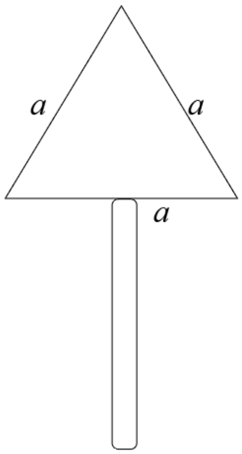
heron's formula

10 Chapter

Exercise 10.1

1. A traffic signal board, indicating 'SCHOOL AHEAD', is an equilateral triangle with side 'a'. Find the area of the signal board, using Heron's formula. If its perimeter is 180 cm, what will be the area of the signal board?

Ans:



Length of the side of traffic signal board $= a$

Perimeter of traffic signal board which is an equilateral triangle $= 3 \times a$

We know that,

$2s =$ Perimeter of the triangle,

So, $2s = 3a$

$$\Rightarrow s = \frac{3}{2}a$$

Area of triangle can be evaluated by Heron's formula:

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

Where,

a, b and c are the sides of the triangle

$$s=\frac{a+b+c}{2}$$

Substituting $s=\frac{3}{2}a$ in Heron's formula, we get:

Area of given triangle:

$$A=\sqrt{\frac{3}{2}a\left(\frac{3}{2}a-a\right)\left(\frac{3}{2}a-a\right)\left(\frac{3}{2}a-a\right)}$$

$$A=\frac{\sqrt{3}}{2}a^2 \quad ......(1)$$

Perimeter of traffic signal board:

$$P=180\,\text{cm}$$

Hence, side of traffic signal board

$$a=\left(\frac{180}{3}\right)$$

$$a=60 \quad ......(2)$$

Substituting Equation (2) in Equation (1), we get:

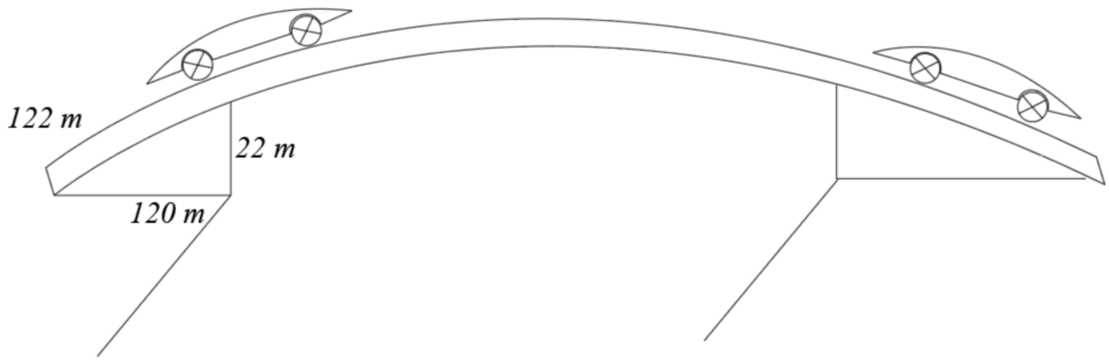
Area of traffic signal board is $A=\frac{\sqrt{3}}{2}(60\,\text{cm})^2$

$$\Rightarrow A=\left(\frac{3600}{4}\sqrt{3}\right)\text{cm}^2$$

$$\Rightarrow A=900\sqrt{3}\,\text{cm}^2$$

Hence, the area of the signal board is $900\sqrt{3}\ \text{cm}^2$.

2. The triangular side walls of a flyover have been used for advertisements. The sides of the walls are $122\text{m}, 22\text{m},$ and 120m. The advertisements yield on earning of Rs. $5000\,\text{per}\,\text{m}^2$ per year. A company hired one of its walls for 3 months. How much rent did it pay?



Ans: The length of the sides of the triangle are (say a, b and c)

$a = 122\text{ m}$

$b = 22\text{ m}$

$c = 120\text{ m}$

Perimeter of triangle = sum of the length of all sides

Perimeter of triangle is:

$P = 122 + 22 + 120$

$P = 264\text{ m}$

We know that,

$2s =$ Perimeter of the triangle,

$2s = 264\text{ m}$

$s = 132\text{ m}$

Area of triangle can be evaluated by Heron's formula:

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

Where,

a, b and c are the sides of the triangle

$$s = \frac{a+b+c}{2}$$

So, in this question,

$$s = \frac{122+22+140}{2}$$

$s = 132\text{ m}$

Substituting values of s a, b, c in Heron's formula, we get:

Area of given triangle $= \left[\sqrt{132(132-122)(132-22)(132-120)}\right]\text{m}^2$

$= \left[\sqrt{132(10)(110)(12)} \right] m^2 = 1320\, m^2$

It is given that:

Rent of $1m^2$ area per year is:

$R = Rs.\ 5000/m^2$

So,

Rent of $1m^2$ area per month will be:

$R = Rs. \frac{5000}{12} / m^2$

Rent of $1320m^2$ area for 3 months:

$R = \left(\frac{5000}{12} \times 3 \times 1320 \right) / m^2$

$\Rightarrow R = Rs. 1650000$

Therefore, the total cost rent that company must pay is Rs. 1650000.

3. The floor of a rectangular hall has a perimeter $250m$. If the cost of painting the four walls at the rate of Rs. $10\, per\, m^2$ is Rs. 15000. find the height of the hall. [Hint: Area of the four walls = Lateral surface area.]

Ans: Let length, breadth, and height of the rectangular hall be $l\, m$, $b\, m$, and $h\, m$ respectively.

Area of four walls is also known as Lateral surface area of cuboid is given as:

$L = 2lh + 2bh$

$\Rightarrow L = 2h(l + b)$

Perimeter of the floor of hall:

$P = 2(l + b)$

It is given that

$P = 250m$

So,

$2(l + b) = 250$

Substituting $2(l + b) = 250$ in equation $L = 2h(l + b)$ we get:

$L = 250hm^2$

Cost of painting per m^2 area:

$C = Rs. 10 / m^2$

Cost of painting $250h\,m^2$ area:

$C = (250h \times 10)$

$\Rightarrow C = 2500h$

However, it is given that the cost of painting the walls is Rs. 15000.

Thus, equating $2500h$ with Rs. 15000 we get:

$2500h = 15000$

$\Rightarrow h = 6m$

Therefore, the height of the hall is $6m$.

4. Find the area of a triangle two sides of which are $18cm$ and $10cm$ and the perimeter is $42cm$.

Ans: Let the length of third side of the triangle be x.

Perimeter of the given triangle:

$P = 42cm$

Let the side of the triangle are a, b and c.

$a = 18cm$

$b = 10cm$

$c = xcm$

Perimeter of the triangle = sum of all sides

$18 + 10 + x = 42$

$\Rightarrow 28 + x = 42$

$\Rightarrow x = 14$

Area of triangle can be evaluated by Heron’s formula:

$A = \sqrt{s(s-a)(s-b)(s-c)}$

Where,

a, b and c are the sides of the triangle

$s = \dfrac{a+b+c}{2}$

$\Rightarrow s = \dfrac{18+10+14}{2}$

$\Rightarrow s = 21cm$

Substituting values of s a, b, c in Heron’s formula, we get:

Area of given triangle:

$$A = \left[\sqrt{21(21-18)(21-10)(21-14)}\right]$$

$$\Rightarrow A = \left[\sqrt{21(3)(11)(7)}\right]$$

$$\Rightarrow A = 21\sqrt{11}\ \text{cm}^2$$

Hence, area of the given triangle is $21\sqrt{11}\ \text{cm}^2$.

5. Sides of a triangle are in the ratio of $12:17:25$ and its perimeter is 540cm. Find its area.

Ans: Let the common ratio between the sides of the given triangle be x.

Therefore, the side of the triangle will be $12x$, $17x$, and $25x$.

It is given that,

Perimeter of this triangle $= 540\text{cm}$

Perimeter = sum of the length of all sides

$12x + 17x + 25x = 540$

$\Rightarrow 54x = 540$

$\Rightarrow x = 10$

Sides of the triangle will be:

$12 \times 10 = 120\text{cm}$

$17 \times 10 = 170\text{cm}$

$25 \times 10 = 250\text{cm}$

Area of triangle can be evaluated by Heron's formula:

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

Where,

a, b and c are the sides of the triangle

$$s = \frac{a+b+c}{2}$$

$$\Rightarrow s = \frac{120+170+250}{2}$$

$$\Rightarrow s = 270\text{cm}$$

Area of given triangle:

$$A = \left[\sqrt{270(270-120)(270-170)(270-250)}\right]$$

$$\Rightarrow A = \left[\sqrt{270(150)(100)(20)}\right]$$

$\Rightarrow A = 9000\text{cm}^2$

Therefore, the area of this triangle is $9000\,\text{cm}^2$.

6. An isosceles triangle has perimeter 30cm and each of the equal sides is 12cm. Find the area of the triangle.

Ans: Let the third side of this triangle be x.

Measure of equal sides is 12cm as the given triangle is an isosceles triangle.

It is given that,

Perimeter of triangle, $P = 30\text{cm}$

Perimeter of triangle = Sum of the sides

$12 + 12 + x = 30$

$\Rightarrow x = 6\text{cm}$

Area of triangle can be evaluated by Heron's formula:

$A = \sqrt{s(s-a)(s-b)(s-c)}$

Where,

a, b and c are the sides of the triangle

$$s = \frac{a+b+c}{2}$$

$$\Rightarrow s = \frac{12+12+6}{2}$$

$\Rightarrow s = 15\text{cm}$

Substituting values of s a, b, c in Heron's formula we get:

$$A = \left[\sqrt{15(15-12)(15-12)(15-6)}\right]$$

$$\Rightarrow A = \left[\sqrt{15(3)(3)(9)}\right]$$

$\Rightarrow A = 9\sqrt{15}\text{cm}^2$

Hence, the area of the given isosceles triangle is $9\sqrt{15}\text{cm}^2$.