equation with respect to R, we get

$$\frac{1}{2}m2v\frac{dv}{dR} - \frac{kQ^2}{2R^2} = 0 \text{ or } \frac{dv}{dR} = \frac{kQ^2}{2mvR^2}$$

Here, $\frac{dv}{dR}$ = slope of v *versus* R graph. It decreases with increasing v and R.

Also, slope $\longrightarrow 0$ as $R \longrightarrow \infty$.

From above conclusions, we can see that the best suited graph is given in option (c).

4. In steady state no current flows through the capacitor.

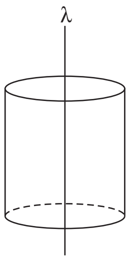
So, the current in circuit $I = \frac{E}{r + r_2}$

$\because$ Potential drop across capacitor = Potential drop across r_2

$$= Ir_2 = \frac{Er_2}{r + r_2}$$

$\therefore$ Stored charge of capacitor, $Q = CV = \frac{CEr_2}{r + r_2}$

5.



Suppose charger per unit length at any instant is λ.

Initial value of λ is suppose λ_0.

Electric field at a distance r at any instant is

$$E = \frac{\lambda}{2\pi\varepsilon r}$$

$$J = \sigma E = \sigma\frac{\lambda}{2\pi\varepsilon r}$$

$$i = \frac{dq}{dt} = J(A) = -J\sigma 2\pi rl$$

$$\frac{d\lambda l}{dt} = -\frac{\lambda}{2\pi\varepsilon r} \times \sigma 2\pi rl \qquad (q = \lambda l)$$

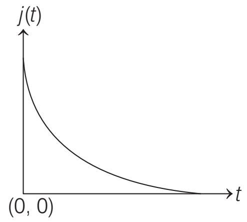
$$\int_{\lambda}^{\lambda}\frac{d\lambda}{\lambda} = -\frac{\sigma}{\varepsilon}\int_{a}^{t} dt$$

$$\Rightarrow \qquad \lambda = \lambda_0 e^{-\frac{\alpha}{\varepsilon}t}$$

$$J = \frac{\sigma}{2\pi\varepsilon r}\lambda = \frac{\sigma\lambda_0}{2\pi\varepsilon r}e^{-\frac{\sigma}{\varepsilon}t} = J_0 e^{-\frac{\sigma}{\varepsilon}t}$$

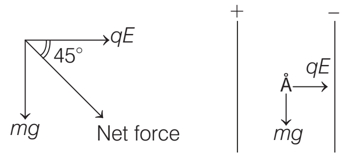
Here, $\quad J_0 = \frac{\sigma\lambda_0}{2\pi\varepsilon r}$

$\therefore$ $J(t)$ decreases exponentially as shown in figure below.



6. At the shown position, net force on both charges is zero. Hence they are in equilibrium. But equilibrium of $+q$ is stable equilibrium. So, it will start oscillations when displaced from this position. These small oscillations are simple harmonic in nature. While equilibrium of $-q$ is unstable. So, it continues to move in the direction of its displacement.

7.



Net force is at 45° from vertical.

$$\therefore \qquad qE = mg \text{ or } \frac{qX}{d} = mg \qquad \left(\because E = \frac{X}{d}\right)$$

$$\text{or} \qquad X = \frac{mgd}{q} = \frac{(1.67 \times 10^{-27})(9.8)(10^{-2})}{(1.6 \times 10^{-19})}$$

$$\approx 1 \times 10^{-9} \text{ V}$$

8. Electrical force per unit area $= \frac{1}{2}\varepsilon_0 E^2$

$$= \frac{1}{2}\varepsilon_0\left(\frac{\sigma}{\varepsilon_0}\right)^2 = \frac{\sigma^2}{2\varepsilon_0}$$

Projected area $= \pi R^2$

$\therefore$ Net electrical force $= \left(\frac{\sigma^2}{2\varepsilon_0}\right)(\pi R^2)$

In equilibrium, this force should be equal to the applied force.

$$\therefore \qquad F = \frac{\pi\sigma^2 R^2}{2\varepsilon_0} \text{ or } F \propto \frac{\sigma^2 R^2}{\varepsilon_0}$$

$\therefore$ The correct option is (a).

9.

$$qE = mg \qquad \ldots\text{(i)}$$

$$6\pi\eta rv = mg$$

$$\frac{4}{3}\pi r^3 \rho g = mg \qquad \ldots\text{(ii)}$$

$$\therefore \qquad r = \left(\frac{3mg}{4\pi\rho g}\right)^{1/3} \qquad \ldots\text{(iii)}$$

Substituting the value of r in Eq. (ii) we get,

$$6\pi\eta v \left(\frac{3mg}{4\pi\,\rho g}\right)^{1/3} = mg$$

or $$(6\pi\eta v)^3 \left(\frac{3mg}{4\pi\rho g}\right) = (mg)^3$$

Again substituting $mg = qE$ we get,

$$(qE)^2 = \left(\frac{3}{4\pi\,\rho g}\right)(6\pi\eta v)^3$$

or $$qE = \left(\frac{3}{4\pi\rho g}\right)^{1/2} (6\pi\eta v)^{3/2}$$

$$\therefore \qquad q = \frac{1}{E}\left(\frac{3}{4\pi\rho g}\right)^{1/2} (6\pi\eta v)^{3/2}$$

Substituting the values we get ,

$$q = \frac{7}{81\pi \times 10^5}\sqrt{\frac{3}{4\pi \times 900 \times 9.8} \times 216\,\pi^3} \times \sqrt{(1.8\times 10^{-5} \times 2 \times 10^{-3})^3}$$

$$= 8.0 \times 10^{-19}\text{ C}$$

10. Distance $BC = AB \sin 60° = (2R)\frac{\sqrt{3}}{2} = \sqrt{3}R$

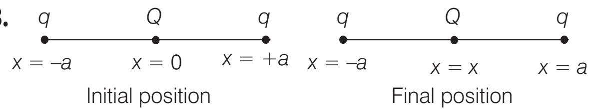
$$\therefore \qquad |Fe_{BC}| = \frac{1}{4\pi\varepsilon_0}\frac{(q/3)(2q/3)}{(\sqrt{3}R)^2} = \frac{q^2}{54\pi\varepsilon_0 R^2}$$

11. Inside the cavity, field at any point is uniform and non-zero. Therefore, correct option is (b).

12. There will be an electric field between two cylinders (using Gauss theorem). This electric field will produce a potential difference.

$\therefore$ Correct answer is (a).

13.



$$U_i = \frac{2KQq}{a} + \frac{K.q.q.}{2a}$$

and $$U_f = KQq\left[\frac{1}{a+x} + \frac{1}{a-x}\right] + \frac{K.q.q.}{2a}$$

Here, $$K = \frac{1}{4\pi\varepsilon_0}$$

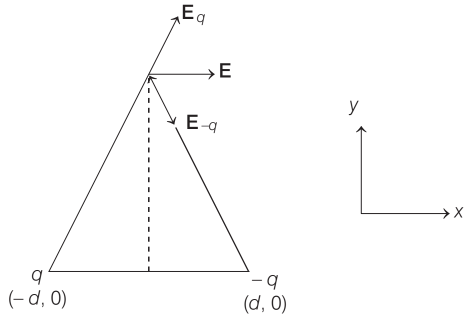
$$\Delta U = U_f - U_i$$

or $$|\Delta U| = \frac{2KQqx^2}{a^3}$$

For $\quad x << a$

$\therefore \quad \Delta U \propto x^2$

14. The diagramatic representation of the given question is shown in figure.



The electrical field **E** at all points on the x-axis will not have the same direction.

For $-d \le x \le d$, electric field is along positive x-axis while for all other points it is along negative x-axis.

The electric field **E** at all points on the y-axis will be parallel to the x-axis (i.e. $\hat{\mathbf{i}}$) [option (c)]

The electrical potential at the origin due to both the charges is zero, hence, no work is done in bringing a test charge from infinity to the origin.

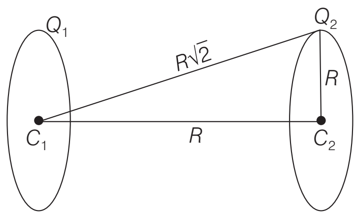
Dipole moment is directed from the $-q$ charge to the $+q$ charge (i.e. $-\hat{\mathbf{i}}$ direction).

15. $$V_{C_1} = V_{Q_1} + V_{Q_2} = \frac{1}{4\,\pi\varepsilon_0}\frac{Q_1}{R} + \frac{1}{4\pi\,\varepsilon_0}\frac{Q_2}{R\sqrt{2}}$$

$$= \frac{1}{4\pi\,\varepsilon_0 R}\left(Q_1 + \frac{Q_2}{\sqrt{2}}\right)$$

Similarly $$V_{C_2} = \frac{1}{4\pi\,\varepsilon_0 R}\left(Q_2 + \frac{Q_1}{\sqrt{2}}\right)$$

$$\therefore \quad \Delta V = V_{C_1} - V_{C_2} = \frac{1}{4\pi\varepsilon_0 R}\left[(Q_1 - Q_2) - \frac{1}{\sqrt{2}}(Q_1 - Q_2)\right]$$



$$= \frac{Q_1 - Q_2}{\sqrt{2}(4\pi\,\varepsilon_0 R)}(\sqrt{2} - 1)$$

$$W = q\Delta V = q\,(Q_1 - Q_2)\,(\sqrt{2}-1)/\sqrt{2}(4\pi\,\varepsilon_0 R)$$

16. Balls will gain positive charge and hence move towards negative plate.

On reaching negative plate, balls will attain negative charge and come back to positive plate.

So on, balls will keep oscillating.

But oscillation is not S.H.M.,

As fore on balls is not $\propto x$.

$\Rightarrow$ (d) is correct.

17. As the balls keep on carrying charge form one plate to another, current will keep on flowing even in steady state. When at bottom plate, if all balls attain charge q,

$$\frac{kq}{r} = V_0 \qquad \left(\because k = \frac{1}{4\pi\varepsilon_0}\right)$$

$$\Rightarrow \qquad q = \frac{V_0 r}{k}$$

Inside cylinder, electric field $E = [V_0 - (-V_0)]h$

$$= 2V_0 h.$$

$\Rightarrow$ Acceleration of each ball,

$$a = \frac{qE}{m} = \frac{2hr}{k\,m}\cdot V_0^2$$

$\Rightarrow$ Time taken by balls to reach other plate,

$$t = \sqrt{\frac{2h}{a}} = \sqrt{\frac{2h.k\,m}{2hrV_0^2}} = \frac{1}{V_0}\sqrt{\frac{k\,m}{r}}$$

If there are n balls, then

Average current, $i_{av} = \dfrac{nq}{t} = n \times \dfrac{V_0 r}{k} \times V_0\sqrt{\dfrac{r}{k\,m}}$

$$\Rightarrow \qquad i_{av} \propto V_0^2$$

18. **List-II**

(1) $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{d^2}$

$\Rightarrow E \propto \dfrac{1}{d^2}$

(2) $E_{\text{axis}} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{2Q(2l)}{d^3}$

$$\Rightarrow \qquad E \propto \frac{1}{d^3}$$

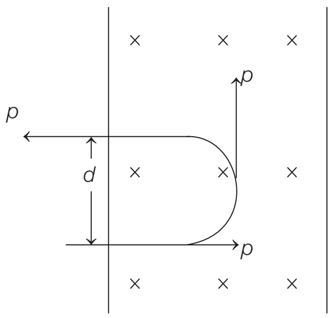
(3) $E = \dfrac{\lambda}{2\pi\varepsilon_0 d} \Rightarrow E \propto \dfrac{1}{d}$

(4) $E = \dfrac{\lambda}{2\pi\varepsilon_0(d-l)} - \dfrac{\lambda}{2\pi\varepsilon_0(d+l)} = \dfrac{\lambda(2l)}{2\pi\varepsilon_0 d^2}$

$\Rightarrow \quad E \propto \dfrac{1}{d^2}$

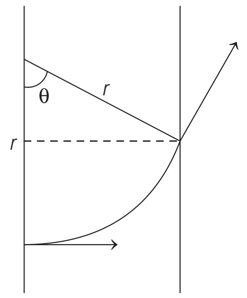
(5) $E = \dfrac{\sigma}{2\varepsilon_0} \Rightarrow$ E is independent of d

20. (a)



$|\Delta \mathbf{P}| = \sqrt{2}p$

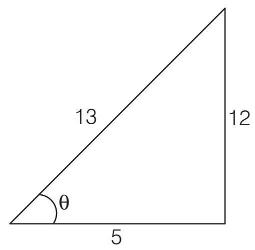
(b) $r(1-\cos\theta) = R$



$$r\sin\theta = \frac{3R}{2} \Rightarrow \frac{\sin\theta}{1-\cos\theta} = \frac{3}{2}$$

$$\frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}} = \frac{3}{2} \Rightarrow \cot\frac{\theta}{2} = \frac{3}{2}$$

$$\Rightarrow \qquad \tan\frac{\theta}{2} = \frac{2}{3} \Rightarrow \tan\theta = \frac{2\left(\frac{2}{3}\right)}{1-\frac{4}{9}} = \frac{\frac{4}{3}}{\frac{5}{9}} = \frac{12}{5}$$



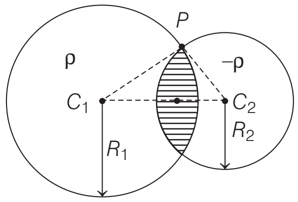
$$\sin\theta = \frac{12}{13}$$

$$r\left(\frac{12}{13}\right) = \frac{3R}{2}; \quad r = \frac{13R}{8} = \frac{P}{QB}; \quad B = \frac{8P}{13QR}$$

(c) $\dfrac{P}{QB} < \dfrac{3R}{2}, \; B > \dfrac{2P}{3QR}$

(d) $r = \dfrac{mv}{QB}, \; d = 2r = \dfrac{2mv}{QB} \Rightarrow d \propto m$

21.



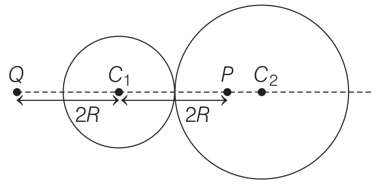
For electrostatic field,

$$\mathbf{E}_P = \mathbf{E}_1 + \mathbf{E}_2 = \frac{\rho}{3\varepsilon_0}\mathbf{C_1P} + \frac{(-\rho)}{3\varepsilon_0}\mathbf{C_1P}$$

$$= \frac{\rho}{3\varepsilon_0}(\mathbf{C_1P} + \mathbf{PC_2})$$

$$\mathbf{E}_P = \frac{\rho}{3\varepsilon_0}\mathbf{C_1C_2}$$

22. **At point *P***



If resultant electric field is zero, then

$$\frac{KQ_1}{4R^2} = \frac{KQ_2}{8R^3} R \quad \Rightarrow \quad \frac{\rho_1}{\rho_2} = 4$$

At point *Q*

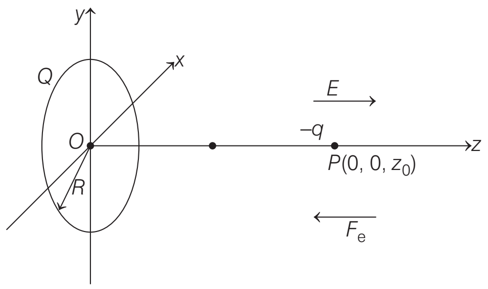
If resultant electric field is zero then

$$\frac{KQ_1}{4R^2} + \frac{KQ_2}{25R^2} = 0$$

$$\frac{\rho_1}{\rho_2} = -\frac{32}{25} \qquad (\rho_1 \text{ must be negative})$$

23. Net torque on $(-q)$ about a point (say P) lying over $+Q$ is zero. Therefore, angular momentum of $(-q)$ about point P should remain constant.

24. Let Q be the charge on the ring, the negative charge $-q$ is released from point $P\ (0, 0, z_0)$. The electric field at P due to the charged ring will be along positive z-axis and its magnitude will be



$$E = \frac{1}{4\pi\varepsilon_0} \frac{Qz_0}{(R^2 + z_0^2)^{3/2}}$$

$E = 0$ at centre of the ring because $z_0 = 0$

Force on charge at P will be towards centre as shown, and its magnitude is

$$F_e = qE = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Qq}{(R^2 + z_0^2)^{3/2}} \cdot z_0 \qquad \ldots\text{(i)}$$

Similarly, when it crosses the origin, the force is again towards centre O.

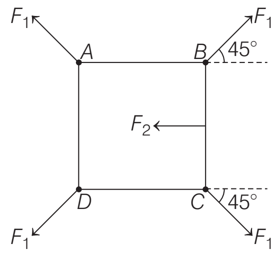
Thus, the motion of the particle is periodic for all values of z_0 lying between 0 and ∞.

Secondly, if $z_0 << R$, $(R^2 + z_0^2)^{3/2} \approx R^3$

$$F_e \approx \frac{1}{4\pi\varepsilon_0} \cdot \frac{Qq}{R^3} \cdot z_0 \qquad \text{[From Eq. (i)]}$$

i.e. the restoring force $F_e \propto -z_0$. Hence, the motion of the particle will be simple harmonic. (Here negative sign implies that the force is towards its mean position.)

25.



F_1 = Net electrostatic force on any one charge due to rest of three charges

$$= \frac{1}{4\pi\varepsilon_0} \frac{q^2}{a^2} \left(\sqrt{2} + \frac{1}{2}\right)$$

F_2 = Surface tension force = γa

If we see the equilibrium of line BC, then

$$2F_1 \cos 45° = F_2$$

or $$\sqrt{2} F_1 = F_2$$

or $$\frac{1}{4\pi\varepsilon_0} \frac{q^2}{a^2} \left(2 + \frac{1}{\sqrt{2}}\right) = \gamma a$$

$\therefore$ $$a^3 = \frac{1}{4\pi\varepsilon_0} \left(2 + \frac{1}{\sqrt{2}}\right) \frac{q^2}{\gamma}$$

or $$a = \left\{\frac{1}{4\pi\varepsilon_0} \left(2 + \frac{1}{\sqrt{2}}\right)\right\}^{1/3} \left[\frac{q^2}{\gamma}\right]^{1/3} = k \left[\frac{q^2}{\gamma}\right]^{1/3}$$

where, $$k = \left\{\frac{1}{4\pi\varepsilon_0} \left(2 + \frac{1}{\sqrt{2}}\right)\right\}^{1/3}$$

Therefore, $N = 3$

Answer is 3.

26. When X-rays fall on the metal ball, some electrons emit from it due to photoelectric effect. The ball thus gets positively charged and on a positively charged ball an electrostatic force in the direction of electric field acts. Due to this force ball gets deflected in the direction of electric field.

27. Mass of negatively charged sphere will be slightly more than the mass of positively charged sphere because some electrons will be given to the negatively charged sphere while some electrons will be taken out from the positively charged sphere.

28. Let q be the charge on the bubble, then

$$V = \frac{Kq}{a} \qquad \left(\text{Here, } K = \frac{1}{4\pi\varepsilon_0}\right)$$

$\therefore$ $$q = \frac{Va}{K}$$

Let after collapsing, the radius of droplet becomes R, then equating the volume, we have

$$(4\pi a^2)\, t = \frac{4}{3} \pi R^3$$

$\therefore \quad R = (3a^2t)^{1/3}$

Now, potential of droplet will be $V' = \dfrac{Kq}{R}$

Substituting the values, we have

$$V' = \frac{(K)\left(\dfrac{Va}{K}\right)}{(3a^2t)^{1/3}} \quad \text{or} \quad V' = V\left(\frac{a}{3t}\right)^{1/3}$$

29. Electric field near a large metallic plate is given by $E = \sigma / \varepsilon_0$. In between the plates the two fields will be in opposite directions. Hence,

$$E_{\text{net}} = \frac{\sigma_1 - \sigma_2}{\varepsilon_0} = E_0 \text{ (say)}$$

Now, $W = (q)$ (potential difference)

$$= q\,(E_0 a \cos 45°) = (q)\left(\frac{\sigma_1 - \sigma_2}{\varepsilon_0}\right)\left(\frac{a}{\sqrt{2}}\right)$$

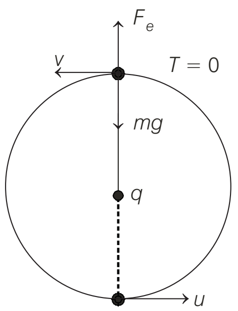
$$= \frac{(\sigma_1 - \sigma_2)\,qa}{\sqrt{2}\varepsilon_0}$$

30. Given : $q = 1\mu\text{C} = 10^{-6}\,\text{C}$

$m = 2 \times 10^{-3}$ kg

and $l = 0.8\,\text{m}$

Let u be the speed of the particle at its lowest point and v its speed at highest point.



At highest point three forces are acting on the particle.

(a) Electrostatic repulsion

$$F_e = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{l^2} \qquad \text{(outwards)}$$

(b) Weight $w = mg$ (inwards)

(c) Tension T (inwards)

$T = 0$, if the particle has just to complete the circle and the necessary centripetal force is provided by $w - F_e$ i.e.

$$\frac{mv^2}{l} = w - F_e$$

or

$$v^2 = \frac{l}{m}\left(mg - \frac{1}{4\pi\varepsilon_0}\frac{q^2}{l^2}\right)$$

$$v^2 = \frac{0.8}{2\times 10^3}\left(2\times 10^{-3}\times 10 - \frac{9.0\times 10^9 \times (10^{-6})^2}{(0.8)^2}\right)\text{m}^2/\text{s}^2$$

or $\quad v^2 = 2.4\,\text{m}^2/\text{s}^2 \qquad \ldots\text{(i)}$

Now, the electrostatic potential energy at the lowest and highest points are equal. Hence, from conservation of mechanical energy.

Increase in gravitational potential energy

= Decrease in kinetic energy.

or $\quad mg\,(2l) = \dfrac{1}{2}m\,(u^2 - v^2)$

or $\quad u^2 = v^2 + 4gl$

Substituting the values of v^2 from Eq. (i), we get

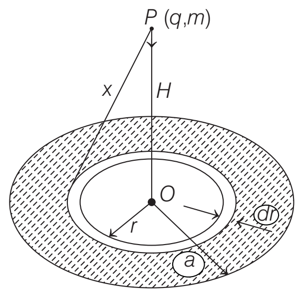
$$u^2 = 2.4 + 4\,(10)\,(0.8) = 34.4\ \text{m}^2/\text{s}^2$$

$\therefore \quad u = 5.86$ m/s

Therefore, minimum horizontal velocity imparted to the lower ball, so that it can make complete revolution, is 5.86 m/s.

31. Potential at a height *H* on the axis of the disc *V* (*P*)

The charge dq contained in the ring shown in figure



$$dq = (2\pi r\,dr)\,\sigma$$

Potential at P due to this ring

$$dV = \frac{1}{4\pi\varepsilon_0}\cdot\frac{dq}{x} \text{ where } x = \sqrt{H^2 + r^2}$$

$$dV = \frac{1}{4\pi\varepsilon_0}\cdot\frac{(2\pi r dr)\sigma}{\sqrt{H^2 + r^2}} = \frac{\sigma}{2\varepsilon_0}\frac{r dr}{\sqrt{H^2 + r^2}}$$

$\therefore$ Potential due to the complete disc

$$V_p = \int_{r=0}^{r=a} dV$$

$$= \frac{\sigma}{2\varepsilon_0}\int_{r=0}^{r=a}\frac{r dr}{\sqrt{H^2 + r^2}}$$

$$V_p = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{a^2 + H^2} - H\right]$$

Potential at centre, (O) will be

$$V_O = \frac{\sigma a}{2\varepsilon_0} \qquad H = 0$$

(a) Particle is released from P and it just reaches point O. Therefore, from conservation of mechanical energy

Decrease in gravitational potential energy = Increase in electrostatic potential energy

($\Delta\text{KE} = 0$ because $K_i = K_f = 0$)

$\therefore\ mgH = q[V_O - V_p]$

or $gH = \left(\frac{q}{m}\right)\left(\frac{\sigma}{2\varepsilon_0}\right)[a - \sqrt{a^2 + H^2} + H]$ ...(i)

$$\frac{q}{m} = \frac{4\varepsilon_0 g}{\sigma}$$

$$\therefore\quad \frac{q\sigma}{2\varepsilon_0 m} = 2g$$

Substituting in Eq. (i), we get

$$gH = 2g\,[a + H - \sqrt{a^2 + H^2}\,]$$

or $\frac{H}{2} = (a + H) - \sqrt{a^2 + H^2}$

or $\sqrt{a^2 + H^2} = a + \frac{H}{2}$

or $a^2 + H^2 = a^2 + \frac{H^2}{4} + aH$

or $\frac{3}{4}H^2 = aH$

or $H = \frac{4}{3}a$

and $H = 0$

$\therefore\ H = (4/3)a$

(b) Potential energy of the particle at height H = Electrostatic potential energy + gravitational potential energy

$\therefore\ U = qV + mgH$

Here V = Potential at height H

$$U = \frac{\sigma q}{2\varepsilon_0}[\sqrt{a^2 + H^2} - H] + mgH \quad \text{...(ii)}$$

At equilibrium position $F = \frac{-dU}{dH} = 0$

Differentiating Eq. (ii) w.r.t. H

or $mg + \sigma\frac{q}{2\,\varepsilon_0}\left[\left(\frac{1}{2}\right)(2H)\frac{1}{\sqrt{a^2 + H^2}} - 1\right] = 0$

$$\left(\because \frac{\sigma q}{2\varepsilon_0} = 2mg\right)$$

$\therefore\ mg + 2mg\left[\frac{H}{\sqrt{a^2 + H^2}} - 1\right] = 0$

or $1 + \frac{2H}{\sqrt{a^2 + H^2}} - 2 = 0$

or $\frac{2H}{\sqrt{a^2 + H^2}} = 1$ or $\frac{H^2}{a^2 + H^2} = \frac{1}{4}$

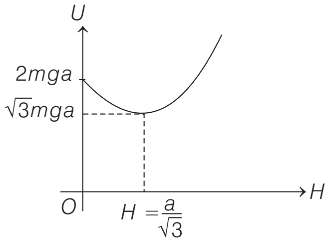
or $3H^2 = a^2$ or $H = \frac{a}{\sqrt{3}}$

From Eq. (ii), we can write

$U - H$ equation as

$U = mg(2\sqrt{a^2 + H^2} - H)$ (Parabolic variation)

$U = 2mga$ at $H = 0$



and $U = U_{\min} = \sqrt{3}mga$ at $H = \frac{a}{\sqrt{3}}$

Therefore, $U - H$ graph will be as shown

Note that at $H = \frac{a}{\sqrt{3}}$, U is minimum.

Therefore, $H = \frac{a}{\sqrt{3}}$ is stable equilibrium position.

32. Capacities of conducting spheres are in the ratio of their radii. Let C_1 and C_2 be the capacities of S_1 and S_2, then

$$\frac{C_2}{C_1} = \frac{R}{r}$$

(a) Charges are distributed in the ratio of their capacities. Let in the first contact, charge acquired by S_2 is q_1. Therefore, charge on S_1 will be $Q - q_1$. Say it is q_1'

$$\therefore\quad \frac{q_1}{q_1'} = \frac{q_1}{Q - q_1} = \frac{C_2}{C_1} = \frac{R}{r}$$

$$\therefore\quad q_1 = Q\left(\frac{R}{R + r}\right) \quad \text{...(i)}$$

In the second contact, S_1 again acquires the same charge Q.

Therefore, total charge in S_1 and S_2 will be

$$Q + q_1 = Q\left(1 + \frac{R}{R + r}\right)$$

This charge is again distributed in the same ratio. Therefore, charge on S_2 in second contact,

$$q_2 = Q\left(1 + \frac{R}{R + r}\right)\left(\frac{R}{R + r}\right) = Q\left[\frac{R}{R + r} + \left(\frac{R}{R + r}\right)^2\right]$$

Similarly, $q_3 = Q\left[\frac{R}{R + r} + \left(\frac{R}{R + r}\right)^2 + \left(\frac{R}{R + r}\right)^3\right]$

and $q_n = Q\left[\frac{R}{R + r} + \left(\frac{R}{R + r}\right)^2 + ... + \left(\frac{R}{R + r}\right)^n\right]$

or $$q_n = Q\frac{R}{r}\left[1-\left(\frac{R}{R+r}\right)^n\right]\left[S_n = \frac{a\,(1-r^n)}{(1-r)}\right] \quad \ldots\text{(ii)}$$

Therefore, electrostatic energy of S_2 after n such contacts

$$= \frac{q_n^2}{2\,(4\pi\varepsilon_0 R)} \text{ or } U_n = \frac{q_n^2}{8\pi\varepsilon_0 R}$$

where, q_n can be written from Eq. (ii).

(b) As $n \to \infty \quad q_\infty = Q\frac{R}{r}$

$$\therefore\ U_\infty = \frac{q_\infty^2}{2C} = \frac{Q^2R^2/r^2}{8\pi\varepsilon_0 R}$$

or $$U_\infty = \frac{Q^2R}{8\,\pi\varepsilon_0 r^2}$$

33. Let q_1 and q_2 be the charges on the two spheres before connecting them.

Then, $q_1 = \sigma(4\pi R^2)$, and $q_2 = \sigma(4\pi)\,(2R)^2 = 16\sigma\pi R^2$

Therefore, total charge (q) on both the spheres is

$$q = q_1 + q_2 = 20\,\sigma\pi R^2$$

Now, after connecting, the charge is distributed in the ratio of their capacities, which in turn depends on the ratio of their radii ($C = 4\pi\varepsilon_0 R$).

$$\therefore \quad \frac{q_1'}{q_2'} = \frac{R}{2R} = \frac{1}{2}$$

$$\therefore \quad q_1' = \frac{q}{3} = \frac{20}{3}\sigma\pi R^2$$

and $$q_2' = \frac{2q}{3} = \frac{40}{3}\sigma\pi R^2$$

Therefore, surface charge densities on the spheres are

$$\sigma_1 = \frac{q_1'}{4\pi R^2} = \frac{(20/3)\sigma\pi R^2}{4\pi R^2} = \frac{5}{3}\sigma$$

and $$\sigma_2 = \frac{q_2'}{4\pi(2R)^2} = \frac{(40/3)\sigma\pi R^2}{16\pi R^2} = \frac{5}{6}\sigma$$

Hence, surface charge density on the bigger sphere is σ_2 i.e. $(5/6)\sigma$.

34. Total charge in the ring is $Q = (2\pi R)\,\lambda$

Potential due to a ring at a distance of x from its centre on its axis is given by $V(x) = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{\sqrt{R^2+x^2}}$

and at the centre is $V_{\text{centre}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$

Using the above formula

$$V_p = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2\pi R\lambda}{\sqrt{R^2+3R^2}} = \frac{\lambda}{4\varepsilon_0}$$

$$V_o = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2\,\pi R\lambda}{R} = \frac{\lambda}{2\varepsilon_0}$$

$$V_o > V_p$$

PD between points O and P is

$$V = V_o - V_p = \frac{\lambda}{2\varepsilon_0} - \frac{\lambda}{4\varepsilon_0} = \frac{\lambda}{4\varepsilon_0}$$

$$\therefore \quad \frac{1}{2}mv^2 \ge qV \text{ or } v \ge \sqrt{\frac{2qV}{m}}$$

or $$v \ge \sqrt{\frac{2q\lambda}{4\varepsilon_0 m}}$$

or $$v \ge \sqrt{\frac{q\lambda}{2\varepsilon_0 m}}$$

Therefore, minimum value of speed v should be

$$v_{\min} = \sqrt{\frac{q\lambda}{2\varepsilon_0 m}}$$

35. (a) In this case the electric field exists from centre of the sphere to infinity. Potential energy is stored in electric field, with energy density

$$u = \frac{1}{2}\varepsilon_0 E^2 \qquad (\because \text{Energy/Volume})$$

(i) Energy stored within the sphere (U_1)

Electric field at a distance r is

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{R^3}\cdot r$$

$$u = \frac{1}{2}\varepsilon_0 E^2 = \frac{\varepsilon_0}{2}\left\{\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{R^3}r\right\}^2$$

Volume of element,

$$dV = (4\pi r^2)\,dr$$

Energy stored in this volume, $dU = u\,(dV)$

$$dU = (4\pi r^2 dr)\frac{\varepsilon_0}{2}\left\{\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{R^3}r\right\}^2$$

$$dU = \frac{1}{8\pi\varepsilon_0}\cdot\frac{Q^2}{R^6}\cdot r^4 dr$$

$$\therefore \quad U_1 = \int_0^R dU = \frac{1}{8\pi\varepsilon_0}\frac{Q^2}{R^6}\int_0^R r^4 dr$$

$$= \frac{Q^2}{40\pi\varepsilon_0 R^6}[r^5]_0^R$$

$$U_1 = \frac{1}{40\pi\varepsilon_0}\cdot\frac{Q^2}{R} \quad \ldots\text{(i)}$$

(ii) Energy stored outside the sphere (U_2)

Electric field at a distance r is

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r^2}$$

$$\therefore \quad u = \frac{1}{2}\varepsilon_0 E^2 = \frac{\varepsilon_0}{2}\left\{\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r^2}\right\}^2$$

$$dU = (4\pi r^2 dr)$$

$$\therefore \quad dU = u \cdot dV$$

$$= (4\pi r^2 dr)\left[\frac{\varepsilon_0}{2}\left(\frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2}\right)^2\right]$$

$$dU = \frac{Q^2}{8\pi\varepsilon_0}\frac{dr}{r^2}$$

$$\therefore \quad U_2 = \int_R^{\infty} dU = \frac{Q^2}{8\pi\varepsilon_0} \cdot \int_R^{\infty} \frac{dr}{r^2}$$

$$U_2 = \frac{Q^2}{8\pi\varepsilon_0 R} \qquad \text{...(ii)}$$

Therefore, total energy of the system is

$$U = U_1 + U_2 = \frac{Q^2}{40\pi\varepsilon_0 R} + \frac{Q^2}{8\pi\varepsilon_0 R}$$

or $$U = \frac{3}{20}\frac{Q^2}{\pi\varepsilon_0 R}$$

(b) Comparing this with gravitational forces, the gravitational potential energy of earth will be

$$U = -\frac{3}{5}\frac{GM^2}{R}$$

by replacing Q^2 by M^2 and $\frac{1}{4\pi\varepsilon_0}$ by G

$$g = \frac{GM}{R^2}$$

$$\therefore \quad G = \frac{gR^2}{M}$$

$$U = \frac{-3}{5}MgR$$

Therefore, energy needed to completely disassemble the earth against gravitational pull amongst its constituent particles will be given by

$$E = |U| = \frac{3}{5}MgR$$

Substituting the values, we get

$$E = \frac{3}{5}(10\text{m/s}^2)(2.5 \times 10^{31}\text{ kg-m})$$

$$E = 1.5 \times 10^{32}\text{ J}$$

(c) This is the case of a charged spherical conductor of radius R, energy of which is given by $= \frac{1}{2}\frac{Q^2}{C}$

or $$U = \frac{1}{2}.\frac{Q^2}{4\pi\varepsilon_0 R} \text{ or } U = \frac{Q^2}{8\pi\varepsilon_0 R}$$