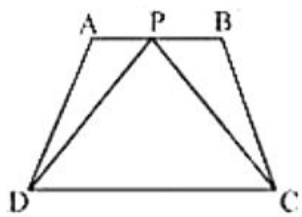
area of parallelograms and triangles

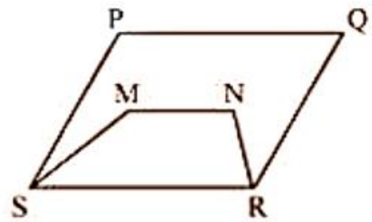
Chapter 9

EXERCISE 9.1

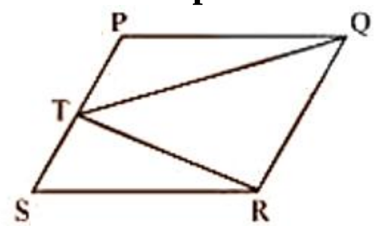
1. Which of the following figures lie on the same base and between the same parallels. In such a case, write the common base and the two parallels.



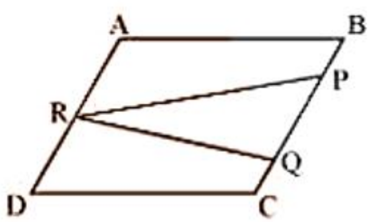
(i)



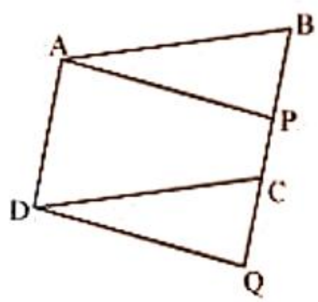
(ii)



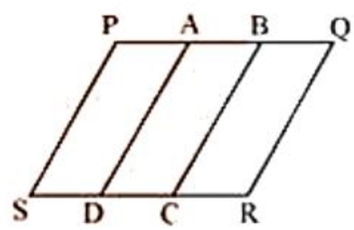
(iii)



(iv)



(v)

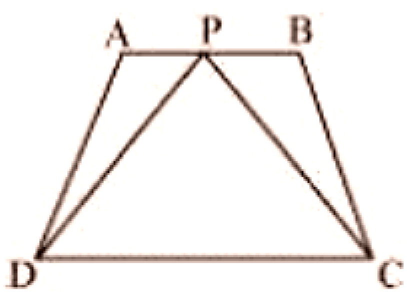


(vi)

Ans: Given: Some figures are given.

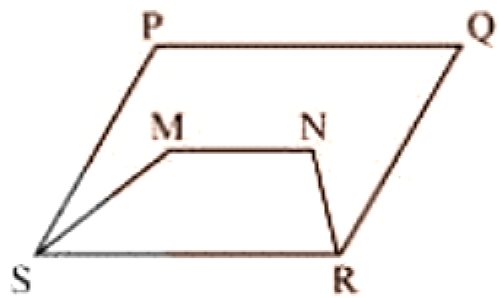
To find: The figures that lie on the same base and between the same parallels

(i)



Yes. The trapezium ABCD and triangle PCD both have the same base CD and are located between the same parallel lines AB and CD.

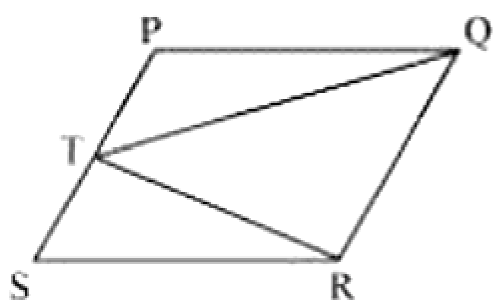
(ii)



No. The parallelogram PQRS and the trapezium MNRS have a same basis RS, as can be shown.

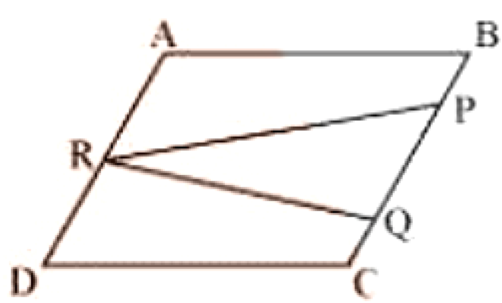
However, the vertices of the parallelogram P and Q and the trapezium M and N do not lie on the same line (i.e., opposed to the same base).

(iii)



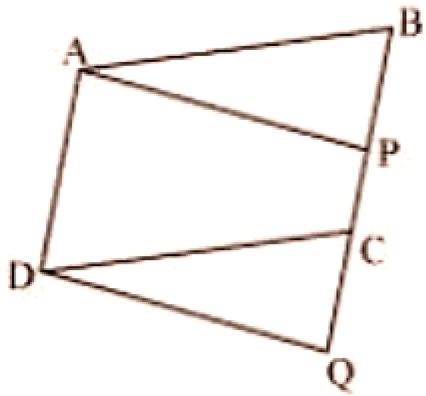
Yes. The parallelogram PQRS and the triangle TQR share the same base QR and are located between the same parallel lines PS and QR.

(iv)



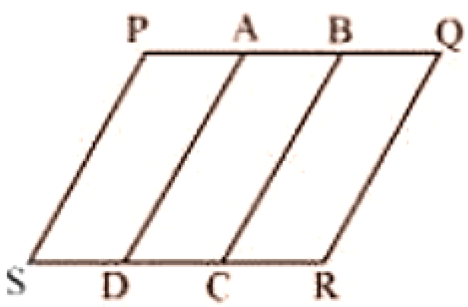
No. The parallelogram ABCD and the triangle PQR are located between the identical parallel lines AD and BC. These, on the other hand, have no common ground.

(v)



Yes. It can be seen that parallelograms ABCD and APQD have the same base AD and are located between the same parallel lines AD and BQ.

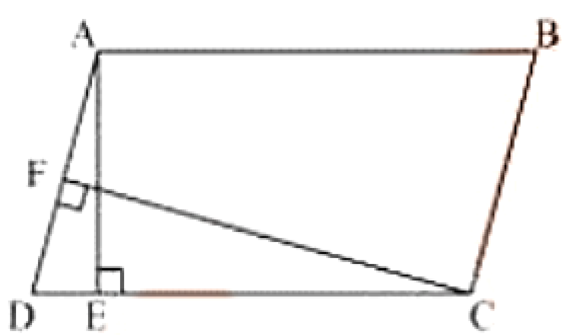
(vi)



No. The parallelograms PBCS and PQRS may be seen resting on the same PS basis. These, on the other hand, do not lay on the same parallel lines.

Exercise (9.2)

1. In the given figure, ABCD is parallelogram, $AE \perp DC$, $CF \perp AD$**. If AB = 16 cm, AE = 8 cm and CF = 10cm, find AD.**



Ans: Given: ABCD is parallelogram, $AE \perp DC$, $CF \perp AD$. If AB = 16 cm, AE = 8 cm and CF = 10cm.

To find: AD.

In parallelogram ABCD, CD = AB = 16 cm

[A parallelogram's opposite sides are equal]

We know that the area of a parallelogram equals the base altitude.

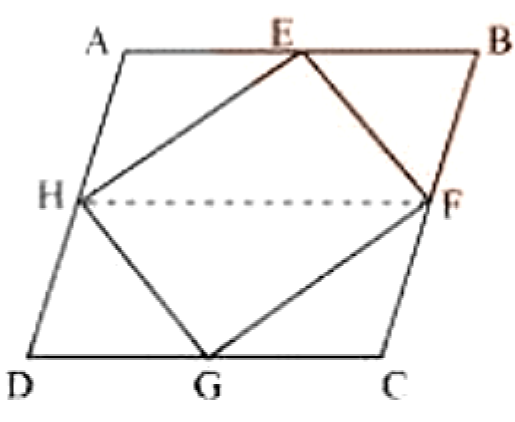
Area of parallelogram $ABCD = CD \times AE = AD \times CF$

$16\ cm \times 8\ cm = AD \times 10\ cm$

$AD = \frac{16(8)}{10} cm = 12.8 cm$

Hence, the length of AD is 12.8 cm.

2. If E, F, G and H are respectively the mid-points of the sides of a parallelogram ABCD show that



$EFGH) = \frac{1}{2} ar\ (ABCD)$

Ans: Given: E, F, G and H are the mid-points of the sides of a parallelogram ABCD

To prove: $ar\ (EFGH) = \frac{1}{2} ar\ (ABCD)$

Let us join HF.

In parallelogram ABCD,

AD = BC and $AD \parallel BC$ (Opposite sides of a parallelogram are equal and parallel)

AB = CD (Opposite sides of a parallelogram are equal)

$\frac{1}{2}AD = \frac{1}{2}BC$ and $AH \parallel BF$

⇒ AH = BF and $AH \parallel BF$ (H and F are the mid-points of AD and BC)

So, ABFH is a parallelogram.

Since ΔHEF and parallelogram ABFH are on same base HF and between same parallels AB and HF,

$\therefore$ Area $(\Delta \text{HEF}) = \frac{1}{2}$ Area (ABFH)…(1)

We know that, it can be proved that

Area $(\Delta \text{HGF}) = \frac{1}{2}$ Area (HDCF)

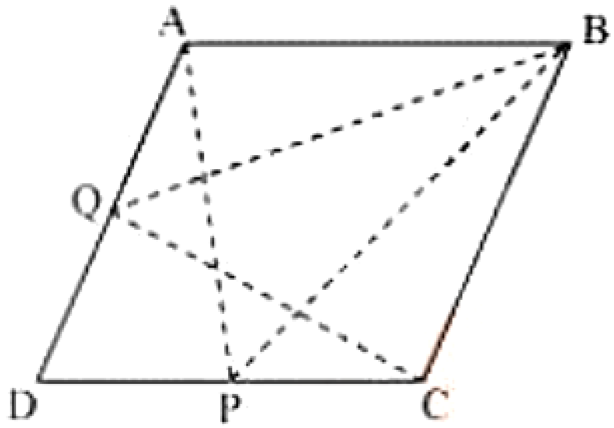
On adding Equations (1) and (2), we obtain

Area $(\Delta \text{HEF}) +$ Area $(\Delta \text{HGF}) = \frac{1}{2}$ Area $(\text{ABFH}) + \frac{1}{2}$ Area (HDCF)

$= \frac{1}{2}[$ Area $(\text{ABFH}) +$ Area $(\text{HDCF})]$

$\Rightarrow$ Area $(\text{EFGH}) = \frac{1}{2}$ Area (ABCD)

3. P and Q are any two points lying on the sides DC and AD respectively of a parallelogram ABCD. Show that $ar\left(APB\right) = ar\left(BQC\right)$**.**



Ans: Given: P and Q are any two points lying on the sides DC and AD respectively of a parallelogram ABCD.

To prove: $ar\left(APB\right) = ar\left(BQC\right)$.

It can be observed that ΔBQC and parallelogram ABCD lie on same base BC and these are between same parallel lines AD and BC.

$\therefore Area\left(\Delta BQC\right) = \frac{1}{2} Area\left(ABCD\right) \ldots (1)$

We know that, ΔAPB and parallelogram ABCD lie on the same base AB and between the same parallel lines AB and DC.

$\therefore Area\left(\Delta APB\right) = \frac{1}{2} Area\left(ABCD\right) \ldots (2)$

Upon (1) and (2, we obtain

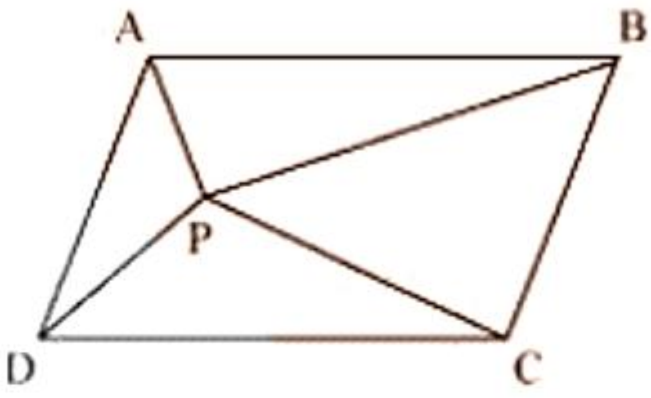
$Area\left(\Delta BQC\right) = Area\left(\Delta APB\right)$

4. In the given figure, P is a point in the interior of a parallelogram ABCD. Show that

(i) $ar\ (APB) + ar\ (PCD) = \frac{1}{2} ar\ (ABCD)$

(ii) $ar\ (APD) + ar\ (PBC) = ar\ (APB) + ar\ (PCD)$

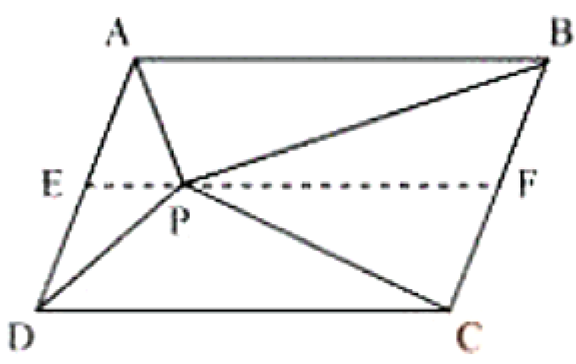
[Hint: Through. P, draw a line parallel to AB]



Ans: Given: P is a point in the interior of a parallelogram ABCD

To prove: (i) $ar\ (APB) + ar\ (PCD) = \frac{1}{2} ar\ (ABCD)$

(ii) $ar\ (APD) + ar\ (PBC) = ar\ (APB) + ar\ (PCD)$



(i) Let us draw a line segment EF, passing through point P and parallel to line segment AB.

In parallelogram ABCD,

$AB \parallel EF$ (By construction) ... (1)

ABCD is a parallelogram.

$\therefore AD \parallel BC$ (Opposite sides of a parallelogram)

$\Rightarrow AE \parallel BF$ ... (2)

Upon (1) and (2, we obtain $AB \parallel EF$, $AE \parallel BF$

So, quadrilateral ABFE is a parallelogram.

It can be observed that ΔAPB and parallelogram ABFE are lying on same base AB and between same parallel lines AB and EF.

$$\therefore Area(\Delta APB) = \frac{1}{2} Area\ (ABFE) \ ...\ (3)$$

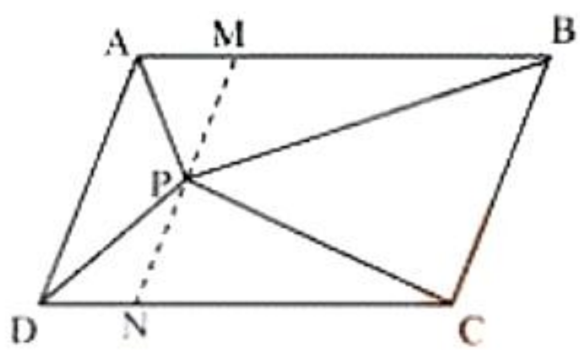
We know that, for ΔPCD and parallelogram EFCD,

$$Area\ (\Delta PCD) = \frac{1}{2}\ Area\ (EFCD) \ ...\ (4)$$

Adding Equations (3) and (4), we obtain

$$Area\ (\Delta APB) + Area\ (\Delta PCD) = \frac{1}{2}\left[Area\ (ABFE) + Area\ (EFCD)\right]$$

$$Area\ (\Delta APB) + Area\ (\Delta PCD) = \frac{1}{2}\ Area\ (ABCD) \ ...\ (5)$$



(ii) Let us draw a line segment MN, passing through point P and parallel to line segment AD. In parallelogram ABCD,

$MN \parallel AD$ (By construction) ... (6)

ABCD is a parallelogram.

$\therefore AB \parallel DC$ (Opposite sides of a parallelogram)

$\Rightarrow AM \parallel DN \ldots (7)$

From Equations (6) and (7), we obtain

$MN \parallel AD, AM \parallel DN$

So, quadrilateral AMND is a parallelogram.

It can be observed that ΔAPD and parallelogram AMND are lying on the same base AD and between the same parallel lines AD and MN.

$\therefore Area\ (\Delta APD) = 12 Area\ (AMND) \ldots (8)$

We know that, for ΔPCB and parallelogram MNCB,

$$Area\ (\Delta PCB) = \frac{1}{2} Area\ (MNCB) \ldots (9)$$

Adding Equations (8) and (9), we obtain

$$Area\ (\Delta APD) + Area\ (\Delta PCB) = \frac{1}{2}\left[Area\ (AMND) + Area\ (MNCB)\right]$$

$$Area\ (\Delta APD) + Area\ (\Delta PCB) = \frac{1}{2} Area\ (ABCD) \ldots (10)$$

upon comparing Equations (5) and (10), we obtain

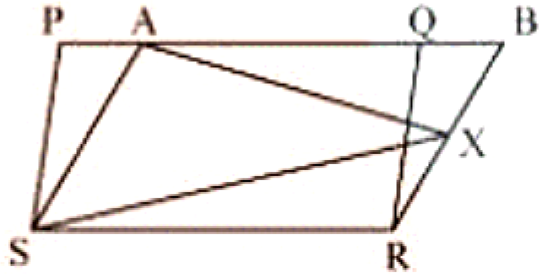
$$Area\ (\Delta APD) + Area\ (\Delta PBC) = Area\ (\Delta APB) + Area\ (\Delta PCD)$$

5. In the given figure, PQRS and ABRS are parallelograms and X is any point on side BR. Show that

(i) ar (PQRS) = ar (ABRS)

(ii) $ar\ (\Delta PXS) = \frac{1}{2} ar\ (PQRS)$

$ar\ (\Delta PXS) = \frac{1}{2} ar\ (PQRS)$



Ans: Given: PQRS and ABRS are parallelograms and X is any point on side BR.

To prove: (i) ar (PQRS) = ar (ABRS)

(ii) $ar\ (\Delta PXS) = \frac{1}{2} ar\ (PQRS)$

(i) It can be observed that parallelogram PQRS and ABRS lie upon same base SR and also, these lie in between same parallel lines SR and PB.

$\therefore Area\ (PQRS) = Area\ (ABRS)\ ...\ (1)$

(ii) Consider ΔAXS and parallelogram ABRS.

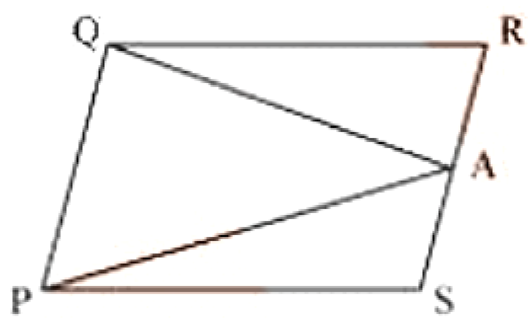
As these lie on the same base and are between the same parallel lines AS and BR,

$\therefore Area\ (\Delta AXS) = \frac{1}{2}\ Area\ (ABRS)\ ...\ (2)$

Upon (1) and (2, we obtain

$Area(\Delta AXS) = \frac{1}{2} Area\ (PQRS)$

6. A farmer was having a field in the form of a parallelogram PQRS. She took any point A on RS and joined it to points P and Q. In how many parts the field is divided? What are the shapes of these parts? The farmer wants to sow wheat and pulses in equal portions of the field separately. How should she do it?



Ans: Given: A farmer had a field that was shaped like a parallelogram PQRS. She connected any point A on RS with points P and Q.

To find: How many pieces does the field have? What are the dimensions of these components? The farmer intends to sow wheat and pulses separately in equal parts of the land. What is the best way for her to go about it?

From the figure, it can be observed that point A divides the field into three parts.

These parts are triangular in shape − $\Delta PSA, \Delta PAQ, \Delta QRA$

Area of ΔPSA + Area of ΔPAQ + Area of ΔQRA = Area of parallelogram PQRS ... (1)

We know that if that parallelogram and a triangle are on same base and between the same parallels, then the area of the triangle is half the area of the given parallelogram.

$$\therefore Area\ (\Delta PAQ) = \frac{1}{2} Area\ (PQRS)\ ...\ (2)$$

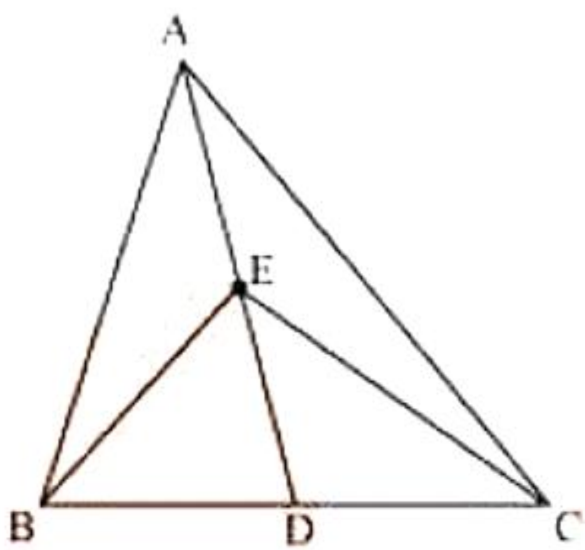
Upon (1) and (2, we obtain

$$Area\ (\Delta PSA) + Area\ (\Delta QRA) = \frac{1}{2} Area\ (PQRS)\ ...\ (3)$$

It is clear that the farmer must seed wheat in the PAQ triangle portion and pulses in the PSA and QRA triangular parts, or wheat in the PSA and QRA triangular parts and pulses in the PAQ triangular parts.

Exercise 9.3

1. In the given figure, E is any point on median AD of a ΔABC. Show that ar (ABE) = ar (ACE)



Ans: Given: E is any point on median AD of a ΔABC.

To prove: ar (ABE) = ar (ACE)

AD is the median of ΔABC. So, it will divide ΔABC into two triangles of equal areas.

$\therefore$ Area $(\Delta ABD) =$ Area $(\Delta ACD) \ldots$ (1)

ED is the median of ΔEBC.

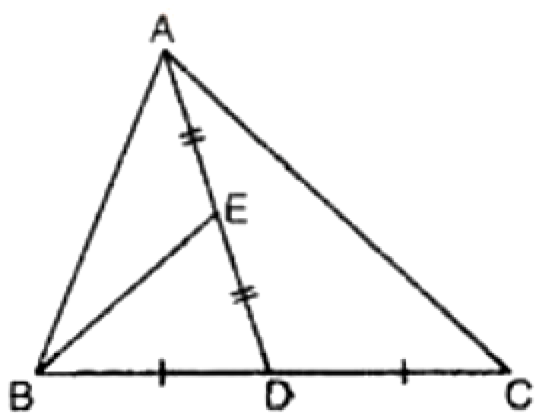
$\therefore$ Area $(\Delta EBD) =$ Area $(\Delta ECD) \ldots (2)$

On subtracting Equation (2) from Equation (1), we obtain

Area $(\Delta ABD) -$ Area $(EBD) =$ Area $(\Delta ACD) -$ Area (ΔECD)

Area $(\Delta ABE) =$ Area (ΔACE)

2. In a triangle ABC, E is the mid-point of median AD. Show that $ar(\Delta BED) = \frac{1}{4} ar(\Delta ABC)$



Ans: Given: A $\Delta ABC, AD$ is the median and E is the mid-point of median AD.

To prove: $ar(\Delta BED) = \frac{1}{4} ar(\Delta ABC)$

To prove: $ar(\Delta BED) = 1/4 \, ar(\Delta ABC)$

Proof: In $\Delta ABC, AD$ is the median.

$\therefore$ ar $(\Delta ABD) = ar(\Delta ADC)$

[$\therefore$ Median divides a Δ into two Δ s of equal area]

$ar(\Delta ABD) = \frac{1}{2} ar(ABC)$

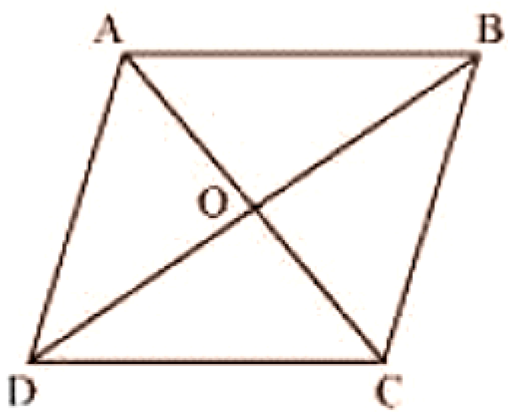
In $\Delta ABD, BE$ is the median.

$ar(\Delta BED) = ar(\Delta BAE)$

$\therefore$ ar $(\Delta BED) = \frac{1}{2} ar(\Delta ABD)$

$= ar(\Delta BED) = \frac{1}{2}\left[\frac{1}{2} ar(\Delta ABC)\right] = \frac{1}{2} ar(\Delta ABC)$

3. Show that the diagonals of a parallelogram divide it into four triangles of equal area.



Ans: Given: Diagonals of a parallelogram.

To prove: Diagonals of a parallelogram divide it into four triangles of equal area.

We know that diagonals of parallelogram bisect each other.

So, O is the mid-point of AC and BD.

BO is the median in ΔABC. So, it will divide it into two triangles of equal areas.

$\therefore Area\ (\Delta AOB) = Area\ (\Delta BOC)\ ...\ (1)$

In ΔBCD, CO is the median.

$\therefore Area\ (\Delta BOC) = Area\ (\Delta COD)\ ...\ (2)$

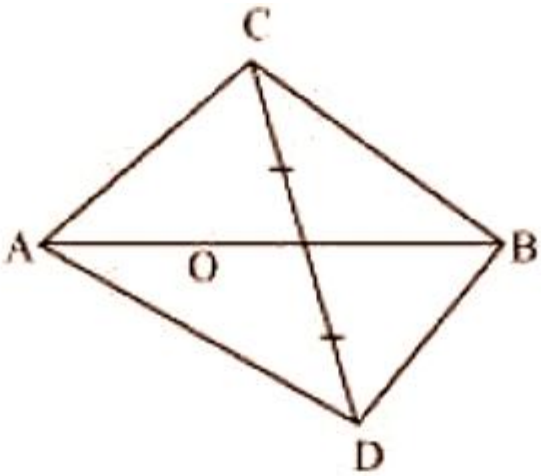
We know that, $Area\ (\Delta COD) = Area(\Delta AOD)$... (3)

From Equations (1), (2), and (3), we obtain

$Area\ (\Delta AOB) = Area\ (\Delta BOC) = Area\ (\Delta COD) = Area\ (\Delta AOD)$

So, it is evident that diagonals of a parallelogram divide it into 4 triangles of equal area.

4. In the given figure, ABC and ABD are two triangles on the same base AB. If line-segment CD is bisected by AB at O, show that ar (ABC) = ar (ABD).



Ans: Given: ABC and ABD are two triangles on the same base AB. If line-segment CD is bisected by AB at O.

To prove: ar (ABC) = ar (ABD).

Consider ΔACD.

Line-segment CD is bisected by AB at O. So, AO is the median of ΔACD.

$\therefore Area(\Delta ACO) = Area(\Delta ADO)$ ... (1)

Considering ΔBCD, BO is the median.

$\therefore Area\ (\Delta BCO) = Area\ (\Delta BDO)$ ... (2)

Adding Equations (1) and (2), we obtain

$Area\ (\Delta ACO) + Area\ (\Delta BCO) = Area\ (\Delta ADO) + Area\ (\Delta BDO)$

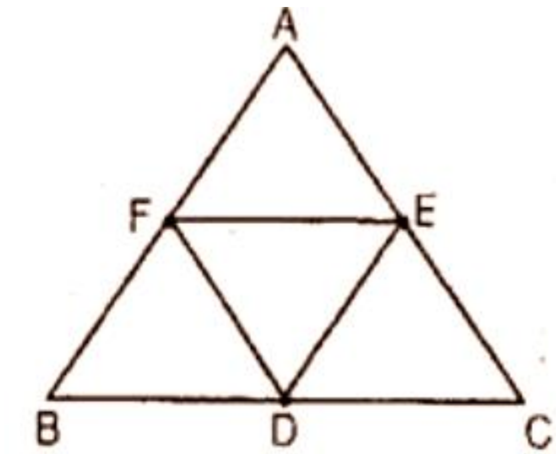
$\Rightarrow Area\ (\Delta ABC) = Area\ (\Delta ABD)$

5. D, E and F are respectively the mid-points of the sides BC, CA and AB of a ΔABC. Show that:

(i) BDEF is a parallelogram.

(ii) $ar\ (DEF) = \frac{1}{4} ar\ (ABC)$

(iii) $ar\ (BDEF) = \frac{1}{2}\ ar\ (ABC)$



Ans: Given: D, E and F are respectively the mid-points of the sides BC, CA and AB of a ΔABC. To prove:

(i) BDEF is a parallelogram.

(ii) $ar\ (DEF) = \frac{1}{4} ar\ (ABC)$

(iii) $ar\ (BDEF) = \frac{1}{2}\ ar\ (ABC)$

(i) F is the mid-point of AB and E is the mid-point of AC.

$\therefore FE \| BC$ and $FE = \frac{1}{2} BD$

Line joining the mid-points of two sides of a triangle is parallel to the third and half of It

$\therefore FE \| BD$ [BD is the part of BC]

And FE = BD

Also, D is the mid-point of BC.

$\therefore BD = \frac{1}{2} BC$

And $FE \| BC$ and FE = BD

Again E is the mid-point of AC and D is the mid-point of BC. $\therefore \mathrm{DE} \parallel \mathrm{AB}$ and $\mathrm{DE} = \frac{1}{2}\mathrm{AB}$

$\mathrm{DE} \parallel \mathrm{AB}[\mathrm{BF}$ is the part of $\mathrm{AB}]$

And $\mathrm{DE} = \mathrm{BF}$

Again F is the mid-point of AB.

$\therefore \mathrm{BF} = \frac{1}{2}\mathrm{AB}$

But $\mathrm{DE} = \frac{1}{2}\mathrm{AB}$

$\therefore \mathrm{DE} = \mathrm{BF}$

Now we have $\mathrm{FE} \parallel \mathrm{BD}$ and $\mathrm{DE} \parallel \mathrm{BF}$

And $\mathrm{FE} = \mathrm{BD}$ and $\mathrm{DE} = \mathrm{BF}$

Therefore, BDEF is a parallelogram

(ii) BDEF is a parallelogram.

$\therefore \mathrm{ar}(\Delta\mathrm{BDF}) = \mathrm{ar}(\Delta\mathrm{DEF})$

[diagonals of parallelogram divides it in two triangles of equal area] DCEF is also parallelogram.

$\therefore$ ar $(\Delta\mathrm{DEF}) = \mathrm{ar}(\Delta\mathrm{DEC}) \ldots\ldots\ldots$ (ii)

Also, AEDF is also parallelogram. $\therefore$ ar $(\Delta\mathrm{AFE}) = \mathrm{ar}(\Delta\mathrm{DEF}) \ldots\ldots\ldots\ldots$ (iii)

From eq. (i), (ii) and (iii),

$ar(\triangle DEF) = ar(\Delta BDF) = ar(\triangle DEC) = ar(\triangle AFE)$………. (iv)

Now, ar $(\triangle ABC) = ar(\triangle DEF) + ar(\triangle BDF) + ar(\triangle DEC) + ar(\Delta AFE)$……….(v)

$ar(\triangle ABC) = ar(\Delta DEF) + ar(\triangle DEF) + ar(\Delta DEF) + ar(\Delta DEF)$

[Using (iv) & (v)]

$ar(\triangle ABC) = 4 \times ar(ADEF)$

$ar(\Delta DEF) = \frac{1}{4}$ ar $(\triangle ABC)$

(iii) ar $(\parallel gmBDEF) = ar(\Delta BDF) + ar(\triangle DEF) = ar(\Delta DEF) + ar(ADEF)$ [Using (iv)]

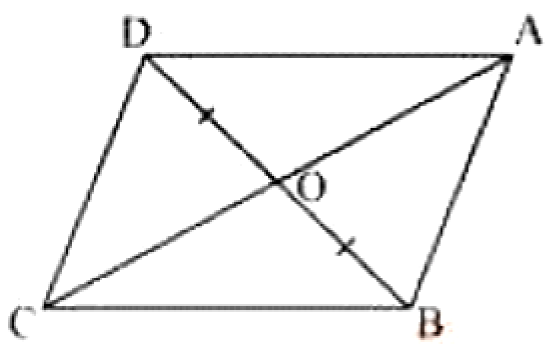
$ar(\parallel gmBDEF) = 2$ ar (ΔDEF)

$ar(\parallel gm BDEF) = 2 \times \frac{1}{4} ar(\triangle ABC)$ AC BD ABCD

O OB = $AB = CD$

$ar(\parallel ggmBDEF) = \frac{1}{2} ar(\triangle ABC)$

6. In the given figure, diagonals and of quadrilateral intersect at such that OD. If , then show that:



(i) ar $(DOC) = ar(AOB)$

(ii) ar (DCB) **= ar** (ACB)

(iii) $DA \parallel CB$ **or** ABCD **is a parallelogram**

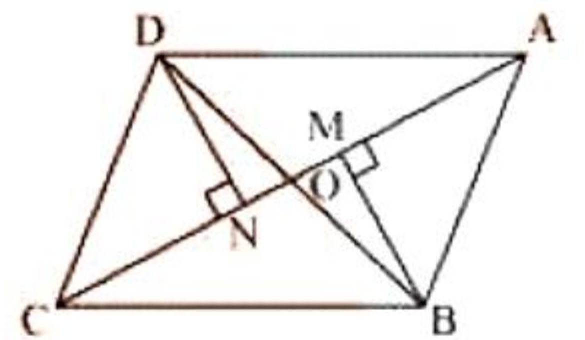
[Hint: From D and B, draw perpendiculars to AC.]

Ans: Given: In the given figure, diagonals AC and BD of quadrilateral $ABCD$ intersect at O such that $OB=$ OD. If $AB=CD$

To prove: (i) ar

(ii) ar ar

(iii) or is a parallelogram



Let us draw $DN \perp AC$ and $BM \perp AC$.

(i) In $\triangle DON$ and $\triangle BOM$,

$\angle DNO=\angle BMO$ (By construction)

$\angle DON=\angle BOM($ Vertically opposite angles $)$ $OD=OB$ (Given)

By AAS congruence rule,

$\triangle DON \cong \triangle BOM$

$DN=BM$

We know that congruent triangles have equal areas. Area $(\triangle DON)=$ Area $(\triangle BOM)$ $\ldots$

In $\triangle DNC$ and $\triangle BMA$,

$\angle DNC=\angle BMA$ (By construction)

$CD=AB$ (given)

$DN=BM[$ Using Equation (1)]

$\therefore \triangle DNC \cong \triangle BMA$ (RHS congruence rule)

$\therefore$ Area $(\triangle DNC) =$ Area $(\Delta BMA) \ldots$ (3)

On adding Equations (2) and (3), we obtain

Area ($(\triangle DON) +$ Area $(\Delta DNC) =$ Area $(\triangle BOM) +$ Area ($\triangle BMA)$

Therefore, Area $(\triangle DOC) =$ Area $(\triangle AOB)$

(ii) We obtained, Area $(\Delta DOC) =$ Area $(\triangle AOB)$

$\therefore$ Area $(\triangle DOC) +$ Area $(\Delta OCB) =$ Area $(\triangle AOB) +$ Area (ΔOCB)

(Adding Area (ΔOCB) to both sides)

$\therefore$ Area $(\triangle DCB) =$ Area $(\triangle ACB)$

(iii) We obtained, Area $(\Delta DCB) =$ Area $(\triangle ACB)$

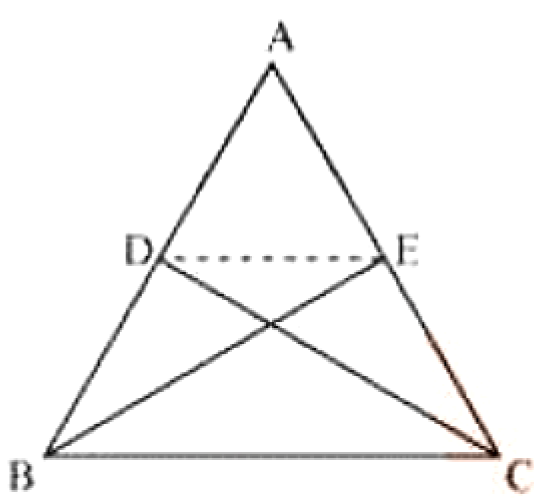
If two triangles have the same base and equal areas, then these will lie between the same parallels.

$DA \parallel CB \quad \ldots$ (4)

In quadrilateral $ABCD$, one pair of opposite sides is equal $(AB = CD)$ and the other pair of opposite sides is parallel (DA $\parallel CB$).

Therefore, $ABCD$ is a parallelogram

7. D **and** E **are points on sides** AB **and** AC **respectively of** $\triangle ABC$ **such that ar** $(DBC) =$ **ar** (EBC)**. Prove that** $DE \parallel BC$**.**



Ans: Given: and are points on sides and respectively of such that ar ar .

To Prove: .

Since $\triangle BCE$ and $\triangle BCD$ are lying on a common base BC and also have equal areas, $\triangle BCE$ and

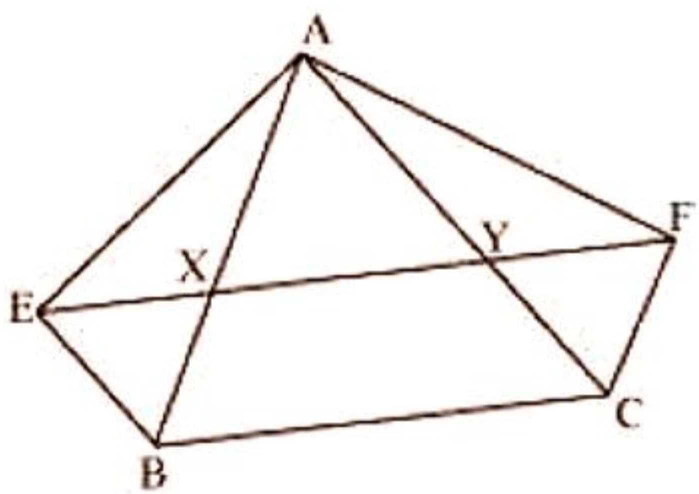
$\triangle BCD$ will lie between the same parallel lines.

$DE \parallel BC$

8. is a line parallel to side of a triangle . If and meet at and respectively, show that ar ar

Ans: Given: XY is a line parallel to side BC of a triangle ABC. If $BE \parallel AC$ and $CF \parallel AB$ meet XY at E and F respectively.

]



It is given in the question that $XY \parallel BC = EY \parallel BC$

$BE \parallel AC = BE \parallel CY$

Therefore, EBCY is a parallelogram.

It is given in the question that

$XY \parallel BC = XF \parallel BC$

$FC \parallel AB = FC \parallel XB$

Therefore, BCFX is a parallelogram.

Parallelograms EBCY and BCFX are on the same base BC and between the same parallels BC and EF.

$\therefore$ Area $(EBCY) = \frac{1}{2}$ Area $(BCFX)$ $\ldots$

Consider parallelogram EBCY and $\triangle$ AEB These lie on the same base BE and are between the same parallels BE and AC.

(2)

$\therefore$ Area $(\triangle ABE) = \frac{1}{2}$ Area (EBCY) $\ldots$

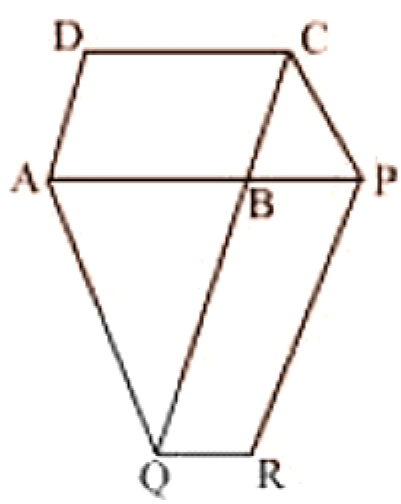
Also, parallelogram ΔCFX and $\triangle ACF$ are on the same base CF and between the same parallels

CF and AB.

$\therefore$ Area $(\Delta ACF) = \frac{1}{2}$ Area $(BCFX)$ $\ldots(3)$

From Equations (1), (2), and (3), we obtain Area $(\triangle ABE) =$ Area $(\triangle ACF)$

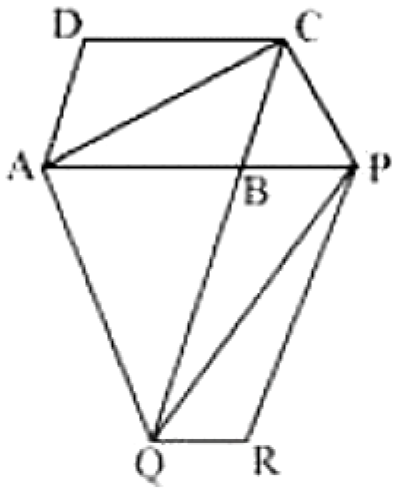
9. The side AB of a parallelogram $ABCD$ is produced to any point P. A line through A and parallel to CP meets CB produced at Q and then parallelogram PBQR is completed (see the following figure). Show that ar $(ABCD) =$ ar (PBQR).



[Hint. Join AC and PQ. Now compare area (ACQ) and area (APQ)]

Ans: Given: The side AB of a parallelogram $ABCD$ is produced to any point P. A line through A and parallel to CP meets CB produced at Q and then parallelogram PBQR is completed.

To prove: ar $(ABCD) =$ ar (PBQR).



Let us join AC and PQ.

$\triangle ACQ$ and $\triangle AQP$ are on the same base AQ and between the same parallels AQ and CP.

Area $(\triangle ACQ) =$ Area $(\triangle APQ)$

Area $(\Delta ACQ) -$ Area $(\triangle ABQ) =$ Area $(\triangle APQ) -$ Area $(\triangle ABQ)$

Area $(\triangle ABC) =$ Area (ΔQBP)

Since AC and PQ are diagonals of parallelograms $ABCD$ and PBQR respectively,

Area $(\triangle ABC) = \frac{1}{2}$ Area $(ABCD)$

Area $(\Delta QBP) = \frac{1}{2}$ Area $(PBQR)$

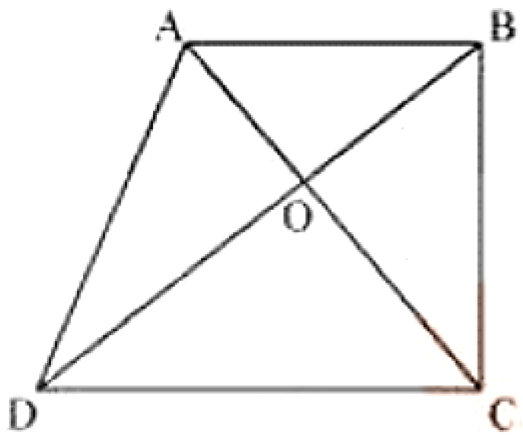
From Equations (1), (2), and (3), we obtain $\frac{1}{2}$ Area $(ABCD) = \frac{1}{2}$ Area (PBQR)

Area $(ABCD) =$ Area (PBQR)

10. Diagonals AC and BD of a trapezium $ABCD$ with $AB \parallel DC$ intersect each other at O. Prove that ar $(AOD) = ar(BOC)$.

Ans: Given: Diagonals AC and BD of a trapezium $ABCD$ with $AB \parallel DC$ intersect each other at O.

To Prove: ar $(AOD) = ar(BOC)$.



It can be observed that $\triangle DAC$ and $\triangle DBC$ lie on the same base DC and between the same parallels AB and CD.

Area $(\triangle DAC) =$ Area $(\triangle DBC)$

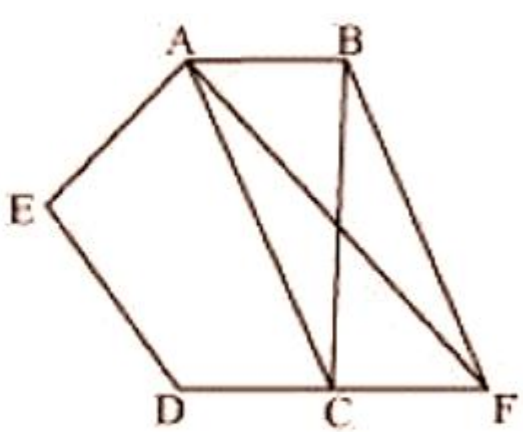
Area $(\Delta DAC) -$ Area $(\triangle DOC) =$ Area $(\triangle DBC) -$ Area $(\triangle DOC)$

Area $(\triangle AOD) =$ Area $(\triangle BOC)$

11. In the given figure, $ABCDE$ is a pentagon A line through B parallel to AC meets DC produced at F. Show that

(i) ar $(ACB) = ar(ACF)$

(ii) ar $(AEDF) =$ **ar** $(ABCDE)$



Ans: Given: ABCDE is a pentagon A line through B parallel to AC meets DC produced at F.

To prove:

(i) ar $(ACB) = ar(ACF)$

(ii) ar $(AEDF) =$ ar $(ABCDE)$

(i) ΔACB and ΔACF lie on the same base AC and are between

The same parallels AC and BF.

Area (ΔACB) = Area (ΔACF)

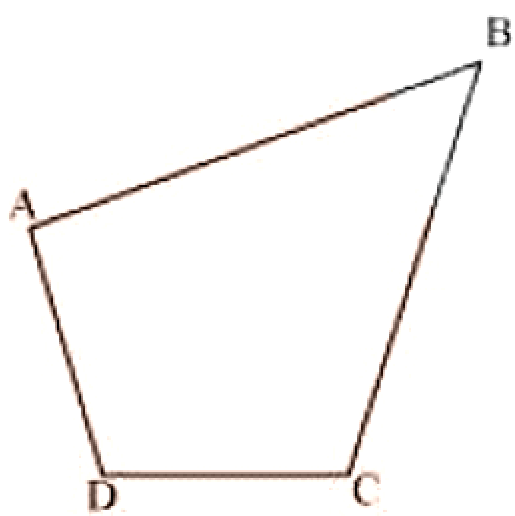
(ii) It can be observed that

Area (ΔACB) = Area (ΔACF)

Area (ΔACB) + Area (ACDE) = Area (ΔACF) + Area (ACDE)

Area (ABCDE) = Area (AEDF)

12. A villager Itwaari has a plot of land of the shape of a quadrilateral. The Gram Panchayat of the village decided to take over some portion of his plot from one of the corners to construct a Health Centre. Itwaari agrees to the above proposal with the condition that he should be given equal amount of land in lieu of his land adjoining his plot so as to form a triangular plot. Explain how this proposal will be implemented.



Ans: Given: Itwaari, a peasant, owns a quadrilateral-shaped parcel of land. The village's Gram Panchayat decided to take over a piece of his property on one of the corners to build a health centre. Itwaari agrees to the aforementioned proposition on the condition that he be given an equal amount of land in exchange for the property adjacent to his allotment, forming a triangle plot.

To prove: Describe how this suggestion will be put into action.

Let quadrilateral ABCD be the original shape of the field.

The proposal may be implemented as follows.

Join diagonal BD and draw a line parallel to BD through point A.

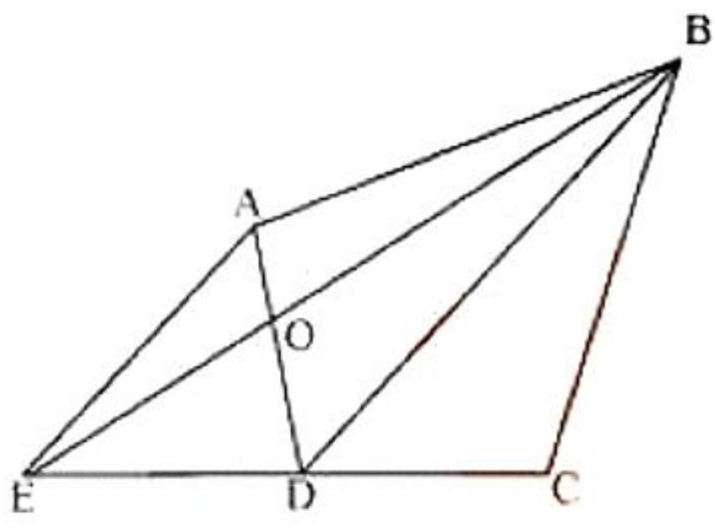
Let it meet the extended side CD of ABCD at point E.

Join BE and AD. Let them intersect each other at O.

Then, portion ΔAOB can be cut from the original field so that the new shape of the field will be ΔBCE. (See figure).

We have to prove that the area of ΔAOB (portion that was cut so as to construct Health Centre)

is equal to the area of ΔDEO (portion added to the field so as to make the area of the new field so formed equal to the area of the original field).



It can be observed that $\triangle DEB$ and $\triangle DAB$ lie on the same base BD and are between the same parallels BD and AE.

Area $(\Delta DEB) =$ Area $(\triangle DAB)$

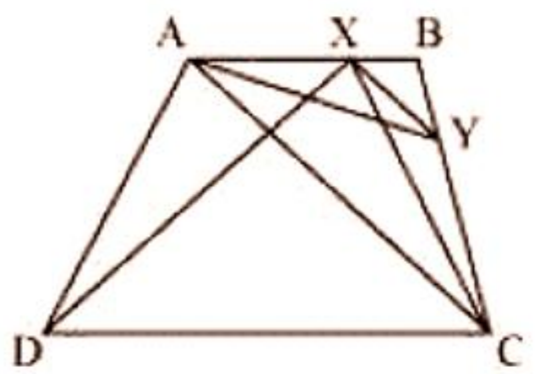
Area $(\Delta DEB) -$ Area $(\triangle DOB) =$ Area $(\triangle DAB) -$ Area $(\triangle DOB)$

Area $(\Delta DEO) =$ Area $(\triangle AOB)$

13. $ABCD$ **is a trapezium with** $AB \parallel DC$**. A line parallel to** AC **intersects** AB **at** X **and** BC **at** Y**. Prove that ar** $(ADX) = ar(ACY)$**. [Hint: Join CX.]**

Ans: Given: $ABCD$ is a trapezium with $AB \parallel DC$. A line parallel to AC intersects AB at X and BC at Y.

To Prove: ar $(ADX) = ar(ACY)$.



It can be observed that $\triangle ADX$ and $\triangle ACX$ lie on the same base AX and are between the same parallels AB and DC.

Area $(\triangle ADX) =$ Area $(\triangle ACX)$

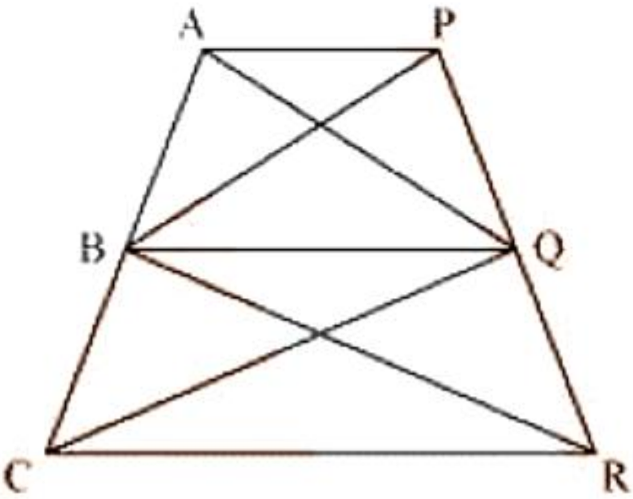
$\triangle ACY$ and $\triangle ACX$ lie on the same base AC and are between the same parallels AC and XY.

Area $(\triangle ACY) =$ Area (ACX)

Upon (1) and (2, we obtain

Area $(\Delta ADX) =$ Area $(\ ACY)$

14. In the given figure, Prove that ar .



Ans: Given: $AP \parallel BQ \parallel CR$

To prove: ar $(AQC) = ar(PBR)$

Since $\triangle ABQ$ and $\triangle PBQ$ lie on the same base BQ and are between the same parallels AP and BQ,

$\therefore$ Area $(\triangle ABQ) =$ Area (ΔPBQ) … (1)

Again, $\triangle BCQ$ and $\triangle BRQ$ lie on the same base BQ and are between the same parallels BQ and CR.

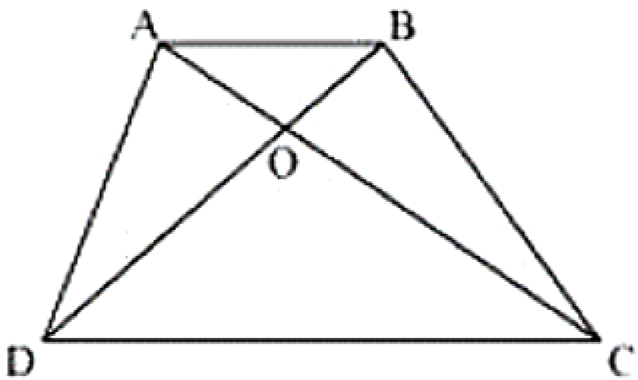
$\therefore$ Area $(\Delta BCQ) =$ Area $(\triangle BRQ)$ …(2)

On adding Equations (1) and (2), we obtain

Area $(\Delta ABQ) +$ Area $(\triangle BCQ) =$ Area $(\Delta PBQ) +$ Area $(\triangle BRQ)$

$\therefore$ Area $(\triangle AQC) =$ Area ($\triangle PBR)$

15. Diagonals AC and BD of a quadrilateral $ABCD$ intersect at O in such a way that ar $(AOD) = \text{ar}$ (BOC). Prove that $ABCD$ is a trapezium.



Ans: Given: Diagonals AC and BD of a quadrilateral $ABCD$ intersect at O in such a way that ar $(AOD) = \text{ar}$ (BOC).

To Prove: $ABCD$ is a trapezium

It is given in the question that

Area $(\triangle AOD) =$ Area $(\triangle BOC)$

Area $(\triangle AOD) +$ Area $(\triangle AOB) =$ Area $(\triangle BOC) +$ Area $(\triangle AOB)$

Area $(\triangle ADB) =$ Area $(\triangle ACB)$

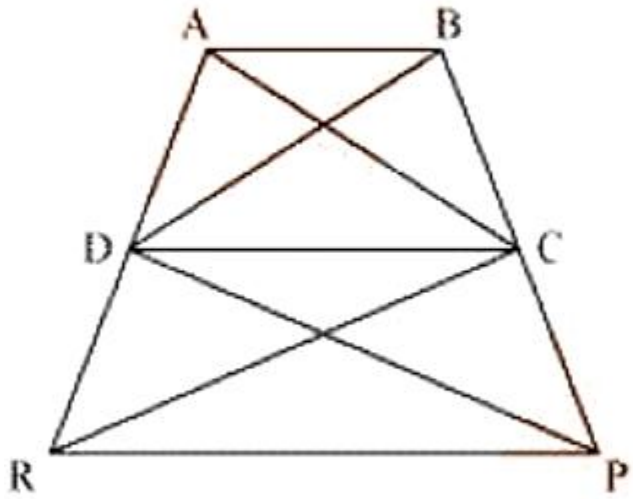
We know that triangles on the same base having areas equal to each other lie between the same

parallels.

Therefore, these triangles, $\triangle ADB$ and $\triangle ACB$, are lying between the same parallels. i.e., $AB \parallel CD$

Therefore, $ABCD$ is a trapezium

16. In the given figure, ar $(DRC) =$ ar (DPC) and ar $(BDP) = ar(ARC)$. Show that both the quadrilaterals $ABCD$ and DCPR are trapeziums.



Ans: Given: ar $(DRC) =$ ar (DPC) and ar $(BDP) = ar(ARC)$.

To prove: Both the quadrilaterals $ABCD$ and DCPR are trapeziums.

As $\triangle DRC$ and $\triangle DPC$ lie on the same base DC and have equal areas, therefore, they must lie between the same parallel lines. $\therefore DC \parallel RP$

Therefore, DCPR is a trapezium It is also given that

Area $(\triangle BDP) =$ Area $(\triangle ARC)$

Area $(\triangle BDP) -$ Area $(\triangle DPC) =$ Area $(\triangle ARC) -$ Area $(\triangle DRC)$

$\therefore$ Area $(\Delta BDC) =$ Area $(\triangle ADC)$

Since $\triangle BDC$ and $\triangle ADC$ are on the same base CD and have equal areas, they must lie between the same parallel lines.

$\therefore AB \parallel CD$

So, ABCD is a trapezium.

Exercise 9.4

1. Parallelogram ABCD and rectangle ABEF are on the same base AB and have equal areas. Show that the perimeter of the parallelogram is greater than that of the rectangle.

Ans: Given: Parallelogram ABCD and rectangle ABEF are on the same base AB and have equal areas.

To prove: The perimeter of the parallelogram is greater than that of the rectangle.

As the parallelogram and the rectangle have the same base and equal area, So, these will also lie between the same parallels.

Consider the parallelogram ABCD and rectangle ABEF as follows.

Here, it can be observed parallelogram ABCD and rectangle ABEF are between same parallels AB and CF.

We know that opposite sides of a parallelogram or a rectangle are of equal lengths.

So,

AB = EF (For rectangle)

AB = CD (For parallelogram)

$\therefore CD = EF$

$\therefore AB + CD = AB + EF \ldots (1)$

Of all the line segments that can be drawn to a given line from a point not lying on it, the perpendicular line segment is the shortest.

$\therefore AF < AD$

And We know that, $BE < BC$

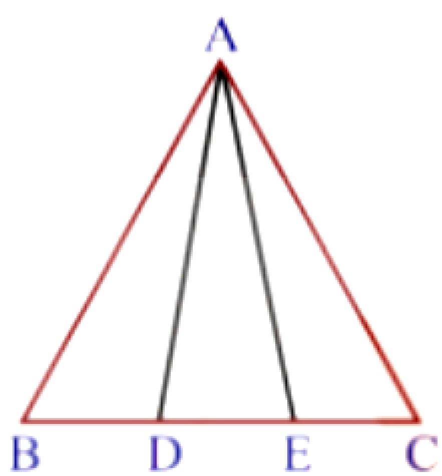
$\therefore AF + BE < AD + BC \ldots (2)$

Upon (1) and (2, we obtain

$AB + EF + AF + BE < AD + BC + AB + CD$

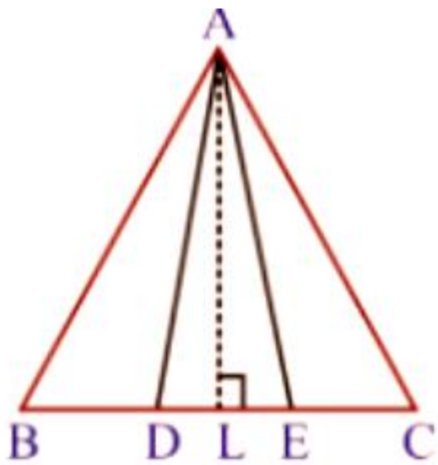
Perimeter of rectangle ABEF < Perimeter of parallelogram ABCD.

2. In the following figure, D and E are two points on BC such that BD = DE = EC. Show that ar (ABD) = ar (ADE) = ar (AEC). Can you now answer the question that you have left in the 'Introduction' of this chapter, whether the field of Budhia has been actually divided into three parts of equal area?



[Remark: Note that by taking BD = DE = EC, the triangle ABC is divided into three triangles ABD, ADE and AEC of equal areas. In the same way, by dividing BC into n equal parts and joining the points of division so obtained to the opposite vertex of BC, you can divide ΔABC into n triangles of equal areas.]

Ans:



Given: D and E are two locations on BC in the diagram below, and BD = DE = EC. Demonstrate that ar (ABD) = ar (ADE) = ar (AEC).

To find: Can you now respond to the question you posed in the 'Introduction' to this chapter, namely, whether the Budhia field has been divided into three equal parts?

Let us draw a line segment $AL \perp BC$.

We know that, Area of a triangle $= \frac{1}{2} \times$ Base $\times$ Altitude Area $(\triangle ADE) = \frac{1}{2} \times DE \times AL$

Area $(\triangle ABD) = \frac{1}{2} \times BD \times AL$

Area $(\Delta AEC) = \frac{1}{2} \times EC \times AL$

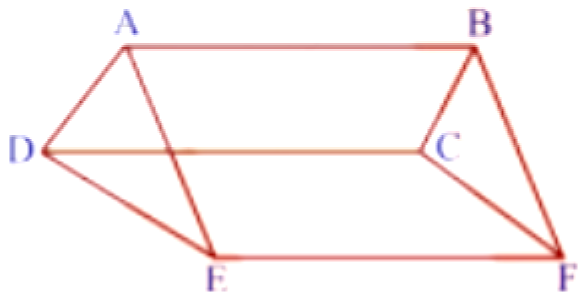
It is given in the question that $DE = BD = EC$

$\frac{1}{2} \times DE \times AL = \frac{1}{2} \times BD \times AL = \frac{1}{2} \times EC \times AL$

Area $(\triangle ADE) =$ Area $(\triangle ABD) =$ Area $(\triangle AEC)$

It can be observed that Budhia has divided her field into 3 equal parts.

3. In the following figure, ABCD, DCFE and ABFE are parallelograms. Show that $ar\ (\Delta ADE) = ar\ (\Delta BCF)$.



Ans: Given: In the following figure, ABCD, DCFE and ABFE are parallelograms.

To prove: $ar\ (\Delta ADE) = ar\ (\Delta BCF)$.

It is given in the question that ABCD is a parallelogram. We know that opposite sides of a parallelogram are equal.

$\therefore AD = BC \ ... (1)$

We know that, for parallelograms DCEF and ABFE, it can be proved that

DE = CF ... (2)

And, EA = FB ... (3)

In ΔADE, ΔBCF,

AD = BC [Using equation (1)]

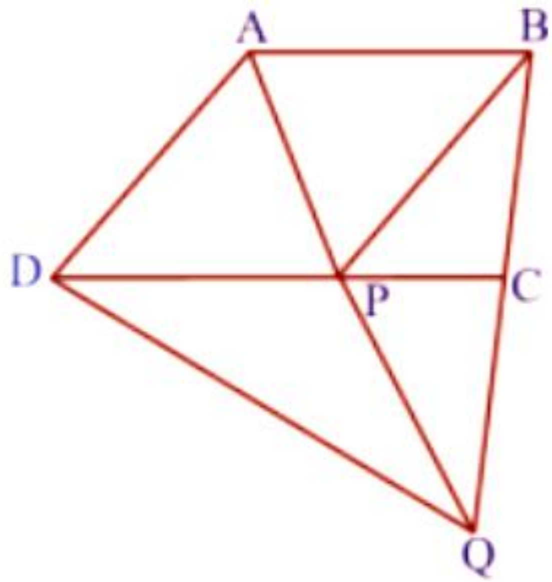
DE = CF [Using equation (2)]

EA = FB [Using equation (3)]

$\therefore \Delta ADE \cong \Delta BCF$ (SSS congruence rule)

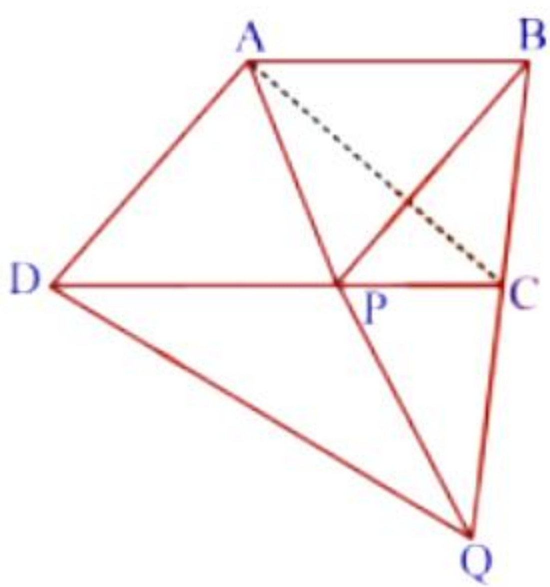
$\therefore Area(\Delta ADE) = Area\ (\Delta BCF)$

4. In the following figure, ABCD is parallelogram and BC is produced to a point Q such that AD = CQ. If AQ intersect DC at P, show that $ar\ (\Delta BPC) = ar\ (\Delta DPQ)$ **. [Hint: Join AC.]**



Ans: Given: In the following figure, ABCD is parallelogram and BC is produced to a point Q such that AD = CQ. If AQ intersect DC at P.

To prove: $ar\ (\Delta BPC) = ar\ (\Delta DPQ)$.



It is given in the question that ABCD is a parallelogram.

$AD \parallel BC$, $AB \parallel DC$ (Opposite sides of a parallelogram are parallel to each other)

Join point A to point C.

Consider $\triangle APC$ and ΔBPC

$\triangle APC$ and $\triangle BPC$ are lying on the same base PC and between the same parallels PC and AB. Therefore,

Area $(\triangle APC) =$ Area $(\triangle BPC) \ldots$ (1)

In quadrilateral ACDQ, it is given that $AD = CQ$

Since $ABCD$ is a parallelogram,

$AD \parallel BC$ (Opposite sides of a parallelogram are parallel) CQ is a line segment which is obtained when line segment BC is produced. $\therefore AD \parallel CQ$

We have, $AC = DQ$ and $AC \parallel DQ$

Hence, ACQD is a parallelogram.

Consider BDCQ and BACQ These are on the same base CQ and between the same parallels CQ and AD.

Therefore, Area $(\Delta DCQ) =$ Area $(\triangle ACQ)$

$\therefore$ Area $(\Delta DCQ) -$ Area $(\Delta PQC) =$ Area $(\Delta ACQ) -$ Area (ΔPQC)

$\therefore$ Area $(\Delta DPQ) =$ Area $(\triangle APC) \ldots (2)$

From equations (1) and (2), we obtain

Area $(\Delta BPC) =$ Area (ΔDPQ)

$(BDE) = \frac{1}{2}$

$(ABC) = 2 \quad (BEC)$

5. In the following figure, ABC and BDE are two equilateral triangles such that D is the mid-point of BC. If AE intersects BC at F, show that

(i) $(BFE) = 2$

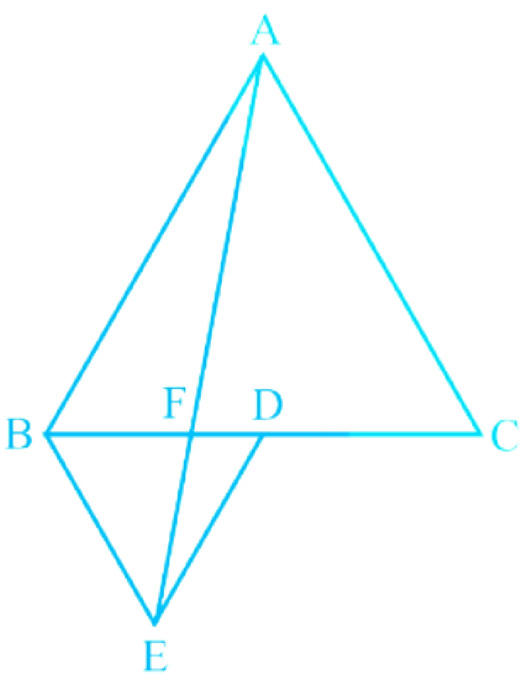
(ii) ar $(FED) = \frac{1}{8}$ **ar (BAE)**

(iii) ar **ar**

(iv) ar (BFE) = ar (AFD)

(v) ar **ar (FED)**

(vi) ar **ar (AFC)**



[Hint: Join EC and AD. Show that $BE \parallel AC$, $DE \parallel AB$ **etc.]**

Ans: Given: In the following figure, ABC and BDE are two equilateral triangles such that D is the mid-point of BC. If AE intersects BC at F.

To prove: (i) $\text{ar(BDE)} = \frac{1}{4}\text{ar(ABC)}$

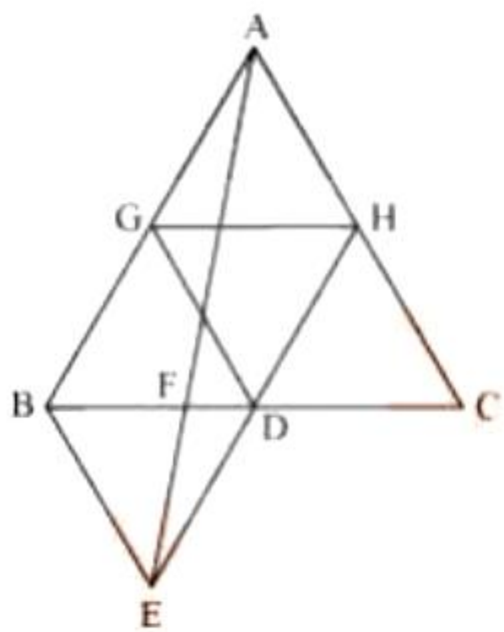
(ii) ar $\text{(BDE)} = \frac{1}{2}$ ar (BAE)

(iii) ar $\text{(ABC)} = 2$ ar (BEC)

(iv) ar (BFE) = ar (AFD)

(v) ar $\text{(BFE)} = 2$ ar (FED)

(



(i) Let G and H be the mid-points of side AB and AC. Line segment GH is joining the mid-points and is parallel to third side. So, BC will be half of length of BC (mid-point theorem).

$\therefore \text{GH} = \frac{1}{2}\text{BC}$ and $\text{GH} \parallel \text{BD}$

$\therefore \text{GH} = \text{BD} = \text{DC}$ and $\text{GH} \parallel \text{BD}$ (D is the mid-point of BC)

Similarly,

- $GD = HC = HA$

- $HD = AG = BG$

Therefore, clearly $\triangle ABC$ is divided into 4 equal equilateral triangles viz $\triangle BGD, \Delta AGH, \triangle DHC$ and ΔGHD

In other words, $\Delta BGD = \frac{1}{4} \Delta ABC$

Now consider $\triangle BDG$ and ΔBDE

$BD = BD$ (Common base) As both triangles are equilateral triangle, we can say $BG = BE$

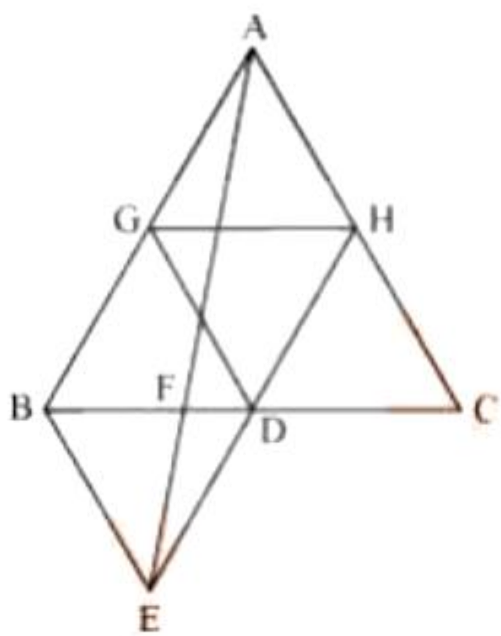
$DG = DE$

Therefore, $\Delta BDG \cong \Delta BDE$[By SSS congruency]

Thus, area $(\triangle BDG) =$ area $(\triangle BDE)$

$ar(\Delta BDE) = \frac{1}{4}$ ar (ΔABC)

Hence proved



(ii) Area ($(\triangle BDE) =$ Area ($\triangle AED)$ (Common base DE and $DE \parallel AB$) Area $(\triangle BDE) -$ Area $(\triangle FED) =$ Area $(\triangle AED) -$ Area $(\triangle FED)$

Area $(\triangle BEF) =$ Area $(\triangle AFD)$

Now, Area $(\triangle ABD) =$ Area $(\triangle ABF) +$ Area $(\triangle AFD)$

Area $(\triangle ABD) =$ Area $(\triangle ABF) +$ Area $(\triangle BEF)$ [From equation (1)] Area $(\triangle ABD) =$ Area $(\triangle ABE)$

AD is the median in $\triangle ABC$.

$$\text{ar}(\triangle ABD) = \frac{1}{2}\text{ar}(\triangle ABC)$$

$$= \frac{4}{2}\text{ar}(\Delta BDE)$$

(As proved earlier)

ar $(\triangle ABD) = 2\,\text{ar}(\Delta BDE)$

(3)

From (2) and (3), we obtain $2\,\text{ar}(\triangle BDE) = \text{ar}(\triangle ABE)$

$$\text{ar (BDE)} = \frac{1}{2} \text{ ar (BAE)}$$

(iii) $\text{ar}(\triangle ABE) = \text{ar}(\triangle BEC)$ (Common base BE and BEI|AC)

ar $(\triangle ABF) + \text{ar}(\Delta BEF) = \text{ar}(\Delta BEC)$

Using equation (1), we obtain

$$\text{ar}(\triangle ABF) + \text{ar}(\triangle AFD) = \text{ar}(\Delta BEC)$$

$$\text{ar}(\triangle ABD) = \text{ar}(\Delta BEC)$$

$$\frac{1}{2}\text{ar}(\triangle ABC) = \text{ar}(ABEC)$$

$$\text{ar}(\quad ABC) = 2\,\text{ar}(\Delta BEC)$$

(iv) It is seen that ΔBDE and ar $\triangle AED$ he on the same base (DE) and between the parallels DE and AB.

$\therefore$ ar $(\Delta BDE) = ar(\Delta AED)$

$\therefore$ ar $(\Delta BDE) - ar(\Delta FED) = ar(\Delta AED) - ar(\Delta FED)$

$\therefore$ ar $(\Delta BFE) = ar(\Delta AFD)$

(v) Let h be the height of vertex E, corresponding to the side BD in $\triangle BDE$. Let H be the height of vertex A, corresponding to the side BC in $\triangle ABC$. In (i), it was shown that ar $(BDE) = \frac{1}{4}$ ar (ABC)

In (iv), it was shown that ar $(\Delta BFE) = ar(\triangle AFD)$.

$\therefore$ ar $(\Delta BFE) = ar(\Delta AFD)$

$= 2$ ar (ΔFED)

Hence,

(vi) $ar\left(\Delta_{AFC)} = ar\left(\Delta_{AFD)} + ar\left(\Delta_{ADC}\right) = 2\,ar\left(\Delta_{FED}\right) + \frac{1}{2}ar\left(\Delta_{ABC}\right)\right.\right.$[using(v)

$= 2$ ar $(\Delta FED) + \frac{1}{2}[4 \times ar(\Delta BDE)]$[Using result of part (i)]

$= 2$ ar $(\Delta FED) + 2ar(\Delta BDE) = 2ar(\Delta FED) + 2$ ar (ΔAED)

[ΔBDE and ΔAED are on the same base and between same parallels] $= 2$ ar $(\Delta FED) + 2[ar(\Delta AFD) + ar(\Delta FED)]$

$= 2ar(\Delta FED) + 2$ ar $\left(\Delta_{AFD}\right) + 2$ ar (ΔFED) [Using (viii)]

$= 4$ ar $(\Delta FED) + 4$ ar (ΔFED)

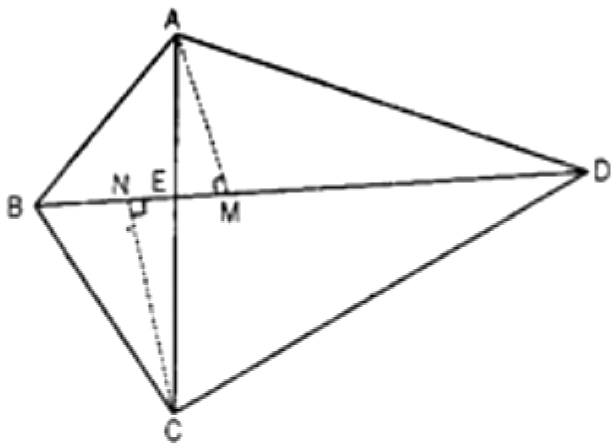
$\Rightarrow ar(\Delta AFC) = 8ar(\Delta FED)$

$\Rightarrow ar(\Delta FED) = \frac{1}{8}ar(\Delta AFC)$

6. Diagonals AC and BD of a quadrilateral ABCD intersect each other at P. Show that [Hint: From A and C, draw perpendiculars to BD]

Ans: Given: A quadrilateral ABCD, in which diagonals AC and BD intersect each other at point E.

To prove: $ar(AED)\ ar(\mathrm{BEC}) = ar(ABE)\ ar(\mathrm{CDE})$



Construction: From A, draw $AM \perp BD$AM BD and from C, draw $CN \perp BD$.

Proof : $\mathrm{ar}(\Delta \mathrm{ABE}) = \frac{1}{2} \times \mathrm{BE} \times \mathrm{AM}$

$\mathrm{ar}(\Delta \mathrm{AED}) = \frac{1}{2} \times \mathrm{DE} \times \mathrm{AM} \ldots\ldots\ldots$

Dividing eq.(ii) by (i), we get, $\frac{\mathrm{ar}(\Delta \mathrm{AED})}{\mathrm{ar}(\Delta \mathrm{ABE})} = \frac{\frac{1}{2} \times \mathrm{DE} \times \mathrm{AM}}{\frac{1}{2} \times \mathrm{BE} \times \mathrm{AM}}$

$\Rightarrow \frac{\mathrm{ar}(\Delta \mathrm{AED})}{\mathrm{ar}(\Delta \mathrm{ABE})} = \frac{\mathrm{DE}}{\mathrm{BE}} \ldots\ldots\ldots\ldots\ldots$ (iii)

Similarly $\frac{\mathrm{ar}(\Delta \mathrm{CDE})}{\mathrm{ar}(\Delta \mathrm{BEC})} = \frac{\mathrm{DE}}{\mathrm{BE}}$ ....(iv)

From eq.(iii) and (iv), we get $\frac{\mathrm{ar}(\Delta \mathrm{AED})}{\mathrm{ar}(\Delta \mathrm{ABE})} = \frac{\mathrm{ar}(\Delta \mathrm{CDE})}{\mathrm{ar}(\Delta \mathrm{BEC})}$

$\Rightarrow \mathrm{ar}(\Delta \mathrm{AED}) \times \mathrm{ar}(\Delta \mathrm{BEC}) = \mathrm{ar}(\Delta \mathrm{ABE}) \times \mathrm{ar}(\Delta \mathrm{CDE})$

Hence proved.

7. P **and** Q **are respectively the mid-points of sides** AB **and** BC **of a triangle** ABC **and** R **is the mid-point of** AP**, show that**

(i) ar $(\mathbf{PRQ}) = \frac{1}{2}\text{ar}(ARC)$

(ii) $\text{ar}(RQC) = \frac{3}{8}\text{ar}(ABC)$

(iii) ar $(PBQ) = \text{ar}(ARC)$

Ans: Given: P and Q are respectively the mid-points of sides AB and BC of a triangle ABC and R is the mid-point of AP,

To prove:

(i) ar $(\mathbf{PRQ}) = \frac{1}{2}\text{ar}(ARC)$

(ii) $\text{ar}(RQC) = \frac{3}{8}\text{ar}(ABC)$

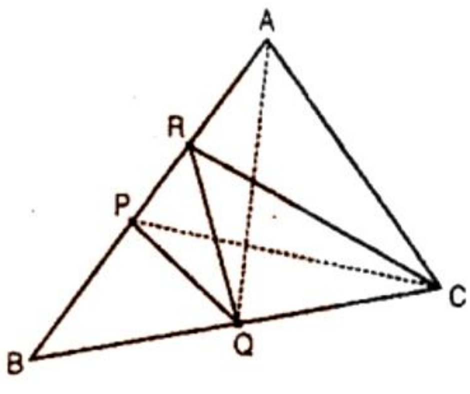
(iii) ar $(PBQ) = \text{ar}(ARC)$

(i) PC is the median of $\triangle ABC$.

$\therefore \text{ar}(\Delta BPC) = \text{ar}(\Delta APC)$……… (i)

RC is the median Δ of APC.

- ar $(\Delta ARC) = \frac{1}{2}$ ar (ΔAPC)……… (ii)



[Median dirides the triangle into two triangles of equal area] PQ is the median of $\triangle BPC$

$\text{ar}(\Delta PQC)=\frac{1}{2}\text{ar}(\Delta BPC)\ldots\ldots..$ (iii)

From eq. (i) and (iii), we get, $\text{ar}(\Delta PQC)=\frac{1}{2}$ ar $(\Delta APC)\ldots\ldots..$ (iv)

From eq. (ii) and (iv), we get, ar $(\Delta PQC)=\text{ar}(\Delta ARC)\ldots\ldots...(v)$

We are given that P and Q are the mid-points of AB and BC respectively. $PQ\parallel AC$ and $PA=\frac{1}{2}AC$

$\Rightarrow$ ar $(\Delta APQ)=\text{ar}(\Delta PQC)\ldots\ldots\ldots$ (vi) [triangles between same parallel are equal in area] From eq. (v) and (vi), we get ar $(\Delta APQ)=a(\Delta ARC)\ldots\ldots..$ (vii)

$\mathbb{R}$ is the mid-point of AP. Therefore RQ is the median of $\triangle APQ$. $-\text{ar}(\Delta PRQ)=\frac{1}{2}\text{ar}(\Delta APQ)\ldots\ldots..$ (viii)

From (ii) and (iii), we get, ar $(\Delta PRQ)=\frac{1}{2}\text{ar}(\Delta ARC)$

(ii) PQ is the median of ΔBPC ar $(\Delta PQC)=\frac{1}{2}$ ar $(\Delta BPC)=\frac{1}{2}\times\frac{1}{2}a(\Delta ABC)=\frac{1}{4}$ ar $(\Delta_{ABC})\ldots\ldots\ldots.(ix)$

Also ar $(\Delta PRC)=\frac{1}{2}$ ar $(\Delta APC)[\text{Using}(r\cdot)]$

$\Rightarrow\text{ar}(\Delta PRC)=\frac{1}{2}\times\frac{1}{2}$ ar $(\Delta_{ABC})=\frac{1}{4}\text{ar}(\Delta_{ABC})\ldots\ldots\ldots.(x)$

Adding eq. (ix) and (x), we get, ar $(\Delta PQC)+\text{ar}(\Delta PRC)=\left(\frac{1}{4}+\frac{1}{4}\right)\text{ar}(\Delta ABC)$

$\Rightarrow$ ar (quad. $PQCR)=\frac{1}{2}$ ar $(\Delta ABC)\ldots\ldots\ldots..(xi)$

Subtracting ar ($\triangle$ PRQ) from the both sides, ar (quad. $PQCR)-\text{ar}(\Delta PRQ)=\frac{1}{2}$ ar $(\Delta ABC)-\text{ar}(\Delta PRQ)$

$\Rightarrow \text{ar}(\Delta\text{RQC}) = \frac{1}{2}\text{ ar }(\Delta\text{ABC}) - \frac{1}{2}\text{ ar }(\Delta\text{ARC})[$ Using result (i)]

$\Rightarrow_{ar}\left(\Delta_{ARC}\right) = \frac{1}{2}\text{ ar }(\Delta ABC) - \frac{1}{2}\times\frac{1}{2}\text{ ar }(\Delta APC)$

$\Rightarrow_{\text{ar}}\left(\Delta_{\text{RQC}}\right) = \frac{1}{2}\text{ ar }\left(\Delta_{\text{ABC}}\right) - \frac{1}{4}\text{ ar }\Delta_{APC}$

$\Rightarrow_{ar}\left(\Delta_{RQC}\right) = \frac{1}{2}\text{ ar }\left(\Delta_{ABC}\right) - \frac{1}{4}\times\frac{1}{2}\text{ ar }\left(\Delta_{ABC}\right)[PC$ is median of $\Delta ABC]$

$\Rightarrow_{ar}\left(\Delta_{RQC}\right) = \frac{1}{2}ar(\Delta ABC) - \frac{1}{8}ar(\Delta ABC)$

$\Rightarrow_{\text{ar}}(\Delta\text{RQC}) = \left(\frac{1}{2} - \frac{1}{8}\right)\times\text{ar}(\Delta\text{ABC})$

$\Rightarrow_{\text{ar}}\left(\Delta_{\text{RQC}}\right) = \frac{3}{8}\text{ ar }\left(\Delta_{\text{ABC}}\right)$

(iii) ar ($\Delta\text{PRQ}) = \frac{1}{2}$ ar ($\triangle\text{ARC})[$ Using result (i)] $\Rightarrow 2\text{ar}(\Delta\text{PRQ}) = \text{ax}(\triangle\text{ARC})$ ..(xii)

ar $(\Delta\text{PRQ}) = \frac{1}{2}$ ar $(\triangle\text{APQ})[\text{RQ}$ is the median of $\Delta_{\text{APQ]}}$……….. (xiii)

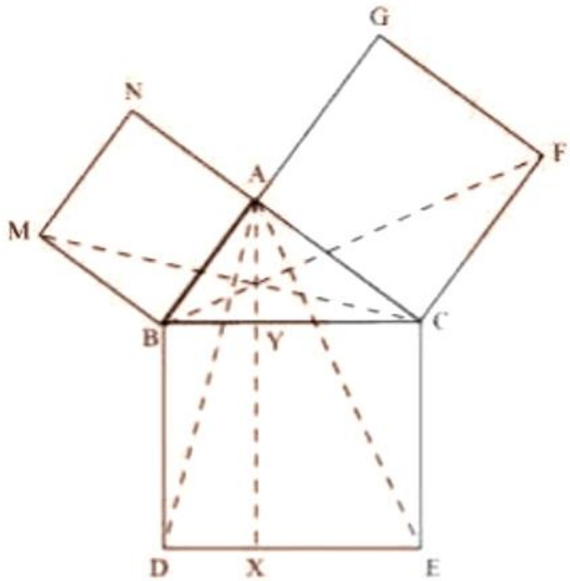
But ar $(\Delta\text{APQ}) =$ ar $(\Delta\text{PQC})[$ Using reason of eq. (vi)] ……… (xiv) From eq. (xiii) and (xiv), we get, ar $(\Delta\text{PRQ}) = \frac{1}{2}$ ar (ΔPQC)………..(xv)

From eq. (x) and (x i), we get,

ar $(\Delta\text{PRQ}) = \frac{1}{2}$ ar (ΔBPQ)……... (xvii)

Now from (xii) and (xvii), we get, $2\times\frac{1}{2}ar(\triangle BPQ) = ar\left(\Delta_{\text{ARC}}\right) \Rightarrow \text{ar}(\Delta\text{BPQ}) = \text{ar}(\Delta\text{ARC})$

8. In the following figure, ABC is a right triangle right angled at A. BCED, ACFG and ABMN are squares on the sides BC, CA and AB respectively. Line segment $AX \perp DE$ meets BC at Y.



(i) $\Delta MBC \cong \Delta ABD$

(ii) $ar(BYXD) = 2\,ar(MBC)$

(iii) $ar(BYXD) = 2\,ar(ABMN)$

(iv) $ar(CYXE) = ar(ACFG)$

(v) $ar(BCED) = ar(ABMN) + ar(ACFG)$

(vi)

(vii)

Note: Result (vii) is the famous Theorem of Pythagoras. You shall learn a simpler proof of this theorem in class X.

Ans: Given: In the following figure, ABC is a right triangle right angled at A. BCED, ACFG and ABMN are squares on the sides BC, CA and AB respectively. Line segment $AX \perp DE$ meets BC at Y.

To prove: (i) $\Delta MBC \cong \Delta ABD$

(ii) $ar(BYXD) = 2\,ar(MBC)$

(iii) $ar(BYXD) = 2\,ar(ABMN)$

(iv) $\Delta FCB \cong \Delta ACE$

(v) $ar(CYXE) = 2\,ar(FCB)$

(vi) $ar(CYXE) = ar(ACFG)$

(vii) $ar(BCED) = ar(ABMN) + ar(ACFG)$

(i) We know that each angle of a square is 90°. Hence, $\angle ABM = \angle DBC = 90^{\circ}$

$\therefore \angle ABM + \angle ABC = \angle DBC + \angle ABC$

$\therefore \angle MBC = \angle ABD$

In $\triangle MBC$ and $\triangle ABD$,

$\angle MBC = \angle ABD$ (Proved abore)

$MB = AB$ (Sides of square ABMIN) $BC = BD$ (Sides of square BCED)

$\therefore \triangle MBC \cong \triangle ABD$ (SAS congruence rule)

(ii) We have $\Delta MBC \cong \Delta ABD$

$\therefore$ ar $(\Delta MBC) = ar(\triangle ABD) \ldots$ (1)

It is given that $AX \perp DE$ and $BD \perp DE$ (Adjacent sides of square BDEC)

$\therefore BD \parallel AX$ (Two lines perpendicular to same line are parallel to each other) $\triangle ABD$ and parallelogram BYXD are on the same base BD and between the same parallels BD and AX. Area $(\Delta YXD) = 2$ Area (ΔMBC) [Using equation (1)] \ldots.. (2)

(iii) $\triangle MBC$ and parallelogram $ABMN$ are lying on the same base MB and between same parallels MB and NC. 2 ar $(\Delta MBC) = ar(ABMN)$

ar $(\Delta YXD) = ar(ABMN)$ [Using equation (2)] ... (3)

(iv) We know that each angle of a square is 90°. $\therefore \angle FCA = \angle BCE = 90^{\circ}$

$\therefore \angle FCA + \angle ACB = \angle BCE + \angle ACB$

$\therefore \angle FCB = \angle ACE$

In $\triangle FCB$ and ΔACE,

$\angle FCB = \angle ACE$

$FC = AC$ (Sides of square ACFG) $CB = CE$ (Sides of square BCED) $\Delta FCB \cong \Delta ACE$ (SAS congruence rule)

(v) It is given that $AX \perp DE$ and $CE \perp DE$ (Adjacent sides of square $BDEC$) Hence, $CE \parallel AX$ (Two lines perpendicular to the same line are parallel to each other) Consider BACE and parallelogram CYXE BACE and parallelogram CYXE are on the same base CE and between the same parallels CE and AX.

$\therefore ar(\Delta YXE) = 2$ ar ($\Delta ACE)\ldots$... (4)

We had proved that

$\therefore \Delta FCB \cong \triangle ACE$

$ar(\Delta FCB) \cong ar(\Delta ACE)\ldots(5)$

On comparing equations (4) and (5), we obtain

$ar(CYXE) = 2$ ar $(\Delta FCB)\ldots(6)$

(vi) Consider BFCB and parallelogram ACFG BFCB and parallelogram ACFG are lying on the same base CF and between the same parallels CF and BG.

$\therefore$ ar $(ACFG) = 2$ ar (ΔFCB)

$\therefore$ ar $(ACFG) = ar(CYXE)[$ Using equation $(6)]\ldots(7)$

(vii) From the figure, it is evident that ar $(\Delta CED) = ar(\Delta YXD) + ar(CYXE)$

$\therefore$ ar $(\Delta CED) = ar(ABMN) + ar(ACFG)$ [Using equations (3) and (7)].