# playing with numbers

16 Chapter

## Exercise 16.1

**1. Find the values of the letters in the following and give reasons for the steps involved.**

$$\begin{array}{r} 3\ A \\ +\ 2\ 5 \\ \hline B\ 2 \end{array}$$

**Ans:**

- The addition of $A$ and $5$ gives $2$ i.e., a number whose one's digit is $2$. This is possible only when the digit $A$ is $7$. In this case, the addition of $A$ $(7)$ and $5$ will give $12$ and thus, $1$ will be the carry for the next step.
- In the next step,

  $1+3+2=6$

  Therefore, the addition is as follows:

  $$\begin{array}{r} 3\ 7 \\ +\ 2\ 5 \\ \hline 6\ 2 \end{array}$$

  Clearly, $B$ is $6$.

  Hence, $A$ and $B$ are $7$ and $6$ respectively.

**2. Find the values of the letters in the following and give reasons for the steps involved.**

$$\begin{array}{r} 4\ A \\ +\ 9\ 8 \\ \hline C\ B\ 3 \end{array}$$

**Ans:**

- The addition of $A$ and $8$ gives $3$ i.e., a number whose one's digit is $3$. This is possible only when digit $A$ is $5$. In this case, the addition of $A$ and $8$ will give $13$ and thus, $1$ will be the carry for the next $1$step.
- In the next step,

  $1+4+9=14$

  Therefore, the addition is as follows:

  $$\begin{array}{r} 4\ 5 \\ +\ 9\ 8 \\ \hline 14\ 3 \end{array}$$

  Clearly, $B$and $C$ are $4$ and respectively.
  Hence, $A$, $B$, and $C$are $5$, $4$, and $1$ respectively.

**3. Find the values of the letters in the following and give reasons for the steps involved.**

$$\begin{array}{r} A\ B \\ +\ 3\ 7 \\ \hline 6\ A \end{array}$$

**Ans:** The addition of $A$ and $3$ is giving $6$. There can be two cases.

**(1) First step is not producing a carry**

- In this case, $A$ comes to be $3$ as $3 + 3 = 6$.
- Considering the first step in which the addition of $B$ and $7$ is giving $A$ (i.e., $3$), $B$ should be a number such that the units digit of this addition comes to be $B$. It is possible only when $B = 6$.
- In this case, $A = 6 + 7 = 13$. However, $A$ is a single digit number. Hence, it is not possible.

**(2) First step is producing a carry**

- In this case, $A$ comes to be $2$ as $1 + 2 + 3 = 6$.
- Considering the first step in which the addition of $B$ and $7$ is giving

$$\begin{array}{r} 2\ 5 \\ +\ 3\ 7 \\ \hline 6\ 2 \end{array}$$

$A$ (i.e., $2$), $B$ should be a number such that the units digit of this addition comes to be $2$. It is possible only when $B = 5$ and $5 + 7 = 12$.
- Hence, the values of $A$ and $B$ are $2$ and $5$ respectively.

**4. Find the values of the letters in the following and give reasons for the steps involved.**

$$\begin{array}{r} A\ B \\ \times\ 3 \\ \hline C\ A\ B \end{array}$$

**Ans:**

- The multiplication of $3$ and $B$ gives a number whose one's digit is $B$ again. Hence, $B$ must be $0$ or $5$.

- Let $B$ is $5$.
  Multiplication of first step $3 \times 5 = 15$
  $1$ will be a carry for the next step.
  We have, $3 \times A + 1 = CA$

  This is not possible for any value of $A$.

  Hence, $B$ must be $0$.

  If $B = 0$, then there will be no carry for the next step. We should obtain, $3 \times A = CA$
- That is, the one's digit of $3 \times A$ should be $A$. This is possible when $A = 5$ or $0$. However, $A$ cannot be $0$ as $AB$ is a two-digit number.
- Therefore, $A$ must be $5$. The multiplication is as follows:

$$\begin{array}{r} 50 \\ \times \quad 3 \\ \hline 150 \end{array}$$

- Hence, the values of $A$, $B$, and $C$ are $5$, $0$, and $1$ respectively.

**5. Find the values of the letters in the following and give reasons for the steps involved.**

$$\begin{array}{r} \mathbf{A\ \ B} \\ \times \quad \mathbf{5} \\ \hline \mathbf{C\ A\ B} \end{array}$$

**Ans:**

- The multiplication of $B$ and $5$ is giving a number whose one's digit is $B$ again. This is possible when $B = 5$ or $B = 0$ only.

- In case of $B=5$, the product,
- $B\times 5=5\times 5=25$

  $2$ will be a carry for the next step.
- We have, $5\times A+2=CA$, which is possible for $A=2$ or $7$

  The multiplication is as follows:

$$\begin{array}{r} 2\ 5 \\ \times\ \ 5 \\ \hline 12\ 5 \end{array} \qquad \begin{array}{r} 7\ \ 5 \\ \times\ \ 5 \\ \hline 3\ 7\ \ 5 \end{array}$$

- If $B=0$,

  $B\times 5=B$

  $\Rightarrow\ 0\times 5=0$
- There will not be any carry in this step. In the next step, $5\times A=CA$.
- It can happen only when $A=5$ or $A=0$

  However, $A$ cannot be $0$ as $AB$ is a two-digit number.
- Hence, $A$ can be $5$ only. The multiplication is as follows:

$$\begin{array}{r} 5\ \ 0 \\ \times\ \ 5 \\ \hline 2\ 5\ \ 0 \end{array}$$

- Hence, there are three possible values of $A$, $B$, and $C$.

(i) $5$, $0$, and $2$ respectively

(ii) $2$, $5$, and $1$ respectively

(iii) $7$, $5$, and $3$ respectively

**6.** **Find the values of the letters in the following and give reasons for the steps involved.**

$$\begin{array}{r} A\ B \\ \times\ \ 5 \\ \hline B\ B\ B \end{array}$$

**Ans:**

The multiplication of $6$ and $B$ gives a number whose one's digit is $B$ again. It is possible only when $B = 0, 2, 4, 6,$ or $8$

- If $B = 0$, then the product will be $0$.
Therefore, this value of $B$ is not possible. If $B = 2$, then $B \times 6 = 12$ and $1$ will be a carry for the next step.
$6A + 1 = BB = 22$
$\Rightarrow 6A = 21$
Hence, any integer value of $A$ is not possible.
- If $B = 6$, then $B \times 6 = 36$ and $3$ will be a carry for the next step.
$6A + 3 = BB = 66$
$\Rightarrow 6A = 63$
Hence, any integer value of $A$ is not possible.
- If $B = 8$, then $B \times 6 = 48$ and $4$ will be a carry for the next step.
$6A + 4 = BB = 88$
$\Rightarrow 6A = 84$
Hence, $A = 14$. However, $A$ is a single digit number. Therefore, this value of $A$ is not possible.
- If $B = 4$, then $B \times 6 = 24$ and $2$ will be a carry for the next step.
$6A + 2 = BB = 44$
$\Rightarrow 6A = 42$
Hence, $A = 7$

- The multiplication is as follows:

$$\begin{array}{r} 7\ 4 \\ \times\ \ 6 \\ \hline 4\,4\ 4 \end{array}$$

Hence, the values of $A$ and $B$ are $7$ and $4$ respectively.

**7. Find the values of the letters in the following and give reasons for the steps involved.**

$$\begin{array}{r} \mathbf{A\ \ 1} \\ \mathbf{+\ 1\ \ B} \\ \hline \mathbf{B\ \ 0} \end{array}$$

**Ans:**

- The addition of $1$ and $B$ gives $0$ i.e., a number whose one's digits is $0$. This is possible only when digit $B$ is $9$.
- In this case, the addition of $1$ and $B$ will give $10$ and thus, $1$ will be the carry for the next step.
- In the next step,

  $1 + A + 1 = B$

  Clearly, $A$ is $7$ as $1 + 7 + 1 = 9 = B$
- Therefore, the addition is as follows:

$$\begin{array}{r} 7\ \ 1 \\ +\ 1\ \ 9 \\ \hline 9\ \ 0 \end{array}$$

Hence, the values of $A$ and $B$ are $7$ and $9$ respectively.

**8.** **Find the values of the letters in the following and give reasons for the steps involved.**

$$\begin{array}{r} 2\,A\,B \\ +\,A\,B\,1 \\ \hline B\,1\,8 \end{array}$$

**Ans:**

- The addition of $B$ and $1$ gives $8$ i.e., a number whose one's digits is $8$. This is possible only when digit $B$ is $7$.
- In this case, the addition of $B$ and $1$ will give $8$. In the next step, $A + B = 1$

  Clearly, $A$ is $4$.

  $4 + 7 = 11$ and 1 will be a carry for the next step.
- In the next step, $1 + 2 + A = B$

  $1 + 2 + 4 = 7$

Therefore, the addition is as follows:

$$\begin{array}{r} 2\,4\,7 \\ +\,4\,7\,1 \\ \hline 7\,1\,8 \end{array}$$

Hence, the values of $A$ and $B$ are $4$ and $7$ respectively.

**9.** **Find the values of the letters in the following and give reasons for the steps involved.**

$$\begin{array}{r} 1\,2\,A \\ +\,6\,A\,B \\ \hline A\,0\,9 \end{array}$$

**Ans:**

- The addition of $A$ and $B$ is giving $9$ i.e., a number whose one's digits is $9$ . The sum can be $9$ only as the sum of two single digit numbers cannot be $19$. Therefore, there will not be any carry in this step.
- In the next step, $2 + A = 0$
  It is possible only when $A = 8$
  $2 + 8 = 10$ and $1$ will be the carry for the next step.
  $1 + 1 + 6 = A$ Clearly, $A$ is $8$. We know that the addition of $A$ and $B$ is giving $9$. As $A$ is $8$, therefore, $B$ is $1$.
- Therefore, the addition is as follows:

$$\begin{array}{r} 1\,2\,8 \\ +\,6\,8\,1 \\ \hline 8\,0\,9 \end{array}$$

Hence, the values of $A$ and $B$ are $8$ and $1$ respectively.

## Exercise 16.2

**1. If $21y5$ is a multiple of $9$, where $y$ is a digit, what is the value of $y$?**

**Ans:**

- If a number is a multiple of $9$, then the sum of its digits will be divisible by $9$.
  Sum of digits of $21y5$:
  $21y5 = 2 + 1 + y + 5$
  $= 8 + y$
- Hence, $8 + y$ should be a multiple of $9$.
  This is possible when $8 + y$ is any one of these numbers $0, 9, 18, 27$, and so on.
- However, since $y$ is a single digit number, this sum can be $9$ only.

  Therefore, $y$ should be $1$ only.

**2. If 31z5 is a multiple of 9, where z is a digit, what is the value of z?**

**You will find that there are two answers for the last problem. Why is this so?**

**Ans:**

- If a number is a multiple of 9, then the sum of its digits will be divisible by 9.
- Sum of digits of 31z5:
  $31z5 = 3 + 1 + z + 5 = 9 + z$
  Hence, $9 + z$ should be a multiple of 9.
- This is possible when $9 + z$ is any one of these numbers 0, 9, 18, 27, and so on.
- However, since z is a single digit number, this sum can be either 9 or 18. Therefore, z should be either 0 or 9.

**3. If 24x is a multiple of 3, where x is a digit, what is the value of x?**

**(Since 24x is a multiple of 3, its sum of digits $6 + x$ is a multiple of 3; so $6 + x$ is one of these numbers: 0, 3, 6, 9, 12, 15, 18…. But since x is a digit, it can only be that $6 + x = 6$ or 9 or 12 or 15. Therefore, $x = 0$ or 3 or 6 or 9. Thus, x can have any of four different values)**

**Ans:**

- Since 24x is a multiple of 3, the sum of its digits is a multiple of 3.
- Sum of digits of $24x = 2 + 4 + x = 6 + x$. Hence, $6 + x$ is a multiple of 3.
- This is possible when $6 + x$ is any one of these numbers 0, 3, 6, 9, and so on … Since x is a single digit number, the sum of the digits can be 6 or 9 or 12 or 15 and thus, the value of x comes to 0 or 3 or 6 or 9. respectively.
- Thus, x can have its value as any of the four different values 0, 3, 6, or 9.

**4. If $31z5$ is a multiple of $3$, where $z$ is a digit, what might be the values of $z$?**

**Ans:**

- Since $31z5$ is a multiple of $3$, the sum of its digits will be a multiple of $3$. Hence, $3 + 1 + z + 5 = 9 + z$ is a multiple of $3$.
- This is possible when $9 + z$ is any one of the following: $0, 3, 6, 9, 12, 15, 18,$ and so on …
- Since $z$ is a single digit number, the value of $9 + z$ can only be $9, 12, 15$ or $18$ and thus, the value of $x$ comes to $0, 3, 6,$ or $9$ respectively.
- Thus, $z$ can have its value as any one of the four different values $0, 3, 6,$ or $9$.