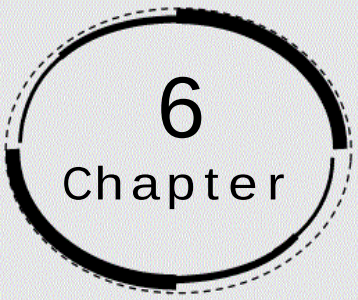
cubes and cube roots



Exercise No: 6.1

1. Which among the following numbers are not perfect cubes?

a) 216

Ans: Prime factorisation of 216 is

2	216
2	108
2	54
3	27
3	9
3	3
	1

$\therefore 216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 2^3 \times 3^3$

Here, as each prime factor 2 and 3 are appearing as many times as a perfect triplet, 216 is a perfect cube.

b) 128

Ans: Prime factorisation of 128 is

2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

$128 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^3 \times 2^3 \times 2$

Here, the prime factor 2 is appearing in two triplets and an extra 2. Thus, 128 is not a perfect cube.

c) 1000

Ans: The prime factorisation of 1000 is

2	1000
2	500

2	250
5	125
5	25
5	5
	1

$1000 = 2 \times 2 \times 2 \times 5 \times 5 \times 5 = 2^2 \times 5^2$

Here, each prime factor is appearing as a perfect triplet, thus, 1000 is a perfect cube.

d) 100

Ans: The prime factorisation of 100 is as follows.

2	100
2	50
5	25
5	5
	1

$100 = 2 \times 2 \times 5 \times 5$

Here, each prime factor is not appearing as a perfect triplet. Thus, 100 is not a perfect cube.

e) 46656

Ans: Prime factorisation of 46656 is

2	46656
2	23328
2	11664
2	5832
2	2916
2	1458
3	729
3	243
3	81
3	27
3	9
3	3
	1

$46656 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

Here, as each prime factor is appearing as a perfect triplet, thus, 46656 is a perfect cube.

The numbers whose factors are not in triplet are not perfect cubes

2. Find the smallest number by which each of the following numbers must be multiplied to obtain a perfect cube.

a) 243

Ans: The prime factorisation of 243 is $243 = 3 \times 3 \times 3 \times 3 \times 3$ Here, two 3s are extra which are not in a triplet. To make 243 a cube, one more 3 is required.

In that case, $243 \times 3 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 729$ is a perfect cube.

Therefore, the smallest natural number by which 243 should be multiplied to make it a perfect cube is 3.

b) 256

Ans: The prime factorisation of $256 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$.

Here, two 2s are extra which are not in a triplet. To make 256 a cube, one more 2 is required. Then, we obtain

$256 \times 2 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 512$, which is a perfect cube.

Therefore, the smallest natural number by which 256 should be multiplied to make it a perfect cube is 2.

c) 72

Ans: $72 = 2 \times 2 \times 2 \times 3 \times 3$

Here, two 3s are extra which are not in a triplet. To make 72 a perfect cube, one more 3 is required.

Thus, we obtain $72 \times 3 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 216$ which is a perfect cube.

Therefore, the smallest natural number by which 72 should be multiplied to make it a perfect cube is 3.

d) 675

Ans: $675 = 3 \times 3 \times 3 \times 5 \times 5$ Here, two 5s are extra which are not in a triplet. To make 675 a perfect cube, one more 5 is required.

Then, we obtain $675 \times 5 = 3 \times 3 \times 3 \times 5 \times 5 \times 5 = 3375$ which is a perfect cube.

Therefore, the smallest natural number by which 675 should be multiplied to make it a perfect cube is 5.

e) 100

Ans: $100 = 2 \times 2 \times 5 \times 5$. Here, two 2s and two 5s are extra which are not in a triplet. To make 100 a cube, we require one more 2 and one more 5.

Then, we obtain $100 \times 2 \times 5 = 2 \times 2 \times 2 \times 5 \times 5 \times 5 = 1000$ which is a perfect cube.
Therefore, the smallest natural number by which 100 should be multiplied to make it a perfect cube is $2 \times 5 = 10$.

3. Find the smallest number by which each of the following numbers must be divided to obtain a perfect cube.

a) 81

Ans: $81 = \underline{3 \times 3 \times 3} \times 3$. Here, one 3 is extra which is not in a triplet.
Dividing 81 by 3, will make it a perfect cube.
Thus, $81 \div 3 = 27 = 3 \times 3 \times 3$ is a perfect cube. Hence, the smallest number by which 81 should be divided to make it a perfect cube is 3.

b) 128

Ans: $128 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times 2$. Here, one 2 is extra which is not in a triplet. If we divide 128 by 2, then it will become a perfect cube.
Thus, $128 \div 2 = 64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$ is a perfect cube. Hence, the smallest number by which 128 should be divided to make it a perfect cube is 2.

c) 135

Ans: $135 = \underline{3 \times 3 \times 3} \times 5$. Here, one 5 is extra which is not in a triplet. If we divide 135 by 5, then it will become a perfect cube.
Therefore, $135 \div 5 = 27 = 3 \times 3 \times 3$ is a perfect cube.
Hence, the smallest number by which 135 should be divided to make it a perfect cube is 5.

d) 192

Ans: $192 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3$.
Here, one 3 is left which is not in a triplet. If we divide 192 by 3, then it will become a perfect cube. Thus, $192 \div 3 = 64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$ is a perfect cube.
Therefore, the smallest number by which 192 should be divided to make it a perfect cube is 3.

e) 704

Ans: $704 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 11$. Here, one 11 is left which is not in a triplet. If we divide 704 by 11, then it will become a perfect cube. Thus, $704 \div 11 = 64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$ is a perfect cube. Therefore, the smallest number by which 704 should be divided to make it a perfect cube is 11.

4. Parikshit makes a cuboid of plasticine of sides 5 cm, 2 cm, 5 cm. How many such cuboids will he need to form a cube?

Ans: Some cuboids of size 5 × 2 × 5 are given. These cuboids when arranged to form a cube, the side of this cube is so formed that it will be a common multiple of the sides (i.e., 5, 2, and 5) of the given cuboid.

Finding the LCM of 5, 2, and 5 we get 10. Thus, a cube of 10 cm side needs to be made. For this arrangement, we have to put 2 cuboids along with its length, 5 along with its width, and 2 along with its height. Therefore, the total cuboids required according to this arrangement = 2 × 5 × 2 = 20 With the help of 20 cuboids of such measures, the required cube is formed.

Otherwise,

Volume of the cube of sides $5\text{cm}, 2\text{cm}, 5\text{cm} = 5\text{cm} \times 2\text{cm} \times 5\text{cm} = (5 \times 5 \times 2)\text{cm}^3$

Here, two 5s and one 2 are extra which are not in a triplet. If we multiply this expression by $2 \times 2 \times 5 = 20$, then it will become a perfect cube. Thus,

$(5 \times 5 \times 2 \times 2 \times 2 \times 5) = (5 \times 5 \times 5 \times 2 \times 2 \times 2) = 1000$ is a perfect cube. Hence, 20 cuboids of 5 cm, 2 cm, 5 cm are required to form a cube.

Exercise No: 6.2

1. Find the cube root of each of the following numbers by prime factorisation method.

a. 64

Ans: Prime factorisation of 512 is

$64 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2}$

Therefore,

$\sqrt[3]{64} = 2 \times 2$

$= 4$

b. 512

Ans: Prime factorisation of 512 is

$512 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2}$

$\therefore \sqrt[3]{512} = 2 \times 2 \times 2 = 8$

c. 10648

Ans: Prime factorisation of 10648 is

$10648 = 2 \times 2 \times 2 \times 11 \times 11 \times 11$

$\therefore \sqrt[3]{10648} = 2 \times 11 = 22$

d. 27000

Ans: Prime factorisation of 27000

$27000 = 2\times2\times2\times3\times3\times3\times5\times5\times5$

$\therefore \sqrt[3]{27000} = 2\times3\times5 = 30$

e. 15625

Ans: Prime factorisation of 15625

$15625 = 5\times5\times5\times5\times5\times5$

$\therefore \sqrt[3]{15625} = 5\times5 = 25$

f. 13824

Ans: Prime factorisation of 13824

$13824 = \underline{2\times2\times2}\times\underline{2\times2\times2}\times\underline{2\times2\times2}\times\underline{3\times3\times3}$

$\therefore \sqrt[3]{13824} = 2\times2\times2\times3 = 24$

g. 110592

Ans: Prime factorisation of 110592

$110592 = \underline{2\times2\times2}\times\underline{2\times2\times2}\times\underline{2\times2\times2}\times\underline{2\times2\times2}\times\underline{3\times3\times3}$

$\therefore \sqrt[3]{110592} = 2\times2\times2\times2\times3 = 48$

h. 46656

Ans: Prime factorisation of 46656

$46656 = \underline{2\times2\times2}\times\underline{2\times2\times2}\times\underline{3\times3\times3}\times\underline{3\times3\times3}$

$\therefore \sqrt[3]{46656} = 2\times2\times3\times3 = 36$

i. 175616

Ans: Prime factorisation of 175616

$175616 = \underline{2\times2\times2}\times\underline{2\times2\times2}\times\underline{2\times2\times2}\times\underline{7\times7\times7}$

$\therefore \sqrt[3]{175616} = 2\times2\times2\times7 = 56$

j. 91125

Ans: Prime factorisation of 91125

$91125 = \underline{3\times3\times3}\times\underline{3\times3\times3}\times\underline{5\times5\times5}$

$\therefore \sqrt[3]{91125} = 3\times3\times5 = 45$

2. State true or false.

a. Cube of any odd number is even.

Ans: False.
Reason: When we find out the cube of an odd number, we will find an odd number as the result because the unit place digit of an odd number is odd and we are multiplying three odd numbers. Therefore, the product will be again an odd number. For example, the cube of 7 (i.e., an odd number) is 343, which is again an odd number.

b. A perfect cube does not end with two zeroes.
Ans: True.
Reason: Perfect cube will end with a certain number of zeroes that are always a perfect multiple of 3. For example, the cube of 10 is 1000 and there are 3 zeroes at the end of it. The cube of 100 is 1000000 and there are 6 zeroes at the end of it.

c. If square of a number ends with 5, then its cube ends with 25.
Ans: False.
Reason: It is not always necessary that if the square of a number ends with 5, then its cube will end with 25. For example, the square of 25 is 625 and 625 has its unit digit as 5. The cube of 25 is 15625. However, the square of 35 is 1225 and also has its unit place digit as 5 but the cube of 35 is 42875 which does not end with 25.

d. There is no perfect cube which ends with 8.
Ans: False.
Reason: There are many cubes which will end with 8. The cubes of all the numbers having their unit place digit as 2 will end with 8. The cube of 12 is 1728 and the cube of 22 is 10648.

e. The cube of a two-digit number may be a three-digit number.
Ans: False.
Reason: The smallest two-digit natural number is 10, and the cube of 10 is 1000 which has 4 digits in it.

f. The cube of a two-digit number may have seven or more digits.
Ans: False.
Reason: The largest two-digit natural number is 99, and the cube of 99 is 970299 which has 6 digits in it. Therefore, the cube of any two-digit number cannot have 7 or more digits in it.

g. The cube of a single digit number may be a single digit number.

Ans: True.
Reason: as the cube of 1 and 2 are 1 and 8 respectively.