surface areas and volumnes

11 Chapter

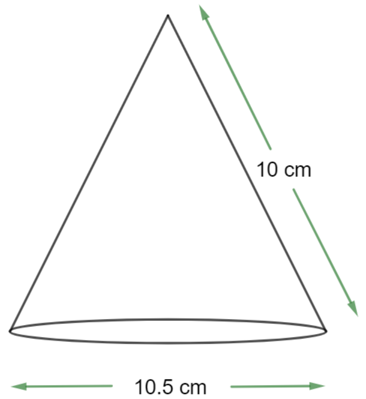
Exercise 11.1

1. Diameter of the base of a cone is $10.5\ \text{cm}$ and its slant height is $10\ \text{cm}$. Find its curved surface area. $\left[\text{Assume } \pi = \dfrac{22}{7}\right]$

Ans: We are given the following:

The slant height (l) of the cone $= 10\ \text{cm}$

The diameter of the base of cone $= 10.5\ \text{cm}$



So, the radius (r) of the base of cone $= \dfrac{10.5}{2}\ \text{cm} = 5.25\ \text{cm}$

The curved surface area of cone, $A = \pi rl$

$$\Rightarrow A = \left(\frac{22}{7} \times 5.25 \times 10\right)\ \text{cm}^2$$

$$\Rightarrow A = (22 \times 0.75 \times 10)\ \text{cm}^2$$

$$\Rightarrow A = 165\ \text{cm}^2$$

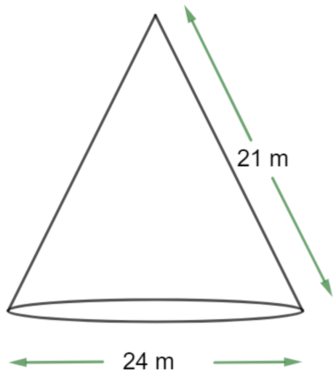
Therefore, the curved surface area of the cone is $165\ \text{cm}^2$.

2. Find the total surface area of a cone, if its slant height is 21 m and diameter of its base is 24 m. $\left[\text{Assume } \pi = \dfrac{22}{7}\right]$

Ans: We are given the following:

The slant height (l) of the cone $= 21 \text{ m}$

The diameter of the base of cone $= 24 \text{ m}$



So, the radius (r) of the base of cone $= \dfrac{24}{2} \text{ m} = 12 \text{ m}$

The total surface area of cone, $A = \pi r(l + r)$

$$\Rightarrow A = \left(\frac{22}{7} \times 12 \times (21 + 12)\right) \text{m}^2$$

$$\Rightarrow A = \left(\frac{22}{7} \times 12 \times 33\right) \text{m}^2$$

$$\Rightarrow A = 1244.57 \text{ m}^2$$

Therefore, the total surface area of the cone is 1244.57 m^2.

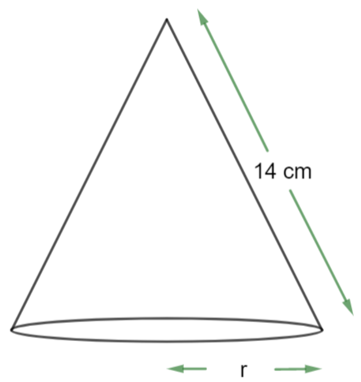
3. Curved surface area of a cone is 308 cm^2 and its slant height is 14 cm. Find

(i) Radius of the base and

Ans:

It is given that the slant height (l) of the cone $= 14 \text{ cm}$

The curved surface area of the cone $= 308 \text{ cm}^2$



Let us assume the radius of base of cone be r.

We know that curved surface area of the cone $= \pi rl$

$\therefore \pi rl = 308 \text{ cm}^2$

$\Rightarrow \left(\frac{22}{7} \times r \times 14\right) \text{cm} = 308 \text{ cm}^2$

$\Rightarrow r = \frac{308}{44} \text{ cm}$

$\Rightarrow r = 7 \text{ cm}$

Hence, the radius of the base is 7 cm.

(ii) Total surface area of the cone. $\left[\textbf{Assume } \boldsymbol{\pi} = \frac{\mathbf{22}}{\mathbf{7}}\right]$

Ans: The total surface area of the cone is the sum of its curved surface area and the area of the base.

Total surface area of cone, $A = \pi rl + \pi r^2$

$\Rightarrow A = \left[308 + \frac{22}{7} \times (7)^2\right] \text{cm}^2$

$\Rightarrow A = [308 + 154] \text{ cm}^2$

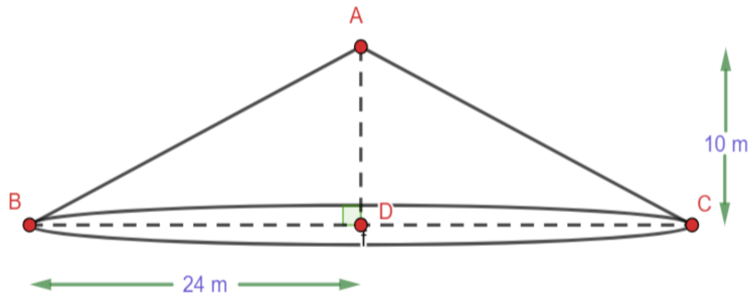
$\Rightarrow A = 462 \text{ cm}^2$

Hence, the total surface area of the cone is 462 cm^2.

4. A conical tent is $\mathbf{10 \text{ m}}$ high and the radius of its base is $\mathbf{24 \text{ m}}$. Find

(i) slant height of the tent

Ans:



From the figure we can say that ABC is a conical tent.
It is given that the height (h) of conical tent $= 10 \text{ m}$
The radius (r) of conical tent $= 24 \text{ m}$
Let us assume the slant height as l.
In ΔABD, we will use Pythagorean Theorem.
$\therefore AB^2 = AD^2 + BD^2$
$\Rightarrow l^2 = h^2 + r^2$
$\Rightarrow l^2 = (10 \text{ m})^2 + (24 \text{ m})^2$
$\Rightarrow l^2 = 676 \text{ m}^2$
$\Rightarrow l = 26 \text{ m}$
The slant height of the tent is 26 m.

(ii) cost of canvas required to make the tent, if cost of 1 m^2 canvas is Rs. 70. $\left[\textbf{Assume } \pi = \frac{22}{7}\right]$

Ans: The curved surface area of the tent, $A = \pi r l$

$$\Rightarrow A = \left(\frac{22}{7} \times 24 \times 26\right) \text{m}^2$$

$$\Rightarrow A = \left(\frac{13728}{7}\right) \text{m}^2$$

It is given that the cost of 1 m^2 of canvas $=$ Rs. 70

So, the cost of $\frac{13728}{7} \text{ m}^2$ canvas $=$ Rs. $\left(\frac{13728}{7} \times 70\right) =$ Rs. 137280

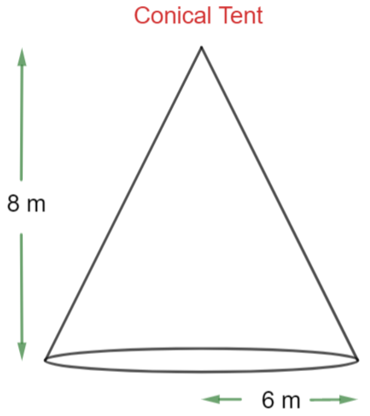
Hence, the cost of canvas required to make the tent is Rs. 137280.

5. What length of tarpaulin 3 m wide will be required to make conical tent of height 8 m and base radius 6 m? Assume that the extra length of material that will be required for stitching margins and wastage in cutting is approximately 20 cm. [Use $\pi = 3.14$]

Ans: We are given the following:

The base radius (r) of tent $= 6\text{ m}$

The height (h) of tent $= 8\text{ m}$



So the slant height of the tent, $l = \sqrt{r^2 + h^2}$

$\Rightarrow l = \left(\sqrt{6^2 + 8^2}\right)\text{ m}$

$\Rightarrow l = \left(\sqrt{100}\right)\text{ m}$

$\Rightarrow l = 10\text{ m}$

The curved surface area of the tent, $A = \pi rl$

$\Rightarrow A = (3.14 \times 6 \times 10)\text{ m}^2$

$\Rightarrow A = 188.4\text{ m}^2$

It is give the width of tarpaulin $= 3\text{ m}$

Let us assume the length of tarpaulin sheet required be x.

It is given that there will be a wastage of 20 cm.

So, the new length of the sheet $= (x - 0.2)\text{ m}$

We know that the area of the rectangular sheet required will be the same as curved surface area of the tent.

$\therefore \left[(x - 0.2) \times 3\right]\text{ m} = 188.4\text{ m}^2$

$\Rightarrow x - 0.2\text{ m} = 62.8\text{ m}$

$\Rightarrow x = 63\text{ m}$

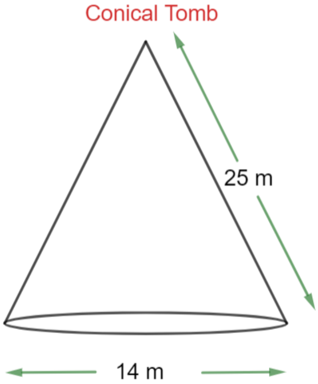
The length of tarpaulin sheet required is 63 m.

6. The slant height and base diameter of a conical tomb are 25 m and 14 m respectively. Find the cost of white-washing its curved surface at the rate of Rs. 210 per 100 m^2. $\left[\text{Assume } \pi = \dfrac{22}{7}\right]$

Ans: We are given the following:

The base radius (r) of tomb $= 7\text{ m}$

The slant height (l) of tomb $= 25 \text{ m}$



The curved surface area of the conical tomb, $A = \pi r l$

$$\Rightarrow A = \left(\frac{22}{7} \times 7 \times 25\right) \text{ m}^2$$

$$\Rightarrow A = 550 \text{ m}^2$$

It is given that the cost of white-washing 1 m^2 area $=$ Rs. 210

So, the cost of white-washing 550 m^2 area $= \text{Rs.} \left(\frac{210}{100} \times 550\right) = \text{Rs. } 1155$

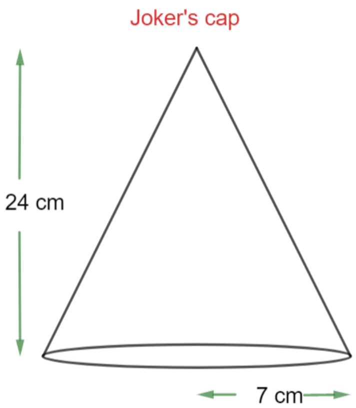
Hence, the cost of white-washing the curved surface area of a conical tomb is Rs. 1155.

7. A joker's cap is in the form of right circular cone of base radius 7 cm and the height 24 cm. Find the area of sheet required to make 10 such caps. $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Ans: We are given the following:

The base radius (r) of conical cap $= 7 \text{ cm}$

The height (h) of conical cap $= 24 \text{ cm}$



So the slant height of the tent, $l = \sqrt{r^2 + h^2}$

$\Rightarrow l = \left(\sqrt{7^2 + 24^2}\right)$ cm

$\Rightarrow l = \left(\sqrt{625}\right)$ cm

$\Rightarrow l = 25$ cm

The curved surface area of one conical cap, $A = \pi rl$

$\Rightarrow A = \left(\frac{22}{7} \times 7 \times 25\right)$ cm^2

$\Rightarrow A = 550$ cm^2

So, the curved surface area of 10 conical caps $= (550 \times 10)$ cm^2 $= 5500$ cm^2

Therefore, the total area of sheet required is 5500 cm^2.

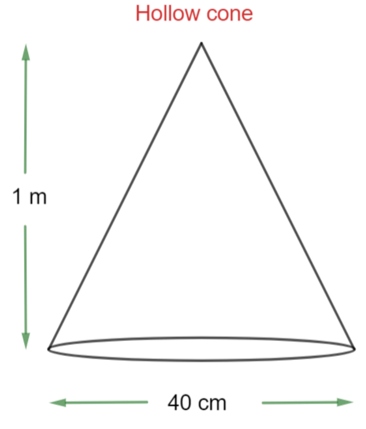
8. A bus stop is barricaded from the remaining part of the road, by using 50 hollow cones made of recycled cardboard. Each cone has a base diameter of 40 cm and height 1 m. If the outer side of each of the cones is to be painted and the cost of painting is Rs. 12 per m^2, what will be the cost of painting all these cones?

$\left[\text{Use } \pi = 3.14 \text{ and take } \sqrt{1.02} = 1.02\right]$

Ans: We are given the following:

The base radius (r) of cone $= \frac{40}{2} = 20$ cm $= 0.2$ m

The height (h) of cone $= 1$ m



So the slant height of the cone, $l = \sqrt{r^2 + h^2}$

$\Rightarrow l = \left(\sqrt{(0.2)^2 + (1)^2}\right)$ m

$\Rightarrow l = \left(\sqrt{1.04}\right)$ m

$\Rightarrow l = 1.02$ m

The curved surface area of one cone, $A = \pi rl$

$\Rightarrow A = (3.14 \times 0.2 \times 1.02)\ m^2$

$\Rightarrow A = 0.64056\ cm^2$

So, the curved surface area of 50 cones $= (50 \times 0.64056)\ m^2 = 32.028\ m^2$

It is given that the cost of painting $1\ m^2$ area $=$ Rs. 12

So, the cost of painting $32.028\ m^2$ area $=$ Rs. $(32.028 \times 12) =$ Rs. 384.336

We can also write the cost approximately as Rs. 384.34.

Therefore, the cost of painting all the hollow cones is Rs. 384.34.

Exercise 11.2

1. Find the surface area of a sphere of radius:

(i) 10.5 cm

Ans: Given radius of the sphere $r = 10.5\ cm$

The surface area of the sphere $A = 4\pi r^2$

$\Rightarrow A = \left[4 \times \frac{22}{7} \times (10.5)^2\right] cm^2$

$\Rightarrow A = (88 \times 1.5 \times 1.5)\ cm^2$

$\Rightarrow A = 1386\ cm^2$

Hence, the surface area of the sphere is $1386\ cm^2$.

(ii) 5.6 cm

Ans: Given radius of the sphere $r = 5.6\ cm$

The surface area of the sphere $A = 4\pi r^2$

$\Rightarrow A = \left[4 \times \frac{22}{7} \times (5.6)^2\right] cm^2$

$\Rightarrow A = (88 \times 0.8 \times 5.6)\ cm^2$

$\Rightarrow A = 394.24\ cm^2$

Hence, the surface area of the sphere is $394.24\ cm^2$.

(iii) 14 cm $\left[\textbf{Assume } \boldsymbol{\pi} = \frac{\mathbf{22}}{\mathbf{7}}\right]$

Ans: Given radius of the sphere $r = 14\ cm$

The surface area of the sphere $A = 4\pi r^2$

$\Rightarrow A = \left[4 \times \frac{22}{7} \times (14)^2\right] cm^2$

$\Rightarrow A = (4 \times 44 \times 14)\ cm^2$

$\Rightarrow A = 2464\ \text{cm}^2$

Hence, the surface area of the sphere is $2464\ \text{cm}^2$.

2. Find the surface area of a sphere of diameter:

(i) 14 cm

Ans: Given diameter of the sphere $= 14$ cm

So, the radius of the sphere $r = \dfrac{14}{2} = 7$ cm

The surface area of the sphere $A = 4\pi r^2$

$$\Rightarrow A = \left[4 \times \frac{22}{7} \times (7)^2\right] \text{cm}^2$$

$$\Rightarrow A = (88 \times 7)\ \text{cm}^2$$

$$\Rightarrow A = 616\ \text{cm}^2$$

Hence, the surface area of the sphere is $616\ \text{cm}^2$.

(ii) 21 cm

Ans: Given diameter of the sphere $= 21$ cm

So, the radius of the sphere $r = \dfrac{21}{2} = 10.5$ cm

The surface area of the sphere $A = 4\pi r^2$

$$\Rightarrow A = \left[4 \times \frac{22}{7} \times (10.5)^2\right] \text{cm}^2$$

$$\Rightarrow A = 1386\ \text{cm}^2$$

Hence, the surface area of the sphere is $1386\ \text{cm}^2$.

(iii) 3.5 m $\left[\text{Assume } \pi = \dfrac{22}{7}\right]$

Ans: Given diameter of the sphere $= 3.5$ m

So, the radius of the sphere $r = \dfrac{3.5}{2} = 1.75$ m

The surface area of the sphere $A = 4\pi r^2$

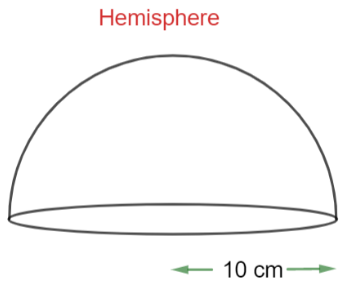
$$\Rightarrow A = \left[4 \times \frac{22}{7} \times (1.75)^2\right] \text{m}^2$$

$$\Rightarrow A = 38.5\ \text{m}^2$$

Hence, the surface area of the sphere is $38.5\ \text{m}^2$.

3. Find the total surface area of a hemisphere of radius 10 cm. $[\text{Use } \pi = 3.14]$

Ans:



Given the radius of hemisphere $r = 10 \text{ cm}$

The total surface area of hemisphere is the sum of its curved surface area and the circular base.

Total surface area of hemisphere $A = 2\pi r^2 + \pi r^2$

$\Rightarrow A = 3\pi r^2$

$\Rightarrow A = \left[3 \times 3.14 \times (10)^2\right] \text{cm}^2$

$\Rightarrow A = 942 \text{ cm}^2$

Hence, the total surface area of the hemisphere is 942 cm^2.

4. The radius of a spherical balloon increases from 7 cm to 14 cm as air is being pumped into it. Find the ratio of surface areas of the balloon in the two cases.

Ans: Given the initial radius of the balloon $r_1 = 10 \text{ cm}$

The final radius of the balloon $r_2 = 14 \text{ cm}$

We have to find the ratio of surface areas of the balloon in the two cases.

The required ratio $R = \dfrac{4\pi {r_1}^2}{4\pi {r_2}^2}$

$\Rightarrow R = \left(\dfrac{r_1}{r_2}\right)^2$

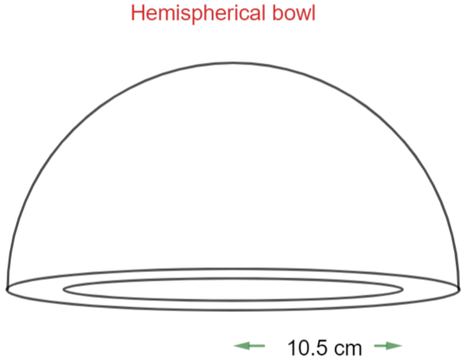
$\Rightarrow R = \left(\dfrac{7}{14}\right)^2$

$\Rightarrow R = \dfrac{1}{4}$

Hence, the ratio of the surface areas of the balloon in both case is $1:4$.

5. A hemispherical bowl made of brass has inner diameter 10.5 cm. Find the cost of tinplating it on the inside at the rate of $\text{Rs. } 16$ per 100 cm^2. $\left[\text{Assume } \pi = \dfrac{22}{7}\right]$

Ans: Given the radius of inner hemispherical bowl $r = \frac{10.5}{2} = 5.25$ cm



The surface area of the hemispherical bowl $A = 2\pi r^2$

$\Rightarrow A = \left[2 \times \frac{22}{7} \times (5.25)^2\right] \text{cm}^2$

$\Rightarrow A = 173.25 \text{ cm}^2$

It is given that the cost of tin-plating 100 cm^2 area $=$ Rs. 16

So, the cost of tin-plating 173.25 cm^2 area $=$ Rs. $\left(\frac{16}{100} \times 173.25\right) =$ Rs. 27.72

Hence, the cost of tin-plating the hemispherical bowl is Rs. 27.72.

6. Find the radius of a sphere whose surface area is 154 cm^2. $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Ans: Let us assume the radius of sphere be r.

We are given the surface area of the sphere, $A = 154 \text{ cm}^2$.

$\therefore 4\pi r^2 = 154 \text{ cm}^2$

$\Rightarrow r^2 = \left(\frac{154 \times 7}{2 \times 22}\right) \text{cm}^2$

$\Rightarrow r = \left(\frac{7}{2}\right) \text{cm}$

$\Rightarrow r = 3.5 \text{ cm}$

Therefore, the radius of the sphere is 3.5 cm.

7. The diameter of the moon is approximately one-fourth of the diameter of the earth. Find the ratio of their surface area.

Ans: Let us assume the diameter of earth be d.

So, the diameter of the moon will be $\frac{d}{4}$.

The radius of the earth $r_1 = \frac{d}{2}$

The radius of the moon $r_2 = \frac{1}{2} \times \frac{d}{2} = \frac{d}{8}$

The ratio of surface area of moon and earth $R = \frac{4\pi r_2^2}{4\pi r_1^2}$

$$\Rightarrow R = \frac{4\pi\left(\frac{d}{8}\right)^2}{4\pi\left(\frac{d}{2}\right)^2}$$

$$\Rightarrow R = \frac{4}{64}$$

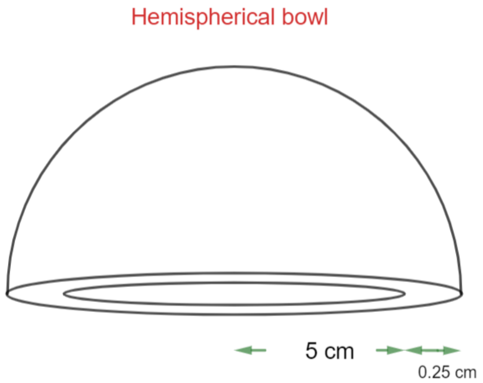
$$\Rightarrow R = \frac{1}{16}$$

Therefore, the ratio of surface area of moon and earth is $1 : 16$.

8. A hemispherical bowl is made of steel, 0.25 cm thick. The inner radius of the bowl is 5 cm. Find the outer curved surface area of the bowl. $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Ans: Given the inner radius $= 5$ cm

The thickness of the bowl $= 0.25$ cm



So, the outer radius of the hemispherical bowl is $r = (5 + 0.25)$ cm $= 5.25$ cm

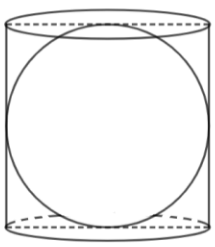
The outer curved surface area of the hemispherical bowl $A = 2\pi r^2$

$$\Rightarrow A = \left[2 \times \frac{2}{7} \times (5.25)^2\right] \text{cm}^2$$

$$\Rightarrow A = 173.25 \text{ cm}^2$$

Therefore, the outer curved surface area of the hemispherical bowl is 173.25 cm^2.

9. A right circular cylinder just encloses a sphere of radius r (see figure). Find

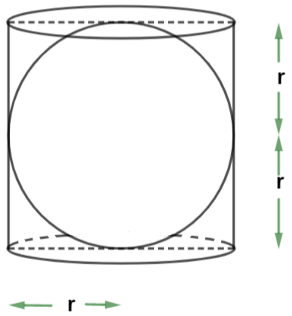


(i) surface area of the sphere,

Ans: The surface area of the sphere is $4\pi r^2$.

(ii) curved surface area of the cylinder,

Ans:



Given the radius of cylinder $= r$

The height of cylinder $= r + r = 2r$

The curved surface area of cylinder $A = 2\pi rh$

$\Rightarrow A = 2\pi r\,(2r)$

$\Rightarrow A = 4\pi r^2$

Therefore the curved surface area of cylinder is $4\pi r^2$.

(iii) ratio of the areas obtained in (i) and (ii).

Ans: The ratio of surface area of the sphere and curved surface area of cylinder

$$R = \frac{4\pi r^2}{4\pi r^2}$$

$$R = \frac{1}{1}$$

Therefore, the required ratio is $1:1$.

Exercise 11.3

1. Find the volume of the right circular cone with

(i) Radius 6 cm, height 7 cm

Ans: It is given the radius of cone $r = 6\text{ cm}$

The height of the cone $h = 7\text{ cm}$

The volume of the cone $V = \frac{1}{3}\pi r^2 h$

$$\Rightarrow V = \left[\frac{1}{3} \times \frac{22}{7} \times (6)^2 \times 7\right]\text{cm}^3$$

$$\Rightarrow V = (12 \times 22)\text{ cm}^3$$

$\Rightarrow V = 264\ cm^3$

The volume of the right circular cone is $264\ cm^3$.

(ii) Radius $3.5\ cm$, height $12\ cm$ $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Ans: It is given the radius of cone $r = 3.5\ cm$

The height of the cone $h = 12\ cm$

The volume of the cone $V = \frac{1}{3}\pi r^2 h$

$\Rightarrow V = \left[\frac{1}{3} \times \frac{22}{7} \times (3.5)^2 \times 12\right] cm^3$

$\Rightarrow V = (1.75 \times 88)\ cm^3$

$\Rightarrow V = 154\ cm^3$

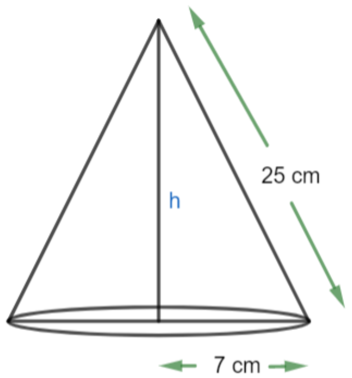
The volume of the right circular cone is $154\ cm^3$.

2. Find the capacity in litres of a conical vessel with

(i) Radius $7\ cm$, slant height $25\ cm$

Ans: It is given the radius of cone $r = 7\ cm$

The slant height of the cone $l = 25\ cm$



So, the height of the cone $h = \sqrt{l^2 - r^2}$

$\Rightarrow h = \sqrt{25^2 - 7^2}\ cm$

$\Rightarrow h = 24\ cm$

The volume of the cone $V = \frac{1}{3}\pi r^2 h$

$\Rightarrow V = \left[\frac{1}{3} \times \frac{22}{7} \times (7)^2 \times 24\right] cm^3$

$\Rightarrow V = (154 \times 8)\ cm^3$

$\Rightarrow V = 1232\ cm^3$

We know that $1000 \text{ cm}^3 = 1$ litre

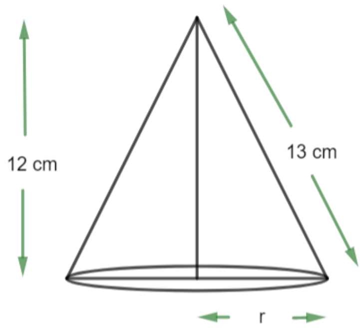
So, the capacity of the conical vessel $= \frac{1232}{1000} = 1.232$ litres

Therefore, the capacity of the conical vessel is 1.232 litres.

(ii) height 12 cm, slant height 13 cm $\left[\textbf{Assume } \boldsymbol{\pi = \frac{22}{7}}\right]$

Ans: It is given the height of cone $h = 12$ cm

The slant height of the cone $l = 13$ cm



So, the radius of the cone $r = \sqrt{l^2 - h^2}$

$\Rightarrow r = \sqrt{13^2 - 12^2}$ cm

$\Rightarrow r = 5$ cm

The volume of the cone $V = \frac{1}{3}\pi r^2 h$

$$\Rightarrow V = \left[\frac{1}{3} \times \frac{22}{7} \times (5)^2 \times 12\right] \text{cm}^3$$

$$\Rightarrow V = \left(4 \times \frac{22}{7} \times 25\right) \text{cm}^3$$

$$\Rightarrow V = \frac{2200}{7} \text{ cm}^3$$

We know that $1000 \text{ cm}^3 = 1$ litre

So, the capacity of the conical vessel $= \frac{2200}{7} \times \frac{1}{1000} = 0.314$ litres

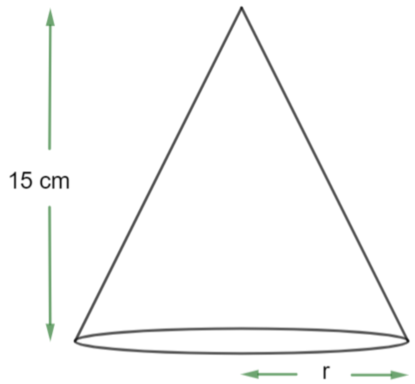
Therefore, the capacity of the conical vessel is 0.314 litres.

3. The height of a cone is 15 cm. It its volume is 1570 cm^3, find the diameter of its base. [Use $\pi = 3.14$]

Ans: It is given the height of cone $h = 12$ cm

Let us assume the radius of the cone be r.

The volume of the cone is $V = 1570 \text{ cm}^3$



We know the formula for the volume of the cone $= \frac{1}{3}\pi r^2 h$

$$\therefore \frac{1}{3}\pi r^2 h = 1570 \text{ cm}^3$$
$$\Rightarrow \left[\frac{1}{3} \times \frac{22}{7} \times (r)^2 \times 12\right] \text{cm} = 1570 \text{ cm}^3$$
$$\Rightarrow r^2 = 100 \text{ cm}^2$$
$$\Rightarrow r = 10 \text{ cm}$$

Diameter of base $= 2r = 20$ cm

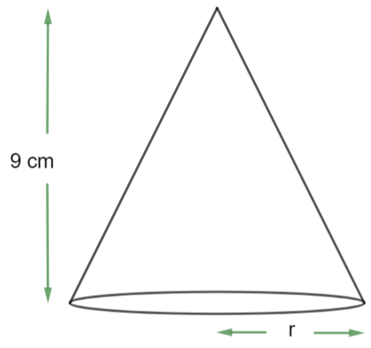
Therefore, the diameter of the cone is 20 cm.

4. If the volume of right circular cone of height 9 cm is 48π cm^3, find the diameter of its base.

Ans: It is given the height of cone $h = 9$ cm

Let us assume the radius of the cone be r.

The volume of the cone is $V = 48\pi \text{ cm}^3$



We know the formula for the volume of the cone $= \frac{1}{3}\pi r^2 h$

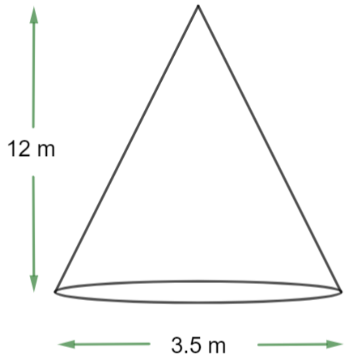
$$\therefore \frac{1}{3}\pi r^2 h = 48\pi \text{ cm}^3$$
$$\Rightarrow \left[\frac{1}{3} \times \pi \times (r)^2 \times 9\right] \text{cm} = 48\pi \text{ cm}^3$$

$\Rightarrow r^2 = 16\text{ cm}^2$
$\Rightarrow r = 4\text{ cm}$
Diameter of base $= 2r = 8\text{ cm}$
Therefore, the diameter of the base of cone is 8 cm.

5. A conical pit of top diameter 3.5 m is 12 m deep. What is the capacity in kilolitres? $\left[\text{Assume } \pi = \dfrac{22}{7}\right]$

Ans: It is given the height of conical pit $h = 12\text{ m}$

The radius of conical pit $r = \dfrac{3.5}{2}\text{ m} = 1.75\text{ m}$



We know the volume of the conical pit $V = \dfrac{1}{3}\pi r^2 h$

$\Rightarrow V = \left[\dfrac{1}{3} \times \dfrac{22}{7} \times (1.75)^2 \times 12\right]\text{ m}^3$

$\Rightarrow V = 38.5\text{ m}^3$

We know that $1\text{ kilolitre} = 1\text{ m}^3$

So, the capacity of the pit $= (38.5 \times 1)$ kilolitres $= 38.5$ kilolitres

Therefore, the capacity of the conical pit is 38.5 kilolitres.

6. The volume of a right circular cone is 9856 cm^3. If the diameter of the base is 28 cm, find

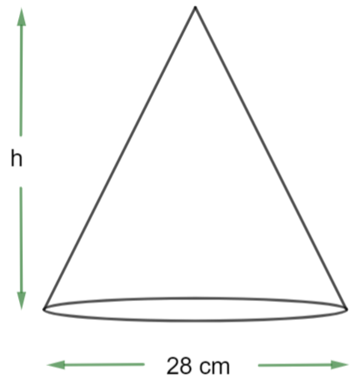
(i) Height of the cone

Ans: It is given the diameter of base of cone $= 28\text{ cm}$

So, the radius $r = \dfrac{28}{2} = 14\text{ cm}$

Let us assume the height of the cone be h.

The volume of the cone is $V = 9856\text{ cm}^3$



We know the formula for the volume of the cone $= \frac{1}{3}\pi r^2 h$

$$\therefore \frac{1}{3}\pi r^2 h = 9856\ \text{cm}^3$$

$$\Rightarrow \left[\frac{1}{3} \times \frac{22}{7} \times (14)^2 \times h\right] \text{cm}^2 = 9856\ \text{cm}^3$$

$$\Rightarrow h = \left(\frac{9856 \times 21}{22 \times 196}\right) \text{cm}$$

$$\Rightarrow h = 48\ \text{cm}$$

Therefore, the height of the cone is $48\ \text{cm}$.

(ii) Slant height of the cone

Ans: The slant height of the cone $l = \sqrt{h^2 + r^2}$

$$\Rightarrow l = \sqrt{48^2 + 14^2}\ \text{cm}$$

$$\Rightarrow l = \sqrt{2304 + 196}\ \text{cm}$$

$$\Rightarrow l = 50\ \text{cm}$$

Therefore, the slant height of the cone is $50\ \text{cm}$.

(iii) Curved surface area of the cone. $\left[\textbf{Assume } \boldsymbol{\pi} = \frac{\mathbf{22}}{\mathbf{7}}\right]$

Ans: The curved surface area of the cone $A = \pi r l$

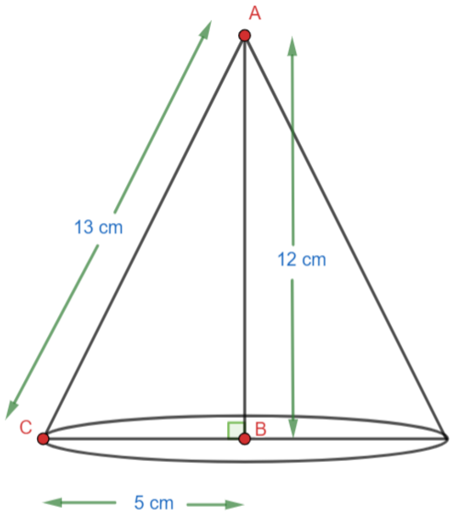
$$\Rightarrow A = \left(\frac{22}{7} \times 14 \times 50\right) \text{cm}^2$$

$$\Rightarrow A = 2200\ \text{cm}^2$$

Therefore, the curved surface area of the cone is $2200\ \text{cm}^2$.

7. A right triangle ΔABC with sides $5\ \text{cm}, 12\ \text{cm}$ and $13\ \text{cm}$ is revolved about the side $12\ \text{cm}$. Find the volume of the solid so obtained.

Ans: We will draw the figure:



If the triangle is revolved about the side 12 cm, we will get a cone with:
Radius $r = 5\text{ cm}$
Slant height $l = 13\text{ cm}$
Height $h = 12\text{ cm}$

We know the volume of the cone $V = \frac{1}{3}\pi r^2 h$

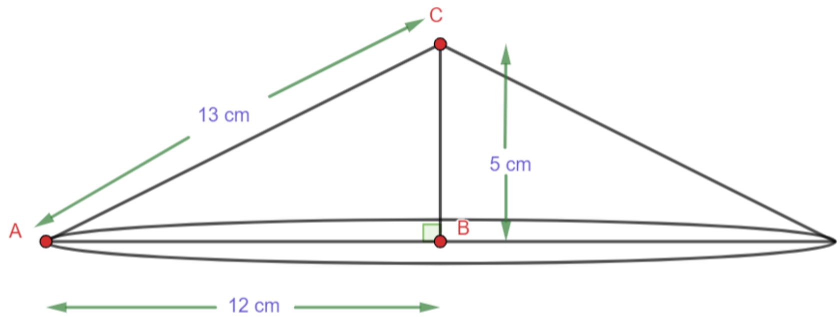
$$\Rightarrow V = \left[\frac{1}{3} \times \pi \times (5)^2 \times 12\right]\text{ cm}^3$$

$$\Rightarrow V = 100\pi\text{ cm}^3$$

Therefore, the volume of the cone will be $100\pi\text{ cm}^3$.

8. If the triangle ΔABC in the Question 7 above is revolved about the side 5 cm, then find the volume of the solid so obtained. Find also the ratio of the volumes of the two solids obtained in Questions 7 and 8.

Ans:



If the triangle is revolved about the side 5 cm, we will get a cone with:
Radius $r = 12\text{ cm}$
Slant height $l = 13\text{ cm}$
Height $h = 5\text{ cm}$

We know the volume of the cone $V = \frac{1}{3}\pi r^2 h$

$\Rightarrow V = \left[\frac{1}{3} \times \pi \times (12)^2 \times 5\right] cm^3$

$\Rightarrow V = 240\pi\ cm^3$

Therefore, the volume of the cone will be $240\pi\ cm^3$.

The ratio of volume of cone from previous question an the one we obtained above

$= \frac{100\pi}{240\pi} = \frac{5}{12} = 5:12$

Therefore, the required ratio is $5:12$.

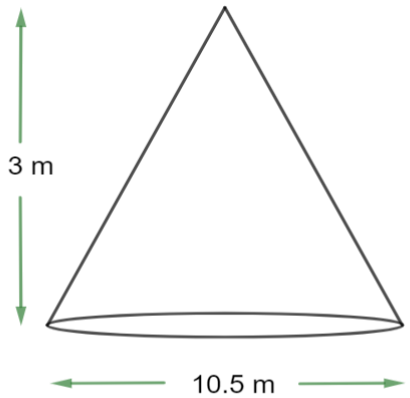
9. A heap of wheat is in the form of a cone whose diameter is $10.5\ m$ and height is $3\ m$. Find its volume. The heap is to be covered by canvas to protect it from rain. Find the area of the canvas required.

$\left[\textbf{Assume } \pi = \frac{22}{7}\right]$

Ans: It is given that diameter of the heap $= 10.5\ m$

So, the radius of heap $r = \frac{10.5}{2} = 5.25\ m$

Height of heap $h = 3\ m$



We know the volume of the cone $V = \frac{1}{3}\pi r^2 h$

$\Rightarrow V = \left[\frac{1}{3} \times \frac{22}{7} \times (5.25)^2 \times 3\right] m^3$

$\Rightarrow V = 86.625\ m^3$

Hence, the volume of heap is $86.625\ m^3$.

The area of canvas required is same as curved surface area of the cone.

$\therefore A = \pi rl$

$\Rightarrow A = \pi r\sqrt{h^2 + r^2}$

$$\Rightarrow A = \frac{22}{7} \times 5.25 \times \sqrt{(3)^2 + (5.25)^2}\ m^2$$

$$\Rightarrow A = \left(\frac{22}{7} \times 5.25 \times 6.05\right) m^2$$

$$\Rightarrow A = 99.825\ m^2$$

Therefore, to protect the heap from the rain, the amount of canvas required is $99.825\ m^2$.

Exercise 11.4

1. Find the volume of the sphere whose radius is
(i) 7 cm
Ans: It is given the radius of sphere $r = 7$ cm

The volume of the sphere $V = \frac{4}{3}\pi r^3$

$$\Rightarrow V = \left[\frac{4}{3} \times \frac{22}{7} \times (7)^3\right] cm^3$$

$$\Rightarrow V = \frac{4312}{3}\ cm^3$$

$$\Rightarrow V = 1437.33\ cm^3$$

Therefore, the volume of the sphere is $1437.33\ cm^3$.

(ii) 0.63 m $\left[\textbf{Assume } \boldsymbol{\pi} = \frac{\mathbf{22}}{\mathbf{7}}\right]$

Ans: It is given the radius of sphere $r = 0.63$ m

The volume of the sphere $V = \frac{4}{3}\pi r^3$

$$\Rightarrow V = \left[\frac{4}{3} \times \frac{22}{7} \times (0.63)^3\right] m^3$$

$$\Rightarrow V = 1.0478\ m^3$$

Therefore, the volume of the sphere is $1.0478\ m^3$.

2. Find the amount of water displaced by a solid spherical ball of diameter
(i) 28 cm
Ans: It is given the diameter of ball $= 28$ cm

So, the radius of ball $r = \frac{28}{2} = 7$ cm

The volume of the ball $V = \frac{4}{3}\pi r^3$

$$\Rightarrow V = \left[\frac{4}{3} \times \frac{22}{7} \times (14)^3\right] cm^3$$

$$\Rightarrow V = 11498\ cm^3$$

Therefore, the volume of the sphere is $11498\ cm^3$.

(ii) 0.21 m $\left[\textbf{Assume } \pi = \frac{22}{7}\right]$

Ans: It is given the diameter of ball $= 0.21$ m

So, the radius of ball $r = \frac{0.21}{2} = 0.105$ m

The volume of the sphere $V = \frac{4}{3}\pi r^3$

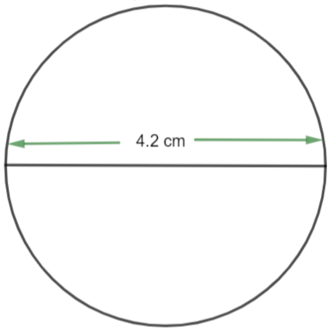
$$\Rightarrow V = \left[\frac{4}{3} \times \frac{22}{7} \times (0.105)^3\right] m^3$$

$$\Rightarrow V = 0.004851\ m^3$$

Therefore, the volume of the sphere is $0.004851\ m^3$.

3. The diameter of a metallic ball is 4.2 cm. What is the mass of the ball, if the density of the metal is 8.9 g per cm³? $\left[\textbf{Assume } \pi = \frac{22}{7}\right]$

Ans: It is given the diameter of metallic ball $= 4.2$ cm



So, the radius of ball $r = \frac{4.2}{2} = 2.1$ cm

The volume of the sphere $V = \frac{4}{3}\pi r^3$

$$\Rightarrow V = \left[\frac{4}{3} \times \frac{22}{7} \times (2.1)^3\right] cm^3$$

$$\Rightarrow V = 38.808\ cm^3$$

We know that $\text{Density} = \dfrac{\text{Mass}}{\text{Volume}}$

$\Rightarrow \text{Mass} = \text{Density} \times \text{Volume}$

$\Rightarrow \text{Mass} = (8.9 \times 38.808)\ \text{g}$

$\Rightarrow \text{Mass} = 345.39\ \text{g}$

Therefore, the mass of the metallic ball is $345.39\ \text{g}$.

4. The diameter of the moon is approximately one-fourth of the diameter of the earth. What fraction of the volume of the earth is the volume of the moon?

Ans: Let us assume the diameter of earth be d.

So, the radius of earth will be $R = \dfrac{d}{2}$.

From the question, we can write the diameter of the moon as $\dfrac{d}{4}$.

So, the radius of moon will be $r = \dfrac{d}{8}$.

The volume of earth $V = \dfrac{4}{3}\pi R^3$

$\Rightarrow V = \dfrac{4}{3}\pi\left(\dfrac{d}{2}\right)^3$

$\Rightarrow V = \dfrac{1}{8} \times \dfrac{4}{3}\pi d^3$

The volume of moon $V' = \dfrac{4}{3}\pi r^3$

$\Rightarrow V' = \dfrac{4}{3}\pi\left(\dfrac{d}{8}\right)^3$

$\Rightarrow V' = \dfrac{1}{512} \times \dfrac{4}{3}\pi d^3$

The ratio of volume of moon and that of earth $= \dfrac{\dfrac{1}{512} \times \dfrac{4}{3}\pi d^3}{\dfrac{1}{8} \times \dfrac{4}{3}\pi d^3} = \dfrac{1}{64}$

So, $\dfrac{\text{Volume of moon}}{\text{Volume of earth}} = \dfrac{1}{64}$

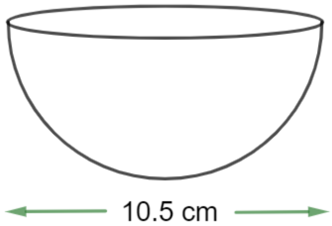
$\Rightarrow \text{Volume of moon} = \left(\dfrac{1}{64}\right) \text{Volume of earth}$

Therefore, the volume of moon is $\frac{1}{64}$ times the volume of earth.

5. How many litres of milk can a hemispherical bowl of diameter 10.5 cm can hold? $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Ans: It is given the diameter of hemispherical bowl $= 10.5\text{ cm}$.

Hemispherical bowl



So, the radius of the bowl $r = \frac{10.5}{2} = 5.25\text{ cm}$.

The volume of the hemispherical bowl $V = \frac{2}{3}\pi r^3$

$\Rightarrow V = \left[\frac{2}{3} \times \frac{22}{7} \times (5.25)^3\right] \text{cm}^3$

$\Rightarrow V = 303.1875\text{ cm}^3$

We know that $1000\text{ cm}^3 = 1$ litre

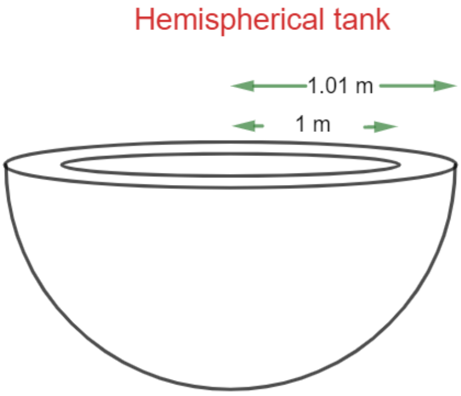
So, the capacity of the bowl $= \frac{303.1875}{1000} = 0.303$ litre

Therefore, the volume of the hemispherical bowl is 0.303 litre.

6. A hemispherical tank is made up of an iron sheet 1 cm thick. If the inner radius is 1 m, then find the volume of the iron used to make the tank. $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Ans: The inner radius of hemispherical tank $r = 1\text{ m}$

The thickness of iron sheet $= 1\text{ cm} = 0.01\text{ m}$.



So, the outer radius of the hemispherical tank $R = (1 + 0.01) = 1.01\text{ m}$

The volume of iron sheet required to make the tank $V = \frac{2}{3}\pi\left(R^3 - r^3\right)$

$$\Rightarrow V = \frac{2}{3} \times \frac{22}{7} \times \left((1.01)^3 - (1)^3\right)\text{ m}^3$$

$$\Rightarrow V = \frac{44}{21} \times (1.030301 - 1)\text{ m}^3$$

$$\Rightarrow V = 0.06348\text{ m}^3$$

Therefore, the volume of iron sheet required to make the hemispherical tank is 0.06348 m^3.

7. Find the volume of a sphere whose surface area is 154 cm^2. $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Ans: Let us assume the radius of the sphere be r.

It is given that the surface area of the sphere $= 154\text{ cm}^2$.

$$\therefore 4\pi r^2 = 154\text{ cm}^2$$

$$\Rightarrow r^2 = \left(\frac{154 \times 7}{4 \times 22}\right)\text{ cm}^2$$

$$\Rightarrow r^2 = \left(\frac{49}{4}\right)\text{ cm}^2$$

$$\Rightarrow r = \left(\frac{7}{2}\right)\text{ cm}$$

$$\Rightarrow r = 3.5\text{ cm}$$

The volume of the sphere $V = \frac{4}{3}\pi r^3$

$$\Rightarrow V = \left[\frac{4}{3} \times \frac{22}{7} \times (3.5)^3\right]\text{ cm}^3$$

$\Rightarrow V = \left[\frac{179 \times 2}{3}\right] cm^3$

$\Rightarrow V = 119.33\ cm^3$

Therefore, the volume of the sphere is $119.33\ cm^3$.

8. A dome of a building is in the form of a hemisphere. From inside, it was w Rs. 498.96 hitewashed at the cost of . If the cost of white-washing is Rs. 2.00 per square meter, find the

(i) Inside surface area of the dome,

Ans: It is given that it costs Rs. 2.00 to whitewash an area $= 1\ m^2$

So, it costs Rs. 498.96 to whitewash an area $= \frac{498.96}{2}\ m^2 = 249.48\ m^2$.

Therefore, the inner surface area of the dome is $249.48\ m^2$.

(ii) Volume of the air inside the dome. $\left[\textbf{Assume } \boldsymbol{\pi} = \frac{\mathbf{22}}{\mathbf{7}}\right]$

Ans: Let us assume the radius of hemispherical dome be r.

We obtained the curved surface area of the inner dome $= 249.48\ m^2$

$\therefore 2\pi^2 = 249.48\ m^2$

$\Rightarrow 2 \times \frac{22}{7} \times r^2 = 249.48\ m^2$

$\Rightarrow r^2 = \left(\frac{249.48 \times 7}{2 \times 22}\right) m^2$

$\Rightarrow r^2 = 39.69\ m^2$

$\Rightarrow r = 6.3\ m$

Volume of hemispherical dome $V = \frac{2}{3}\pi r^3$

$\Rightarrow V = \left[\frac{2}{3} \times \frac{22}{7} \times (6.3)^3\right] m^3$

$\Rightarrow V = 523.908\ m^3$

$\Rightarrow V = 523.9\ \ m^3 (\text{approximately})$

Therefore, the volume of air inside the hemispherical dome is $523.9\ m^3$.

9. Twenty-seven solid iron spheres, each of radius r and surface area S are melted to form a sphere with surface area S'. Find the

(i) radius r' of the new sphere,

Ans: It is given the radius of one iron sphere $= r$.

The volume of one iron sphere $= \frac{4}{3}\pi r^3$

So, the volume of 27 iron spheres $= 27 \times \frac{4}{3}\pi r^3$

These spheres are melted to form one big sphere.
Let us assume the radius of this new sphere be r'.

The volume of new iron sphere $= \frac{4}{3}\pi r'^3$

We can now equate the volumes.

$\Rightarrow \frac{4}{3}\pi r'^3 = 27 \times \frac{4}{3}\pi r^3$

$\Rightarrow r'^3 = 27r^3$

$\Rightarrow r' = 3r$

Therefore, the radius of new sphere is $3r$.

(ii) ratio of S and S'.

Ans: The surface area of an iron sphere of r is $S = 4\pi r^2$.

The surface area of an iron sphere of r' is $S' = 4\pi r'^2$.

$\Rightarrow S' = 4\pi(3r)^2$

$\Rightarrow S' = 36\pi r^2$

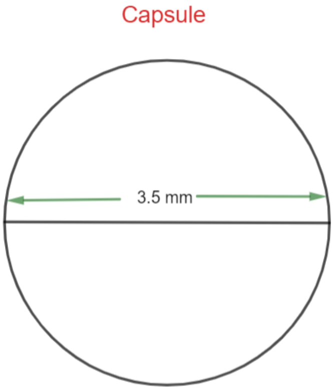
The ratio of $\frac{S}{S'} = \frac{4\pi r^2}{36\pi r^2} = \frac{1}{9} = 1:9$

Therefore, the required ratio is $1:9$.

10. A capsule of medicine is in the shape of a sphere of diameter 3.5 mm. How much medicine $\left(\text{in mm}^3\right)$ is needed to fill this capsule? $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Ans: It is given that the diameter of capsule $= 3.5$ mm.

So, the radius will be $r = \left(\frac{3.5}{2}\right) = 1.75$ mm.



Volume of spherical capsule $V = \frac{4}{3}\pi r^3$

$\Rightarrow V = \left[\frac{4}{3} \times \frac{22}{7} \times (1.75)^3\right] \text{mm}^3$

$\Rightarrow V = 22.458 \text{ mm}^3$

$\Rightarrow V = 22.46 \text{ mm}^3 (\text{approx})$

Hence, the amount of medicine required to fill the capsule is 22.46 mm^3.