linear equation in two variables

4 Chapter

Exercise 4.1

1. Construct a linear equation in two variables to express the following statement.
The cost of a textbook is twice the cost of an exercise book.

Ans: Let the cost of a textbook be x rupees and the cost of an exercise book be y rupees.

The given statement: Cost of Notebook is twice the cost of Pen.

So, in order to form a linear equation,

the cost of the textbook $=2\times$ the cost of an exercise book.

$\Rightarrow x=2y$

$\Rightarrow x-2y=0$.

2. Determine the values of a, b, c from the following linear equations by expressing each of them in the standard form $ax+by+c=0$.

(i) $2x+3y=9.\overline{35}$

Ans: The given linear equation is

$2x+3y=9.\overline{35}$

Subtracting $9.\overline{35}$ from both sides of the equation gives

$2x+3y-9.\overline{35}=0$

Now, by comparing the above equation with the standard form of the linear equation, $ax+by+c=0$, the values of $a, b,$ and c are obtained as

$a=2$,

$b=3$, and

$c = -9.\overline{35}$

(ii) $\mathbf{x - \frac{y}{5} - 10 = 0}$

Ans: The given linear equation is

$$x - \frac{y}{5} - 10 = 0$$

Now, by comparing the above equation with the standard form of the linear equation, $ax+by+c=0$, the values of $a, b,$ and c are obtained as

$a = 1$,

$b = -\frac{1}{5}$, and

$c = -10$.

(iii) $\mathbf{-2x+3y=6}$

Ans: The given linear equation is

$$-2x+3y=6$$

Subtracting 6 from both sides of the equation gives

$$-2x+3y-6=0$$

Now, by comparing the above equation with the standard form of the linear equation, $ax+by+c=0$, the values of $a, b,$ and c are obtained as

$a = -2$,

$b = 3$, and

$c = -6$.

(iv) $\mathbf{x=3y}$

Ans: The given linear equation can be written as

$$1x = 3y$$

Subtracting $3y$ from both sides of the equation gives

$1x-3y+0=0$

Now, by comparing the above equation with the standard form of the linear equation $ax+by+c=0$, the values of $a, b,$ and c are obtained as

$a=1$,

$b=-3$, and

$c=0$.

(v) 2x = -5y

Ans: The given linear equation is

$2x=-5y$.

Adding $5y$ on both sides of the equation gives

$2x+5y+0=0$.

Now, by comparing the above equation with the standard form of the linear equation, $ax+by+c=0$, the values of $a, b,$ and c are obtained as

$a=2$,

$b=5$, and

$c=0$.

(vi) 3x+2=0

Ans: The given linear equation is

$3x+2=0$.

Rewriting the equation gives

$3x+0y+2=0$

Now, by comparing the above equation with the standard form of linear equation $ax+by+c=0$, the values of $a, b,$ and c are obtained as

$a=3$,

$b=0$, and

$c=2$.

(vii) $y-2=0$

Ans: The given linear equation is

$y-2=0$

The equation can be expressed as

$0x+1y-2=0$

Now, by comparing the above equation with the standard form of the linear equation, $ax+by+c=0$, the values of $a, b,$ and c are obtained as

$a=0$,

$b=1$, and

$c=-2$.

(viii) $5=2x$

Ans: The given linear equation is

$5=2x$.

The equation can be written as

$-2x+0y+5=0$.

Now, by comparing the above equation with the standard form of the linear equation $ax+by+c=0$, the values of $a, b,$ and c are obtained as

$a=-2$,

$b=0$, and

$c=5$.

Exercise 4.2

1. Complete the following statement by choosing the appropriate answer and explain why it should be chosen?
$y=3x+5$ has ___________.

(a) A unique solution,

(b) Only two solutions,

(c) Infinitely many solutions.

Ans: Observe that, $y = 3x+5$ is a linear equation.

Now, note that, for $x = 0$, $y = 0+5=5$.

So, $(0,5)$ is a solution of the given equation.

If $x=1$, then $y = 3\times1+5 = 8$.

That is, $(1,8)$ is another solution of the equation.

Again, when $y = 0$, $x = -\frac{5}{3}$.

Therefore, $\left(-\frac{5}{3},0\right)$ is another solution of the equation.

Thus, it is noticed that for different values of x and y, different solutions are obtained for the given equation.

So, there are countless different solutions exist for the given linear equation in two variables. Therefore, a linear equation in two variables has infinitely many solutions.

Hence, option (c) is the correct answer.

2. Determine any four solutions for each of equations given below.
(i) $2x + y = 7$.

Ans: The given equation

$2x+y = 7$ is a linear equation.

Solving the equation for y gives

$y=7-2x$.

Now substitute $x=0,1,2,3$ in succession into the above equation.

For $x=0$,

$2(0)+y=7$

$\Rightarrow y=7$

So, one of the solutions obtained is $(x,y)=(0,7)$.

For $x=1$,

$2(1)+y=7$

$\Rightarrow y=5$

Therefore, another solution obtained is $(x,y)=(1,5)$.

For $x=2$,

$2(2)+y=7$

$\Rightarrow y=3$

That is, a solution obtained is $(x,y)=(3,1)$.

Also, for $x=3$,

$2(3)+y=7$

$\Rightarrow y=1$

So, another one solution is $(x,y)=(3,1)$.

Thus, four solutions obtained for the given equations are $(0,7)$, $(1,5)$, $(2,3)$, $(3,1)$.

(ii) $\pi x+y=9$.

Ans: The given equation

$\pi x+y=9$ …… (a)

is a linear equation in two variables.

By transposing, the above equation (a) can be written as

$y=9-\pi x$.

Now substitute $x=0,1,2,3$ in succession into the above equation.

For $x=0$,

$y=9-\pi(0)$

$\Rightarrow y=9$

Therefore, one of the solutions obtained is $(x,y)=(0,9)$.

For $x=1$,

$y=9-\pi(1)$

$\Rightarrow y=9-\pi$.

So, another solution obtained is $(x,y)=(1,9\text{-}\pi)$.

For $x=2$,

$y=9-\pi(2)$

$\Rightarrow y=9-2\pi$

That is, another solution obtained is $(x,y)=(2,9\text{-}2\pi)$.

Also, for $x=3$,

$y=9-\pi(3)$

$\Rightarrow y=9-3\pi$.

Therefore, another one solution is $(x,y)=(3,9\text{-}3\pi)$.

Thus, four solutions obtained for the given equations are $(0,9)$, $(1,9,\text{-}\pi)$, $(2,9\text{-}2\pi)$, $(3,9\text{-}3\pi)$.

(iii) $\mathbf{x=4y}$.

Ans: The given equation

$x=4y$ is a linear equation in two variables.

By transposing, the above equation can be written as

$y=\frac{x}{4}$.

Now substitute $x=0,1,2,3$ in succession into the above equation.

For $x=0$,

$$y=\frac{0}{4}=0.$$

Therefore, one of the solutions is $(x,y)=(0,0)$.

For $x=1$,

$$y=\frac{1}{4}.$$

So, another solution of the given equation is $(x,y)=\left(1,\frac{1}{4}\right)$.

For $x=2$,

$$y=\frac{2}{4}=\frac{1}{2}.$$

That is, another solution obtained is $(x,y)=\left(2,\frac{1}{2}\right)$.

Also, for $x=3$,

$$y=\frac{3}{4}.$$

Therefore, another one solution is $(x,y)=\left(3,\frac{3}{4}\right)$.

Thus, four solutions obtained for the given equations are $(0,0)$, $\left(1,\frac{1}{4}\right)$, $\left(2,\frac{1}{2}\right)$, $\left(3,\frac{3}{4}\right)$.

3. Identify the actual solutions of the linear equation $x-2y=4$ from each of the following solutions.

(i) $(0,2)$

Ans: Substituting $x=0$ and $y=2$ in the Left-hand-side of the equation $x-2y=4$ gives

$$\begin{aligned} x\text{-}2y &= 0-2(2) \\ &= -4 \\ &\neq 4. \end{aligned}$$

Therefore, Left-hand-side is not equal Right-hand-side of the given equation for $(x,y)=(0,2)$.

Hence, $(0,2)$ is not a solution of the equation $x\text{-}2y=4$.

(ii) $\mathbf{(2,0)}$

Ans: Substituting $x=2$ and $y=0$ in the Left-hand-side of the equation $x\text{-}2y=4$ gives

$$\begin{aligned} x\text{-}2y &= 2-2(0) \\ &= 2 \\ &\neq 4. \end{aligned}$$

Therefore, Left-hand-side is not equal Right-hand-side of the given equation for $(x,y)=(2,0)$.

Hence, $(2,0)$ is not a solution of the equation $x\text{-}2y=4$.

(iii) $\mathbf{(4,0)}$

Ans: Substituting $x=4$ and $y=0$ in the Left-hand-side of the equation $x\text{-}2y=4$ gives

$$\begin{aligned} x\text{-}2y &= 4-2(0) \\ &= 4. \end{aligned}$$

Therefore, Left-hand-side is equal Right-hand-side of the given equation for $(x,y)=(4,0)$.

Hence, $(4,0)$ is a solution of the equation $x\text{-}2y=4$.

(iv) $\mathbf{\left(\sqrt{2},4\sqrt{2}\right)}$

Ans: Substituting $x=\sqrt{2}$ and $y=4\sqrt{2}$ in the Left-hand-side of the equation $x\text{-}2y=4$ gives

$$\begin{aligned} x\text{-}2y &= \sqrt{2} - 2\left(4\sqrt{2}\right) \\ &= \sqrt{2} - 8\sqrt{2} \\ &= -7\sqrt{2} \\ &\neq 4. \end{aligned}$$

Therefore, Left-hand-side is not equal Right-hand-side of the given equation for $\left(x,y\right) = \left(\sqrt{2}, 4\sqrt{2}\right)$.

Hence, $\left(\sqrt{2}, 4\sqrt{2}\right)$ is not a solution of the equation $x\text{-}2y=4$.

(v) $\left(1,1\right)$

Ans: Substituting $x = 1$ and $y = 1$ in the Left-hand-side of the equation $x\text{-}2y=4$ gives

$$\begin{aligned} x\text{-}2y &= 1 - 2\left(1\right) \\ &= 1 - 2 \\ &= -1 \\ &\neq 4. \end{aligned}$$

Therefore, Left-hand-side is not equal Right-hand-side of the given equation for $\left(x,y\right) = \left(1,1\right)$.

Hence, $\left(1,1\right)$ is not a solution of the equation $x\text{-}2y=4$.

4. If $\left(x,y\right) = \left(2,1\right)$ is a solution of the equation $2x+3y=k$, then what is the value of k?

Ans: By substituting $x = 2$, $y = 1$ and into the equation

$2x+3y=k$ gives

$2\left(2\right)+3\left(1\right)=k$

$\Rightarrow 4+3=k$

$\Rightarrow k=7$.

Hence, the value of k is 7.