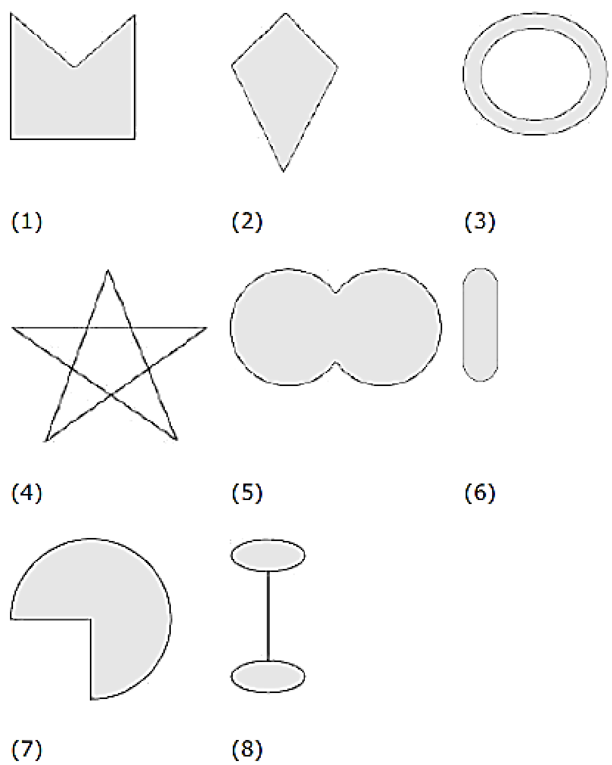
understanding quadrilaterlas

3
Chapter

Exercise 3.1

1. Given here are some figures.



Classify each of them on the basis of following.

a. Simple curve

Ans: Given: the figures (1) to (8)

We need to classify the given figures as simple curves.

We know that a curve that does not cross itself is referred to as a simple curve.

Therefore, simple curves are 1, 2, 5, 6, 7.

b. Simple closed curve

Ans: Given: the figures (1) to (8)

We need to classify the given figures as simple closed curves.

We know that a simple closed curve is one that begins and ends at the same point without crossing itself.

Therefore, simple closed curves are 1, 2, 5, 6, 7.

c. Polygon

Ans: Given: the figures (1) to (8)

We need to classify the given figures as polygon.

We know that any closed curve consisting of a set of sides joined in such a way that no two segments cross is known as a polygon.

Therefore, the polygons are 1, 2.

d. Convex polygon

Ans: Given: the figures (1) to (8)

We need to classify the given figures as convex polygon.

We know that a closed shape with no vertices pointing inward is called a convex polygon.

Therefore, the convex polygon is 2.

e. Concave polygon

Ans: Given: the figures (1) to (8)

We need to classify the given figures as concave polygon.

We know that a polygon with at least one interior angle greater than 180 degrees is called a concave polygon.

Therefore, the concave polygon is 1.

2. What is a regular polygon? State the name of a regular polygon of:

(i) 3 sides **(ii) 4 sides** **(iii) 6 sides**

Ans: A regular polygon is a polygon which has all its side equal and so all angles are equal.

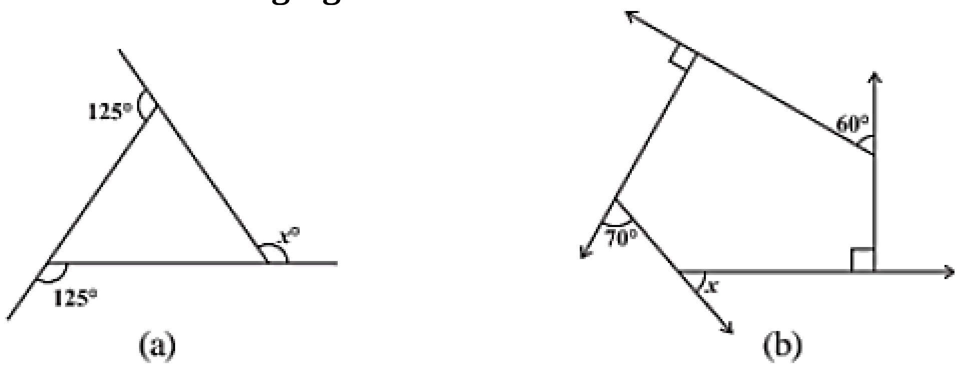
(i) 3sides: It is an equilateral triangle.

(ii) 4 sides: It is a square.

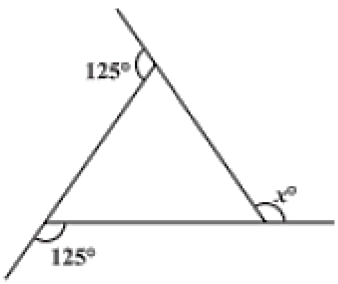
(iii) 6 sides: It is a hexagon.

Exercise-3.2

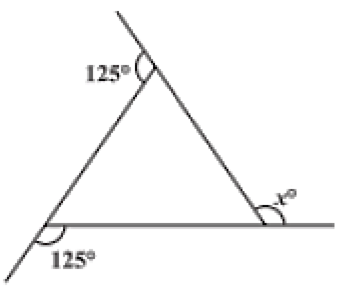
1. Find x in the following figures.



a.



Ans: Given:



We need to find the value of x.

We know that the sum of all exterior angles of a polygon is 360°.

Thus.

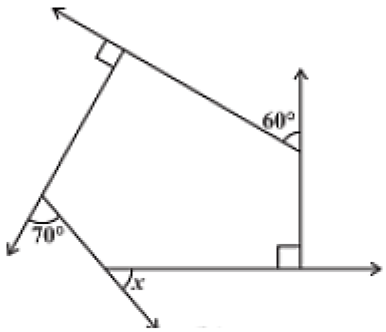
$$x+125^\circ+125^\circ=360^\circ$$

$$\Rightarrow x+250^\circ=360^\circ$$

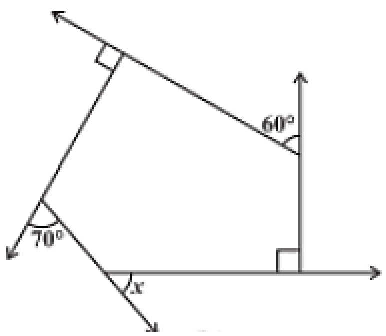
$$\Rightarrow x=360^\circ-250^\circ$$

$$\Rightarrow x=110^\circ$$

b.



Ans: Given:



We need to find the value of x.

We know that the sum of all exterior angles of a polygon is 360°.

Thus,

$x+90^\circ+60^\circ+90^\circ+70^\circ=360^\circ$

$\Rightarrow x+310^\circ=360^\circ$

$\Rightarrow x=360^\circ-310^\circ$

$\Rightarrow x=50^\circ$

2. Find the measure of each exterior angle of a regular polygon of
i. 9 sides

Ans: Given: a regular polygon with 9 sides

We need to find the measure of each exterior angle of the given polygon.

We know that all the exterior angles of a regular polygon are equal.

The sum of all exterior angle of a polygon is 360°.

Formula Used: $\text{Exterior angle} = \dfrac{360^\circ}{\text{Number of sides}}$

Therefore,

Sum of all angles of given regular polygon $= 360^\circ$

Number of sides $= 9$

Therefore, measure of each exterior angle will be

$= \dfrac{360^\circ}{9}$

$= 40^\circ$

ii. 15 sides

Ans:

Given: a regular polygon with 15 sides

We need to find the measure of each exterior angle of the given polygon.

We know that all the exterior angles of a regular polygon are equal.

The sum of all exterior angle of a polygon is 360°.

Therefore,

Sum of all angles of given regular polygon $= 360^\circ$

Number of sides $= 15$

Formula Used: $\text{Exterior angle} = \dfrac{360^\circ}{\text{Number of sides}}$

Therefore, measure of each exterior angle will be

$= \dfrac{360^\circ}{15}$

$= 24^\circ$

3. How many sides does a regular polygon have if the measure of an exterior angle is 24°?

Ans: Given: A regular polygon with each exterior angle 24°

We need to find the number of sides of given polygon.

We know that sum of all exterior angle of a polygon is 360°.

Formula Used: $\text{Number of sides} = \dfrac{360^\circ}{\text{Exterior angle}}$

Thus,

Sum of all angles of given regular polygon $= 360^\circ$

Each angle measure $= 24^\circ$

Therefore, number of sides of given polygon will be

$= \dfrac{360^\circ}{24^\circ}$

$= 15^\circ$

4. How many sides does a regular polygon have if each of its interior angles is 165°?

Ans: Given: A regular polygon with each interior angle 165°

We need to find the sides of the given regular polygon.

We know that sum of all exterior angle of a polygon is 360°.

Formula Used: $\text{Number of sides} = \dfrac{360^\circ}{\text{Exterior angle}}$

$\text{Exterior angle} = 180^\circ - \text{Interior angle}$

Thus,

Each interior angle $= 165^\circ$

So, measure of each exterior angle will be

$= 180^\circ - 165^\circ$

$= 15^\circ$

Therefore, number of sides of polygon will be

$= \dfrac{360^\circ}{15^\circ}$

$=24$

5. (a) Is it possible to have a regular polygon with measure of each exterior angle as 22°?

Ans: Given: A regular polygon with each exterior angle 22°

We need to find if it is possible to have a regular polygon with given angle measure.

We know that sum of all exterior angle of a polygon is 360°. The polygon will be possible if 360° is a perfect multiple of exterior angle.

Thus,

$\frac{360^\circ}{22^\circ}$ does not give a perfect quotient.

Thus, 360° is not a perfect multiple of exterior angle. So, the polygon will not be possible.

(b) Can it be an interior angle of a regular polygon? Why?

Ans: Given: Interior angle of a regular polygon $=22^\circ$

We need to state if it can be the interior angle of a regular polygon.

We know that sum of all exterior angle of a polygon is 360°. The polygon will be possible if 360° is a perfect multiple of exterior angle.

And, $\text{Exterior angle} = 180^\circ - \text{Interior angle}$

Thus, Exterior angle will be

$=180^\circ - 22^\circ$

$=158^\circ$

$\frac{158^\circ}{22^\circ}$ does not give a perfect quotient.

Thus, 158° is not a perfect multiple of exterior angle. So, the polygon will not be possible.

6. (a) What is the minimum interior angle possible for a regular polygon?

Ans: Given: A regular polygon

We need to find the minimum interior angle possible for a regular polygon.

A polygon with minimum number of sides is an equilateral triangle.

So, number of sides $=3$

We know that sum of all exterior angle of a polygon is 360°.

And,

$$\text{Exterior angle} = \frac{360^{\circ}}{\text{Number of sides}}$$

Thus, Maximum Exterior angle will be

$$= \frac{360^{\circ}}{3}$$

$$= 120^{\circ}$$

We know, $\text{Interior angle} = 180^{\circ} - \text{Exterior angle}$

Therefore, minimum interior angle will be

$$= 180^{\circ} - 120^{\circ}$$

$$= 60^{\circ}$$

(b) What is the maximum exterior angel possible for a regular polygon?

Ans: Given: A regular polygon

We need to find the maximum exterior angle possible for a regular polygon.

A polygon with minimum number of sides is an equilateral triangle.

So, number of sides $=3$

We know that sum of all exterior angle of a polygon is 360°.

And,

$$\text{Exterior angle} = \frac{360^{\circ}}{\text{Number of sides}}$$

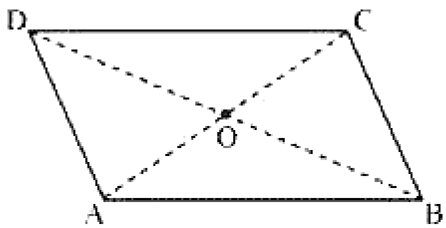
Therefore, Maximum Exterior angle possible will be

$= \frac{360^\circ}{3}$

$= 120^\circ$

Exercise 3.3

1. Given a parallelogram ABCD. Complete each statement along with the definition or property used.



i. AD **=...**

Ans: Given: A parallelogram $ABCD$

We need to complete each statement along with the definition or property used.

We know that opposite sides of a parallelogram are equal.

Hence, $AD = BC$

ii. $\angle DCB =$ **...**

Ans: Given: A parallelogram $ABCD$

We need to complete each statement along with the definition or property used. $ABCD$ is a parallelogram, and we know that opposite angles of a parallelogram are equal.

Hence, $\angle DCB = \angle DAB$

iii. $OC = ...$

Ans: Given: A parallelogram $ABCD$.

We need to complete each statement along with the definition or property used.

$ABCD$ is a parallelogram, and we know that diagonals of parallelogram bisect each other.

Hence, $OC = OA$

iv. $m\angle DAB + m\angle CDA = ...$

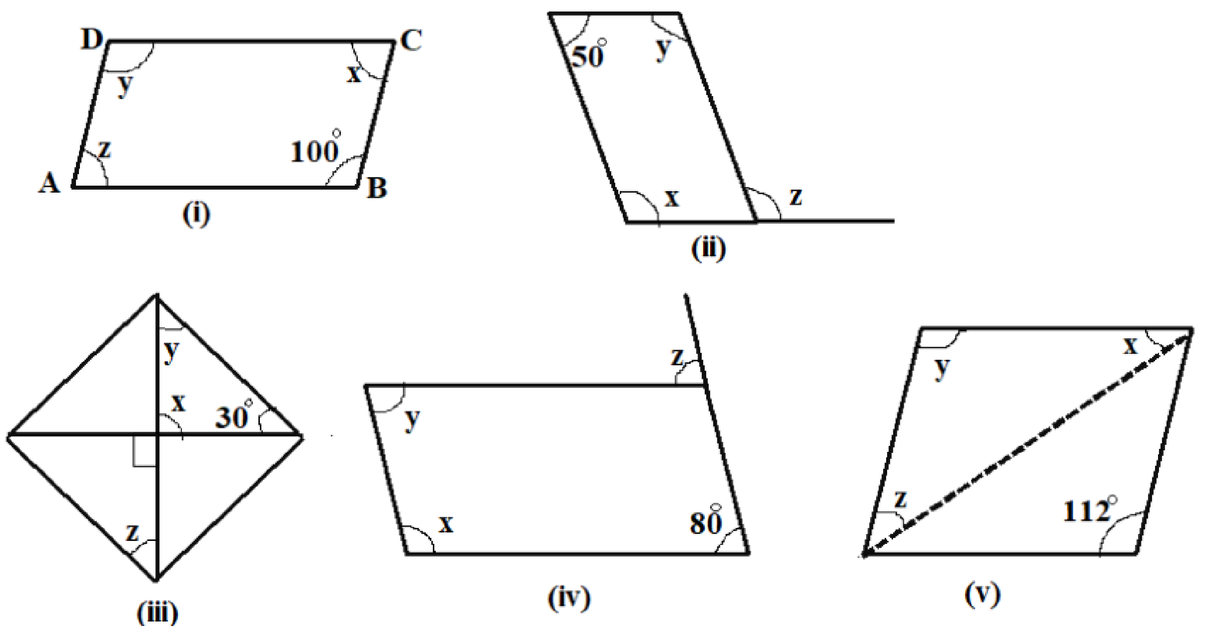
Ans: Given: A parallelogram $ABCD$.

We need to complete each statement along with the definition or property used.

$ABCD$ is a parallelogram, and we know that adjacent angles of a parallelogram are supplementary to each other.

Hence, $m\angle DAB + m\angle CDA = 180^\circ$

2. Consider the following parallelograms. Find the values of the unknowns x, y, z.



(i)

Ans: Given: A parallelogram $ABCD$

We need to find the unknowns x, y, z

The adjacent angles of a parallelogram are supplementary.

Therefore, $x + 100^\circ = 180^\circ$

$x = 80^\circ$

Also, the opposite angles of a parallelogram are equal.

Hence, $z = x = 80^\circ$ and $y = 100^\circ$

(ii)

Ans: Given: A parallelogram.

We need to find the values of x,y,z

The adjacent pairs of a parallelogram are supplementary.

Hence, $50^\circ + y = 180^\circ$

$y = 130^\circ$

Also, $x = y = 130^\circ$ (opposite angles of a parallelogram are equal)

And, $z = x = 130^\circ$ (corresponding angles)

(iii)

Ans: Given: A parallelogram

We need to find the values of x,y,z

$x = 90^\circ$ (Vertically opposite angles)

Also, by angle sum property of triangles

$x + y + 30^\circ = 180^\circ$

$y = 60^\circ$

Also, $z = y = 60^\circ$ (alternate interior angles)

(iv) Ans:

Given: A parallelogram

We need to find the values of x, y, z

Corresponding angles between two parallel lines are equal.

Hence, $z = 80^\circ$

Also, $y = 80^\circ$ (opposite angles of parallelogram are equal)

In a parallelogram, adjacent angles are supplementary

Hence, $x + y = 180^\circ$

$x = 180^\circ - 80^\circ$

$x = 100^\circ$

v.

Ans: Given: A parallelogram

We need to find the values of x, y, z

As the opposite angles of a parallelogram are equal, therefore, $y = 112^\circ$

Also, by using angle sum property of triangles

$x + y + 40^\circ = 180^\circ$

$x + 152^\circ = 180^\circ$

$x = 28^\circ$

And $z = x = 28^\circ$ (alternate interior angles)

3. Can a quadrilateral $ABCD$ be a parallelogram if

(i) $\angle D + \angle B = 180^\circ$?

Ans: Given: A quadrilateral $ABCD$

We need to find whether the given quadrilateral is a parallelogram.

For the given condition, quadrilateral $ABCD$ may or may not be a parallelogram.

For a quadrilateral to be parallelogram, the sum of measures of adjacent angles should be 180° and the opposite angles should be of same measures.

(ii) $AB = DC = 8\ cm, AD = 4\ cm$ and $BC = 4.4\ cm$

Ans: Given: A quadrilateral $ABCD$

We need to find whether the given quadrilateral is a parallelogram.

As, the opposite sides AD and BC are of different lengths, hence the given quadrilateral is not a parallelogram.

(iii) $\angle A = 70^\circ$ **and** $\angle C = 65^\circ$

Ans: Given: A quadrilateral $ABCD$

We need to find whether the given quadrilateral is a parallelogram.

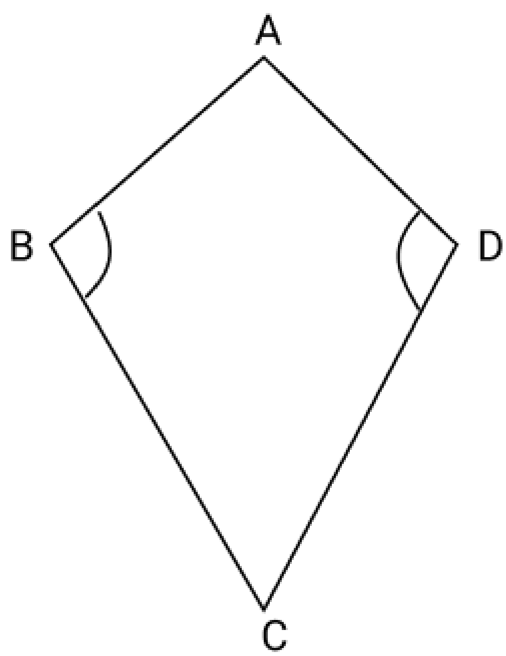
As, the opposite angles have different measures, hence, the given quadrilateral is a parallelogram.

4. Draw a rough figure of a quadrilateral that is not a parallelogram but has exactly two opposite angles of equal measure.

Ans: Given: A quadrilateral.

We need to draw a rough figure of a quadrilateral that is not a parallelogram but has exactly two opposite angles of equal measure.

A kite is a figure which has two of its interior angles, $\angle B$ and $\angle D$ of same measures. But the quadrilateral $ABCD$ is not a parallelogram as the measures of the remaining pair of opposite angles are not equal.



5. The measures of two adjacent angles of a parallelogram are in the ratio 3:2. Find the measure of each of the angles of the parallelogram.

Ans: Given: A parallelogram with adjacent angles in the ratio 3:2

We need to find the measure of each of the angles of the parallelogram.

Let the angles be $\angle A = 3x$ and $\angle B = 2x$

As the sum of measures of adjacent angles is 180° for a parallelogram.

$\angle A + \angle B = 180^\circ$

$3x + 2x = 180^\circ$

$5x = 180^\circ$

$x = 36^\circ$

$\angle A = \angle C = 3x = 108^\circ$ and $\angle B = \angle D = 2x = 72^\circ$ (Opposite angles of a parallelogram are equal).

Hence, the angles of a parallelogram are $108^\circ, 72^\circ, 108^\circ$ and 72°.

6. Two adjacent angles of a parallelogram have equal measure. Find the measure of each of the angles of the parallelogram.

Ans: Given: A parallelogram with two equal adjacent angles.

We need to find the measure of each of the angles of the parallelogram.

The sum of adjacent angles of a parallelogram are supplementary.

$\angle A + \angle B = 180^\circ$

$2\angle A = 180^\circ$

$\angle A = 90^\circ$

$\angle B = \angle A = 90^\circ$

Also, opposite angles of a parallelogram are equal

Therefore,

$\angle C = \angle A = 90^\circ$

$\angle D = \angle B = 90^\circ$

Hence, each angle of the parallelogram measures 90°.

7. The adjacent figure $HOPE$ is a parallelogram. Find the angle measures x, y and z. State the properties you use to find them.

Ans: Given: A parallelogram $HOPE$.

We need to find the measures of angles x, y, z and also state the properties used to find these angles.

$\angle y = 40^\circ$ (Alternate interior angles)

And $\angle z + 40^\circ = 70^\circ$ (corresponding angles are equal)

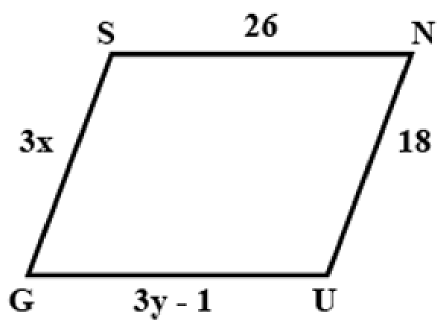
$\angle z = 30^\circ$

Also, $x + z + 40^\circ = 180^\circ$ (adjacent pair of angles)

$x = 110^\circ$

8. The following figures $GUNS$ and $RUNS$ are parallelograms. Find x and y. (Lengths are in cm).

(i)



Ans: Given: Parallelogram $GUNS$.

We need to find the measures of x and y.

$GU = SN$ (Opposite sides of a parallelogram are equal).

$3y - 1 = 26$

$3y = 27$

$y = 9$

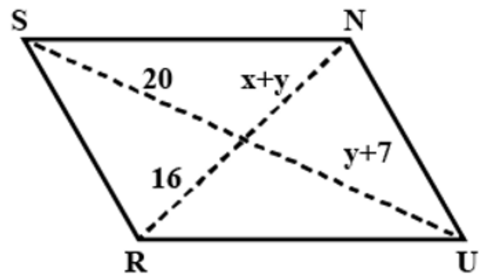
Also, $SG = NU$

Therefore,

$3x = 18$

$x = 3$

(ii)



Ans: Given: Parallelogram RUNS

We need to find the value of x and y.

The diagonals of a parallelogram bisect each other, therefore,

$y + 7 = 20$

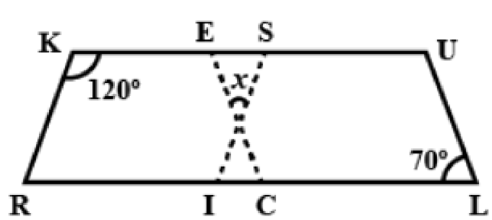
$y = 13$

$x + y = 16$

$x + 13 = 16$

$x = 3$

9. In the above figure both RISK and CLUE are parallelograms. Find the value of x.



Ans: Given: Parallelograms RISK and CLUE

We need to find the value of x.

As we know that the adjacent angles of a parallelogram are supplementary, therefore,

In parallelogram RISK

$\angle RKS + \angle ISK = 180^\circ$

$120^\circ + \angle ISK = 180^\circ$

As the opposite angles of a parallelogram are equal, therefore,

In parallelogram $CLUE$,

$\angle ULC = \angle CEU = 70^\circ$

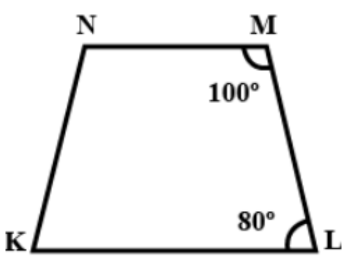
Also, the sum of all the interior angles of a triangle is 180°

Therefore,

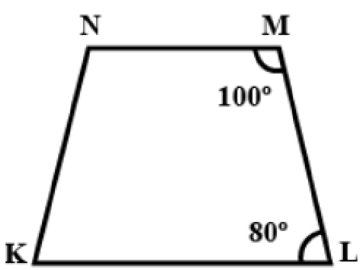
$x + 60^\circ + 70^\circ = 180^\circ$

$x = 50^\circ$

10. Explain how this figure is a trapezium. Which of its two sides are parallel?



Ans: Given:



We need to explain how the given figure is a trapezium and find its two sides that are parallel.

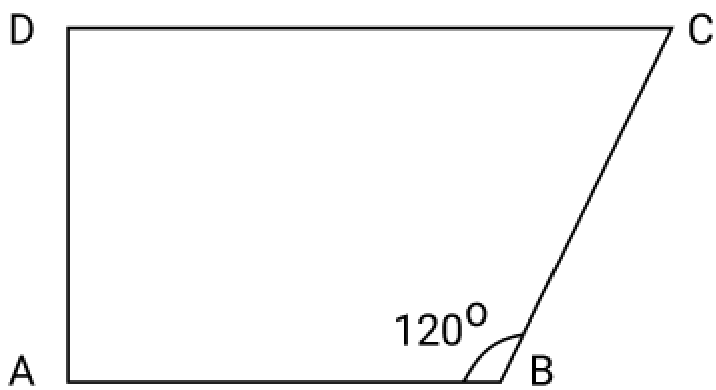
If a transversal line intersects two specified lines in such a way that the sum of the angles on the same side of the transversal equals 180°, the two lines will be parallel to each other.

Here, $\angle NML = \angle MLK = 180^\circ$

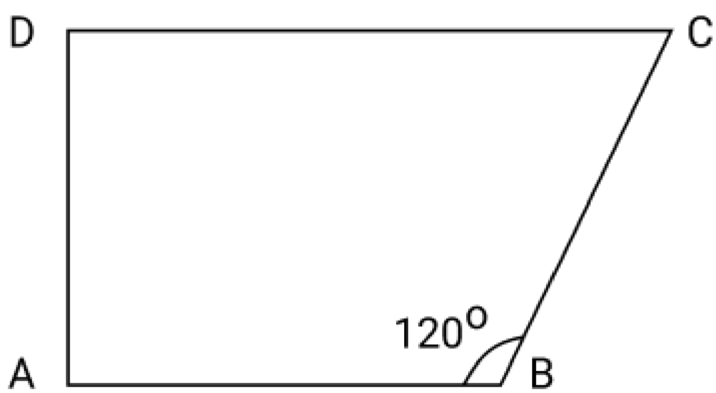
Hence, $NM \| LK$

Hence, the given figure is a trapezium.

11. Find $m\angle C$ in the following figure if $AB \parallel CD$.



Ans: Given: $AB \parallel CD$ and quadrilateral



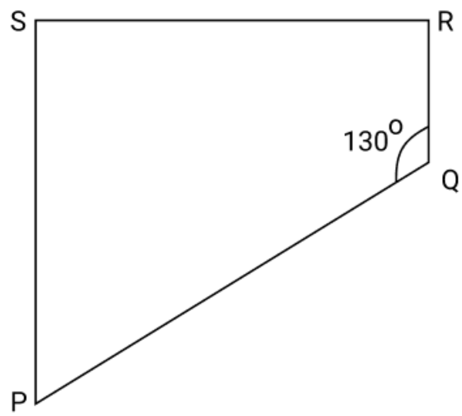
We need to find the measure of $\angle C$

$\angle B + \angle C = 180^{\circ}$ (Angles on the same side of transversal).

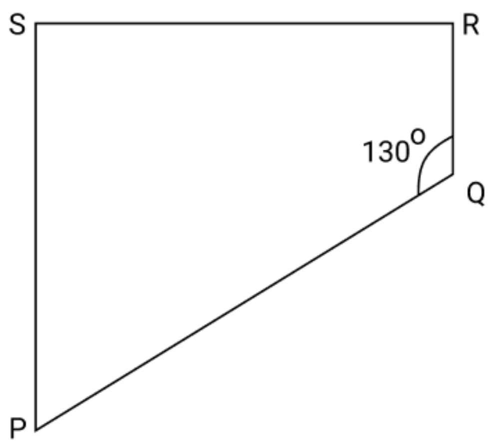
$120^{\circ} + \angle C = 180^{\circ}$

$\angle C = 60^{\circ}$

12. Find the measure of $\angle P$ and $\angle S$, if $SP \parallel RQ$ in the following figure. (If you find $m\angle R$, is there more than one method to find $m\angle P$?)



Ans: Given: $SP \parallel RQ$ and



We need to find the measure of $\angle P$ and $\angle S$.

The sum of angles on the same side of transversal is 180°.

$\angle P + \angle Q = 180^\circ$

$\angle P + 130^\circ = 180^\circ$

$\angle P = 50^\circ$

Also,

$\angle R + \angle S = 180^\circ$

$90^\circ + \angle S = 180^\circ$

$\angle S = 90^\circ$

Yes, we can find the measure of $m\angle P$ by using one more method.

In the question, $m\angle R$ and $m\angle Q$ are given. After finding $m\angle S$ we can find $m\angle P$ by using angle sum property.

Exercise-3.4

1: State whether True or False.

(a) All rectangles are squares.

(b) All rhombuses are parallelograms.

(c) All squares are rhombuses and also rectangles.

(d) All squares are not parallelograms.

(e) All kites are rhombuses.

(f) All rhombuses are kites.

(g) All parallelograms are trapeziums.

(h) All squares are trapeziums.

Ans: (a) False. All squares are rectangles but all rectangles are not squares.

(b) True. Opposite sides of a rhombus are equal and parallel to each other.

(c) True. All squares are rhombuses as all sides of a square are of equal lengths. All squares are also rectangles as each internal angle measures 90°

(d) False. All squares are parallelograms as opposite sides are equal and parallel.

(e) False. A kite does not have all sides of the same length.

(f) True. A rhombus also has two distinct consecutive pairs of sides of equal length.

(g) True. All parallelograms have a pair of parallel sides.

(h) True. All squares have a pair of parallel sides.

2: Identify all the quadrilaterals that have

(a) four sides of equal length

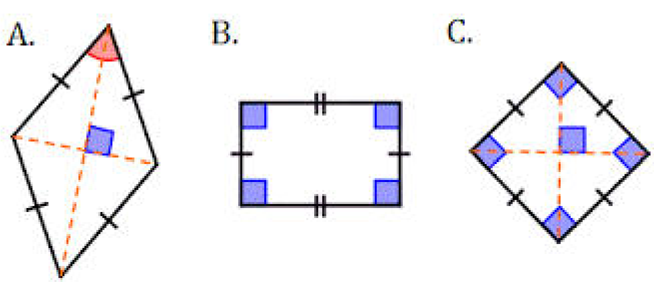
(b) four right angles

Ans: In geometry, a quadrilateral can be defined as a closed, two-dimensional shape that has four straight sides.

Fig A shows Rhombus, B shows a rectangle, C shows a square.

(a) Quadrilaterals that have four sides of equal length are Rhombus and squares.

(b) Quadrilaterals that have four right angles are Rectangle and squares.



3: Explain how a square is.

(i) a quadrilateral (ii) a parallelogram (iii) a rhombus (iv) a rectangle.

Ans: (i) a quadrilateral

Solution A square is a quadrilateral, as it has four equal sides.

(ii) a parallelogram

Solution A square is a parallelogram, as it contains the pairs of opposite sides equal.

(iii) a rhombus

Solution A square is a rhombus, as it has four equal sides and diagonals bisect at 90^0.

(iv) a rectangle

Solution A square is a rectangle, as it has each adjacent angle at 90^0 and opposite sides are equal.

4: Name the quadrilaterals whose diagonals.

(i) bisect each other (ii) are perpendicular bisectors of each other (iii) are equal

Ans: (i) Bisects each other: Diagonals of a parallelogram, rhombus, square and rectangle.

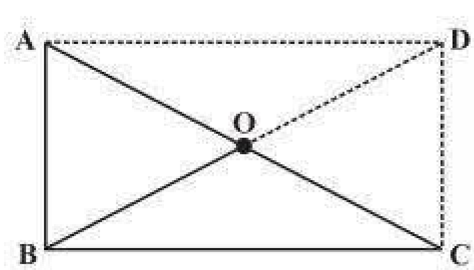
(ii) Are perpendicular bisectors of each other: Diagonals of rhombus and square → perpendicular bisectors.

(iii) Are equal: Diagonals of rectangle and square are equal.

5: Explain why a rectangle is a convex quadrilateral.

Ans: In a rectangle, there are two diagonals, both lying in the interior of the rectangle. Hence, it is a convex quadrilateral.

6: ABC is a right-angled triangle and O is the midpoint of the side opposite to the right angle. Explain why O is equidistant from A, B, and C. (The dotted lines are drawn additionally to help you).



Ans: Correct option is A)

Between Δ AOD & Δ BOC we have AO=CO (given),BO=OD (by Construction).

∠ AOD =∠ BOC....Vertically opposite angle

∴ By SAS test Δ AOD & Δ BOC are congruent.

So AD=BC....(i)

similarly, between Δ AOB & Δ DOC we have AO=CO (given), BO=OD (by Construction)

∠ AOB =∠ DOC

∴ By SAS test Δ AOB & Δ DOC are congruent.

So AB=DC.....(ii)

Also ∠ ABC=90o....(iii)

∴ From (i) & (ii) & (iii) we conclude that ABCD is a rectangle.

So the diagonals AC & BD are equal and bisect each other at O.

∴ OA=OB=OC=OD.

i.e O is equidistant from A, B & C.