quadrilaterals

8 Chapter

EXERCISE 8.1

1. If the diagonals of a parallelogram are equal, then show that it is a rectangle.

Ans:

Given: Diagonals of the parallelogram are the same.

To prove: It is a rectangle.

Consider ABCD be the given parallelogram.

Now we need to show that ABCD is a rectangle, by proving that one of its interior angles is .

In ΔABC and ΔDCB,

AB = DC (side opposite to the parallelogram are equal)

BC = BC (in common)

AC = DB (Given)

$\therefore \Delta ABC \cong \Delta DCB$

(By SSS Congruence rule)

$\Rightarrow \angle ABC = \angle DCB$

The sum of the measurements of angles on the same side of a transversal is known to be 180^o

.Hence, ABCD is a rectangle because it is a parallelogram with a 90^o inner angle.

i.e.

$OA = OC,\ OB = OD$, and .

In order to prove ABCD a rhombus, we need to prove ABCD is the parallelogram and all sides of the ABCD are the same.

In ΔAOD

and ΔCOD,

OA = OC (Diagonals of the parallelogram bisect each other)

$\angle AOD = \angle COD$

(Given)

OD = OD (Common)

$\therefore \Delta AOD \cong \Delta COD$

(By SAS congruence rule)

$\therefore AD = CD \ldots$ (1)

Similarly, it can be proved that

$AD = AB$ and $CD = BC \ldots$ (2)

From Equations (1) and (2),

$AB = BC = CD = AD$

To conclude, ABCD is a parallelogram because the opposite sides of the quadrilateral ABCD are equal. ABCD is a rhombus because all of the sides of a parallelogram ABCD are equal.

2. Show that the diagonals of a square are equal and bisect each other at right angles.

Ans:

Given: A square is given.

To find: The diagonals of a square are the same and bisect each other at 90^o

Consider ABCD be a square.

Consider the diagonals AC and BD intersect each other at a point O.

We must first show that the diagonals of a square are equal and bisect each other at right angles,

$\mathrm{AC} = \mathrm{BD}, \mathrm{OA} = \mathrm{OC}, \mathrm{OB} = \mathrm{OD}$, and .

In ΔABC and ΔDCB,

$AB = DC$

(Sides of the square are equal)

$\angle ABC = \angle DCB$

(All the interior angles are of the value 90^o)

$BC = CB$

(Common side)

$\therefore \Delta ABC \cong \Delta DCB$

(By SAS congruency)

$\therefore AC = DB$

(By CPCT)

Hence, the diagonals of a square are equal in length.

In ΔAOB and ΔCOD,

$\angle AOB = \angle COD$

(Vertically opposite angles)

$\angle ABO = \angle CDO$

(Alternate interior angles)

AB = CD (Sides of a square are always equal)

$\therefore \Delta AOB \cong \Delta COD$

(By AAS congruence rule)

$\therefore AO = CO$ and $OB = OD$

(By CPCT)

As a result, the diagonals of a square are bisected.

In ΔAOB and ΔCOB,

Because we already established that diagonals intersect each other,

$AO = CO$

$AB = CB$

(Sides of a square are equal)

$BO = BO$

(Common)

$\therefore \Delta AOB \cong \Delta COB$

(By SSS congruency)

$\therefore \angle AOB = \angle COB$

(By CPCT)

However, (Linear pair)

As a result, the diagonals of a square are at right angles to each other.

3. **Diagonal AC of a parallelogram ABCD is bisecting $\angle A$ (see the given figure). Show that**

(i) It is bisecting $\angle C$ also,

(ii) ABCD is a rhombus

Ans:

Given: Diagonal AC of a parallelogram ABCD is bisecting $\angle A$

To find: (i) It is bisecting $\angle C$ also,

(ii) ABCD is a rhombus

(i) ABCD is a parallelogram.

$\angle DAC = \angle BCA$

(Alternate interior angles) ... (1)

And,

$\angle BAC = \angle DCA$

(Alternate interior angles) ... (2)

However, it is given that AC is bisecting $\angle A$

$\angle DAC = \angle BAC$ ... (3)

From Equations (1), (2), and (3), we obtain

$\angle DAC = \angle BCA = \angle BAC = \angle DCA$ ... (4)

$\angle DCA = \angle BCA$

Hence, AC is bisecting $\angle C$

(ii) From Equation (4), we obtain

$\angle DAC = \angle DCA$

$DA = DC$

(Side opposite to equal angles are equal)

However,

$DA = BC$ and $AB = CD$

(Opposite sides of a parallelogram)

$AB = BC = CD = DA$

As a result, ABCD is a rhombus.

4. ABCD is a rectangle in which diagonal AC bisects $\angle A$ as well as $\angle C$. Show that:

(i) ABCD is a square

Ans: It is given that ABCD is a rectangle.

$\angle A = \angle C$

$\Rightarrow \frac{1}{2}\angle A = \frac{1}{2}\angle C$ (AC bisects $\angle A$ and $\angle C$)

$\Rightarrow \angle DAC = \frac{1}{2}\angle DCA$

CD = DA (Sides that are opposite to the equal angles are also equal)

Also,

$DA = BC$ and $AB = CD$

(Opposite sides of the rectangle are same)

$AB = BC = CD = DA$

ABCD is a rectangle with equal sides on all sides.

Hence, ABCD is a square.

(ii) Diagonal BD bisects $\angle B$ as well as $\angle D$.

Ans:

Let us now join BD.

In ΔBCD,

$BC = CD$

(Sides of a square are equal to each other)

$\angle CDB = \angle CBD$

(Angles opposite to equal sides are equal)

However,

$\angle CDB = \angle ABD$

(Alternate interior angles for $AB \parallel CD$)

$\angle CBD = \angle ABD$ BD bisects $\angle B.$

Also,

$\angle CBD = \angle ADB$

(Alternate interior angles for $BC \parallel AD$)

$\angle CDB = \angle ABD$ BD bisects $\angle D$ and $\angle B$

.

5. In parallelogram ABCD, two points P and Q are taken on diagonal BD such that DP = BQ (see the given figure). Show that:

(i) $\Delta APD \cong \Delta CQB$

Ans: (i) In ΔAPD and ΔCQB,

$\angle ADP = \angle CBQ$ (Alternate interior angles for $BC \parallel AD$)

$AD = CB$ (Opposite sides of the parallelogram ABCD)

$DP = BQ$ (Given)

$\therefore \Delta APD \cong \Delta CQB$ (Using SAS congruence rule)

(ii) $AP = CQ$

Ans: As we had observed that $\Delta APD \cong \Delta CQB$,

$\therefore AP = CQ$ (CPCT)

(iii) $\Delta AQB \cong \Delta CPD$

Ans: In ΔAQB and ΔCPD,

$\angle ABQ = \angle CDP$ (Alternate interior angles for $AB \parallel CD$)

$AB = CD$ (Opposite sides of parallelogram ABCD)

$BQ = DP$ (Given)

$\therefore \Delta AQB \cong \Delta CPD$ (Using SAS congruence rule)

(iv) $AQ = CP$

Ans: Since we had observed that

$\Delta AQB \cong \Delta CPD$,

$\therefore AQ = CP$ (CPCT)

(v) APCQ is a parallelogram

Ans: From the result obtained in (ii) and (iv),

$AQ = CP$ and $AP = CQ$

APCQ is a parallelogram because the opposite sides of the quadrilateral are equal.

6. ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD (See the given figure). Show that

(i) $\Delta APB \cong \Delta CQD$

Ans: In ΔAPB and ΔCQD,

$\angle APB = \angle CQD$ (Each 90°)

$AB = CD$

(The opposite sides of a parallelogram ABCD)

$\angle ABP = \angle CDQ$ (Alternate interior angles for $AB \parallel CD$)

$\therefore \Delta APB \cong \Delta CQD$ (By AAS congruency)

(ii) $AP = CQ$

Ans: By using

$\therefore \Delta APB \cong \Delta CQD$, we obtain

$AP = CQ$ (By CPCT)

7. ABCD is a trapezium in which AB || CD and AD = BC (see the given figure). Show that

[Hint: Extend AB and draw a line through C parallel to DA intersecting AB produced at E.]

(i) ∠A = ∠B

Ans: $AD = CE$ (Opposite sides of parallelogram AECD)

However,

$AD = BC$ (Given)

Therefore,

$BC = CE$

$\angle CEB = \angle CBE$ (Angle opposite to the equal sides are also equal)

Consideing parallel lines AD and CE. AE is the transversal line for them.

(Angles on a same side of transversal)

(Using the relation ∠CEB = ∠CBE) ... (1)

However, (Linear pair angles) ... (2)

From Equations (1) and (2), we obtain

$\angle A = \angle B$

(ii) ∠C = ∠D

Ans: $AB \parallel CD$ (Angles on a same side of the transversal)

Also,

$\angle C + \angle B = 180^{\circ}$ (Angles on a same side of a transversal)

$\therefore \angle A + \angle D = \angle C + \angle B$

However,

$\angle A = \angle B$ [Using the result obtained in (i)]

$\therefore \angle C = \angle D$

(iii) $\Delta ABC \cong \Delta BAD$

Ans: In ΔABC and ΔBAD,

$AB = BA$ (Common side)

$BC = AD$ (Given)

$\angle B = \angle A$ (Proved before)

$\therefore \Delta ABC \cong \Delta BAD$ (SAS congruence rule)

(iv) diagonal AC = diagonal BD

Ans: We had seen that,

$\Delta ABC \cong \Delta BAD$

$\therefore AC = BD$ (By CPCT)

Exercise - 8.2

1. ABCD is a quadrilateral in which P, Q, R and S are mid-points of the sides AB, BC, CD and DA (see the given figure). AC is a diagonal. Show that:

(i) $SR \parallel AC$ **and** $SR = \frac{1}{2} AC$

Ans: In ΔADC, S and R are the mid-points of sides AD and CD respectively.

In a triangle, the line segment connecting the midpoints of any two sides is parallel to and half of the third side.

$\therefore SR \parallel AC$ and $SR = \frac{1}{2} AC$ ... (1)

(ii) PQ = SR

Ans: In ΔABC, P and Q are mid-points of sides AB and BC respectively. Therefore, by using midpoint theorem,

$PQ \parallel AC$ and $PQ = \frac{1}{2} AC$ ... (2)

Using Equations (1) and (2), we obtain

$PQ \parallel SR$ and $PQ = \frac{1}{2} SR$ ... (3)

$\therefore PQ = SR$

(iii) PQRS is a parallelogram.

Ans: From Equation (3), we obtained

$PQ \parallel SR$ and $PQ = SR$

Clearly, one pair of quadrilateral PQRS opposing sides is parallel and equal.

PQRS is thus a parallelogram.

2. ABCD is a rhombus and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rectangle.

Ans:

Given: ABCD is a rhombus and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively.

To find: Quadrilateral PQRS is a rectangle

In ΔABC, P and Q are the mid-points of sides AB and BC respectively.

$PQ \parallel AC$, $PQ = \frac{1}{2} AC$ (Using mid-point theorem) ... (1)

In ΔADC,

R and S are the mid-points of CD and AD respectively.

$RS \parallel AC$, $RS = \frac{1}{2} AC$ (Using mid-point theorem) ... (2)

From Equations (1) and (2), we obtain

$PQ \parallel RS$ and $PQ = RS$

It is a parallelogram because one pair of opposing sides of quadrilateral PQRS is equal and parallel to each other. At position O, the diagonals of rhombus ABCD should cross.

In quadrilateral OMQN,

$MQ \parallel ON$ ($PQ \parallel AC$)

$QN \parallel OM$ ($QR \parallel BD$)

Hence , OMQN is a parallelogram.

$\therefore \angle MQN = \angle NOM$
$\therefore \angle PQR = \angle NOM$

Since,

$\angle NOM = 90^\circ$ (Diagonals of the rhombus are perpendicular to each other)

$\therefore \angle PQR = 90^\circ$

Clearly, PQRS is a parallelogram having one of its interior angles as .

So , PQRS is a rectangle.

3. ABCD is a rectangle and P, Q, R and S are mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rhombus.

Ans:

Given: ABCD is a rectangle and P, Q, R and S are mid-points of the sides AB, BC, CD and DA respectively.

To prove: The quadrilateral PQRS is a rhombus.

Let us join AC and BD.

In ΔABC,

P and Q are the mid-points of AB and BC respectively.

$\therefore PQ \parallel AC$ and $PQ = \frac{1}{2} AC$ (Mid-point theorem) ... (1)

Similarly in ΔADC,

$SR \parallel AC$, $SR = \frac{1}{2} AC$ (Mid-point theorem) ... (2)

Clearly,

$PQ \parallel SR$ and $PQ = SR$

It is a parallelogram because one pair of opposing sides of quadrilateral PQRS is equal and parallel to each other.

$\therefore PS \parallel QR$, $PS = QR$ (Opposite sides of parallelogram) ... (3)

In ΔBCD, Q and R are the mid-points of side BC and CD respectively.

$\therefore QR \parallel BD$, $QR = \frac{1}{2} BD$ (Mid-point theorem) ... (4)

Also, the diagonals of a rectangle are equal.

$\therefore AC = BD \ldots(5)$

By using Equations (1), (2), (3), (4), and (5), we obtain

$PQ = QR = SR = PS$

So , PQRS is a rhombus

4. ABCD is a trapezium in which $AB \parallel DC$, BD is a diagonal and E is the mid - point of AD. A line is drawn through E parallel to AB intersecting BC at F (see the given figure). Show that F is the mid-point of BC.

Ans:

Given: ABCD is a trapezium in which $AB \parallel DC$, BD is a diagonal and E is the mid - point of AD. A line is drawn through E parallel to AB intersecting BC at F.

To prove: F is the mid-point of BC.

Let EF intersect DB at G.

We know that a line traced through the mid-point of any side of a triangle and parallel to another side bisects the third side by the reverse of the mid-point theorem.

In ΔABD,

$EF \parallel AB$ and E is the mid-point of AD.

Hence , G will be the mid-point of DB.

As

$EF \parallel AB$, $AB \parallel CD$,

$\therefore EF \parallel CD$ (Two lines parallel to the same line are parallel)

In ΔBCD,

$GF \parallel CD$ and G is the mid-point of line BD. So , by using converse of mid-point theorem, F is the mid-point of BC.

5. In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively (see the given figure). Show that the line segments AF and EC trisect the diagonal BD.

Ans:

Given: In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively To prove: The line segments AF and EC trisect the diagonal BD.

ABCD is a parallelogram.

$AB \parallel CD$

And hence,

$AE \parallel FC$

Again, AB = CD (Opposite sides of parallelogram ABCD)

$\frac{1}{2}AB = \frac{1}{2}CD$

$AE = FC$ (E and F are mid-points of side AB and CD)

In quadrilateral AECF, one pair of the opposite sides (AE and CF) is parallel and same to each other. So , AECF is a parallelogram.

$\therefore AF \parallel EC$ (Opposite sides of a parallelogram)

In ΔDQC, F is the mid-point of side DC and

$FP \parallel CQ$ (as $AF \parallel EC$).

So , by using the converse of mid-point theorem, it can be said that P is the mid-point of DQ.

$\therefore DP = PQ$ ... (1)

Similarly, in

ΔAPB, E is the mid-point of side AB and

$EQ \parallel AP$ (as $AF \parallel EC$).

As a result, the reverse of the mid-point theorem may be used to say that Q is the mid-point of PB.

$\therefore PQ = QB$ ... (2)

From Equations (1) and (2),

$DP = PQ = BQ$

Hence, the line segments AF and EC trisect the diagonal BD.

6. **ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D. Show that**

(i) D is the mid-point of AC

Ans: In ΔABC,

It is given that M is the mid-point of AB and

$MD \parallel BC$.

Therefore, D is the mid-point of AC. (Converse of the mid-point theorem)

(ii) MD $\perp$ AC

Ans: As $DM \parallel CB$ and AC is a transversal line for them, therefore,

(Co-interior angles)

(iii) $CM = MA = \frac{1}{2}AB$

Ans: Join MC.

In ΔAMD and ΔCMD,

$AD = CD$ (D is the mid-point of side AC)

$\angle ADM = \angle CDM$ (Each)

DM = DM (Common)

$\therefore \Delta AMD \cong \Delta CMD$ (By SAS congruence rule)

Therefore,

$AM = CM$ (By CPCT)

However,

$AM = \frac{1}{2}AB$ (M is mid-point of AB)

Therefore, it is said that

$$CM = AM = \frac{1}{2}AB$$