circles

Chapter 9

Exercise 9.1

1: Recall that two circles are congruent if they have the same radii. Prove that equal chords of congruent circles subtend equal angles at their centres.

Ans: Let us consider a circle with center O and two equal chords of a circle AB and CD.

We need to prove that $\angle AOB = \angle COD$

In $\triangle$ AOB and COD, we have
AO=CO (Radius of the circle)

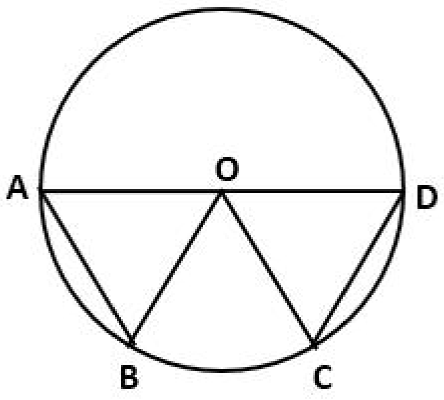
BO=DO (Radius of the circle)

AB=CD (Equal chords)

By SAS criterion of congruence, we have

$\triangle AOB \cong \triangle COD$

$\Rightarrow \angle AOB = \angle COD$



2: Prove that if chords of congruent circles subtend equal angles at their centre, then the chords are equal.

Ans: In △AOB and △PXQ

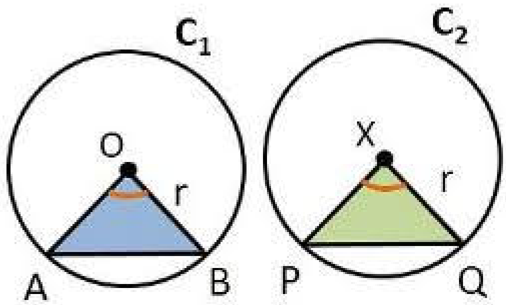
AO=PX [Radius of congruent circles are equal]

∠AOB=∠PXQ [Given]

BO=QX [Radius of congruent circles are equal]

△AOB≅△PXQ [SAS congruence rule]

∴AB=PQ [CPCT]



EXERCISE 9.2

1: Two circles of radii 5 cm and 3 cm intersect at two points and the distance between their centres is 4 cm. Find the length of the common chord.

Ans: Let the common chord be AB and P and Q be the centers of the two circles.

$\therefore$ AP = 5cm and AQ = 3cm.

PQ = 4cm

given

Now, segPQ $\perp$ chord AB

$\therefore$ AR = RB = $\frac{1}{2}$AB....perpendicular from center to the chord, bisects the chord

Let $PR = xcm$, so $RQ = (4 - x)cm$

In Δ ARP,

$AP^2 = AR^2 + PR^2$

$AR^2 = 5^2 - x^2$

In Δ ARQ,

$AQ^2 = AR^2 + QR^2$

$AR^2 = 3^2 - (4 - x)^2$

$\therefore 5^2 - x^2 = 3^2 \quad (4 - x)^2$

from (1) \& (2)

$25 - x^2 = 9 - (16 - 8x + x^2)$

$25 - x^2 = -7 + 8x - x^2$

$32 = 8x$

$\therefore x = 4$

Substitute in eq(1) we get,

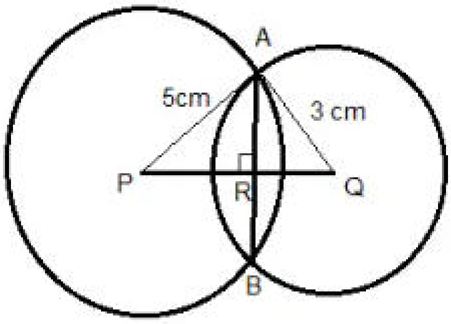
$AR^2 = 25 - 16 = 9$

$\therefore AR = 3cm.$

$\therefore AB = 2 \times AR = 2 \times 3$

$\therefore AB = 6cm.$

So, length of common chord AB is 6cm



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2: If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of other chord.

Ans: Drop a perpendicular from O to both chords AB and CD

In △OMP and △ONP

As chords are equal, perpendicular from centre would also be equal.

OM=ON

OP is common.

$\angle OMP = \angle ONP = 90^{O}$

△OMP ≅ △ONP (RHS Congruence)

PM=PN ....(1)

AM=BM (Perpendicular from centre bisects the chord)

Similarly ,CN=DN

As AB=CD

AB−AM=CD−DN

BM=CN   ....(2)

From (1) and (2)

BM−PM=CN−PN

BM−PM=CN−PN

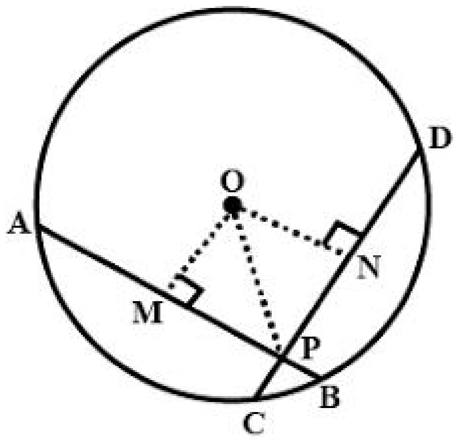
PB=PC

AM=DN   (Half the length of equal chords are equal)

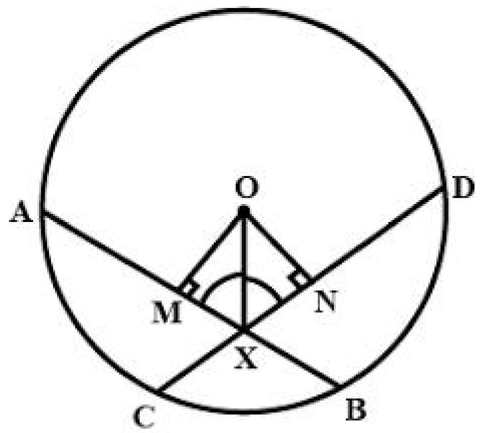
AM+PM=DN+PN

AP=PD

Therefore , PB=PC and AP=PD is proved.



3: If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersection to the centre makes equal angles with the chords.



Ans: In △OMX and △ONX,

∠OMX=∠ONX=90°

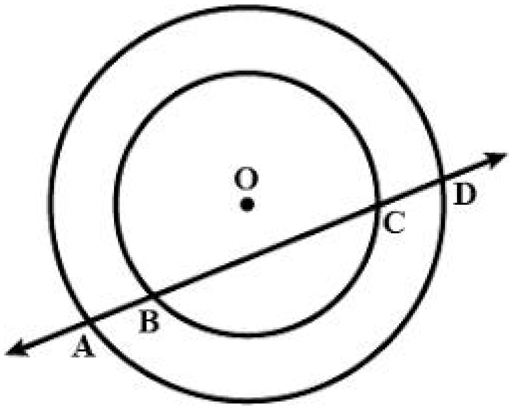
OX=OX(common)
OM=ON where AB and CD are equal chords and equal chords are equidistant from the centre.

△OMX≅△ONX by RHS congruence rule.

∴∠OXM=∠OXN

i.e.,∠OXA=∠OXD
Hence proved.

4: If a line intersects two concentric circles (circles with the same centre) with centre O at A,B,C and D, prove that AB=CD (see figure)



Ans: We know that, OA=OD and OB=OC.

They are radius of respective circles.

In ΔOBC, we know that OB = OC, so ∠OBC =∠OCB

∴∠OCD =∠OBA

In ΔOAD, we know that OA= OD, so ∠OAD =∠ODA

Since, ∠OCD =∠OBA and ∠OAD =∠ODA, we get ∠AOB in ΔOAB is equal to ∠COD in ΔOCD.

∴ From SAS congruency, we can say that ΔOAB and ΔOCD are congruent.

So, AB=CD.

5: Three girls Reshma, Salma and Mandip are playing a game by standing on a circle of radius 5 m drawn in a park. Reshma throws a ball to Salma, Salma to Mandip, Mandip to Reshma. If the distance between Reshma and Salma and between Salma and Mandip is 6 m each, what is the distance between Reshma and Mandip?

Ans: Draw perpendicular OA and OB on RS and SM respectively

$AR = AS = \frac{1}{6} = 3m$

$OR = OS = OM =$ 5m. (Radii of the circle) ln

$OA^2 + AR^2 = OR^2$

$OA^1 + (3m^2) = (5m)^2$

$OA^2 = (25 - 9)m^2 = 16m^2$

$OA = 4m$

ORSM will be a kite (OR $= OM$ and $RS = SM$). We know that diagonals of a

kite are perpendicular and the diagonal common to both the isosceles traingle is bisected by another diagonal

$\therefore \angle$RCS will be of 90° and RC = CM

Area of $\triangle ORS = \frac{1}{2} \times OA \times RS$

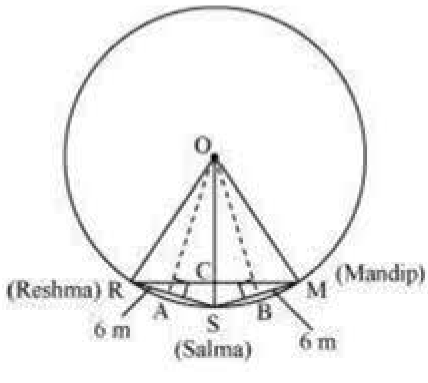
$\frac{1}{2} \times RC \times OS = \frac{1}{2} \times 4 \times 6$

$RC \times 5 = 24$

RC = 4.8

RM = 2RC = 2(4.8) = 9.6

Therefore, the distance between Reshma and Mandip is 9.6m.



6: A circular park of radius 20m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in his hands to talk to each other. Find the length of the string of each phone

Ans: Let Ankur be represented as A, Syed as S and David as D.

The boys are sitting at an equal distance.

Hence, $\triangle$ASD is an equilateral triangle.Let the radius of the circular park be r meters.

$\therefore$OS=r=20m.

Let the length of each side of △ASD be x meters.

Draw AB⊥SD

∴SB=BD= $\frac{1}{2}$ SD= $\frac{x}{2}$ m

In $\triangle ABS, \angle B = 90°$

By Pythagoras theorem,

$$AS^2 = AB^2 + BS^2$$

$$\therefore AB^2 = AS^2 - BS^2 = x^2 - (\frac{x}{2})^2 = \frac{3x^2}{4}$$

$$\therefore AB = \frac{\sqrt{3}x}{2} m$$

Now, $AB = AO + OB$

$$OB = AB - AO$$

$$OB = \left(\frac{\sqrt{3}x}{2} - 20\right) m$$

In $\triangle OBS$,

$$OS^2 = OB^2 + SB^2$$

$$20^2 = (\frac{\sqrt{3}x}{2} - 20)^2 + (\frac{x}{2})^2$$

$$400 = \frac{3}{4}x^2 + 400 - 2(20)(\frac{\sqrt{3}x}{2}) + \frac{x^2}{4}$$

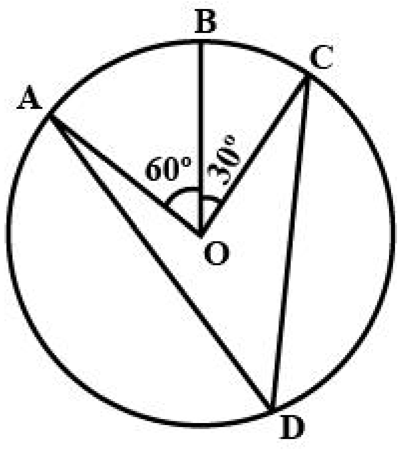
$$0 = x^2 - 20\sqrt{3}x$$

$$\therefore x = 20\sqrt{3}m$$

The length of the string of each phone is $20\sqrt{3}$m.

EXERCISE 9.3

1: In figure A,B and C are three points on a circle with centre O such that ∠BOC=30º and ∠AOB=60º
. If D is a point on the circle other than the arc ABC, find ∠ADC.



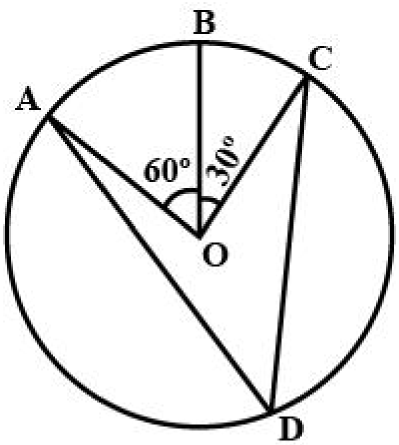
Ans: Correct option is C)

According to theorem of circles ∠AOC=2∠ADC .....(i)

∠AOC=∠AOB+∠BOC=60° +30° =90° from equation(i)

90° =2∠ADC

∠ADC=45°



2: A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc.

Ans: Consider the figure.

Given,
AB is equal to the radius of the circle.

In △OAB,

OA=OB=AB= radius of the circle.

Thus, △OAB is an equilateral triangle.

and ∠AOC=60°.

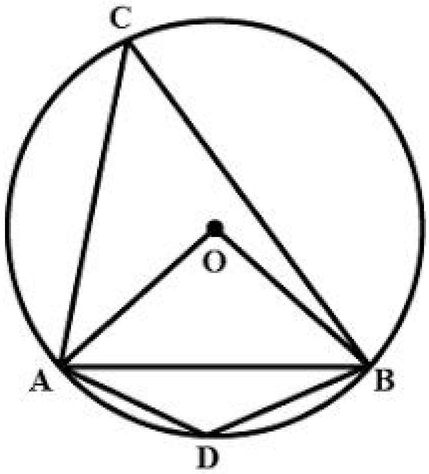
Also, ∠ACB= $\frac{1}{2}$∠AOB= $\frac{1}{2}$ ×60°=30°.

Since, ACBD is a cyclic quadrilateral,

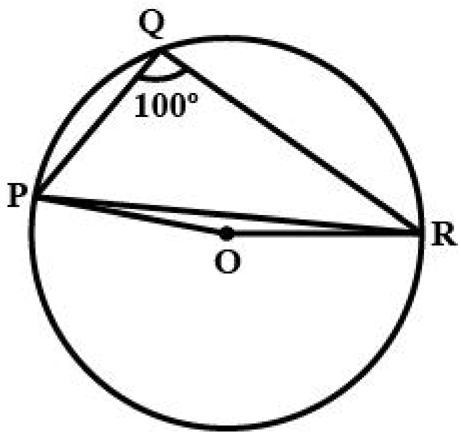
∠ACB+∠ADB=180° ....[Opposite angles of cyclic quadrilateral are supplementary]

⇒∠ADB=180°−30°=150°.

Thus, angle subtend by the chord at a point on the minor arc and also at a point on the major arc are 150° and 30°, respectively.



3: In the figure,∠PQR=100°, where P,Q and R are points on a circle with centre O. Find ∠OPR.
arc are 150° and 30°, respectively.



Ans: Given:
$\angle PQR = 100°$

$\angle POR = 2 \times \angle PQR = 2 \times 100° = 200°$

$\therefore \angle POR = 360° - 200° = 160°$

In ΔOPR,

$\Rightarrow OP = OR$ ...Radii of the circle

$\Rightarrow \angle OPR = \angle ORP$

Now,

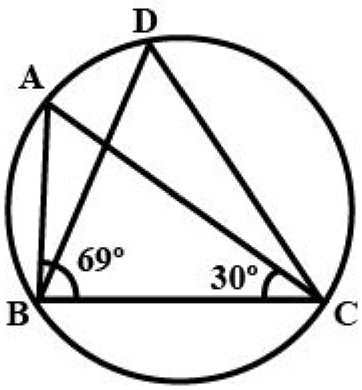
$\angle OPR+\angle ORP+\angle POR=180°$ ...Sum of the angles in a triangle

$\Rightarrow \angle OPR+\angle OPR+160°=180°$

$\Rightarrow 2\angle OPR=180°-160°$

$\Rightarrow \angle OPR=10°$

4: In the figure, $\angle ABC=69^o, \angle ACB=31^o$, find $\angle BDC$.



Ans: $\angle BAC=\angle BDC$ ...Angles in the segment of the circle

In ΔABC,

$\Rightarrow \angle BAC+\angle ABC+\angle ACB=180°$ ...Sum of the angles in a triangle

$\Rightarrow \angle BAC+69°+31°=180°$

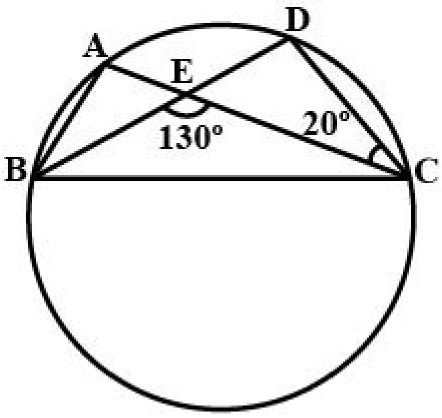
$\Rightarrow \angle BAC=180°-100°$

$\Rightarrow \angle BAC=80°$

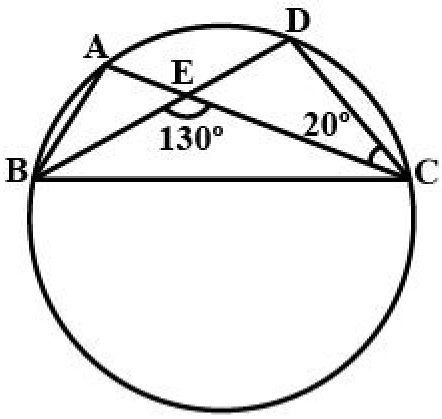
Thus, $\angle BDC=80°$

5: In figure, A,B,C and D are four points on a circle. AC and BD intersect at a point E such that $\angle BEC=130_o$ and $\angle ECD=20_o$. Find $\angle BAC$.

5: In figure, A,B,C and D are four points on a circle. AC and BD intersect at a point E



Ans: Correct option is A)
Here, ∠BEC+∠DEC=180° (Linear pairs are complimentary) 130°+∠DEC=180° ∠DEC=50° In ΔDEC, ∠DEC+∠ECD+∠CDE=180° ...(Angle sum property) 50° +20° +∠CDE=180° ∠CDE=110° By theorem of circles, ∠BAC=∠CDB ∠BAC=∠CDE=110°



6: ABCD is a cyclic quadrilateral whose diagonals intersect at a point E. If ΔDBC=70°, ΔBAC is 30° , find ∠ BCD. Further if AB=BC, find ∠ ECD.

Ans: For chord CD,

∠CBD=∠CAD ...Angles in same segment

∠CAD=70°

∠BAD=∠BAC+∠CAD=30°+70°=100°

∠BCD+∠BAD=180° ...Opposite angles of a cyclic quadrilateral

⇒∠BCD+100°=180°

$\Rightarrow \angle BCD = 80°$

In $\triangle ABC$

AB=BC (given)

$\angle BCA = \angle CAB$ ...Angles opposite to equal sides of a triangle

$\angle BCA = 30°$

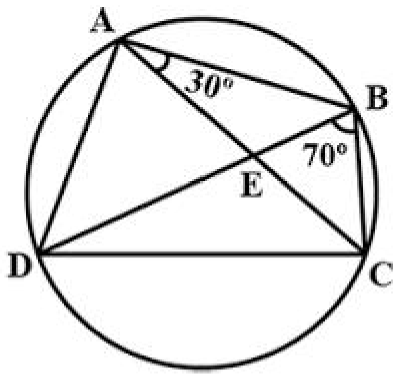
Also, $\angle BCD = 80°$

$\angle BCA + \angle ACD = 80°$

$\Rightarrow 30° + \angle ACD = 80°$

$\angle ACD = 50°$

$\angle ECD = 50°$



7: If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, prove that it is a rectangle.

Ans: Given that,

A cyclic quadrilateral .

ABCD,AC and BD$ are diameters of the circle where they meet at center O of the circle.

To prove:ABCD is a rectangle.

Proof: In triangle ΔAOD and ΔBOC,

OA=OC (both are radii of same circle)

$\angle AOD = \angle BOC$ (vert.opp$\angle$S)

OD=OB(both are radii of same circle)

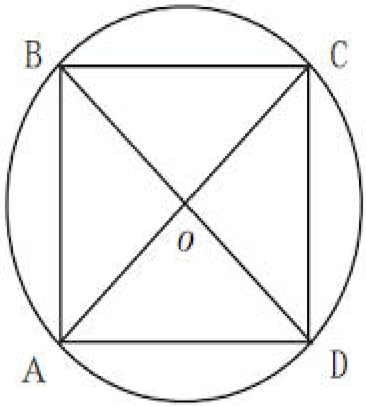
$\therefore \Delta AOD \cong \Delta BOC \Rightarrow AD = BC$(C.P.C.T)

Similarly,by taking ΔAOB and ΔCOD,AB=DC

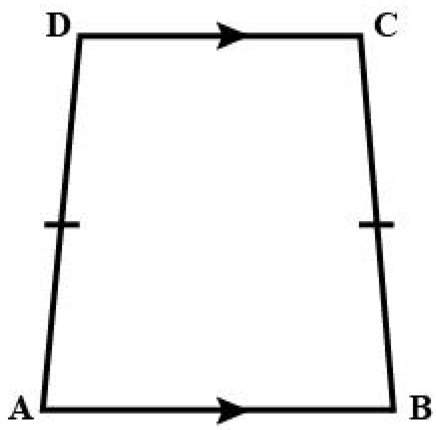
Also, $\angle BAD = \angle ABC = \angle BCD = \angle ADC = 900$

(angle in a semicircle)

$\therefore$ ABCD is a rectangle.



8: If the non-parallel sides of a trapezium are equal, prove that it is cyclic.



Ans: Given:
ABCD is a trapezium where non-parallel sides AD and BC are equal.

Construction:
DM and CN are perpendicular drawn on AB from D and C, respectively.

To prove:
ABCD is cyclic trapezium.

Proof:
In △DAM and △CBN,

AD=BC ...[Given]

∠AMD=∠BNC ...[Right angles]

DM=CN ...[Distance between the parallel lines]

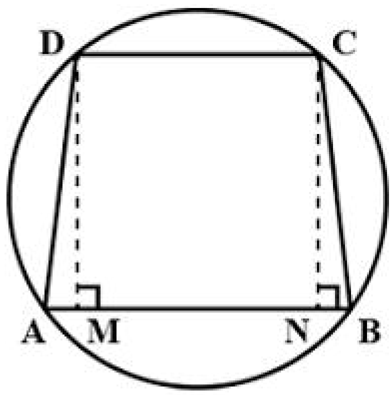
Therefore, $\triangle DAM \cong \triangle CBN$ by RHS congruence condition.

Now, $\angle A = \angle B$ ...[by CPCT]

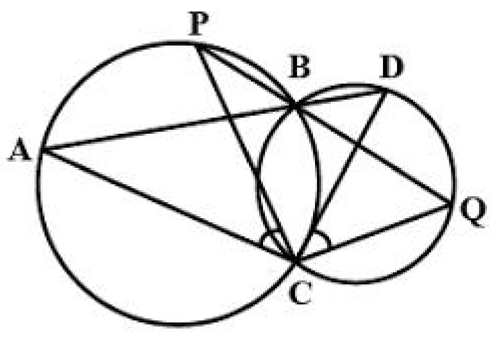
Also, $\angle B + \angle C = 180°$ ....[Sum of the co-interior angles]

$\Rightarrow \angle A + \angle C = 180°$.

Thus, ABCD is a cyclic quadrilateral as the sum of the pair of opposite angles is 180°.



9: Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A,D and P,Q respectively (see figure). Prove that $\angle ACP = \angle QCD$.



Ans: $\Rightarrow$ Chords AP and DQ are joined.

$\Rightarrow$ For chord AP,

$\Rightarrow \angle PBA = \angle ACP$ ...Angles in the same segment --- (i)

⇒For chord DQ,

⇒∠DBQ=∠QCD ...Angles in same segment --- (ii)

⇒ABD and PBQ are line segments intersecting at B.

⇒∠PBA=∠DBQ ...Vertically opposite angles --- (iii)

By the equations (i), (ii) and (iii),

∠ACP=∠QCD

10: If circles are drawn taking two sides of a triangle as diameters, prove that the point of intersection of these circles lie on the third side.

Ans: Given,

Two circles are drawn on the sides AB and AC of the triangle

ABC as diameters. The circles intersected at D.

Construction: AD is joined.

To prove: D lies on BC. We have to prove that BDC is a straight line.

Proof:

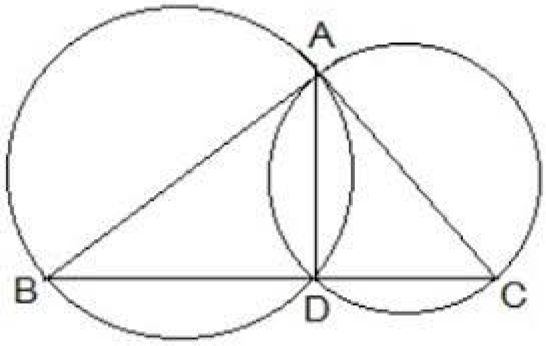
∠ADB=∠ADC=90° ...Angle in the semi circle

Now,

∠ADB+∠ADC=180°

⇒∠BDC is straight line.

Thus, D lies on the BC.



11: ABC and ADC are two right triangles with common hypotenuse AC. Prove that ∠CAD=∠CBD.

Ans: Given,

AC is the common hypotenuse. ∠B=∠D=90°.

To prove,

∠CAD=∠CBD

Proof:

Since, ∠ABC and ∠ADC are 90°.

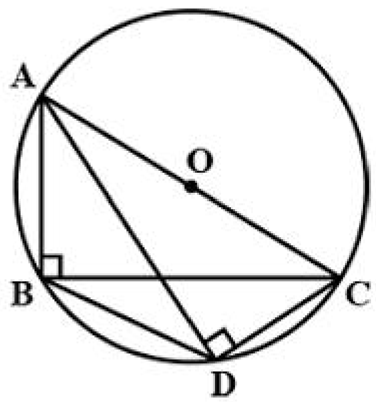
These angles are in the semi-circle.

Thus, both the triangles are lying in the semi-circle and AC is the diameter of the circle.

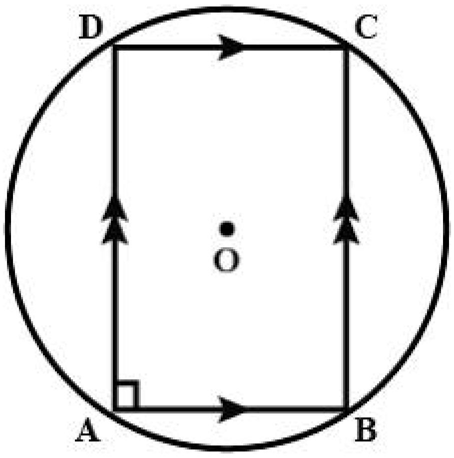
⇒ Points A,B,C and D are concyclic.

Thus, CD is the chord.

⇒∠CAD=∠CBD ...Angles in the same segment of the circle



12: Prove that a cyclic parallelogram is a rectangle.



Ans: Given,

ABCD is a cyclic parallelogram.

To prove,
ABCD is a rectangle.

Proof:
$\angle 1+\angle 2=180°$ ...Opposite angles of a cyclic parallelogram

Also, Opposite angles of a cyclic parallelogram are equal.
Thus,

$\angle 1=\angle 2$

$\Rightarrow \angle 1+\angle 1=180°$

$\Rightarrow \angle 1=90°$

One of the interior angle of the parallelogram is right angled. Thus,
ABCD is a rectangle.