euclid's geometry

5 Chapter

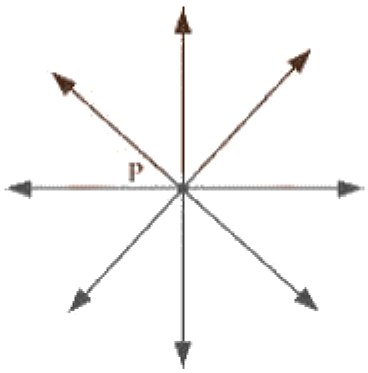
Exercise: 5.1

1. **Which of the following statements are true and which are false? Give reasons for your answers.**

(a) Only one line can pass through a single point.

Ans. False.

Since through a single point 'P' below, infinite number of lines can pass. In the following figure, it can be seen that there are infinite numbers of lines passing through a single point P.



(b) There are an infinite number of lines which pass through two distinct points.

Ans.

False.

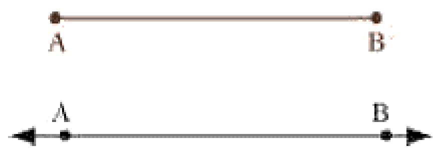
Since through two distinct points, only one line can pass. In the following figure, it can be seen that there is only one single line that can pass through two distinct points P and Q.

(c)A terminated line can be produced indefinitely on both the sides.

Ans.

True.

A terminated line can be produced indefinitely on both the sides. Let AB be a terminated line. It can be seen that it can be produced indefinitely on both the sides.



(d)If two circles are equal, then their radii are equal.

Ans.

True.

If two circles are equal, then their centre and circumference will coincide and hence, the radii will also be equal.

(e) In the following figure, if AB = PQ and PQ = XY, then AB = XY.

Ans.

True.

It is given that AB and XY are two terminated lines (Line Segments) and both are equal to a third line PQ.

Euclid's first axiom states that things which are equal to the same thing are equal to one another.

Therefore, the lines AB =PQ and PQ = XY, Hence AB = XY will be equal to each other.

2. **Give a definition for each of the following terms. Are there other terms that need to be defined first? What are they, and how might you define them?**

   (a) Parallel lines

   (b) Perpendicular lines

   (c) Line segment

   (d) Radius of a circle

   (e) Square.

   Ans.

For the desired definition, we need the following terms:

- **Point**

➢ A small dot made by a sharp pencil on a sheet paper gives an idea about a point

➢ A point has no dimension, it has only a position.

- **Line**

➢ A straight crease obtained by folding a paper, a straight string pulled at its two ends, the edge of a ruler are some close examples of a geometrical line.

➢ The basic concept about a line is that it should be straight and that it should extend in definitely in both the directions.

- **Plane**

➢ The surface of a smooth wall or the surface of a sheet of paper are close examples of plane.

- **Ray**

➢ A part of line l which has only one end- point A and contains the point B is called a ray AB

- **Angle**

➢ An angle is the union of two non- collinear rays with common initial point.

- **Circle**

➢ A circle is the set of all those points in a plane whose distance from a fixed point remains constant.

➢ The fixed point is called the centre of the circle.

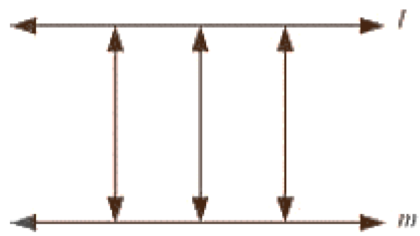
• **Quadrilateral.**

➢ A closed figure made of four line segments is called a quadrilateral

(a) Parallel Lines

• If the perpendicular distance between two lines is always constant, then these are called parallel lines.

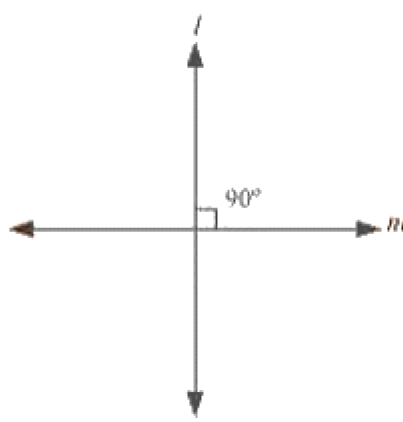
• In other words, the lines which never intersect each other are called parallel lines. • To define parallel lines, we must know about point, lines, and distance between the lines and the point of intersection.



(b) Perpendicular lines

• If two lines intersect each other at 90° , then these are called perpendicular lines.

• We are required to define line and the angle before defining perpendicular lines.

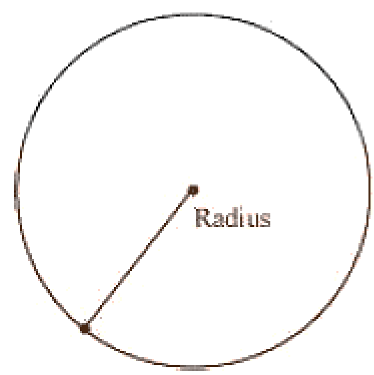


(c) Line segment

• A straight line drawn from any point to any other point is called as line segment.
• To define a line segment, we must know about point and line segment

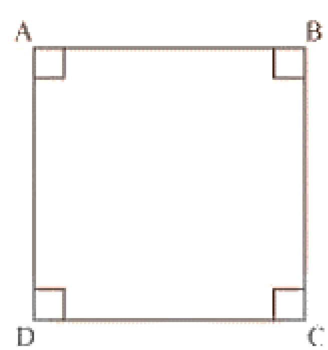
(d) Radius of a circle

• It is the distance between the centers of a circle to any point lying on the circle.
• To define the radius of a circle, we must know about point and circle.



(e) Square

• A square is a quadrilateral having all sides of equal length and all angles of same measure, i.e. 90°.
• To define square, we must know about quadrilateral, side, and angle.



3. Consider the two 'postulates' given below:

(i) Given any two distinct points A and B, there exists a third point C, which is between A and B.
(ii) There exists at least three points that are not on the same line. Do these postulates contain any undefined terms?
Are these postulates consistent?
Do they follow from Euclid's postulates? Explain.

Ans.

• There are various undefined terms in the given postulates.

• The given postulates are consistent because they refer to two different situations.

• Also, it is impossible to deduce any statement that contradicts any well-known axiom and postulate.

• These postulates do not follow from Euclid's postulates.

• They follow from the axiom, "Given two distinct points, there is a unique line that passes through them".

4. **If a point C lies between two points A and B such that AC = BC, then prove that AC = 1 2 AB. Explain by drawing the figure.**

Ans.

From the Figure, Given that,

AC = BC

Point C lies between two points A and B

To Prove: AC = $\frac{1}{2}$AB

Proof:

Consider AC = BC

Adding AC to both the sides of the above Equation,

AC + AC = BC + AC …Equation (1)

2 AC = BC +AC

Here, (BC + AC) coincides with AB. It is known that things which coincide with one another are equal to one another.

$\therefore$ BC + AC = AB … Equation (2)

It is also known that things which are equal to the same thing are equal to one another. Therefore, from Equations (1) and (2), we obtain

AC + AC = AB

2 AC = AB

$\therefore$ AC = $\frac{1}{2}$ AB

5. **In the above question, point C is called a mid-point of line segment AB, prove that every line segment has one and only one mid-point.**

Ans.

From the figure,

Given,

Let there be two mid-points, C and D.

C is the mid-point of AB.

To Prove:

Every line segment has one and only one mid-point.

Proof:

Let us assume, D be another mid- point of AB.

Therefore AD = DB ... Equation (1)

But it is given that C is the mid- point of AB.

Therefore AC = CB ... Equation (2)

Subtracting Equation (1) from Equation (2) we get

AC – AD = CB – DB

DC = – DC

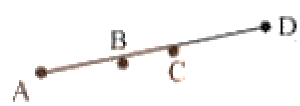
2 DC = 0

DC = 0

Therefore C and D coincides.

Thus, every line segment has one and only one mid- point.

6. In the following figure, if AC = BD, then prove that AB = CD.



Ans.

From the figure, it can be observed that

AC = AB + BC

BD = BC + CD

Given,

AC = BD

To Prove:

AB = CD

Proof:

AB + BC = BC + CD ….Equation (1)

According to Euclid’s axiom, when equals are subtracted from equals, the remainders are also equal.

Subtracting BC from Equation (1), we obtain

AB + BC − BC = BC + CD − BC

AB = CD

Hence it is proved.

7. **Why is Axiom 5, in the list of Euclid’s axioms, considered a ‘universal truth?**

   Ans.

   Axiom 5 states that the whole is greater than the part.

   This axiom is known as a universal truth because it holds true in any field, and not just in the field of mathematics.

   Let us take two cases – one in the field of mathematics, and one other than that.

I. **Case I:**

a. Let t represent a whole quantity and only a, b, c are parts of it.
b. t = a + b + c o Clearly, t will be greater than all its parts a, b, and c.
c. Therefore, it is rightly said that the whole is greater than the part.

II. **Case II**:

a. Let us consider the continent Asia. o
b. Then, let us consider a country India which belongs to Asia.
c. India is a part of Asia and it can also be observed that Asia is greater than India.
d. That is why we can say that the whole is greater than the part.
e. This is true for anything in any part of the world and is thus a universal truth.