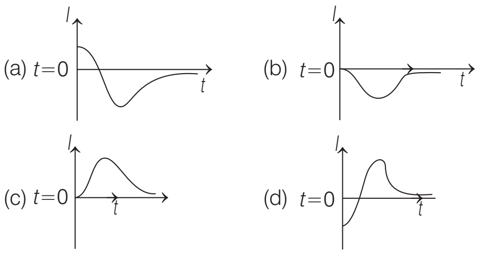
16

Electromagnetic Induction and Alternating Current

Topic 1 Magnetic Flux and Induced EMF by Change in Flux

Objective Questions I (Only one correct option)

1. A very long solenoid of radius R is carrying current $I(t) = kte^{-\alpha t}$ $(k > 0)$, as a function of time $(t \geq 0)$. Counter clockwise current is taken to be positive. A circular conducting coil of radius $2R$ is placed in the equatorial plane of the solenoid and concentric with the solenoid. The current induced in the outer coil is correctly depicted, as a function of time, by **(Main 2019, 9 April II)**



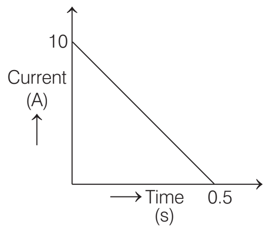
2. A 10 m long horizontal wire extends from North-East to South-West. It is falling with a speed of 5.0 ms^{-1} at right angles to the horizontal component of the earth's magnetic field of $0.3 \times 10^{-4} \text{ Wb/m}^2$. The value of the induced emf in wire is **(Main 2019, 12 Jan II)**

(a) 1.5×10^{-3} V (b) 1.1×10^{-3} V
(c) 0.3×10^{-3} V (d) 2.5×10^{-3} V

3. The self-induced emf of a coil is 25 V. When the current in it is changed at uniform rate from 10 A to 25 A in 1s, the change in the energy of the inductance is **(Main 2019, 10 Jan II)**

(a) 437.5 J (b) 740 J
(c) 637.5 J (d) 540 J

4. In a coil of resistance 100 Ω, a current is induced by changing the magnetic flux through it as shown in the figure. The magnitude of change in flux through the coil is **(2017 Main)**

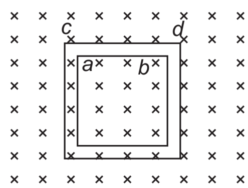


(a) 225 Wb (b) 250 Wb (c) 275 Wb (d) 200 Wb

5. A circular loop of radius 0.3 cm lies parallel to a much bigger circular loop of radius 20 cm. The centre of the smaller loop is on the axis of the bigger loop. The distance between their centres is 15 cm. If a current of 2.0 A flows through the bigger loop, then the flux linked with smaller loop is **(2013 Main)**

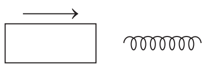
(a) 9.1×10^{-11} Wb (b) 6×10^{-11} Wb
(c) 3.3×10^{-11} Wb (d) 6.6×10^{-9} Wb

6. The figure shows certain wire segments joined together to form a coplanar loop. The loop is placed in a perpendicular magnetic field in the direction going into the plane of the figure. The magnitude of the field increases with time. I_1 and I_2 are the currents in the segments ab and cd. Then, **(2009)**

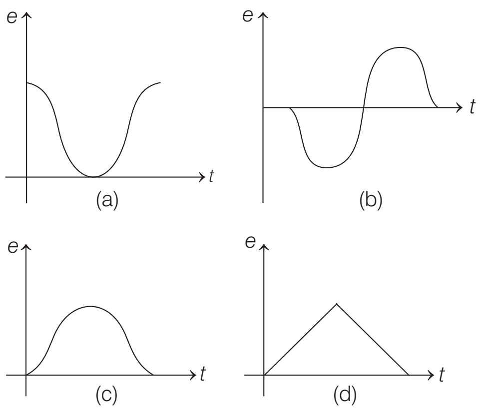


(a) $I_1 > I_2$
(b) $I_1 < I_2$
(c) I_1 is in the direction ba and I_2 is in the direction cd.
(d) I_1 is in the direction ab and I_2 is in the direction dc.

7. The variation of induced emf (e) with time (t) in a coil if a



short bar magnet is moved along its axis with a constant velocity is best represented as **(2004, 2M)**

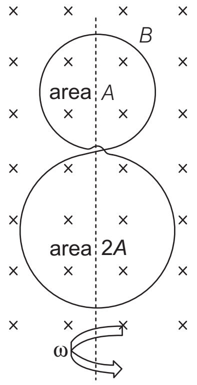


8. A circular loop of radius R, carrying current I, lies in x-y plane with its centre at origin. The total magnetic flux through x-y plane is **(1999, 2M)**

(a) directly proportional to I.
(b) directly proportional to R.
(c) inversely proportional to R.
(d) zero.

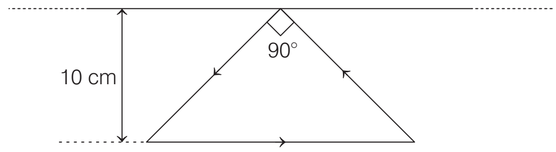
Objective Questions II (One or more correct option)

9. A circular insulated copper wire loop is twisted to form two loops of area A and $2A$ as shown in the figure. At the point of crossing the wires remain electrically insulated from each other. The entire loop lies in the plane (of the paper). A uniform magnetic field **B** points into the plane of the paper. At $t = 0$, the loop starts rotating about the common diameter as axis with a constant angular velocity ω in the magnetic field. Which of the following options is/are correct? **(2017 Adv.)**



(a) The emf induced in the loop is proportional to the sum of the areas of the two loops.
(b) The rate of change of the flux is maximum when the plane of the loops is perpendicular to plane of the paper.
(c) The net emf induced due to both the loops is proportional to $\cos \omega t$.
(d) The amplitude of the maximum net emf induced due to both the loops is equal to the amplitude of maximum emf induced in the smaller loop alone.

10. A conducting loop in the shape of a right angled isosceles triangle of height 10 cm is kept such that the 90° vertex is very close to an infinitely long conducting wire (see the figure). The wire is electrically insulated from the loop. The hypotenuse of the triangle is parallel to the wire. The current in the triangular loop is in counterclockwise direction and increased at a constant rate of $10\,\text{As}^{-1}$. Which of the following statement(s) is (are) true? **(2016 Adv.)**



(a) There is a repulsive force between the wire and the loop.
(b) If the loop is rotated at a constant angular speed about the wire, an additional emf of $\left(\frac{\mu_0}{\pi}\right)$ volt is induced in the wire.
(c) The magnitude of induced emf in the wire is $\left(\frac{\mu_0}{\pi}\right)$ volt.
(d) The induced current in the wire is in opposite direction to the current along the hypotenuse.

11. A current carrying infinitely long wire is kept along the diameter of a circular wire loop, without touching it. The correct statement(s) is(are) **(2012)**

(a) The emf induced in the loop is zero if the current is constant.
(b) The emf induced in the loop is finite if the current is constant.
(c) The emf induced in the loop is zero if the current decreases at a steady rate.
(d) The emf induced in the loop is finite if the current decreases at a steady rate.

Fill in the Blank

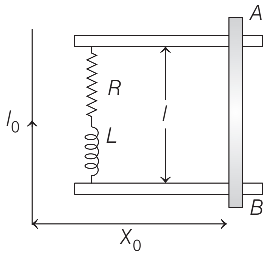
12. In a straight conducting wire, a constant current is flowing from left to right due to a source of emf. When the source is switched off, the direction of the induced current in the wire will be...... **(1993, 1M)**

True/False

13. A coil of metal wire is kept stationary in a non-uniform magnetic field. An emf is induced in the coil. **(1986, 3M)**

Analytical & Descriptive Questions

14. A metal bar AB can slide on two parallel thick metallic rails separated by a distance l. A resistance R and an inductance L are connected to the rails as shown in the figure. A long straight wire, carrying a constant current I_0 is placed in the plane of the rails as shown. The bar AB is held at rest at a distance x_0 from the long wire. At $t = 0$, it made to slide on the rails away from the wire. Answer the following questions. **(2002, 5M)**

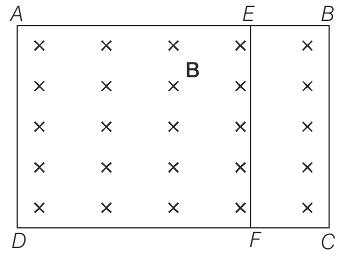


(a) Find a relation among i, $\frac{di}{dt}$ and $\frac{d\phi}{dt}$, where i is the current in the circuit and ϕ is the flux of the magnetic field due to the long wire through the circuit.

(b) It is observed that at time $t = T$, the metal bar AB is at a distance of $2x_0$ from the long wire and the resistance R carries a current i_1. Obtain an expression for the net charge that has flown through resistance R from $t = 0$ to $t = T$.

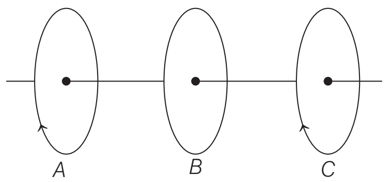
(c) The bar is suddenly stopped at time T. The current through resistance R is found to be $i_1/4$ at time $2T$. Find the value of L/R in terms of the other given quantities.

15. A rectangular frame $ABCD$, made of a uniform metal wire, has a straight connection between E and F made of the same wire, as shown in figure. $AEFD$ is a square of side 1 m and $EB = FC = 0.5$ m. The entire circuit is placed in a steadily increasing uniform magnetic field directed into the plane of the paper and normal to it.

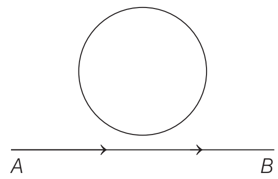


The rate of change of the magnetic field is 1 T/s. The resistance per unit length of the wire is 1 Ω/m. Find the magnitudes and directions of the currents in the segments AE, BE and EF. **(1993, 5M)**

16. Three identical closed coils A, B and C are placed with their planes parallel to one another. Coils A and C carry equal currents as shown in figure. Coils B and C are fixed in position and coil A is moved towards B with uniform motion. Is there any induced current in B ? If no, give reasons. If yes, mark the direction of the induced current in the diagram. **(1982, 2M)**



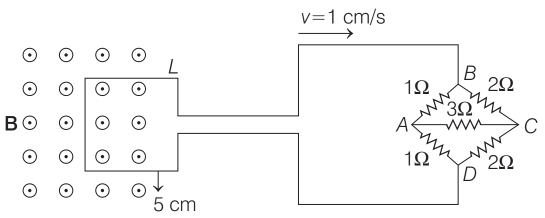
17. A current from A to B is increasing in magnitude. What is the direction of induced current, in the loop as shown in the figure? **(1979, 6M)**



Topic 2 Motional and Rotational EMF

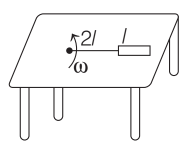
Objective Questions I (Only one correct option)

1 The figure shows a square loop L of side 5 cm which is connected to a network of resistances. The whole setup is moving towards right with a constant speed of 1 cm s^{-1}. At some instant, a part of L is in a uniform magnetic field of 1 T, perpendicular to the plane of the loop. If the resistance of L is 1.7 Ω, the current in the loop at that instant will be close to **(Main 2019, 12 April I)**



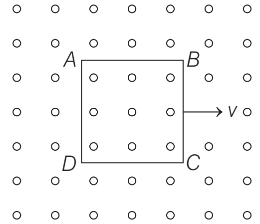
(a) 60 μA (b) 170 μA
(c) 150 μA (d) 115 μA

2. A metallic rod of length l is tied to a string of length $2l$ and made to rotate with angular speed ω on a horizontal table with one end of the string fixed. If there is a vertical magnetic field B in the region, the emf induced across the ends of the rod is **(2013 Main)**



(a) $\frac{2B\omega l^3}{2}$ (b) $\frac{3B\omega l^3}{2}$ (c) $\frac{4B\omega l^2}{2}$ (d) $\frac{5B\omega l^2}{2}$

3. A metallic square loop $ABCD$ is moving in its own plane with velocity v in a uniform magnetic field perpendicular to its plane as shown in the figure. Electric field is induced **(2001, 2M)**

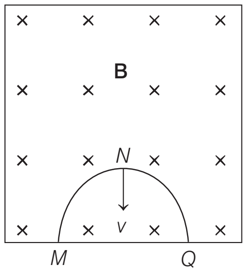


(a) in AD, but not in BC
(b) in BC, but not in AD
(c) neither in AD nor in BC
(d) in both AD and BC

4. A metal rod moves at a constant velocity in a direction perpendicular to its length. A constant uniform magnetic field exists in space in a direction perpendicular to the rod as well as its velocity. Select the correct statement (s) from the following. **(1998, 2M)**

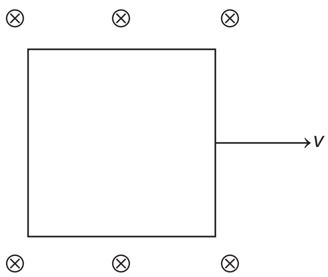
(a) The entire rod is at the same electric potential.
(b) There is an electric field in the rod.
(c) The electric potential is highest at the centre of the rod and decrease towards its ends.
(d) The electric potential is lowest at the centre of the rod and increases towards its ends.

5. A thin semicircular conducting ring of radius R is falling with its plane vertical in a horizontal magnetic induction **B**. At the position MNQ the speed of the ring is v and the potential difference developed across the ring is **(1996, 2M)**



(a) zero
(b) $Bv\pi R^2/2$ and M is at higher potential
(c) πBRv and Q is at higher potential
(d) $2RBv$ and Q is at higher potential

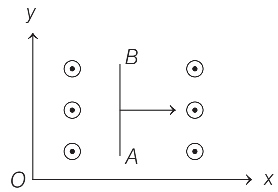
6. A conducting square loop of side L and resistance R moves in its plane with a uniform velocity v perpendicular to one of its sides. A magnetic induction B, constant in time and space, pointing perpendicular to and into the plane of the loop exists everywhere.The current induced in the loop is **(1989, 2M)**



(a) BLv/R clockwise
(b) BLv/R anti-clockwise
(c) $2BLv/R$ anti-clockwise
(d) zero

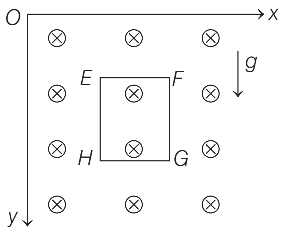
True/ False

7. A conducting rod AB moves parallel to the X-axis in a uniform magnetic field pointing in the positive z-direction. The end A of the rod gets positively charged. **(1987, 2M)**



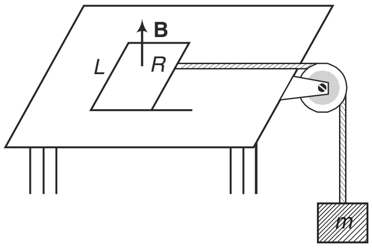
Analytical & Descriptive Questions

8. A magnetic field $B = (B_0\, y/a)\hat{\mathbf{k}}$ is acting into the paper in the $+z$ direction. B_0 and a are positive constants. A square loop $EFGH$ of side a, mass m and resistance R in x-y plane starts falling under the influence of gravity. Note the directions of x and y in the figure. Find **(1999, 10M)**



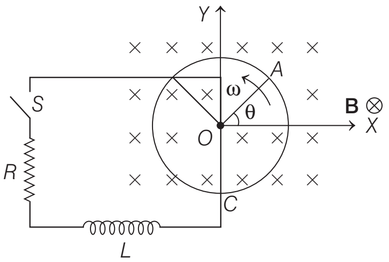
(a) the induced current in the loop and indicate its direction.
(b) the total Lorentz force acting on the loop and indicate its direction.
(c) an expression for the speed of the loop $v(t)$ and its terminal velocity.

9. A pair of parallel horizontal conducting rails of negligible resistance shorted at one end is fixed on a table. The distance between the rails is L. A conducting massless rod of resistance R can slide on the rails frictionlessly. The rod is tied to a massless string which passes over a pulley fixed to the edge of the table. A mass m tied to the other end of the string hangs vertically. A constant magnetic field B exists perpendicular to the table. If the system is released from rest. Calculate : **(1997, 5M)**



(a) the terminal velocity achieved by the rod, and
(b) the acceleration of the mass at the instant when the velocity of the rod is half the terminal velocity.

10. A metal rod OA and mass m and length r kept rotating with a constant angular speed ω in a vertical plane about horizontal axis at the end O. The free end A is arranged to slide without friction along a fixed conducting circular ring in the same plane as that of rotation. A uniform and constant magnetic induction **B** is applied perpendicular and into the plane of rotation as shown in figure. An inductor L and an external resistance R are connected through a switch S between the point O and a point C on the ring to form an electrical circuit. Neglect the resistance of the ring and the rod. Initially, the switch is open. **(1995, 10M)**



(a) What is the induced emf across the terminals of the switch?

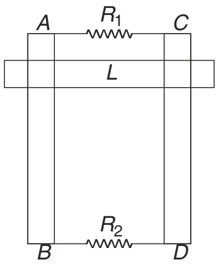
(b) The switch S is closed at time $t = 0$.

(i) Obtain an expression for the current as a function of time.

(ii) In the steady state, obtain the time dependence of the torque required to maintain the constant angular speed. Given that the rod OA was along the positive x-axis at $t = 0$.

11. Two parallel vertical metallic rails AB and CD are separated by 1 m. They are connected at two ends by resistances R_1 and R_2 as shown in figure. A horizontal metallic bar of mass 0.2 kg slides without friction vertically down the rails under the action of gravity.

There is a uniform horizontal magnetic field of 0.6 T perpendicular to the plane of the rails. It is observed that when the terminal velocity is attained, the powers dissipated in R_1 and R_2 are 0.76 W and 1.2 W respectively. Find the terminal velocity of the bar L and the values of R_1 and R_2. **(1994, 6M)**



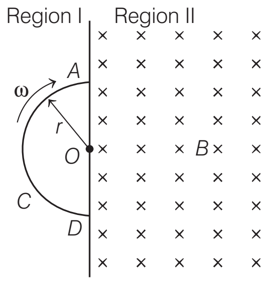
12. Space is divided by the line AD into two regions. Region I is field free and the region II has a uniform magnetic field B directed into the plane of the paper. ACD is a semicircular conducting loop of radius r with centre at O, the plane of the loop being in the plane of the paper.

The loop is now made to rotate with a constant angular velocity ω about an axis passing through O and perpendicular to the plane of the paper. The effective resistance of the loop is R. **(1985, 6M)**

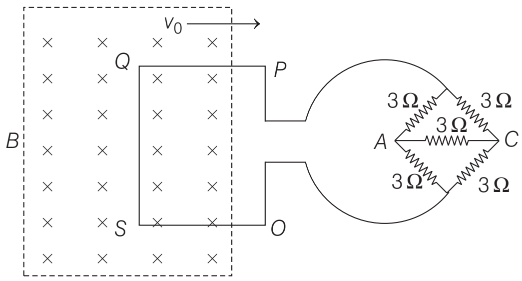
(a) Obtain an expression for the magnitude of the induced current in the loop.

(b) Show the direction of the current when the loop is entering into the region II.

(c) Plot a graph between the induced current and the time of rotation for two periods of rotation.



13. A square metal wire loop of side 10 cm and resistance 1 Ω is moved with a constant velocity v_0 in a uniform magnetic field of induction $B = 2\ \text{Wb/m}^2$ as shown in the figure. The magnetic field lines are perpendicular to the plane of the loop (directed into the paper). The loop is connected to a network of resistors each of value 3 Ω.



What should be the speed of the loop so as to have a steady current of 1mA in the loop? Give the direction of current in the loop. **(1983, 6M)**

14. The two rails of a railway track, insulated from each other and the ground, are connected to a millivoltmeter. What is the reading of the millivoltmeter when a train travels at a speed of 180 km/h along the track given that the vertical components of earth's magnetic field is $0.2 \times 10^{-4}\ \text{Wb/m}^2$ and the rails are separated by 1 m? Track is South to North. **(1981, 4M)**

Topic 3 Self and Mutual Inductance

Objective Questions I (Only one correct option)

1. A transformer consisting of 300 turns in the primary and 150 turns in the secondary gives output power of 2.2 kW. If the current in the secondary coil is 10 A, then the input voltage and current in the primary coil are **(Main 2019, 10 April I)**

(a) 440 V and 5 A (b) 220 V and 20 A
(c) 220 V and 10 A (d) 440 V and 20 A

2. Two coils P and Q are separated by some distance. When a current of 3 A flows through coil P, a magnetic flux of 10^{-3} Wb passes through Q. No current is passed through Q. When no current passes through P and a current of 2 A passes through Q, the flux through P is **(Main 2019, 9 April II)**

(a) 6.67×10^{-3} Wb (b) 6.67×10^{-4} Wb
(c) 3.67×10^{-3} Wb (d) 3.67×10^{-4} Wb

3. The total number of turns and cross-section area in a solenoid is fixed. However, its length L is varied by adjusting the separation between windings. The inductance of solenoid will be proportional to **(Main 2019, 9 April I)**

(a) $1/L$ (b) L^2 (c) L (d) $1/L^2$

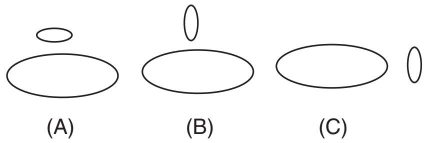
4. A copper wire is wound on a wooden frame, whose shape is that of an equilateral triangle. If the linear dimension of each side of the frame is increased by a factor of 3, keeping the number of turns of the coil per unit length of the frame the same, then the self-inductance of the coil **(Main 2019, 11 Jan II)**

(a) increases by a factor of 3.
(b) decreases by a factor of $9\sqrt{3}$.
(c) increases by a factor of 27.
(d) decreases by a factor of 9.

5 There are two long coaxial solenoids of same length l. The inner and outer coils have radii r_1 and r_2 and number of turns per unit length n_1 and n_2, respectively. The ratio of mutual inductance to the self-inductance of the inner coil is **(Main 2019, 11 Jan I)**

(a) $\frac{n_2}{n_1} \cdot \frac{r_1}{r_2}$ (b) $\frac{n_2}{n_1} \cdot \frac{r_2^2}{r_1^2}$ (c) $\frac{n_2}{n_1}$ (d) $\frac{n_1}{n_2}$

6. Two circular coils can be arranged in any of the three situations shown in the figure. Their mutual inductance will be **(2001, S)**



(a) maximum in situation (A).
(b) maximum in situation (B).
(c) maximum in situation (C).
(d) the same in all situations.

7. A small square loop of wire of side l is placed inside a large square loop of wire of side $L (L >> l)$. The loops are coplanar and their centres coincide. The mutual inductance of the system is proportional to **(1998, 2M)**

(a) l/L (b) l^2/L (c) L/l (d) L^2/l

Objective Questions II (One or more correct option)

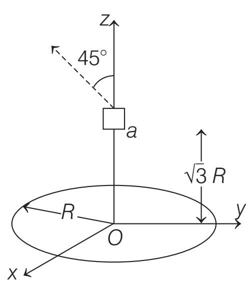
8. Two different coils have self-inductances $L_1 = 8$ mH and $L_2 = 2$ mH. The current in one coil is increased at a constant rate. The current in the second coil is also increased at the same constant rate. At a certain instant of time, the power given to the two coils is the same. At that time, the current, the induced voltage and the energy stored in the first coil are i_1, V_1 and W_1 respectively. Corresponding values for the second coil at the same instant are i_2, V_2 and W_2 respectively. Then **(1994, 2M)**

(a) $\frac{i_1}{i_2} = \frac{1}{4}$ (b) $\frac{i_1}{i_2} = 4$ (c) $\frac{W_1}{W_2} = \frac{1}{4}$ (d) $\frac{V_1}{V_2} = 4$

Integer Answer Type Question

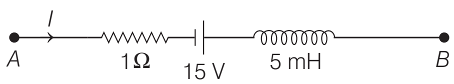
9. A circular wire loop of radius R is placed in the x-y plane centred at the origin O. A square loop of side a ($a << R$) having two turns is placed with its centre at $z = \sqrt{3}\,R$ along the axis of the circular wire loop, as shown in figure. The plane of the square loop makes an angle of 45° with respect to the Z-axis.

If the mutual inductance between the loops is given by $\frac{\mu_0 a^2}{2^{p/2} R}$, then the value of p is **(2012)**



Fill in the Blank

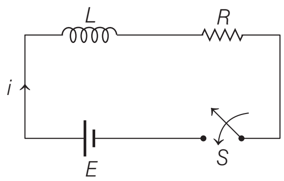
10. The network shown in figure is part of a complete circuit. If at a certain instant the current (I) is 5A and is decreasing at a rate of 10^3 A/s, then $V_B - V_A =$ ....V **(1997, 1M)**



Topic 4 *L-R* Circuits and *L-C* Oscillations

Objective Questions I (Only one correct option)

1. Consider the L-R circuit shown in the figure. If the switch S is closed at $t = 0$, then the amount of charge that passes through the battery between $t = 0$ and $t = \frac{L}{R}$ is **(Main 2019, 12 April II)**

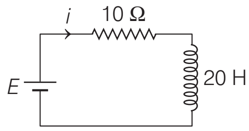


(a) $\frac{2.7EL}{R^2}$ (b) $\frac{EL}{2.7R^2}$ (c) $\frac{7.3EL}{R^2}$ (d) $\frac{EL}{7.3R^2}$

2. A coil of self inductance 10 mH and resistance $0.1\,\Omega$ is connected through a switch to a battery of internal resistance $0.9\,\Omega$. After the switch is closed, the time taken for the current to attain 80% of the saturation value is [Take, $\ln 5 = 1.6$] **(Main 2019, 10 April II)**

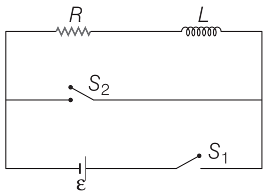
(a) 0.002 s (b) 0.324 s
(c) 0.103 s (d) 0.016 s

3. A 20 H inductor coil is connected to a 10 ohm resistance in series as shown in figure. The time at which rate of dissipation of energy (Joule's heat) across resistance is equal to the rate at which magnetic energy is stored in the inductor, is **(Main 2019, 10 Jan I)**

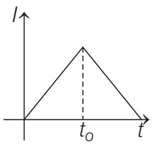


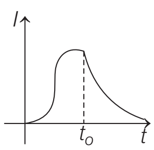
(a) $\frac{2}{\ln 2}$ (b) $\frac{1}{2}\ln 2$ (c) $2\ln 2$ (d) $\ln 2$

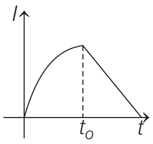
4. In the circuit shown,

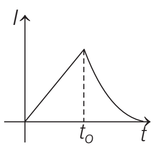


The switch S_1 is closed at time $t = 0$ and the switch S_2 is kept open. At some later time (t_0), the switch S_1 is opened and S_2 is closed. The behaviour of the current I as a function of time 't' is given by **(Main 2019, 11 Jan I)**

(a)


(b)


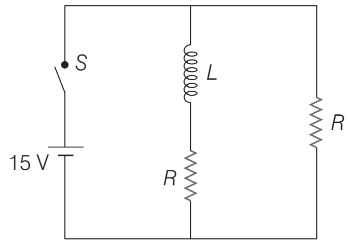
(c)


(d)


5. A series AC circuit containing an inductor (20 mH), a capacitor (120 μF) and a resistor (60 Ω) is driven by an AC source of 24 V/50 Hz. The energy dissipated in the circuit in 60 s is **(Main 2019, 9 Jan II)**

(a) 3.39×10^3 J (b) 5.65×10^2 J
(c) 2.26×10^3 J (d) 5.17×10^2 J

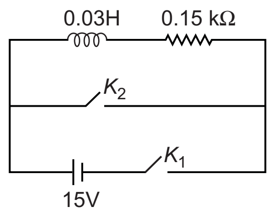
6. In the figure shown, a circuit contains two identical resistors with resistance $R = 5\Omega$ and an inductance with $L = 2$ mH. An ideal battery of 15 V is connected in the circuit.



What will be the current through the battery long after the switch is closed? **(2019 Main, 12 Jan I)**

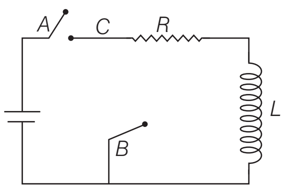
(a) 6 A (b) 3 A (c) 5.5 A (d) 7.5 A

7. An inductor ($L = 0.03$ H) and a resistor ($R = 0.15\,k\Omega$) are connected in series to a battery of 15V EMF in a circuit shown below. The key K_1 has been kept closed for a long time. Then at $t = 0$, K_1 is opened and key K_2 is closed simultaneously. At $t = 1$ms, the current in the circuit will be ($e^5 \cong 150$) **(2015 Main)**



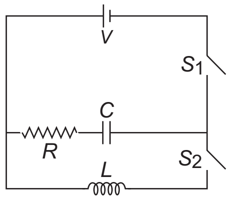
(a) 100 mA (b) 67 mA (c) 0.67mA (d) 6.7 mA

8. In the circuit shown here, the point C is kept connected to point A till the current flowing through the circuit becomes constant. Afterward, suddenly point C is disconnected from point A and connected to point B at time $t = 0$. Ratio of the voltage across resistance and the inductor at $t = L/R$ will be equal to **(2014 Main)**



(a) $\frac{e}{1-e}$ (b) 1 (c) -1 (d) $\frac{1-e}{e}$

9. In a L-C-R circuit as shown below, both switches are open initially. Now, switch S_1 and S_2, are closed. (q is charge on the capacitor and $\tau = RC$ is capacitance time constant). Which of the following statement is correct? **(2013 Main)**



(a) Work done by the battery is half of the energy dissipated in the resistor

(b) At $t = \tau$, $q = CV/2$

(c) At $t = 2\tau$, $q = CV\,(1 - e^{-2})$

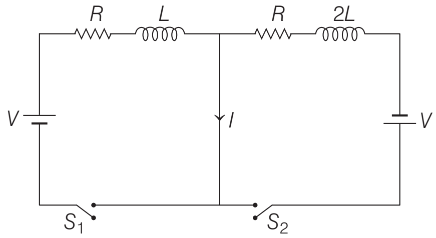
(d) At $t = \tau/2$, $q = CV\,(1 - e^{-1})$

10. A coil of inductance 8.4 mH and resistance 6 Ω is connected to a 12 V battery. The current in the coil is 1A at approximately the time **(1999, 2M)**

(a) 500 s (b) 20 s (c) 35 ms (d) 1 ms

Objective Question II (One or More than One)

11. In the figure below, the switches S_1 and S_2 are closed simultaneously at $t = 0$ and a current starts to flow in the circuit. Both the batteries have the same magnitude of the electromotive force (emf) and the polarities are as indicated in the figure. Ignore mutual inductance between the inductors. The current I in the middle wire reaches its maximum magnitude $I_{\max}$ at time $t = \tau$. Which of the following statements is (are) true ? **(2018 Adv.)**



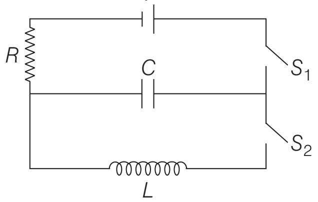
(a) $I_{\max} = \dfrac{V}{2R}$ (b) $I_{\max} = \dfrac{V}{4R}$

(c) $\tau = \dfrac{L}{R}\ln 2$ (d) $\tau = \dfrac{2L}{R}\ln 2$

Passage Based Questions

Passage

The capacitor of capacitance C can be charged (with the help of a resistance R) by a voltage source V, by closing switch S_1 while keeping switch S_2 open. The capacitor can be connected in series with an inductor L by closing switch S_2 and opening S_1.



12. Initially, the capacitor was uncharged. Now, switch S_1 is closed and S_2 is kept open. If time constant of this circuit is τ, then **(2006, 6M)**

(a) after time interval τ, charge on the capacitor is $CV/2$

(b) after time interval 2τ, charge on the capacitor is $CV(1 - e^{-2})$

(c) the work done by the voltage source will be half of the heat dissipated when the capacitor is fully charged

(d) after time interval 2τ, charge on the capacitor is $CV(1 - e^{-1})$

13. After the capacitor gets fully charged, S_1 is opened and S_2 is closed so that the inductor is connected in series with the capacitor. Then, **(2006, 6M)**

(a) at $t = 0$, energy stored in the circuit is purely in the form of magnetic energy.

(b) at any time $t > 0$, current in the circuit is in the same direction.

(c) at $t > 0$, there is no exchange of energy between the inductor and capacitor.

(d) at any time $t > 0$, maximum instantaneous current in the circuit may be $V\sqrt{\dfrac{C}{L}}$.

14. If the total charge stored in the LC circuit is Q_0, then for $t \ge 0$, **(2006, 6M)**

(a) the charge on the capacitor is $Q = Q_0 \cos\left(\dfrac{\pi}{2} + \dfrac{t}{\sqrt{LC}}\right)$

(b) the charge on the capacitor is $Q = Q_0 \cos\left(\dfrac{\pi}{2} - \dfrac{t}{\sqrt{LC}}\right)$

(c) the charge on the capacitor is $Q = -LC\,\dfrac{d^2Q}{dt^2}$

(d) the charge on the capacitor is $Q = -\dfrac{1}{\sqrt{LC}}\,\dfrac{d^2Q}{dt^2}$

Fill in the Blank

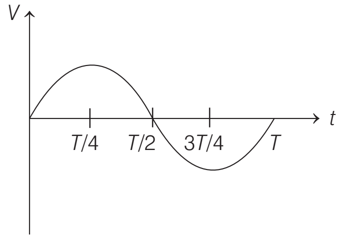
15. A uniformly wound solenoidal coil of self-inductance 1.8×10^{-4} H and resistance 6 Ω is broken up into two identical coils. These identical coils are then connected in parallel across a 15 V battery of negligible resistance. The time constant for the current in the circuit is …… s and the steady state current through the battery is … A. **(1989, 2M)**

Integer Answer Type Question

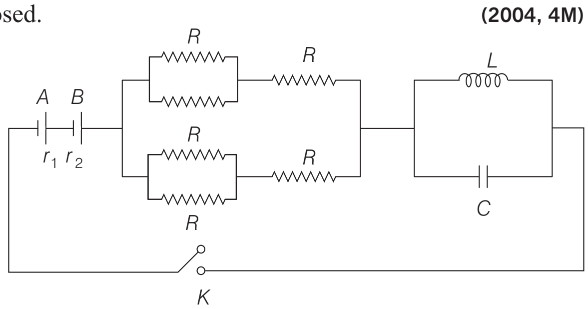
16. Two inductors L_1 (inductance 1mH, internal resistance 3Ω) and L_2 (inductance 2 mH, internal resistance 4Ω), and a resistor R (resistance 12 Ω) are all connected in parallel across a 5V battery. The circuit is switched on at time $t = 0$. The ratio of the maximum to the minimum current $(I_{\max}/I_{\min})$ drawn from the battery is **(2016 Adv.)**

Analytical & Descriptive Questions

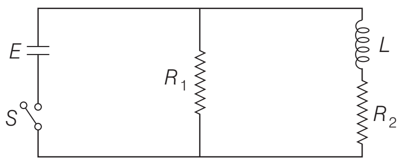
17. In an L-R series circuit, a sinusoidal voltage $V = V_0 \sin \omega t$ is applied. It is given that $L = 35$ mH, $R = 11\ \Omega$, $V_{rms} = 220$ V, $\omega / 2\pi = 50$ Hz and $\pi = 22/7$. Find the amplitude of current in the steady state and obtain the phase difference between the current and the voltage. Also plot the variation of current for one cycle on the given graph. **(2004, 4M)**



18. In the circuit shown A and B are two cells of same emf E but different internal resistances r_1 and $r_2 (r_1 > r_2)$ respectively. Find the value of R such that the potential difference across the terminals of cell A is zero, a long time after the key K is closed. **(2004, 4M)**



19. An inductor of inductance $L = 400$ mH and resistors of resistances $R_1 = 2\Omega$ and $R_2 = 2\Omega$ are connected to a battery of emf $E = 12$ V as shown in the figure. The internal resistance of the battery is negligible. The switch S is closed at time $t = 0$.



What is the potential drop across L as a function of time? After the steady state is reached, the switch is opened. What is the direction and the magnitude of current through R_1 as a function of time? **(2001, 5M)**

20. An inductor of inductance 2.0 mH is connected across a charged capacitor of capacitance 5.0 μF and the resulting L-C circuit is set oscillating at its natural frequency. Let Q denote the instantaneous charge on the capacitor and I, the current in the circuit. It is found that the maximum value of Q is 200 μC. **(1998, 8M)**

(a) When $Q = 100\ \mu$C, what is the value of $|dI/dt|$?

(b) When $Q = 200\ \mu$C, what is the value of I?

(c) Find the maximum value of I.

(d) When I is equal to one-half its maximum value, what is the value of $|Q|$?

21. A solenoid has an inductance of 10 H and a resistance of 2 Ω. It is connected to a 10 V battery. How long will it take for the magnetic energy to reach 1/4 of its maximum value? **(1996, 3M)**

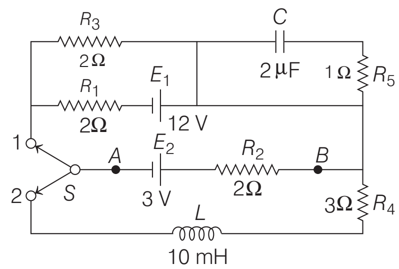
22. A circuit containing a two position switch S is shown in figure. **(1991, 4+4M)**

(a) The switch S is in position 1. Find the potential difference $V_A - V_B$ and the rate of production of joule heat in R_1.

(b) If now the switch S is put in position 2 at $t = 0$. Find

(i) steady current in R_4 and

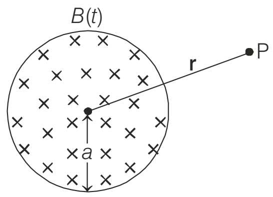
(ii) the time when current in R_4 is half the steady value. Also, calculate the energy stored in the inductor L at that time.



Topic 5 Induced Electric Field

Objective Question I (Only one correct option)

1. A uniform but time-varying magnetic field $B(t)$ exists in a circular region of radius a and is directed into the plane of the paper as shown. The magnitude of the induced electric field at point P at a distance r from the centre of the circular region **(2000, 2M)**



(a) is zero (b) decreases as $1/r$
(c) increases as r (d) decreases as $1/r^2$

Topic 6 Alternating Current

Objective Questions I (Only one correct option)

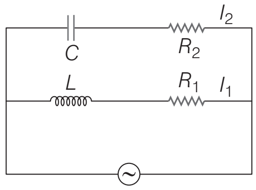
1. A circuit connected to an AC source of emf $e = e_0 \sin(100t)$ with t in seconds, gives a phase difference of $\frac{\pi}{4}$ between the emf e and current i. Which of the following circuits will exhibit this? **(Main 2019, 8 Apr II)**

(a) RC circuit with $R = 1\,k\Omega$ and $C = 1\,\mu F$
(b) RL circuit with $R = 1\,k\Omega$ and $L = 1\,mH$
(c) RC circuit with $R = 1\,k\Omega$ and $C = 10\,\mu F$
(d) RL circuit with $R = 1\,k\Omega$ and $L = 10\,mH$

2. An alternating voltage $V(t) = 220 \sin 100\pi t$ volt is applied to a purely resistive load of $50\,\Omega$. The time taken for the current to rise from half of the peak value to the peak value is **(Main 2019, 8 April I)**

(a) 5 ms (b) 2.2 ms (c) 7.2 ms (d) 3.3 ms

3.



In the above circuit, $C = \frac{\sqrt{3}}{2}\,\mu F$, $R_2 = 20\,\Omega$, $L = \frac{\sqrt{3}}{10}$ H and $R_1 = 10\,\Omega$. Current in L-R_1 path is I_1 and in C-R_2 path is I_2. The voltage of AC source is given by $V = 200\sqrt{2}\sin(100t)$ volts. The phase difference between I_1 and I_2 is **(Main 2019, 12 Jan II)**

(a) 30º (b) 60º (c) 0º (d) 90º

4. For an R-L-C circuit driven with voltage of amplitude v_m and frequency $\omega_0 = \frac{1}{\sqrt{LC}}$, the current exhibits resonance. The quality factor, Q is given by **(2018 Main)**

(a) $\frac{CR}{\omega_0}$ (b) $\frac{\omega_0 L}{R}$ (c) $\frac{\omega_0 R}{L}$ (d) $\frac{R}{\omega_0 C}$

5. In an AC circuit, the instantaneous emf and current are given by $e = 100 \sin 30\,t$, $i = 20 \sin\left(30\,t - \frac{\pi}{4}\right)$

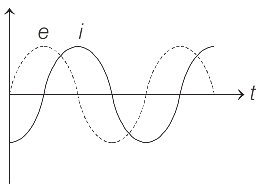
In one cycle of AC, the average power consumed by the circuit and the wattless current are, respectively **(2018 Main)**

(a) 50 , 0 (b) 50 , 10 (c) $\frac{1000}{\sqrt{2}}$, 10 (d) $\frac{50}{\sqrt{2}}$, 0

6. An AC voltage source of variable angular frequency ω and fixed amplitude V_0 is connected in series with a capacitance C and an electric bulb of resistance R (inductance zero). When ω is increased, **(2010)**

(a) the bulb glows dimmer
(b) the bulb glows brighter
(c) total impedance of the circuit is unchanged
(d) total impedance of the circuit increases

7. When an AC source of emf $e = E_0 \sin(100t)$ is connected across a circuit, the phase difference between the emf e and the current i in the circuit is observed to be $\frac{\pi}{4}$ ahead, as shown in the diagram.

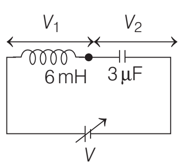


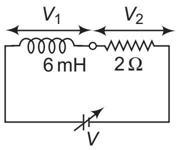
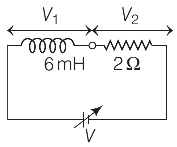
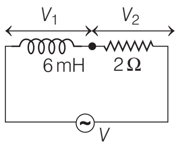
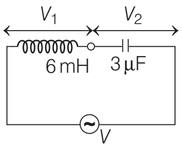
If the circuit consists possibly only of R-C or R-L or L-C in series, find the relationship between the two elements. **(2003, 2M)**

(a) $R = 1\,k\Omega$, $C = 10\,\mu F$ (b) $R = 1\,k\Omega$, $C = 1\,\mu F$
(c) $R = 1\,k\Omega$, $L = 10$ H (d) $R = 1\,k\Omega$, $L = 1$H

Match the Column

8. You are given many resistances, capacitors and inductors. These are connected to a variable DC voltage source (*the first two circuits*) or an AC voltage source of 50 Hz frequency (*the next three circuits*) in different ways as shown in **Column II**. When a current I (*steady state for DC or rms for AC*) flows through the circuit, the corresponding voltage V_1 and V_2 (*indicated in circuits*) are related as shown in **Column I**. **(2010)**

	Column I		Column II
(A)	$I \neq 0, V_1$ is proportional to I	(p)	

	Column I		Column II
(B)	$I \neq 0, V_2 > V_1$	(q)	
(C)	$V_1 = 0, V_2 = V$	(r)	
(D)	$I \neq 0, V_2$ is proportional to I	(s)	
		(t)	

Objective Questions II (One or more correct option)

9. The instantaneous voltages at three terminals marked X, Y and Z are given by $V_X = V_0 \sin \omega t$,

$$V_Y = V_0 \sin\left(\omega t + \frac{2\pi}{3}\right) \text{ and } V_Z = V_0 \sin\left(\omega t + \frac{4\pi}{3}\right).$$

An ideal voltmeter is configured to read rms value of the potential difference between its terminals. It is connected between points X and Y and then between Y and Z. The reading(s) of the voltmeter will be **(2017 Adv.)**

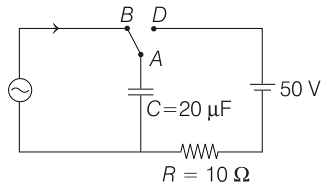
(a) $(V_{YZ})_{\text{rms}} = V_0\sqrt{\frac{1}{2}}$

(b) $(V_{XY})_{\text{rms}} = V_0\sqrt{\frac{3}{2}}$

(c) independent of the choice of the two terminals

(d) $(V_{XY})_{\text{rms}} = V_0$

10. At time $t = 0$, terminal A in the circuit shown in the figure is connected to B by a key and an alternating current $I(t) = I_0 \cos(\omega t)$, with $I_0 = 1\,\text{A}$ and $\omega = 500\ \text{rad s}^{-1}$ starts flowing in it with the initial direction shown in the figure. At $t = 7\pi/6\omega$, the key is switched from B to D. Now onwards only A and D are connected. A total charge Q flows from the battery to charge the capacitor fully. If $C = 20\,\mu\text{F}$, $R = 10\,\Omega$ and the battery is ideal with emf of 50 V, identify the correct statement(s). **(2014 Adv.)**



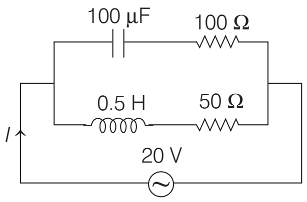
(a) Magnitude of the maximum charge on the capacitor before $t = \frac{7\pi}{6\omega}$ is 1×10^{-3} C

(b) The current in the left part of the circuit just before $t = \frac{7\pi}{6\omega}$ is clockwise

(c) Immediately after A is connected to D, the current in R is 10 A

(d) $Q = 2 \times 10^{-3}$ C

11. In the given circuit, the AC source has $\omega = 100$ rad/s. Considering the inductor and capacitor to be ideal, the correct choice(s) is(are) **(2012)**



(a) the current through the circuit, I is 0.3 A

(b) the current through the circuit, I is $0.3\sqrt{2}$ A

(c) the voltage across 100 Ω resistor = $10\sqrt{2}$ V

(d) the voltage across 50 Ω resistor = 10 V

12. A series R-C circuit is connected to AC voltage source. Consider two cases; (A) when C is without a dielectric medium and (B) when C is filled with dielectric of constant 4. The current I_R through the resistor and voltage V_C across the capacitor are compared in the two cases. Which of the following is/are true? **(2011)**

(a) $I_R^A > I_R^B$
(b) $I_R^A < I_R^B$
(c) $V_C^A > V_C^B$
(d) $V_C^A < V_C^B$

Integer Answer Type Question

13. A series R-C combination is connected to an AC voltage of angular frequency $\omega = 500$ rad/s. If the impedance of the R-C circuit is $R\sqrt{1.25}$, the time constant (in millisecond) of the circuit is **(2011)**

Topic 7 Miscellaneous Problems

Objective Questions I (Only one correct option)

1. A solid metal cube of edge length 2 cm is moving in a positive Y-direction at a constant speed of 6 m/s. There is a uniform magnetic field of 0.1 T in the positive Z-direction. The potential difference between the two faces of the cube perpendicular to the X-axis is **(Main 2019, 10 Jan I)**

(a) 2 mV (b) 12 mV
(c) 6 mV (d) 1 mV

2. A power transmission line feeds input power at 2300 V to a step-down transformer with its primary windings having 4000 turns. The output power is delivered at 230 V by the transformer. If the current in the primary of the transformer is 5A and its efficiency is 90%, the output current would be **(Main 2019, 9 Jan II)**

(a) 45 A (b) 50 A (c) 25 A (d) 35 A

3. An arc lamp requires a direct current of 10 A at 80 V to function. If it is connected to a 220 V (rms), 50 Hz AC supply, the series inductor needed for it to work is close to **(2016 Main)**

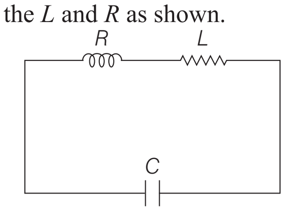
(a) 80 H (b) 0.08 H (c) 0.044 H (d) 0.065 H

4. Arrange the following electromagnetic radiations in the order of increasing energy. **(2016 Main)**

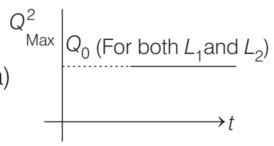
A. Blue light B. Yellow light
C. X-ray D. Radio wave

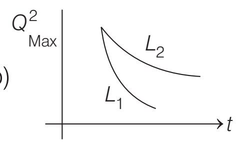
(a) D, B, A, C (b) A, B, D, C
(c) C, A, B, D (d) B, A, D, C

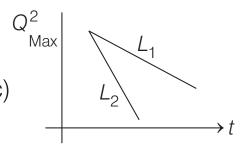
5. An L-C-R circuit is equivalent to a damped pendulum. In an L-C-R circuit, the capacitor is charged to Q_0 and then connected to the L and R as shown.

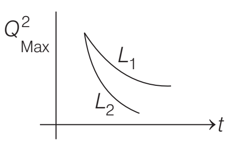


If a student plots graphs of the square of maximum charge (Q^2_{Max}) on the capacitor with time (t) for two different values L_1 and L_2 ($L_1 > L_2$) of L, then which of the following represents this graph correctly? (plots are schematic and not drawn to scale) **(2015 Main)**

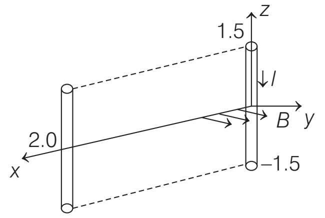
(a)


(b)


(c)


(d)


6. A conductor lies along the z-axis at $-1.5 \le z < 1.5$ m and carries a fixed current of 10.0 A in $-a_z$ direction (see figure). For a field $\mathbf{B} = 3.0 \times 10^{-4} e^{-0.2x} a_y$ T, find the power required to move the conductor at constant speed to $x = 2.0$ m, $y = 0$ in 5×10^{-3} s.
Assume parallel motion along the x-axis. **(2014 Main)**

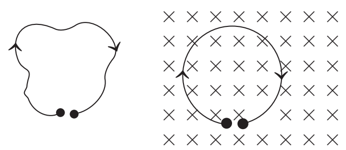


(a) 1.57 W (b) 2.97 W
(c) 14.85 W (d) 29.7 W

7. The amplitude of a damped oscillator decreases to 0.9 times its original magnitude is 5 s. In another 10 s, it will decrease to α times its original magnitude, where α equals **(2013 Main)**

(a) 0.7 (b) 0.81
(c) 0.729 (d) 0.6

8. A thin flexible wire of length L is connected to two adjacent fixed points and carries a current I in the clockwise direction, as shown in the figure. When the system is put in a uniform magnetic field of strength B going into the plane of the paper, the wire takes the shape of a circle. The tension in the wire is **(2010)**



(a) IBL (b) $\frac{IBL}{\pi}$
(c) $\frac{IBL}{2\pi}$ (d) $\frac{IBL}{4\pi}$

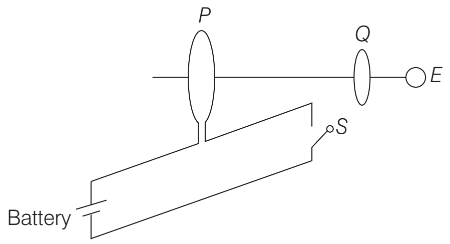
9. An infinitely long cylinder is kept parallel to an uniform magnetic field B directed along positive Z-axis. The direction of induced current as seen from the Z-axis will be

(a) clockwise of the +ve Z-axis **(2005, 2M)**
(b) anti-clockwise of the +ve Z-axis
(c) zero
(d) along the magnetic field

10. A short-circuited coil is placed in a time varying magnetic field. Electrical power is dissipated due to the current induced in the coil. If the number of turns were to be quadrupled (four times) and the wire radius halved, the electrical power dissipated would be **(2002, 2M)**

(a) halved (b) the same
(c) doubled (d) quadrupled

11. As shown in the figure, P and Q are two coaxil conducting loops separated by some distance. When the switch S is closed, a clockwise current I_p flows in P (as seen by E) and an induced current I_{Q_1} flows in Q. The switch remains closed for a long time. When S is opened, a current I_{Q_2} flows in Q. Then the direction I_{Q_1} and I_{Q_2} (as seen by E) are **(2002, 2M)**



(a) respectively clockwise and anti-clockwise
(b) both clockwise
(c) both anti-clockwise
(d) respectively anti-clockwise and clockwise

12. A coil of wire having finite inductance and resistance has a conducting ring placed co-axially within it. The coil is connected to a battery at time $t = 0$, so that a time dependent current $I_1(t)$ starts flowing through the coil. If $I_2(t)$ is the current induced in the ring and $B(t)$ is the magnetic field at the axis of the coil due to $I_1(t)$, then as a function of time ($t > 0$), the product $I_2(t)\, B(t)$ **(2000, 2M)**
(a) increases with time
(b) decreases with time
(c) does not vary with time
(d) passes through a maximum

13. Two identical circular loops of metal wire are lying on a table without touching each other. Loop A carries a current which increases with time. In response, the loop B **(1999, 2M)**
(a) remains stationary
(b) is attracted by the loop A
(c) is repelled by the loop A
(d) rotates about its CM, with CM fixed

Assertion and Reason

Mark your answer as

(a) If Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I

(b) If Statement I is true, Statement II is true; Statement II is not a correct explanation for Statement I

(c) If Statement I is true; Statement II is false

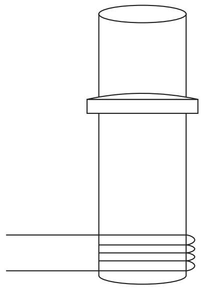
(d) If Statement I is false; Statement II is true

14. **Statement I** A vertical iron rod has a coil of wire wound over it at the bottom end. An alternating current flows in the coil. There is a conducting ring round the rod as shown in the figure. The ring can float at a certain height above the rod. **(2007, 3M)**

Statement II In the above situation, a current is induced in the ring which interacts with the horizontal component of the magnetic field to produce an average force in the upward direction.

Passage Based Questions

Passage 1



Consider a simple RC circuit as shown in Figure 1.

Process 1 In the circuit, the switch S is closed at $t = 0$ and the capacitor is fully charged to voltage V_0 (i.e. charging continues for time $T >> RC$). In the process, some dissipation (E_D) occurs across the resistance R. The amount of energy finally stored in the fully charged capacitor is E_c.

Process 2 In a different process the voltage is first set to $\frac{V_0}{3}$ and maintained for a charging time $T >> RC$. Then, the voltage is raised to $\frac{2V_0}{3}$ without discharging the capacitor and again maintained for a time $T >> RC$. The process is repeated one more time by raising the voltage to V_0 and the capacitor is charged to the same final voltage V_0 as in process 1.

These two processes are depicted in Figure 2. **(2017 Adv.)**

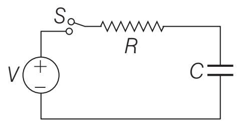


Figure 1

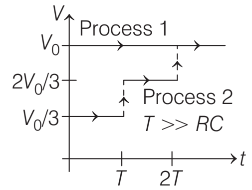


Figure 2

15. In process 1, the energy stored in the capacitor E_C and heat dissipated across resistance E_D are related by
(a) $E_C = E_D \ln 2$
(b) $E_C = E_D$
(c) $E_C = 2E_D$
(d) $E_C = \frac{1}{2}E_D$

16. In process 2, total energy dissipated across the resistance E_D is
(a) $E_D = \frac{1}{3}\left(\frac{1}{2}CV_0^2\right)$
(b) $E_D = 3\left(\frac{1}{2}CV_0^2\right)$
(c) $E_D = 3CV_0^2$
(d) $E_D = \frac{1}{2}CV_0^2$

Passage 2

A thermal power plant produces electric power of 600 kW at 4000 V, which is to be transported to a place 20 km away from the power plant for consumers' usage. It can be transported either directly with a cable of large current carrying capacity or by using a combination of step-up and step-down transformers at the two ends. The drawback of the direct transmission is the large energy dissipation. In the method using transformers, the dissipation is much smaller. In this method, a step-up transformer is used at the plant side so that the current is reduced to a smaller value.

At the consumers' end, a step-down transformer is used to supply power to the consumers at the specified lower voltage. It is reasonable to assume that the power cable is purely resistive and the transformers are ideal with a power factor unity. All the current and voltages mentioned are rms values. **(2013 Adv.)**

17. If the direct transmission method with a cable of resistance $0.4\ \Omega\ \text{km}^{-1}$ is used, the power dissipation (in %) during transmission is
(a) 20 (b) 30
(c) 40 (d) 50

18. In the method using the transformers, assume that the ratio of the number of turns in the primary to that in the secondary in the step-up transformer is 1 : 10. If the power to the consumers has to be supplied at 200V, the ratio of the number of turns in the primary to that in the secondary in the step-down transformer is
(a) 200 : 1 (b) 150 : 1 (c) 100 : 1 (d) 50 : 1

Passage 3

A point charge Q is moving in a circular orbit of radius R in the x-y plane with an angular velocity ω. This can be considered as equivalent to a loop carrying a steady current $\frac{Q\omega}{2\pi}$. A uniform magnetic field along the positive z-axis is now switched on, which increases at a constant rate from 0 to B in one second. Assume that the radius of the orbit remains constant. The applications of the magnetic field induces an emf in the orbit. The induced emf is defined as the work done by an induced electric field in moving a unit positive charge around a closed loop. It is known that, for an orbiting charge, the magnetic dipole moment is proportional to the angular momentum with a proportionality constant γ. **(2013 Adv.)**

19. The magnitude of the induced electric field in the orbit at any instant of time during the time interval of the magnetic field change is
(a) $\frac{BR}{4}$ (b) $\frac{-BR}{2}$ (c) BR (d) $2BR$

20. The change in the magnetic dipole moment associated with the orbit, at the end of the time interval of the magnetic field change, is
(a) γBQR^2 (b) $-\gamma\frac{BQR^2}{2}$ (c) $\gamma\frac{BQR^2}{2}$ (d) γBQR^2

Passage 4

Modern trains are based on Maglev technology in which trains are magnetically leviated, which runs its EDS Maglev system. There are coils on both sides of wheels. Due to motion of train, current induces in the coil of track which levitate it. This is in accordance with Lenz's law. If trains lower down then due to Lenz's law a repulsive force increases due to which train gets uplifted and if it goes much high, then there is a net downward force due to gravity. The advantage of Maglev train is that there is no friction between the train and the track, thereby reducing power consumption and enabling the train to attain very high speeds.

Disadvantage of Maglev train is that as it slows down the electromagnetic forces decreases and it becomes difficult to keep it leviated and as it moves forward according to Lenz's law, there is an electromagnetic drag force. **(2006, 6M)**

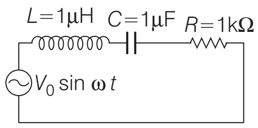
21. What is the advantage of this system ?
(a) No friction hence no power consumption
(b) No electric power is used
(c) Gravitation force is zero
(d) Electrostatic force draws the train

22. What is the disadvantage of this system ?
(a) Train experiences upward force according to Lenz's law
(b) Friction froce create a drag on the train
(c) Retardation
(d) By Lenz's law train experience a drag

23. Which force causes the train to elevate up ?
(a) Electrostatic force
(b) Time varying electric field
(c) Magnetic force
(d) Induced electric field

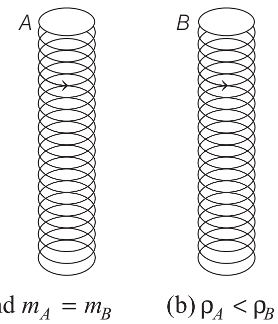
Objective Questions II (One or more correct option)

24. In the circuit shown, $L = 1\mu\text{H}$, $C = 1\mu\text{F}$ and $R = 1\,\text{k}\Omega$. They are connected in series with an AC source $V = V_0 \sin \omega t$ as shown. Which of the following options is/are correct? **(2017 Adv.)**



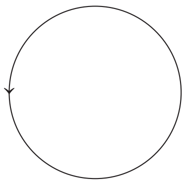
(a) At $\omega \sim 0$, the current flowing through the circuit becomes nearly zero
(b) The frequency at which the current will be in phase with the voltage is independent of R
(c) The current will be in phase with the voltage if $\omega = 10^4\ \text{rads}^{-1}$
(d) At $\omega >> 10^6\ \text{rads}^{-1}$, the circuit behaves like a capacitor

25. Two metallic rings A and B, identical in shape and size but having different resistivities ρ_A and ρ_B, are kept on top of two identical solenoids as shown in the figure. When current I is switched on in both the solenoids in identical manner, the rings A and B jump to heights h_A and h_B, respectively, with $h_A > h_B$. The possible relation(s) between their resistivities and their masses m_A and m_B is (are) **(2009)**



(a) $\rho_A > \rho_B$ and $m_A = m_B$ (b) $\rho_A < \rho_B$ and $m_A = m_B$
(c) $\rho_A > \rho_B$ and $m_A > m_B$ (d) $\rho_A < \rho_B$ and $m_A < m_B$

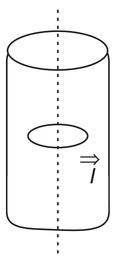
26. A field line is shown in the figure. This field cannot represent **(2006, 5M)**



(a) magnetic field (b) electrostatic field
(c) induced electric field (d) gravitational field

Integer Answer Type Question

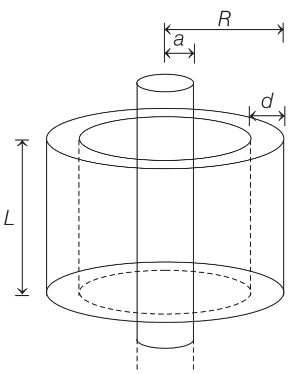
27. A long circular tube of length 10 m and radius 0.3 m carries a current I along its curved surface as shown. A wire-loop of resistance 0.005 Ω and of radius 0.1 m is placed inside the tube with its axis coinciding with the axis of the tube.



The current varies as $I = I_0 \cos 300\, t$ where I_0 is constant. If the magnetic moment of the loop is $N \mu_0 I_0 \sin(300t)$, then N is **(2011)**

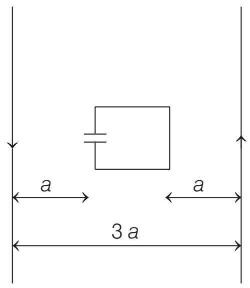
Analytical & Descriptive Questions

28. A long solenoid of radius a and number of turns per unit length n is enclosed by cylindrical shell of radius R, thickness d $(d << R)$ and length L.



A variable current $i = i_0 \sin \omega t$ flows through the solenoid. If the resistivity of the material of cylindrical shell is ρ, find the induced current in the shell. **(2005, 4M)**

29. Two infinitely long parallel wires carrying currents $I = I_0 \sin \omega t$ in opposite directions are placed a distance $3a$ apart. A square loop of side a of negligible resistance with a capacitor of capacitance C is placed in the plane of wires as shown. Find the maximum current in the square loop. Also, sketch the graph showing the variation of charge on the upper plate of the capacitor as a function of time for one complete cycle taking anti-clockwise direction for the current in the loop as positive. **(2003, 4M)**

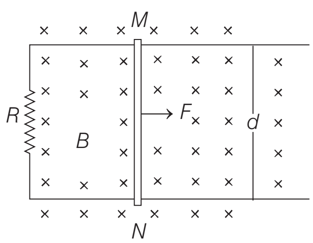


30. A thermocol vessel contains 0.5 kg of distilled water at 30°C. A metal coil of area $5 \times 10^{-3}\, m^2$, number of turns 100, mass 0.06 kg and resistance 1.6 Ω is lying horizontally at the bottom of the vessel. A uniform time varying magnetic field is setup to pass vertically through the coil at time $t = 0$. The field is first increased from 0 to 0.8 T at a constant rate between 0 and 0.2 s and then decreased to zero at the same rate between 0.2 and 0.4 s.

The cycle is repeated 12000 times. Make sketches of the current through the coil and the power dissipated in the coil as a function of time for the first two cycles. Clearly indicate the magnitudes of the quantities on the axes. Assume that no heat is lost to the vessel or the surroundings. Determine the final temperature of the water under thermal equilibrium. Specific heat of metal = 500 J/kg-K and the specific heat of water = 4200 J/kg-K. Neglect the inductance of coil. **(2000, 10M)**

31. An infinitesimally small bar magnet of dipole moment **M** is pointing and moving with the speed v in the positive x-direction. A small closed circular conducting loop of radius a and negligible self-inductance lies in the y-z plane with its centre at $x = 0$, and its axis coinciding with the X-axis. Find the force opposing the motion of the magnet, if the resistance of the loop is R. Assume that the distance x of the magnet from the centre of the loop is much greater than a. **(1997C, 5M)**

32. Two long parallel horizontal rails, a distance d apart and each having a resistance λ per unit length, are joined at one end by a resistance R. A perfectly conducting rod MN of mass m is free to slide along the rails without friction (see figure). There is a uniform magnetic field of induction B normal to the plane of the paper and directed into the paper. A variable force F is applied to the rod MN such that, as the rod moves, a constant current i flows through R.



Find the velocity of the rod and the applied force F as functions of the distance x of the rod from R.v **(1988, 6M)**

Answers

Topic 1

1. (d) **2.** (a) **3.** (a) **4.** (b)
5. (a) **6.** (d) **7.** (b) **8.** (d)
9. (b,d) **10.** (a,c) **11.** (a, c)
12. Left to right or zero **13.** F and T
14. (a) $\frac{d\phi}{dt} = iR + L\frac{di}{dt}$ (b) $\frac{1}{R}\left[\frac{\mu_0 I_0 l}{2\pi}(\ln(2) - Li_1)\right]$ (c) $\frac{T}{\ln(4)}$
15. $\frac{7}{22}$ A (E to A), $\frac{6}{22}$ A (B to E), $\frac{1}{22}$ A (F to E)
16. Yes, in the direction opposite to A.
17. Clockwise

Topic 2

1. (b) **2.** (d) **3.** (d) **4.** (b)
5. (d) **6.** (d) **7.** T
8. (a) $i = \frac{B_0 av}{R}$, anti-clockwise (b) $\mathbf{F} = -\frac{B_0^2 a^2 v}{R}\hat{\mathbf{j}}$
(c) $v = \frac{g}{K}(1 - e^{-kT})$, where $K = \frac{B_0^2 a^2}{mR}$, $v_t = \frac{g}{K} = \frac{gmR}{B_0^2 a^2}$
9. (a) $v_T = \frac{mgR}{B^2 L^2}$ (b) $a = \frac{g}{2}$
10. (a) $e = \frac{B\omega r^2}{2}$
(b) (i) $i = \frac{B\omega r^2}{2R}\left[1 - e^{-\left(\frac{R}{L}\right)t}\right]$ (ii) $\tau_{net} = \frac{B^2\omega r^4}{4R} + \frac{mgr}{2}\cos\omega t$
11. $v = 1$ m/s, $R_1 = 0.47\ \Omega$, $R_2 = 0.3\ \Omega$
12. (a) $\frac{1}{2}\frac{Br^2\omega}{R}$ (b) anti-clockwise (c) see the solution
13. 0.02 m/s, direction of induced current is clockwise
14. 1 mV

Topic 3

1. (a) **2.** (b) **3.** (a) **4.** (c)
5. (c) **6.** (a) **7.** (b) **8.** (a, c, d)
9. 7 **10.** 15

Topic 4

1. (b) **2.** (d) **3.** (c) **4.** (b)
5. (d) **6.** (a) **7.** (c) **8.** (c)
9. (c)
10. (d) **11.** (b, d) **12.** (b) **13.** (d)
14. (c) **15.** 3×10^{-5}, 10 **16.** (8)
17. 20 A, $\frac{\pi}{4}$ **18.** $R = \frac{4}{3}(r_1 - r_2)$
19. $12\,e^{-5t}$ V, $6e^{-10t}$ A (clockwise)
20. (a) 10^4 A/s (b) zero (c) 2.0 A (d) 1.732×10^{-4} C
21. 3.465 s
22. (a) –5V, 24.5 W (b) (i) 0.6 A (ii) 1.386×10^{-3}s, 4.5×10^{-4}J

Topic 5

1. (b)

Topic 6

1. (c) **2.** (d) **3.** (a) **4.** (b)
5. (c) **6.** (b) **7.** (a)
8. A $\rightarrow$ r, s, t; B $\rightarrow$ q, r, s, t; C $\rightarrow$ q, p; D $\rightarrow$ q, r, s, t
9. (b, c) **10.** (c, d) **11.** (a, c)
12. (b, c) **13.** 4

Topic 7

1. (b) **2.** (a) **3.** (d) **4.** (a)
5. (d) **6.** (b) **7.** (c) **8.** (c)
9. (c) **10.** (d) **11.** (d) **12.** (d)
13. (c) **14.** (a) **15.** (b) **16.** (a)
17. (b) **18.** (a) **19.** (b) **20.** (b)
21. (a) **22.** (d) **23.** (c) **24.** (a, b)
25. (b, d) **26.** (b, d) **27.** 6
28. $i = \frac{\mu_0 Ldna^2 I_0\omega\cos\omega t}{2\rho R}$ **29.** $i_{max} = \frac{\mu_0 a\,CI_0\omega^2\ln(2)}{\pi}$
30. 35.6°C **31.** $F = \frac{21}{4}\frac{\mu_0^2 M^2 a^4 v}{Rx^8}$ (repulsion)
32. $v = \frac{(R + 2\lambda x)\,i}{Bd}$, $F = \frac{2\lambda i^2 m}{B^2 d^2}(R + 2\lambda x)^2 + idB$

Hints & Solutions

Topic 1 Magnetic Flux and Induced EMF by change in Flux

1. Magnetic flux associated with the outer coil is

$$\phi_{\text{outer}} = \mu_0 \pi NR \cdot I = \mu_0 N\pi R(kte^{-\alpha t})$$
$$= Cte^{-\alpha t}$$

where,

$$C = \mu_0 N\pi Rk = \text{constant}$$

Induced emf,

$$e = \frac{-d\phi_{\text{outer}}}{dt} = Ce^{-\alpha t} + (-\alpha C\, t\, e^{-\alpha t})$$
$$= Ce^{-\alpha t}(1-\alpha t)$$

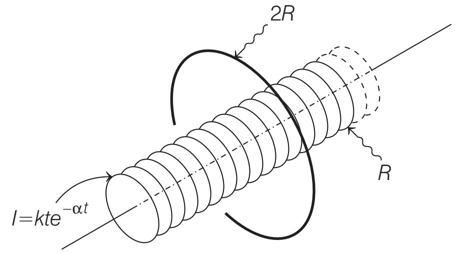
$\therefore$ Induced current, $I = \dfrac{e}{\text{Resistance}}$

$\Rightarrow$ At $t = 0$, $I = -$ ve

$\therefore$ The correct graph representing this condition is given in option (d).

Alternate Solution

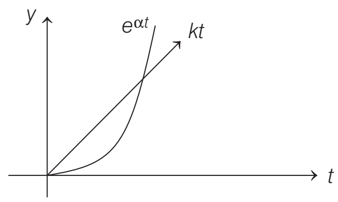
Given solenoid is shown below as,



At $t = 0$, current in solenoid

$$= I(t=0) = k(0)\,e^{-\alpha \cdot 0} = 0$$

Graph of $e^{\alpha t}$ and kt *versus* time can be shown as,



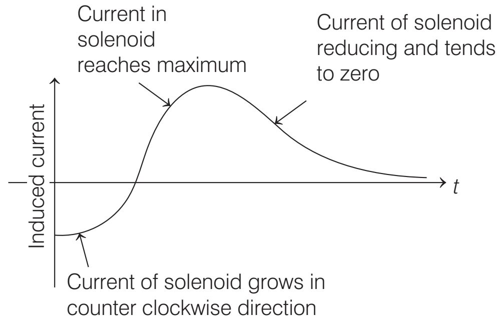
As, $\quad I = \dfrac{kt}{e^{\alpha t}}$;

Initially, $\quad kt > e^{\alpha t}$

So, current is increasing in magnitude.

Finally, after a short time $kt < e^{\alpha t}$. So, current is decreasing in magnitude.

But in both cases, it remains positive or counter clockwise. So, current induced is at first anti-clockwise (following Lenz's law) and then it becomes clockwise and finally reduces to zero as $t \longrightarrow \infty$.



So, correct graph of induced current is

2. Wire falls perpendicularly to horizontal component of earth's magnetic field, so induced electromotive force $(\varepsilon) = Blv$

Substituting the given values, we get

$$\varepsilon = 0.3 \times 10^{-4} \times 10 \times 5 = 1.5 \times 10^{-3}\ \text{V}$$

3. Energy stored in an inductor of inductance L and current I is given by

$$E = \frac{1}{2}LI^2$$

When current is being changed from I_1 to I_2, change in energy will be

$$\Delta E = E_2 - E_1 = \frac{1}{2}LI_2^2 - \frac{1}{2}LI_1^2 \qquad \ldots\text{(i)}$$

As I_1 and I_2 are given, we need to find value of L.

Now, induced emf in a coil is

$$\varepsilon = L\frac{dI}{dt}$$

Here, $\quad \varepsilon = 25\,\text{V}$,

$$dI = I_2 - I_1 = (25-10) = 15\ \text{A and } dt = 1\ \text{s}$$

$\Rightarrow \quad 25 = L \times \dfrac{15}{1}$ or $L = \dfrac{25}{15} = 5/3\,\text{H}$

Putting values of L, I_1 and I_2 in Eq. (i), we get

$$\Delta E = \frac{1}{2} \times \frac{5}{3} \times [25^2 - 10^2] = \frac{1}{2} \times \frac{5}{3} \times 525 \quad \Delta E = 437.5\ \text{J}$$

4. Induced constant, $I = \dfrac{e}{R}$

Here, e = induced emf $= \dfrac{d\phi}{dt}$

$$I = \frac{1}{R} = \left(\frac{d\phi}{dt}\right) \cdot \frac{1}{R}$$
$$d\phi = IRdt$$
$$\phi = \int IRdt$$

$\therefore$ Here, R is constant

$\therefore \quad \phi = R\int Idt$

$$\int I \cdot dt = \text{Area under } I\text{-}t \text{ graph}$$

$$= \frac{1}{2} \times 10 \times 0.5 = 2.5$$

$$\therefore \quad \phi = R \times 2.5 = 100 \times 2.5 = 250 \text{ Wb.}$$

5. Magnetic field at the centre of smaller loop

$$B = \frac{\mu_0 i R_2^2}{2(R_2^2 + x^2)^{3/2}}$$

Area of smaller loop $S = \pi R_1^2$

$\therefore$ Flux through smaller loop $\phi = BS$

Substituting the values, we get, $\phi \approx 9.1 \times 10^{-11}$ Wb

6. Cross $\otimes$ magnetic field passing from the closed loop is increasing. Therefore, from Lenz's law induced current will produce dot $\odot$ magnetic field. Hence, induced current is anti-clockwise.

7. Polarity of emf will be opposite in the two cases while entering and while leaving the coil. Only in option (b) polarity is changing.

8. Total magnetic flux passing through whole of the x-y plane will be zero, because magnetic lines form a closed loop. So, as many lines will move in $-z$ direction same will return to $+z$ direction from the x-y plane.

9. The net magnetic flux through the loops at time t is

$$\phi = B(2A - A)\cos \omega t = BA \cos \omega t$$

So, $\left|\frac{d\phi}{dt}\right| = B\omega A \sin \omega t$

$\therefore \left|\frac{d\phi}{dt}\right|$ is maximum when $\phi = \omega t = \pi/2$

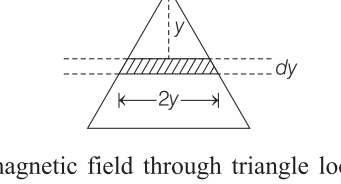
The emf induced in the smaller loop,

$$\varepsilon_{\text{smaller}} = -\frac{d}{dt}(BA \cos \omega t) = B\omega A \sin \omega t$$

$\therefore$ Amplitude of maximum net emf induced in both the loops

= Amplitude of maximum emf induced in the smaller loop alone.

10. By reciprocity theorem of mutual induction, it can be assumed that current in infinite wire is varying at 10A/s and EMF is induced in triangular loop.



Flux of magnetic field through triangle loop, if current in infinite wire is ϕ, can be calculated as follows

$$d\phi = \frac{\mu_0 i}{2\pi y} \cdot 2y\,dy$$

$$d\phi = \frac{\mu_0 i}{\pi} dy$$

$$\phi = \frac{\mu_0 i}{\pi}\left(\frac{l}{\sqrt{2}}\right)$$

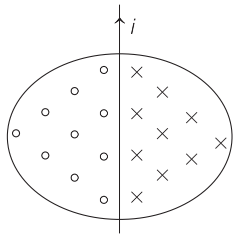
$$\Rightarrow \quad \text{EMF} = \left|\frac{d\phi}{dt}\right| = \frac{\mu_0}{\pi}\left(\frac{l}{\sqrt{2}}\right) \cdot \frac{di}{dt}$$

$$= \frac{\mu_0}{\pi}(10\text{cm})\left(10\frac{\text{A}}{\text{s}}\right) = \frac{\mu_0}{\pi} \text{ volt}$$

If we assume the current in the wire towards right then as the flux in the loop increases we know that the induced current in the wire is counter clockwise. Hence, the current in the wire is towards right.

Field due to triangular loop at the location of infinite wire is into the paper. Hence, force on infinite wire is away from the loop. By cylindrical symmetry about infinite wire, rotation of triangular loop will not cause any additional EMF.

11.



Due to the current in the straight wire, net magnetic flux from the circular loop is zero. Because in half of the circle, magnetic field is inwards and in other half, magnetic field is outwards. Therefore, change in current will not cause any change in magnetic flux from the loop. Therefore, induced emf under all conditions through the circular loop is zero.

12. When source is switched-off, left to right current decreases to zero. Therefore, from Lenz's law, induced current will oppose the change i.e. it will be from left to right if there is some inductance in the circuit, otherwise it will be zero.

13. If field is non-uniform from position point of view, no emf will be induced. If it is non-uniform from time point of view, emf will be induced.

14. (a) Applying Kirchhoff's second law, we get

$$\frac{d\phi}{dt} - iR - L\frac{di}{dt} = 0$$

or $$\frac{d\phi}{dt} = iR + L\frac{di}{dt} \quad \ldots(\text{i})$$

This is the desired relation between i, $\frac{di}{dt}$ and $\frac{d\phi}{dt}$.

(b) Eq. (i) can be written as

$$d\phi = iRdt + Ldi$$

Integrating, we get

$$\Delta\phi = R\,\Delta q + Li_1$$

$$\Delta q = \frac{\Delta\phi}{R} - \frac{Li_1}{R} \quad \ldots(\text{ii})$$

Here, $\Delta\phi = \phi_f - \phi_i = \int_{x=2x_0}^{x=x_0} \frac{\mu_0}{2\pi}\frac{I_0}{x} l\,dx = \frac{\mu_0 I_0 l}{2\pi} \ln(2)$

So, from Eq. (ii) charge flown through the resistance upto time $t = T$, when current i_1, is

$$\Delta q = \frac{1}{R}\left[\frac{\mu_0 I_0 l}{2\pi}\ln(2) - Li_1\right]$$

(c) This is the case of current decay in an L-R circuit. Thus,

$$i = i_0 e^{-t/\tau_L} \quad \ldots(\text{iii})$$

Here, $i = \frac{i_1}{4}$, $i_0 = i_1$, $t = (2T - T) = T$ and $\tau_L = \frac{L}{R}$

Substituting these values in Eq. (iii), we get

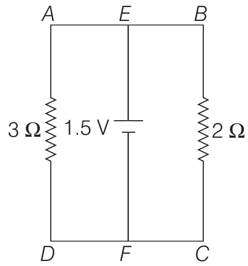
$$\tau_L = \frac{L}{R} = \frac{T}{\ln 4}$$

15. Induced emf in two loops $AEFD$ and $EBCF$ would be

$$e_1 = \left|\frac{d\phi_1}{dt}\right| = S_1\left(\frac{dB}{dt}\right) = (1\times 1)\,(1)\text{ V} = 1\text{ V}$$

Similarly, $$e_2 = \left|\frac{d\phi_2}{dt}\right| = S_2\left(\frac{dB}{dt}\right) = (0.5\times 1)\,(1)\text{ V} = 0.5\text{ V}$$

Now, since the magnetic field is increasing, the induced current will produce the magnetic field in ⊗ direction. Hence, e_1 and e_2 will be applied as shown in the figure.



Kirchhoff's first law at junction F gives

$$i_1 = i + i_2 \quad \ldots(\text{i})$$

Kirchhoff's second law in loop $FEADF$ gives

$$3i_1 + i = 1 \quad \ldots(\text{ii})$$

Kirchhoff's second law in loop $FEBCF$ gives

$$2i_2 - i = 0.5 \quad \ldots(\text{iii})$$

Solving Eqs. (i), (ii) and (iii), we get

$$i_1 = \left(\frac{7}{22}\right)\text{A} \quad \text{and} \quad i_2 = \left(\frac{6}{22}\right)\text{A}$$

and $\quad i = (1/22)\,\text{A}$

Therefore, current in segment AE is (7/22) A from E to A, current in segment BE is 6/22 A from B to E and current in segment EF is (1/22) A from F to E.

16. Due to the current in A, a magnetic field is from right to left. When A is moved towards B, magnetic lines passing through B (from right to left) will increase, i.e. magnetic flux passing through B will increase. Therefore, current will be induced in B. The induced current will have such a direction that it gives a magnetic field opposite to that, passing through B due to current in A. Therefore, induced current in B will be in opposite direction of current in A.

17. Magnetic field due to straight wire passing through the wire loop will be perpendicular to paper outwards. With increase in current in straight wire, outwards magnetic field through the loop will increase. Therefore, from Lenz's law, inward magnetic field will be produced by the induced current. Hence, induced current is clockwise.

Topic 2 Motional and Rotational EMF

1. Induced emf in the conductor of length L moving with velocity of 1 cm/s in the magnetic field of 1T is given by

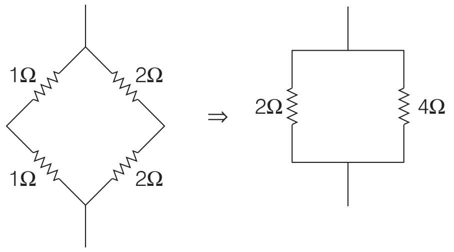
$$V = BLv \quad \ldots(\text{i})$$

If equivalent resistance of the circuit is R_{eq}, then current in the loop will be

$$i = \frac{V}{R_{eq}} = \frac{BLv}{R_{eq}} \quad \ldots(\text{ii})$$

Now, given network is a balanced Wheatstone bridge $\left(\frac{P}{Q} = \frac{R}{S}\right)$.

So, equivalent resistance of the Wheatstone bridge is



$$R_W = \frac{2\times 4}{2+4} = \frac{8}{6} = \frac{4}{3}\Omega$$

Again, resistance of conductor is 1.7 Ω.

So, effective resistance will be

$$R_{eq} = \frac{4}{3} + 1.7 = \frac{4}{3} + \frac{17}{10}$$

$$R_{eq} = \frac{40+51}{30} = \frac{91}{30} \simeq 3\Omega$$

By putting given values of R_{eq}, B and v in Eq. (ii), we have

$$i = \frac{(1)\,(5\times 10^{-2})\times 10^{-2}}{3}$$

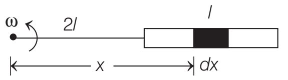
[here, $L = 5\times 10^{-2}$ m, $v = 1\,\text{cm/s} = 10^{-2}$m/s]

$$i = \frac{5\times 10^{-4}}{3} = 1.67\times 10^{-4}\,\text{A}$$

$$i = 167\,\mu\text{A} \approx 170\,\mu\text{A}$$

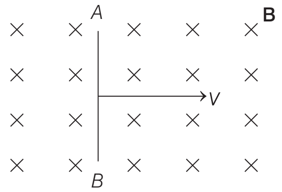
2. $e = \int_{2l}^{3l} (\omega x)\,B\,dx = B\omega\,\frac{[(3l)^2 - (2l)^2]}{2}$

$$= \frac{5Bl^2\omega}{2}$$



3. Electric field will be induced in both AD and BC.

4. A motional emf, $e = Blv$ is induced in the rod. Or we can say a potential difference is induced between the two ends of the rod AB, with A at higher potential and B at lower potential. Due to this potential difference, there is an electric field in the rod.



5. Induced motional emf in MNQ is equivalent to the motional emf in an imaginary wire MQ i.e.

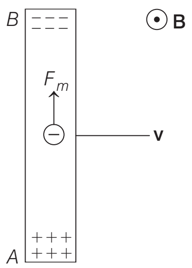
$$e_{MNQ} = e_{MQ} = Bvl = Bv(2R) \quad [l = MQ = 2R]$$

Therefore, the potential difference developed across the ring is $2RBv$ with Q at higher potential.

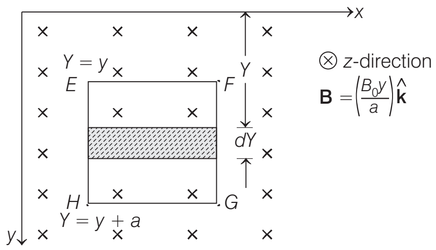
6. Net change in magnetic flux passing through the coil is zero.

$\therefore$ Current (or emf) induced in the loop is zero.

7. Magnetic force on free electrons will be towards B. Therefore, at B, there is excess of electrons (means negative charge) and at A, there is defficiency of electrons (means positive charge).



8. When the side EF is at a distance y from the X-axis, magnetic flux passing through the loop is



$$\phi = \int d\phi = \int_{Y=y}^{Y=y+a} \frac{B_0 Y}{a}(a\,dY)$$

$$\phi = \frac{B_0}{2}[(y+a)^2 - y^2]$$

(a) Induced emf is

$$e = \left|\frac{-d\phi}{dt}\right| = \left|-\frac{B_0}{2}[2(y+a) - 2y]\frac{dy}{dt}\right|$$

$$e = B_0 a \frac{dy}{dt} \Rightarrow e = B_0 va$$

where, $v = \frac{dy}{dt}$ = speed of loop

$\therefore$ Induced current, $i = \frac{e}{R} = \frac{B_0 av}{R}$

Direction $|\mathbf{B}| \propto y$ i.e. as the loop comes down ⊗ magnetic field passing through the loop increases, therefore the induced current will producer ⊙, magnetic field or the induced current in the loop will be **counter-clockwise.**

Alternate solution (of part a)

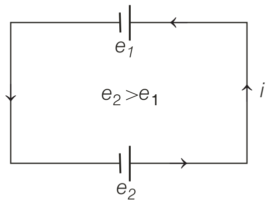
Motional emf in EH and $FG = 0$ as $\mathbf{v} || \mathbf{l}$

Motional emf in EF is $e_1 = \left(\frac{B_0 y}{a}\right)(a)v = B_0 yv \quad (\because e = Blv)$

Similarly, motional emf in GH will be

$$e_2 = \left\{\frac{B_0(y+a)}{a}\right\}(a)(v) = B_0(a+y)v$$

Polarities of e_1 and e_2 are shown in adjoining figures.



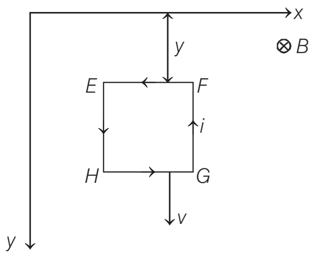
Net emf, $\quad e = e_2 - e_1$

$$e = B_0 av$$

$$\therefore \quad i = \frac{e}{R} = \frac{B_0 av}{R}$$

and direction of current will be counter-clockwise.

(b) Total Lorentz force on the loop :3



We have seen in part (a) that induced current passing through the loop (when its speed is v) is

$$i = \frac{B_0 av}{R}$$

Now, magnetic force on EH and FG are equal in magnitude and in opposite directions, hence they cancel each other and produce no force on the loop.

$$F_{EF} = \left(\frac{B_0 av}{R}\right)(a)\left(\frac{B_0 y}{a}\right) \quad \text{(downwards)}$$

$$(F = ilB) = \frac{B_0{}^2 avy}{R}$$

and $$F_{GH} = \left(\frac{B_0 av}{R}\right)(a)\left(\frac{B_0 (y+a)}{a}\right) \quad \text{(upwards)}$$

$$= \left(\frac{B_0{}^2 av}{R}\right)(y+a)$$

$$F_{GH} > F_{EF}$$

$\therefore$ Net Lorentz force on the loop

$$\mathbf{F} = F_{GH} - F_{EF} = \frac{B_0^2 a^2 v}{R} \quad \text{(upwards)}$$

$$\mathbf{F} = -\frac{B_0^2 a^2 v}{R}\hat{\mathbf{j}}$$

(c) Net force on the loop will be

F = weight − Lorentz force (downwards)

or $$F = mg - \frac{B_0^2 a^2 v}{R}$$

or $$m\left(\frac{dv}{dt}\right) = mg - \left(\frac{B_0^2 a^2}{R}\right)v$$

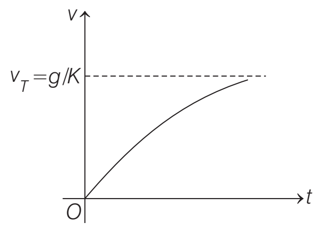
$\therefore$ $$\frac{dv}{dt} = g - \left(\frac{B_0^2 a^2}{mR}\right)v = g - Kv$$

where, $K = \frac{B_0^2 a^2}{mR}$ = constant

or $$\frac{dv}{g - Kv} = dt$$

or $$\int_0^v \frac{dv}{g - Kv} = \int_0^t dt$$

This equation gives $v = \frac{g}{K}(1 - e^{-Kt})$



Here, $$K = \left(\frac{B_0^2 a^2}{mR}\right)$$

i.e. speed of the loop is increasing exponentially with time t. Its terminal velocity will be

$$v_T = \frac{g}{K} = \left(\frac{mgR}{B_0^2 a^2}\right)$$

at $t \to \infty$

9. (a) Let v be the velocity of the wire (as well as block) at any instant of time t.

Motional emf, $e = BvL$

Motional current, $i = \frac{e}{r} = \frac{BvL}{R}$

and magnetic force on the wire

$$F_m = iLB = \frac{vB^2L^2}{R}$$

Net force in the system at this moment will be

$$F_{\text{net}} = mg - F_m = mg - \frac{vB^2L^2}{R}$$

or $$ma = mg - \frac{vB^2L^2}{R}$$

$$a = g - \frac{vB^2L^2}{mR} \quad \dots\text{(i)}$$

Velocity will acquire its terminal value i.e. $v = v_T$ when F_{net} or acceleration (a) of the particle becomes zero.

Thus, $$0 = g - \frac{v_T B^2 L^2}{mR}$$

or $$v_T = \frac{mgR}{B^2L^2}$$

(b) When $$v = \frac{v_T}{2} = \frac{mgR}{2B^2L^2}$$

Then from Eq. (i), acceleration of the block,

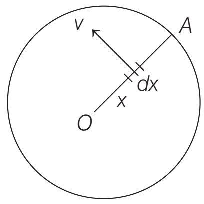
$$a = g - \left(\frac{mgR}{2B^2L^2}\right)\left(\frac{B^2L^2}{mR}\right) = g - \frac{g}{2}$$

or $$a = \frac{g}{2}$$

10. (a) Consider a small element of length dx of the rod OA situated at a distance x from O.

Speed of this element, $v = x\omega$

Therefore, induced emf developed across this element in uniform magnetic field B



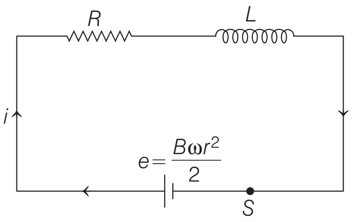
$$de = (B)(x\omega)dx \quad (\because e = Bvl)$$

Hence, total induced emf across OA,

$$e = \int_{x=0}^{x=r} de = \int_0^r B\omega x dx = \frac{B\omega r^2}{2} \Rightarrow e = \frac{B\omega r^2}{2}$$

(b) (i) A constant emf or PD, $e = \frac{B\omega r^2}{2}$ is induced across O and A.

The equivalent circuit can be drawn as shown in the figure.



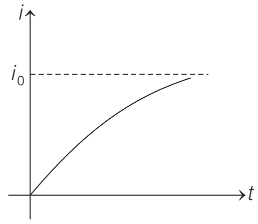
Switch S is closed at time $t = 0$. Therefore, it is case of growth of current in an L-R circuit. Current at any time t is given by

$$i = i_0(1 - e^{-t/\tau_L}),\ i_0 = \frac{e}{R} = \frac{B\omega r^2}{2R}$$

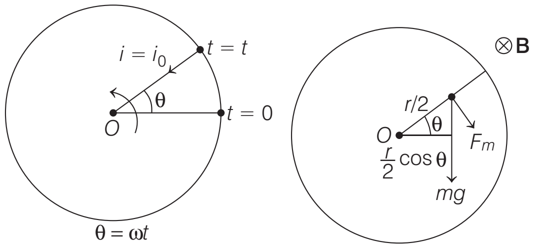
$$\tau_L = L/R$$

$$i = \frac{B\omega r^2}{2R}\left[1 - e^{-\left(\frac{R}{L}\right)t}\right]$$

The i-t graph will be as follows



(ii) At constant angular speed, net torque $= 0$



The steady state current will be $i = i_0 = \dfrac{B\omega r^2}{2R}$

From right hand rule, we can see that this current would be inwards (from circumference to centre) and corresponding magnetic force F_m will be in the direction shown in figure and its magnitude is given by

$$F_m = (i)(r)(B) = \frac{B^2\omega r^3}{2R} \qquad (\because F_m = ilB)$$

Torque of this force about centre O is

$$\tau_{F_m} = F_m \cdot \frac{r}{2} = \frac{B^2\omega r^4}{4R} \qquad \text{(clockwise)}$$

Similarly, torque of weight (mg) about centre O is

$$\tau_{mg} = (mg)\frac{r}{2}\cos\theta = \frac{mgr}{2}\cos\omega t \qquad \text{(clockwise)}$$

Therefore, net torque at any time t (after steady state condition is achieved) about centre O will be

$$\tau_{\text{net}} = \tau_{F_m} + \tau_{mg}$$

$$= \frac{B^2\omega r^4}{4R} + \frac{mgr}{2}\cos\omega t \qquad \text{(clockwise)}$$

Hence, the external torque applied to maintain a constant angular speed is $\tau_{\text{ext}} = \dfrac{B^2\omega r^4}{4R} + \dfrac{mgr}{2}\cos\omega t$ (but in anti-clockwise direction).

Note that for $\dfrac{\pi}{2} < \theta < \dfrac{3\pi}{2}$, torque of weight will be anti-clockwise, the sign of which is automatically adjusted because $\cos\theta$ = negative for $\dfrac{\pi}{2} < \theta < \dfrac{3\pi}{2}$.

11. Let the magnetic field be perpendicular to the plane of rails and inwards ⊗ . If v be the terminal velocity of the rails, then potential difference across E and F would be BvL with E at lower potential and F at higher potential. The equivalent circuit is shown in figure (2). In figure (2)

$$i_1 = \frac{e}{R_1} \qquad \ldots \text{(i)}$$

$$i_2 = \frac{e}{R_2} \qquad \ldots \text{(ii)}$$

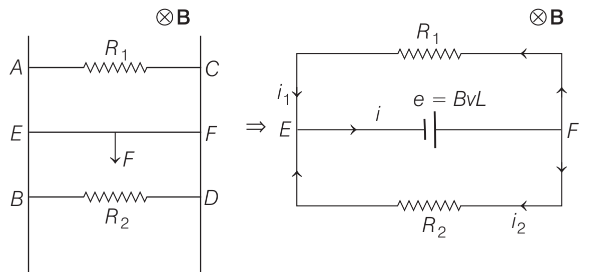
Power dissipated in R_1 is 0.76 W

Therefore $\quad ei_1 = 0.76\ \text{W} \qquad \ldots$ (iii)

Similarly, $\quad ei_2 = 1.2\ \text{W} \qquad \ldots$ (iv)

Now, the total current in bar EF is

$$i = i_1 + i_2 \qquad \text{(from } E \text{ to } F) \ldots \text{(v)}$$

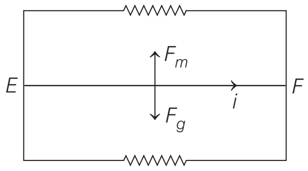


(1) (2)

Under equilibrium condition, magnetic force (F_m) on bar EF = weight (F_g) of bar EF

i.e. $\quad F_m = F_g \quad$ or $\quad iLB = mg \qquad \ldots$(vi)

From Eq. (vi) $\quad i = \dfrac{mg}{LB} = \dfrac{(0.2)(9.8)}{(1.0)(0.6)}$



or $\quad i = 3.27$ A

Multiplying Eq. (v) by e, we get

$$ei = ei_1 + ei_2$$

$$= (0.76 + 1.2) \quad \text{[From Eqs. (iii) and (iv)]}$$
$$= 1.96 \text{ W}$$
$$e = \frac{1.96}{i} \text{ V} = \frac{1.96}{3.27}$$

or $\quad e = 0.6 \text{ V}$

But since $\quad e = BvL$

$$v = \frac{e}{BL} = \frac{(0.6)}{(0.6)(1.0)} = 1.0 \text{ m/s}$$

Hence, terminal velocity of bar is 1.0 m/s.

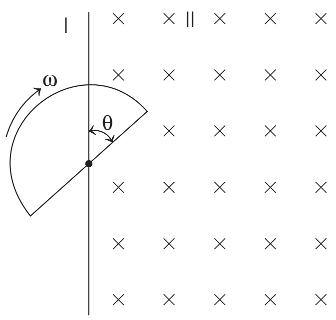
Power in R_1 is 0.76 W

$$\therefore \quad 0.76 = \frac{e^2}{R_1} \Rightarrow \therefore R_1 = \frac{e^2}{0.76} = \frac{(0.6)^2}{0.76}$$
$$= 0.47\ \Omega \Rightarrow R_1 = 0.47\ \Omega$$

Similarly, $\quad R_2 = \dfrac{e^2}{1.2} = \dfrac{(0.6)^2}{1.2} = 0.3\ \Omega$

$$R_2 = 0.3\ \Omega$$

12. (a) At time $t : \theta = \omega t$



$\therefore$ Flux passing through coil $\phi = BS \cos 0°$

or $\quad \phi = B\left(\dfrac{\theta}{2\pi}\right)(\pi r^2)$

or $\quad \phi = \left(\dfrac{Br^2}{2}\right)\theta = \left(\dfrac{Br^2}{2}\right)\omega t$

Magnitude of induced emf

$$e = \frac{d\phi}{dt} = \frac{B\omega r^2}{2}$$

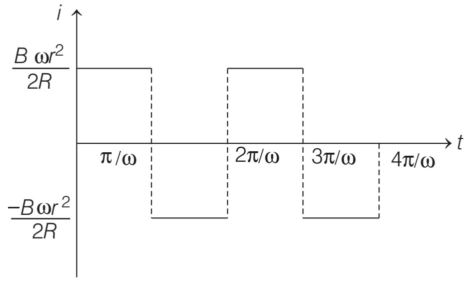
$\therefore$ Magnitude of induced current

$$i = \frac{e}{R} = \frac{B\omega r^2}{2R}$$

(b) When the loop enters in region II, magnetic field in cross direction passing through the loop is increasing. Hence, from the Lenz's law, induced current will produce magnetic field in dot direction or the current will be anti-clockwise.

(c) For half rotation $\left(t = \dfrac{T}{2} = \dfrac{\pi}{\omega}\right)$, current in the loop will be of constant magnitude $i = \dfrac{B\omega r^2}{2R}$ and anti-clockwise.

In the next half rotation when loop comes out of region II current will be clockwise, but again magnitude is constant. So, taking anti-clockwise current as the positive, i-t graph for two rotations will be as under.



13. Given network forms a balanced Wheatstone's bridge. The net resistance of the circuit is therefore $3\Omega + 1\Omega = 4\Omega$. Emf of the circuit is Bv_0l. Therefore, current in the circuit would be

$$i = \frac{Bv_0l}{R} \quad \text{or} \quad v_0 = \frac{iR}{Bl}$$
$$= \frac{(1 \times 10^{-3})(4)}{2 \times 0.1} = 0.02 \text{ m/s}$$

Cross magnetic field passing through the loop is decreasing. Therefore, induced current will produce magnetic field in cross direction. Or direction of induced current is clockwise.

14. Potential difference between the two rails :
$V = Bvl$ (When **B**, **v** and **l** all are mutually perpendicular to each other)

$$= (0.2 \times 10^{-4})\left(180 \times \frac{5}{18}\right)(1)$$
$$= 10^{-3} \text{ V} = 1 \text{ mV}$$

Topic 3 Self and Mutual Inductance

1. Given, Number of turns in primary, $N_1 = 300$

Number of turns in secondary, $N_2 = 150$

Output power, $P_2 = 2.2 \text{ kW} = 2.2 \times 10^3$ W

Current in secondary coil, $I_2 = 10$ A

Output power, $P_2 = I_2 V_2$

$$\Rightarrow \quad V_2 = \frac{P_2}{I_2} = \frac{2.2 \times 10^3}{10} = 220 \text{V} \quad \ldots \text{(i)}$$

We know that,

$$\frac{N_1}{N_2} = \frac{\text{Input voltage}}{\text{Output voltage}} = \frac{V_1}{V_2} \Rightarrow V_1 = \left(\frac{N_1}{N_2}\right)V_2$$

$$\Rightarrow \quad V_1 = \left(\frac{300}{150}\right) \times (220 \text{ V}) \quad \text{[using Eq. (i)]}$$

$$V_1 = 440 \text{ V} \quad \ldots \text{(ii)}$$

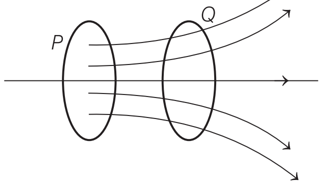
Again, $\quad \dfrac{V_1}{V_2} = \dfrac{I_2}{I_1}$

$$\Rightarrow \quad I_1 = \left(\frac{V_2}{V_1}\right) I_2 = \frac{220}{440} \times 10$$

[using Eqs. (i) and (ii)]

$$\Rightarrow \quad I_1 = 5\text{A}$$

2. As, coefficient of mutual induction is same for both coils



$$\therefore \quad M_{PQ} = M_{QP}$$

$$\Rightarrow \quad \frac{N_P \phi_{PQ}}{I_Q} = \frac{N_Q \phi_{QP}}{I_P} \quad \ldots \text{(i)}$$

Here, $N_P = N_Q = 1$,

$\phi_{PQ} = ?, \phi_{QP} = 10^{-3}$ Wb

$I_Q = 2\text{A}, I_P = 3\text{A}$

Substituting values in Eq (i), we get

$$\phi_{PQ} = \frac{I_Q \cdot \phi_{QP}}{I_P} = \frac{2}{3} \times 10^{-3}$$

$$= 0.667 \times 10^{-3} = 6.67 \times 10^{-4} \text{ Wb}$$

3. *(a)* Self inductance L_{sol} of a solenoid is given by

$$L_{\text{sol}} = \mu_0 n^2 \pi r^2 L$$

(Here, $n = N / L$ and L = length of solenoid)

or $$L_{\text{sol}} = \frac{\mu_0 N^2 \pi r^2}{L}$$

Clearly, $$L_{\text{sol}} \propto \frac{1}{L}$$

($\because$ All other parameters are fixed)

NOTE We can determine expression of L as follows

$$\phi = NBA = L_{\text{sol}} I$$

But for a solenoid, $B = \mu_0 n I$, $A = \pi r^2$

$$\therefore \quad L_{\text{sol}} I = \mu_0 n I \pi r^2 N$$

or $$L_{\text{sol}} = \mu_0 n^2 \pi r^2 L = \mu_0 \frac{N^2}{L} \pi r^2$$

4. Self-inductance of a coil is given by the relation

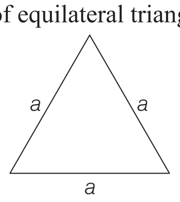
$$L = \mu_0 n^2 A \cdot l$$

where, n is number of turns per unit length. Shape of the wooden frame is equilateral triangle.

$\therefore$ Area of equilateral triangle,

$$A = \frac{\sqrt{3}}{4} a^2$$

(where, a is side of equilateral triangle)



$\therefore$ Self-inductance, $L = \mu_0 n^2 \left[\frac{\sqrt{3}}{4} a^2\right] . l$

Here, $l = 3a \times N$ (where, N is total number of turns)

$$\therefore \quad L = \mu_0 n^2 \left[\frac{\sqrt{3}}{4} a^2\right] \times 3aN \text{ or } L \propto a^3$$

When each side of frame is increased by a factor 3 keeping the number of turns per unit length of the frame constant.

Then, $a' = 3a$

$$\therefore \quad L' \propto (a')^3 \text{ or } L' \propto (3a)^3$$

or $$L' \propto 27a^3 \text{ or } L' = 27L$$

5. Mutual inductance for a coaxial solenoid of radius r_1 and r_2 and number of turns n_1 and n_2, respectively is given as,

$M = \mu_0 n_1 n_2 \pi r_1^2 l$ (for internal coil of radius r_1)

Self inductance for the internal coil,

$$L = \mu_0 n_1^2 \pi r_1^2 l$$

$$\therefore \quad \frac{M}{L} = \frac{n_1 n_2}{n_1^2} = \frac{n_2}{n_1}$$

6. When current flows in any of the coils, the flux linked with the other coil will be maximum in the first case. Therefore, mutual inductance will be maximum in case (A).

7. Magnetic field produced by a current i in a large square loop at its centre,

$$B \propto \frac{i}{L} \quad \text{say} \quad B = K \frac{i}{L}$$

$\therefore$ Magnetic flux linked with smaller loop,

$$\phi = B \cdot S$$

$$\phi = \left(K \frac{i}{L}\right)(l^2)$$

Therefore, the mutual inductance

$$M = \frac{\phi}{i} = K \frac{l^2}{L} \quad \text{or} \quad M \propto \frac{l^2}{L}$$

NOTE Dimensions of self inductance (L) or mutual inductance (M) are [Mutual inductance] = [Self inductance]

= [μ_0] [length]

Similarly, dimensions of capacitance are

[capacitance] = [ε_0] [length]

From this point of view, options (b) and (d) may be correct.

8. From Faraday's law, the induced voltage

$V \propto L$, if rate of change of current is constant $\left(V = -L\frac{di}{dt}\right)$

$$\therefore \quad \frac{V_2}{V_1} = \frac{L_2}{L_1} = \frac{2}{8} = \frac{1}{4} \quad \text{or} \quad \frac{V_1}{V_2} = 4$$

Power given to the two coils is same, i.e.

$$V_1 i_1 = V_2 i_2 \quad \text{or} \quad \frac{i_1}{i_2} = \frac{V_2}{V_1} = \frac{1}{4}$$

Energy stored, $W = \frac{1}{2} L i^2$

$$\therefore \quad \frac{W_2}{W_1} = \left(\frac{L_2}{L_1}\right)\left(\frac{i_2}{i_1}\right)^2 = \left(\frac{1}{4}\right)(4)^2$$

or $$\frac{W_1}{W_2} = \frac{1}{4}$$

9. If I current flows through the circular loop, then magnetic flux at the location of square loop is

$$B = \frac{\mu_0 I R^2}{2(R^2 + Z^2)^{3/2}}$$

Substituting the value of $Z\ (= \sqrt{3}R)$,

we have $\quad B = \dfrac{\mu_0 I}{16R}$

Now, total flux through the square loop is

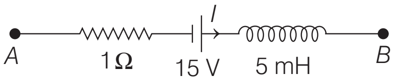
$$\phi_T = NBS\cos\theta = (2)\left(\frac{\mu_0 T}{16R}\right)a^2\cos 45^\circ$$

Mutual inductance,

$$M = \frac{\phi_T}{I} = \frac{\mu_0 a^2}{2^{7/2}R}$$

$\therefore \quad p = 7$

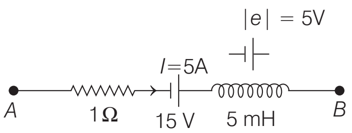
10. $\dfrac{di}{dt} = 10^3$ A/s



$\therefore$ Induced emf across inductance, $|e| = L\dfrac{di}{dt}$

$$|e| = (5\times10^{-3})(10^3)\ \text{V} = 5\ \text{V}$$

Since, the current is decreasing, the polarity of this emf would be so as to increase the existing current. The circuit can be redrawn as



Now, $\quad V_A - 5 + 15 + 5 = V_B$

$\therefore \quad V_A - V_B = -15$ V

or $\quad V_B - V_A = 15$ V

Topic 4 *L-R* Circuits and *L-C* Oscillations

1. In an L-R circuit, current during charging is given by

$$I = I_0\left(1 - e^{-\frac{R}{L}t}\right)$$

where, $\quad I_0 = \dfrac{E}{R}$ = saturation current

So, we have $\dfrac{dq}{dt} = I = I_0\left(1 - e^{-\frac{R}{L}t}\right)$

$$\Rightarrow \quad dq = I_0\left(1 - e^{-\frac{R}{L}t}\right)dt$$

So, charge q that passes through battery from time $t = 0$ to $t = \dfrac{L}{R}$ is obtained by integrating the above equation within the specified limits, i.e.

$$q = \int_0^Q dq = \int_{t=0}^{t=\frac{L}{R}} I_0\left(1 - e^{-\frac{R}{L}t}\right)dt$$

$$= I_0\left[\left(t - \frac{1}{\left(-\frac{R}{L}\right)}\cdot e^{-\frac{R}{L}t}\right)\right]_0^{\frac{L}{R}}$$

$$= \frac{E}{R}\left[\left\{\frac{L}{R} + \frac{L}{Re'}\right\} - \left\{0 + \frac{L}{R}\right\}\right] = \frac{E}{R}\times\frac{L}{Re} = \frac{EL}{R^2 e}$$

$$\Rightarrow \quad q = \frac{EL}{2.7R^2} \qquad [\because e \approx 2.72]$$

2. **Key Idea** In an L-R circuit, current during charging of inductor is given by

$$i = i_0\left(1 - e^{-\frac{R}{L}\cdot t}\right)$$

where, i_0 = saturation current.

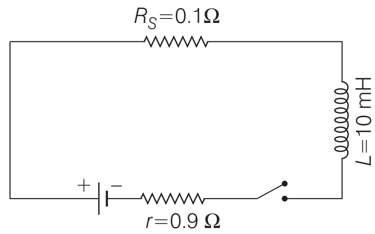
In given circuit,

Inductance of circuit is

$$L = 10\ \text{mH} = 10\times10^{-3}\ \text{H}$$

Resistance of circuit is

$$R = (R_s + r) = 0.1 + 0.9 = 1\Omega$$



Now, from

$$i = i_0\left(1 - e^{-\frac{R}{L}\cdot t}\right) \qquad \text{...(i)}$$

Given, $\quad i = 80\%$ of i_0

$$\Rightarrow \quad i = \frac{80\,i_0}{100} = 0.8\,i_0$$

Substituting the value of i in Eq. (i), we get

$$0.8 = 1 - e^{-\frac{R}{L}\cdot t} \Rightarrow e^{-\frac{R}{L}t} = 0.2 \Rightarrow e^{\frac{R}{L}t} = 5$$

$$\Rightarrow \ln(e)^{\frac{R}{L}t} = \ln 5 \quad \Rightarrow \frac{R}{L}t = \ln 5$$

$$\Rightarrow \quad t = \frac{L}{R}\cdot\ln(5) = \frac{10\times10^{-3}}{1}\times\ln(5)$$

$$= 10\times10^{-3}\times1.6$$

$$= 1.6\times10^{-2}\ \text{s} = 0.016\ \text{s}$$

3. Given circuit is a series L-R circuit

In an L-R circuit, current increases as

$$i = \frac{E}{R}\left(1 - e^{\frac{-R}{L}t}\right)$$

Now, energy stored in inductor is

$$U_L = \frac{1}{2}Li^2$$

where, L = self inductance of the coil
and energy dissipated by resistor is

$$U_R = i^2R$$

Given, rate of energy stored in inductor is equal to the rate of energy dissipation in resistor. So, after differentiating, we get

$$iL\frac{di}{dt} = i^2R \quad \Rightarrow \frac{di}{dt} = \frac{R}{L}i$$

$$\Rightarrow \qquad \frac{E}{R}\cdot\frac{R}{L}e^{-\frac{R}{L}t} = \frac{R}{L}\cdot\frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

$$\Rightarrow \qquad 2e^{-\frac{R}{L}t} = 1$$

$$\Rightarrow \qquad e^{-\frac{R}{L}t} = \frac{1}{2}$$

Taking log on both sides, we have

$$\Rightarrow \qquad \frac{-R}{L}t = \ln\left(\frac{1}{2}\right) \Rightarrow \frac{R}{L}t = \ln 2$$

$$\Rightarrow \qquad t = \frac{L}{R}\ln 2 = \frac{20}{10}\ln 2 \Rightarrow t = 2\ln 2$$

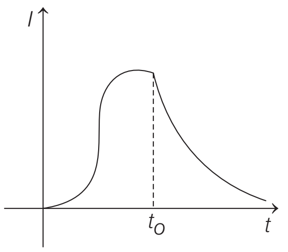
4. Initially in the given RL circuit with a source, when S_1 is closed and S_2 is open at $t \le t_0$.

$$I_1 = \frac{V}{R}\left[1 - \exp\left(\frac{-R}{L}t\right)\right]$$

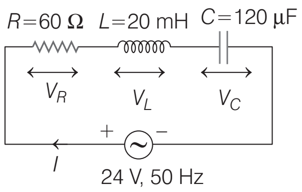
In this case, inductor L is charging.
When switch S_2 is closed and S_1 is open (after $t > t_0$), the inductor will be discharged through resistor.
In this case ($t > t_0$),

$$I_2 = \frac{V}{R}\exp\left[-\frac{R}{L}(t - t_0)\right]$$

Thus, the variation of I with t approximately is shown below



5. The given series R-L-C circuit is shown in the figure below.



Here,
V_R = potential across resistance (R)
V_L = potential across inductor (L) and
V_C = potential across capacitor (C)
Impedance of this series circuit is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \qquad \dots\text{(i)}$$

$$\because \qquad X_L = \omega L = (2\pi f)(L)$$
$$= 2\pi \times 50 \times 20 \times 10^{-3}\,\Omega$$
$$X_L = 6.28\Omega \qquad \dots\text{(ii)}$$

and
$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$
$$= \frac{1}{2\pi \times 50 \times 120 \times 10^{-6}} = \frac{250}{3\pi}\Omega \qquad \dots\text{(iii)}$$

and
$$X_L - X_C = \left(6.28 - \frac{250}{3\pi}\right) = -20.23\,\Omega \qquad \dots\text{(iv)}$$

RMS value of current in circuit is

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{24}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$I_{\text{rms}} = \frac{24}{\sqrt{60^2 + (-20.23)^2}} = \frac{24}{63.18}$$

$$I_{\text{rms}} = 0.379\text{ A}$$

Therefore, energy dissipated is

$$= I_{\text{rms}}^2 \times R \times t$$
$$E = (0.379)^2 \times 60 \times 60$$
or
$$= 517.10 = 5.17 \times 10^2\text{ J}$$

6. After a sufficiently long time, in steady state, resistance offered by inductor is zero. So, circuit is reduced to

∴ Current in circuit is

$$I = \frac{E}{R_{\text{eq}}} = \frac{15}{\left(\frac{5 \times 5}{5 + 5}\right)}$$
$$= \frac{15 \times 2}{5} = 6\text{ A}$$

7. Steady state current i_0 was already flowing in the L-R circuit when K_1 was closed for a long time. Here,

$$i_0 = \frac{V}{R} = \frac{15\text{ V}}{150\,\Omega} = 0.1\text{ A}$$

Now, K_1 is opened and K_2 is closed. Therefore, this i_0 will decrease exponentially in the $L - R$ circuit. Current i at time t will be given by $i = i_0 e^{\frac{-t}{\tau_L}}$

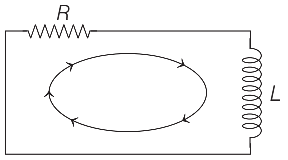
where, $\tau_L = \dfrac{L}{R} \;\Rightarrow\; \therefore\; i = i_0 e^{\frac{-Rt}{L}}$

Substituting the values, we have

$$i = (0.1)e^{\frac{-(0.15 \times 10^3)(10^{-3})}{(0.03)}}$$

$$= (0.1)(e^{-5}) = \frac{0.1}{150} = 6.67 \times 10^{-4}\,\text{A}$$
$$= 0.67\,\text{mA}$$

8. After connecting C to B hanging the switch, the circuit will act like an L-R discharging circuit.



Applying Kirchhoff's loop equation,

$$V_R + V_L = 0 \quad \Rightarrow \quad V_R = -V_L \quad \Rightarrow \quad \frac{V_R}{V_L} = -1$$

9. For charging of capacitor $q = CV\,(1 - e^{t/\tau})$

At $t = 2\tau$

$$q = CV\,(1 - e^{-2})$$

10. The current-time (i-t) equation in L-R circuit is given by [Growth of current in L-R circuit]

$$i = i_0\,(1 - e^{-t/\tau_L}) \qquad \ldots\text{(i)}$$

where, $\quad i_0 = \frac{V}{R} = \frac{12}{6} = 2\,\text{A}$

and $\quad \tau_L = \frac{L}{R} = \frac{8.4 \times 10^{-3}}{6} = 1.4 \times 10^{-3}\,\text{s}$

and $\quad i = 1\,\text{A}$ (given)

$\quad t = ?$

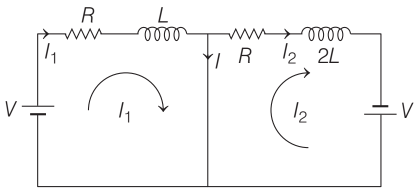
Substituting these values in Eq. (i), we get

$$t = 0.97 \times 10^{-3}\,\text{s}$$

or $\quad t = 0.97$ ms

$\quad t \approx 1\,\text{ms}$

11. $$I_1 = \frac{V}{R}\left(1 - e^{-\frac{tR}{L}}\right)$$



$$I_2 = \frac{V}{R}\left(1 - e^{-\frac{tR}{2L}}\right)$$

From principle of superposition,

$$I = I_1 - I_2 \Rightarrow I = \frac{V}{R} e^{-\frac{tR}{2L}}\left(1 - e^{-\frac{tR}{2L}}\right) \qquad \ldots\text{(i)}$$

I is maximum when $\frac{dI}{dt} = 0$, which gives

$$e^{-\frac{tR}{2L}} = \frac{1}{2} \text{ or } t = \frac{2L}{R}\ln 2$$

Substituting this time in Eq. (i), we get $I_{\max} = \frac{V}{4R}$

12. Charge on capacitor at time t is

$$q = q_0\,(1 - e^{-t/\tau})$$

Here, $\quad q_0 = CV$ and $t = 2\tau$

$\therefore \quad q = CV\,(1 - e^{-2\tau/\tau}) = CV\,(1 - e^{-2})$

13. From conservation of energy,

$$\frac{1}{2}LI_{\max}^2 = \frac{1}{2}CV^2$$

$$\therefore \quad I_{\max} = V\sqrt{\frac{C}{L}}$$

14. Comparing the L-C oscillations with normal SHM, we get

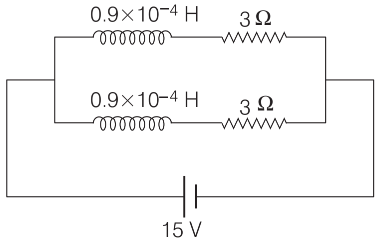
$$\frac{d^2Q}{dt^2} = -\omega^2 Q$$

Here, $\quad \omega^2 = \frac{1}{LC}$

$$\therefore \quad Q = -LC\,\frac{d^2Q}{dt^2}$$

15. Inductance of the circuit $L = \frac{0.9 \times 10^{-4}}{2} = 0.45 \times 10^{-4}\,\text{H}$

(in parallel)



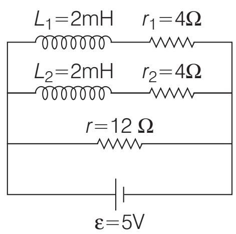
Resistance of the circuit $R = 3/2 = 1.5\,\Omega$ (in parallel)

$\therefore \quad \tau_L$ (time constant) $= \frac{L}{R} = 3.0 \times 10^{-5}$s

Steady state current in the circuit through the battery

$$i_0 = \frac{V}{R} = \frac{15}{1.5} = 10\text{A}$$

16.



$$I_{\max} = \frac{\varepsilon}{R} = \frac{5}{12}\text{A (Initially at } t = 0)$$

$$I_{\min} = \frac{\varepsilon}{R_{eq}} = \varepsilon\left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{R}\right) \text{ (finally in steady state)}$$

$$= 5\left(\frac{1}{3} + \frac{1}{4} + \frac{1}{12}\right) = \frac{10}{3}\text{A}$$

$$\frac{I_{\max}}{I_{\min}} = 8$$

17. Inductive reactance,

$$X_L = \omega L = (50)(2\pi)(35 \times 10^{-3}) \approx 11\Omega$$

Impedence, $Z = \sqrt{R^2 + X_L^2} = \sqrt{(11)^2 + (11)^2} = 11\sqrt{2}\ \Omega$

Given, $V_{rms} = 220\ \text{V}$

Hence, amplitude of voltage,

$$V_0 = \sqrt{2}V_{rms} = 220\sqrt{2}\ \text{V}$$

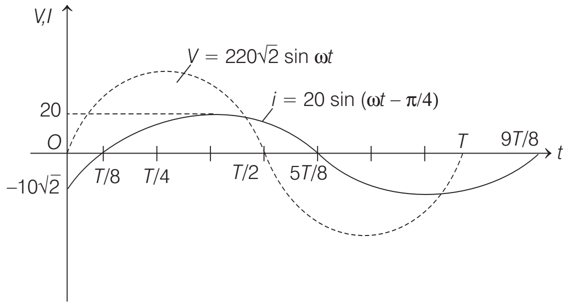
$\therefore$ Amplitude of current, $i_0 = \frac{V_0}{Z} = \frac{220\sqrt{2}}{11\sqrt{2}}$ or $i_0 = 20\ \text{A}$

Phase difference, $\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{11}{11}\right) = \frac{\pi}{4}$

In L-R circuit, voltage leads the current. Hence, instantaneous current in the circuit is

$$i = (20\text{A})\sin(\omega t - \pi/4)$$

Corresponding i-t graph is shown in figure.



18. After a long time, resistance across an inductor becomes zero while resistance across capacitor becomes infinite. Hence, net external resistance,

$$R_{net} = \frac{\frac{R}{2} + R}{2} = \frac{3R}{4}$$

Current through the batteries, $i = \dfrac{2E}{\frac{3R}{4} + r_1 + r_2}$

Given that potential across the terminals of cell A is zero.

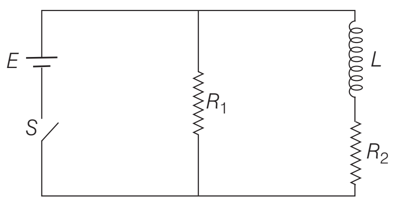
$\therefore$ $E - ir_1 = 0$

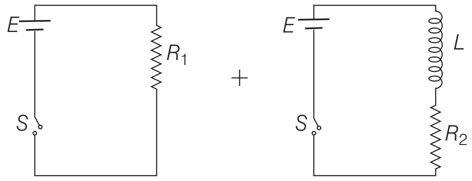
or $E - \left(\dfrac{2E}{3R/4 + r_1 + r_2}\right) r_1 = 0$

Solving this equation, we get, $R = \frac{4}{3}(r_1 - r_2)$

19. (a) Given, $R_1 = R_2 = 2\ \Omega$, $E = 12\ \text{V}$

and $L = 400\ \text{mH} = 0.4\ \text{H}$. Two parts of the circuit are in parallel with the applied battery. So, the upper circuit can be broken as





(a) (b)

Now, refer Fig. (b)

This is a simple L-R circuit, whose time constant

$$\tau_L = L/R_2 = \frac{0.4}{2} = 0.2\ \text{s}$$

and steady state current

$$i_0 = \frac{E}{R_2} = \frac{12}{2} = 6\ \text{A}$$

Therefore, if switch S is closed at time $t = 0$, then current in the circuit at any time t will be given by

$$i(t) = i_0(1 - e^{-t/\tau_L})$$

$$i(t) = 6(1 - e^{-t/0.2})$$

$$= 6(1 - e^{-5t}) = i \qquad \text{(say)}$$

Therefore, potential drop across L at any time t is

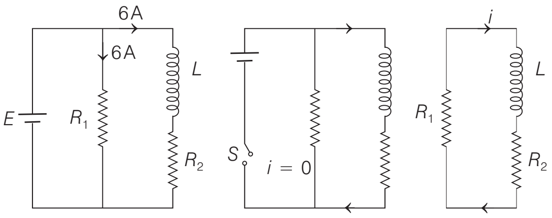
$$V = \left|L\frac{di}{dt}\right| = L(30e^{-5t}) = (0.4)(30)e^{-5t}$$

or $V = 12e^{-5t}\ \text{V}$

(b) The steady state current in L or R_2 is

$$i_0 = 6\ \text{A}$$

Now, as soon as the switch is opened, current in R_1 is reduced to zero immediately. But in L and R_2 it decreases exponentially. The situation is as follows



Steady state condition (c) | $t = 0$ S is open (d) | $t = t$ (e)

Refer figure (e) :

Time constant of this circuit would be

$$\tau_L' = \frac{L}{R_1 + R_2} = \frac{0.4}{(2+2)} = 0.1\ \text{s}$$

$\therefore$ Current through R_1 at any time t is

$$i = i_0 e^{-t/\tau_L'} = 6e^{-t/0.1} \text{ or } i = 6e^{-10t}\ \text{A}$$

Direction of current in R_1 is as shown in figure or clockwise.

20. This is a problem of L-C oscillations. Charge stored in the capacitor oscillates simple harmonically as

$$Q = Q_0 \sin(\omega t \pm \phi)$$

Here, Q_0 = maximum value of $Q = 200\mu C = 2\times10^{-4}$ C

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(2\times10^{-3})(5.0\times10^{-6})}} = 10^4 s^{-1}$$

Let at $t = 0, Q = Q_0$, then

$$Q(t) = Q_0 \cos\omega t \quad \ldots(i)$$

$$I(t) = \frac{dQ}{dt} = -Q_0\omega \sin\omega t \text{ and} \quad \ldots(ii)$$

$$\frac{dI(t)}{dt} = -Q_0\omega^2 \cos(\omega t) \quad \ldots(iii)$$

(a) $Q = 100\mu C$ or $\frac{Q_0}{2}$ at $\cos\omega t = \frac{1}{2}$ or $\omega t = \frac{\pi}{3}$

At $\cos(\omega t) = \frac{1}{2}$, from Eq. (iii) :

$$\left|\frac{dI}{dt}\right| = (2.0\times10^{-4} C)(10^4 s^{-1})^2\left(\frac{1}{2}\right)$$

$$\left|\frac{dI}{dt}\right| = 10^4 \text{ A/s}$$

(b) $Q = 200\,\mu C$ or Q_0 when $\cos(\omega t) = 1$ i.e. $\omega t = 0, 2\pi \;\ldots$

At this time $I(t) = -Q_0\omega \sin\omega t$

or $I(t) = 0$ $(\sin 0° = \sin 2\pi = 0)$

(c) $I(t) = -Q_0\omega \sin\omega t$

$\therefore$ Maximum value of I is $Q_0\omega$.

$$I_{max} = Q_0\omega = (2.0\times10^{-4})(10^4)$$

$$I_{max} = 2.0 \text{ A}$$

(d) From energy conservation,

$$\frac{1}{2}LI_{max}^2 = \frac{1}{2}LI^2 + \frac{1}{2}\frac{Q^2}{C}$$

or $$Q = \sqrt{LC(I_{max}^2 - I^2)}$$

$$I = \frac{I_{max}}{2} = 1.0 \text{ A}$$

$\therefore$ $$Q = \sqrt{(2.0\times10^{-3})(5.0\times10^{-6})(2^2 - 1^2)}$$

$$Q = \sqrt{3}\times10^{-4} C \text{ or } Q = 1.732\times10^{-4} \text{ C}$$

21. $U = \frac{1}{2}Li^2$ i.e. $U \propto i^2$

U will reach $\frac{1}{4}$th of its maximum value when current is reached half of its maximum value. In L-R circuit, equation of current growth is written as

$$i = i_0(1 - e^{-t/\tau_L})$$

Here, i_0 = Maximum value of current

τ_L = Time constant = L/R

$$\tau_L = \frac{10 \text{ H}}{2\,\Omega} = 5 \text{ s}$$

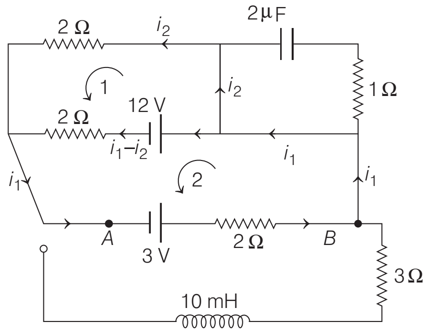
Therefore, $i = i_0/2 = i_0(1 - e^{-t/5})$

or $\frac{1}{2} = 1 - e^{-t/5}$ or $e^{-t/5} = \frac{1}{2}$

or $-t/5 = \ln\left(\frac{1}{2}\right)$ or $t/5 = \ln(2) = 0.693$

$\therefore$ $t = (5)(0.693)$ or $t = 3.465$ s

22. (a) In steady state, no current will flow through capacitor. Applying Kirchhoff's second law in loop 1



$$-2i_2 + 2(i_1 - i_2) + 12 = 0$$

$\therefore$ $2i_1 - 4i_2 = -12$

or $i_1 - 2i_2 = -6$ $\ldots(i)$

Applying Kirchhoff's second law in loop 2

$$-12 - 2(i_1 - i_2) + 3 - 2i_1 = 0$$

$\therefore$ $4i_1 - 2i_2 = -9$ $\ldots(ii)$

Solving Eqs. (i) and (ii), we get

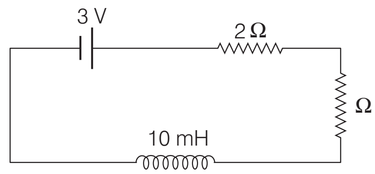
$$i_2 = 2.5 \text{ A} \quad \text{and} \quad i_1 = -1 \text{ A}$$

Now, $V_A + 3 - 2i_1 = V_B$

or $V_A - V_B = 2i_1 - 3 = 2(-1) - 3 = -5$ V

$$P_{R_1} = (i_1 - i_2)^2 R_1 = (-1 - 2.5)^2\,(2) = 24.5 \text{ W}$$

(b) **In position 2 :** Circuit is as under



(i) Steady current in R_4 :

$$i_0 = \frac{3}{3+2} = 0.6 \text{ A}$$

(ii) Time when current in R_4 is half the steady value

$$t_{1/2} = \tau_L\,(\ln 2) = \frac{L}{R}\ln(2)$$

$$= \frac{(10\times10^{-3})}{5}\ln(2)$$

$$= 1.386\times10^{-3} \text{ s}$$

$$U = \frac{1}{2}Li^2 = \frac{1}{2}(10\times10^{-3})(0.3)^2$$

$$= 4.5\times10^{-4} \text{ J}$$

Topic 5 Induced Electric Field

1. $$\int \mathbf{E}\cdot d\mathbf{l} = \left|\frac{d\phi}{dt}\right| = S\left|\frac{dB}{dt}\right|$$

or $$E(2\pi r) = \pi a^2\left|\frac{dB}{dt}\right|$$

For $r \geq a$,

$\therefore$ $$E = \frac{a^2}{2r}\left|\frac{dB}{dt}\right|$$

$\therefore$ Induced electric field $\propto 1/r$

For $r \leq a$,

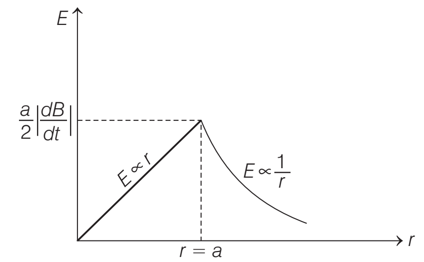
$$E(2\pi r) = \pi r^2\left|\frac{dB}{dt}\right|$$

or $$E = \frac{r}{2}\left|\frac{dB}{dt}\right|$$

or $$E \propto r$$

At $r = a$, $$E = \frac{a}{2}\left|\frac{dB}{dt}\right|$$

Therefore, variation of E with r (distance from centre) will be as follows



Topic 6 Alternating Currents

1. Given, phase difference, $\phi = \frac{\pi}{4}$

As we know, for R-L or R-C circuit,

$$\tan\phi = \frac{\text{Capacitive reactance } (X_C) \text{ or inductive reactance } (X_L)}{\text{Resistance (R)}}$$

$$\Rightarrow \quad \tan\frac{\pi}{4} = \frac{X_C \text{ or } X_L}{R}$$

$$1 = \frac{X_C \text{ or } X_L}{R} \Rightarrow R = X_C \text{ or } X_L$$

Also, given $e = e_0 \sin(100t)$

Comparing the above equation with general equation of emf, i.e. $e = e_0 \sin\omega t$, we get

$$\omega = 100\,\text{rad/s} = 10^2\,\text{rad/s}$$

Now, checking option wise,

For R-C circuit, with

$$R = 1\text{k}\Omega = 10^3\,\Omega \text{ and } C = 1\mu\text{F} = 10^{-6}\,\text{F}$$

So, $$X_C = \frac{1}{\omega C} = \frac{1}{10^2 \times 10^{-6}} = 10^4\,\Omega \Rightarrow R \neq X_C$$

For R - L circuit, with

$$R = 1\text{k}\Omega = 10^3\,\Omega$$

and $$L = 1\text{mH} = 10^{-3}\,\text{H}$$

So, $X_L = \omega L = 10^2 \times 10^{-3} = 10^{-1}\,\Omega \Rightarrow R \neq X_L$

For R - C circuit, with

$$R = 1\text{k}\Omega = 10^3\,\Omega$$

and $$C = 10\,\mu\text{F} = 10 \times 10^{-6}\,\text{F} = 10^{-5}\,\text{F}$$

So, $$X_C = \frac{1}{10^2 \times 10^{-5}} = 10^3\,\Omega \Rightarrow R = C$$

For R - L circuit, with

$$R = 1\text{k}\Omega = 10^3\,\Omega$$

and $$L = 10\,\text{mH} = 10 \times 10^{-3}\,\text{H} = 10^{-2}\,\text{H}$$

$$X_L = 10^2 \times 10^{-2} = 1\,\Omega \Rightarrow R \neq X_L$$

Alternate Solution

Since, $\tan\frac{\pi}{4} = 1 = \frac{X_C \text{ or } X_L}{R}$

$\therefore$ For R-C circuit, we have

$$1 = \frac{1}{C\omega R} \text{ or } \omega = \frac{1}{CR} \quad \text{...(i)}$$

Similarly, for R-L circuit, we have

$$1 = \frac{\omega L}{R} \Rightarrow \omega = \frac{R}{L} \quad \text{...(ii)}$$

It is given in the question that, $\omega = 100$ rad/s

Thus, again by substituting the given values of R, C or L option wise in the respective Eqs. (i) and (ii), we get that only for option (c),

$$\omega = \frac{1}{CR} = \frac{1}{10 \times 10^{-6} \times 10^3} \text{ or } \omega = 100\,\text{rad/s}$$

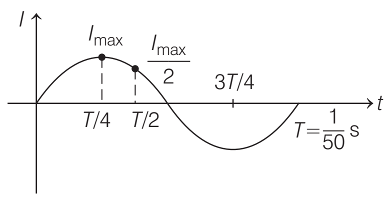
2. In an AC resistive circuit, current and voltage are in phase.

So, $$I = \frac{V}{R}$$

$$\Rightarrow \quad I = \frac{220}{50}\sin(100\pi t) \quad \text{...(i)}$$

$\therefore$ Time period of one complete cycle of current is

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{100\,\pi} = \frac{1}{50}\,\text{s}$$



So, current reaches its maximum value at

$$t_1 = \frac{T}{4} = \frac{1}{200}\,\text{s}$$

When current is half of its maximum value, then from Eq. (i), we have

$$I = \frac{I_{\max}}{2} = I_{\max}\sin(100\pi t_2)$$

$$\Rightarrow \sin(100\pi t_2) = \frac{1}{2} \Rightarrow 100\pi t_2 = \frac{5\pi}{6}$$

So, instantaneous time at which current is half of maximum value is $t_2 = \frac{1}{120}$ s

Hence, time duration in which current reaches half of its maximum value after reaching maximum value is

$$\Delta t = t_2 - t_1 = \frac{1}{120} - \frac{1}{200} = \frac{1}{300} \text{ s} = 3.3 \text{ ms}$$

3. Phase difference between I_2 and V, i.e. $C - R_2$ circuit is given by

$$\tan\phi = \frac{X_C}{R_2} \quad \Rightarrow \quad \tan\phi = \frac{1}{C\omega R_2}$$

Substituting the given values, we get

$$\tan\phi = \frac{1}{\frac{\sqrt{3}}{2} \times 10^{-6} \times 100 \times 20} = \frac{10^3}{\sqrt{3}}$$

$\therefore \phi_1$, is nearly 90°.

Phase difference between I_1 and V, i.e. in $L - R_1$ circuit is given by

$$\tan\phi_2 = -\frac{X_L}{R_1} = -\frac{L\omega}{R}$$

Substituting the given values, we get

$$\tan\phi_2 = -\frac{\frac{\sqrt{3}}{10} \times 100}{10} = -\sqrt{3}$$

As, $\tan\phi_2 = -\sqrt{3} \Rightarrow \phi_2 = 120°$

Now, phase difference between I_1 and I_2 is

$$\Delta\phi = \phi_2 - \phi_1 = 120° - 90° = 30°$$

4. $$Q = \frac{\omega_1 - \omega_2}{\omega_0}$$

Here, $\omega_1 - \omega_2 = \text{bandwidth} = \frac{R}{L}$

Substituting the values, we get

$$Q = \frac{\omega_0 L}{R}$$

Alternative solution $\frac{\omega_0 L}{R}$ is the only dimensionless quantity, hence it must be the quality factor.

5. $$\langle P \rangle = e_{\text{rms}}\, i_{\text{rms}} \cos\phi = \left(\frac{e_0}{\sqrt{2}}\right)\left(\frac{i_0}{\sqrt{2}}\right)\cos\phi$$

$$= \frac{e_0 i_0}{2}\cos\phi = \frac{(100)(20)\cos\frac{\pi}{4}}{2} = \frac{1000}{\sqrt{2}} \text{ W}$$

Wattless current $= I_{\text{rms}} \sin\phi = \frac{20}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = 10\text{A}$

6. Impedance, $Z = \sqrt{R^2 + X_c^2}$, $I_{\text{rms}} = \frac{V_{\text{rms}}}{Z}$, $P = I^2{}_{\text{rms}} R$

where, $X_C = \frac{1}{\omega C}$

As ω is increased, X_C will be decrease or Z will be decrease. Hence, I_{rms} or P will increase. Therefore, bulb glows brighter.

7. As the current i leads the emf e by $\pi/4$, it is an R-C circuit.

$$\tan\phi = \frac{X_C}{R}$$

or $$\tan\frac{\pi}{4} = \frac{\frac{1}{\omega C}}{R}$$

$\therefore$ $$\omega CR = 1$$

As $$\omega = 100 \text{ rad/s}$$

The product of C-R should be $\frac{1}{100}\text{s}^{-1}$

Option (a) satisfy this condition.

8. **In circuit (p)** I can't be non zero in steady state.

In circuit (q) $V_1 = 0$ and $V_2 = 2I = V$ (also)

In circuit (r) $V_1 = X_L I = (2\pi f L) I$

$$= (2\pi \times 50 \times 6 \times 10^{-3})I = 1.88\, I$$

$$V_2 = 2I$$

In circuit (s) $V_1 = X_L I = 1.88I$

$$V_2 = X_C I = \left(\frac{1}{2\pi fC}\right) I$$

$$= \left(\frac{1}{2\pi \times 50 \times 3 \times 10^{-6}}\right) I = (1061)\, I$$

In circuit (t)

$$V_1 = IR = (1000)\, I$$

$$V_2 = X_C I = (1061)\, I$$

Therefore, the correct options are as under

(A) $\rightarrow$ r, s, t (B) $\rightarrow$ q, r, s, t

(C) $\rightarrow$ q or p, q (D) $\rightarrow$ q, r, s, t

9. $$V_{XY} = V_0 \sin\left(\omega t + \frac{2\pi}{3}\right) - V_0 \sin\omega t$$

$$= V_0 \sin\left(\omega t + \frac{2\pi}{3}\right) + V_0 \sin(\omega t + \pi)$$

$$\Rightarrow \quad \phi = \pi - \frac{2\pi}{3} = \frac{\pi}{3}$$

$$\Rightarrow \quad V_0' = 2V_0 \cos\left(\frac{\pi}{6}\right) = \sqrt{3}\, V_0$$

$$\Rightarrow \quad V_{XY} = \sqrt{3} V_0 \sin(\omega t + \phi)$$

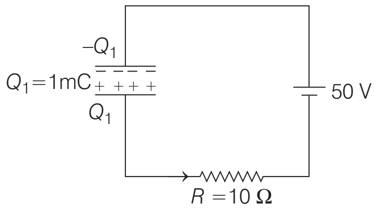
$$\Rightarrow \quad (V_{XY})_{\text{rms}} = (V_{YZ})_{\text{rms}} = \sqrt{\frac{3}{2}}\, V_0$$

10. $$\frac{dQ}{dt} = I$$

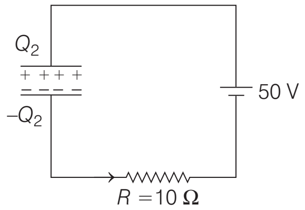
$$\Rightarrow \quad Q = \int I\, dt = \int (I_0 \cos\omega t)\, dt$$

$$\therefore \quad Q_{\max} = \frac{I_0}{\omega} = \frac{1}{500} = 2 \times 10^{-3} \text{ C}$$

Just after switching



In steady state



At $t = \dfrac{7\pi}{6\omega}$ or $\omega t = \dfrac{7\pi}{6}$

Current comes out to be negative from the given expression. So, current is anti-clockwise. Charge supplied by source from $t = 0$ to $t = \dfrac{7\pi}{6\omega}$

$$Q = \int_0^{\frac{7\pi}{6\omega}} \cos(500t)\, dt = \left[\frac{\sin 500t}{500}\right]_0^{\frac{7\pi}{6\omega}} = \frac{\sin\frac{7\pi}{6}}{500} = -1\ \text{mC}$$

Apply Kirchhoff's loop law, just after changing the switch to position D

$$50 + \frac{Q_1}{C} - IR = 0$$

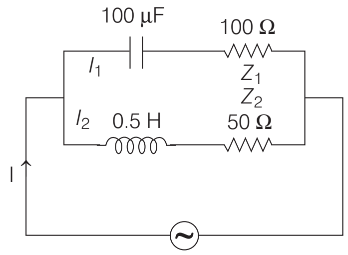
Substituting the values of Q_1, C and R, we get

$$I = 10\ \text{A}$$

In steady state $Q_2 = CV = 1\ \text{mC}$

$\therefore$ Net charge flown from battery = 2 mC

11.



***Circuit* 1**

$$X_C = \frac{1}{\omega C} = 100\ \Omega$$

$$\therefore \quad Z_1 = \sqrt{(100)^2 + (100)^2} = 100\sqrt{2}\ \Omega$$

$$\phi_1 = \cos^{-1}\left(\frac{R_1}{Z_1}\right) = 45°$$

In this circuit, current leads the voltage.

$$I_1 = \frac{V}{Z_1} = \frac{20}{100\sqrt{2}} = \frac{1}{5\sqrt{2}}\ \text{A}$$

$$V_{100\ \Omega} = (100)\, I_1 = (100)\frac{1}{5\sqrt{2}}\ \text{V} = 10\sqrt{2}\ \text{V}$$

***Circuit* 2**

$$X_L = \omega L = (100)(0.5) = 50\ \Omega$$

$$Z_2 = \sqrt{(50)^2 + (50)^2} = 50\sqrt{2}\ \Omega$$

$$\phi_2 = \cos^{-1}\left(\frac{R_2}{Z_2}\right) = 45°$$

In this circuit, voltage leads the current.

$$I_2 = \frac{V}{Z_2} = \frac{20}{50\sqrt{2}} = \frac{\sqrt{2}}{5}\ \text{A}$$

$$V_{50\ \Omega} = (50)\, I_2 = 50\left(\frac{\sqrt{2}}{5}\right) = 10\sqrt{2}\ \text{V}$$

Further, I_1 and I_2 have a mutual phase difference of 90°.

$$\therefore \quad I = \sqrt{I_1^2 + I_2^2} = 0.34$$

12. $Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \left(\dfrac{1}{\omega C}\right)^2}$

In case (b), capacitance C will be more. Therefore, impedence Z will be less. Hence, current will be more.

Further, $V_C = \sqrt{V^2 - V_R^2} = \sqrt{V^2 - (IR)^2}$

In case (b), since current I is more.

Therefore, V_C will be less.

13. $Z = \sqrt{R^2 + X_C^2} = R\sqrt{1.25}$

$\therefore R^2 + X_C^2 = 1.25\, R^2$ or $X_C = \dfrac{R}{2}$ or $\dfrac{1}{\omega C} = \dfrac{R}{2}$

$\therefore$ Time constant $= CR = \dfrac{2}{\omega} = \dfrac{2}{500}\ \text{s} = 4\ \text{ms}$

Topic 7 Miscellaneous Problems

1. Potential difference between opposite faces of cube is

$$V = \text{induced emf} = B\, l\, v$$

where,

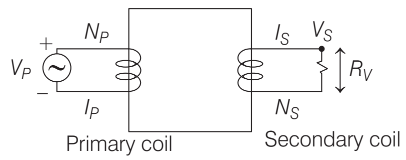
B = magnetic field = 0.1 T,

l = distance between opposite faces of cube

$= 2\ \text{cm} = 2 \times 10^{-2}$ m and v = speed of cube $= 6\ \text{ms}^{-1}$.

Hence, $V = 0.1 \times 2 \times 10^{-2} \times 6 = 12\ \text{mV}$

2. For a transformer, there are two circuits which have N_p and N_s (number of coil turns), I_p and I_S (currents) respectively as shown below.



Here, input voltage, $V_p = 2300\ \text{V}$

Number of turns in primary coil, $N_P = 4000$

Output voltage, $V_S = 230$ volt

Output power, $P_S = V_S \cdot I_S$

Input power, $P_P = V_P I_P$

$\therefore$ The efficiency of the transformer is

$$\eta = \frac{\text{Output (secondary) power}}{\text{Input (primary) power}}$$

$$\Rightarrow \quad \eta = \frac{V_S \cdot I_S}{V_P \cdot I_P} \times 100$$

$$\Rightarrow \quad \eta = \frac{(230)\,(I_S)}{(2300)\,(5)} \times 100$$

$$90 = \frac{230\, I_S}{(2300) \times 5} \times 100$$

$$\Rightarrow \quad I_S = 45\,\text{A}$$

3. Resultant voltage, $V^2 = V_R^2 + V_L^2 \Rightarrow 220^2 = 80^2 + V_L^2$

Solving, we get

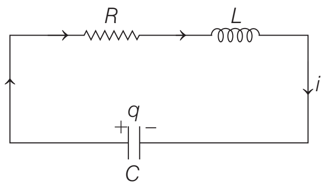
$$V_L \approx 205 \text{ V}$$

$$X_L = \frac{V_L}{I} = \frac{205}{10} = 20.5\,\Omega = \omega L$$

$$\therefore \quad L = \frac{20.5}{2\pi \times 50} = 0.065\,\text{H}$$

4. Remembering based question. Therefore, no solution is required.

5.



At a general time t, suppose charge on capacitor is q and current in the circuit is i, then applying Kirchhoff's loop law, we have

$$\frac{q}{c} - iR - L\frac{di}{dt} = 0$$

Putting $\quad i = -\dfrac{dq}{dt}$ and $\dfrac{di}{dt} = -\dfrac{d^2q}{dt^2}$

In the above equation, we have

$$\frac{d^2q}{dt^2} + \frac{R}{L}\left(\frac{dq}{dt}\right) + \frac{q}{LC} = 0 \qquad \text{...(i)}$$

Comparing this equation with the standard differential equation of damped oscillation,

$$\frac{d^2X}{dt^2} + \frac{b}{m}\frac{dX}{dt} + \frac{K}{m}X = 0$$

Which has a general solution of amplitude, $A = A_0 e^{\frac{-bt}{2m}}$

The general solution of Eq. (i) will be

$$Q_{\max} = Q_0 e^{\frac{-Rt}{2L}} \text{ or } Q_{\max}^2 = Q_0^2 e^{\frac{-Rt}{L}}$$

Hence, $Q_{\max}^2$ *versus* time graph is exponentially decreasing graph. Lesser the value of self inductance, faster will be the damping.

6. When force exerted on a current carrying conductor

$$F_{\text{ext}} = BIL$$

$$\text{Average power} = \frac{\text{Work done}}{\text{Time taken}}$$

$$P = \frac{1}{t}\int_0^2 F_{\text{ext.}} \cdot dx = \frac{1}{t}\int_0^2 B(x) IL\, dx$$

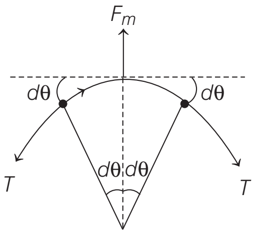
$$= \frac{1}{5 \times 10^{-3}} \int_0^2 3 \times 10^{-4} e^{-0.2x} \times 10 \times 3\, dx$$

$$= 9\,[1 - e^{-0.4}] = 9\left[1 - \frac{1}{e^{0.4}}\right]$$

$$= 2.967 \approx 2.97\,\text{W}$$

7. Amplitude decreases exponentially. In 5 s, it remains 0.9 times. Therefore, in total 15 s it will remains (0.9) (0.9) (0.9) = 0.729 times its original value.

8.



$$L = 2\pi R$$

$$\therefore \quad R = L/2\pi$$

$$2T \sin(d\theta) = F_m$$

For small angles, $\sin(d\theta) \approx d\theta$

$$\therefore \quad 2T\,(d\theta) = I\,(dL)\,B \sin 90^\circ = I\,(2R \cdot d\theta) \cdot B$$

$$\therefore \quad T = IRB = \frac{ILB}{2\pi}$$

9. In uniform magnetic field, change in magnetic flux is zero. Therefore, induced current will be zero.

10. Power, $P = e^2/R$

Here, $\quad e = \text{induced emf} = -\left(\dfrac{d\phi}{dt}\right)$ where, $\phi = NBA$

$$e = -NA\left(\frac{dB}{dt}\right)$$

Also, $\quad R \propto \dfrac{l}{r^2}$

where, R = resistance, r = radius, l = length.

$$\therefore \quad P \propto N^2 r^2 \Rightarrow \therefore \frac{P_2}{P_1} = 4$$

11. When switch S is closed magnetic field lines passing through Q increases in the direction from right to left. So, according to Lenz's law, induced current in Q i.e. I_{Q_1} will flow in such a direction, so that the magnetic field lines due to I_{Q_1} passes from left to right through Q.

This is possible when I_{Q_1} flows in anti-clockwise direction as seen by E. Opposite is the case when switch S is opened i.e. I_{Q_2} will be clockwise as seen by E.

12. The equations of $I_1(t)$, $I_2(t)$ and $B(t)$ will take the following forms :

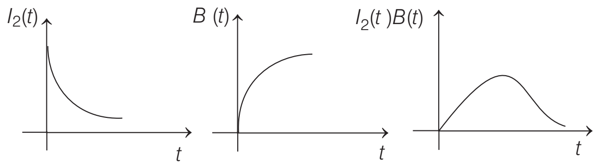
$$I_1(t) = K_1(1 - e^{-k_2 t}) \rightarrow \text{current growth in } L\text{-}R \text{ circuit}$$

$$B(t) = K_3(1 - e^{-k_2 t}) \rightarrow B(t) \propto I_1(t)$$

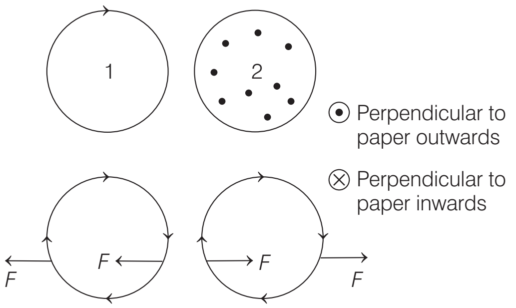
$$I_2(t) = K_4 e^{-k_2 t}$$

$$\left[I_2(t) = \frac{e_2}{R} \text{ and } e_2 \propto \frac{dI_1}{dt} : e_2 = -M\frac{dI_1}{dt}\right]$$

Therefore, the product $I_2(t)B(t) = K_5\, e^{-k_2 t}(1 - e^{-k_2 t})$. The value of this product is zero at $t = 0$ and $t = \infty$. Therefore, the product will pass through a maximum value. The corresponding graphs will be as follows :



13. For understanding, let us assume that the two loops are lying in the plane of paper as shown. The current in loop 1 will produce U magnetic field in loop 2. Therefore, increase in current in loop 1 will produce an induced current in loop 2 which produces ⊗ magnetic field passing through it i.e. induced current in loop 2 will also be clockwise as shown in the figure.



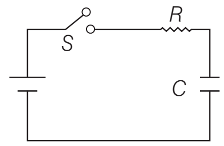
The loops will now repel each other as the currents at the nearest and farthest points of the two loops flow in opposite directions.

14. The induced current in the ring will interact with horizontal component of magnetic field and both will repel each other. This repulsion will balance the weight of ring.

Hence, option (a) is correct.

15. When switch is closed for a very long time capacitor will get fully charged and charge on capacitor will be $q = CV$

Energy stored in capacitor, $E_C = \frac{1}{2}CV^2$ …(i)



Work done by a battery, $W = Vq = VCV = CV^2$

Energy dissipated across resistance

E_D = (work done by battery) − (energy stored)

$E_D = CV^2 - \frac{1}{2}CV^2 = \frac{1}{2}CV^2$ …(ii)

From Eqs. (i) and (ii)

$$E_D = E_C$$

16. **For process (1)**

Charge on capacitor $= \frac{CV_0}{3}$

Energy stored in capacitor $= \frac{1}{2}C\frac{V_0^2}{9} = \frac{CV_0^2}{18}$

Work done by battery $= \frac{CV_0}{3} \times \frac{V}{3} = \frac{CV_0^2}{9}$

$\therefore$ Heat loss $= \frac{CV_0^2}{9} - \frac{CV_0^2}{18} = \frac{CV_0^2}{18}$

For process (2)

Charge on capacitor $= \frac{2\,CV_0}{3}$

Extra charge flow through battery $= \frac{CV_0}{3}$

Work done by battery $= \frac{CV_0}{3} \cdot \frac{2V_0}{3} = \frac{2CV_0^2}{9}$

Final energy stored in capacitor $= \frac{1}{2}C\left(\frac{2V_0}{3}\right)^2 = \frac{4CV_0^2}{18}$

Energy stored in process 2 $= \frac{4CV_0^2}{18} - \frac{CV_0^2}{18} = \frac{3CV_0^2}{18}$

Heat loss in process (2) = work done by battery in process (2) − energy stored in capacitor process (2)

$$= \frac{2CV_0^2}{9} - \frac{3CV_0^2}{18} = \frac{CV_0^2}{18}$$

For process (3) Charge on capacitor $= CV_0$

Extra charge flown through battery $= CV_0 - \frac{2CV_0}{3} = \frac{CV_0}{3}$

Work done by battery in this process $= \left(\frac{CV_0}{3}\right)(V_0) = \frac{CV_0^2}{3}$

Final energy stored in capacitor $= \frac{1}{2}CV_0^2$

Energy stored in this process $= \frac{1}{2}CV_0^2 - \frac{4CV_0^2}{18} = \frac{5CV_0^2}{18}$

Heat loss in process (3) $= \frac{CV_0^2}{3} - \frac{5CV_0^2}{18} = \frac{CV_0^2}{18}$

Now, total heat loss $(E_D) = \frac{CV_0^2}{18} + \frac{CV_0^2}{18} + \frac{CV_0^2}{18} = \frac{CV_0^2}{6}$

Final energy stored in capacitor $= \frac{1}{2}CV_0^2$

So, we can say that $E_D = \frac{1}{3}\left(\frac{1}{2}CV_0^2\right)$

17. $$P = Vi$$

$$\therefore \quad i = \frac{P}{V} = \frac{600 \times 10^3}{4000} = 150\text{ A}$$

Total resistance of cables, $R = 0.4 \times 20 = 8\ \Omega$

$\therefore$ Power loss in cables $= i^2R = (150)^2(8)$

$= 180000\text{ W} = 180\text{ kW}$

This loss is 30% of 600 kW.

18. During step-up, $\frac{N_p}{N_s} = \frac{V_p}{V_s}$ or $\frac{1}{10} = \frac{4000}{V_s}$

or $V_s = 40000\text{ V}$

In step down transformer,

$$\frac{N_p}{N_s} = \frac{V_p}{V_s} = \frac{40000}{200} = \frac{200}{1}$$

19. The induced electric field is given by,

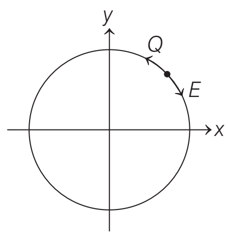
$$\oint \mathbf{E}\cdot\mathbf{dl} = -\frac{d\phi}{dt} \text{ or } El = -s\left(\frac{dB}{dt}\right)$$

$$\therefore \quad E(2\pi R) = -(\pi R^2)(B) \text{ or } E = -\frac{BR}{2}$$

20. $$\frac{M}{L} = \frac{Q}{2m}$$

$$\therefore \quad M = \left(\frac{Q}{2m}\right)L \quad \Rightarrow \quad M \propto L, \text{ where } \gamma = \frac{Q}{2m}$$

$$= \left(\frac{Q}{2m}\right)(I\omega) = \left(\frac{Q}{2m}\right)(mR^2\omega) = \frac{Q\omega R^2}{2}$$



Induced electric field is opposite direction. Therefore,

$$\omega' = \omega - \alpha t$$

$$\alpha = \frac{\tau}{I} = \frac{(QE)R}{mR^2}$$

$$= \frac{(Q)\left(\frac{BR}{2}\right)R}{mR^2} = \frac{QB}{2m}$$

$$\therefore \quad \omega' = \omega - \frac{QB}{2m}\cdot 1 = \omega - \frac{QB}{2m}$$

$$M_f = \frac{Q\omega' R^2}{2} = Q\left(\omega - \frac{QB}{2m}\right)\frac{R^2}{2}$$

$$\therefore \quad \Delta M = M_f - M_i = -\frac{Q^2BR^2}{4m}$$

$$M = -\gamma\frac{QBR^2}{2} \qquad \left(\because \gamma = \frac{Q}{2m}\right)$$

24. At $\omega \approx 0$, $X_C = \frac{1}{\omega C} = \infty$. Therefore, current is nearly zero.

Further at resonance frequency, current and voltage are in phase. This resonance frequency is given by

$$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-6} \times 10^{-6}}} = 10^6 \text{ rad / s}$$

We can see that this frequency is independent of R.

Further, $X_L = \omega L$, $X_C = \frac{1}{\omega C}$

At, $\omega = \omega_r = 10^6$ rad/s, $X_L = X_C$.

For $\omega > \omega_r$, $X_L > X_C$. So, circuit is inductive.

25. Induced emf $e = -\frac{d\phi}{dt}$. For identical rings induced emf will be same. But currents will be different. Given $h_A > h_B$. Hence, $v_A > v_B$ as $\left(h = \frac{v^2}{2g}\right)$.

If $\rho_A > \rho_B$, then, $I_A < I_B$. In this case given condition can be fulfilled if $m_A < m_B$.

If $\rho_A < \rho_B$, then $I_A > I_B$. In this case given condition can be fulfilled if $m_A \le m_B$.

26. Electrostatic and gravitational field do not make closed loops.

27. Take the circular tube as a long solenoid. The wires are closely wound. Magnetic field inside the solenoid is

$$B = \mu_0 ni$$

Here, n = number of turns per unit length

$\therefore ni$ = current per unit length

In the given problem $ni = \frac{I}{L}$

$$\therefore \quad B = \frac{\mu_0 I}{L}$$

Flux passing through the circular coil is

$$\phi = BS = \left(\frac{\mu_0 I}{L}\right)(\pi r^2)$$

Induced emf $\quad e = -\frac{d\phi}{dt} = -\left(\frac{\mu_0\pi r^2}{L}\right)\cdot\frac{dI}{dt}$

Induced current, $i = \frac{e}{R} = -\left(\frac{\mu_0\pi r^2}{LR}\right)\cdot\frac{dI}{dt}$

Magnetic moment, $M = iA = i\pi r^2$

$$\text{or} \quad M = -\left(\frac{\mu_0\pi^2 r^4}{LR}\right)\cdot\frac{dI}{dt} \qquad \text{...(i)}$$

Given, $\quad I = I_0\cos(300t)$

$$\therefore \qquad \frac{dI}{dt} = -300 I_0 \sin(300t)$$

Substituting in Eq. (i), we get

$$M = \left(\frac{300\,\pi^2 r^4}{LR}\right) \mu_0 I_0 \sin(300\,t)$$

$$\therefore \qquad N = \frac{300\,\pi^2 r^4}{LR}$$

Substituting the values, we get

$$N = \frac{300\,(22/7)^2\,(0.1)^4}{(10)\,(0.005)} = 5.926 \text{ or } N \simeq 6$$

28. Outside the solenoid, net magnetic field is zero. It can be assumed only inside the solenoid and equal to $\mu_0 nI$.

$$\text{Induced } e = -\frac{d\phi}{dt} = -\frac{d}{dt}(\mu_0 nI\pi a^2)$$

$$\text{or} \qquad |e| = (\mu_0 n\pi a^2)(I_0\omega \cos \omega t)$$

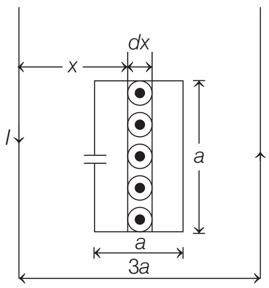
$$\text{Resistance of the cylindrical vessel, } R = \frac{\rho l}{s} = \frac{\rho(2\pi R)}{Ld}$$

$$\therefore \text{ Induced current } i = \frac{|e|}{R} = \frac{\mu_0 L d n a^2 I_0 \omega \cos \omega t}{2\rho R}$$

29. (a) For an elemental strip of thickness dx at a distance x from left wire, net magnetic field (due to both wires)

$$B = \frac{\mu_0}{2\pi}\frac{I}{x} + \frac{\mu_0}{2\pi}\frac{I}{3a-x} \qquad \text{(outwards)}$$

$$= \frac{\mu_0 I}{2\pi}\left(\frac{1}{x} + \frac{1}{3a-x}\right)$$



Magnetic flux in this strip,

$$d\phi = BdS = \frac{\mu_0 I}{2\pi}\left(\frac{1}{x} + \frac{1}{3a-x}\right) a\,dx$$

$$\therefore \text{ Total flux, } \phi = \int_a^{2a} d\phi = \frac{\mu_0 Ia}{2\pi}\int_a^{2a}\left(\frac{1}{x} + \frac{1}{3a-x}\right)dx$$

$$\text{or} \qquad \phi = \frac{\mu_0 Ia}{\pi}\ln(2)$$

$$\phi = \frac{\mu_0 a \ln(2)}{\pi}(I_0 \sin \omega t) \qquad \ldots\text{(i)}$$

Magnitude of induced emf,

$$e = \left|-\frac{d\phi}{dt}\right| = \frac{\mu_0 a I_0 \omega \ln(2)}{\pi}\cos \omega t = e_0 \cos \omega t$$

$$\text{where,} \quad e_0 = \frac{\mu_0 a I_0 \omega \ln(2)}{\pi}$$

Charge stored in the capacitor,

$$q = Ce = Ce_0 \cos \omega t \qquad \ldots\text{(ii)}$$

and current in the loop

$$i = \frac{dq}{dt} = C\omega\, e_0 \sin \omega t \qquad \ldots\text{(iii)}$$

$$i_{max} = C\omega e_0 = \frac{\mu_0 a I_0 \omega^2 C \ln(2)}{\pi}$$

(b) Magnetic flux passing through the square loop

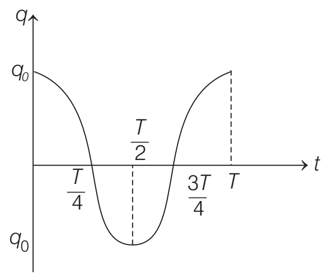
$$\phi \propto \sin \omega t \qquad \text{[From Eq. (i)]}$$

i.e. u magnetic field passing through the loop is increasing at $t = 0$. Hence, the induced current will produce ⊗ magnetic field (from Lenz's law). Or the current in the circuit at $t = 0$ will be clockwise (or negative as per the given convention). Therefore, charge on upper plate could be written as,

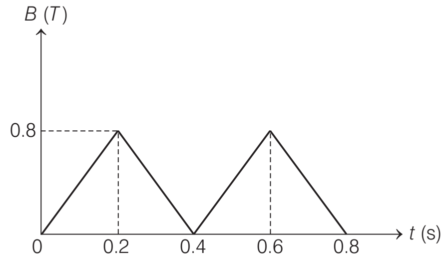
$$q = +q_0 \cos \omega t \qquad \text{[From Eq. (ii)]}$$

$$\text{Here,} \quad q_0 = Ce_0 = \frac{\mu_0 a C I_0 \omega \ln(2)}{\pi}$$

The corresponding q-t graph is shown in figures,



30. Magnetic field (B) varies with time (t) as shown in figure.



$$\left|\frac{dB}{dt}\right| = \frac{0.8}{0.2} = 4 \text{ T/s}$$

Induced emf in the coil due to change in magnetic flux passing through it, $e = \left|\frac{d\phi}{dt}\right| = NA\left|\frac{dB}{dt}\right|$

Here, A = Area of coil = 5×10^{-3} m^2

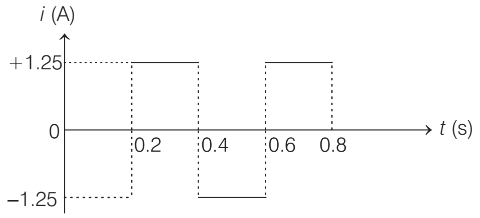
N = Number of turns = 100

Substituting the values, we get $e = (100)\,(5 \times 10^{-3})(4) = 2$ V

Therefore, current passing through the coil

$$i = \frac{e}{R} \text{ or } i = \frac{2}{1.6} = 1.25 \text{ A}$$

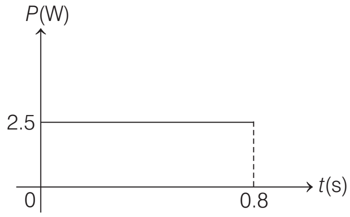
NOTE That from 0 to 0.2 s and from 0.4 s to 0.6 s, magnetic field passing through the coil increases, while during the time 0.2s to 0.4s and from 0.6s to 0.8s magnetic field passing through the coil decreases. Therefore, direction of current through the coil in these two time intervals will be opposite to each other. The variation of current (i) with time (t) will be as follows :



Power dissipated in the coil is

$$P = i^2R = (1.25)^2\ (1.6) = 2.5\ \text{W}$$

Power is independent of the direction of current through the coil. Therefore, power (P) *versus* time (t) graph for first two cycles will be as under :



Total heat obtained in 12,000 cycles will be

$$H = P.t\ = (2.5)(12000)(0.4) = 12000\ \text{J}$$

This heat is used in raising the temperature of the coil and the water. Let θ be the final temperature. Then

$$H = m_w s_w(\theta - 30) + m_c s_c(\theta - 30)$$

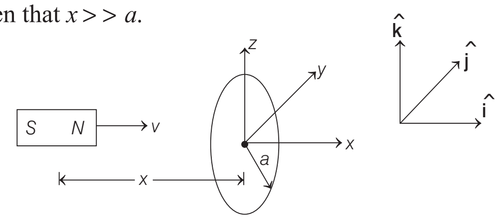
Here, m_w = mass of water = 0.5 kg
s_w = specific heat of water = 4200 J/kg-K
m_c = mass of coil = 0.06 kg
and s_c = specific heat of coil = 500 J/kg-K

Substituting the values, we get

$$12000 = (0.5)\,(4200)\,(\theta - 30) + (0.06)\,(500)\,(\theta - 30)$$

or $\theta = 35.6°$ C

31. Given that $x >> a$.



Magnetic field at the centre of the coil due to the bar magnet is

$$B = \frac{\mu_0}{4\pi}\frac{2M}{x^3} = \frac{\mu_0}{2\pi}\frac{M}{x^3}$$

Due to this, magnetic flux linked with the coil will be,

$$\phi = BS = \frac{\mu_0}{2\pi}\frac{M}{x^3}(\pi a^2) = \frac{\mu_0 Ma^2}{2x^3}$$

$\therefore$ Induced emf in the coil, due to motion of the magnet is

$$e = \frac{-d\phi}{dt} = -\left(\frac{\mu_0 Ma^2}{2}\right)\frac{d}{dt}\left(\frac{1}{x^3}\right)$$

$$= \frac{\mu_0 Ma^2}{2}\left(\frac{3}{x^4}\right)\frac{dx}{dt} = \frac{3}{2}\frac{\mu_0 Ma^2}{x^4}v \quad \left(\because \frac{dx}{dt} = v\right)$$

Therefore, induced current in the coil is

$$i = \frac{e}{R} = \frac{3}{2}\frac{\mu_0 Ma^2}{Rx^4}v$$

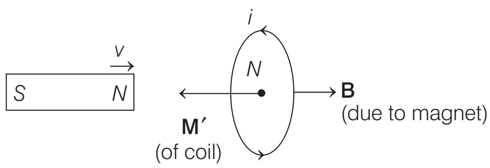
Magnetic moment of the coil due to this induced current will be,

$$M' = iS = \frac{3}{2}\frac{\mu_0 Ma^2}{Rx^4}v\,(\pi a^2) \Rightarrow M' = \frac{3}{2}\frac{\mu_0 \pi Ma^4 v}{Rx^4}$$

Potential energy of $\mathbf{M'}$ in $\mathbf{B}$ will be

$$U = -M'B\cos 180°$$

$$U = M'B = \frac{3}{2}\frac{\mu_0 \pi Ma^4 v}{Rx^4}\left(\frac{\mu_0}{2\pi}\cdot\frac{M}{x^3}\right)$$



$$U = \frac{3}{4}\frac{\mu_0^2 M^2 a^4 v}{R}\frac{1}{x^7} \Rightarrow F = -\frac{dU}{dx} = \frac{21}{4}\frac{\mu_0^2 M^2 a^4 v}{Rx^8}$$

Positive sign of F implies that there will be a repulsion between the magnet and the coil.

32. Total resistance of the circuit as function of distance x from resistance R is $R_{\text{net}} = R + 2\lambda x$

Let v be velocity of rod at this instant, then motional emf induced across the rod, $e = Bvd$

$\therefore$ Current $i = \dfrac{e}{R_{\text{net}}} = \dfrac{Bvd}{R + 2\lambda x} \Rightarrow v = \dfrac{(R + 2\lambda x)\,i}{Bd}$

Net force on the rod, $F_{\text{net}} = m\dfrac{dv}{dt} = \dfrac{2\lambda im}{Bd}(R + 2\lambda x)\cdot\dfrac{dx}{dt}$

but $\dfrac{dx}{dt} = v = \dfrac{(R + 2\lambda x)\,i}{Bd}$

$$F_{\text{net}} = \frac{2\lambda i^2 m}{B^2 d^2}(R + 2\lambda x)^2$$

This net force is equal to $F - F_m$ where $F_m = idB$

$$\therefore \quad F = F_{\text{net}} + F_m = \frac{2\lambda i^2 m}{B^2 d^2}(R + 2\lambda x)^2 + idB$$