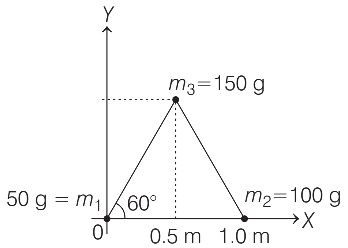
5

Centre of Mass

Topic 1 Centre of Mass

Objective Questions I (Only one correct option)

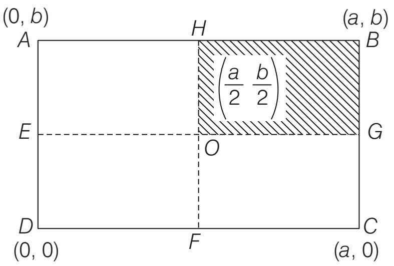
1. Three particles of masses 50 g, 100 g and 150 g are placed at the vertices of an equilateral triangle of side 1 m (as shown in the figure). The (x, y) coordinates of the centre of mass will be **(2019 Main, 12 April II)**



(a) $\left(\frac{\sqrt{3}}{4}\text{ m}, \frac{5}{12}\text{ m}\right)$ (b) $\left(\frac{7}{12}\text{ m}, \frac{\sqrt{3}}{8}\text{ m}\right)$

(c) $\left(\frac{7}{12}\text{ m}, \frac{\sqrt{3}}{4}\text{ m}\right)$ (d) $\left(\frac{\sqrt{3}}{8}\text{ m}, \frac{7}{12}\text{ m}\right)$

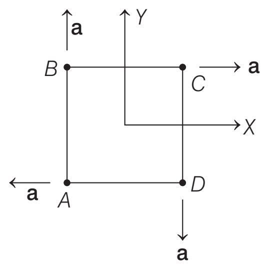
2. A uniform rectangular thin sheet $ABCD$ of mass M has length a and breadth b, as shown in the figure. If the shaded portion $HBGO$ is cut-off, the coordinates of the centre of mass of the remaining portion will be **(2019 Main, 8 April II)**



(a) $\left(\frac{2a}{3}, \frac{2b}{3}\right)$ (b) $\left(\frac{5a}{12}, \frac{5b}{12}\right)$

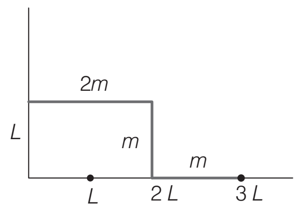
(c) $\left(\frac{3a}{4}, \frac{3b}{4}\right)$ (d) $\left(\frac{5a}{3}, \frac{5b}{3}\right)$

3. Four particles A, B, C and D with masses $m_A = m$, $m_B = 2m$, $m_C = 3m$ and $m_D = 4m$ are at the corners of a square. They have accelerations of equal magnitude with directions as shown. The acceleration of the centre of mass of the particles (in ms^{-2}) is **(2019 Main, 8 April I)**



(a) $\frac{a}{5}(\hat{\mathbf{i}} - \hat{\mathbf{j}})$ (b) $a(\hat{\mathbf{i}} + \hat{\mathbf{j}})$ (c) zero (d) $\frac{a}{5}(\hat{\mathbf{i}} + \hat{\mathbf{j}})$

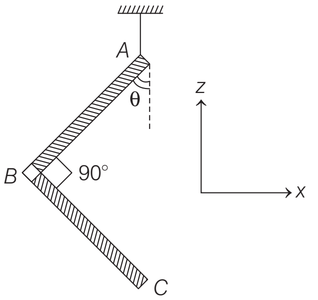
4. The position vector of the centre of mass $\mathbf{r}_{\text{cm}}$ of an asymmetric uniform bar of negligible area of cross-section as shown in figure is **(2019 Main, 12 Jan I)**



(a) $\mathbf{r} = \frac{13}{8}L\,\hat{\mathbf{x}} + \frac{5}{8}L\,\hat{\mathbf{y}}$ (b) $\mathbf{r} = \frac{11}{8}L\,\hat{\mathbf{x}} + \frac{3}{8}L\,\hat{\mathbf{y}}$

(c) $\mathbf{r} = \frac{3}{8}L\,\hat{\mathbf{x}} + \frac{11}{8}L\,\hat{\mathbf{y}}$ (d) $\mathbf{r} = \frac{5}{8}L\,\hat{\mathbf{x}} + \frac{13}{8}L\,\hat{\mathbf{y}}$

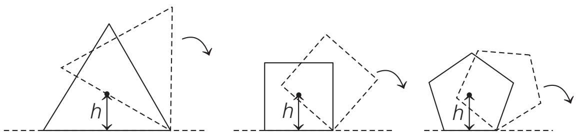
5. An L-shaped object made of thin rods of uniform mass density is suspended with a string as shown in figure. If $AB = BC$ and the angle is made by AB with downward vertical is θ, then **(2019 Main, 9 Jan)**



(a) $\tan\theta = \frac{2}{\sqrt{3}}$ (b) $\tan\theta = \frac{1}{2\sqrt{3}}$

(c) $\tan\theta = \frac{1}{2}$ (d) $\tan\theta = \frac{1}{3}$

6. Consider regular polygons with number of sides $n = 3, 4, 5$ …… as shown in the figure. The centre of mass of all the polygons is at height h from the ground. They roll on a horizontal surface about the leading vertex without slipping and sliding as depicted. The maximum increase in height of the locus of the centre of mass for each each polygon is Δ. Then, Δ depends on n and h as **(2017 Adv.)**



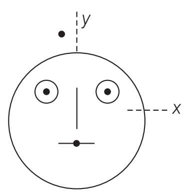
(a) $\Delta = h\sin^2\left(\frac{\pi}{n}\right)$ (b) $\Delta = h\sin\left(\frac{2\pi}{n}\right)$

(c) $\Delta = h\tan^2\left(\frac{\pi}{2n}\right)$ (d) $\Delta = h\left(\frac{1}{\cos\left(\frac{\pi}{n}\right)} - 1\right)$

7. Distance of the centre of mass of a solid uniform cone from its vertex is z_0. If the radius of its base is R and its height is h, then z_0 is equal to **(2015 Main)**

(a) $\frac{3h}{4}$ (b) $\frac{h^2}{4R}$ (c) $\frac{5h}{8}$ (d) $\frac{3h^2}{8R}$

8. Look at the drawing given in the figure, which has been drawn with ink of uniform line-thickness. The mass of ink used to draw each of the two inner circles, and each of the two line segments is m. The mass of the ink used to draw the outer circle is $6\ m$. The coordinates of the centres of the different parts are : outer circle (0, 0), left inner circle $(-a, a)$, right inner circle (a, a), vertical line (0,0) and horizontal line $(0, -a)$. The y-coordinate of the centre of mass of the ink in this drawing is **(2009)**



(a) $\frac{a}{10}$ (b) $\frac{a}{8}$ (c) $\frac{a}{12}$ (d) $\frac{a}{3}$

9. Two blocks of masses 10 kg and 4 kg are connected by a spring of negligible mass and placed on a frictionless horizontal surface. An impulse gives a velocity of 14 m/s to the heavier block in the direction of the lighter block. The velocity of the centre of mass is **(2002, S)**

(a) 30 m/s (b) 20 m/s
(c) 10 m/s (d) 5 m/s

10. Two particles A and B initially at rest, move towards each other by mutual force of attraction. At the instant when the speed of A is v and the speed of B is $2v$, the speed of the centre of mass of the system is **(1982, 3M)**

(a) $3\ v$ (b) v
(c) $1.5\ v$ (d) zero

Assertion and Reason

Mark your answer as

(a) If Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I

(b) If Statement I is true, Statement II is true; Statement II is not a correct explanation for Statement I

(c) If Statement I is true; Statement II is false

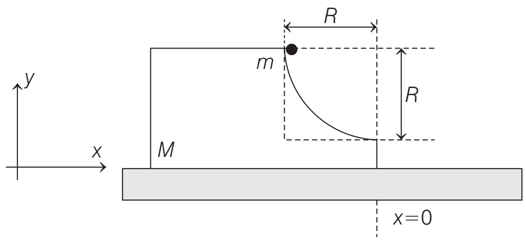
(d) If Statement I is false; Statement II is true

11. **Statement I** If there is no external torque on a body about its centre of mass, then the velocity of the centre of mass remains constant.

Statement II The linear momentum of an isolated system remains constant. **(2007, 3M)**

Objective Questions II (One or more correct option)

12. A block of mass M has a circular cut with a frictionless surface as shown. The block rests on the horizontal frictionless surfaced of a fixed table. Initially the right edge of the block is at $x = 0$, in a coordinate system fixed to the table. A point mass m is released from rest at the topmost point of the path as shown and it slides down. When the mass loses contact with the block, its position is x and the velocity is v. At that instant, which of the following option is/are correct? **(2017 Adv.)**



(a) The velocity of the point mass m is $v = \sqrt{\frac{2gR}{1+\frac{m}{M}}}$

(b) The x component of displacement of the centre of mass of the block M is $-\frac{mR}{M+m}$

(c) The position of the point mass is $x = -\sqrt{2}\dfrac{mR}{M+m}$

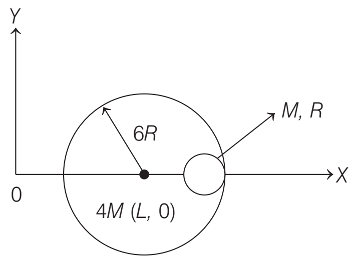
(d) The velocity of the block M is $V = -\dfrac{m}{M}\sqrt{2gR}$

True/False

13. Two particles of mass 1 kg and 3 kg move towards each other under their mutual force of attraction. No other force acts on them. When the relative velocity of approach of the two particles is 2 m/s, their centre of mass has a velocity of 0.5 m/s. When the relative velocity of approach becomes 3 m/s, the velocity of the centre of mass is 0.75 m/s. **(1989, 2M)**

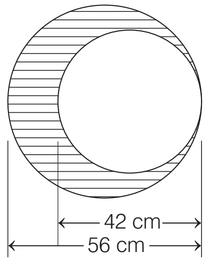
Analytical & Descriptive Questions

14. A small sphere of radius R is held against the inner surface of a larger sphere of radius $6R$. The masses of large and small spheres are $4M$ and M respectively. This arrangement is placed on a horizontal table. There is no friction between any surfaces of contact. The small sphere is now released. Find the coordinates of the centre of the larger sphere when the smaller sphere reaches the other extreme position. **(1996, 3 M)**



15. A ball of mass 100 g is projected vertically upwards from the ground with a velocity of 49 m/s. At the same time, another identical ball is dropped from a height of 98 m to fall freely along the same path as that followed by the first ball. After some time, the two balls collide and stick together and finally fall to the ground. Find the time of flight of the masses. **(1985, 8M)**

16. A circular plate of uniform thickness has a diameter of 56 cm. A circular portion of diameter 42 cm is removed from one edge of the plate as shown in figure.
Find the position of the centre of mass of the remaining portion. **(1980)**



Topic 2 Linear Momentum, Mechanical Energy and Their Conservation

Objective Question I (Only one correct option)

1. A person of mass M is sitting on a swing to length L and swinging with an angular amplitude θ_0. If the person stands up when the swing passes through its lowest point, the work done by him, assuming that his centre of mass moves by a distance $l (l << L)$, is close to **(2019 Main, 12 April I)**

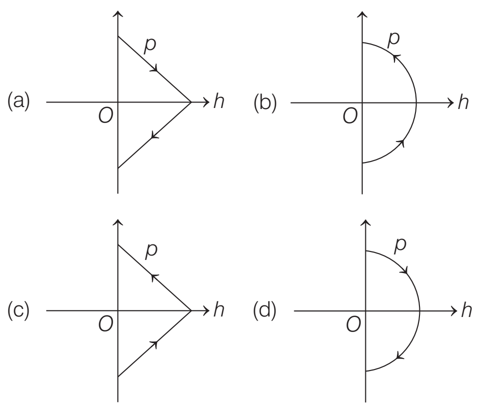
(a) $Mgl\,(1-\theta_0^2)$ (b) $Mgl\,(1+\theta_0^2)$

(c) Mgl (d) $Mgl\left(1+\dfrac{\theta_0^2}{2}\right)$

2. A wedge of mass $M = 4$ m lies on a frictionless plane. A particle of mass m aproaches the wedge with speed v. There is no friction between the particle and the plane or between the particle and the wedge. The maximum height climbed by the particle on the wedge is given by **(2019 Main, 9 April II)**

(a) $\dfrac{2v^2}{7g}$ (b) $\dfrac{v^2}{g}$ (c) $\dfrac{2v^2}{5g}$ (d) $\dfrac{v^2}{2g}$

3. A ball is thrown vertically up (taken as + Z-axis) from the ground. The correct momentum-height (p-h) diagram is **(2019 Main, 9 April I)**



4. A particle of mass m is moving in a straight line with momentum p. Starting at time $t = 0$, a force $F = kt$ acts in the same direction on the moving particle during time interval T, so that its momentum changes from p to $3p$. Here, k is a constant. The value of T is **(JEE Main 2019, 11 Jan Shift II)**

(a) $\sqrt{\dfrac{2p}{k}}$ (b) $2\sqrt{\dfrac{p}{k}}$ (c) $\sqrt{\dfrac{2k}{p}}$ (d) $2\sqrt{\dfrac{k}{p}}$

5. If the resultant of all the external forces acting on a system of particles is zero, then from an inertial frame, one can surely say that **(2009)**

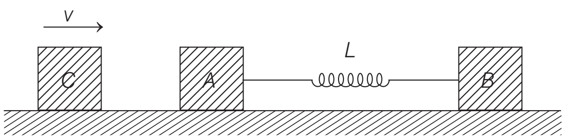
(a) linear momentum of the system does not change in time
(b) kinetic energy of the system does not change in time
(c) angular momentum of the system does not change in time
(d) potential energy of the system does not change in time

6. A particle moves in the x-y plane under the influence of a force such that its linear momentum is $\mathbf{p}(t) = A[\hat{\mathbf{i}}\cos(kt) - \hat{\mathbf{j}}\sin(kt)]$, where, A and k are constants. The angle between the force and the momentum is **(2007, 3M)**

(a) 0° (b) 30° (c) 45° (d) 90°

Objective Question II (One or more correct option)

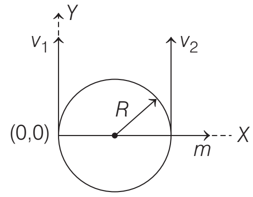
7. Two blocks A and B each of mass m, are connected by a massless spring of natural length L and spring constant k. The blocks are initially resting on a smooth horizontal floor with the spring at its natural length, as shown in figure. A third identical block C, also of mass m, moves on the floor with a speed v along the line joining A and B, and collides elastically with A. Then **(1993, 2M)**



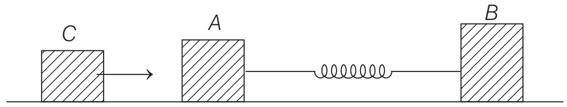
(a) the kinetic energy of the A-B system, at maximum compression of the spring, is zero
(b) the kinetic energy of the A-B system, at maximum compression of the spring, is $mv^2/4$
(c) the maximum compression of the spring is $v\sqrt{(m/k)}$
(d) the maximum compression of the spring is $v\sqrt{\dfrac{m}{2k}}$

Analytical & Descriptive Questions

8. A particle of mass m, moving in a circular path of radius R with a constant speed v_2 is located at point $(2R, 0)$ at time $t = 0$ and a man starts moving with a velocity v_1 along the positive Y-axis from origin at time $t = 0$. Calculate the linear momentum of the particle w.r.t. man as a function of time. **(2003, 2M)**

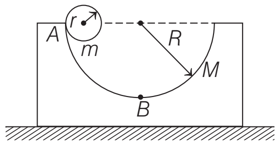


9. Two bodies A and B of masses m and $2m$ respectively are placed on a smooth floor. They are connected by a spring. A third body C of mass m moves with velocity v_0 along the line joining A and B and collides elastically with A as shown in figure. At a certain instant of time t_0 after collision, it is found that the instantaneous velocities of A and B are the same. Further at this instant the compression of the spring is found to be x_0. Determine (a) the common velocity of A and B at time t_0 and (b) the spring constant. **(1984, 6M)**



10. A block of mass M with a semicircular track of radius R, rests on a horizontal frictionless surface. A uniform cylinder of radius r and mass m is released from rest at the top point A (see fig.). The cylinder slips on the semicircular frictionless track.

(a) How far has the block moved when the cylinder reaches the bottom (point B) of the track ?
(b) How fast is the block moving when the cylinder reaches the bottom of the track? **(1983, 7M)**



11. When a ball is thrown up, the magnitude of its momentum decreases and then increases. Does this violate the conservation of momentum principle? **(1979)**

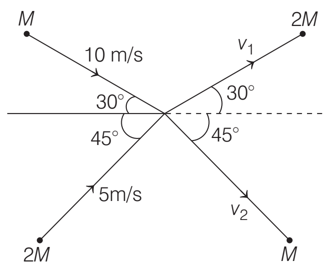
Topic 3 Impulse, Explosions and Collisions

Objective Questions I (Only one correct option)

1. A man (mass = 50kg) and his son (mass = 20kg) are standing on a frictionless surface facing each other. The man pushes his son, so that he starts moving at a speed of 0.70 ms^{-1} with respect to the man. The speed of the man with respect to the surface is **(2019 Main, 12 April I)**

(a) 0.28 ms^{-1} (b) 0.20 ms^{-1} (c) 0.47 ms^{-1} (d) 0.14 ms^{-1}

2. Two particles of masses M and $2M$, moving as shown, with speeds of 10 m/s and 5 m/s, collide elastically at the origin. After the collision, they move along the indicated directions with speed v_1 and v_2 are nearly **(2019 Main, 10 April I)**



(a) 6.5 m/s and 3.2 m/s (b) 3.2 m/s and 6.3 m/s
(c) 3.2 m/s and 12.6 m/s (d) 6.5 m/s and 6.3 m/s

3. A particle of mass m is moving with speed $2v$ and collides with a mass $2m$ moving with speed v in the same direction. After collision, the first mass is stopped completely while the second one splits into two particles each of mass m, which move at angle 45° with respect to the original direction.

The speed of each of the moving particle will be **(2019 Main, 9 April II)**

(a) $\sqrt{2}\,v$ (b) $\frac{v}{\sqrt{2}}$ (c) $\frac{v}{(2\sqrt{2})}$ (d) $2\sqrt{2}\,v$

4. A body of mass 2 kg makes an elastic collision with a second body at rest and continues to move in the original direction but with one-fourth of its original speed. What is the mass of the second body ? **(2019 Main, 9 April I)**

(a) 1.5 kg (b) 1.2 kg (c) 1.8 kg (d) 1.0 kg

5. A body of mass m_1 moving with an unknown velocity of $v_1\hat{\mathbf{i}}$, undergoes a collinear collision with a body of mass m_2 moving with a velocity $v_2\hat{\mathbf{i}}$. After collision, m_1 and m_2 move with velocities of $v_3\hat{\mathbf{i}}$ and $v_4\hat{\mathbf{i}}$, respectively.

If $m_2 = 0.5m_1$ and $v_3 = 0.5\,v_1$, then v_1 is **(2019 Main, 8 April II)**

(a) $v_4 + v_2$ (b) $v_4 - \frac{v_2}{4}$

(c) $v_4 - \frac{v_2}{2}$ (d) $v_4 - v_2$

6. An α-particle of mass m suffers one-dimensional elastic collision with a nucleus at rest of unknown mass. It is scattered directly backwards losing 64% of its initial kinetic energy. The mass of the nucleus is **(2019 Main, 12 Jan II)**

(a) 1.5 m (b) 4 m (c) 3.5 m (d) 2 m

7. A satellite of mass M is in a circular orbit of radius R about the centre of the earth. A meteorite of the same mass falling towards the earth collides with the satellite completely inelastically. The speeds of the satellite and the meteorite are the same just before the collision. The subsequent motion of the combined body will be **(2019 Main, 12 Jan I**

(a) in the same circular orbit of radius R

(b) in an elliptical orbit

(c) such that it escapes to infinity

(d) in a circular orbit of a different radius

8. A simple pendulum is made of a string of length l and a bob of mass m, is released from a small angle θ_0. It strikes a block of mass M, kept on a horizontal surface at its lowest point of oscillations, elastically. It bounces back and goes up to an angle θ_1. Then, M is given by **(2019 Main, 12 Jan I)**

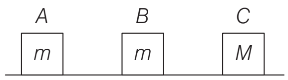
(a) $m\left(\frac{\theta_0 + \theta_1}{\theta_0 - \theta_1}\right)$ (b) $\frac{m}{2}\left(\frac{\theta_0 - \theta_1}{\theta_0 + \theta_1}\right)$

(c) $m\left(\frac{\theta_0 - \theta_1}{\theta_0 + \theta_1}\right)$ (d) $\frac{m}{2}\left(\frac{\theta_0 + \theta_1}{\theta_0 - \theta_1}\right)$

9. A piece of wood of mass 0.03 kg is dropped from the top of a 100 m height building. At the same time, a bullet of mass 0.02 kg is fired vertically upward with a velocity 100 ms^{-1} from the ground. The bullet gets embedded in the wood. Then, the maximum height to which the combined system reaches above the top of the building before falling below is (Take, $g = 10\,ms^{-2}$) **(2019 Main, 10 Jan I)**

(a) 20 m (b) 30 m (c) 10 m (d) 40 m

10. Three blocks A, B and C are lying on a smooth horizontal surface as shown in the figure. A and B have equal masses m while C has mass M. Block A is given an initial speed v towards B due to which it collides with B perfectly inelastically. The combined mass collides with C, also perfectly inelastically $\frac{5}{6}$th of the initial kinetic energy is lost in whole process. What is value of $\frac{M}{m}$? **(2019 Main, 9 Jan I)**



(a) 4 (b) 2 (c) 3 (d) 5

11. In a collinear collision, a particle with an initial speed v_0 strikes a stationary particle of the same mass. If the final total kinetic energy is 50% greater than the original kinetic energy, the magnitude of the relative velocity between the two particles after collision, is **(2018 Main)**

(a) $\frac{v_0}{\sqrt{2}}$ (b) $\frac{v_0}{4}$ (c) $\sqrt{2}\,v_0$ (d) $\frac{v_0}{2}$

12. It is found that, if a neutron suffers an elastic collinear collision with deuterium at rest, fractional loss of its energy is P_d; while for its similar collision with carbon nucleus at rest, fractional loss of energy is P_c. The values of P_d and P_c are respectively **(2018 Main)**

(a) (0, 1) (b) (.89, .28) (c) (.28, .89) (d) (0, 0)

13. A particle of mass m moving in the x-direction with speed $2v$ is hit by another particle of mass $2m$ moving in the y-direction with speed v. If the collision is perfectly inelastic, the percentage loss in the energy during the collision is close to **(2015 Main)**

(a) 50 % (b) 56 % (c) 62 % (d) 44 %

14. This question has statement I and statement II. Of the four choices given after the statements, choose the one that best describes the two statements. **(2013 Main)**

Statement I A point particle of mass m moving with speed v collides with stationary point particle of mass M. If the maximum energy loss possible is given as $f\left(\frac{1}{2}mv^2\right)$, then $f = \left(\frac{m}{M+m}\right)$.

Statement II Maximum energy loss occurs when the particles get stuck together as a result of the collision.

(a) Statement I is true, Statement II is true, and Statement II is the correct explanation of Statement I

(b) Statement I is true, Statement II is true, but Statement II is not the correct explanation of Statement I

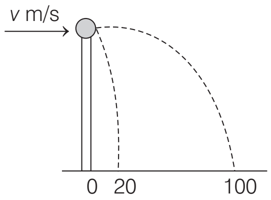
(c) Statement I is true, Statement II is false

(d) Statement I is false, Statement II is true

15. A particle of mass m is projected from the ground with an initial speed u_0 at an angle α with the horizontal. At the highest point of its trajectory, it makes a completely inelastic collision with another identical particle, which was thrown vertically upward from the ground with the same initial speed u_0. The angle that the composite system makes with the horizontal immediately after the collision is **(2013 Adv.)**

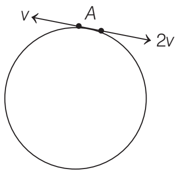
(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{4}+\alpha$ (c) $\frac{\pi}{4}-\alpha$ (d) $\frac{\pi}{2}$

16. A ball of mass 0.2 kg rests on a vertical post of height 5 m. A bullet of mass 0.01 kg, travelling with a velocity v m/s in a horizontal direction, hits the centre of the ball. After the collision, the ball and bullet travel independently. The ball hits the ground at a distance of 20 m and the bullet at a distance of 100 m from the foot of the post. The initial velocity v of the bullet is **(2011)**



(a) 250 m/s (b) $250\sqrt{2}$ m/s (c) 400 m/s (d) 500 m/s

17. Two small particles of equal masses start moving in opposite directions from a point A in a horizontal circular orbit. Their tangential velocities are v and $2v$ respectively, as shown in the figure. Between collisions, the particles move with constant speeds. After making how many elastic collisions, other than that at A, these two particles will again reach the point A? **(2009)**



(a) 4 (b) 3 (c) 2 (d) 1

18. Two particles of masses m_1 and m_2 in projectile motion have velocities $\mathbf{v}_1$ and $\mathbf{v}_2$ respectively at time $t=0$. They collide at time t_0. Their velocities become $\mathbf{v}_1'$ and $\mathbf{v}_2'$ at time $2t_0$ while still moving in air. The value of $|(m_1\mathbf{v}_1' + m_2\mathbf{v}_2') - (m_1\mathbf{v}_1 + m_2\mathbf{v}_2)|$ is **(2001, S)**

(a) zero (b) $(m_1+m_2)g\,t_0$

(c) $2\,(m_1+m_2)g\,t_0$ (d) $\frac{1}{2}(m_1+m_2)\,gt_0$

19. An isolated particle of mass m is moving in horizontal plane (x-y), along the X-axis, at a certain height above the ground. It suddenly explodes into two fragment of masses $m/4$ and $3m/4$. An instant later, the smaller fragment is at $y=+15$ cm. The larger fragment at this instant is at **(1997 C, 1M)**

(a) $y=-5$ cm (b) $y=+20$ cm
(c) $y=+5$ cm (d) $y=-20$ cm

20. A shell is fired from a cannon with a velocity v (m/s) at an angle θ with the horizontal direction. At the highest point in its path it explodes into two pieces of equal mass. One of the pieces retraces its path to the cannon and the speed (m/s) of the other piece immediately after the explosion is

(a) $3\,v\cos\theta$ (b) $2\,v\cos\theta$ **(1986, 2M)**

(c) $\frac{3}{2}\,v\cos\theta$ (d) $\sqrt{\frac{3}{2}}\,v\cos\theta$

21. A ball hits the floor and rebounds after an inelastic collision. In this case, **(1986, 2M)**

(a) the momentum of the ball just after the collision is the same as that just before the collision
(b) the mechanical energy of the ball remains the same in the collision
(c) the total momentum of the ball and the earth is conserved
(d) the total mechanical energy of the ball and the earth is conserved

Numerical Value

22. A ball is projected from the ground at an angle of 45° with the horizontal surface. It reaches a maximum height of 120 m and returns to the ground. Upon hitting the ground for the first time, it loses half of its kinetic energy. Immediately after the bounce, the velocity of the ball makes an angle of 30° with the horizontal surface. The maximum height it reaches after the bounce, in metres, is .... **(2018 Adv.)**

Assertion and Reason

Mark your answer as

(a) If Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) If Statement I is true, Statement II is true; Statement II is not a correct explanation for Statement I
(c) If Statement I is true; Statement II is false
(d) If Statement I is false; Statement II is true

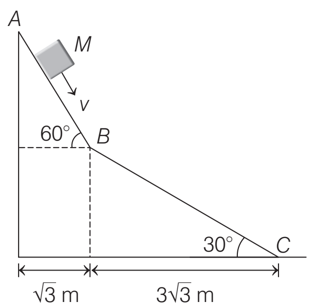
23. **Statement I** In an elastic collision between two bodies, the relative speed of the bodies after collision is equal to the relative speed before the collision.

Statement II In an elastic collision, the linear momentum of the system is conserved. **(2007, 3M)**

Passage Based Questions

Passage 1

A small block of mass M moves on a frictionless surface of an inclined plane, as shown in figure. The angle of the incline suddenly changes from 60° to 30° at point B. The block is initially at rest at A. Assume that collisions between the block and the incline are totally inelastic ($g = 10\,\text{m/s}^2$).



24. The speed of the block at point B immediately after it strikes the second incline is **(2008, 4M)**

(a) $\sqrt{60}$ m/s (b) $\sqrt{45}$ m/s (c) $\sqrt{30}$ m/s (d) $\sqrt{15}$ m/s

25. The speed of the block at point C, immediately before it leaves the second incline is **(2008, 4M)**

(a) $\sqrt{120}$ m/s (b) $\sqrt{105}$ m/s (c) $\sqrt{90}$ m/s (d) $\sqrt{75}$ m/s

26. If collision between the block and the incline is completely elastic, then the vertical (upward) component of the velocity of the block at point B, immediately after it strikes the second incline is **(2008, 4M)**

(a) $\sqrt{30}$ m/s (b) $\sqrt{15}$ m/s (c) zero (d) $-\sqrt{15}$ m/s

Objective Questions II (One or more correct option)

27. A point mass of 1 kg collides elastically with a stationary point mass of 5 kg. After their collision, the 1 kg mass reverses its direction and moves with a speed of $2\ \text{ms}^{-1}$. Which of the following statement(s) is/are correct for the system of these two masses? **(2010)**

(a) Total momentum of the system is 3 kg-ms^{-1}

(b) Momentum of 5 kg mass after collision is 4 kg-ms^{-1}

(c) Kinetic energy of the centre of mass is 0.75 J

(d) Total kinetic energy of the system is 4 J

28. Two balls, having linear momenta $\mathbf{p}_1 = p\,\hat{\mathbf{i}}$ and $\mathbf{p}_2 = -\,p\,\hat{\mathbf{i}}$, undergo a collision in free space. There is no external force acting on the balls. Let $\mathbf{p}'_1$ and $\mathbf{p}'_2$ be their final momenta. The following option figure (s) is (are) not allowed for any non-zero value of $p, a_1, a_2, b_1, b_2, c_1$ and c_2. **(2008, 4M)**

(a) $\mathbf{p}'_1 = a_1\hat{\mathbf{i}} + b_1\hat{\mathbf{j}} + c_1\hat{\mathbf{k}},\ \mathbf{p}'_2 = a_2\hat{\mathbf{i}} + b_2\hat{\mathbf{j}}$

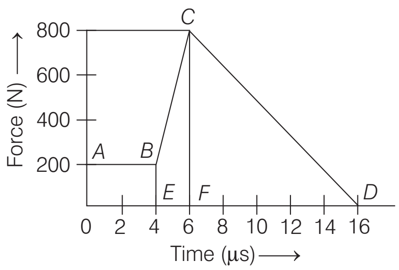
(b) $\mathbf{p}'_1 = c_1\hat{\mathbf{k}},\ \mathbf{p}'_2 = c_2\hat{\mathbf{k}}$

(c) $\mathbf{p}'_1 = a_1\hat{\mathbf{i}} + b_1\hat{\mathbf{j}} + c_1\hat{\mathbf{k}},\ \mathbf{p}'_2 = a_2\hat{\mathbf{i}} + b_2\hat{\mathbf{j}} - c_1\hat{\mathbf{k}}$

(d) $\mathbf{p}'_1 = a_1\hat{\mathbf{i}} + b_1\hat{\mathbf{j}},\ \mathbf{p}'_2 = a_2\hat{\mathbf{i}} + b_1\hat{\mathbf{j}}$

Fill in the Blanks

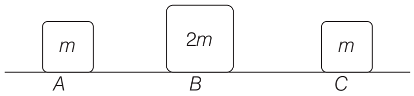
29. The magnitude of the force (in Newtons) acting on a body varies with time t (in microseconds) as shown in the figure. AB, BC and CD are straight line segments. The magnitude of the total impulse of the force on the body from $t = 4\ \mu\text{s}$ to $t = 16\ \mu\text{s}$ is ……… N-s. **(1994, 2M)**



30. A particle of mass $4m$ which is at rest explodes into three fragments. Two of the fragments each of mass m are found to move with a speed v each in mutually perpendicular directions. The total energy released in the process of explosion is ……… . **(1987, 2M)**

Integer Answer Type Questions

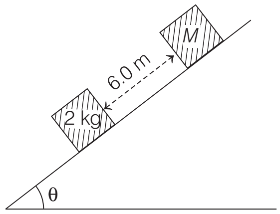
31. Three objects A, B and C are kept in a straight line on a frictionless horizontal surface. These have masses m, $2m$ and m, respectively. The object A moves towards B with a speed $9\ \text{ms}^{-1}$ and makes an elastic collision with it. Thereafter, B makes completely inelastic collision with C. All motions occur on the same straight line. Find the final speed (in ms^{-1}) of the object C. **(2009)**



Analytical & Descriptive Questions

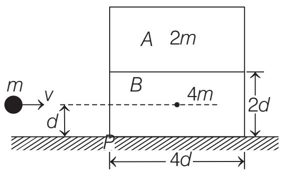
32. Two blocks of mass 2 kg and M are at rest on an inclined plane and are separated by a distance of 6.0 m as shown. The coefficient of friction between each block and the inclined plane is 0.25. The 2 kg block is given a velocity of 10.0 m/s up the inclined plane. It collides with M, comes back and has a velocity of 1.0 m/s when it reaches its initial position. The other block M after the collision moves 0.5 m up and comes to rest. Calculate the coefficient of restitution between the blocks and the mass of the block M.

[Take $\sin\theta \approx \tan\theta = 0.05$ and $g = 10\ \text{m/s}^2$] **(1999, 10M)**



33. A block A of mass $2m$ is placed on another block B of mass $4m$ which in turn is placed on a fixed table. The two blocks have a same length $4d$ and they are placed as shown in figure. The coefficient of friction (both static and kinetic) between the block B and table is μ. There is no friction between the two blocks. A small object of mass m moving horizontally along a line passing through the centre of mass (CM) of the block B and perpendicular to its face with a speed v collides elastically with the block B at a height d above the table. **(1991, 4 + 4M)**

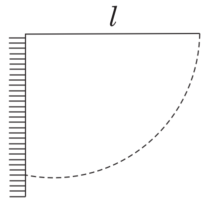
(a) What is the minimum value of v (call it v_0) required to make the block A to topple?



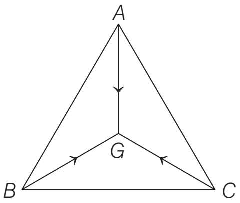
(b) If $v = 2\,v_0$, find the distance (from the point P in the figure) at which the mass m falls on the table after collision. (Ignore the role of friction during the collision.)

34. An object of mass 5 kg is projected with a velocity of 20 m/s at an angle of 60° to the horizontal. At the highest point of its path, the projectile explodes and breaks up into two fragments of masses 1 kg and 4 kg. The fragments separate horizontally after the explosion. The explosion releases internal energy such that the kinetic energy of the system at the highest point is doubled. Calculate the separation between the two fragments when they reach the ground. **(1990, 8M)**

35. A simple pendulum is suspended from a peg on a vertical wall. The pendulum is pulled away from the wall to a horizontal position (see fig.) and released. The ball hits the wall, the coefficient of restitution being $\frac{2}{\sqrt{5}}$. What is the minimum number of collisions after which the amplitude of oscillations becomes less than 60 degrees ? **(1987, 7M)**

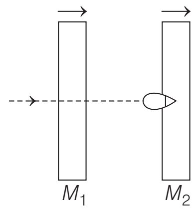


36. Three particles A, B and C of equal mass move with equal speed v along the medians of an equilateral triangle as shown in figure. They collide at the centroid G of the triangle. After the collision, A comes to rest, B retraces its path with the speed v. What is the velocity of C? **(1982, 2M)**



37. A body of mass 1 kg initially at rest, explodes and breaks into three fragments of masses in the ratio 1 : 1 : 3. The two pieces of equal mass fly-off perpendicular to each other with a speed of 30 m/s each. What is the velocity of the heavier fragment ? **(1981, 3M)**

38. A 20 g bullet pierces through a plate of mass $M_1 = 1$ kg and then comes to rest inside a second plate of mass $M_2 =$ 2.98 kg as shown in the figure. It is found that the two plates initially at rest, now move with equal velocities. Find the percentage loss in the initial velocity of the bullet when it is between M_1 and M_2. Neglect any loss of material of the plates due to the action of bullet. Both plates are lying on smooth table. **(1979)**



39. A body of mass m moving with a velocity v in the x-direction collides with another body of mass M moving in the y-direction with a velocity V. They coalesce into one body during collision. Find

(a) the direction and magnitude of the momentum of the composite body.

(b) the fraction of the initial kinetic energy transformed into heat during the collision. **(1978)**

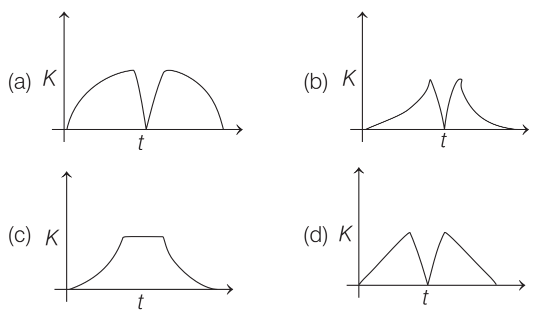
Topic 4 Miscellaneous Problems

Objective Questions I (Only one correct option)

1. A body of mass 1 kg falls freely from a height of 100 m on a platform of mass 3 kg which is mounted on a spring having spring constant $k = 1.25 \times 10^6$ N/m. The body sticks to the platform and the spring's maximum compression is found to be x. Given that $g = 10\,\text{ms}^{-2}$, the value of x will be close to **(JEE Main 2019, 11 Jan Shift I)**

(a) 8 cm (b) 4 cm
(c) 40 cm (d) 80 cm

2. A tennis ball is dropped on a horizontal smooth surface. It bounces back to its original position after hitting the surface. The force on the ball during the collision is proportional to the length of compression of the ball. Which one of the following sketches describes the variation of its kinetic energy K with time t most appropriately? The figures are only illustrative and not to the scale. **(2014 Adv.)**

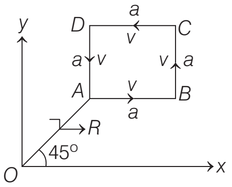


3. A pulse of light of duration 100 ns is absorbed completely by a small object initially at rest. Power of the pulse is 30 mV and the speed of light is 3×10^8 ms^{-1}. The final momentum of the object is **(2013 Adv.)**

(a) 0.3×10^{-17} kg-ms^{-1} (b) 1.0×10^{-17} kg-ms^{-1}
(c) 3.0×10^{-17} kg-ms^{-1} (d) 9.0×10^{-17} kg-ms^{-1}

Objective Questions II (One or more correct option)

4. A particle of mass m is moving along the side of a square of side a, with a uniform speed v in the xy-plane as shown in the figure. **(2016 Main)**



Which of the following statements is **false** for the angular momentum **L** about the origin?

(a) $\mathbf{L} = \frac{-mv}{\sqrt{2}} R\hat{\mathbf{k}}$ when the particle is moving from A to B

(b) $\mathbf{L} = mv\left(\frac{R}{\sqrt{2}} - a\right)\hat{\mathbf{k}}$ when the particle is moving from C to D

(c) $\mathbf{L} = mv\left(\frac{R}{\sqrt{2}} + a\right)\hat{\mathbf{k}}$ when the particle is moving from B to C

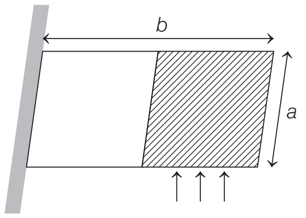
(d) $\mathbf{L} = \frac{mv}{\sqrt{2}} R\hat{\mathbf{k}}$ when the particle is moving from D to A

Numerical Value

5. A solid horizontal surface is covered with a thin layer of oil. A rectangular block of mass $m = 0.4$ kg is at rest on this surface. An impulse of 1.0 N s is applied to the block at time $t = 0$, so that it starts moving along the X-axis with a velocity $v(t) = v_0 e^{-t/\tau}$, where v_0 is a constant and $\tau = 4$ s. The displacement of the block, in metres, at $t = \tau$ is .... (Take, $e^{-1} = 0.37$). **(2018 Adv.)**

Analytical & Descriptive Questions

6. There is a rectangular plate of mass M kg of dimensions $(a \times b)$. The plate is held in horizontal position by striking n small balls uniformly each of mass m per unit area per unit time. These are striking in the shaded half region of the plate. The balls are colliding elastically with velocity v. What is v?

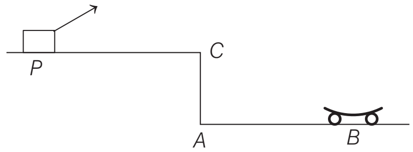


It is given $n = 100$, $M = 3$ kg, $m = 0 \cdot 01$ kg; $b = 2$ m; $a = 1$ m; $g = 10\ \text{m/s}^2$. **(2006, 6M)**

7. Two point masses m_1 and m_2 are connected by a spring of natural length l_0. The spring is compressed such that the two point masses touch each other and then they are fastened by a string. Then the system is moved with a velocity v_0 along positive x-axis. When the system reaches the origin the string breaks ($t = 0$). The position of the point mass m_1 is given by $x_1 = v_0 t - A(1 - \cos \omega t)$ where A and ω are constants. Find the position of the second block as a function of time. Also, find the relation between A and l_0. **(2003, 4M)**

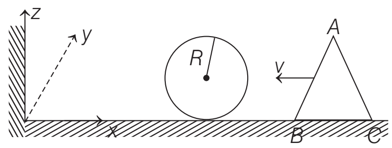
8. A car P is moving with a uniform speed of $5\sqrt{3}$ m/s towards a carriage of mass 9 kg at rest kept on the rails at a point B as shown in figure. The height AC is 120 m. Cannon balls of 1 kg are fired from the car with an initial velocity 100 m/s at an angle 30° with the horizontal. The first cannon ball hits the stationary carriage after a time t_0 and sticks to it. Determine t_0. At t_0, the second cannon ball is fired. Assume that the resistive force between the rails and the carriage is constant and ignore the vertical motion of the carriage throughout.

If the second ball also hits and sticks to the carriage, what will be the horizontal velocity of the carriage just after the second impact? **(2011, 10M)**



9. A cylindrical solid of mass 10^{-2} kg and cross-sectional area 10^{-4}m^2 is moving parallel to its axis (the x-axis) with a uniform speed of 10^3 m/s in the positive direction. At $t = 0$, its front face passes the plane $x = 0$. The region to the right of this plane is filled with stationary dust particles of uniform density $10^{-3}\ \text{kg/m}^3$. When a dust particle collides with the face of the cylinder, it sticks to its surface. Assuming that the dimensions of the cylinder remain practically unchanged and that the dust sticks only to the front face of the cylinder find the x-coordinate of the front of the cylinder at $t = 150$ s. **(1998, 5M)**

10. A wedge of mass m and triangular cross-section ($AB = BC = CA = 2R$) is moving with a constant velocity $(-v\hat{\mathbf{i}})$ towards a sphere of radius R fixed on a smooth horizontal table as shown in the figure.



The wedge makes an elastic collision with the fixed sphere and returns along the same path without any rotation. Neglect all friction and suppose that the wedge remains in contact with the sphere for a very short time Δt during which the sphere exerts a constant force $\mathbf{F}$ on the wedge. **(1998, 8M)**

(a) Find the force $\mathbf{F}$ and also the normal force $\mathbf{N}$ exerted by the table on the wedge during the time Δt.

(b) Let h denote the perpendicular distance between the centre of mass of the wedge and the line of action of F. Find the magnitude of the torque due to the normal force $\mathbf{N}$ about the centre of the wedge during the interval Δt.

11. A uniform thin rod of mass M and length L is standing vertically along the Y-axis on a smooth horizontal surface, with its lower end at the origin (0, 0). A slight disturbance at $t = 0$ causes the lower end to slip on the smooth surface along the positive X-axis, and the rod starts falling. **(1993, 1+5M)**

(a) What is the path followed by the centre of mass of the rod during its fall?

(b) Find the equation of the trajectory of a point on the rod located at a distance r from the lower end. What is the shape of the path of this point ?

Answers

Topic 1

1. (c)	**2.** (b)	**3.** (a)	**4.** (a)
5. (d)	**6.** (d)	**7.** (a)	**8.** (a)
9. (c)	**10.** (d)	**11.** (d)	**12.** (a)
13. F	**14.** $(L + 2R, 0)$	**15.** 6.53 s	

16. 9 cm from centre of bigger circle (leftwards)

Topic 2

1. (b)	**2.** (c)	**3.** (d)	**4.** (b)
5. (a)	**6.** (d)	**7.** (b, d)	

8. $-mv_2 \sin\frac{v_2}{R}t\,\hat{\mathbf{i}} + m\left(v_2 \cos\frac{v_2}{R}t - v_1\right)\hat{\mathbf{j}}$

9. (a) $\frac{v_0}{3}$ (b) $\frac{2mv_0^2}{3x_0^2}$ **10.** $\frac{m(R-r)}{M+m}, m\sqrt{\frac{2g(R-r)}{M(M+m)}}$

11. No

Topic 3

1. (b)	**2.** (d)	**3.** (d)	**4.** (b)	
5. (d)	**6.** (b)	**7.** (b)	**8.** (a)	
9. (d)	**10.** (a)	**11.** (c)	**12.** (b)	
13. (b)	**14.** (d)	**15.** (a)	**16.** (d)	
17. (c)	**18.** (c)	**19.** (a)	**20.** (a)	
21. (c)	**22.** (30)	**23.** (d)	**24.** (b)	**25.** (b)

26. (c)	**27.** (a, c)	**28.** (a, d)	**29.** 5×10^{-3}
30. $\frac{3}{2}mv^2$	**31.** 4	**32.** $e = 0.84, M = 15.12$ kg	
33. (a) $\frac{5}{2}\sqrt{6\mu gd}$ (b) $6d\sqrt{3\mu}$		**34.** 44.25 m	**35.** 4

36. Velocity of C is v in a direction opposite to Velocity of B.

37. $10\sqrt{2}$ m/s at 45° **38.** 25%

39. (a) At an angle $\tan^{-1}\left(\frac{MV}{mv}\right)$ with positive x-axis. Magnitude is $\sqrt{(mv)^2 + (MV)^2}$ (b) Fraction $= \frac{Mm(v^2 + V^2)}{(M+m)(mv^2 + MV^2)}$

Topic 4

1. (*)	**2.** (b)	**3.** (b)	**4.** (b, d)
5. (6.30)	**6.** 10 m/s		

7. $x_2 = v_0 t + \frac{m_1}{m_2}A(1 - \cos\omega t),\ l_0 = \left(\frac{m_1}{m_2} + 1\right)A$

8. 12s, 15.75 m/s **9.** 10^5 m

10. (a) $\frac{2mv}{\sqrt{3}\Delta t}(\sqrt{3}\hat{\mathbf{i}} - \hat{\mathbf{k}}), \left(\frac{2mv}{\sqrt{3}\Delta t} + mg\right)\hat{\mathbf{k}}$ (b) $\frac{4mv}{\sqrt{3}\Delta t}h$

11. (a) Straight line (b) $\frac{x^2}{\left(\frac{L}{2} - r\right)^2} + \frac{y^2}{r^2} = 1$, ellipse

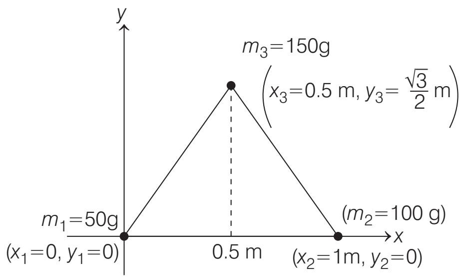
Hints & Solutions

Topic 1 Centre of Mass

1. The height of equilateral Δ is

$$h = y_3 = \sqrt{(1)^2 - (0.5)^2} = \sqrt{3}/2\,\text{m}$$

Thus, coordinates of three masses are (0, 0), (1, 0) and $\left(0.5, \frac{\sqrt{3}}{2}\right)$



Using, $X_{CM} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$,

we have

$$X_{CM} = \frac{50 \times 0 + 100 \times 1 + 150 \times 0.5}{50 + 100 + 150}$$

$$= \frac{175}{300} = \frac{7}{12}\text{m}$$

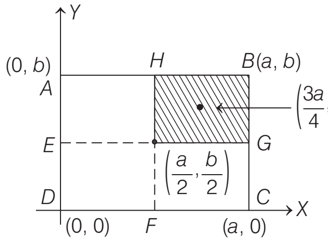
Similarly,

$$Y_{CM} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}$$

$$= \frac{50 \times 0 + 100 \times 0 + 150 \times \frac{\sqrt{3}}{2}}{50 + 100 + 150}$$

$$= \frac{\sqrt{3}}{4}\text{ m}$$

2. The given rectangular thin sheet $ABCD$ can be drawn as shown in the figure below,



Here,

Area of complete lamina, $A_1 = ab$

Area of shaded part of lamina $= \frac{a}{2} \times \frac{b}{2} = \frac{ab}{4}$

(x_1, y_1) = coordinates of centre of mass of complete lamina

$$= \left(\frac{a}{2}, \frac{a}{2}\right)$$

(x_2, y_2) = coordinates of centre of mass of shaded part of lamina $= \left(\frac{3a}{4}, \frac{3a}{4}\right)$

$\therefore$ Using formula for centre of mass, we have

$$X_{CM} = \frac{A_1 x_1 - A_2 x_2}{A_1 - A_2}$$

$$= \frac{ab\left(\frac{a}{2}\right) - \frac{ab}{4}\left(\frac{3a}{4}\right)}{ab - \frac{ab}{4}} = \frac{\frac{8a^2b - 3a^2b}{16}}{\frac{3ab}{4}} = \frac{5a}{12}$$

Similarly, $Y_{CM} = \frac{A_1 y_1 - A_2 y_2}{A_1 - A_2}$

$$= \frac{ab\left(\frac{b}{2}\right) - \frac{ab}{4}\left(\frac{3b}{4}\right)}{ab - \frac{ab}{4}} = \frac{5b}{12}$$

The coordinate of the centre of mass is $\left(\frac{5a}{12}, \frac{5b}{12}\right)$.

Alternate Solution

Let m be the mass of entire rectangular lamina.

So, the mass of the shaded portion of lamina $= \frac{m}{4}$

Using the relation,

$$X_{CM} = \frac{m_1 x_1 - m_2 x_2}{m_1 - m_2}, \text{ we get}$$

$$X_{CM} = \frac{m\left(\frac{a}{2}\right) - \frac{m}{4}\left(\frac{3a}{4}\right)}{m - \frac{m}{4}} = \frac{\frac{a}{2} - \frac{3a}{16}}{\frac{3}{4}} = \frac{5a}{12}$$

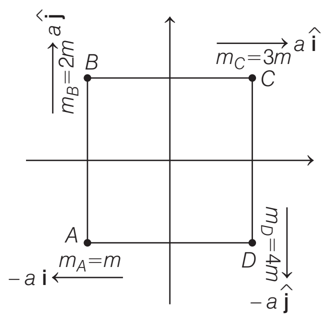
Similarly, $Y_{CM} = \frac{m_1 y_1 - m_2 y_2}{m_1 - m_2}$, we get

$$Y_{CM} = \frac{m\left(\frac{a}{2}\right) - \frac{m}{4}\left(\frac{3b}{4}\right)}{m - \frac{m}{4}} = \frac{\frac{b}{2} - \frac{3b}{16}}{\frac{3}{4}} = \frac{5b}{12}$$

$\therefore$ The coordinates of the centre of mass of remaining portion will be $\left(\frac{5a}{12}, \frac{5b}{12}\right)$.

3. For a system of discrete masses, acceleration of centre of mass (CM) is given by

$$\mathbf{a}_{CM} = \frac{m_A \mathbf{a}_A + m_B \mathbf{a}_B + m_C \mathbf{a}_C + m_D \mathbf{a}_D}{m_A + m_B + m_C + m_D}$$



where, $m_A = m, m_B = 2m, \quad m_C = 3m \quad$ and $\quad m_D = 4m$, $|\mathbf{a}_A| = |\mathbf{a}_B| = |\mathbf{a}_C| = |\mathbf{a}_D| = a$ (according to the question)

$$a_{CM} = \frac{-ma\hat{\mathbf{i}} + 2ma\hat{\mathbf{j}} + 3ma\hat{\mathbf{i}} - 4ma\hat{\mathbf{j}}}{m + 2m + 3m + 4m}$$

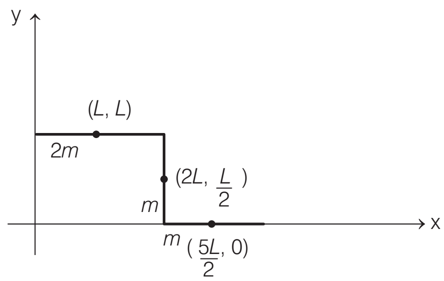
$$= \frac{2a\hat{\mathbf{i}} - 2a\hat{\mathbf{j}}}{10} = \frac{a}{5} \cdot (\hat{\mathbf{i}} - \hat{\mathbf{j}})\, \text{ms}^{-2}$$

4. Coordinates of centre of mass (COM) are given by

$$X_{COM} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

and $$Y_{COM} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3}$$

For given system of rods, masses and coordinates of centre of rods are as shown.



So, $X_{COM} = \left(\frac{2mL + m2L + m\frac{5L}{2}}{4\,m}\right) = \frac{13}{8} L$

and $Y_{COM} = \frac{2mL + m \times \frac{L}{2} + m \times 0}{4\,m} = \frac{5L}{8}$

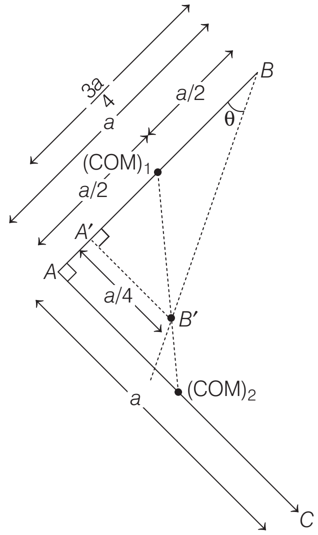
So, position vector of COM is

$$r_{com} = X_{COM}\,\hat{\mathbf{x}} + Y_{COM}\,\hat{\mathbf{y}}$$

$$= \frac{13}{8} L\hat{\mathbf{x}} + \frac{5}{8} L\hat{\mathbf{y}}$$

5. **Key Idea** The centre of mass of a thin rod of uniform density lies at its centre.

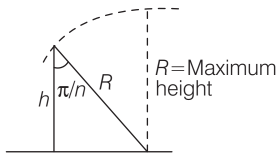
The given system of rods can be drawn using geometry as,



where, $(\text{COM})_1$ and $(\text{COM})_2$ are the centre of mass of both rods AB and AC, respectively.
So, in $\Delta A'BB'$,

$$\tan\theta = \frac{A'B'}{A'B} = \frac{\frac{a}{4}}{\frac{3a}{4}} = \frac{1}{3} \text{ or } \tan\theta = \frac{1}{3}$$

6.



$$\cos\left(\frac{\pi}{n}\right) = \frac{h}{R}$$

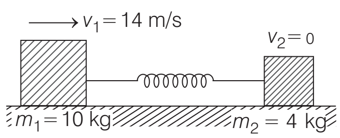
$$\Delta = R - h = \frac{h}{\cos\left(\frac{\pi}{n}\right)} - h = h\left[\frac{1}{\cos\left(\frac{\pi}{n}\right)} - 1\right]$$

7. Centre of mass of uniform solid cone of height h is at a height of $\frac{h}{4}$ from base. Therefore from vertex it's $\frac{3h}{4}$.

8. $$y_{CM} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3 + m_4 y_4 + m_5 y_5}{m_1 + m_2 + m_3 + m_4 + m_5}$$

$$= \frac{(6m)(0) + (m)(a) + m(a) + m(0) + m(-a)}{6m + m + m + m + m} = \frac{a}{10}$$

9.



$$v_{CM} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{10 \times 14 + 4 \times 0}{10 + 4} = \frac{140}{14} = 10\,\text{m/s}$$

10. Net force on centre of mass is zero. Therefore, centre of mass always remains at rest.

11. If a force is applied at centre of mass of a rigid body, its torque about centre of mass will be zero, but acceleration will be non-zero. Hence, velocity will change.

12. Δx_{cm} of the block and point mass system = 0

$\therefore \quad m(x + R) + Mx = 0$

where, x is displacement of the block.
Solving this equation, we get

$$x = -\frac{mR}{M + m}$$

From conservation of momentum and mechanical energy of the combined system

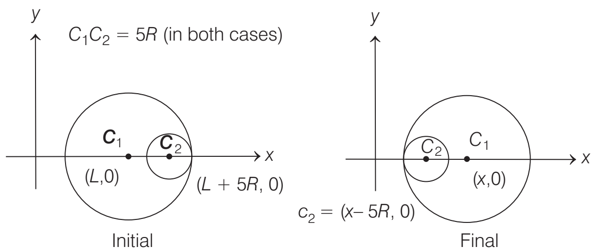
$$0 = mv - MV \Rightarrow mgR = \frac{1}{2}mv^2 + \frac{1}{2}MV^2$$

Solving these two equations, we get

$$\therefore \quad v = \sqrt{\frac{2gR}{1 + \frac{m}{M}}}$$

13. Since, net force on the system is zero. Velocity of centre of mass will remain constant.

14. Since, all the surfaces are smooth, no external force is acting on the system in horizontal direction. Therefore, the centre of mass of the system in horizontal direction remains stationary.



x-coordinate of CM initially will be given by

$$x_i = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$= \frac{(4M)(L) + M(L + 5R)}{4M + M} = (L + R) \quad \ldots\text{(i)}$$

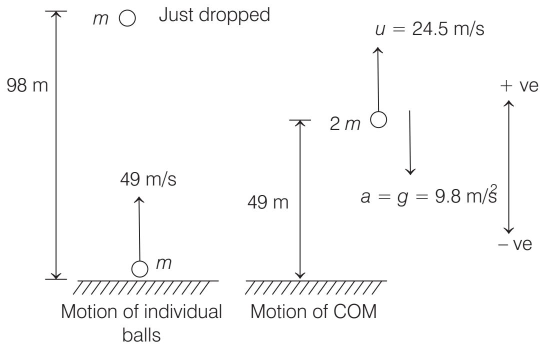
Let $(x, 0)$ be the coordinates of the centre of large sphere in final position. Then, x-coordinate of CM finally will be

$$x_f = \frac{(4M)(x) + M(x - 5R)}{4M + M} = (x - R) \quad \ldots\text{(ii)}$$

Equating Eqs. (i) and (ii), we have $x = L + 2R$

Therefore, coordinates of large sphere, when the smaller sphere reaches the other extreme position, are $(L + 2R, 0)$.

15. Both the balls are of equal masses. Therefore, their centre of mass is at height $98/2 = 49\,\text{m}$ from ground. Acceleration of both the balls is g downwards. Therefore, acceleration of their centre of mass is also g downwards. Further, initially one ball has a velocity 49 m/s.



While the other is at rest. Therefore, initial velocity of their centre of mass is 49/2 = 24.5 m/s upwards. So, looking the motion of their centre of mass. Let it strikes after time t with the ground. Putting the proper values with sign in equation.

$$s = ut + \frac{1}{2}at^2$$

$$-49 = 24.5\,t - 4.9\,t^2$$

or $$t^2 - 5t - 10 = 0$$

or $$t = \frac{5 \pm \sqrt{25+40}}{2}$$

The positive value of t from this equation comes out to be, 6.53 s. Therefore, time of flight of the balls is 6.53 s.

16. Suppose r_1 be the distance of centre of mass of the remaining portion from centre of the bigger circle, then

$$A_1 r_1 = A_2 r_2$$

$$r_1 = \left(\frac{A_2}{A_1}\right) r_2$$

$$= \frac{\pi(42)^2}{\pi[(56)^2 - (42)^2]} \times 7 = 9\,\text{cm}$$

Topic 2 Linear Momentum, Mechanical Energy and Their Conservation

1. Initially, centre of mass is at distance L from the top end of the swing. It shifts to $(L-l)$ distance when the person stands up on the swing.

$\therefore$ Using angular momentum conservation law, if v_0 and v_1 are the velocities before standing and after standing of the person, then

$$Mv_0 L = Mv_1(L-l)$$

$$\Rightarrow \quad v_1 = \left(\frac{L}{L-l}\right)v_0 \quad \ldots\text{(i)}$$

Now, total work done by (person + gravitation) system will be equal to the change in kinetic energy of the person, i.e.

$$W_g + W_p = \text{KE}_1 - \text{KE}_0$$

$$\Rightarrow \quad -Mgl + W_p = \frac{1}{2}Mv_1^2 - \frac{1}{2}Mv_0^2$$

$$\Rightarrow \quad W_p = Mgl + \frac{1}{2}M(v_1^2 - v_0^2)$$

$$= Mgl + \frac{1}{2}M\left[\left(\frac{L}{L-l}\right)^2 v_0^2 - v_0^2\right] \quad \text{[from Eq. (i)]}$$

$$= Mgl + \frac{1}{2}M\,v_0^2\left[\left(\frac{L-l}{L}\right)^{-2} - 1\right]$$

$$= Mgl + \frac{1}{2}Mv_0^2\left[\left(1 - \frac{l}{L}\right)^{-2} - 1\right]$$

$$= Mgl + \frac{1}{2}Mv_0^2\left[\left(1 + \frac{2l}{L}\right) - 1\right]$$

[using $(1+x)^n = 1 + nx$ as higher terms can be neglected, if $n << 1$]

$$\Rightarrow \quad W_p = Mgl + \frac{1}{2}Mv_0^2 \times \frac{2l}{L}$$

or $$W_p = Mgl + Mv_0^2\,\frac{l}{L} \quad \ldots\text{(ii)}$$

Here, $$v_0 = WA = \left(\sqrt{\frac{g}{L}}\right)(\theta_0 L)$$

$$\Rightarrow \quad v_0 = \theta_0\sqrt{gL}$$

$\therefore$ Using this value of v_0 in Eq.(ii), we get

$$W_p = Mgl + M\theta_0^2\,gL \cdot \frac{l}{L}$$

$$\Rightarrow \quad W_p = Mgl\,[1 + \theta_0^2]$$

2. **Key Idea** Since, the ground is frictionless, so whe the particle will collide and climb over the wedge, then the wedge will also move. Thus, by using conservation laws for momentum and energy, maximum height climbed by the particle can be calculated.

Initial condition can be shown in the figure below

As mass m collides with wedge, let both wedge and mass move with speed v'. Then,

By applying linear momentum conservation, we have

Initial momentum of the system =

Final momentum of the system

$$mv + 0 = (m + 4m)\,v'$$

$$\Rightarrow \quad v' = \frac{v}{5} \quad \ldots\text{(i)}$$

Now, if m rises upto height h over wedge, then by applying conservation of mechanical energy, we have

Initial energy of the system = Final energy of the system

$$\frac{1}{2}mv^2 + 0 = \frac{1}{2}mv'^2 + mgh + \frac{1}{2}(4m)v'^2$$

$$mv^2 = (m + 4m)v'^2 + 2mgh$$

$$\Rightarrow \quad v^2 = 5v'^2 + 2gh$$

$$\Rightarrow \quad v^2 = \frac{1}{5}v^2 + 2gh \quad \text{[using Eq. (i)]}$$

$$\Rightarrow \quad \frac{4}{5}v^2 = 2gh \Rightarrow h = \frac{2v^2}{5g}$$

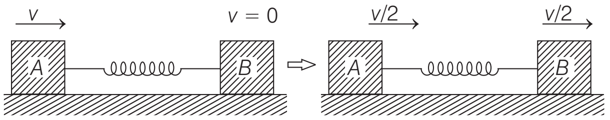
3. When a ball is thrown vertically upward, then the acceleration of the ball,
a = acceleration due to gravity (g) (acting in the downward direction).
Now, using the equation of motion,
$$v^2 = u^2 - 2gh$$
or $$h = \frac{-v^2 + u^2}{2g} \quad \ldots\text{(i)}$$
As we know, momentum, $p = mv$ or $v = p/m$
So, substituting the value of v in Eq. (i), we get
$$h = \frac{u^2 - (p/m)^2}{2g}$$
As we know that, at the maximum height, velocity of the ball thrown would be zero.
So, for the flight when the ball is thrown till is reaches the maximum height (h).
$v \rightarrow$ changes from u to 0
$\Rightarrow p \rightarrow$ changes from mu to 0
Similarly, when it reacher it's initial point, then
$h \rightarrow$ changes from h_{max} to 0
Also, $p \rightarrow$ changes from 0 to some values.
Thus, these conditions are only satisfied in the plot given in option (d).

4. Here,
$$F = kt$$
When $t = 0$, linear momentum = p
When $t = T$, linear momentum = $3p$
According to Newton's second law of motion,
applied force, $F = \frac{dp}{dt}$
or $dp = F \cdot dt$
or $dp = kt \cdot dt$
Now, integrate both side with proper limit
$$\int_p^{3p} dp = k\int_0^T t\,dt \text{ or } [p]_p^{3p} = k\left[\frac{t^2}{2}\right]_0^T$$
or $(3p - p) = \frac{1}{2}k[T^2 - 0]$
or $T^2 = \frac{4p}{k}$ or $T = 2\sqrt{\frac{p}{k}}$

5. On a system of particles if, $\Sigma \mathbf{F}_{ext.} = 0$
then, $\mathbf{p}_{system}$ = constant
No other conclusions can be drawn.

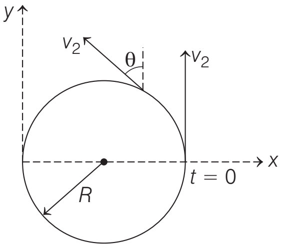
6.
$$\mathbf{F} = \frac{d\mathbf{p}}{dt} = -kA \sin(kt)\,\hat{\mathbf{i}} - kA\cos(kt)\,\hat{\mathbf{j}}$$
$$\mathbf{p} = A\cos(kt)\,\hat{\mathbf{i}} - A\sin(kt)\,\hat{\mathbf{j}}$$
Since, $\mathbf{F} \cdot \mathbf{p} = 0$
$\therefore$ Angle between $\mathbf{F}$ and $\mathbf{p}$ should be 90°.

7. After collision between C and A, C stops while A moves with speed of C i.e. v [in head on elastic collision, two equal masses exchange their velocities]. At maximum compression, A and B will move with same speed $v/2$ (From conservation of linear momentum).



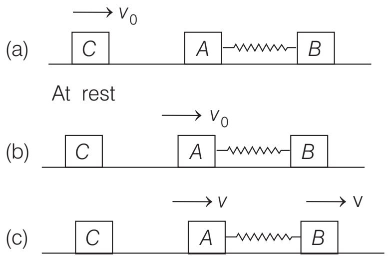
Let x be the maximum compression in this position.
$\therefore$ KE of A-B system at maximum compression
$$= \frac{1}{2}(2m)\left(\frac{v}{2}\right)^2 \text{ or } K_{max} = mv^2/4$$
From conservation of mechanical energy in two positions shown in above figure
or $$\frac{1}{2}mv^2 = \frac{1}{4}mv^2 + \frac{1}{2}kx^2$$
$$\frac{1}{2}kx^2 = \frac{1}{4}mv^2$$
$\Rightarrow$ $$x = v\sqrt{\frac{m}{2k}} \text{ (Maximum compression)}$$

8. Angular speed of particle about centre of the circle



$$\omega = \frac{v_2}{R}, \theta = \omega t = \frac{v_2}{R}t$$
$$\mathbf{v}_p = (-v_2 \sin\theta\,\hat{\mathbf{i}} + v_2\cos\theta\,\hat{\mathbf{j}})$$
or $$\mathbf{v}_p = \left(-v_2 \sin\frac{v_2}{R}t\,\hat{\mathbf{i}} + v_2\cos\frac{v_2}{R}t\,\hat{\mathbf{j}}\right)$$
and $$\mathbf{v}_m = v_1\hat{\mathbf{j}}$$
$\therefore$ Linear momentum of particle w.r.t. man as a function of time is
$$\mathbf{L}_{pm} = m(\mathbf{v}_p - \mathbf{v}_m)$$
$$= m\left[\left(-v_2\sin\frac{v_2}{R}t\right)\hat{\mathbf{i}} + \left(v_2\cos\frac{v_2}{R}t - v_1\right)\hat{\mathbf{j}}\right]$$

9. (a) Collision between A and C is elastic and mass of both the blocks is same. Therefore, they will exchange their velocities i.e. C will come to rest and A will be moving will velocity v_0. Let v be the common velocity of A and B, then from conservation of linear momentum, we have



$$m_A v_0 = (m_A + m_B)\, v \quad \text{or} \quad m v_0 = (m + 2m)\, v \quad \text{or} \quad v = \frac{v_0}{3}$$

(b) From conservation of energy, we have

$$\frac{1}{2} m_A v_0^2 = \frac{1}{2}(m_A + m_B)\, v^2 + \frac{1}{2} k x_0^2$$

or $$\frac{1}{2} m v_0^2 = \frac{1}{2}(3m)\left(\frac{v_0}{3}\right)^2 + \frac{1}{2} k x_0^2$$

or $$\frac{1}{2} k x_0^2 = \frac{1}{3} m v_0^2 \quad \text{or} \quad k = \frac{2 m v_0^2}{3 x_0^2}$$

10. (a) The centre of mass of $M + m$ in this case will not move in horizontal direction. Let M moves towards left by a distance x then m will move towards right by a distance $R - r - x$ (with respect to ground). For centre of mass not to move along horizontal we should have

$$Mx = m\,(R - r - x), \quad x = \frac{m\,(R - r)}{M + m}$$

(b) Let v_1 be the speed of m towards right and v_2 the speed of M towards left. From conservation of linear momentum.

$$m v_1 = M v_2 \quad \ldots\text{(i)}$$

From conservation of mechanical energy

$$mg\,(R - r) = \frac{1}{2} m v_1^2 + \frac{1}{2} M v_2^2 \quad \ldots\text{(ii)}$$

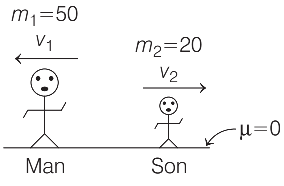
Solving these two equations, we get

$$v_2 = m\sqrt{\frac{2g\,(R - r)}{M\,(M + m)}}$$

11. No, the situation given does not violate conservation of linear momentum. Because linear momentum of ball-earth system remains constant.

Topic 3 Impulse, Explosions and Collisions

1. The given situation can be shown as below



After collision

Using momentum conservation law,

$$(\text{Total momentum})_{\text{before collision}}$$
$$= (\text{Total momentum})_{\text{after collision}}$$
$$(m_1 \times 0) + (m_2 \times 0) = m_1 \mathbf{v}_1 + m_2\, \mathbf{v}_2$$
$$0 = m_1(-v_1)\hat{\mathbf{i}} + m_2 v_2 \hat{\mathbf{i}}$$

$\Rightarrow \quad m_1 v_1 = m_2 v_2$

$\Rightarrow \quad 50 v_1 = 20 v_2$

$\Rightarrow \quad v_2 = 2.5 v_1 \quad \ldots\text{(i)}$

Again, relative velocity = 0.70 m/s

But from figure, relative velocity = $v_1 + v_2$

$\therefore \quad v_1 + v_2 = 0.7 \quad \ldots\text{(ii)}$

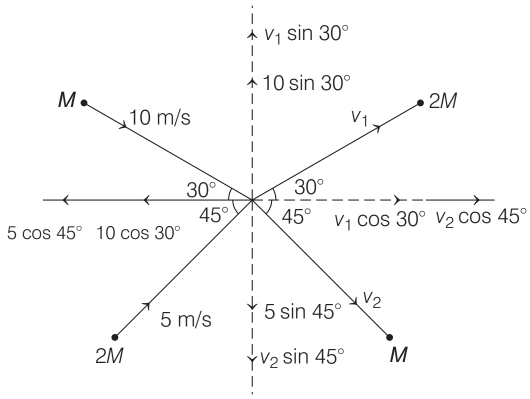
From Eqs. (i) and (ii), we get

$$v_1 + 2.5 v_1 = 0.7$$

$\Rightarrow \quad v_1(3.5) = 0.7$

$$v_1 = \frac{0.7}{3.5} = 0.20\ \text{m/s}$$

2. The given condition can be drawn as shown below



Applying linear momentum conservation law in x-direction, we get

Initial momentum = Final momentum

$$(M \times 10 \cos 30^\circ) + (2M \times 5 \cos 45^\circ)$$
$$= (M \times v_2 \cos 45^\circ) + (2M \times v_1 \cos 30^\circ)$$

$$\Rightarrow \left(M \times 10 \times \frac{\sqrt{3}}{2}\right) + \left(2M \times 5 \times \frac{1}{\sqrt{2}}\right)$$
$$= \left(M \times v_2 \times \frac{1}{\sqrt{2}}\right) + \left(2M \times v_1 \times \frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow \quad 5\sqrt{3} + 5\sqrt{2} = \frac{v_2}{\sqrt{2}} + v_1\sqrt{3} \quad \ldots\text{(i)}$$

Similarly, applying linear momentum conservation law in y-direction, we get

$$(M \times 10 \sin 30^\circ) - (2M \times 5 \sin 45^\circ)$$
$$= (M \times v_2 \sin 45^\circ) - (2M \times v_1 \sin 30^\circ)$$

$$\Rightarrow \left(M \times 10 \times \frac{1}{2}\right) - \left(2M \times 5 \times \frac{1}{\sqrt{2}}\right)$$
$$= \left(M \times v_2 \times \frac{1}{\sqrt{2}}\right) - \left(2M \times v_1 \times \frac{1}{2}\right)$$

$$\Rightarrow \quad 5 - 5\sqrt{2} = \frac{v_2}{\sqrt{2}} - v_1 \quad \ldots \text{(ii)}$$

Subtracting Eq. (ii) from Eq. (i), we get

$$(5\sqrt{3} + 5\sqrt{2}) - (5 - 5\sqrt{2}) = \left(\frac{v_2}{\sqrt{2}} + v_1\sqrt{3}\right) - \left(\frac{v_2}{\sqrt{2}} - v_1\right)$$

$$\Rightarrow \quad 5\sqrt{3} + 10\sqrt{2} - 5 = v_1\sqrt{3} + v_1$$

$$\Rightarrow \quad v_1 = \left(\frac{5\sqrt{3} + 10\sqrt{2} - 5}{1 + \sqrt{3}}\right) = \frac{8.66 + 14.142 - 5}{1 + 1.732}$$

$$= \frac{17.802}{2.732} \Rightarrow v_1 = 6.516 \text{ m/s} \approx 6.5 \text{ m/s} \quad \ldots \text{(iii)}$$

Substituting the value from Eq. (iii) in Eq. (i), we get

$$5\sqrt{3} + 5\sqrt{2} = \frac{v_2}{\sqrt{2}} + 6.51 \times \sqrt{3}$$

$$\Rightarrow \quad v_2 = (5\sqrt{3} + 5\sqrt{2} - 6.51 \times \sqrt{3})\sqrt{2}$$

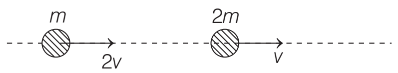
$$v_2 = (8.66 + 7.071 - 11.215)\, 1.414$$

$$\Rightarrow \quad v_2 = 4.456 \times 1.414$$

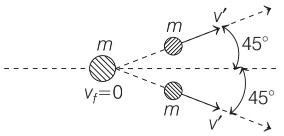
$$\Rightarrow \quad v_2 \approx 6.3 \text{ m/s}$$

3. According to the questions,

Initial condition,



Final condition,



As we know that, in collision, linear momentum is conserved in both x and y directions separately.

So, $(p_x)_{\text{initial}} = (p_x)_{\text{final}}$

$$m(2v) + 2m(v) = 0 + mv'\cos 45^\circ + mv'\cos 45^\circ$$

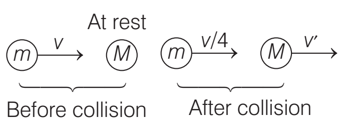
$$\Rightarrow \quad 4mv = \frac{2m}{\sqrt{2}}v' \Rightarrow v' = 2\sqrt{2}v$$

So, each particle will move with a speed of $2\sqrt{2}\,v$.

4. **Key Idea** For an elastic collision, coefficient of restitution (e), i.e. the ratio of relative velocity of separation after collision to the relative velocity of approach before collision is 1.

Given, mass of small body, $m = 2$ kg

Given situation is as shown



Using momentum conservation law for the given system,

$$(\text{Total momentum})_{\text{before collision}} = (\text{Total momentum})_{\text{after collision}}$$

$$\Rightarrow \quad m(v) + M(0) = m\left(\frac{v}{4}\right) + M(v') \ldots \text{(i)}$$

$\because \; e = 1$ and we know that,

$$e = -\frac{v_2 - v_1}{u_2 - u_1}$$

$$\Rightarrow \quad 1 = -\frac{v' - v/4}{0 - v}$$

$$\Rightarrow \quad v = v' - v/4$$

or $\quad v' = 5v/4 \quad \ldots \text{(ii)}$

Using value from Eq. (ii) into Eq. (i), we get

$$mv = \frac{mv}{4} + M\left(\frac{5v}{4}\right)$$

$$\Rightarrow \quad m\left(v - \frac{v}{4}\right) = M\left(\frac{5v}{4}\right)$$

$$\Rightarrow \quad \frac{3}{4}mv = \frac{5}{4}Mv$$

$$M = \frac{3}{5}m = \frac{3}{5} \times 2 = 1.2 \text{ kg}$$

5. **Key Idea** Total linear momentum is conserved in all collisions, i.e. the initial momentum of the system is equal to final momentum of the system.

Given,

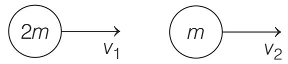
$$m_2 = 0.5m_1 \quad \Rightarrow \quad m_1 = 2m_2$$

Let $m_2 = m$, then, $m_1 = 2m$

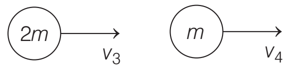
Also, $v_3 = 0.5v_1$

Given situation of collinear collision is as shown below

Before collision,



After collision,



$\therefore$ According to the conservation of linear momentum,

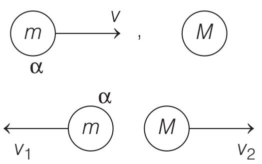
Initial momentum = Final momentum

$$m_1v_1\hat{\mathbf{i}} + m_2v_2\hat{\mathbf{i}} = m_1v_3\hat{\mathbf{i}} + m_2v_4\hat{\mathbf{i}}$$

$$\Rightarrow 2mv_1\hat{\mathbf{i}} + mv_2\hat{\mathbf{i}} = 2m(0.5v_1)\hat{\mathbf{i}} + mv_4\hat{\mathbf{i}}$$

$$\Rightarrow v_4 = v_1 + v_2 \quad \Rightarrow \quad v_1 = v_4 - v_2$$

6. We have following collision, where mass of α particle = m and mass of nucleus = M



Let α particle rebounds with velocity v_1, then

Given; final energy of α = 36% of initial energy

$$\Rightarrow \quad \frac{1}{2}mv_1^2 = 0.36 \times \frac{1}{2}mv^2$$

$$\Rightarrow \quad v_1 = 0.6\,v \qquad \text{...(i)}$$

As unknown nucleus gained 64% of energy of α, we have

$$\frac{1}{2}Mv_2^2 = 0.64 \times \frac{1}{2}mv^2$$

$$\Rightarrow \quad v_2 = \sqrt{\frac{m}{M}} \times 0.8\,v \qquad \text{...(ii)}$$

From momentum conservation, we have

$$mv = Mv_2 - mv_1$$

Substituting values of v_1 and v_2 from Eqs. (i) and (ii), we have

$$mv = M\sqrt{\frac{m}{M}} \times 0.8v - m \times 0.6v$$

$$\Rightarrow \quad 1.6mv = \sqrt{mM} \times 0.8v$$

$$\Rightarrow \quad 2m = \sqrt{mM}$$

$$\Rightarrow \quad 4m^2 = mM \quad \Rightarrow \quad M = 4m$$

7 According to the given condition in the question, after collision the mass of combined system is doubled. Also, this system would be displaced from its circular orbit.

So, the combined system revolves around centre of mass of 'system + earth' under action of a central force.

Hence, orbit must be elliptical.

8 Pendulum's velocity at lowest point just before striking mass m is found by equating it's initial potential energy (PE) with final kinetic energy (KE).

Initially, when pendulum is released from angle θ_0 as shown in the figure below,

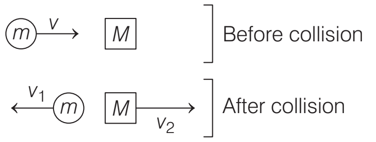
We have,

$$mgh = \frac{1}{2}mv^2$$

Here, $\quad h = l - l\cos\theta_0$

So, $\quad v = \sqrt{2gl\,(1 - \cos\theta_0)} \qquad \text{...(i)}$

With velocity v, bob of pendulum collides with block. After collision, let v_1 and v_2 are final velocities of masses m and M respectively as shown



Then if pendulum is deflected back upto angle θ_1, then

$$v_1 = \sqrt{2gl\,(1 - \cos\theta_1)} \qquad \text{...(ii)}$$

Using definition of coefficient of restitution to get

$$e = \frac{|\text{velocity of separation}|}{|\text{velocity of approach}|}$$

$$1 = \frac{v_2 - (-v_1)}{v - 0} \Rightarrow v = v_2 + v_1 \qquad \text{...(iii)}$$

From Eqs. (i), (ii) and (iii), we get

$$\Rightarrow \sqrt{2gl(1 - \cos\theta_0)} = v_2 + \sqrt{2gl(1 - \cos\theta_1)}$$

$$\Rightarrow \quad v_2 = \sqrt{2gl}\,(\sqrt{1 - \cos\theta_0} - \sqrt{1 - \cos\theta_1}) \qquad \text{...(iv)}$$

According to the momentum conservation, initial momentum of the system = final momentum of the system

$$\Rightarrow \quad mv = Mv_2 - mv_1$$

$$\Rightarrow Mv_2 = m(v + v_1)$$

$$Mv_2 = m\sqrt{2gl}\,(\sqrt{1 - \cos\theta_0} + \sqrt{1 - \cos\theta_1})$$

Dividing Eq. (v) and Eq. (iv), we get

$$\Rightarrow \frac{M}{m} = \frac{\sqrt{1 - \cos\theta_0} + \sqrt{1 - \cos\theta_1}}{\sqrt{1 - \cos\theta_0} - \sqrt{1 - \cos\theta_1}}$$

$$= \frac{\sqrt{\sin^2\left(\frac{\theta_0}{2}\right)} + \sqrt{\sin^2\left(\frac{\theta_1}{2}\right)}}{\sqrt{\sin^2\left(\frac{\theta_0}{2}\right)} - \sqrt{\sin^2\left(\frac{\theta_1}{2}\right)}}$$

$$\frac{M}{m} = \frac{\sin\left(\frac{\theta_0}{2}\right) + \sin\left(\frac{\theta_1}{2}\right)}{\sin\left(\frac{\theta_0}{2}\right) - \sin\left(\frac{\theta_1}{2}\right)}$$

For small θ_0, we have

$$\frac{M}{m} = \frac{\frac{\theta_0}{2} + \frac{\theta_1}{2}}{\frac{\theta_0}{2} - \frac{\theta_1}{2}}$$

$$\text{or} \quad M = m\left(\frac{\theta_0 + \theta_1}{\theta_0 - \theta_1}\right)$$

9.

> **Key Idea** As bullet gets embedded in the block of wood so, it represents a collision which is perfectly inelastic and hence only momentum of the system is conserved.

Velocity of bullet is very high compared to velocity of wooden block so, in order to calculate time for collision, we take relative velocity nearly equal to velocity of bullet.

So, time taken for particles to collide is

$$t = \frac{d}{v_{rel}} = \frac{100}{100} = 1\text{s}$$

Speed of block just before collision is;

$$v_1 = gt = 10 \times 1 = 10\ \text{ms}^{-1}$$

Speed of bullet just before collision is

$$v_2 = u - gt$$

$$= 100 - 10 \times 1 = 90\ \text{ms}^{-1}$$

Let v = velocity of bullet + block system, then by conservation of linear momentum, we get

$-(0.03 \times 10) + (0.02 \times 90) = (0.05)\, v$

$\Rightarrow \quad v = 30 \text{ ms}^{-1}$

Now, maximum height reached by bullet and block is

$$h = \frac{v^2}{2g} \Rightarrow h = \frac{30 \times 30}{2 \times 10}$$

$\Rightarrow \quad h = 45$ m

$\therefore$ Height covered by the system from point of collision = 45 m

Now, distance covered by bullet before collision in 1 sec.

$$= 100 \times 1 - \frac{1}{2} \times 10 \times 1^2 = 95 \text{ m}$$

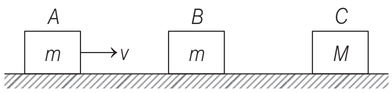
Distance of point of collision from the top of the building

$= 100 - 95 = 5$ m

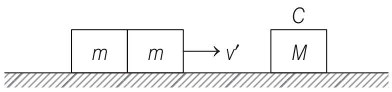
$\therefore$ Maximum height to which the combined system reaches above the top of the building before falling below $= 45 - 5 = 40$ m

10. **Key Idea** For a perfectly inelastic collision, the momentum of the system remains conserved but there is some of loss of kinetic energy. Also, after collision the objects of the system are stuck to each other and move as a combined system.

Initially, block A is moving with velocity v as shown in the figure below,

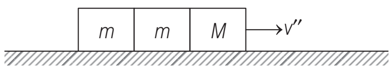


Now, A collides with B such that they collide inelastically. Thus, the combined mass (say) move with the velocity 'v' as shown below,



Then, if this combined system is collided inelastically again with the block C.

So, now the velocity of system be v'' as shown below.



Thus, according to the principle of conservation of momentum,

initial momentum of the system

= final momentum of the system

$\Rightarrow \quad mv = (2m + M)v''$

or $\quad v'' = \left(\frac{mv}{2m+M}\right)$ … (i)

Initial kinetic energy of the system,

$$(\text{KE})_i = \frac{1}{2} mv^2 \quad \text{…(ii)}$$

Final kinetic energy of the system, $(\text{KE})_f$

$$= \frac{1}{2}(2m + M)(v'')^2$$

$$= \frac{1}{2}(2m + M)\left(\frac{mv}{2m + M}\right)^2 \quad [\because \text{ using Eq. (i)}]$$

$$= \frac{1}{2} \cdot \frac{v^2 m^2}{(2m+M)} \quad \text{…(iii)}$$

Dividing Eq. (iii) and Eq. (ii), we get

$$\frac{(\text{KE})_f}{(\text{KE})_i} = \frac{\frac{\frac{1}{2}m^2 v^2}{(2m+M)}}{\frac{1}{2}mv^2} = \frac{m}{2m + M} \quad \text{… (iv)}$$

It is given that $\frac{5}{6}$th of $(\text{KE})_i$ is lost in this process.

$\Rightarrow \quad (\text{KE})_f = \frac{1}{6}(\text{KE})_i$

$\Rightarrow \quad \frac{(\text{KE})_f}{(\text{KE})_i} = \frac{1}{6}$ … (v)

Comparing Eq. (iv) and Eq. (v), we get

$$\frac{m}{2m + M} = \frac{1}{6} \Rightarrow 6m = 2m + M$$

$$4m = M \Rightarrow \frac{M}{m} = 4$$

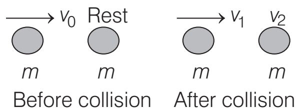
11. From conservation of linear momentum,

$$mv_0 + 0 = mv_1 + mv_2$$

$$v_0 = v_1 + v_2 \quad \text{…(i)}$$

Further, $\quad K_f = \frac{3}{2}K$,

$$\therefore \quad \frac{3}{2}\left[\frac{1}{2}mv_0^2\right] = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$$



$$\frac{3}{2}v_0^2 = v_1^2 + v_2^2 \quad \text{…(ii)}$$

Solving Eqs. (i) and (ii),

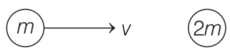
$$v_1 = \frac{v_0}{2}(1 + \sqrt{2})$$

$$v_2 = \frac{v_0}{2}(1 - \sqrt{2})$$

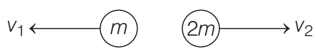
$$v_{\text{rel}} = v_1 - v_2$$

$$\frac{v_0}{2}[1 + \sqrt{2} - 1 + \sqrt{2}] = \frac{v_0}{2} \times 2\sqrt{2} = \sqrt{2}v_0$$

12. **Case I** Just before Collision,



Just after collision



From momentum conservation,

$$2v_2 - v_1 = v$$

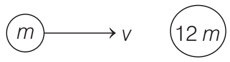
From the definition of e (= 1 for elastic collision)

$$v_2 + v_1 = v \quad \Rightarrow \quad 3v_2 = 2v$$

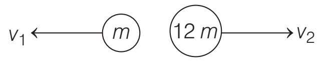
$$v_2 = \frac{2v}{3} \quad \Rightarrow \quad v_1 = \frac{v}{3}$$

$$p_d = \frac{\frac{1}{2}mv^2 - \frac{1}{2}mv_1^2}{\frac{1}{2}mv^2} = \frac{1 - \frac{1}{9}}{1} = \frac{8}{9} = 0.89$$

Case II Just before Collision



Just after Collision,



From momentum conservation,

$$12v_2 - v_1 = v$$

From the definition of e(= 1 for elastic collision),

$$v_2 + v_1 = v$$

$$13v_2 = 2v$$

$$v_2 = \frac{2v}{13} \quad \Rightarrow \quad v_1 = v - \frac{2v}{13} = \frac{11v}{13}$$

$\Rightarrow$

$$p_c = \frac{\frac{1}{2}mv^2 - \frac{1}{2}mv_1^2}{\frac{1}{2}mv^2} = \frac{1 - \frac{121}{169}}{1} = \frac{48}{169} = 0.28$$

13. In all type of collisions, momentum of the system always remains constant. In perfectly inelastic collision, particles stick together and move with a common velocity.

Let this velocity is $\mathbf{v}_c$. Then,

initial momentum of system = final momentum of system

or $\quad m(2v)\hat{\mathbf{i}} + 2m(v)\hat{\mathbf{j}} = (m + 2m)\mathbf{v}_c$

$\therefore \quad \mathbf{v}_c = \frac{2}{3}(v\hat{\mathbf{i}} + v\hat{\mathbf{j}})$

$$|\mathbf{v}_c| \text{ or } v_c \text{ or speed} = \sqrt{\left(\frac{2}{3}v\right)^2 + \left(\frac{2}{3}v\right)^2} = \frac{2\sqrt{2}}{3}v$$

Initial kinetic energy

$$K_i = \frac{1}{2}(m)(2v)^2 + \frac{1}{2}(2m)(v)^2 = 3mv^2$$

Final kinetic energy

$$K_f = \frac{1}{2}(3m)\left(\frac{2\sqrt{2}}{3}v\right)^2 = \frac{4}{3}mv^2$$

$$\text{Fractional loss} = \left(\frac{K_i - K_f}{K_i}\right) \times 100$$

$$= \left[\frac{(3mv^2) - \left(\frac{4}{3}mv^2\right)}{(3mv^2)}\right] \times 100 = 56\%$$

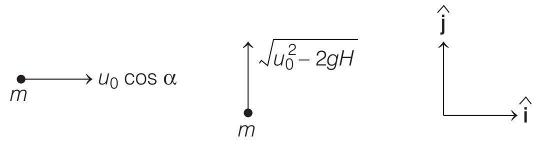
14. Maximum energy loss

$$= \frac{p^2}{2m} - \frac{p^2}{2(m + M)} \qquad \left(\because \text{KE} = \frac{p^2}{2m}\right)$$

Before collision the mass m and after collision the mass is $m + M$

$$= \frac{p^2}{2m}\left[\frac{M}{(m + M)}\right] = \frac{1}{2}mv^2\left\{\frac{M}{m + M}\right\} \qquad \left(f = \frac{M}{m + M}\right)$$

15. From momentum conservation equation, we have,



$$\mathbf{p}_i = \mathbf{p}_f$$

$$\therefore \quad m(u_0 \cos\alpha)\hat{\mathbf{i}} + m(\sqrt{u_0^2 - 2gH})\hat{\mathbf{j}} = (2m)\mathbf{v} \quad \text{...(i)}$$

$$H = \frac{u_0^2 \sin^2\alpha}{2g} \quad \text{...(ii)}$$

From Eqs. (i) and (ii)

$$\mathbf{v} = \frac{u_0 \cos\alpha}{2}\hat{\mathbf{i}} + \frac{u_0 \cos\alpha}{2}\hat{\mathbf{j}}$$

Since both components of $\mathbf{v}$ are equal. Therefore, it is making 45° with horizontal.

16. $\quad R = u\sqrt{\frac{2h}{g}} \Rightarrow 20 = v_1\sqrt{\frac{2 \times 5}{10}}$

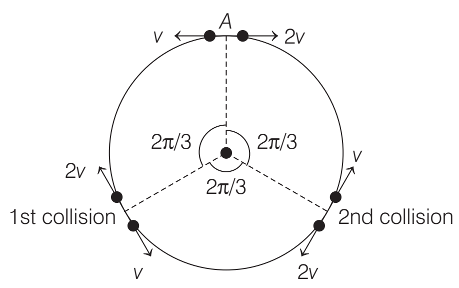
and $\quad 100 = v_2\sqrt{\frac{2 \times 5}{10}}$

$\Rightarrow \quad v_1 = 20\,\text{m/s},\ v_2 = 100\,\text{m/s}.$

Applying momentum conservation just before and just after the collision

$(0.01)(v) = (0.2)(20) + (0.01)(100)\ v = 500\,\text{m/s}$

17. At first collision one particle having speed $2v$ will rotate 240° $\left(\text{or } \frac{4\pi}{3}\right)$ while other particle having speed v will rotate 120° $\left(\text{or } \frac{2\pi}{3}\right)$. At first collision, they will exchange their velocities. Now, as shown in figure, after two collisions they will again reach at point A.



18. $|(m_1 \mathbf{v}_1' + m_2 \mathbf{v}_2') - (m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2)|$

$= |\text{change in momentum of the two particles}|$

$= |\text{External force on the system}| \times \text{time interval}$

$= (m_1 + m_2)\, g\, (2t_0) = 2\,(m_1 + m_2)\, g t_0$

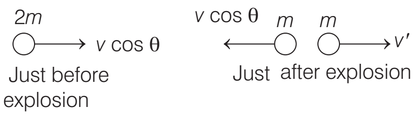
19. Before explosion, particle was moving along x-axis *i.e.,* it has no y-component of velocity. Therefore, the centre of mass will not move in y-direction or we can say $y_{CM} = 0$.

Now, $$y_{CM} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2}$$

Therefore, $$0 = \frac{(m/4)(+15) + (3m/4)(y)}{(m/4 + 3m/4)}$$

or $y = -5$ cm

20. Let v' be the velocity of second fragment. From conservation of linear momentum,



$$2m\,(v\cos\theta) = mv' - m\,(v\cos\theta)$$

$\therefore$ $$v' = 3v\cos\theta$$

21. In an inelastic collision, only momentum of the system may remain conserved. Some energy can be lost in the form of heat, sound etc.

22. $\because$ $H = \dfrac{u^2 \sin^2 45°}{2g} = 120 \text{ m} \Rightarrow \dfrac{u^2}{4g} = 120 \text{ m}$

If speed is v after the first collision, then speed should remain $\dfrac{1}{\sqrt{2}}$ times, as kinetic energy has reduced to half.

$\Rightarrow$ $$v = \frac{u}{\sqrt{2}}$$

$\therefore$ $$h_{max} = \frac{v^2 \sin^2 30°}{2g} = \frac{(u/\sqrt{2})^2 \sin^2 30°}{2g}$$

$$= \left(\frac{u^2/4g}{4}\right) = \frac{120}{4} = 30$$

23. In case of elastic collision, coefficient of restitution $e = 1$.

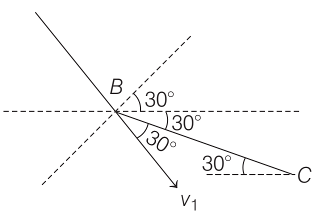
Magnitude of relative velocity of approach

= Magnitude of relative velocity of separation.

But relative speed of approach

$\neq$ Relative speed of separation

24. Between A and B, height fallen by block



$$h_1 = \sqrt{3} \tan 60° = 3 \text{ m.}$$

$\therefore$ Speed of block just before striking the second incline,

$$v_1 = \sqrt{2gh_1} = \sqrt{2 \times 10 \times 3} = \sqrt{60} \text{ ms}^{-1}$$

In perfectly inelastic collision, component of v_1 perpendicular to BC will become zero, while component of v_1 parallel to BC will remain unchanged.

$\therefore$ Speed of block B immediately after it strikes the second incline is,

$$v_2 = \text{component of } v_1 \text{ along } BC$$

$$= v_1 \cos 30° = (\sqrt{60})\left(\frac{\sqrt{3}}{2}\right)$$

$$= \sqrt{45} \text{ ms}^{-1}$$

25. Height fallen by the block from B to C

$$h_2 = 3\sqrt{3} \tan 30° = 3 \text{ m}$$

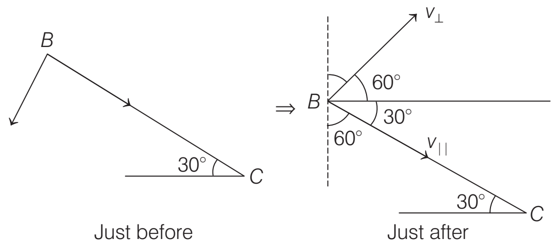
Let v_3 be the speed of block, at point C, just before it leaves the second incline, then :

$$v_3 = \sqrt{v_2^2 + 2gh_2}$$

$$= \sqrt{45 + 2 \times 10 \times 3}$$

$$= \sqrt{105} \text{ ms}^{-1}$$

26. In elastic collision, compound of v_1 parallel to BC will remain unchanged, while component perpendicular to BC will remain unchanged in magnitude but its direction will be reversed.



$$v_{11} = v_1 \cos 30°$$

$$= (\sqrt{60})\left(\frac{\sqrt{3}}{2}\right) = \sqrt{45} \text{ ms}^{-1}$$

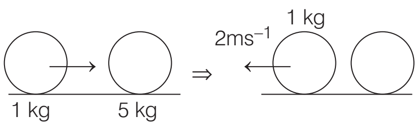
$$v_{\perp} = v_1 \sin 30° = (\sqrt{60})\left(\frac{1}{2}\right) = \sqrt{15} \text{ ms}^{-1}$$

Now vertical component of velocity of block

$$v = v_{\perp} \cos 30° - v_{||} \cos 60°$$

$$= (\sqrt{15})\left(\frac{\sqrt{3}}{2}\right) - (\sqrt{45})\left(\frac{1}{2}\right) = 0$$

27.



$$v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_1 + \left(\frac{2m_2}{m_1 + m_2}\right) v_2$$

$$-2 = \left(\frac{1-5}{1+5}\right) v_1 + 0 \qquad \text{(as } v_2 = 0\text{)}$$

$$\therefore \quad v_1 = 3 \text{ ms}^{-1}$$

$$v_2{}' = \left(\frac{m_2 - m_1}{m_1 + m_2}\right) v_2 + \left(\frac{2m_1}{m_1 + m_2}\right) v_1 = 1 \text{ ms}^{-1}$$

$$P_{CM} = P_i = (1)(3) = 3 \text{ kg - m/s}$$

$$P_5{}' = (5)(1) = 5 \text{ kg - m/s}$$

$$K_{CM} = \frac{p_{CM}^2}{2M_{CM}} = \frac{9}{2 \times 6} = 0.75 \text{ J}$$

$$K_{total} = \frac{1}{2} \times 1 \times (3)^2 = 4.5 \text{J}$$

28. Initial momentum of the system $\mathbf{p}_1 + \mathbf{p}_2 = 0$

$\therefore$ Final momentum $\mathbf{p}_1' + \mathbf{p}_2'$ should also be zero.

Option (b) is allowed because if we put $c_1 = -c_2 \neq 0$, $\mathbf{p}_1' + \mathbf{p}_2'$ will be zero. Similary, we can check other options.

29. Impulse = $\int F dt$ = area under F-t graph

$\therefore$ Total impulse from $t = 4\,\mu$s to $t = 16\,\mu$s

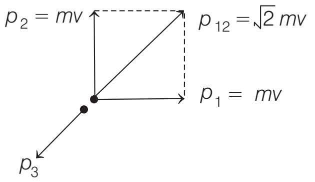
= Area $EBCD$

= Area of trapezium $EBCF$ + Area of triangle FCD

$$= \frac{1}{2}(200 + 800)\, 2 \times 10^{-6} + \frac{1}{2} \times 800 \times 10 \times 10^{-6}$$

$$= 5 \times 10^{-3} \text{ N-s}$$

30. From conservation of linear momentum p_3 should be $\sqrt{2}\, mv$ in a direction opposite to $\mathbf{p}_{12}$ (resultant of $\mathbf{p}_1$ and $\mathbf{p}_2$). Let v' be the speed of third fragment, then



$$(2m)\, v' = \sqrt{2} mv$$

$$\therefore \quad v' = \frac{v}{\sqrt{2}}$$

$\therefore$ Total energy released is,

$$E = 2\left(\frac{1}{2} mv^2\right) + \frac{1}{2}(2m) v'^2$$

$$= mv^2 + m\left(\frac{v}{\sqrt{2}}\right)^2 = \frac{3}{2} mv^2$$

31. After elastic collision,

$$v'_A = \left(\frac{m - 2m}{m + 2m}\right)(9) + \frac{2(2m)}{m + 2m}(0) = -3 \text{ms}^{-1}$$

Now, from conservation of linear momentum after all collisions are complete,

$$m(+9 \text{ ms}^{-1}) = m(-3 \text{ ms}^{-1}) + 3m(v_C) \text{ or } v_C = 4 \text{ ms}^{-1}$$

32. Let v_1 = velocity of block 2 kg just before collision,

v_2 = velocity of block 2 kg just after collision,

and v_3 = velocity of block M just after collision.

Applying work-energy theorem

(change in kinetic energy = work done by all the forces) at different stages as shown in figure.

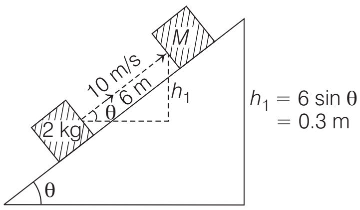


Figure 1.

$$\Delta KE = W_{friction} + W_{gravity}$$

$$\left[\frac{1}{2} m \{v_1^2 - (10)^2\}\right] = -6\mu\, mg \cos\theta - mgh_1$$

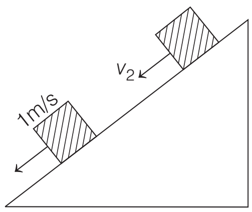
or $\quad v_1^2 - 100 = -2[6\mu g \cos\theta + gh_1]$

$$\cos\theta = \sqrt{1 - \sin^2\theta} = \sqrt{1 - (0.05)^2} \approx 0.99$$

$$\therefore v_1^2 = 100 - 2[(6)(0.25)(10)(0.99) + (10)(0.3)]$$

$$\Rightarrow \quad v_1 \approx 8 \text{m/s}$$

Figure 2. $\quad \Delta KE = W_{friction} + W_{gravity}$



$$\frac{1}{2} m[(1)^2 - (v_2^2)] = -6\mu\, mg \cos\theta + mgh_1$$

or $\quad 1 - v_2^2 = 2[-6\mu\, g \cos\theta + gh_1]$

$$= 2[-(6)(0.25)(10)(0.99) + (10)(0.3)]$$

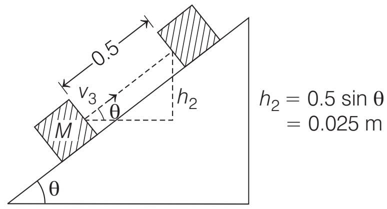
$$= -23.7$$

$\therefore \quad v_2^2 = 24.7$

or $\quad v_2 \approx 5$ m/s

Figure 3. $\Delta KE = W_{friction} + W_{gravity}$

$$\frac{1}{2} M[0 - v_3^2] = -(0.5)(\mu)(M) g \cos\theta - Mgh_2$$



or $\quad -v_3^2 = -\mu\, g\cos\theta - 2gh_2$

or $\quad v_3^2 = (0.25)(10)(0.99) + 2(10)(0.025)$

or $\quad v_3^2 = 2.975 \quad \Rightarrow \quad \therefore \quad v_3 \approx 1.72\,\text{m/s}$

Now,

(a) Coefficient of restitution

$$= \frac{\text{Relative velocity of separation}}{\text{Relative velocity of approach}}$$

$$= \frac{v_2 + v_3}{v_1} = \frac{5 + 1.72}{8} = \frac{6.72}{8} \quad \text{or} \quad e \approx 0.84$$

(b) Applying conservation of linear momentum before and after collision

$$2v_1 = Mv_3 - 2v_2$$

$$\therefore \quad M = \frac{2(v_1 + v_2)}{v_3}$$

$$= \frac{2(8+5)}{1.72} = \frac{26}{1.72}$$

$$M \approx 15.12\,\text{kg}$$

33. If v_1 and v_2 are the velocities of object of mass m and block of mass $4m$, just after collision then by conservation of momentum,

$$mv = mv_1 + 4mv_2, \text{ i.e. } v = v_1 + 4v_2 \qquad \ldots\text{(i)}$$

Further, as collision is elastic

$$\frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}4mv_2^2, \text{ i.e. } v^2 = v_1^2 + 4v_2^2 \qquad \ldots\text{(ii)}$$

Solving, these two equations we get either

$$v_2 = 0 \quad \text{or} \quad v_2 = \frac{2}{5}v$$

Therefore, $\quad v_2 = \frac{2}{5}v$

Substituting in Eq. (i) $v_1 = \frac{3}{5}v$

when $v_2 = 0$, $v_1 = v_2$, but it is physically unacceptable.

(a) Now, after collision the block B will start moving with velocity v_2 to the right. Since, there is no friction between blocks A and B, the upper block A will stay at its position and will topple if B moves a distance s such that

$$s > 2d \qquad \ldots\text{(iii)}$$

However, the motion of B is retarded by frictional force $f = \mu\,(4m + 2m)g$ between table and its lower surface. So, the distance moved by B till it stops

$$0 = v_2^2 - 2\left(\frac{6\mu mg}{4m}\right)s, \text{ i.e. } s = \frac{v_2^2}{3\mu g}$$

Substituting this value of s in Eq. (iii), we find that for toppling of A

or $\quad v_2^2 > 6\,\mu g d$

$$\frac{2}{5}v > \sqrt{6\mu g d} \quad [\text{as } v_2 = 2v/5]$$

i.e. $\quad v > \frac{5}{2}\sqrt{6\,\mu g d}$

or $\quad v_{\min} = v_0 = \frac{5}{2}\sqrt{6\,\mu g d}$

(b) If $v = 2v_0 = 5\sqrt{6\mu g d}$, the object will rebound with speed

$v_1 = \frac{3}{5}v = 3\sqrt{6\,\mu g d}$

and as time taken by it to fall down

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2d}{g}} \qquad [\text{as } h = d]$$

The horizontal distance moved by it to the left of P in this time $x = v_1 t = 6d\sqrt{3\mu}$

NOTE

- Toppling will take place if line of action of weight does not pass through the base area in contact.
- v_1 and v_2 can be obtained by using the equations of head on elastic collision

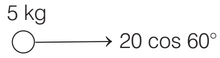
$$v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_2 + \left(\frac{2m_2}{m_1 + m_2}\right)v_2$$

and $$v_2' = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)v_2 + \left(\frac{2m_1}{m_1 + m_2}\right)v_1$$

34. Let v_1 and v_2 be the velocities after explosion in the directions shown in figure. From conservation of linear momentum, we have

$$5\,(20\cos 60^\circ) = 4v_1 - 1\times v_2$$

or $\quad 4v_1 - v_2 = 50 \qquad \ldots\text{(i)}$



Just before explosion

Just after explosion

Further, it is given that, kinetic energy after explosion becomes two times.

Therefore,

$$\frac{1}{2}\times 4\times v_1^2 + \frac{1}{2}\times 1\times v_2^2 = 2\left[\frac{1}{2}\times 5\times(20\cos 60^\circ)^2\right]$$

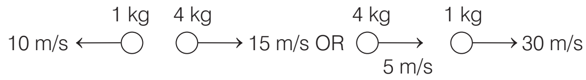
or $\quad 4v_1^2 + v_2^2 = 1000 \qquad \ldots\text{(ii)}$

Solving Eqs. (i) and (ii), we have

$$v_1 = 15\,\text{m/s}, \quad v_2 = 10\,\text{m/s}$$

or $\quad v_1 = 5\,\text{m/s} \quad \text{and} \quad v_2 = -30\,\text{m/s}.$

In both the cases relative velocity of separation in horizontal direction is 25 m/s.



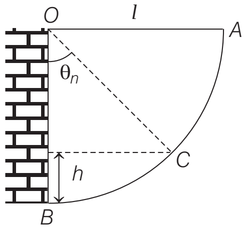
$\therefore\ x = 25t$ = distance between them when they strike the ground.

Here, $t = \frac{T}{2}$ (T = time of flight of projectile)

$$= \frac{u \sin\theta}{g} = \frac{20 \sin 60^\circ}{9.8} = 1.77\text{s}$$

$\therefore$ $x = 25 \times 1.77 \text{ m} = 44.25\text{m}$

35. As shown in figure initially when the bob is at A, its potential energy is mgl. When the bob is released and it strikes the wall at B, its potential energy mgl is converted into its kinetic energy. If v be the velocity with which the bob strikes the wall, then



$$mgl = \frac{1}{2}mv^2 \text{ or } v = \sqrt{(2gl)} \quad \ldots\text{(i)}$$

Speed of the bob after rebounding (first time)

$$v_1 = e\sqrt{(2gl)} \quad \ldots\text{(ii)}$$

The speed after second rebound is $v_2 = e^2\sqrt{(2gl)}$

In general after n rebounds, the speed of the bob is

$$v_n = e^n\sqrt{(2gl)} \quad \ldots\text{(iii)}$$

Let the bob rises to a height h after n rebounds. Applying the law of conservation of energy, we have

$$\frac{1}{2}mv_n^2 = mgh$$

$$\therefore \quad h = \frac{v_n^2}{2g} = \frac{e^{2n} \cdot 2gl}{2g} = e^{2n} \cdot l = \left(\frac{2}{\sqrt{5}}\right)^{2n} \cdot l = \left(\frac{4}{5}\right)^n l \quad \ldots\text{(iv)}$$

If θ_n be the angle after n collisions, then

$$h = l - l\cos\theta_n = l(1 - \cos\theta_n)$$
...(v)

From Eqs. (iv) and (v), we have

$$\left(\frac{4}{5}\right)^n l = l(1 - \cos\theta_n) \text{ or } \left(\frac{4}{5}\right)^n = (1 - \cos\theta_n)$$

For θ_n to be less than 60°, i.e. $\cos\theta_n$ is greater than 1/2, i.e. $(1 - \cos\theta_n)$ is less than 1/2, we have

$$\left(\frac{4}{5}\right)^n < \left(\frac{1}{2}\right)$$

This condition is satisfied for $n = 4$.

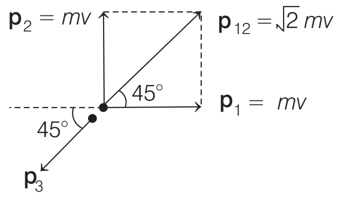
$\therefore$ Required number of collisions = 4.

36. Before collision, net momentum of the system was zero. No external force is acting on the system. Hence, momentum after collision should also be zero. A has come to rest. Therefore, B and C should have equal and opposite momenta or velocity of C should be v in opposite direction of velocity of B.

37. From conservation of linear momentum $\mathbf{p}_3$ should be equal and opposite to $\mathbf{p}_{12}$ (resultant of $\mathbf{p}_1$ and $\mathbf{p}_2$). So, let v' be the velocity of third fragment, then

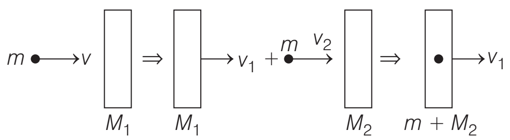
$$(3m)\,v' = \sqrt{2}mv \quad \Rightarrow \quad \therefore \quad v' = \frac{\sqrt{2}}{3}v$$

Here, $v = 30$ m/s (given) $\Rightarrow v' = \frac{\sqrt{2}}{3} \times 30 = 10\sqrt{2}$ m/s



This velocity is at 45° as shown in figure.

38. Applying conservation of linear momentum twice. We have



$$mv = M_1v_1 + mv_2 \quad \ldots\text{(i)}$$

$$mv_2 = (M_2 + m)\,v_1 \quad \ldots\text{(ii)}$$

Solving Eqs. (i) and (ii), we get

$$\frac{v_2}{v} = \frac{M_2 + m}{M_1 + M_2 + m}$$

Substituting the values of $m : M_1$ and M_2 we get percentage of velocity retained by bullet

$$\frac{v_2}{v} \times 100 = \left(\frac{2.98 + 0.02}{1 + 2.98 + 0.02}\right) \times 100 = 75\%$$

$\therefore$ % loss = 25%

39. (a) From conservation of linear momentum, momentum of composite body

$$\mathbf{p} = (\mathbf{p}_i)_1 + (\mathbf{p}_i)_2 = (mv)\,\hat{\mathbf{i}} + (MV)\,\hat{\mathbf{j}}$$

$$\therefore \quad |\mathbf{p}| = \sqrt{(mv)^2 + (MV)^2}$$

Let it makes an angle α with positive X-axis, then

$$\alpha = \tan^{-1}\left(\frac{p_y}{p_x}\right) = \tan^{-1}\left(\frac{MV}{mv}\right)$$

(b) Fraction of initial kinetic energy transformed into heat during collision

$$= \frac{K_f - K_i}{K_i} = \frac{K_f}{K_i} - 1 = \frac{p^2/2(M+m)}{\frac{1}{2}mv^2 + \frac{1}{2}MV^2} - 1$$

$$= \frac{(mv)^2 + (MV)^2}{(M+m)(mv^2 + MV^2)} - 1$$

$$= \frac{Mm\,(v^2 + V^2)}{(M+m)(mv^2 + MV^2)}$$

Topic 4 Miscellaneous Problems

1 Initial compression of the spring,

$$mg = k\left(\frac{x_0}{100}\right) \quad (x_0 \text{ in cm})$$

$$\Rightarrow \quad x_0 = \frac{3 \times 10 \times 100}{1.25 \times 10^6} = \frac{3}{1250}$$

Which is very small and can be neglected.

Applying conservation of momentum before and after the collision i.e., momentum before collision = momentum after collision.

$m \times \sqrt{2gh} = (m + M)\, v$ ($\because$ velocity of the block just before the collision is

$$v^2 - 0^2 = 2gh$$

or $$v = \sqrt{2gh})$$

After, substituting the given values, we get

$$1 \times \sqrt{2 \times 10 \times 100} = 4v \text{ or } 4v = 20\sqrt{5}$$

so $$v = 5\sqrt{5} \text{ m / s}$$

Let this be the maximum velocity, then for the given system, using

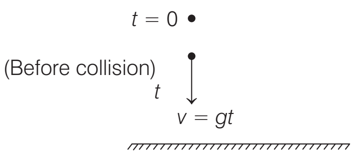
$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

$$\therefore \quad \frac{1}{2} \times 4 \times 125 = \frac{1}{2} \times 1.25 \times 10^6 \times \left(\frac{x}{100}\right)^2$$

$$\Rightarrow \quad 4 = 10^4 \times \frac{x^2}{10^4} \text{ or } x = 2 \text{ cm}$$

$\therefore$ No option given is correct.

2.



$$K = \frac{1}{2} mg^2t^2$$

$K \propto t^2$ Therefore, K-t graph is parabola.

During collision, retarding force is just like the spring force ($F \propto x$), therefore kinetic energy first decreases to elastic potential energy and then increases.

3. Final momentum of object $= \dfrac{\text{Power} \times \text{time}}{\text{Speed of light}}$

$$= \frac{30 \times 10^{-3} \times 100 \times 10^{-9}}{3 \times 10^8}$$

$$= 1.0 \times 10^{-17} \text{ kg-m/s}$$

4. We can apply $\mathbf{L} = m(\mathbf{r} \times \mathbf{v})$ for different parts.

For example :

In part (a), coordinates of A are $\left(\dfrac{R}{\sqrt{2}}, \dfrac{R}{\sqrt{2}}\right)$

Therefore, $$\mathbf{r} = \frac{R}{\sqrt{2}}\hat{\mathbf{i}} + \frac{R}{\sqrt{2}}\hat{\mathbf{j}} \text{ and } \mathbf{v} = v\hat{\mathbf{i}}$$

So, substituting in $\mathbf{L} = m(\mathbf{r} \times \mathbf{v})$ we get,

$$\mathbf{L} = -\frac{mvR}{\sqrt{2}}\hat{\mathbf{k}}$$

Hence, option (a) is correct. Similarly, we can check other options also.

5. Linear impulse, $J = mv_0$

$$\therefore \quad v_0 = \frac{J}{m} = 2.5 \text{ m/s}$$

$$\therefore \quad v = v_0 e^{-t/\tau}$$

$$\frac{dx}{dt} = v_0 e^{-t/\tau} \qquad \int_0^x dx = v_0 \int_0^\tau e^{-t/\tau} dt$$

$$x = v_0 \left[\frac{e^{-t/\tau}}{-\frac{1}{\tau}}\right]_0^\tau$$

$$x = 2.5\,(-4)\,(e^{-1} - e^0)$$

$$= 2.5(-4)\,(0.37 - 1)$$

$$x = 6.30 \text{ m}$$

6. $F = \dfrac{\Delta p}{\Delta t} = n \times \left(a \times \dfrac{b}{2}\right) \times (2mv)$

Equating the torque about hinge side, we have

$$n \times \left(a \times \frac{b}{2}\right) \times (2mv) \times \frac{3b}{4} = Mg \times \frac{b}{2}$$

Substituting the given values, we get

$$v = 10 \text{ m/s}$$

7. (a) $$x_1 = v_0\, t - A\,(1 - \cos \omega t)$$

$$x_{CM} = \frac{m_1\, x_1 + m_2\, x_2}{m_1 + m_2} = v_0\, t$$

$$\therefore \quad x_2 = v_0\, t + \frac{m_1}{m_2} A\,(1 - \cos \omega t)$$

(b) $$a_1 = \frac{d^2x_1}{dt^2} = -\omega^2\, A \cos \omega t$$

The separation $x_2 - x_1$ between the two blocks will be equal to l_0 when $a_1 = 0$

or $\cos \omega t = 0$

$$x_2 - x_1 = \frac{m_1}{m_2} A\,(1 - \cos \omega t) + A\,(1 - \cos \omega t)$$

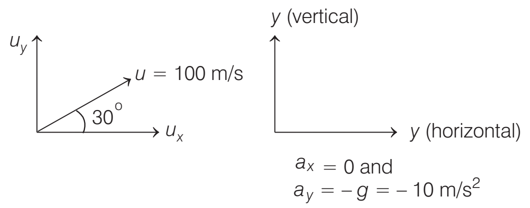
or $$l_0 = \left(\frac{m_1}{m_2} + 1\right) A \;(\because \cos \omega t = 0)$$

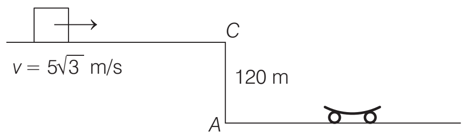
Thus, the relation between l_0 and A is,

$$l_0 = \left(\frac{m_1}{m_2} + 1\right) A$$

8. (a) 100 m/s velocity of the ball is relative to ground.

[Unless and until it is mentioned in the question, the velocity is always relative to ground]





Horizontal component of velocity of cannon ball,

$$u_x = u\cos 30°$$

or $$u_x = (100) \times \frac{\sqrt{3}}{2} = 50\sqrt{3}\,\text{m/s}$$

and vertical component of its velocity,

$$u_y = u\sin 30°$$

$$u_y = 100 \times \frac{1}{2} = 50\,\text{m/s}$$

Vertical displacement of the ball when it strikes the carriage is – 120 m or

$$s_y = u_y\, t + \frac{1}{2}a_y\, t^2 \Rightarrow -120 = (50\,t) + \left(\frac{1}{2}\right)(-10)t^2$$

$$\Rightarrow t^2 - 10t - 24 = 0 \Rightarrow t = 12\text{s} \quad \text{or} \quad -2\text{s}$$

Ignoring the negative time, we have

$$t_0 = 12\text{s}$$

(b) When it strikes the carriage, its horizontal component of velocity is still $50\sqrt{3}$ m/s. It strikes to the carriage. Let v_2 be the velocity of (carriage + ball) system after collision. Then, applying conservation of linear momentum in horizontal direction

(mass of ball) (horizontal component of its velocity before collision) = (mass of ball + carriage) (v_2)

$$\therefore \quad (1\,\text{kg})(50\sqrt{3}\text{ m/s}) = (10\,\text{kg})(v_2)$$

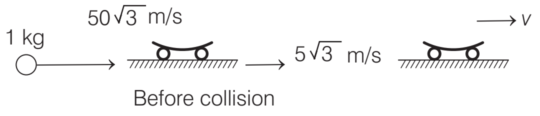
$$\therefore \quad v_2 = 5\sqrt{3}\text{ m/s}$$

The second ball is fired when the first ball strikes the carriage i.e. after 12 s. In these 12 s, the car will move forward a distance of $12v_1$ or $60\sqrt{3}$ m.

The second ball also takes 12 s to travel a vertical displacement of – 120 m. This ball will strike the carriage only when the carriage also covers the same distance of $60\sqrt{3}$ m in these 12 s. This is possible only when resistive forces are zero because velocity of car (v_1) = velocity of carriage after first collision. $(v_2) = 5\sqrt{3}$ m/s.

Hence, at the time of second collision

Horizontal component of velocity of ball = $50\sqrt{3}$ m/s and horizontal velocity of carriage + first ball = $5\sqrt{3}$ m/s. Let v be the desired velocity of carriage after second collision. Then, conservation of linear momentum in horizontal direction gives



$$11v = (1)(50\sqrt{3}) + (10)(5\sqrt{3})$$

$$= 100\sqrt{3}$$

$$\therefore \quad v = \frac{100\sqrt{3}}{11}\text{ m/s}$$

or $v = 15.75$ m/s

In this particular problem, values are so adjusted that even if we take the velocity of ball with respect to car, we get the same results of both the parts, although the method will be wrong.

9. Given, $m_0 = 10^{-2}$ kg, $A = 10^{-4}$ m², $v_0 = 10^3$ m/s

and $\rho_{dust} = \rho = 10^{-3}$ kg/m³.

$$m = m_0 + \text{mass of dust collected so far}$$

$$= m_0 + Ax\rho_{dust}$$

or $$m = m_0 + Ax\rho$$

The linear momentum at $t = 0$ is

$$p_0 = m_0 v_0$$

and momentum at $t = t$ is

$$p_t = mv = (m_0 + Ax\rho)\, v$$

From law of conservation of momentum

$$p_0 = p_t$$

$$\therefore \quad m_0 v_0 = (m_0 + Ax\rho)\, v$$

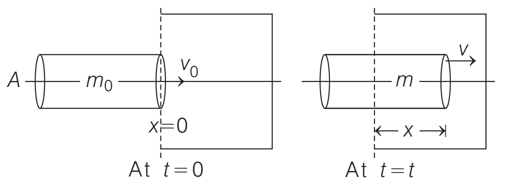
$$\therefore \quad m_0 v_0 = (m_0 + Ax\rho)\frac{dx}{dt}$$

or $$(m_0 + A\rho x)\, dx = m_0 v_0\, dt$$

or $$\int_0^x (m_0 + A\rho x)\, dx = \int_0^{150} m_0 v_0\, dt$$

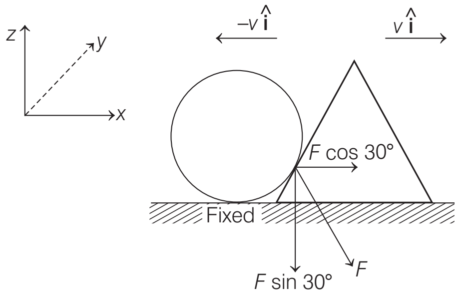
$$\Rightarrow \quad \left(m_0 x + A\rho\frac{x^2}{2}\right)_0^x = (m_0 v_0 t)_0^{150}$$

Hence, $$m_0 x + A\rho\frac{x^2}{2} = 150\, m_0 v_0$$



Solving this quadratic equation and substituting the values of m_0 A, ρ and v_0, we get positive value of x as 10^5m. Therefore, $x = 10^5$m.

10. (a) (i) Since, the collision is elastic, the wedge will return with velocity $v\,\hat{\mathbf{i}}$.



Now, linear impulse in x-direction
= change in momentum in x-direction.

$\therefore\ (F\cos 30^\circ)\,\Delta t = mv - (-mv) = 2mv$

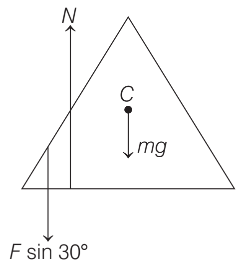
$$\therefore \quad F = \frac{2mv}{\Delta t \cos 30^\circ} = \frac{4mv}{\sqrt{3}\,\Delta t} \Rightarrow F = \frac{4mv}{\sqrt{3}\,\Delta t}$$

$$\therefore \quad \mathbf{F} = (F\cos 30^\circ)\,\hat{\mathbf{i}} - (F\sin 30^\circ)\,\hat{\mathbf{k}}$$

$$\text{or} \quad \mathbf{F} = \left(\frac{2\,mv}{\Delta t}\right)\hat{\mathbf{i}} - \left(\frac{2\,mv}{\sqrt{3}\Delta t}\right)\hat{\mathbf{k}}$$

(ii) Taking the equilibrium of wedge in vertical z-direction during collision.

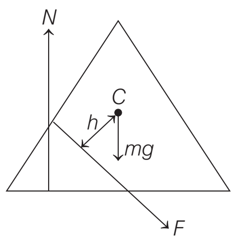
$$N = mg + F\sin 30^\circ \Rightarrow N = mg + \frac{2\,mv}{\sqrt{3}\,\Delta t}$$



or in vector form

$$\mathbf{N} = \left(mg + \frac{2mv}{\sqrt{3}\Delta t}\right)\hat{\mathbf{k}}$$

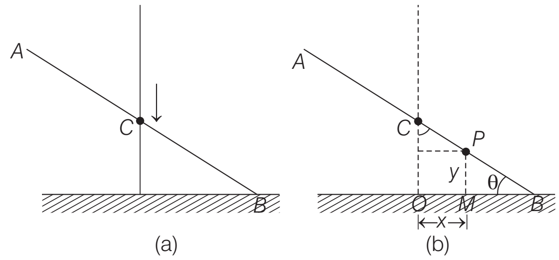
(b) For rotational equilibrium of wedge [about (CM)] anticlockwise torque of F = clockwise torque due to N



$\therefore$ Magnitude of torque of N about CM = magnitude of torque of F about CM

$$= F\,.h\,|\tau_N| = \left(\frac{4\,mv}{\sqrt{3}\,\Delta t}\right)h$$

11. (a) Since, only two forces are acting on the rod, its weight Mg (vertically downwards) and a normal reaction N at point of contact B (vertically upwards).



No horizontal force is acting on the rod (surface is smooth).

Therefore, CM will fall vertically downwards towards negative Y-axis i.e. the path of CM is a straight line.

(b) Refer figure (b). We have to find the trajectory of a point $P\,(x, y)$ at a distance r from end B.

$$CB = L/2$$

$$\therefore \quad OB = (L/2)\cos\theta;$$

$$MB = r\cos\theta$$

$$\therefore \quad x = OB - MB$$

$$= \cos\theta\,\{(L/2 - r)\}$$

$$\text{or} \quad \cos\theta = \frac{x}{\{(L/2) - r\}} \quad \text{...(i)}$$

$$\text{Similarly,} \quad y = r\sin\theta$$

$$\text{or} \quad \sin\theta = \frac{y}{r} \quad \text{...(ii)}$$

Squaring and adding Eqs. (i) and (ii), we get

$$\sin^2\theta + \cos^2\theta = \frac{x^2}{\{(L/2) - r\}^2} + \frac{y^2}{r^2}$$

$$\frac{x^2}{\{(L/2) - r\}^2} + \frac{y^2}{r^2} = 1 \quad \text{...(iii)}$$

This is an equation of an ellipse. Hence, path of point P is an ellipse whose equation is given by (iii).