squares and square roots

5 Chapter

Exercise 5.1

1. What will be the unit digit of the square of the following numbers?

i. 81

Ans: It is known to us that, the square of the number having unit place digit as ‘a’, will end up with the unit digit of ‘$a\times a$’.

Now, in the given number, the unit’s place digit is 1,its square will end with the unit digit of the multiplication $\left(1\times1=1\right)$ i.e., 1.

ii. 272

Ans: It is known to us that, the square of the number having unit place digit as ‘a’, will end up with the unit digit of ‘$a\times a$’.

Now, in the given number, the unit’s place digit is 2, its square will end with the unit digit of the multiplication $\left(2\times2=4\right)$i.e., 4.

iii. 799

Ans: It is known to us that, the square of the number having unit place digit as ‘a’, will end up with the unit digit of ‘$a\times a$’.

Now, in the given number, the unit’s place digit is 9, its square will end with the unit digit of the multiplication $\left(9\times9=81\right)$i.e., 1.

iv. 3853

Ans: It is known to us that, the square of the number having unit place digit as ‘a’, will end up with the unit digit of ‘$a\times a$’.

Now, in the given number, the unit’s place digit is 3, its square will end with the unit digit of the multiplication $\left(3\times3=9\right)$ i.e., 9.

v. 1234

Ans: It is known to us that, the square of the number having unit place digit as ‘a’, will end up with the unit digit of ‘$a\times a$’.

Now, in the given number, the unit’s place digit is 4,its square will end with the unit digit of the multiplication $\left(4\times4=16\right)$ i.e., 6.

vi. 26387

Ans: It is known to us that, the square of the number having unit place digit as ‘a’, will end up with the unit digit of ‘$a \times a$’.
Now, in the given number, the unit’s place digit is 7, its square will end with the unit digit of the multiplication $(7 \times 7 = 49)$ i.e., 9.

vii. 52698
Ans: It is known to us that, the square of the number having unit place digit as ‘a’, will end up with the unit digit of ‘$a \times a$’.
Now, in the given number, the unit’s place digit is 8, its square will end with the unit digit of the multiplication $(8 \times 8 = 64)$ i.e., 4.

viii. 99880
Ans: It is known to us that, the square of the number having unit place digit as ‘a’, will end up with the unit digit of ‘$a \times a$’.
Now, in the given number, the unit’s place digit is 0,its square will have two zeroes at the end. Hence, the unit digit of the square of the given number is 0.

ix. 12796
Ans: It is known to us that, the square of the number having unit place digit as ‘a’, will end up with the unit digit of ‘$a \times a$’.
Now, in the given number, the unit’s place digit is 6, its square will end with the unit digit of the multiplication $(6 \times 6 = 36)$ i.e., 6.

x. 55555
Ans: It is known to us that, the square of the number having unit place digit as ‘a’, will end up with the unit digit of ‘$a \times a$’.
Now, in the given number, the unit’s place digit is 5, its square will end with the unit digit of the multiplication $(5 \times 5 = 25)$ i.e., 5.

2. Give reason why the following numbers are not perfect squares.
i. 1057
Ans: The square of numbers may end with any one of the digits $0, 1, 4, 5, 6$, or 9. Also, a perfect square has even number of zeroes at the end of it.
We can see that 1057 has its unit place digit as 7.
Hence, 1057 cannot be a perfect square.

ii. 23453
Ans: The square of numbers may end with any one of the digits $0, 1, 4, 5, 6$, or 9 Also, a perfect square has even number of zeroes at the end of it.
We can see that 23453 has its unit place digit as 3.

Hence, 23453 cannot be a perfect square.

iii. 7928
Ans: The square of numbers may end with any one of the digits $0, 1, 4, 5, 6$, or 9 Also, a perfect square has even number of zeroes at the end of it.

We can see that 7928 has its unit place digit as 8.
Hence, 7928 cannot be a perfect square.

iv. 222222

Ans: The square of numbers may end with any one of the digits $0, 1, 4, 5, 6,$ or 9 Also, a perfect square has even number of zeroes at the end of it.
We can see that 222222 has its unit place digit as 2.
Hence, 222222 cannot be a perfect square.

v. 64000

Ans: The square of numbers may end with any one of the digits $0, 1, 4, 5, 6,$ or 9 Also, a perfect square has even number of zeroes at the end of it.
We can see that 64000 has three zeroes at the end of it.
Since a perfect square cannot end with odd number of zeroes, therefore, 64000 is not a perfect square.

vi. 89722

Ans: The square of numbers may end with any one of the digits $0, 1, 4, 5, 6,$ or 9 Also, a perfect square has even number of zeroes at the end of it.
We can see that 89722 has its unit place digit as 2.
Hence, 89722 cannot be a perfect square.

vii. 222000

Ans: The square of numbers may end with any one of the digits $0, 1, 4, 5, 6,$ or 9 Also, a perfect square has even number of zeroes at the end of it.
We can see that 222000 has three zeroes at the end of it.
Since a perfect square cannot end with odd number of zeroes, therefore, 222000 is not a perfect square.

viii. 505050

Ans: The square of numbers may end with any one of the digits $0, 1, 4, 5, 6,$ or 9 Also, a perfect square has even number of zeroes at the end of it.
We can see that 505050 has three zeroes at the end of it.
Since a perfect square cannot end with odd number of zeroes, therefore, 505050 is not a perfect square.

3. The squares of which of the following would be odd numbers?

(i) 431 (ii) 2826 (iii) 7779 (iv) 82004

Ans: Correct options are A) and C)
The square of a number with odd number at units place is odd and square of a . number with even number at units place is even. Hence, Square of 431 and 7779 will be odd.

4. Find the missing digits after observing the following pattern.

$\mathbf{11^2 = 121}$

$\mathbf{101^2 = 10201}$

$\mathbf{1001^2 = 1002001}$

$\mathbf{100001^2 = 1...2...1}$

$\mathbf{10000001^2 = ...}$

Ans: It can be observed from the given pattern that after doing the square of the number, there are same number of zeroes before the digit and same number of zeroes after the digit as there are in the original number.

So, the square of the number 100001 will have four zeroes before 2 and four zeroes after 2.

Similarly, the square of the number 10000001 will have six zeroes before 2 and six zeroes after 2.

Hence,

$100001^2 = 10000200001$

$10000001^2 = 100000020000001$

5. Find the missing number after observing the following pattern.

$\mathbf{11^2 = 121}$

$\mathbf{101^2 = 10201}$

$\mathbf{10101^2 = 102030201}$

$\mathbf{1010101^2 = ...}$

$\mathbf{...^2 = 10203040504030201}$

Ans: It can be observed from the given pattern that:

- the square of the numbers has odd number of digits
- the first and the last digit of the square of the numbers is 1
- the square of the numbers is symmetric about the middle digit

Since there are four 1 in 1010101, so the square of this number will have natural numbers up to 4 with 0 in between every consecutive number and then making the number symmetric about 4

That is, $1010101^2 = 1020304030201$

Now, 10203040504030201 has natural numbers up to 5 and the number is symmetric about.

So, the number whose square is 10203040504030201, is 101010101

That is, $101010101^2 = 10203040504030201$

Hence,

$1010101^2 = 1020304030201$

$101010101^2 = 10203040504030201$

6. Find the missing numbers using the given pattern.

$1^2+2^2+2^2=3^2$

$2^2+3^2+6^2=7^2$

$3^2+4^2+12^2=13^2$

$4^2+5^2+...^2=21^2$

$5^2+...^2+30^2=31^2$

$6^2+7^2+...^2=...^2$

Ans: It can be observed from the given pattern that:

- The third number in the addition is the product of first two numbers.
- The R.H.S can be obtained by adding to the third number.

That is, in the first three patterns, it can be observed that

$$1^2+2^2+(1\times 2)^2=(2+1)^2$$

$$2^2+3^2+(2\times 3)^2=(6+1)^2$$

$$3^2+4^2+(3\times 4)^2=(12+1)^2$$

Hence, according to the pattern, the missing numbers are as follows:

$$4^2+5^2+\underline{20}^2=21^2$$

$$5^2+\underline{6}^2+30^2=31^2$$

$$6^2+7^2+\underline{42}^2=\underline{43}^2$$

7. Find the sum without adding.

i. 1+3+5+7+9

Ans: Since, the sum of first n odd natural numbers is n^2.

So, the sum of the first five odd natural numbers is $(5)^2=25$

Thus, $1+3+5+7+9=(5)^2=25$

ii. 1+3+5+7+9+11+13+15+17+19

Ans: Since, the sum of first n odd natural numbers is n^2.

So, the sum of the first ten odd natural numbers is $(10)^2=100$

Thus, $1+3+5+7+9+11+13+15+17+19=(10)^2=100$

iii. 1+3+5+7+9+11+13+15+17+19+21+23

Ans: Since, the sum of first n odd natural numbers is n^2.

So, the sum of the first twelve odd natural numbers is $(12)^2=144$

Thus, $1+3+5+7+9+11+13+15+17+19+21+23=(12)^2=144$

8.

i. Express 49 as the sum of 7 odd numbers.

Ans: Since, the sum of first n odd natural numbers is n^2.

We know that $49=(7)^2$

$49=$ Sum of 7 odd natural numbers

Hence, $49=1+3+5+7+9+11+13$

ii. Express 121 as the sum of 11 odd numbers.

Ans: Since, the sum of first n odd natural numbers is n^2.

We know that $121=(11)^2$

$121=$ Sum of 11 odd natural numbers

Hence, $121=1+3+5+7+9+11+13+15+17+19+21$

9. How many numbers lie between squares of the following numbers?

i. 12 and 13

Ans: Between the squares of the numbers n and (n+1), there will be 2n numbers.

So, there will be $2\times12=24$ numbers between $(12)^2$ and $(13)^2$.

ii. 25 and 26

Ans: Between the squares of the numbers n and (n+1), there will be 2n numbers.

So, there will be $2\times25=50$ numbers between $(25)^2$ and $(26)^2$.

iii. 99 and 100

Ans: Between the squares of the numbers n and (n+1), there will be 2n numbers.

So, there will be $2\times99=198$ numbers between $(99)^2$ and $(100)^2$.

Exercise 5.2

1. Find the square of the following numbers.

i. 32

Ans:

$32=30+2$

$(32)^2=(30+2)^2$

Since, $(a+b)^2=a^2+2ab+b^2$

So, $(30+2)^2=30^2+2\times30\times2+2^2$

$=900+120+4$

$=1024$

ii. 35

Ans:

$35=30+5$

$(35)^2=(30+5)^2$

Since, $(a+b)^2=a^2+2ab+b^2$

So, $(30+5)^2 = 30^2 + 2\times 30\times 5 + 5^2$

$= 900 + 300 + 25$

$= 1225$

iii. 86

Ans:

$86 = 80 + 6$

$(86)^2 = (80+6)^2$

Since, $(a+b)^2=a^2+2ab+b^2$

So, $(80+6)^2 = 80^2 + 2\times 80\times 6 + 6^2$

$= 6400 + 960 + 36$

$= 7396$

iv. 93

Ans:

$93 = 90 + 3$

$(93)^2 = (90+3)^2$

Since, $(a+b)^2=a^2+2ab+b^2$

So, $(90+3)^2 = 90^2 + 2\times 90\times 3 + 3^2$

$= 8100 + 540 + 9$

$= 8649$

v. 71

Ans:

$71 = 70 + 1$

$(71)^2 = (70+1)^2$

Since, $(a+b)^2=a^2+2ab+b^2$

So, $(70+1)^2 = 70^2 + 2\times 70\times 1 + 1^2$

$= 4900 + 140 + 1$

$= 5041$

vi. 46

Ans:

$46 = 40 + 6$

$(46)^2 = (40+6)^2$

Since, $(a+b)^2=a^2+2ab+b^2$

So, $(40+6)^2 = 40^2 + 2\times 40\times 6 + 6^2$

$= 1600 + 480 + 36$

$= 2116$

2. Write a Pythagoras triplet whose one number is

i. 6

Ans: We know that 2m, m^2-1, m^2+1 is the Pythagoras triplet for any natural number m >1

It is given that one number in the triplet is 6.

If we take m^2-1=6, then we get $m^2 = 7$

And $m = \sqrt{7}$ which is not an integer.

Similarly, if we take m^2+1=6, then we get $m^2 = 5$

And $m = \sqrt{5}$ which is not an integer.

So let $2m = 6$

Then we get, $m = 3$

Now, $m^2 - 1 = 3^2 - 1$

$= 9 - 1$

$= 8$

Similarly, $m^2 + 1 = 3^2 + 1$

$= 9 + 1$

$= 10$

Therefore $(6,8,10)$ is the Pythagoras triplet.

ii. 14

Ans: We know that 2m, m^2-1, m^2+1 is the Pythagoras triplet for any natural number m >1

It is given that one number in the triplet is 14.

If we take m^2-1=14, then we get m^2-1=6

And $m = \sqrt{15}$ which is not an integer.

Similarly, if we take m^2+1=14, then we get m^2+1=14

And $m = \sqrt{13}$ which is not an integer.

So let 2m=14

Then we get, m=7

Now, m^2-1=7^2-1

$= 49 - 1$

$= 48$

Similarly, m^2+1=7^2+1

$= 49 + 1$

$= 50$

Therefore $(14, 48, 50)$ is the Pythagoras triplet.

iii. 16

Ans: We know that 2m, m^2-1, m^2+1 is the Pythagoras triplet for any natural number m >1

It is given that one number in the triplet is 16.

If we take m^2-1=16, then we get m^2=17

And $m = \sqrt{17}$ which is not an integer.

Similarly, if we take m^2+1=16, then we get m^2=15

And $m=\sqrt{15}$ which is not an integer.
So let 2m=16
Then we get, m=8
Now, $m^2-1=8^2-1$
$=64-1$
$=63$
Similarly, $m^2+1=8^2+1$
$=64+1$
$=65$
Therefore $(16,63,65)$ is the Pythagoras triplet.

iv. 18
Ans: We know that 2m, m^2-1, m^2+1 is the Pythagoras triplet for any natural number m >1
It is given that one number in the triplet is 18.
If we take $m^2-1=18$, then we get $m^2=19$
And $m=\sqrt{19}$ which is not an integer.
Similarly, if we take $m^2+1=18$, then we get $m^2=17$
And $m=\sqrt{17}$ which is not an integer.
So let 2m=18
Then we get, m=9
Now, $m^2-1=9^2-1$
$=81-1$
$=80$
Similarly, $m^2+1=9^2-1$
$=81+1$
$=82$
Therefore $(18,80,82)$ is the Pythagoras triplet.

Exercise 5.3

1. What could be the possible 'one's' digits of the square root of each of the following numbers?
i. 9801
Ans: We know that the one's digit of the square root of the number ending with 1 can be 1 or 9.
Thus, the possible one's digit of the square root of 9801 is either 1 or 9.

ii. 99856
Ans: We know that the one's digit of the square root of the number ending with 6 can be 6 or 4.
Thus, the possible one's digit of the square root of 99856 is either 6 or 4.

iii. 998001

Ans: We know that the one's digit of the square root of the number ending with 1 can be 1 or 9.

Thus, the possible one's digit of the square root of 998001 is either 1 or 9.

iv. 657666025

Ans: We know that the one's digit of the square root of the number ending with 5 will be 5 .

Thus, the only possible one's digit of the square root of 657666025 is 5 .

2. Find the numbers which are surely not perfect squares without doing any calculations.

i. 153

Ans: The perfect square of numbers may end with any one of the digits 0, 1 ,4, 5, 6, or 9. Also, a perfect square has even number of zeroes at the end of it, if any.

We can see that 153 has its unit place digit as 3.

Hence, 153 cannot be a perfect square.

ii. 257

Ans: The perfect square of numbers may end with any one of the digits 0, 1 ,4, 5, 6, or 9. Also, a perfect square has even number of zeroes at the end of it, if any.

We can see that 257 has its unit place digit as 7.

Hence, 257 cannot be a perfect square.

iii. 408

Ans: The perfect square of numbers may end with any one of the digits 0, 1 ,4, 5, 6, or 9. Also, a perfect square has even number of zeroes at the end of it, if any.

We can see that 408 has its unit place digit as 8.

Hence, 408 cannot be a perfect square.

iv. 441

Ans: The perfect square of numbers may end with any one of the digits 0, 1 ,4, 5, 6, or 9. Also, a perfect square has even number of zeroes at the end of it, if any.

We can see that 441 has its unit place digit as 1.

Hence, 441 is a perfect square.

3. Find the square roots of 100 and 169 by the method of repeated subtraction.

Ans: It is already known to us that the sum of the first n odd natural numbers is n^2.

For $\sqrt{100}$

1) $100 - 1 = 99$
2) $99 - 3 = 96$
3) $96 - 5 = 91$
4) $91 - 7 = 84$
5) $84 - 9 = 75$
6) $75 - 11 = 64$
7) $64 - 13 = 51$
8) $51 - 15 = 36$
9) $36 - 17 = 19$
10) $19 - 19 = 0$

After subtracting successive odd numbers from 1 to 100, we are getting a 0 at the 10^{th} step.

Hence, $\sqrt{100} = 10$

For $\sqrt{169}$

1) $169 - 1 = 168$
2) $168 - 3 = 165$
3) $165 - 5 = 160$
4) $160 - 7 = 153$
5) $153 - 9 = 144$
6) $144 - 11 = 133$
7) $133 - 13 = 120$
8) $120 - 15 = 105$
9) $105 - 17 = 88$
10) $88 - 19 = 69$
11) $69 - 21 = 48$
12) $48 - 23 = 25$
13) $25 - 25 = 0$

After subtracting successive odd numbers from 1 to 169, we are getting a 0 at the 13^{th} step.

Hence, $\sqrt{169} = 13$

4. Find the square roots of the following numbers by Prime Factorisation Method.

i. 729

Ans: The factorization of 729 is as follows:

3	729
3	243
3	81
3	27
3	9
3	3
	1

$729 = \underline{3 \times 3} \times \underline{3 \times 3} \times \underline{3 \times 3}$

$\sqrt{729} = 3 \times 3 \times 3$

So, $\sqrt{729} = 27$

ii. 400

Ans: The factorization of 400 is as follows:

2	400
2	200
2	100
2	50
5	25
5	5
	1

$400 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{5 \times 5}$

$\sqrt{400} = 2 \times 2 \times 5$

So, $\sqrt{400} = 20$

iii. 1764

Ans: The factorization of 1764 is as follows:

2	1764
2	882
3	441
3	147

7	49
7	7
	1

$$1764 = \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{7 \times 7}$$

$$\sqrt{1764} = 2 \times 3 \times 7$$

So, $\sqrt{1764} = 42$

iv. 4096

Ans: The factorization of 4096 is as follows:

2	4096
2	2048
2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

$$4096 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2}$$

$$\sqrt{4096} = 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

So, $\sqrt{4096} = 64$

v. 7744

Ans: The factorization of 7744 is as follows:

2	7744
2	3872

2	1936
2	968
2	484
2	242
11	121
11	11
	1

$7744 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{11 \times 11}$

$\sqrt{7744} = 2 \times 2 \times 2 \times 11$

So, $\sqrt{7744} = 88$

vi. 9604

Ans: The factorization of 9604 is as follows:

2	9604
2	4802
7	2401
7	343
7	49
7	7
	1

$9604 = \underline{2 \times 2} \times \underline{7 \times 7} \times \underline{7 \times 7}$

$\sqrt{9604} = 2 \times 7 \times 7$

So, $\sqrt{9604} = 98$

vii. 5929

Ans: The factorization of 5929 is as follows:

7	5929
7	847
11	121
11	11

	1

$5929 = \underline{7 \times 7} \times \underline{11 \times 11}$

$\sqrt{5929} = 7 \times 11$

So, $\sqrt{5929} = 77$

viii. 9216

Ans: The factorization of 9216 is as follows:

2	9216
2	4608
2	2304
2	1152
2	576
2	288
2	144
2	72
2	36
2	18
3	9
3	3
	1

$9216 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3}$

$\sqrt{9216} = 2 \times 2 \times 2 \times 2 \times 2 \times 3$

So, $\sqrt{9216} = 96$

ix. 529

Ans: The factorization of 529 is as follows:

23	529
23	23
	1

$529 = \underline{23 \times 23}$

So, $\sqrt{529} = 23$

x. 8100

Ans: The factorization of 8100 is as follows:

2	8100
2	4050
3	2025
3	675
3	225
3	75
5	25
5	5
	1

$8100 = \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{3 \times 3} \times \underline{5 \times 5}$

$\sqrt{8100} = 2 \times 3 \times 3 \times 5$

So, $\sqrt{8100} = 90$

5: For each of the following numbers, find the smallest whole number by which it should be multiplied so as to get a perfect square number. Also find the square root of the square number so obtained.

i. 252

Ans: The factorization of 252 is as follows:

2	252
2	126
3	63
3	21
7	7
	1

Here, $252 = \underline{2 \times 2} \times \underline{3 \times 3} \times 7$

We can see that 7 is not paired

So, we have to multiply 252 by 7 to get a perfect square.

The new number will be $252 \times 7 = 1764$

$1764 = \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{7 \times 7}$ which is a perfect square

$\sqrt{1764} = 2 \times 3 \times 7$

So, $\sqrt{1764} = 42$

ii. 180

Ans: The factorization of 180 is as follows:

2	180
2	90
3	45
3	15
5	5
	1

Here, $180 = \underline{2 \times 2} \times \underline{3 \times 3} \times 5$

We can see that 5 is not paired

So, we have to multiply 180 by 5 to get a perfect square.

The new number will be $180 \times 5 = 900$

$900 = \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{5 \times 5}$ which is a perfect square

$\sqrt{900} = 2 \times 3 \times 5$

So, $\sqrt{900} = 30$

iii. 1008

Ans: The factorization of 1008 is as follows:

2	1008
2	504
2	252
2	126
3	63
3	21
7	7
	1

Here, $1008 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3} \times 7$

We can see that 7 is not paired

So, we have to multiply 1008 by 7 to get a perfect square.
The new number will be $1008 \times 7 = 7056$
$7056 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{7 \times 7}$ which is a perfect square
$\sqrt{7056} = 2 \times 2 \times 3 \times 7$
So, $\sqrt{7056} = 84$

iv. 2028
Ans: The factorization of 2028 is as follows:

2	2028
2	1014
3	507
13	169
13	13
	1

Here, $2028 = \underline{2 \times 2} \times 3 \times \underline{13 \times 13}$
We can see that 3 is not paired
So, we have to multiply 2028 by 3 to get a perfect square.
The new number will be $2028 \times 3 = 6084$
$6084 = \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{13 \times 13}$ which is a perfect square
$\sqrt{6084} = 2 \times 3 \times 13$
So, $\sqrt{6084} = 78$

v. 1458
Ans: The factorization of 1458 is as follows:

2	1458
3	729
3	243
3	81
3	27
3	9
3	3
	1

Here, $1458 = 2 \times \underline{3 \times 3} \times \underline{3 \times 3} \times \underline{3 \times 3}$

We can see that 2 is not paired

So, we have to multiply 1458 by 2 to get a perfect square.

The new number will be $1458 \times 2 = 2916$

$2916 = \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{3 \times 3} \times \underline{3 \times 3}$ which is a perfect square

$\sqrt{2916} = 2 \times 3 \times 3 \times 3$

So, $\sqrt{2916} = 54$

vi. 768

Ans: The factorization of 768 is as follows:

2	768
2	384
2	192
2	96
2	48
2	24
2	12
2	6
3	3
	1

Here, $768 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times 3$

We can see that 3 is not paired

So, we have to multiply 768 by 3 to get a perfect square.

The new number will be $768 \times 3 = 2304$

$2304 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3}$ which is a perfect square

$\sqrt{2304} = 2 \times 2 \times 2 \times 2 \times 3$

So, $\sqrt{2304} = 48$

6. For each of the following numbers, find the smallest whole number by which it should be divided so as to get a perfect square. Also find the square root of the square number so obtained.

i. 252

Ans: The factorization of 252 is as follows:

2	252

2	126
3	63
3	21
7	7
	1

Here, $252 = \underline{2 \times 2} \times \underline{3 \times 3} \times 7$

We can see that 7 is not paired

So, we have to divide 252 by 7 to get a perfect square.

The new number will be $252 \div 7 = 36$

$36 = \underline{2 \times 2} \times \underline{3 \times 3}$ which is a perfect square

$\sqrt{36} = 2 \times 3$

So, $\sqrt{36} = 6$

ii. 2925

Ans: The factorization of 2925 is as follows:

3	2925
3	975
5	325
5	65
13	13
	1

Here, $2925 = \underline{3 \times 3} \times \underline{5 \times 5} \times 13$

We can see that 13 is not paired

So, we have to divide 2925 by 13 to get a perfect square.

The new number will be $2925 \div 13 = 225$

$225 = \underline{3 \times 3} \times \underline{5 \times 5}$ which is a perfect square

$\sqrt{225} = 3 \times 5$

So, $\sqrt{225} = 15$

iii. 396

Ans: The factorization of 396 is as follows:

2	396

2	198
3	99
3	33
11	11
	1

Here, $396 = \underline{2 \times 2} \times \underline{3 \times 3} \times 11$
We can see that $11$is not paired
So, we have to divide 396 by 11 to get a perfect square.
The new number will be $396 \div 11 = 36$
$36 = \underline{2 \times 2} \times \underline{3 \times 3}$ which is a perfect square
$\sqrt{36} = 2 \times 3$
So, $\sqrt{36} = 6$

iv. 2645
Ans: The factorization of 2645 is as follows:

5	2645
23	529
23	23
	1

Here, $2645 = 5 \times \underline{23 \times 23}$
We can see that $5$is not paired
So, we have to divide 2645 by 5 to get a perfect square.
The new number will be $2645 \div 5 = 529$
$529 = \underline{23 \times 23}$ which is a perfect square
So, $\sqrt{529} = 23$

v. 2800
Ans: The factorization of 2800 is as follows:

2	2800
2	1400
2	700
2	350

5	175
5	35
7	7
	1

Here, $2800 = \underline{2\times2}\times\underline{2\times2}\times\underline{5\times5}\times7$

We can see that 7 is not paired

So, we have to divide 2800 by 7 to get a perfect square.

The new number will be $2800 \div 7 = 400$

$400 = \underline{2\times2}\times\underline{2\times2}\times\underline{5\times5}$ which is a perfect square

$\sqrt{400} = 2\times2\times5$

So, $\sqrt{400} = 20$

vi. 1620

Ans: The factorization of 1620 is as follows:

2	1620
2	810
3	405
3	135
3	45
3	15
5	5
	1

Here, $1620 = \underline{2\times2}\times\underline{3\times3}\times\underline{3\times3}\times5$

We can see that 5 is not paired

So, we have to divide 1620 by 5 to get a perfect square.

The new number will be $1620 \div 5 = 324$

$324 = \underline{2\times2}\times\underline{3\times3}\times\underline{3\times3}$ which is a perfect square

$\sqrt{324} = 2\times3\times3$

So, $\sqrt{324} = 18$

7. The students of Class VIII of a school donated Rs 2401 in all, for Prime Minister's National Relief Fund. Each student donated as many rupees as the number of students in the class. Find the number of students in the class.

Ans: According to the question, each student donated as many rupees as the number of students in the class.

We can find the number of students in the class by doing the square root of the total amount donated by the students of Class VIII.

Total amount donated by students is Rs. 2401

Then, the number of students in the class will be $\sqrt{2401}$

$\sqrt{2401}=\sqrt{\underline{7\times7}\times\underline{7\times7}}$

$=7\times7$

$=49$

Thus, there are total 49 students in the class.

8. 2025 plants are to be planted in a garden in such a way that each row contains as many plants as the number of rows. Find the number of rows and the number of plants in each row.

Ans: According to the question, the plants are be planted in a garden in such a way that each row contains as many plants as the number of rows.

So, the number of rows will be equal to the number of plants in each row.

Hence,

The number of rows $\times$ Number of plants in each row $=$Total number of plants

The number of rows $\times$ Number of plants in each row $=$ 2025

The number of rows $\times$ The number of rows $=$ 2025

The number of rows $=\sqrt{2025}$

$\sqrt{2025}=\sqrt{\underline{5\times5}\times\underline{3\times3}\times\underline{3\times3}}$

$=5\times3\times3$

$=45$

Thus, the number of rows $=45$ and the number of plants in each row $=45$.

9. Find the smallest square number that is divisible by each of the numbers $4,9$ and 10.

Ans: We know that the number that is perfectly divisible by each one of $4,9$ and 10 is their L.C.M

So, taking the L.C.M of these numbers

2	4,9,10
2	2,9,5
3	1,9,5

3	$1,3,5$
5	$1,1,5$
	$1,1,1$

$\text{L.C.M} = 2 \times 2 \times 3 \times 3 \times 5$
$= 180$
It can be clearly seen that 5 cannot be paired.
Therefore, we have to multiply 180 by 5 in order to get a perfect square.
Thus, the smallest square number divisible by $4,9$ and $10 = 180 \times 5 = 900$

10. Find the smallest square number that is divisible by each of the numbers $8,15$ and 20.

Ans: We know that the number that is perfectly divisible by each one of $8,15$ and 20 is their L.C.M
So, taking the L.C.M of these numbers

2	$8,15,20$
2	$4,15,10$
2	$2,15,5$
3	$1,15,5$
5	$1,5,5$
	$1,1,1$

$\text{L.C.M} = 2 \times 2 \times 2 \times 3 \times 5$
$= 120$
It can be clearly seen that the prime factors $2, 3$ and 5 cannot be paired.
Therefore, we have to multiply 120 by $2, 3$ and 5 in order to get a perfect square.
Thus, the smallest square number divisible by $8,15$ and 20 is $120 \times 2 \times 3 \times 5$ $= 3600$

Exercise 5.4

1. Find the square root of each of the following numbers by division method.
i. 2304
Ans: The square root of 2304 by division method is calculated as follows:

	48
4	$\overline{23}\,\overline{04}$ -16

88	704 704
	0

Hence, $\sqrt{2304} = 48$

ii. 4489

Ans: The square root of 4489 by division method is calculated as follows:

	67
6	$\overline{44}\,\overline{89}$ -36
127	889 889
	0

Hence, $\sqrt{4489} = 67$

iii. 3481

Ans: The square root of 3481 by division method is calculated as follows:

	59
5	$\overline{34}\,\overline{81}$ -25
109	981 981
	0

Hence, $\sqrt{3481} = 59$

iv. 529

Ans: The square root of 529 by division method is calculated as follows:

	23
2	$\overline{5}\,\overline{29}$ -4
43	129 129
	0

Hence, $\sqrt{529} = 23$

v. 3249

Ans: The square root of 3249 by division method is calculated as follows:

	57
5	$\overline{32}\,\overline{49}$ -25
107	749 749
	0

Hence, $\sqrt{3249} = 57$

vi. 1369

Ans: The square root of 1369 by division method is calculated as follows:

	37
3	$\overline{13}\,\overline{69}$ -9
67	469 469
	0

Hence, $\sqrt{1369} = 37$

vii. 5776

Ans: The square root of 5776 by division method is calculated as follows:

	76
7	$\overline{57}\,\overline{76}$ -49
146	$\overline{57}\,\overline{76}$ -49
	0

Hence, $\sqrt{5776} = 76$

viii. 7921

Ans: The square root of 7921 by division method is calculated as follows:

	89
8	$\overline{79}\,\overline{21}$ -64
169	1521 1521
	0

Hence, $\sqrt{7921} = 89$

ix. 576

Ans: The square root of 576 by division method is calculated as follows:

	24
2	$\overline{5}\,\overline{76}$ -4
44	176 176
	0

Hence, $\sqrt{576} = 24$

x. 1024

Ans: The square root of 1024 by division method is calculated as follows:

	32
3	$\overline{10}\,\overline{24}$ -9
62	124 124
	0

Hence, $\sqrt{1024} = 32$

xi. 3136

Ans: The square root of 3136 by division method is calculated as follows:

	56
5	$\overline{31}\,\overline{36}$ -25
106	636 636
	0

Hence, $\sqrt{3136} = 56$

xii. 900

Ans: The square root of 900 by division method is calculated as follows:

	30
3	$\overline{9}\,\overline{00}$ -9
60	00 00

	0

Hence, $\sqrt{900} = 30$

2. Find the number of digits in the square root of each of the following numbers (without any calculation).

i. 64

Ans: In order to find the number of digits in the square root, without any calculation, we have to place the bars on the given number

After placing bars, we get

$64 = \overline{64}$

We can see that there is only one bar. So, the square root of 64 will have only one digit.

ii. 144

Ans: In order to find the number of digits in the square root, without any calculation, we have to place the bars on the given number

After placing bars, we get

$144 = \overline{1}\,\overline{44}$

We can see that there are two bars. So, the square root of 144 will have two digits.

iii. 4489

Ans: In order to find the number of digits in the square root, without any calculation, we have to place the bars on the given number

After placing bars, we get

$4489 = \overline{44}\,\overline{89}$

We can see that there are two bars. So, the square root of 4489 will have two digits.

iv. 27225

Ans: In order to find the number of digits in the square root, without any calculation, we have to place the bars on the given number

After placing bars, we get

$27225 = \overline{2}\,\overline{72}\,\overline{25}$

We can see that there are three bars. So, the square root of 27225 will have three digits.

v. 390625

Ans: In order to find the number of digits in the square root, without any calculation, we have to place the bars on the given number

After placing bars, we get

$390625 = \overline{39}\,\overline{06}\,\overline{25}$

We can see that there are three bars. So, the square root of 390625 will have three digits.

3. Find the square root of the following decimal numbers.

i. 2.56

Ans: The square root of 2.56 by division method is calculated as follows:

	1.6
1	$\overline{2}.\overline{56}$ -1
26	156 156
	0

Hence, $\sqrt{2.56} = 1.6$

ii. 7.29

Ans: The square root of 7.29 by division method is calculated as follows:

	2.7
2	$\overline{7}.\overline{29}$ -4
47	329 329
	0

Hence, $\sqrt{7.29} = 2.7$

iii. 51.84

Ans: The square root of 51.84 by division method is calculated as follows:

	7.2
7	$\overline{51}.\overline{84}$ -49
142	284 284
	0

Hence, $\sqrt{51.84} = 7.2$

iv. 42.25

Ans: The square root of 42.25 by division method is calculated as follows:

	6.5
6	$\overline{42}.\overline{25}$ -36
125	625 625
	0

Hence, $\sqrt{42.25} = 6.5$

v. 31.36

Ans: The square root of 31.36 by division method is calculated as follows:

	5.6
5	$\overline{31}.\overline{36}$ -25
106	636 636
	0

Hence, $\sqrt{31.36} = 5.6$

4. Find the least number which must be subtracted from each of the following numbers so as to get a perfect square. Also find the square root of the perfect square so obtained.

i. 402

Ans: The square root of 402 by division method is calculated as follows:

	20
2	$\overline{4}\overline{02}$ -4
40	02 00
	2

We are getting a remainder 2.
This means that, the square of 20 is less than 402 by 2.
So, we must subtract 2 from 402 in order to get a perfect square.
Hence, the required perfect square is $402 - 2 = 400$
The square root of the perfect square obtained is $\sqrt{400} = 20$

ii. 1989

Ans: The square root of 1989 by division method is calculated as follows:

	44
4	$\overline{19}\,\overline{89}$ -16
84	389 336
	53

We are getting a remainder 53.
This means that, the square of 44 is less than 1989 by 53.
So, we must subtract 53 from 1989 in order to get a perfect square.
Hence, the required perfect square is $1989-53=1936$
The square root of the perfect square obtained is $\sqrt{1936}=44$

iii. 3250

Ans: The square root of 3250 by division method is calculated as follows:

	57
5	$\overline{32}\,\overline{50}$ -25
107	750 749
	1

We are getting a remainder 1 .
This means that, the square of 57 is less than 3250 by 1.
So, we must subtract 1 from 3250 in order to get a perfect square.
Hence, the required perfect square is $3250-1=3249$
The square root of the perfect square obtained is $\sqrt{3249}=57$

iv. 825

Ans: The square root of 825 by division method is calculated as follows:

	28
2	$\overline{8}\,\overline{25}$ -4
48	425 384
	41

We are getting a remainder 41.
This means that, the square of 28 is less than 825 by 41.
So, we must subtract 41 from 825 in order to get a perfect square.
Hence, the required perfect square is $825 - 41 = 784$
The square root of the perfect square obtained is $\sqrt{784} = 28$

v. 4000
Ans: The square root of 4000 by division method is calculated as follows:

	63
6	$\overline{40}\,\overline{00}$ -36
123	400 369
	31

We are getting a remainder 31.
This means that, the square of 63 is less than 4000 by 31.
So, we must subtract 31 from 4000 in order to get a perfect square.
Hence, the required perfect square is $4000 - 31 = 3969$
The square root of the perfect square obtained is $\sqrt{3969} = 63$

5. Find the least number which must be added to each of the following numbers so as to get a perfect square. Also find the square root of the perfect square so obtained.
i. 525
Ans: The square root of 4000 by division method is calculated as follows:

	22
2	$\overline{5}\,\overline{25}$ -4
42	125 84
	41

We are getting a remainder 41.
This means that, the square of 63 is less than 4000.
The next number is 23 and its square is $23^2 = 529$
So, the number that should be added to 525 is $23^2 - 525 = 529 - 525 = 4$
Hence, the required perfect square is $525 + 4 = 529$

The square root of the perfect square obtained is $\sqrt{529}=23$

ii. 1750

Ans: The square root of 1750 by division method is calculated as follows:

	41
4	$\overline{17}\,\overline{50}$ -16
81	150 81
	69

We are getting a remainder 69.

This means that, the square of 41 is less than 1750.

The next number is 42 and its square is $42^2=1764$

So, the number that should be added to 1750 is $42^2-1750=1764-1750=14$

Hence, the required perfect square is $1750+14=1764$

The square root of the perfect square obtained is $\sqrt{1764}=42$

iii. 252

Ans: The square root of 252 by division method is calculated as follows:

	15
1	$\overline{2}\,\overline{52}$ -1
25	152 125
	27

We are getting a remainder 27.

This means that, the square of 15 is less than 252.

The next number is 16 and its square is $16^2=256$

So, the number that should be added to 252 is $16^2-252=256-252=4$

Hence, the required perfect square is $252+4=256$

The square root of the perfect square obtained is $\sqrt{256}=16$

iv. 1825

Ans: The square root of 1825 by division method is calculated as follows:

	42

4	$\overline{18}\,\overline{25}$ -16
82	225 164
	61

We are getting a remainder 61.
This means that, the square of 42 is less than 1825
The next number is 43 and its square is $43^2 = 1849$
So, the number that should be added to 1825 is $43^2 - 1825 = 1849 - 1825 = 24$
Hence, the required perfect square is $1825 + 24 = 1849$
The square root of the perfect square obtained is $\sqrt{1849} = 43$

v. 6412
Ans: The square root of 6412 by division method is calculated as follows:

	80
8	$\overline{64}\,\overline{12}$ -64
160	012 0
	12

We are getting a remainder 12.
This means that, the square of 80 is less than 6412
The next number is 81 and its square is $81^2 = 6561$
So, the number that should be added to 6412 is $81^2 - 6412 = 6561 - 6412 = 149$
Hence, the required perfect square is $6412 + 149 = 6561$
The square root of the perfect square obtained is $\sqrt{6561} = 81$

6. Find the length of the side of a square whose area is 441m^2.
Ans: Let us consider that the side of the square be x m in length
Then the area of the square is $(x)^2 = 441\ m^2$
We get x×
Calculating the square root of 441 using long division method as follows:

	21
2	$\overline{4}\,\overline{41}$ -4

41	041 41
	0

We get x=21 m
Therefore, the length of the side of the square is 21 m.

7. In a right triangle ABC, $\angle B = 90^o$

a) Find AC if AB=6 cm, BC=8 cm

Ans: It is given that triangle ABC is right angled at B
So, on applying Pythagoras Theorem, we get
$AC^2 = AB^2 + BC^2$
$AC^2 = 6^2 + 8^2$
$= 36 + 64$
$= 100$
$AC = \sqrt{10 \times 10}$
Thus, AC=10 cm.

b) Find AB if AC=13 cm, BC=5 cm

Ans: It is given that triangle ABC is right angled at B
So, on applying Pythagoras Theorem, we get
$AC^2 = AB^2 + BC^2$
$AB^2 = AC^2 - BC^2$
$AB^2 = 13^2 - 5^2$
$= 169 - 25$
$= 144$
$AB = \sqrt{12 \times 12}$
Thus, AB=12 cm.

8. A gardener has 1000 plants. He wants to plant these in such a way that the number of rows and the number of columns remain same. Find the minimum number of plants he needs more for this.

Ans: According to the question the gardener has 1000 plants and the number of rows and columns are same.
Our aim is to find the minimum number of plants that he needs so that after planting them, the number of rows and columns are same.
This means that we have to find the number that should be added to 1000 to make it a perfect square.
The square root of 1000 by long division method is calculated as follows:

	31
3	$\overline{10}\,\overline{00}$ -9
61	100 61
	39

We are getting a remainder 39.
This means that, the square of 31 is less than 1000
The next number is 32 and its square is $32^2 = 1024$
So, the number that should be added to 1000 is $32^2 - 1000 = 1024 - 1000 = 24$
Hence, the required number of plants is 24.

9. These are 500 children in a school. For a P.T. drill they have to stand in such a manner that the number of rows is equal to number of columns. How many children would be left out in this arrangement?
Ans: According to the given question, there are 500 children in the school and they have to stand for a PT drill in such a way that the number of rows and columns are equal.
We have to calculate the number of children that are left out in this arrangement.
This means that we have to find the number that should be subtracted from 500 in order to make it a perfect square.
The square root of 1000 by long division method is calculated as follows:

	22
2	$\overline{5}\,\overline{00}$ -4
42	100 84
	16

We are getting a remainder 16.
This means that, the square of 22 is less than 500 by 16
So, we must subtract 16 from 500 in order to get a perfect square.
Hence, the required perfect square is $500 - 16 = 484$
Thus, 16 children will be left out of this arrangement.