number system

Chapter 1

Exercise 1.1

1: Is zero a rational number? Can you write it in the form $\frac{p}{q}$ where p and q are integers and $q \neq 0$?

Ans: Explain whether zero is rational or not

A number is said to be a rational number if it can be written in the form of $\frac{p}{q}$,

where p and q are integers and $q \neq 0$.

- We can write 0 in the form of $\frac{0}{1}, \frac{0}{2}, \frac{0}{3}, \frac{0}{4}, \frac{0}{-1}, \frac{0}{-2}, \frac{0}{-3}, \frac{0}{-4}$

So, we can observe that 0 can be written in $\frac{p}{q}$, and it satisfies the condition.

Hence, 0 is a rational number.

2. Write any 6 rational numbers between 3 and 4.

Ans: It is known that there are infinitely many rational numbers between any two numbers. Since we need to find 6 rational numbers between 3 and 4, so multiply and divide the numbers by 7 (or by any number greater than 6)

Then it gives,

$$3 = 3 \times \frac{7}{7} = \frac{21}{7}$$

$$4 = 4 \times \frac{7}{7} = \frac{28}{7}$$

Hence, 6 rational numbers found between 3 and 4 are $\frac{22}{7}, \frac{23}{7}, \frac{24}{7}, \frac{25}{7}, \frac{26}{7}, \frac{27}{7}$.

3. Write any five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.

Ans: It is known that there are infinitely many rational numbers between any two numbers.

Since here we need to find five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$, so multiply and divide by 6 (or by any number greater than 5).

Then it gives,

$$\frac{3}{5} = \frac{3}{5} \times \frac{6}{6} = \frac{18}{30},$$

$$\frac{4}{5} = \frac{4}{5} \times \frac{6}{6} = \frac{24}{30}.$$

Hence, 5 rational numbers found between $\frac{3}{5}$ and $\frac{4}{5}$ are $\frac{19}{30}, \frac{20}{30}, \frac{21}{30}, \frac{22}{30}, \frac{23}{30}$.

4. Verify all the statements given below and state whether they are true or false. Show proper reasons for your answers.

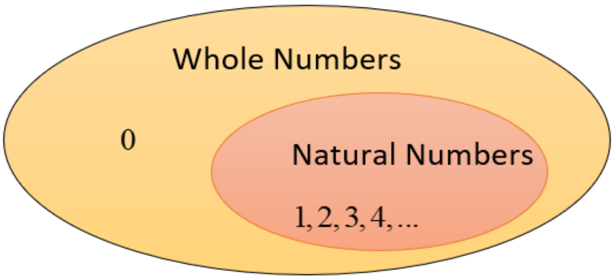
i. Statement: Every natural number is a whole number.

Ans: Write the whole numbers and natural numbers in a separate manner.

It is known that the whole number series is $0, 1, 2, 3, 4, 5......$ and

the natural number series is $1, 2, 3, 4, 5......$

Therefore, it is concluded that all the natural numbers lie in the whole number series as represented in the diagram given below.



Thus, it is concluded that every natural number is a whole number.

Hence, the given statement is true.

ii. Statement: Every integer is a whole number.

Ans: Write the integers and whole numbers in a separate manner.

It is known that integers are those rational numbers that can be expressed in the form of $\frac{p}{q}$, where $q=1$.

Now, the series of integers is like $0,\pm 1,\pm 2,\pm 3,\pm 4,\ldots$.

But the whole numbers are $0,1,2,3,4,\ldots$.

Therefore, it is seen that all the whole numbers lie within the integer numbers, but the negative integers are not included in the whole number series.

Thus, it can be concluded from here that every integer is not a whole number.

Hence, the given statement is false.

iii. Statement: Every rational number is a whole number.

Ans: Write the rational numbers and whole numbers in a separate manner.

It is known that rational numbers are the numbers that can be expressed in the form $\frac{p}{q}$, where $q\neq 0$ and the whole numbers are represented as $0,1,2,3,4,5,\ldots$

Now, notice that every whole number can be expressed in the form of $\frac{p}{q}$ as $\frac{0}{1},\ \frac{1}{1},\ \frac{2}{1},\ \frac{3}{1},\ \frac{4}{1},\ \frac{5}{1},\ldots$

Thus, every whole number is a rational number, but all the rational numbers are not whole numbers. For example, $\frac{1}{2},\frac{1}{3},\frac{1}{4},\frac{1}{5},\ldots$ are not whole numbers.

Therefore, it is concluded from here that every rational number is not a whole number.

Hence, the given statement is false.

Exercise 1.2

1. Verify all the statements given below and state whether they are true or false. Give proper reasons for your answers.

i. Every irrational number is a real number.

Ans: Write the irrational numbers and the real numbers in a separate manner.

- The irrational numbers are the numbers that cannot be represented in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

  For example, $\sqrt{2}, 3\pi,\ .011011011...$ are all irrational numbers.

- The real number is the collection of both the rational numbers and irrational numbers.

  For example, $0, \pm\frac{1}{2}, \pm\sqrt{2}, \pm\pi, ...$ are all real numbers.

Thus, it is concluded that every irrational number is a real number.

Hence, the given statement is true.

ii. Every point on the number line is of the form $\sqrt{m}$, where m is a natural number.

Ans: Consider points on a number line to represent negative as well as positive numbers.

Observe that, positive numbers on the number line can be expressed as $\sqrt{1}, \sqrt{1.1}, \sqrt{1.2}, \sqrt{1.3}, ...$, but any negative number on the number line cannot be expressed as $\sqrt{-1}, \sqrt{-1.1}, \sqrt{-1.2}, \sqrt{-1.3}, ...$, because these are not real numbers.

Therefore, it is concluded from here that every number point on the number line is not of the form $=\sqrt{m}$, where m is a natural number.

Hence, the given statement is false.

iii. Every real number is an irrational number.

Ans: Write the irrational numbers and the real numbers in a separate manner.

- The irrational numbers are the numbers that cannot be represented in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

  For example, $\sqrt{2}, 3\pi,$ $.011011011...$ are all irrational numbers.

- Real numbers are the collection of rational numbers (Ex: $\frac{1}{2}, \frac{2}{3}, \frac{3}{5}, \frac{5}{7}, \ldots\ldots$) and the irrational numbers (Ex: $\sqrt{2}, 3\pi,$ $.011011011...$).

Therefore, it can be concluded that every irrational number is a real number, but every real number cannot be an irrational number.

Hence, the given statement is false.

2. Are the square roots of all positive integer numbers irrational? If not, provide an example of the square root of a number that is not an irrational number.

Ans: Square root of every positive integer does not give an integer.

For example: $\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{6}, ...$ are not integers, and hence these are irrational numbers. But $\sqrt{4}$ gives ± 2 , these are integers and so, $\sqrt{4}$ is not an irrational number.

Therefore, it is concluded that the square root of every positive integer is not an irrational number.

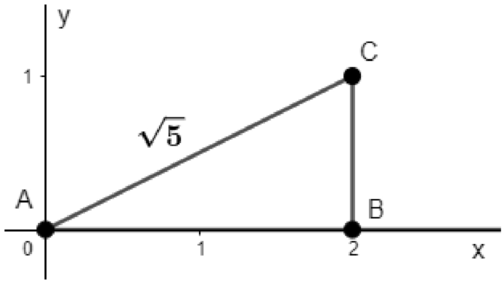
3. Represent $\sqrt{5}$ on the number line.

Ans: Follow the procedures to get $\sqrt{5}$ on the number line.

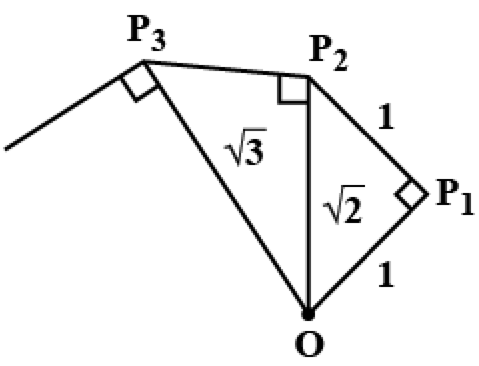
- Firstly, Draw a line segment AB of 2 unit on the number line.
- Secondly, draw a perpendicular line segment BC at B of 1 units.
- Thirdly, join the points C and A, to form a line segment AC.
- Fourthly, apply the Pythagoras Theorem as

$$AC^2 = AB^2 + BC^2$$
$$AC^2 = 2^2 + 1^2$$
$$AC^2 = 4 + 1 = 5$$
$$AC = \sqrt{5}$$

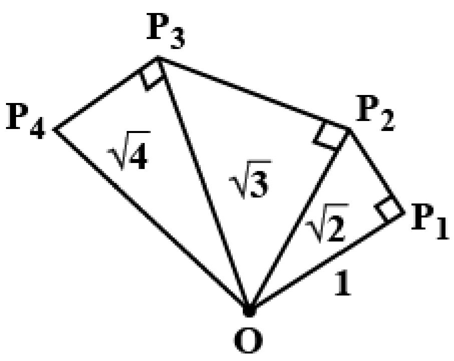
- Finally, draw the arc ACD, to find the number $\sqrt{5}$ on the number line as given in the diagram below.



4: Classroom activity(Constructing the 'square root spiral'): Take a large sheet of paper and construct the 'square root spiral' in the following fashion. Start with a point O and draw a line segment OP_1 of unit length. Draw a line segment P_1P_2 perpendicular to OP_1 of unit length. Now draw a line segment P_2P_3 perpendicular to OP_2. Then draw a line segment P_3P_4 perpendicular to OP_3. Continuing in this manner, you can get the line segment $P_{n-1}P_n$ by drawing a line segment of unit length perpendicular to OP_{n-1}. In this manner, you will have created the points $P_2, P_3, \ldots, P_n, \ldots$, and joined them to create a beautiful spiral depicting $\sqrt{2}, \sqrt{3}, \sqrt{4}, \ldots$



Ans: Starting with point O, draw a line segment OP_1 of unit length.
Now draw a line segment P_1P_2 perpendicular to OP_1 again of unit length.
Here always we need to keep in mind that the measure of the line segment would always be a constant, i.e., 1 unit.
Now again daw a line segment P_2P_3 perpendicular to OP_2 of unit length.
Again draw a line segment P_3P_4 perpendicular to OP_3.
In this way, we have formed a figure as shown in figure.



Exercise 1.3

1. Convert the following numbers in decimal form and state what kind of decimal expansion each has:

i. $\frac{36}{100}$

Ans: Divide 36 by 100.

$$\begin{array}{r} 0.36 \\ 100 \overline{\smash{)}\ 36} \\ \underline{-0} \\ 360 \\ \underline{-300} \\ 600 \\ \underline{-600} \\ \underline{0} \end{array}$$

So, $\frac{36}{100} = 0.36$ and it is a terminating decimal number.

ii. $\mathbf{\frac{1}{11}}$

Ans: Divide 1 by 11.

$$
\begin{array}{r|l}
11 & \overline{\;\;\;\;\;\;\;1\;\;}\;\; 0.0909.... \\
& \underline{-0} \\
& \;10 \\
& \underline{-0} \\
& \;100 \\
& \underline{-99} \\
& \;\;\;10 \\
& \underline{-0} \\
& \;\;100 \\
& \underline{-99} \\
& \;\;\;\;1
\end{array}
$$

It is noticed that while dividing 1 by 11, in the quotient 09 is repeated.

So, $\frac{1}{11} = 0.0909.....$ or

$\frac{1}{11} = 0.\overline{09}$ and it is a non-terminating and recurring decimal number.

iii. $\mathbf{4\frac{1}{8}}$

Ans: $4\frac{1}{8} = 4 + \frac{1}{8} = \frac{32+1}{8} = \frac{33}{8}$

Divide 33 by 8.

$$
\begin{array}{r|l}
8 & \overline{\;33\;}\;\; 4.125 \\
& \underline{-32} \\
& \;\;10
\end{array}
$$

$$\begin{array}{r}
\underline{-8} \\
20 \\
\underline{-16} \\
40 \\
\underline{-40} \\
\underline{0}
\end{array}$$

Notice that, after dividing 33 by 8, the remainder is found as 0.

So, $4\frac{1}{8} = 4.125$ and it is a terminating decimal number.

iv. $\mathbf{\frac{3}{13}}$

Ans: Divide 3 by 13.

$$\begin{array}{r|r}
 & 0.230769 \\
13 & 3 \\
 & \underline{-0} \\
 & 30 \\
 & \underline{-26} \\
 & 40 \\
 & \underline{-39} \\
 & 10 \\
 & \underline{-0} \\
 & 100 \\
 & \underline{-91} \\
 & 90 \\
 & \underline{-78} \\
 & 120 \\
 & \underline{-117} \\
 & \underline{3}
\end{array}$$

It is observed that while dividing 3 by 13, the remainder is found as 3 and that is repeated after each 6 continuous divisions.

So, $\frac{3}{13} = 0.230769.......$ or

$\frac{3}{13} = 0.\overline{230769}$ and it is a non-terminating and recurring decimal number.

v. $\mathbf{\frac{2}{11}}$

Ans: Divide 2 by 11.

$$
\begin{array}{r l}
 & 0.1818..... \\
11 \overline{)} & 2 \\
 & \underline{-0} \\
 & 20 \\
 & \underline{-11} \\
 & 90 \\
 & \underline{-88} \\
 & 20 \\
 & \underline{-11} \\
 & 90 \\
 & \underline{-88} \\
 & \underline{2}
\end{array}
$$

It can be noticed that while dividing 2 by 11, the remainder is obtained as 2 and then 9, and these two numbers are repeated infinitely as remainders.

So, $\frac{2}{11} = 0.1818.....$ or

$\frac{2}{11} = 0.\overline{18}$ and it is a non-terminating and recurring decimal number.

vi. $\mathbf{\frac{329}{400}}$

Ans: Divide 329 by 400.

$$
\begin{array}{r}
0.8225 \\
400\overline{)329} \\
-0 \\
\hline
3290 \\
-3200 \\
\hline
900 \\
-800 \\
\hline
1000 \\
-800 \\
\hline
2000 \\
-2000 \\
\hline
\underline{0}
\end{array}
$$

It can be seen that while dividing 329 by 400, the remainder is obtained as 0.

So, $\frac{329}{400} = 0.8225$ and is a terminating decimal number.

2. If $\frac{1}{7} = 0.142857...$, then predict the decimal expansions of $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$ without calculating the long division?

Ans: Note that, $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}$ and $\frac{6}{7}$ can be rewritten as $2 \times \frac{1}{7}$, $3 \times \frac{1}{7}$, $4 \times \frac{1}{7}$, $5 \times \frac{1}{7}$, and $6 \times \frac{1}{7}$

Substituting the value of $\frac{1}{7} = 0.142857$, gives

$$2 \times \frac{1}{7} = 2 \times 0.142857... = 0.285714...$$

$$3 \times \frac{1}{7} = 3 \times 0.428571... = 0.428571...$$

$$4 \times \frac{1}{7} = 4 \times 0.142857... = 0.571428...$$

$5\times\frac{1}{7}=5\times0.71425... \quad =0.714285...$

$6\times\frac{1}{7}=6\times0.142857...=0.857142...$

So, the values of $\frac{2}{7},\ \frac{3}{7},\ \frac{4}{7},\ \frac{5}{7}$ and $\frac{6}{7}$ obtained without performing long division are $\frac{2}{7}=0.\overline{285714}$

$\frac{3}{7}=0.\overline{428571}$

$\frac{4}{7}=0.\overline{571428}$

$\frac{5}{7}=0.\overline{714285}$

$\frac{6}{7}=0.\overline{857142}$

3. Convert the following decimal numbers into the form of $\frac{p}{q}$, where p and q are integers and $q\neq0$.

i. $0.\overline{6}$

Ans: Let $x=0.\overline{6}$

$\Rightarrow x=0.6666$ ……. (1)

Multiplying both sides of the equation (1) by 10, gives

$10x=0.6666\times10$

o$10x=6.6666$ …… (2)

Subtracting the equation (1) from (2), gives

$10x=6.6666.....$

$\underline{-x=0.6666.....}$

$9x=6$

$9x = 6$

$x = \frac{6}{9} = \frac{2}{3}$

So, the decimal number becomes

$0.\overline{6} = \frac{2}{3}$ and it is in the required $\frac{p}{q}$ form.

ii. $0.4\overline{7}$

Ans: Let $x = 0.4\overline{7}$

$\Rightarrow x = 0.47777.....$ ……(a)

Multiplying both sides of the equation (a) by 10, gives

$10x = 4.7777.....$ ……(b)

Subtracting the equation (a) from (b), gives

$$\begin{aligned} 10x &= 4.7777..... \\ \underline{-x} &\underline{= 0.4777.....} \\ 9x &= 4.3 \end{aligned}$$

Therefore,

$x = \frac{4.3}{9} \times \frac{10}{10}$

$\Rightarrow x = \frac{43}{90}$

So, the decimal number becomes

$0.4\overline{7} = \frac{43}{90}$ and it is in the required $\frac{p}{q}$ form.

iii. $0.\overline{001}$

Ans: Let $x = 0.\overline{001} \Rightarrow$ …… (1)

Since the number of recurring decimal number is 3, so multiplying both sides of the equation (1) by 1000, gives

$1000 \times x = 1000 \times 0.001001.....$ $\qquad \ldots\ldots$ (2)

Subtracting the equation (1) from (2) gives

$$\begin{array}{l} 1000x = 1.001001..... \\ \underline{\quad -x = 0.001001.....} \\ 999x = 1 \end{array}$$

$$\Rightarrow x = \frac{1}{999}$$

Hence, the decimal number becomes

$0.\overline{001} = \frac{1}{999}$ and it is in the $\frac{p}{q}$ form.

4. Represent the nonterminating decimal number 0.99999..... into the form of $\frac{p}{q}$. Did you expect this type of answer? Explain why the answer is appropriate.

Ans: Let $x = 0.99999.....$ $\qquad$ .... (a)

Multiplying by 10 both sides of the equation (a), gives

$10x = 9.9999.....$ $\qquad \ldots\ldots$ (b)

Now, subtracting the equation (a) from (b), gives

$$\begin{array}{l} 10x = 9.99999..... \\ \underline{-x = 0.99999.....} \\ 9x = 9 \end{array}$$

$$\Rightarrow x = \frac{9}{9}$$

$$\Rightarrow x = 1.$$

So, the decimal number becomes

$0.99999... = \frac{1}{1}$ which is in the $\frac{p}{q}$ form.

Yes, for a moment we are amazed by our answer, but when we observe that $0.9999.........$ is extending infinitely, then the answer makes sense.

Therefore, there is no difference between 1 and $0.9999.........$ and hence these two numbers are equal.

5. Find the maximum number of digits in the recurring block of digits in the decimal expansion of $\frac{1}{17}$ by performing the long division.

Ans: Here the number of digits in the recurring block of $\frac{1}{17}$ is to be determined.

So, let us calculate the long division to obtain the recurring block of $\frac{1}{17}$.

Dividing 1 by 17 gives

$$
\begin{array}{r|l}
 & 0.0588235294117647.... \\
\hline
17 & 1 \\
 & \underline{-0} \\
 & 10 \\
 & \underline{-0} \\
 & 100 \\
 & \underline{-85} \\
 & 150 \\
 & \underline{-136} \\
 & 140 \\
 & \underline{-136} \\
 & 40 \\
 & \underline{-34} \\
 & 60 \\
 & \underline{-85} \\
 & 150 \\
 & \underline{-136} \\
 & 140
\end{array}
$$

$$
\begin{array}{r}
\underline{-136} \\
40 \\
\underline{-34} \\
60 \\
\underline{-51} \\
90 \\
\underline{-85} \\
50 \\
\underline{-34} \\
160 \\
\underline{-153} \\
70 \\
\underline{-68} \\
20 \\
\underline{-17} \\
30 \\
\underline{-17} \\
130 \\
\underline{-119} \\
110 \\
\underline{-102} \\
80 \\
\underline{-68} \\
120 \\
\underline{-119} \\
\underline{1}
\end{array}
$$

Thus, it is noticed that while dividing 1 by 17, we found 16 number of digits in the repeating block of decimal expansion that will continue to be 1 after going through 16 continuous divisions.

Hence, it is concluded that $\frac{1}{17} = 0.0588235294117647.....$ or

$\frac{1}{17}=0.\overline{0588235294117647}$ and it is a recurring and non-terminating decimal number.

6. Observe at several examples of rational numbers in the form $\frac{p}{q}(q \neq 0)$, where p and q are integers with H.C.F between them is 1 and having terminating decimal representations. Guess the property that q must satisfy?

Ans: Let us consider the examples of such rational numbers $\frac{5}{2},\frac{5}{4},\frac{2}{5},\frac{2}{10},\frac{5}{16}$ of the form $\frac{p}{q}$ which have terminating decimal representations.

$\frac{5}{2}=2.5$

$\frac{5}{4}=1.25$

$\frac{2}{5}=0.4$

$\frac{2}{10}=0.2$

$\frac{5}{16}=0.3125$

In each of the above examples, it can be noticed that the denominators of the rational numbers have powers of $2,5$ or both.

So, q must satisfy the form either 2^m, or 5^n, or both $2^m \times 5^n$ (where $m=0,1,2,3.....$ and $n=0,1,2,3.....$) in the form of $\frac{p}{q}$.

7. Give examples of three numbers whose decimal representations are non-terminating and non-recurring.

Ans: All the irrational numbers are non-terminating and non-recurring, because irrational numbers do not have any representations of the form of $\frac{p}{q}$ $(q \neq 0)$, where p and q are integers. For example:

$\sqrt{2} = 1.41421.....,$

$\sqrt{3} = 1.73205...$

$\sqrt{7} = 2.645751....$

are the numbers whose decimal representations are non-terminating and non-recurring.

8. Write any three irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$.

Ans: Converting $\frac{5}{7}$ and $\frac{9}{11}$ into the decimal form gives

$\frac{5}{7} = 0.714285.....$ and

$\frac{9}{11} = 0.818181.....$

Therefore, 3 irrational numbers that are contained between $0.714285......$ and $0.818181.....$

are:

$0.73073007300073......$

$0.74074007400074......$

$0.76076007600076......$

Hence, three irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$ are

$0.73073007300073......$

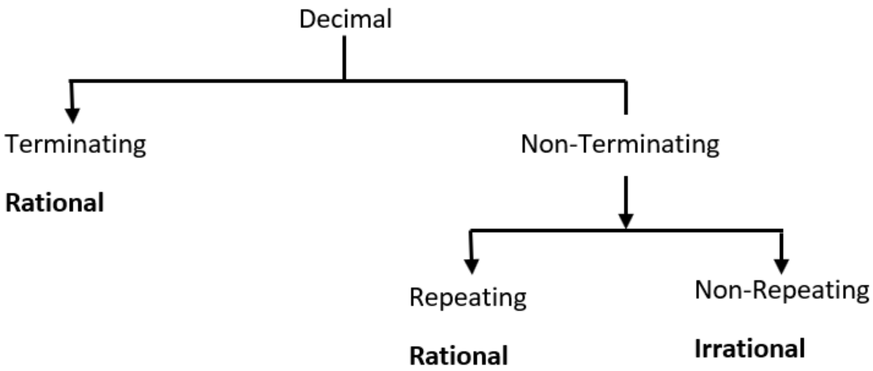
$0.74074007400074......$

$0.76076007600076......$

9. Classify the following numbers and state whether it is rational or irrational:

(i) $\sqrt{23}$

Ans: The following diagram reminds us of the distinctions among the types of rational and irrational numbers.



After evaluating the square root gives

$\sqrt{23} = 4.795831.....$, which is an irrational number.

(ii) $\sqrt{225}$

Ans: After evaluating the square root gives

$\sqrt{225} = 15$, which is a rational number.

That is, $\sqrt{225}$ is a rational number.

(iii) 0.3796

Ans: The given number is 0.3796. It is terminating decimal.

So, 0.3796 is a rational number.

(iv) 7.478478

Ans: The given number is $7.478478\ldots$.

It is a non-terminating and recurring decimal that can be written in the $\frac{p}{q}$ form.

Let $\quad x = 7.478478....$ $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ ......(a)

Multiplying the equation (a) both sides by 100 gives

$\Rightarrow 1000x = 7478.478478.....$ $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ ......(b)

Subtracting the equation (a) from (b), gives

$$1000x = 7478.478478....$$
$$\underline{\quad -x = \quad 7.478478....}$$
$$999x = 7471$$
$$999x = 7471$$
$$x = \frac{7471}{999}$$

Therefore, $7.478478..... = \frac{7471}{999}$, which is in the form of $\frac{p}{q}$

So, $7.478478...$ is a rational number.

(v) 1.101001000100001.....

Ans: The given number is $1.101001000100001....$

It can be clearly seen that the number $1.101001000100001....$ is a non-terminating and non recurring decimal and it is known that non-terminating non-recurring decimals cannot be written in the form of $\frac{p}{q}$.

Hence, the number $1.101001000100001....$ is an irrational number.

Exercise 1.4

1: Classify the following number as rational or irrational:

(i) $\mathbf{2 - \sqrt{5}}$

(ii) $\mathbf{(3+\sqrt{23}) - \sqrt{23}}$

(iii) $\mathbf{\frac{2\sqrt{7}}{7\sqrt{7}}}$

(iv) $\mathbf{\frac{1}{\sqrt{2}}}$

(v) 2π

Ans: (i) $2-\sqrt{5}$:

Since $\sqrt{5}$ is an irrational number, $2-\sqrt{5}$ is also an irrational number.

(ii) $(3+\sqrt{23})-\sqrt{23}$:

$(3+\sqrt{\mathbf{23}})-\sqrt{\mathbf{23}}=3+\sqrt{\mathbf{23}}-\sqrt{\mathbf{23}}=3$, which is a rational number.

(iii) $\frac{\mathbf{2\sqrt{7}}}{\mathbf{7\sqrt{7}}}=\frac{2}{7}$:

$\frac{\mathbf{2\sqrt{7}}}{\mathbf{7\sqrt{7}}}=\frac{2}{7}$, which is a rational number.

(iv) $\frac{\mathbf{1}}{\sqrt{\mathbf{2}}}$:

Since 2 is an irrational number, $\frac{\mathbf{1}}{\sqrt{\mathbf{2}}}$**,**which is also an irrational number.

(v) 2π:

Since π is an irrational number, 2π is also an irrational number

(iv) $\frac{\mathbf{1}}{\sqrt{\mathbf{2}}}$:

Since 2 is an irrational number, $\frac{\mathbf{1}}{\sqrt{\mathbf{2}}}$**,**which is also an irrational number.

(v) 2π:

Since π is an irrational number, 2π is also an irrational number

2: Simplify each of the following e x pressions:

(i) $(3+\sqrt{3})(2+\sqrt{2})$

(ii) $(3+\sqrt{3})(3-\sqrt{3})$

(iii) $(\sqrt{5}+\sqrt{2})^2$

(iv) $(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})$

Ans: We know that, (A+B)(C+D)=A×(C+D)+B×(C+D)

=AC+AD+BC+CD

(i) $(3+\sqrt{3})(2+\sqrt{2}) = (6+2\sqrt{3}+3\sqrt{2}+\sqrt{6})$

(ii) $(\sqrt{5}+\sqrt{2})^2=(5+2+2\sqrt{10}) = (7+2\sqrt{10})$

(iii) $(3+\sqrt{3})(3-\sqrt{3}) = (9-3) = 6$

(iv) $(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2}) = (5-2) = 3$

3: Recall, π is defined as the ratio of the circumference (say c) of a circle to its diameter (say d). That is, π=c/d. This seems to contradict the fact that π is irrational. How will you resolve this contradiction?

Ans: When we measure a length with a scale or any other device, we only get an approximate rational value.

Therefore we may not realise that c or d is irrational.

Circumference c or the perimeter of a circle is given by 2πr,
where r is the radius of the circle
π is approximated as 3.14 or $\frac{22}{7}$
Also diameter(longest chord of circle) of the circle is equal to 2r.

Hence, c = (2πr),d = 2r $\Rightarrow \frac{c}{d} = \pi$

This is analogous to the approximated value of $\frac{22}{7}$ which though looks like a rational number of the form $\frac{p}{q}$ (q! = 0)

But when computed corresponds to a real value of ~3.14.

And real numbers consists of irrational numbers.

Hence, there is no contradiction in the equation = $\frac{c}{d}$.

4: Represent $\sqrt{9.9}$ on the number line.

Ans: Steps:

1) Draw a line segment A B of length 9.3 units.

2) Extend the line by 1 unit more such that BC=1 unit .

3) Find the midpoint of AC.

4) Draw a line BD perpendicular to A B and let it intersect the semicircle at point D.

5) Draw an arc DE such that BE=BD.

Therefore, BE $=\sqrt{9.9}$ units

5: Rationalise the denominators of the following:

(i) $\frac{1}{\sqrt{7}}$

(ii) $\frac{1}{\sqrt{7}-\sqrt{6}}$

(iii) $\frac{1}{\sqrt{5}+\sqrt{2}}$

(iv) $\frac{1}{\sqrt{7}-2}$

Ans: (1) $\frac{1}{\sqrt{7}}=\frac{1}{\sqrt{7}}\times\frac{\sqrt{7}}{\sqrt{7}}$ (multiplying and dividing by $\sqrt{7}$)

$=\frac{\sqrt{7}}{7}$

(2) $\frac{1}{\sqrt{7}+\sqrt{6}}=\frac{1}{\sqrt{7}+\sqrt{6}}\times\frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}}$ (multiplying and dividing by $\sqrt{7}+\sqrt{6}$)

$=\frac{\sqrt{7}+\sqrt{6}}{7-6}+\sqrt{7}+\sqrt{6}$

(3) $\frac{1}{\sqrt{5}+\sqrt{2}}=\frac{1}{\sqrt{5}+\sqrt{2}}\times\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}$ (multiplying and dividing by $\sqrt{5}-\sqrt{2}$)

$=\frac{\sqrt{5}-\sqrt{2}}{5-2}=\frac{\sqrt{5}-\sqrt{2}}{3}$

(4) $\frac{1}{\sqrt{7}-2}=\frac{1}{\sqrt{7}-2}\times\frac{\sqrt{7}-2}{\sqrt{7}-2}$ (multiplying and dividing by $\sqrt{7}-2$)

$=\frac{\sqrt{7}-2}{7-2}=\frac{\sqrt{7}-2}{3}$

Exercise 1.5

1: Find : (i) $64^{\frac{1}{2}}$ (ii) $32^{\frac{1}{5}}$ (iii) $125^{\frac{1}{3}}$

Ans: i) $(64)^{\frac{1}{2}}$

Express 64 in terms of 8

$=(8^2)^{\frac{1}{2}}$ $=8^{2\times\frac{1}{2}}=8^1=8$

ii) $(32)^{\frac{1}{5}}$

Express 32 in terms of 2

$=(2^5)^{\frac{1}{5}}=2^{5\times\frac{1}{2}}=2^1=2$

iii) $(125)^{\frac{1}{3}}$

Express 125 in terms of 5

$=(5^2)^{\frac{1}{3}}=2^{3\times\frac{1}{2}}=5^1=5$

2: Find: (i) $9^{\frac{3}{2}}$ (ii) $32^{\frac{2}{5}}$ (iii) $16^{\frac{3}{4}}$ (iv) $125^{\frac{-1}{3}}$

Ans: (i) $(9)^{\frac{3}{2}}$

Express 9 in terms of 3

$=(3^2)^{\frac{3}{2}}=3^{2\times\frac{3}{2}} = 3^3 = 27$

(ii) $32^{\frac{2}{5}}$

Express 32 in terms of 2

$= (2^5)^{\frac{2}{5}} = 2^{5\times\frac{2}{5}} = 2^2 = 4$

(iii) $\mathbf{16^{\frac{3}{4}}}$

Express 16 in terms of 4

$=(2^4)^{\frac{3}{4}} = 2^{4\times\frac{3}{4}} = 2^3 = 8$

(iv) $\mathbf{125^{\frac{-1}{3}}}$

Express 125 in terms of 5

$=(5^3)^{\frac{-1}{3}} = 5^{3\times\frac{-1}{3}} = 5^{-1} = \frac{1}{5}$

3: (i) $\mathbf{2^{\frac{3}{2}} . 2^{\frac{1}{5}}}$ **(ii)** $\mathbf{\left(\frac{1}{3^3}\right)^7}$ **(iii)** $\mathbf{\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}}}$ **(iv)** $\mathbf{7^{\frac{1}{2}} . 8^{\frac{1}{2}}}$

Ans: (i) $2^{\frac{3}{2}} . 2^{\frac{1}{5}}$

By using property $X^a. X^b = X^{a+b}$

$= 2^{\frac{3}{2}+\frac{1}{5}} = 2^{\frac{17}{10}}$

(ii) $\left(\frac{1}{3^3}\right)^7$

By using property $\left(\frac{X}{y}\right)^a = \frac{X^a}{y^b}$

$= \frac{1}{(3^3)^7}$

$= \frac{1}{3^{21}}$...... [$\because (xa)^b = x^{ab}$]

(iii) $\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}}$

By using property $\frac{X^a}{X^b} = X^{a\text{-}d}$

$= 11^{\frac{1}{2}+\frac{1}{4}} = 11^{\frac{1}{4}}$

(iv) $7^{\frac{1}{2}}.8^{\frac{1}{2}}$

$=56^{\frac{1}{2}}$ ...... [$\because x^a y^a = (xy)^a$]