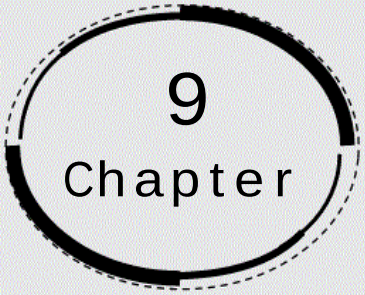
algebraic expressions and identities



Exercise – 8.1

1. Add the following:

(i) ab - bc, bc - ca, ca - ab

Ans:

12a - 9ab + 5b - 3

Therefore, the sum of the given expressions is o.

(ii) a - b + ab, b - c + bc, c - a + ac

Ans:

$$\begin{array}{rrrr} & a & -b & +ab \\ & & +b & -c+bc \\ + & -a & & +c \quad + \text{ ac} \\ \hline & & \text{ab} & + \text{ bc} + \text{ ac} \end{array}$$

Thus the sum of given expressions is ab + bc + ac

(iii) $2p^2q^2 - 3pq + 4, 5 + 7pq - 3p^2q^2$

Ans:

$$\begin{array}{rrrr} & 2p^2q^2 & - 3pq & + 4 \\ + & - 3p^2q^2 & + 7pq & + 5 \\ \hline & - p^2q^2 & + 4pq & + 9 \end{array}$$

Therefore, the sum of given expressions is $- p^2q^2 + 4pq + 9$

(iv) $\mathbf{l^2 + m^2, m^2 + n^2, n^2 + l^2, 2lm + 2mn + 2nl}$

Ans:

$l^2 + m^2$

$+ \quad m^2 + n^2$

$+ \quad l^2 + n^2$

$+ \quad 2lm + 2mn + 2nl$

$2\ l\ 2 + 2m^2 + 2n^2 + 2lm + 2mn + 2nl$

.Therefore, the sum of the given expressions is $2l2 + 2m^2 + 2n^2 + 2lm + 2mn + 2nl$

2. Solve the following:

(i) Subtract $\mathbf{4a - 7ab + 3b + 12}$ **from** $\mathbf{12a - 9ab + 5b - 3}$

Ans:

$12a - 9ab + 5b - 3$

$4a - 7ab + 3b + 12$

$(-) \quad (+) \quad (-)(-)$

$8a - 2ab + 2b - 15$

(ii) Subtract $\mathbf{3xy + 5yz - 7zx}$ **from** $\mathbf{5xy - 2yz - 2zx + 10xyz}$

Ans:

5xy - 2yz - 2zx + 10xyz

3xy + 5yz - 7zx

(-)(-) (+)

2xy - 7yz + 5zx + 10xyz

(iii) Subtract $\mathbf{4p\,2q - 3pq + 5pq2 - 8p + 7q - 10}$ **from**
$\mathbf{18 - 3p - 11q + 5pq - 2pq2 + 5p\,2q}$

Ans:

$18 - 3p - 11q + 5pq - 2pq^2 + 5p^2q$

$- 10 - 8p + 7q - 3pq + 5pq^2 + 4p^2q$

$$\frac{(+)(+)(-)(+)(-) \quad (-)}{28 + 5p - 18q + 8pq - 7pq^2 + p^2q}$$

Exercise – 8.2

1. Find the product of the following pairs of monomials.

(i) **4, 7p**

Ans: $4 \times 7p = 4 \times 7 \times p = 28p$

(ii) $\frac{\text{first monomial} \quad \uparrow}{\text{second monomial} \quad -}$

Ans: $-4p \times 7p = -4 \times p \times 7 \times p = (-4 \times 7) \times (p \times p) = -28p^2$

(iii) - 4p, 7pq

Ans: $-4p \times 7pq = -4 \times p \times 7 \times p \times q = (-4 \times 7) \times (p \times p \times q) = -28p^2q$

(iv) $\mathbf{4p^3}$ **, - 3p**

Ans: $4p^3 \times -3p = 4 \times (-3) \times p \times p \times p \times p = -12p^4$

(v) **4p, 0**

Ans: $4p \times 0 = 4 \times p \times 0 = 0$

2. Find the areas of rectangles with the following pairs of monomials as their lengths and breadths respectively.

$(\mathbf{p, q}); (\mathbf{10m, 5n}); (\mathbf{20x^2, 5y^2}); (\mathbf{4x, 3x^2}); (\mathbf{3mn, 4np})$

Ans: We know that,

Area of rectangle = length x breadth

Area of 1st rectangle = p x q = pq

Area of 2nd rectangle = $10m \times 5n = 10 \times 5 \times m \times n = 50mn$

Area of 3rd rectangle = $20x^2 \times 5y^2 = 20 \times 5 \times x^2 \times y^2 = 100x^2y^2$

Area of 4th rectangle = $4x \times 3x^2 = 4 \times 3 \times x \times x^2 = 12x^3$

Area of 5th rectangle $= 3mn \times 4np = 3 \times 4 \times m \times n \times n \times p = 12mn^2p$

3. Complete the table of products.

first monomial → / second monomial ↓	**2x**	**-5y**	**$3x^2$**	**-4xy**	**$7x^2y$**	**$-9x^2y$**
2x	**$4x^2$**	**…**	**..**	**…**	**…**	**…**
-5y	**…**	**…**	**$15x^2$**	**…**	**…**	**…**
$3x^2$	**…**	**…**	**…**	**…**	**…**	**…**
-4xy	**…**	**…**	**…**	**…**	**…**	**…**
$7x^2y$	**…**	**…**	**…**	**…**	**…**	**…**
$-9x^2y^2$	**…**	**…**	**…**	**…**	**…**	**…**

Ans:

The table can be completed as follows.

first monomial → / second monomial ↓	$2x$	$-5y$	$3x^2$	$4xy$	$7x^2y$	$-9x^2y$
$2x$	$4x^2$	$-10xy$	$6x^2$	$-8x^2y$	$14x^3y$	$-18x^3y^2$
$-5y$	$-10xy$	$25y^2$	$-15x^2$	$20xy^2$	$-35x^2y^2$	$45x^2y^3$
$3x^2$	$6x^3$	$-15x^2y$	$9x^4$	$-12x^3$	$21x^4y$	$-27x^4y^2$
$-4xy$	$-8x^2y$	$20xy^2$	$-12x^3y$	$16x^2y^2$	$-28x^3y^2$	$36x^3y^3$
$7x^2y$	$14x^3y$	$-35x^2y^2$	$21x^4y$	$-28x^3y^2$	$49x^4y^2$	$-63x^4y^3$
$-9x^2y^2$	$-18x^3y^2$	$45x^2y^3$	$-27x^4y^2$	$36x^3y^3$	$-63x^4y^3$	$81x^4y^4$

4. Obtain the volume of rectangular boxes with the following length, breadth and height respectively.

(i) $\mathbf{5a, 3a^2, 7a^4}$

Ans: We know that

Volume= length x breadth x height

Volume $= 5a \times 3a^2 \times 7a^4 = 105a^7$

(ii) $\mathbf{2p, 4q, 8r}$

Ans: We know that

Volume = length x breadth x height

Volume = $2p \times 4q \times 8r = 64pqr$

(iii) $\mathbf{xy, 2x^2y, 2xy^2}$

Ans: We know that

Volume = length x breadth x height

Volume = $xy \times 2x^2y \times 2xy^2 = 4x^4y^4$

(iv) a, 2b, 3c

Ans: We know that

Volume = length x breadth x height

Volume = $a \times 2b \times 3c = 6abc$

5. Obtain the product of

(i) xy, yz, zx

Ans: $xy \times yz \times zx = x^2\ y^2\ z^2$

(ii) a, - a^2, a^3

Ans: $a \times (-a^2) \times a^3 = -a^6$

(iii) 2, 4y, $8y^2$, $16y^3$

Ans: $2 \times 4y \times 8y^2 \times 16y^3 = 1024y^6$

(iv) a, 2b, 3c, 6abc

Ans: $a \times 2b \times 3c \times 6abc = 36a^2b^2c^2$

(v) m, - mn, mnp

Ans: $m \times (-mn) \times mnp = -m^3n^2$

Exercise – 8.3

1. Carry out the multiplication of the expressions in each of the following pairs.

(i) 4p, q + r

Ans: $(4p)\times(q + r) = (4p\times q) + (4p\times r) = 4pq + 4pr$

(ii) ab, a - b

Ans: $(ab)\times(a - b) = (ab\times a) + [ab\times(-b)] = a^2b - ab^2$

(iii) a + b, $7a^2b^2$

Ans: $(a + b)\times(7a\,2\,b\,2) = (a\times 7a^2b^2) + (b\times 7a^2b^2) = 7a^3b^2 + 7a^2b^3$

(iv) a^2 - 9, 4a

Ans: $(a^2 - 9)\times(4a) = (a^2\times 4a) + (-9)\times(4a) = 4a^3 - 36a$

(v) pq + qr + rp, 0

Ans: $(pq + qr + rp)\times 0 = (pq\times 0) + (qr\times 0) + (rp\times 0) = 0$

2. Complete the table

--	First expression	Second expression	Product
a)	a	b+c+d	-
b)	x+y-5	5xy	-
c)	p	$6p^2 - 7p + 5$	-
d)	$4p^2q^2$	$p^2 - q^2$	-
e)	a+b+c	abc	-

Ans: The table can be completed as follows:

---	First expression	Second expression	Product
a)	a	b+c+d	ab+ac+ad
b)	x+y-5	5xy	$5x^2y+5xy^2-25xy$
c)	p	$6p^2 - 7p + 5$	$6p^3-7p^2+5p$
d)	$4p^2q^2$	$p^2 - q^2$	$4p^4q^2-4p^2q^4$
e)	a+b+c	abc	$a^2bc+ab^2c+abc^2$

3. Find the product:

(i) $\left(\mathbf{a^2}\right)\times\left(\mathbf{2a^{22}}\right)\times\left(\mathbf{4a^{26}}\right)$

Ans: $\left(a^2\right)\times\left(2a^{22}\right)\times\left(4a^{26}\right)=2\times4\times a^2\times a^{22}\times a^{26} = 8a^{50}$

(ii) $\left(\frac{\mathbf{2}}{\mathbf{3}}\mathbf{xy}\right)\times\left(\frac{\mathbf{-9}}{\mathbf{10}}\mathbf{x^2y^2}\right)$

Ans: $\left(\frac{2}{3}xy\right)\times\left(\frac{-9}{10}x^2y^2\right) = \left(\frac{2}{3}\right)\times\left(\frac{-9}{10}\right)\times x\times y\times x^2\times y^2 = \frac{-3}{5}x^3y^3$

(iii) $\left(-\frac{\mathbf{10}}{\mathbf{3}}\mathbf{pq^3}\right)\times\left(\frac{\mathbf{6}}{\mathbf{5}}\mathbf{p^3q}\right)$

Ans: $\left(-\frac{10}{3}pq^3\right)\times\left(\frac{6}{5}p^3q\right) = \left(\frac{-10}{3}\right)\times\left(\frac{6}{5}\right)\times pq^3\times p^3q = -4p^4q^4$

(iv) $\mathbf{x\times x^2\times x^3\times x^4}$

Ans: $x\times x^2\times x^3\times x^4 = x^{10}$

4. Solve the following

(i) Simplify $\mathbf{3x\left(4x-5\right)+3}$ **and find its values for**

(a) $\mathbf{x = 3}$

Ans: $3x\left(4x - 5\right) + 3 = 12x^2 - 15x + 3$

For $x = 3, 12x^2 - 15x + 3 = 12\left(3\right)^2 - 15\left(3\right) + 3$

$= 108 - 45 + 3$

$= 66$

(b) $\mathbf{x = \frac{1}{2}}$

Ans:

For $x = \frac{1}{2}, 12x^2 - 15x + 3 = 12\left(\frac{1}{2}\right)^2 - 15\left(\frac{1}{2}\right) + 3$

$= 3 - \frac{15}{2} + 3$

$= 6 - \frac{15}{2} = \frac{12 - 15}{2} = \frac{-3}{2}$

(ii) $\mathbf{a\left(a^2 + a + 1\right) + 5}$ and find its value for

(a) $\mathbf{a = 0}$

Ans: For $a = 0, a^3 + a^2 + a + 5 = 0 + 0 + 0 + 5 = 5$

(b) $\mathbf{a = 1}$

Ans: For $a = 1, a^3 + a^2 + a + 5 = \left(1\right)^3 + \left(1\right)^2 + 1 + 5$

$= 1 + 1 + 1 + 5 = 8$

(c) $\mathbf{a = -1}$

Ans: For $a = -1, a^3 + a^2 + a + 5 = \left(-1\right)^3 + \left(-1\right)^2 + \left(-1\right) + 5$

$= -1 + 1 - 1 + 5 = 4$

5. Solve the following

(i) Add: $p(p - q), q(q - r)$ and $r(r - p)$

Ans:

First expression $= p(p - q) = p^2 - pq$

Second expression $= q(q - r) = q^2 - qr$

Third expression $= r(r - p) = r^2 - pr$

Adding the three expressions, we obtain

$$\begin{array}{rrrr} & p^2 & - pq & \\ + & & q^2 - qr & \\ + & & & r^2 - pr \\ \hline & p^2 - pq & + q^2 - qr & + r^2 - pr \end{array}$$

Therefore, the sum is $p^2 - pq + q^2 - qr + r^2 - pr$

(ii) Add: $2x(z - x - y)$ and $2y(z - y - x)$

Ans:

First expression $= 2x(z - x - y) = 2xz - 2x^2 - 2xy$

Second expression $= 2y(z - y - x) = 2yz - 2y^2 - 2yx$

Adding the two expressions, we obtain

$$\begin{array}{rrrr} & 2xz & - 2x^2 & - 2xy \\ + & & - 2yx & + 2yz - 2y^2 \\ \hline & 2xz & - 2x^2 - 4xy & + 2yz - 2y^2 \end{array}$$

Therefore, the sum is $2xz - 2x^2 - 4xy + 2yz - 2y^2$

(iii) Subtract $3l(l - 4m + 5n)$ from $4l(10n - 3m + 2l)$

Ans:

$3l\left(l - 4m + 5n\right) = 3l^2 - 12lm + 15ln$

$4l\left(10n - 3m + 2l\right) = 40ln - 12lm + 8l^2$

Subtracting these expressions, we obtain

$8l^2 - 12lm + 40ln$

$3l^2 - 12lm + 15ln$

$(-) \quad (+) \quad (-)$

$5l^2 \qquad + 25ln$

Therefore, the result is $5l^2 + 25ln$

(iv) Subtract $3a\left(a + b + c\right) - 2b\left(a - b + c\right)$ from $4c\left(-a + b + c\right)$

Ans:

$+ 4c^2 \qquad - 4ac + 4bc$

$3a^2 + 2b^2 \qquad + ab + 3ac - 2bc$

$(-) \quad (-) \qquad (-) \quad (-) \quad (+)$

$-3a^2 - 2b^2 + 4c^2 + ab - 7ac + 6bc$

Therefore, the result is $-3a^2 - 2b^2 + 4c^2 + ab - 7ac + 6bc$

Exercise – 8.4

1. Multiply the binomials.

(i) $(2x + 5)$ and $(4x - 3)$

Ans: $(2x + 5) \times (4x - 3) = 2x \times (4x - 3) + 5 \times (4x - 3)$

$= 8x^2 - 6x + 20x - 15$

$= 8x2 + 14x - 15$ (By adding like terms)

(ii) (y - 8) and (3y - 4)

Ans: $(y-8) \times (3y-4) = y \times (3y-8) - 8 \times (3y-4)$

$= 3y^2 - 4y - 24y + 32$

$= 3y^2 - 28y + 32$ (By adding like terms)

(iii) (2.5l - 0.5m) and (2.5l + 0.5m)

Ans: $(2.5l - 0.5m)(2.5l + 0.5m) = 2.5l \times (2.5l + 0.5m) - 0.5m(2.5l + 0.5m)$

$= 6.25l^2 + 1.25lm - 1.25lm - 0.25m^2$

$= 6.25l^2 - 0.25m^2$

(iv) (a + 3b) and (x + 5)

Ans: $(a + 3b) \times (x + 5) = a \times (x+5) + 3b \times (x + 5)$

$= ax + 5a + 3bx + 15b$

(v) $(2pq + 3q^2)$ and $(3pq - 2q^2)$

Ans: $(2pq + 3q^2) \times (3pq - 2q^2) = 2pq \times (3pq - 2q^2) + 3q^2 \times (3pq - 2q^2)$

$= 6p2q^2 - 4pq^3 + 9pq^3 - 6q^4$

$= 6p2q^2 + 5pq^3 - 6q^4$

(vi) $\left(\frac{3}{4}a^2 + 3b^2\right)$ **and** $4\left(a^2 - \frac{2}{3}b^2\right)$

Ans: $\left(\frac{3}{4}a^2 + 3b^2\right) \times \left[4\left(a^2 - \frac{2}{3}b^2\right)\right] = \left(\frac{3}{4}a^2 + 3b^2\right) \times \left(4a^2 - \frac{8}{3}b^2\right)$

$= \frac{3}{4}a^2 \times \left(4\alpha^2 - \frac{8}{3}b^2\right) + 3b^2 \times \left(4a^2 - \frac{8}{3}b^2\right)$

$= 3a^4 - 2a^2b^2 + 12b^2a^2 - 8b^4$

$= 3a^4 + 10a^2b^2 - 8b^4$

2. Find the product.

(i) (5 - 2x)(3 + x)

Ans: $(5 - 2x)(3 + x) = 5(3 + x) - 2x(3 + x)$

$= 15 + 5x - 6x - 2x^2$

$= 15 - x - 2x^2$

(ii) (x + 7y)(7x - y)

Ans: $(x + 7y)(7x - y) = x(7x - y) + 7y(7x - y)$

$= 7x^2 - xy + 49xy - 7y^2$

$= 7x^2 + 48xy - 7y^2$

(iii) $(a^2 + b)(a + b^2)$

Ans: $(a^2 + b)(a + b^2) = a^2(a + b^2) + b(a + b^2)$

$= a^3 + a^2b^2 + ab + b^3$

(iv) $(p^2 - q^2)(2p + q)$

Ans: $(a - b)(a + b) + (b - c)(b + c) + (c - a)(c + a) = 0$

$= 2p^3 + p^2q - 2pq^2 - q^3$

3. Simplify.

(i) $(x^2 - 5)(x + 5) + 25$

Ans: $(x^2 - 5)(x + 5) + 25$

$x^2(x + 5) - 5(x + 5) + 25$

$= x^3 + 5x^2 - 5x - 25 + 25$

$= x^3 + 5x^2 - 5x$

(ii) $(a^2 + 5)(b^3 + 3) + 5$

Ans: $(a^2 + 5)(b^3 + 3) + 5$

$= a^2(b^3 + 3) + 5(b^3 + 3) + 5$

$= a^2b^3 + 3a^2 + 5b^3 + 15 + 5$

$= a^2b^3 + 3a^2 + 5b^3 + 20$

(iii) $(t + s^2)(t^2 - s)$

Ans: $(t + s^2)(t^2 - s)$

$= t(t^2 - s) + s^2(t^2 - s)$

$= t^3 - st + s^2t^2 - s^3$

(iv) $(a + b)(c - d) + (a - b)(c + d) + 2(ac + bd)$

Ans: $(a + b)(c - d) + (a - b)(c + d) + 2(ac + bd)$

$= a(c - d) + b(c - d) + a(c + d) - b(c + d) + 2(ac + bd)$

$= ac - ad + bc - bd + ac + ad - bc - bd + 2ac + 2bd$

$= (ac + ac + 2ac) + (ad - ad) + (bc - bc) + (2bd - bd - bd)$

$= 4ac$

(v) $(x + y)(2x + y) + (x + 2y)(x - y)$

Ans: $(x + y)(2x + y) + (x + 2y)(x - y)$

$= x(2x + y) + y(2x + y) + x(x - y) + 2y(x - y)$

$= 2x^2 + xy + 2xy + y^2 + x^2 - xy + 2xy - 2y^2$

$= (2x^2 + x^2) + (y^2 - 2y^2) + (xy + 2xy - xy + 2xy)$

$= 3x^2 - y^2 + 4xy$

(vi) $(x + y)(x^2 - xy + y^2)$

Ans: $(x + y)(x^2 - xy + y^2)$

$= x(x^2 - xy + y^2) + y(x^2 - xy + y^2)$

$= x^3 - x^2y + xy^2 + x^2y - xy^2 + y^3$

$= x^3 + y^3 + (xy^2 - xy^2) + (x^2y - x^2y)$

$= x^3 + y^3$

(vii) $(1.5x - 4y)(1.5x + 4y + 3) - 4.5x + 12y$

Ans: $(1.5x - 4y)(1.5x + 4y + 3) - 4.5x + 12y$

$= 1.5x(1.5x + 4y + 3) - 4y(1.5x + 4y + 3) - 4.5x + 12y$

$= 2.25x^2 + 6xy + 4.5x - 6xy - 16y^2 - 12y - 4.5x + 12y$

$= 2.25x^2 + (6xy - 6xy) + (4.5x - 4.5x) - 16y^2 + (12y - 12y)$

$= 2.25x^2 - 16y^2$

(viii) (a + b + c)(a + b - c)

Ans: $(a + b + c)(a + b - c)$

$= a(a + b - c) + b(a + b - c) + c(a + b - c)$

$= a^2 + ab - ac + ab + b^2 - bc + ca + bc - c^2$

$= a^2 + b^2 - c^2 + (ab + ab) + (bc - bc) + (ca - ca)$

$= a^2 + b^2 - c^2 + 2ab$