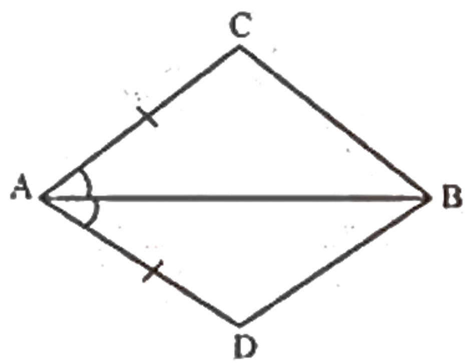
triangles

7 Chapter

Exercise 7.1

1. In quadrilateral $ACBD$, $AC = AD$ and AB bisects $\angle A$ (See the given figure). Show that $\Delta ABC \cong \Delta ABD$. What can you say about BC and BD?



Ans: Since AB is bisector of $\angle 1CAD$

Therefore, $\angle 1CAB = \angle 1DAB$ 1

Now, $CA = DA$ [Given]

$\angle 1CAB = \angle 1DAB$ [From (1)]

$1AB = AB$ [Common]

$\therefore \Delta 1ABC \cong \Delta 1ABD \otimes$ [By SAS]

And so, $1BD = BC$ [CPCT]

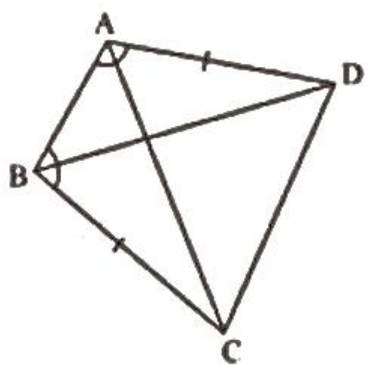
Hence, we can state that BD and BC are equal in lengths.

2. ABCD is a quadrilateral in which $2AD = BC$ and $\angle 2DAB = \angle 2CBA$(See the given figure). Prove that

i. $\Delta\ 2ABD \cong \Delta\ 2BAC$

ii. $2BD = AC$

iii. $\angle 2ABD = \angle 2BAC$



Ans: (i) In $1\Delta ABD$ and $1\Delta BAC$,

$1AD = BC$ [Given]

$\angle 1BAD = \angle 1ABC$ [Given]

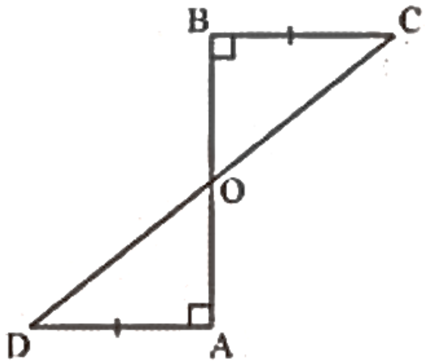
$1AB = BA$ [Common]

$\therefore \Delta\ 1ABD \cong \Delta\ 1BAC$ [SAS]

(ii) $1BD = AC$ [CPCT]

(iii) $\angle 1ABD = \angle 1BAC$ [CPCT]

3. AD and BC are equal perpendiculars to a line segment AB (See the given figure). Show that CD bisects AB.



Ans: If AO = OB, then we can say that CD bisects AB.

To prove: CD bisects AB

Proof: $\because$ BC and AD are both perpendicular to the same line segment AB.

$\therefore$ We can say that $1BC \parallel 1AD$ …(1)

Now, In $\Delta 1AOD$ and $\Delta 1BOC$,

$AD = BC$ [Given]

$\angle DAO = \angle CBO$ [Both right angles]

$\angle ADO = \angle BCO$ [Alternate interior angles, form 1]

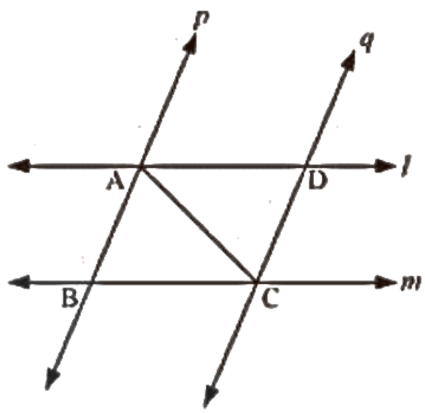
$\therefore \Delta AOD \cong \Delta BOC$ [ASA]

$\therefore AO = BO$ [CPCT]

$\Rightarrow$ CD bisects AB

Hence proved.

4. l and m are two parallel lines intersected by another pair of parallel lines p and q (see the given figure). Show that $\Delta\ 2ABC \cong \Delta\ 2CDA$



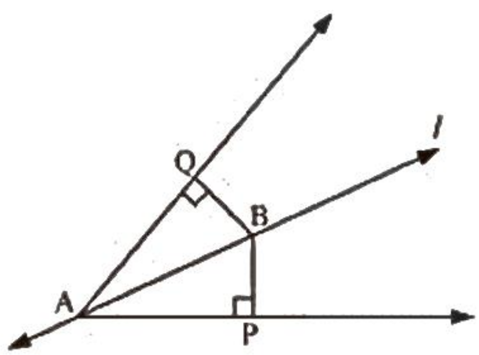
Ans: In ΔABC and ΔCDA,

$\angle BAC = \angle DCA$	[Alternate interior angles]
$\angle BCA = \angle DAC$	[Alternate interior angles]
$AC = CA$	[Common]
$\therefore \Delta ABC \cong \Delta CDA$	[ASA]

5. Line l is the bisector of an angle $\angle 2A$ and B is any point on l. BP and BQ are perpendiculars from B to the arms of $\angle 2A$ (see the given figure). Show that:

(i) $\Delta\ 2APB \cong \Delta\ 2AQB$

(ii) $2BP = BQ$



Ans: Since line l is the bisector of $\angle A$

Therefore, $\angle QAB = \angle PAB$ …(1)

Now In ΔAPB and ΔAQB,

$\angle QAB = \angle PAB$ [From 1]

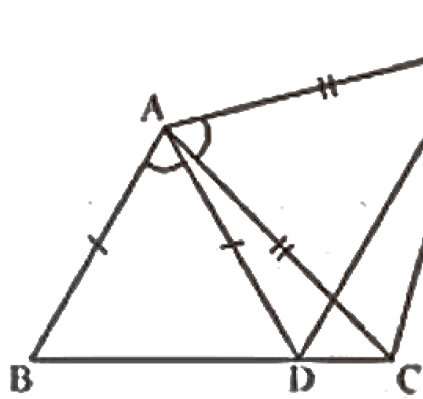
$\angle AQB = \angle APB$ [Both right angled]

AB = AB [Common]

$\therefore \Delta APB \cong \Delta AQB$ [AAS]

And so, $BP = BQ$ [CPCT]

6. In the given figure, $2AC = AE$, $2AB = AD$ and $\angle 2BAD = \angle 2EAC$. Show that BC=DE.



Ans: Since $\angle BAD = \angle EAC$

$\Rightarrow \angle BAD + \angle DAC = \angle EAC + \angle DAC$

$\Rightarrow \angle BAC = \angle DAE$ … (1)

Now in ΔBAC and ΔDAE,

$AB = DA$ [Given]

$\angle BAC = \angle DAE$ [From 1]

$AC = AE$ [Given]

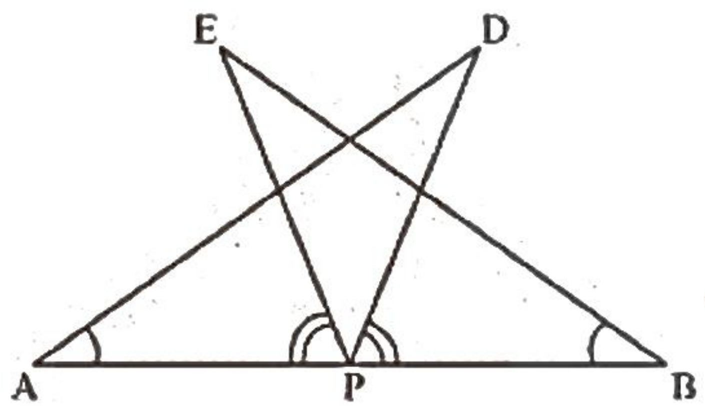
$\therefore \Delta BAC \cong \Delta DAE$ [By SAS]

And so, $BC = DE$ [By CPCT]

7. AB is a line segment and P is its mid-point. D and E are points on the same side of AB such that $\angle 2BAD = \angle 2ABE$ and $\angle \mathbf{EPA} = \angle \mathbf{DPB}$ (See the given figure). Show that

(i) $\Delta \mathbf{DAP} \cong \Delta \mathbf{EBP}$

(ii) $\mathbf{AD} = \mathbf{BE}$



Ans: Since, $\angle EPA = \angle DPB$

$\Rightarrow \angle EPA + \angle EPD = \angle DPB + \angle EPD$

$\Rightarrow \angle DPA = \angle EPB$ ...(1)

Now, in ΔDAP and ΔEPB,

$\angle DAP = \angle EPB$ [Given]

$AP = BP$ [P is the midpoint of AB]

$\angle DPA = \angle EPB$ [From 1]

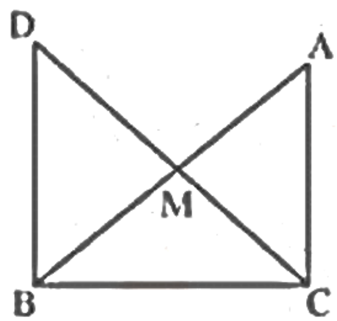
8. In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B (see the given figure). Show that:

(i) $\Delta\ 2AMC \cong \Delta\ 2BMD$

(ii) $\angle\ 2DBC$ **is a right angle**

(iii) $\Delta\ 2DBC \cong \Delta\ 2ACB$

(iv) $2CM = \frac{21}{22} 2AB$



Ans:

(i) In ΔAMC and ΔBMD,

$AM = BM$ [M is midpoint AB]

$CM = DM$ [Given] …(1)

$\angle AMC = \angle DMB$ [Vertically opposite angles]

$\therefore \Delta AMC \cong \Delta BMD$ [By SAS]

$\therefore \angle ACM = \angle BDM$ [CPCT]

$\therefore AC = DB$ [CPCT] …(2)

(ii) Since $\angle ACM = \angle BDM$

Also, $\angle ACM$ and $\angle BDM$ are alternate interior angles.

$\therefore AC \parallel BD$

Now,

$\angle DBC + \angle ACB = 180^\circ$ [Co interior angles]

$\Rightarrow \angle DBC + 90^\circ = 180^\circ$ [$\angle ACB = 90^\circ$]

$\Rightarrow \angle DBC = 90^\circ$

(iii) In ΔDBC and ΔACB,

$DB = AC$ [From 2]

$\angle DBC = \angle ACB$ [Each 90°]

$BC = CB$ [Common]

$\therefore \Delta DBC \cong \Delta ACB$ [By SAS]

(iv) $\therefore DC = AB$ [CPCT]

$\Rightarrow 2CM = AB$ [From 1]

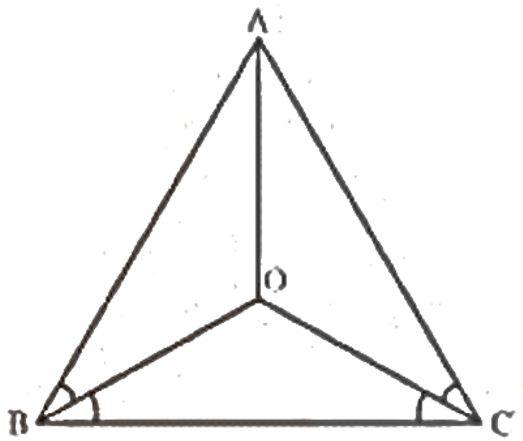
$\Rightarrow CM = \frac{1}{2} AB$

Hence Proved.

Exercise 7.2

1. In an isosceles triangle ABC, with AB = AC, the bisectors of ∠B and ∠C intersect each other at O. Join A to O. Show that:

(i) OB = OC (ii) AO bisects ∠A



Ans: We know, $\angle ABC = \angle ACB$ [Equal angles of isosceles triangle]

$\Rightarrow \frac{1}{2}\angle ABC = \frac{1}{2} ACB$

$\Rightarrow \angle OBC = \angle OCB$

$\therefore OB = OC$ [Sides opposite to equal angles of Isosceles triangle are equal]

Now, In ΔABO and ΔACO,

$AB = AC$ [Equal sides of Isosceles triangle]

$OB = OC$ [Proved above]

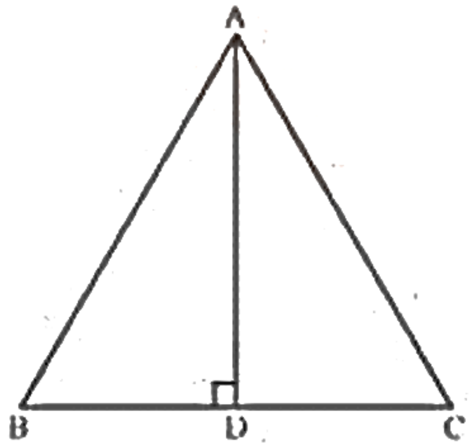
AO=AO [Common]

$\therefore \Delta ABO \cong \Delta ACO$ [By SSS]

$\therefore \angle BAO = \angle CAO$ [CPCT]

$\therefore$ AO bisects angle A.

2. In ΔABC, AD is the perpendicular bisector of BC (see the given figure). Show that ΔABC is an isosceles triangle in which AB = AC.



Ans:

$\because$ AD is perpendicular bisector of BC,

$\therefore BD = DC$

& $\angle ADB = \angle ADC$ [Each 90°]

Now, in ΔABD and ΔACD,

$AD = AD$ [Common]

$\angle ADB = \angle ADC$ [Proved above]

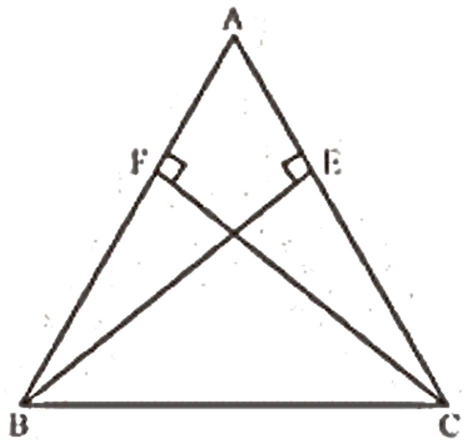
$BD = CD$ [Proved above]

$\therefore \Delta ADB \cong \Delta ACD$ [by SAS]

$\therefore AB = AC$ [CPCT]

So, ΔABC is an isosceles triangle with $AB = AC$

3. ABC is an isosceles triangle in which altitudes BE and CF are drawn to equal sides AC and AB respectively (see the given figure). Show that these altitudes are equal.



Ans: In ΔABE and ΔACF,

$\angle AEB = \angle AFE$ [Each 90°]

$\angle BAE = \angle CAF$ [Common]

$AB = AC$ [Given]

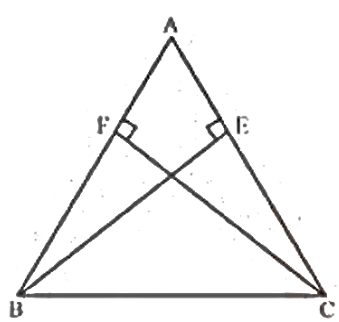
$\therefore \Delta ABE \cong \Delta ACF$ [By AAS]

$\therefore BE = CF$ [CPCT]

4. ABC is a triangle in which altitudes BE and CF to sides AC and AB are equal (see the given figure). Show that

(i) $\Delta\ 2ABE \cong \Delta\ 2ACF$

(ii) $2AB = AC$**, i.e., ABC is an isosceles triangle**



Ans: In ΔABE and ΔACF,

$\angle BAE = \angle CAF$ [Common]

$\angle BEA = \angle CFA$ [Each 90°]

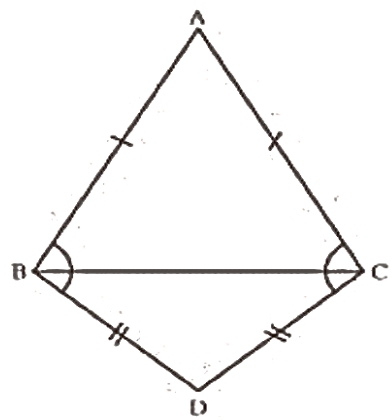
$BE = CF$ [Given]

$\therefore \Delta ABE \cong \Delta ACF$ [by AAS]

$\therefore AB = AC$ [CPCT]

And therefore, ΔABC is an isosceles triangle with AB=AC.

5. ABC and DBC are two isosceles triangles on the same base BC (see the given figure). Show that ∠ABD = ∠ACD.



Ans: Since AB=AC, and DB=DC,

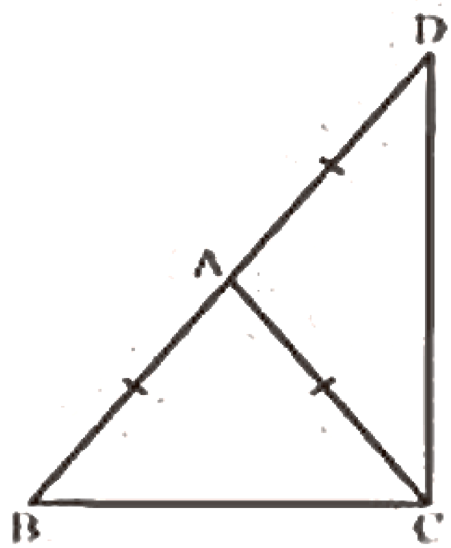
$\therefore$ ABDC is a quadrilateral with adjacent sides being equal.

$\Rightarrow$ ABDC is a kite.

We know that one pair of opposite (obtuse) angles of kite are equal.

Hence, $\angle ABD = \angle ACD$

6. ΔABC is an isosceles triangle in which AB = AC. Side BA is produced to D such that AD = AB (see the given figure). Show that ∠BCD is a right angle.



Ans: In ΔABC

$\angle ABC = \angle ACB$ [angles opposite to equal sides of a triangle] … (1)

Similarly, in ΔADC

$\angle ADC = \angle ACD$ [angles opposite to equal sides of a triangle] … (2)

Now since BD is a straight line,

$\angle BAC + \angle CAD = 180^{\circ}$ … (3)

And we know,

$\angle BAC = \angle ADC + \angle ACD$ [Exterior angle of a triangle = sum of opposite interior angles]

$\Rightarrow \angle BAC = 2\angle ACD$ [From 2] … (4)

Similarly,

$\angle CAD = \angle ABC + \angle ACB$

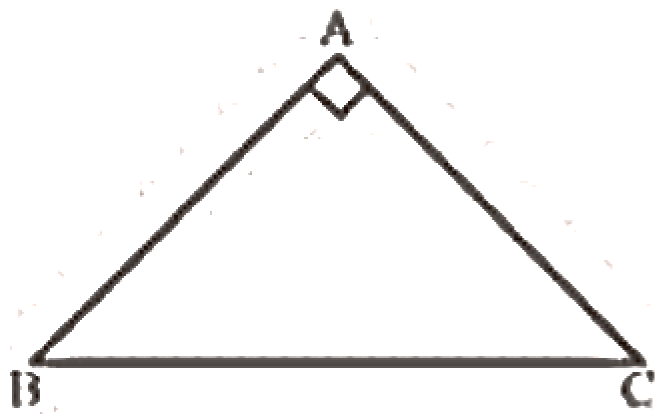
$\Rightarrow \angle CAD = 2\angle ACB$ [From 1] … (5)

$\therefore 2\angle ACB + 2\angle ACD = 180^\circ$ [From 3, 4 and 5]

$\Rightarrow \angle ACB + \angle ACD = 90^\circ \Rightarrow \angle BCD = 90^\circ$

Hence proved.

7. ABC is a right-angled triangle in which ∠A = 90° and AB = AC. Find ∠B and ∠C.



Ans: Since AB = AC,

$\angle B = \angle C$ [Angles opposite to equal sides of a triangle]

Now, we know

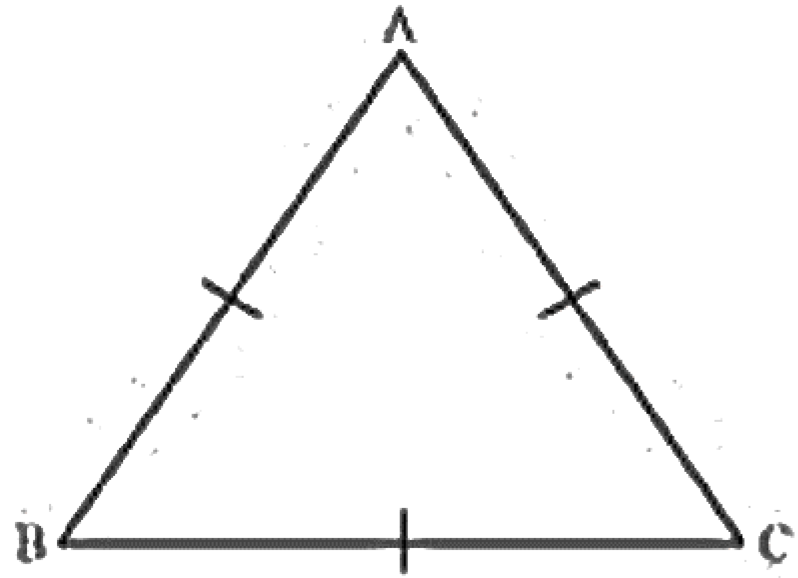
$\angle A + \angle B + \angle C = 180^\circ$ [Angle sum property]

$\Rightarrow \angle A + 2\angle C = 180^\circ \Rightarrow 90^\circ + 2\angle C = 180^\circ \Rightarrow 2\angle C = 90^\circ \Rightarrow \angle C = 45^\circ$

$\therefore \angle B = 45^\circ$ Hence $\angle B$ and $\angle C$ are each 45°

8. Show that the angles of an equilateral triangle are 60º each.

Ans:



In $\angle B = \angle C$ [Angles opposite to equal sides of a triangle]

Similarly,

$\angle A = \angle B$ [Angles opposite to equal sides of a triangle]

$\Rightarrow \angle A = \angle B = \angle C$

Now, we know that

$\angle A + \angle B + \angle C = 180^\circ$ [Angle sum property]

$\Rightarrow \angle A + \angle A + \angle A = 180^\circ \quad \Rightarrow 3\angle A = 180^\circ \quad \Rightarrow \angle A = 60^\circ$

$\therefore \angle A = \angle B = \angle C = 60^\circ$

Hence proved.

Exercise 7.3

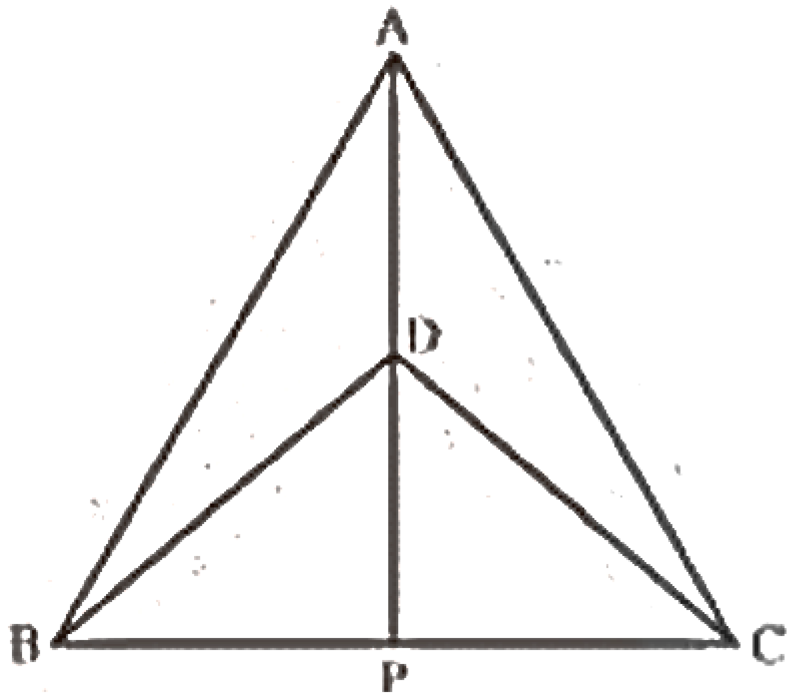
1. ΔABC and ΔDBC are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC (see the given figure). If AD is extended to intersect BC at P, show that

(i) $\Delta\ 2ABD \cong \Delta\ 2ACD$

(ii) $\Delta\ 2ABP \cong \Delta\ 2ACP$

(iii) AP bisects $\angle\ 2A$ **as well as** $\angle\ 2D$

(iv) AP is the bisector of BC



Ans:

(i) In ΔABD and ΔACD,

$AB = AC$ [Equal sides of isosceles triangle]

$DB = DC$ [Equal sides of isosceles triangle]

AD=AD [Common]

$\therefore \Delta ABD \cong \Delta ACD$ [By SSS]

$\Rightarrow \angle BAD = \angle CAD$ [CPCT]

$\Rightarrow \angle BAP = \angle CAP$ … (1)

And $\angle ADB = \angle ADC$ [CPCT] … (2)

(ii) In ΔABP and ΔACP

$AB = AC$ [Equal sides of isosceles triangle]

$\angle BAP = \angle CAP$ [From 1]

$AP = AP$ [Common]

$\therefore \Delta ABP \cong \Delta ACP$ [By SAS]

$\therefore BP = CP$ [CPCT] … (3)

Similarly, $\angle APB = \angle APC$ [CPCT] … (4)

(iii) AP is bisector of $\angle A$ [From 1]

Now, since AP is a line segment

$\therefore \angle ADB + \angle BDP = 180^\circ$ … (5)

Similarly, $\angle ADC + \angle CDP = 180^\circ$ … (6)

Comparing equations 2, 5 and 6 we can say that

$\angle BDP = \angle CDP$

$\therefore$ AP bisects $\angle D$

Hence AP bisects both $\angle A$ and $\angle D$

(iv) We know,

$\angle APB + \angle APC = 180^\circ$

$\Rightarrow \angle APB + \angle APB = 180^\circ$ [From 4]

$\Rightarrow \angle APB = 90^\circ$ … (7)

From equations 3 and 7 we can say that,

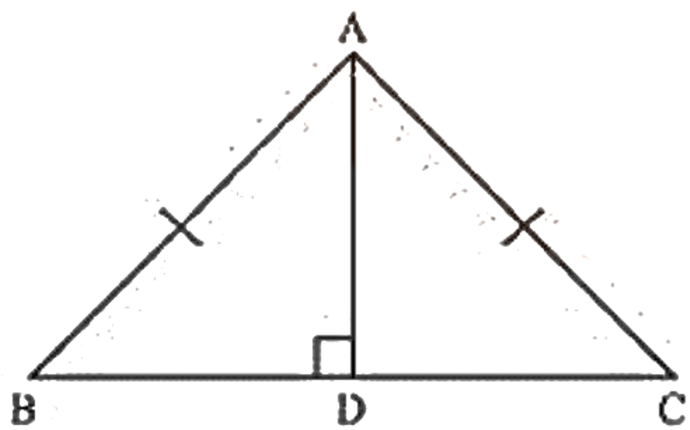
AP is perpendicular bisector of BC.

2. AD is an altitude of an isosceles triangles ABC in which AB = AC. Show that

(i) AD bisects BC

(ii) AD bisects $\angle 2A$

Ans:



In ΔADB and ΔADC

$AB = AC$	[Given]
$AD = AD$	[Common]
$\angle ADB = \angle ADC$	[Each 90°]
$\therefore \Delta ADB \cong \Delta ADC$	[By RHS]
$\Rightarrow BD = DC$	[CPCT]

Therefore, AD is bisector of BC

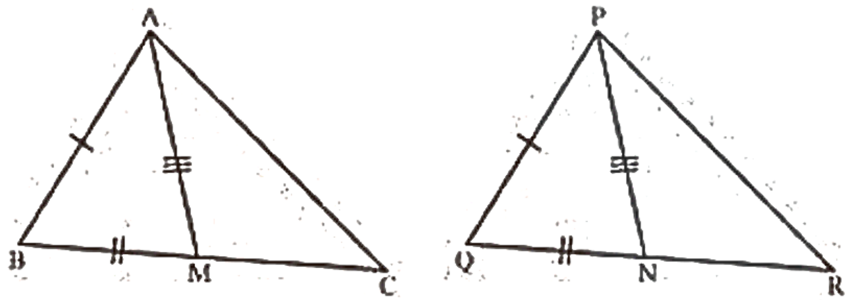
Similarly, $\angle BAD = \angle CAD$ [CPCT]

Therefore, AD bisects $\angle A$ as well.

3. Two sides AB and BC and median AM of one triangle ABC are respectively equal to sides PQ and QR and median PN of ΔPQR (see the given figure). Show that:

(i) $\Delta\ 2ABM \cong \Delta\ 2PQN$

(ii) $\Delta\ 2ABC \cong \Delta\ 2PQR$



Ans: (i) We know that $BC = QR$ … (1)

Now, Since AM is median of ΔABC,

$\Rightarrow BM = \frac{1}{2}BC$ … (2)

Similarly, PN is median of ΔPQR,

$\Rightarrow QN = \frac{1}{2}QR$ … (3)

From equations 1, 2 and 3, we can say that,

$BM = QN$ … (4)

Now in ΔABM and ΔPQN

$AB = PQ$ [Given]

$BM = QN$ [From 4]

AM=PN [Given]

$\therefore \Delta ABM \cong \Delta PQN$ [By SSS]

$\Rightarrow \angle ABM = \angle AQN$ [CPCT]

$\Rightarrow \angle ABC = \angle PQR$ … (5)

(ii) Now in ΔABC and ΔPQR,

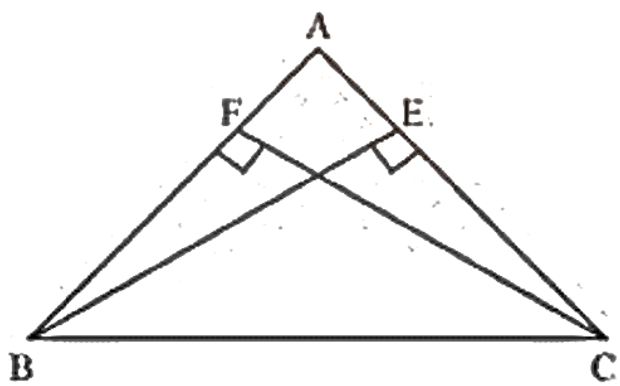
$AB = PQ$ [Given]

$\angle ABC = \angle PQR$ [From 5]

$BC = QR$ [Given]

4. BE and CF are two equal altitudes of a triangle ABC. Using RHS congruence rule, prove that the triangle ABC is isosceles.

Ans:



In ΔBEC and ΔCFB

$BE = CF$ [Given]

$\angle BEC = \angle CFB$ [Each 90°]

BC=CB [Common]

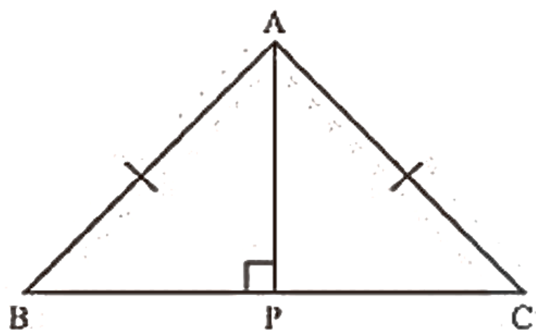
$\therefore \Delta BEC \cong \Delta CFB$ [By RHS]

$\Rightarrow \angle BCE = \angle CBF$ [CPCT]

$\therefore AB = AC$ [Sides opposite to equal angles of a triangle are equal]

Therefore, ΔABC is an isosceles triangle.

5. ABC is an isosceles triangle with AB = AC. Draw AP ⊥ BC to show that ∠B= ∠C.



Ans: In ΔABP and ΔACP

$AB = AC$ [Given]

$AP = AP$ [Common]

$\angle APB = \angle APC$ [Each 90°]

$\therefore \Delta ABP \cong \Delta ACP$ [By RHS]

$\Rightarrow \angle ABP = \angle ACP$ [CPCT]