polynomials

2
Chapter

Exercise 2.1

Question 1:
Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer.

(i) $4x^2 - 3x + 7$
(ii) $y^2 + \sqrt{2}$
(iii) $3\sqrt{t} + t\sqrt{2}$
(iv) $y + \frac{2}{y}$
(v) $y + 2y^{-1}$

Solution 1:
i) $4x^2 - 3x + 7$
One variable is involved in given polynomial which is 'x'
Therefore, it is a polynomial in one variable 'x'.

(ii) $y^2 + \sqrt{2}$
One variable is involved in given polynomial which is 'y'
Therefore, it is a polynomial in one variable 'y'.

(iii) $3\sqrt{t} + t\sqrt{2}$
No. It can be observed that the exponent of variable t in term $3\sqrt{t}$ is $\frac{1}{2}$, which is nota whole number. Therefore, this expression is not a polynomial.
(iv) $y + \frac{2}{y}$
$= y + 2y^{-1}$

The power of variable 'y' is -1 which is not a whole number.
Therefore, it is not a polynomial in one variable

No. It can be observed that the exponent of variable y in term $\frac{2}{y}$ is -1, which is not a whole number. Therefore, this expression is not a polynomial.

(v) $x^{10}+y^{3}+t^{50}$

In the given expression there are 3 variables which are 'x, y, t' involved.

Therefore, it is not a polynomial in one variable.

Question 2:

Write the coefficients of x^2 in each of the following:

(i) $2+x^2+x$

(ii) $2-x^2+x^3$

(iii) $\frac{\pi}{2}x^2+x$

(iv) $\sqrt{2}x-1$

Solution 2:

(i) $2+x^2+x^3$

$=2+1(x^2)+x$

The coefficient of x^2 is 1.

(ii) $2-x^2+x^3$

$=2-1(x^2)+x$

The coefficient of x^2 is -1.

(iii) $\frac{\pi}{2}x^2+x$

The coefficient x^2 of is $\frac{\pi}{2}$.

(iv) $\sqrt{2}x-1=0x^2+\sqrt{2}x-1$

The coefficient of x^2 is 0.

Question 3:

Give one example each of a binomial of degree 35, and of a monomial of degree 100.

Solution 3 :

Binomial of degree 35 means a polynomial is having

1. Two terms
2. Highest degree is 35

Example: $x^{35}+x^{34}$

Monomial of degree 100 means a polynomial is having

1. One term
2. Highest degree is 100

Example : x^{100}.

Question 4:

Write the degree of each of the following polynomials:

(i) $5x^3+4x^2+7x$

(ii) $4-y^2$

(iii) $5t-\sqrt{7}$

(iv) 3

Solution 4:

Degree of a polynomial is the highest power of the variable in the polynomial.

(i) $5x^3+4x^2+7x$

Highest power of variable 'x' is 3. Therefore, the degree of this polynomial is 3

(ii) $4-y^2$

Highest power of variable 'y' is 2. Therefore, the degree of this polynomial is 2.

(iii) $5t-\sqrt{7}$

Highest power of variable 't' is 1. Therefore, the degree of this polynomial is 1.

(iv) 3

This is a constant polynomial. Degree of a constant polynomial is always 0.

Question 5: Classify the following as linear, quadratic and cubic polynomial:

(i) x^2+x

(ii) $x-x^3$

(iii) $y+y^2+4$

(iv) $1+x$
(v) 3t
(vi) r^2
(vii) $7x^2 \quad 7x^3$

Solution 5:
Linear polynomial – whose variable power is ‘1’

Quadratic polynomial - whose variable highest power is ‘2’
Cubic polynomial- whose variable highest power is ‘3’

(i) x^2+x is a quadratic polynomial as its highest degree is 2.
(ii) $x-x^3$ is a cubic polynomial as its highest degree is 3.
(iii) $y+y^2+4$ is a quadratic polynomial as its highest degree is 2.
(iv) 1 + x is a linear polynomial as its degree is 1.
(v) 3t is a linear polynomial as its degree is 1.
(vi) r^2 is a quadratic polynomial as its degree is 2.
(vii) $7x^2 \quad 7x^3$ is a cubic polynomial as highest its degree is 3.

Exercise 2.2

Question 1:
Find the value of the polynomial at $5x-4x^2+3$ at

(i) x = 0
(ii) x = −1
(iii) x = 2

Solution 1:

(i) $p(x)=5x-4x^2+3$

$p(0)=5(0)-4(0)^2+3 = 3$

(ii) $p(x)=5x-4x^2+3$

$p(-1)=5(-1)-4(-1)^2+3$
$= -5-4(1)+3=-6$

(iii) $p(x)=5x-4x^2+3$

$p(2)=5(2)-4(2)^2+3 = 10-16+3=-3$

Question 2:

Find p(0), p(1) and p(2) for each of the following polynomials:

(i) $p(y) = y^2 - y + 1$

(ii) $p(t) = 2 + t + 2t^2 - t3$

(iii) $p(x) = x^3$

(iv) $p(x) = (x - 1)(x + 1)$

Solution 2:

(i) $p(y) = y^2 - y + 1$

- $p(0) = (0)^2 - (0) + 1 = 1$
- $p(1) = (1)^2 - (1) + 1 = 1 - 1 + 1 = 1$
- $p(2) = (2)^2 - (2) + 1 = 4 - 2 + 1 = 3$

(ii) $p(t) = 2 + t + 2t^2 - t^3$

- $p(0) = 2 + 0 + 2(0)^2 - (0)^3 = 2$
- $p(1) = 2 + (1) + 2(1)^2 - (1)^3 = 2 + 1 + 2 - 1 = 4$
- $p(2) = 2 + 2 + 2(2)^2 - (2)^3$

$= 2 + 2 + 8 - 8 = 4$

(iii) $p(x) = x^3$

- $p(0) = (0)^3 = 0$
- $p(1) = (1)^3 = 1$
- $p(2) = (2)^3 = 8$

(v) $p(x) = (x - 1)(x + 1)$

- $p(0) = (0 - 1)(0 + 1) = (-1)(1) = -1$
- $p(1) = (1 - 1)(1 + 1) = 0(2) = 0$
- $p(2) = (2 - 1)(2 + 1) = 1(3) = 3$

Question 3:

Verify whether the following are zeroes of the polynomial, indicated against them.

(i) $p(x)=3x+1, x=-\frac{1}{3}$

(ii) $p(x)=5x-\pi, x=\frac{4}{5}$

(iii) $p(x)=x^2-1, x=1,-1$

(iv) $p(x)=(x+1)(x-2), x=-1,2$

(v) $p(x)=x^2, x=0$

(vi) $p(x)=lm+m, x=-\frac{m}{l}$

(vii) $p(x)=3x^2-1, x=-\frac{1}{\sqrt{3}},\frac{2}{\sqrt{3}}$

(viii) $p(x)=2x+1, x=\frac{1}{2}$

Solution 3:

(i) If $x=-\frac{1}{3}$ is a zero of given polynomial p(x) = 3x + 1, then $p\left(-\frac{1}{3}\right)$ should be 0.

Here, $p\left(-\frac{1}{3}\right)=3\left(-\frac{1}{3}\right)+1$ = -1+1=0

Therefore, is a zero of the given polynomial.

(ii) If $x=\frac{4}{5}$ is a zero of polynomial p(x) = 5x − π, then $p\left(\frac{4}{5}\right)$ should be 0.

Here, $p\left(\frac{4}{5}\right)=5\left(\frac{4}{5}\right)$-π = 4 – π

As $p\left(\frac{4}{5}\right)\neq 0$

Therefore, $x=\frac{4}{5}$ is not a zero of the given polynomial.

(iii) If x = 1 and x = −1 are zeroes of polynomial $p(x) = x^2 - 1$, then p(1) and p(−1)should be 0.

Here, $p(1) = (1)^2 - 1 = 0$, and

$p(-1) = (-1)^2 - 1 = 0$

Hence, x = 1 and −1 are zeroes of the given polynomial.

(iv) If x = −1 and x = 2 are zeroes of polynomial p(x) = (x +1) (x − 2), then p(−1) and p(2)should be 0.

Here, p(−1) = (−1 + 1) (−1 − 2) = 0(−3) = 0, and

p(2) = (2 + 1) (2 − 2) = 3(0) = 0

Therefore, x = −1 and x = 2 are zeroes of the given polynomial.

(v) If x = 0 is a zero of polynomial $p(x) = x^2$, then p(0) should be zero.

Here, $p(0) = (0)^2 = 0$

Hence, x = 0 is a zero of the given polynomial.

(vi) If $p\left(\frac{-m}{l}\right)$ is a zero of polynomial p(x) = lx + m, then $p\left(\frac{-m}{l}\right)$ should be 0.

Here, $p\left(\frac{-m}{l}\right) = l\left(\frac{-m}{l}\right) + m = -m + m = 0$

Therefore, $x = \frac{-m}{l}$ is a zero of the given polynomial.

(vii) If $x = \frac{-1}{\sqrt{3}}$ and $x = \frac{2}{\sqrt{3}}$ are zeroes of polynomial $p(x) = 3x^2 - 1$, then

$p\left(\frac{-1}{\sqrt{3}}\right)$ and $p\left(\frac{2}{\sqrt{3}}\right)$ should be 0.

Here, $p\left(\frac{-1}{\sqrt{3}}\right)=3\left(\frac{-1}{\sqrt{3}}\right)^2-1=3\left(\frac{1}{3}\right)-1=1-1=0$, and

$$p\left(\frac{2}{\sqrt{3}}\right)=3\left(\frac{2}{\sqrt{3}}\right)^2-1=3\left(\frac{4}{3}\right)-1=4-1=3$$

Hence, $x=\frac{-1}{\sqrt{3}}$ is a zero of the given polynomial.

However, $x=\frac{2}{\sqrt{3}}$ is not a zero of the given polynomial.

(viii) If $x=\frac{1}{2}$ is a zero of polynomial p(x) = 2x + 1, then $p\left(\frac{1}{2}\right)$ should be 0.

Here, $p\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right) + 1 = 1 + 1 = 2$

As $p\left(\frac{1}{2}\right)\neq 0$,

Therefore, $x=\frac{1}{2}$ is not a zero of the given polynomial.

Question 4:

Find the zero of the polynomial in each of the following cases:

(i) p(x) = x + 5

(ii) p(x) = x – 5

(iii) p(x) = 2x + 5

(iv) p(x) = 3x – 2

(v) p(x) = 3x

(vi) p(x) = ax, a ≠ 0

(vii) p(x) = cx + d, c ≠ 0, c, d are real numbers.

Solution 4:

Zero of a polynomial is that value of the variable at which the value of thepolynomial is obtained as 0.

(i) p(x) = x + 5
Let p(x) = 0
x + 5 = 0
x = − 5
Therefore, for x = −5, the value of the polynomial is 0 and hence, x = −5 is a zero of the given polynomial.

(ii) p(x) = x − 5
Let p(x) = 0
x − 5 = 0
x = 5
Therefore, for x = 5, the value of the polynomial is 0 and hence, x = 5 is a zero of the given polynomial.

(iii) p(x) = 2x + 5
Let p(x) = 0
2x + 5 = 0
2x = − 5

$$x = -\frac{5}{2}$$

Therefore, for $x = -\frac{5}{2}$, the value of the polynomial is 0 and hence, $x = -\frac{5}{2}$ is a zero of the given polynomial.

(iv) p(x) = 3x – 2

p(x) = 0
3x − 2 = 0
Therefore, for $x = \frac{2}{3}$, the value of the polynomial is 0 and hence, $x = \frac{2}{3}$ is a zero of the given polynomial.

(v) $p(x) = 3x$
Let $p(x) = 0$
$3x = 0$
$x = 0$
Therefore, for $x = 0$, the value of the polynomial is 0 and hence, $x = 0$ is a zero of the given polynomial.

(vi) $p(x) = ax$
Let $p(x) = 0$
$ax = 0$
$x = 0$
Therefore, for $x = 0$, the value of the polynomial is 0 and hence, $x = 0$ is a zero of the given polynomial.

(vii) $p(x) = cx + d$
Let $p(x) = 0$
$cx + d = 0$

$$x = \frac{-d}{c}$$

Therefore, for $x = \frac{-d}{c}$, the value of the polynomial is 0 and hence, $x = \frac{-d}{c}$ is a zero of the given polynomial.

Exercise 2.3

Question 1:

Determine which of the following polynomials has (x+1) a factor:

(i) $x^3 + x^2 + x + 1$

(ii) $x^4 + x^3 + x^2 + x + 1$

(iii) $x^4 + 3x^3 + 3x^2 + x + 1$

(iv) $x^3 - x^2 - (2+\sqrt{2})x + \sqrt{2}$

Solution:

Apply remainder theorem

$x + 1 = 0$

$x = -1$

Put the value of $x = -1$ in all equations.

(i) $x^3 + x^2 + x + 1 = (-1)^3 + (-1)^2 + (-1) + 1 = -1 + 1 - 1 + 1 = 0$

Then $x + 1$ is the factor of equation

(ii) $x^4 + x^3 + x^2 + x + 1 = (-1)^4 + (-1)^3 + (-1)^2 + (-1) + 1 = 1 - 1 + 1 - 1 + 1 = 1$

This is not zero.Then $x + 1$ is not the factor of equation

(iii) $x^4 + 3x^3 + 3x^2 + x + 1 = (-1)^4 + 3(-1)^3 + 3(-1)^2 + (-1) + 1 = 1$

This is not zero.Then $x + 1$ is not the factor of equation

(iv) $x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2} = (-1)^3 - (-1)^2 - (2 + \sqrt{2})(-1) + \sqrt{2} = -1 - 1 + 2 - \sqrt{2} + \sqrt{2} = 0$

This is zero. Then x + 1 is the factor of equation

Question 2:

Use the Factor Theorem to determine whether g(x) is a factor of p(x) in each of the following cases:

(i) $p(x) = 2x^3 + x^2 - 2x - 1$, $g(x) = x + 1$

(ii) $p(x) = x^3 + 3x^2 + 3x + 1$, $g(x) = x + 2$

(iii) $p(x) = x3 - 4x2 + x + 6$, $g(x) = x - 3$

Solution:

(i) Apply factor theorem

$x + 1 = 0$

So $x = -1$

$2x^3 + x^2 - 2x - 1$

Replace x by −1, we get

$2(-1)^3 + (-1)^2 - 2(-1) - 1 = -2 + 1 + 2 - 1 = 0$

Reminder is 0 so that x+1 is a factor of $2x^3 + x^2 - 2x - 1$

(ii) Apply factor theorem

$x + 2 = 0$

So $x = -2$

$x^3 + 3x^2 + 3x + 1$

Replace x by −2, we get

$(-2)^3 + 3(-2)^2 + 3(-2) + 1 = -8 + 12 - 6 + 1 = 1$

Reminder is 1 so that x + 2 is not a factor of $x^3 + 3x^2 + 3x + 1$

(iii) Apply factor theorem

$x - 3 = 0$

So $x = 3$

$x^3 - 4x^2 + x + 6$

Replace x by 3, we get

$(3)^3 - 4(3)^2 + (3) - 1 = 27 - 36 + 3 + 6 = 0$

Reminder is 0 so that $x - 3$ is a factor of $x^3 - 4x^2 + x + 6$

Question 3:

Find the value of k, if $x - 1$ is a factor of p(x) in each of the following cases:

(i) $p(x) = x^2 + x + k$

(ii) $p(x) = 2x^2 + kx + \sqrt{2}$

(iii) $p(x) = kx^2 - \sqrt{2}x + 1$

(iv) $p(x) = kx^2 - 3x + k$

Solution:

If $x - 1$ is the factor of the equation then put $x = 1$ in these equation

(i) $x^2 + x + k = 0 \Rightarrow (1)^2 + 1 + k = 0 \Rightarrow 1 + 1 + k = 0 \Rightarrow k = -2$

(ii) $2x^2 + kx + \sqrt{2} = 0 \Rightarrow 2(1)^2 + k(1) + \sqrt{2} = 0 \Rightarrow 2 + k + \sqrt{2} = 0 \Rightarrow k = -2 - \sqrt{2}$

(iii) $kx^2 - \sqrt{2}\,x + 1 = 0 \Rightarrow k(1)^2 - \sqrt{2}(1) + 1 = 0 \Rightarrow k - \sqrt{2} + 1 = 0 \Rightarrow k = \sqrt{2} - 1$

(iv) $kx^2 - 3x + k = 0 \Rightarrow k(1)^2 - 3(1) + k = 0 \Rightarrow k - 3 + k = 0 \Rightarrow 2k = -3 \Rightarrow k = -\frac{3}{2}$

Question 4:

Factorise:

(i) $12x^2 - 7x - 1$

(ii) $2x^2 + 7x + 3$

(iii) $6x^2 + 5x - 6$

(iv) $3x^2 - x - 4$

Solution:

(i) $12x^2 - 7x + 1$

$= 12x^2 - 4x - 3x + 1$

$= 4x(3x - 1) - 1(3x - 1)$

$= (4x - 1)(3x - 1)$

(ii) $2x^2 + 7x + 3$

$= 2x^2 + 6x + x + 3$

$= 2x(x + 3) + 1(x + 3)$

$= (2x + 1)(x + 3)$

(iii) $6x^2 + 5x - 6$

$= 6x^2 + 9x - 4x - 6$

$= 3x(2x + 3) - 2(2x + 3)$

$= (3x - 2)(2x + 3)$

(iv)) $3x^2 - x - 4$

$= 3x^2 - 4x + 3x - 4$

$= x(3x - 4) + 1(3x - 4)$

$=(3x - 4)(x + 1)$

Question 5:

Factorise:

(i) $x^3 - 2x^2 - x + 2$

(ii) $x^3 - 3x^2 - 9x - 5$

(iii) $x^3 + 13x^2 + 32x + 20$

(iv) $2y^3 + y^2 - 2y - 1$

Solution

(i) $x^3 - 2x^2 - x + 2$

$= x^2(x - 2) - 1(x - 2)$

$= (x - 2)(x^2 - 1)$

$= (x - 2)(x + 1)(x - 1)$

(ii) $x^3 - 3x^2 - 9x - 5$

$= x^3 - 5x^2 + 2x^2 - 10x + x - 5$

$= x^2(x - 5) + 2x(x - 5) + 1(x - 5)$

$= (x - 5)(x^2 + 2x + 1)$

$= (x - 5)(x + 1)(x + 1)$

(iii) $x^3 + 13x^2 + 32 + 20$

Let us put $x = 1, 2, -1, -2$ and check if they satisfy the equation.

$x = -2$ satisfies the equation.

Dividing the equation by $(x + 2)$, we get

$\Rightarrow x^3 + 13x^2 + 32 + 20$

$= (x+2)(x^2 + 11x + 10)$

$= (x + 2)(x^2 + x + 10x + 10)$

$= (x + 2)[x(x + 1) + 10(x + 1)]$

$= (x + 2)(x + 1)(x + 10)$

(iv) $2y^3 + y^2 - 2y - 1$

$= 2y^3 + 2y^2 - y^2 - y - y - 1$

$= 2y^2(y + 1) - y(y + 1) - 1(y + 1)$

$= (y + 1)\{2y^2 - y - 1\}$

$= (y + 1)\{2y^2 - 2y + y - 1\}$

$= (y + 1)\{2y(y - 1) + 1(y - 1)\}$

$= (y - 1)(y + 1)(2y + 1)$

Exercise 2.4

Question 1:

Use suitable identities to find the following products:

(i) $(x+4)(x+10)$

(ii) $(x+8)(x-10)$

(iii) $(3x+4)(3x-5)$

(iv) $(y^2 + \frac{2}{3})(y^2 - \frac{2}{3})$

(v) $(3-2x)(3+2x)$

Solution:

We use identities
$(x + a)(x + b) = x^2 + (a + b)x + ab$ and $x^2 - y^2 = (x+y)(x-y)$

(i) $(x + 4)(x + 10)$
$= x^2 + (4 + 10)x + (4)(10)$
$= x^2 + 14x + 40$

(ii) $(x + 8)(x - 10)$
$= x^2 + (8 - 10) + (8)(-10)$
$= x^2 - 2x - 80$

(iii) $(3x + 4)(3x + 5)$

$= (3x)^2 + (4 - 5)3x + (4)(-5)$

$= 9x^2 - 3x - 20$

(iv) $(y^2 + \frac{2}{3})(y^2 - \frac{2}{3})$

$= (y^2)^2 - (\frac{2}{3})^2$

$= y^4 - \frac{9}{4}$

(v) $(3 - 2x)(3 + 2x)$

$= (3)^2 - (2x)^2 = 9 - 4x^2$

Question 2:

Evaluate the following products without multiplying directly:

(i) 103×107

(ii) 95×96

(iii) 104×96

Solution:

(i) 103×107

$= (100 + 3)(100 + 7)$

$= (100)^2 + (3 + 7)(100) + (3)(7)$

$= 10000 + 1000 + 21 = 11021$

(ii) 95×96

$= (90 + 5)(90 + 6) = (90)^2 + (5 + 6)(90) + (5)(6)$

$= 8100 + 990 + 30 = 9120$

(iii) 104×96

$= (100 + 4)(100 - 4)$

$= (100)2 - (4)^2$

$= 10000 - 16 = 9984$

Question 3:

Factorise the following using appropriate identities:

(i) $9x^2 + 6xy + y^2$

(ii) $4y^2 - 4y + 1$

(iii) $x^2 - \frac{y^2}{100}$

Solution

(i) $9x^2 + 6xy + y^2 = (3x)^2 + 2(3x)(y) + (y)^2 = (3x+y)^2$

(ii) $4y^2 - 4y + 1 = (2y)^2 - 2(2y)(1) + 1 = (2y-1)^2$

(iii) $x^2 - \frac{y^2}{100} = (x)^2 - (\frac{y}{10})^2 = (x - \frac{y}{10})(x + \frac{y}{10})$

Question 4:

Expand each of the following, using suitable identities:

(i) $(x + 2y + 4z)^2$

(ii) $(2x - y + z)^2$

(iii) $(-2x + 3y + 2z)^2$

(iv) $(3a - 7b - c)^2$

(v) $(-2x + 5y - 3z)^2$

(vi) $[\frac{1}{4}a - \frac{1}{2}b + 1]^2$

Solution:

We know,

$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$

(i) $(x+2y+4z)^2$
$= x^2 + (2y)^2 + (4z)^2 + 2(x)(2y) + 2(2y)(4z) + 2(x)(4z)$
$= x^2 + 4y^2 + 16z^2 + 4xy + 16yz + 8xz$

(ii) $(2\mathrm{x} - \mathrm{y} + \mathrm{z})^2$
$= (2\mathrm{x})^2 + (-\mathrm{y})^2 + (\mathrm{z})^2 + 2(2\mathrm{x})(-\mathrm{y}) + 2(-\mathrm{y})(\mathrm{z}) + 2(2\mathrm{x})(\mathrm{z})$
$= 4\mathrm{x}^2 + \mathrm{y}^2 + \mathrm{z}^2 - 4\mathrm{xy} - 2\mathrm{yz} + 4\mathrm{xz}$

(iii) $(-2x+3y+2z)^2$
$= (-2x)^2 + (3y)^2 + (2z)^2 + 2(-2x)(3y) + 2(3y)(2z) + 2(-2x)(2z)$
$= 4x^2 + 9y^2 + 4z^2 - 12xy + 12yz - 8xz$

(iv) $(3a - 7b - c)^2$
$= (3a)^2 + (-7b)^2 + (-c)^2 + 2(3a)(-7b) + 2(-7b)(-c) + 2(3a)(-c)$
$= 9a^2 + 49b^2 + c^2 - 42ab + 14bc - 6ac$

(v) $(-2x+5y-3z)^2$
$= (-2x)^2 + (5y)^2 + (-3z)^2 + 2(-2x)(5y) + 2(5y)(-3z) + 2(-3z)(-2x)$
$= 4x^2 + 25y^2 + 9z^2 - 20xy - 30yz + 12xz$

(iv) $(\frac{1}{4}a - \frac{1}{2}b + 1)^2$
$= (\frac{1}{4}\mathrm{a})^2 + (-\frac{1}{2}\mathrm{b})^2 + (1)^2 + 2(\frac{1}{4}\mathrm{a} \times -\frac{1}{2}\mathrm{b}) + 2(-\frac{1}{2}\mathrm{b} \times 1) + 2(\frac{1}{4}\mathrm{a} \times 1)$
$= \frac{1}{16}a^2 + \frac{1}{4}b^2 + 1 - \frac{1}{4}ab - b + \frac{1}{2}a$

Question 5:

Factorise:

(i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

(ii) $2x^2 + y^z + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$

Solution:

We know that

$(x^2 + y^2 + z^2 + 2xy + 2yz + zx) = (x + y + z)^2$

(i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

$= (2x)^2 + (3y)^2 + (-4z)^2 + 2(2x)(3y) + 2(3y)(-4z) + 2(-4z)(2x)$

$= (2x + 3y - 4z)^2 = (2x + 3y - 4z)(2x + 3y - 4z)$

(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8zx$

$= (-\sqrt{2}x) + (y)^2 + (\sqrt{2}z) + 2(-\sqrt{2}x)(y) + 2(y)(2\sqrt{2}z) + 2(2\sqrt{2}z)(y)$

$= (-\sqrt{2}x + y + 2\sqrt{2}z)^2 = (-\sqrt{2}x + y + 2\sqrt{2}z)(-\sqrt{2}x + y + 2\sqrt{2}z)$

Question 6:

Write the following cubes in expanded from:

(i) $(2\mathrm{x} + 1)^3$

(ii) $(2a - 3b)^3$

(iii) $[\frac{3}{2}\mathrm{x} + 1]^3$

(iv) $[\mathrm{x} - \frac{2}{3}\mathrm{y}]^3$

Solution:

(i) $(2x+1)^3 = (2x)^3 + (1)^3 - 3(2x)(1)(2x+1)$
$= 8x^3 + 1 + 6x(2x+1) = 8x^3 + 12x^2 + 6x + 1$

(ii) $(2a-3b)^3 = (2a)^3 - (3b)^3 + 3(2a)(3b)(2a-3b)$
$= 8a^3 + 27b^3 - 18ab(2a-3b) = 8a^3 - 27b^3 - 36a^2b + 54ab^2$

(iii) $(\frac{3}{2}x+1)^3 = (\frac{3}{2})x^3 + (1)^3 + 3(\frac{3}{2}x)\ (1)\ (\frac{3}{2}x+1)$
$= \frac{27}{8}x^3 + 1 + \frac{9}{2}(\frac{3}{2}+1) = \frac{27}{8}x^3 + \frac{27}{4}x^2 + \frac{9}{2}x + 1$

(iv)$[x - \frac{2}{3}y]^3 \quad = x^3 - 3 \times x^2 \times \frac{2}{3}y + 3 \times x \times (\frac{2}{3}y)^2 + (\frac{2}{3}y)^3$
$= x^3 - 2x^2y + \frac{4}{3}xy^2 + \frac{8}{27}y^3$

Question 7:

Evaluate the following using suitable identities:

(i) $(99)^3$

(ii) $(102)^3$

(iii) $(998)^3$

Solution:

(i) $99^3 = (100-1)^3 = (100)^3 + (-1)^3 - 3(100)(1)(100-1)$
$= 1000000 - 1 - 300(100-1) = 100000 - 1 - 30000 + 300 = 970299$

(ii) $102^3 = (100+2)^3 = (100)^3 + (2)^3 + 3(100)(2)(100+2)$
$= 1000000 + 8 + 600(100+2) = 1000000 + 8 + 60000 + 1200 = 1061208$

(iii) $998^3 = (1000-2)^3 = (1000)^3 - (2)^3 - 3(1000)(2)(1000-2)$
$= 1000000000 - 8 + 6000(1000-2) = 1000000000 - 8 - 6000000 + 1200 =$
994011992

Question 8:

Factorise each of the following:

(i) $8a^3 + b^3 + 12a^b + 6ab^2$

(ii) $8a^3 - b^3 - 12a^2b + 6ab^2$

(iii) $27 - 125a^3 - 135a + 225a^2$

(iv) $64a^3 - 27b^3 \frac{1}{-} 144a^2b + 108ab^2$

(v) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

Solution:

We know that

$(a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$ and $(a - b)^3 = a^3 + b^3 - 3a^2b - 3ab^2$

(i) $8a^3 + b^3 + 12a^2b + 6ab^2 = (2a)^3 + (b)^3 + 3(2a)^2(b) + 3(2a)(b)^2 = (2a + b)^3$

(ii) $8a^3 + b^3 - 12a^2b - 6ab^2 = (2a)^3 + (b)^3 - 3(2a)^2(b) - 3(2a)(b)^2 = (2a - b)^3$

(iii) $27 - 125\mathrm{a}^3 - 135\mathrm{a} + 225\mathrm{a}^2 = (3)^3 + (-5\mathrm{a})^3 - 3(3)^2(-5\mathrm{a}) - 3(3)(-5\mathrm{a})^2$

$= (3 - 5\mathrm{a})^3$

(iv) $64\mathrm{a}^3 - 27\mathrm{b}^3 - 144\mathrm{a}^2\mathrm{b} + 108\mathrm{abb}^2 = (4\mathrm{a})^3 + (-3\mathrm{b})^3 + 3(4\mathrm{a})^2(-3\mathrm{b}) + 3(4\mathrm{a})(-3\mathrm{b})^2$

$= (4a - 3b)^3$

(v) $27p^3 + \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

$= (3p)^3(-\frac{1}{6})^3 + 3(3p)(-\frac{1}{6})^2 + 3(3p)(-\frac{1}{6})^2$

$= (3p - \frac{1}{6})^3$

Question 9:

Verify $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$ using some non-zero positive integers and check by actual multiplication. Can you call theses as identities?

Solution:

To prove: $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

Consider the right hand side (RHS) and expand it as follows:

$(x - y)(x^2 + xy + y^2) = x^3 + x^2y + xy^2 - yx^2 - xy^2 - y^3$

$= (x^3 - y^3) + (x^2y + xy^2 + x^2y - xy^2) = x^3 - y^3 =$ LHS

Hence proved. Yes, we can call it as an identity: For example:

Let us take $\mathrm{x} = 2$ and $\mathrm{y} = 1$ in $\mathrm{x}^3 - \mathrm{y}^3 = (\mathrm{x} - \mathrm{y})(\mathrm{x}^2 + \mathrm{xy} + \mathrm{y}^2)$ then the LHS and RHS will be equal as shown below:

$2^3 - 1^3 = 7$ and

$(2 - 1)(2^2 + (2 \times 1) + 1^2) = 1(5 + 2) = 1 \times 7 = 7$

Therefore, LHS $=$ RHS

Hence, $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$ can be used as an identity.

Question 10:

Factorise each of the following:

(i) $27y^3 + 125z^3$

(ii) $64\mathrm{m}^3 - 343\mathrm{n}^3$

Solution:

(i) $27y^3 + 125z^3 = (3y)^3 + (5z)^3 = (3y + 5z)\{(3y)^2 - (3y)(5z) + (5z)^2\}$

$= (3y + 5z)(9y^2 - 15yz + 25z^2)$

(ii) $64m^3 - 343n^3 = (4m)^3 - (7n)^3 = (4m - 7n)\{(4m)^2 + (4m)(7n) + (7n)^2\}$

$= (4m - 7n)(16m^2 + 28mn + 49n^2)$

Question 11:

Factorise :

$27x^3 + y^3 + z^3 - 9xyz$

Solution:

$27x^3 + y^3 + z^3 - 9xyz$
$= (3x)^3 + y^3 + z^3 - 9xyz$
$= (3x)^3 + y^3 + z^3 - 3 \times 3x \times y \times z$
using identity
$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
Putting $\mathrm{a} = 3\mathrm{x}, \mathrm{b} = \mathrm{y}, \mathrm{c} = \mathrm{z}$
$= (3x + y + z)(9x^2 + y^2 + z^2 - 3xy - yz - 3zx).$

Question 12:

Verify that $x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z)[(x - y)^2 + (y - z) + (z - x)^2]$

Solution:

We have, L.H.S. $= x^3 + y^3 + z^3 - 3xyz$

$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$ [by poynomial identity]

$= \frac{1}{2}(x + y + z)(2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2zx)$

$= \frac{1}{2}[(x + y + z)(x - y)^2 + (y - z)^2 + (z - x)^2]$

Hence proved.

Question 13:

If $x + y + z = 0$ show that $x^3 + y^3 + z^3 = 3xyz$

Solution:

Given $\mathrm{x} + \mathrm{y} + \mathrm{z} = 0$
$\Rightarrow x + y = -z$
Cubing on both sides
$(x + y)^3 = (-z)^3$
$\Rightarrow x^3 + y^3 + 3x^2y + 3xy^2 = -z^3$
$\Rightarrow x^3 + y^3 + 3xy(x + y) = -z^3$
$\Rightarrow x^3 + y^3 + 3xy(-z) = -z^3$
$\Rightarrow x^3 + y^3 - 3xyz = -z^3$
$\Rightarrow x^3 + y^3 + z^3 = 3xyz$

Question 14:

Without actually calculating the cubes, find the value of the following: $-$

$((-12)^3 + (7)^3 + (5)^3)$

Solution:

Correct option is A)

$(-12)^3 + (7)^3 + (5)^3$

Consider, $a = -12 \;\; b = 7 \;\; c = 5$

' Then,'

$a + b + c = (-12) + 7 + 5 = 0$

Using, $a^3 + b^3 + c^3 = 3abc$, If $a + b + c = 0$

So, $(\_12) + 7^3 + 5^3$

$= 3 \times (-12) \times 7 \times 5$

$= -1260$

Question 15:

Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given :

(i) Area: $25a^2 - 35a + 12$

(ii) Area: $35y^2 + 13y - 12$

Solution:

(i) Area $= 25a^2 - 35a + 12$

$\Rightarrow 25a^2 - 15a - 20a + 12$

$\Rightarrow 5a(5a - 3) - 4(5a - 3)$

$\Rightarrow (5a - 3)(5a - 4)$

Hence, Area $= (5a - 3)(5a - 4)$

We know, Area $=$ Length $\times$ breadth

Hence, there are two possibilities

Case 1 : Length $= (5a - 3)$ and Breadth $= (5a - 4)$

Case 2 : Length $= (5a - 4)$ and Breadth $= (5a - 3)$

(ii) Area $= 35y^2 + 13y - 12$

$\Rightarrow 35y^2 + 28y - 15y - 12$

$\Rightarrow 7y(5y + 4) - 3(5y + 4)$

$\Rightarrow (5y + 4)(7y - 3)$

Hence, Area $= (5y + 4)(7y - 3)$

We know, Area $=$ Length $\times$ breadth

Hence, there are two possibilities

Case 1: Length $= (5y + 4)$ and Breadth $= (7y - 3)$

Case 2: Length $= (7y - 3)$ and Breadth $= (5y + 4)$

Question 16:

What are the possible expressions for the dimensions of the cuboids whose volume are given?

Volume: $12ky^2 + 8ky - 20k$

Solution:

here function of volume is in yk

so, here Function $v(k, y) = 12ky^2 + 8ky - 20k$

$v(k, y) = 4k(3y^2) + 4k(2y) + 4k(-5)$

$= 4k(3y^2 + 2y - 5)$

$= 4k(3y^2 + 5y - 3y - 5)$

$= 4k[(y)(3y + 5) + (-1)(3y + 5)]$

$= (4k)(y - 1)(3y + 5)$

so, the dimension of the cuboids will be $4k, y - 13y + 5$