direct and inverse proportions

11 Chapter

Exercise 11.1

1. Following are the car parking charges near a railway station up to 4 hrs Rs 60, 8 hrs Rs 100, 12 hrs Rs 140, and 24 hrs Rs 180. Check if the parking charges are in direct proportion to the parking time.

Ans: We know the charges will be in direct proportion to the parking time when each of their ratios will be the same.

We can form a table by using the given information as –

Table for car parking charges

Time taken for parking (hrs)	4	8	12	24
Charges (Rs)	60	100	140	180

Therefore, the ratio of the parking charges to the time taken for parking will be $= \frac{\text{charge}}{\text{time taken}}$.

Hence,

$\frac{60}{4} = 15,$

$\Rightarrow \frac{100}{8} = \frac{25}{2},$

$\Rightarrow \frac{140}{12} = \frac{35}{3},$

$\Rightarrow \frac{180}{24} = \frac{15}{2}.$

As, we can observe that the ratios are not the same. Hence, we can conclude that the parking charges are not in direct proportion to the parking time.

2. A mixture of paint is prepared by mixing 1 part of red pigments with 8 parts of base. In the following table, find the parts of base that need to be added.

Parts of red pigment	1	4	7	12	20
Parts of base	8	-	-	-	-

Ans: Given we have a mixture of paint that is prepared by mixing 1 part of red pigments with 8 parts of base. We can observe that they are in a direct proportion. Therefore,

Let a_1, a_2, a_3, and a_4 denote the parts of base of the respective parts of red pigment.

Hence, we have –

$$\frac{8}{1} = \frac{a_1}{4}$$

$$\Rightarrow a_1 = 32.$$

Similarly, $\frac{8}{1} = \frac{a_2}{7}$

$$\Rightarrow a_2 = 56,$$

$$\frac{8}{1} = \frac{a_3}{12}$$

$$\Rightarrow a_3 = 96, \text{ and}$$

$$\frac{8}{1} = \frac{a_4}{20}$$

$$\Rightarrow a_4 = 160.$$

Therefore, now the given table will be as –

Parts of red pigment	1	4	7	12	20
Parts of base	8	32	56	96	160

3. In Question 2 above, if 1 part of a red pigment requires 75mL of base, how much red pigment should we mix with 1800mL of base?

Ans: Let us assume that a denotes the parts of red pigment required. We know that parts of base are in direct proportion of parts of red pigment.

Therefore,

$$\frac{75}{1} = \frac{1800}{a}$$

$$\Rightarrow a = \frac{1800}{75}$$

$$\Rightarrow a = 24.$$

Hence, the table will be as –

Parts of red pigment	1	24
Parts of base	75	1800

4. A machine in a soft drink factory fills 840 bottles in six hours. How many bottles will it fill in five hours?

Ans: Given we have a soft drink factory that fills 840 bottles in six hours. Let us assume that the number of bottles to be filled in five hours be x.

Therefore, we can conclude that both the parameters are in direct proportion.

Hence,

$$\frac{840}{6} = \frac{x}{5}$$

$$\Rightarrow x = \frac{840 \times 5}{6}$$

$\Rightarrow x = 700$

Therefore, the number of bottles to be filled in five hours will be 700.

5. A photograph of a bacteria enlarged $50,000$ times attains a length of $5cm$. What is the actual length of the bacteria? If the photograph is enlarged $20,000$ times only, what would be its enlarged length?

Ans: Let us assume that the actual length of the bacteria is $a\,cm$ and let us assume that its enlarged length will be $b\,cm$. The given information implies that the parameters are in direct proportion.

Hence, we can compute a table as –

Length of bacteria (cm)	5	a	b
Number of times bacteria enlarged	50000	1	20000

Therefore, the actual length of the bacteria when the photograph was not enlarged will be –

$$\frac{5}{50000} = \frac{a}{1}$$

$$\Rightarrow a = \frac{1}{10000}$$

$$\Rightarrow a = 10^{-4} cm$$

Now, we will calculate the enlarged length of the bacteria when photograph is enlarged $20,000$ times.

$$\frac{5}{50000} = \frac{b}{20000}$$

$$\Rightarrow b = 2cm.$$

Therefore, the actual length of the bacteria is $0.0001\,cm$ and the enlarged length is $2cm$.

6. In a model of a ship, the mast is $9\,\text{cm}$ high, while the mast of the actual ship is $12\,\text{m}$ high. If the length of the ship is $28\,\text{m}$, how long is the model ship?

Ans: Let us assume that the length of the model ship is $x\,\text{cm}$. We can observe that the mast and length of the ship are directly proportional to each other. Therefore,

$$\frac{12}{9}=\frac{28}{x}$$

$$\Rightarrow x=\frac{28\times 9}{12}$$

$$\Rightarrow x=21\,\text{cm}.$$

Hence, the length of the model ship will be of $21\,\text{cm}$.

7. Suppose $2\,\text{kg}$ of sugar contains 9×10^6 crystals. How many sugar crystals are there in –

Ans:

i. $5\,\text{kg}$ of sugar?

Let us assume that the $5\,\text{kg}$ of sugar contains x number of crystals.

As, weight of sugar is directly proportional to the number of crystals it contains. Therefore,

$$\frac{2}{9\times10^6}=\frac{5}{x}$$

$$\Rightarrow x=\frac{9\times10^6\times5}{2}$$

$$\Rightarrow x=2.25\times10^7$$

Hence, $5\,\text{kg}$ of sugar contains 2.25×10^7 crystals of sugar.

ii. $1.2\,\text{kg}$ of sugar?

Similarly, let us assume that the $1.2\,\text{kg}$ of sugar contains x number of crystals.

As, weight of sugar is directly proportional to the number of crystals it contains. Therefore,

$$\frac{2}{9\times10^6}=\frac{1.2}{x}$$

$$\Rightarrow x=\frac{9\times10^6\times1.2}{2}$$

$$\Rightarrow x=5.4\times10^6$$

Hence, $1.2\,\text{kg}$ of sugar contains 5.4×10^6 crystals of sugar.

8. Rashmi has a road map with a scale of $1\,\text{cm}$ representing $18\,\text{km}$. She drives on a road for $72\,\text{km}$. What would be her distance covered in the map?

Ans: Given we have a road map with a scale of $1\,\text{cm}$ representing $18\,\text{km}$. Let us assume that the distance covered by her is $x\,\text{cm}$. We can observe that the scale is directly proportional to the distance she covers.

Therefore,

$$\frac{18}{1}=\frac{72}{x}$$

$$\Rightarrow x=4\,\text{cm}.$$

Hence, the distance covered by her represents on the map as $4\,\text{cm}$.

9. A $5\,\text{m}\,60\,\text{cm}$ high vertical pole casts a shadow $3\,\text{m}\,20\,\text{cm}$ long. Find at the same time –

i. the length of the shadow cast by another pole $10\,\text{m}\,50\,\text{cm}$ high

Ans: Given we have a vertical pole of height $5\,\text{m}\,60\,\text{cm}$ which casts a shadow of $3\,\text{m}\,20\,\text{cm}$. Let us assume that the length of shadow cast by pole $10\,\text{m}\,50\,\text{cm}$

high is $x\ m$. As, we know that the height of the pole is directly proportional to the length of shadow.

Therefore,

$$\frac{5.60}{3.20} = \frac{10.50}{x}$$

$$\Rightarrow x = \frac{10.50 \times 3.20}{5.60}$$

$$\Rightarrow x = 6$$

Hence, the length of the shadow cast by another pole $10\ m\ 50\ cm$ high is $6\ m$.

ii. the height of a pole which casts a shadow 5m long.

Ans: Let us assume that the height of the pole is $x\ m$ which casts a shadow $5\ m$ long. As, we know that the height of the pole is directly proportional to the length of shadow.

Therefore,

$$\frac{5.60}{3.20} = \frac{x}{5}$$

$$\Rightarrow x = \frac{5 \times 5.60}{3.20}$$

$$\Rightarrow x = 8.75$$

Hence, the height of the pole is $8\ m\ 75\ cm$ which casts a shadow $5\ m$ long.

10. A loaded truck travels $14\ km$ in 25 minutes. If the speed remains the same, how far can it travel in 5 hours?

Ans: Let us assume the distance travelled by truck in 5 hours is $x\ km$. As, we know that the distance travelled by the loaded truck is directly proportional to the time.

Therefore,

$$\frac{14}{25} = \frac{x}{5 \times 60}$$

$$\Rightarrow x = \frac{5 \times 60 \times 14}{25}$$

$$\Rightarrow x = 168 \text{ km}$$

Hence, the distance travelled by the loaded truck in 5 hours is 168 km.

Exercise 11.2

1. Which of the following are in inverse proportion?

(i) The number of workers on a job and the time to complete the job.

Ans: Since, the number of workers increases, it reduces the time to complete the job. Therefore, we can conclude that this is an inverse proportion.

(ii) The time taken for a journey and the distance travelled in a uniform speed.

Ans: We know, if we travel at a uniform speed then the distance travelled can be more if we have taken more time. This implies that they are in direct proportion and not in inverse proportion. Hence, we can conclude that this is not in inverse proportion.

(iii) Area of cultivated land and the crop harvested.

Ans: We know, the more cultivated land we will have, the greater number of crops can be harvested. This implies that they are in direct proportion and not in inverse proportion. Therefore, we can conclude that this is not in inverse proportion.

(iv) The time taken for a fixed journey and the speed of the vehicle.

Ans: When we travel with the vehicle at high speed, there is a possibility that we can complete the journey in less time than expected. This implies that this is in inverse proportion.

(v) The population of a country and the area of land per person.

Ans: As the population of a country increases, the area of the land per person will decrease because more people will be requiring the area for themselves. Therefore, this is in inverse proportion.

2. In a Television game show, the prize money of Rs 1,00,000 is to be divided equally amongst the winners. Complete the following table and find whether the prize money given to an individual winner is directly or inversely proportional to the number of winners?

Numbers of winners	1	2	4	5	8	10	20
Prize of each winner	100000	50000	….	….	….	….	….

Ans: We know, the more the number of winners will be, the lesser the prize money each winner gets. This implies that these are in inverse proportion.

Now, let us assume the prize for each winner as a_1, a_2, a_3, a_4 and a_5.

Therefore,

$1 \times 100000 = 4 \times a_1$

$\Rightarrow a_1 = \frac{100000}{4}$

$\Rightarrow a_1 = \text{Rs } 25000$

Similarly,

$1 \times 100000 = 5 \times a_2$

$\Rightarrow a_2 = \frac{100000}{5}$

$\Rightarrow a_2 = \text{Rs } 20000$

Now, we will calculate a_3

$1 \times 100000 = 8 \times a_3$

$$\Rightarrow a_1 = \frac{100000}{8}$$

$$\Rightarrow a_3 = \text{Rs}\,12500$$

Now, we will calculate a_4

$$1 \times 100000 = 10 \times a_4$$

$$\Rightarrow a_4 = \frac{100000}{10}$$

$$\Rightarrow a_4 = \text{Rs}\,10000$$

Now, we will calculate a_5

$$1 \times 100000 = 20 \times a_5$$

$$\Rightarrow a_5 = \frac{100000}{20}$$

$$\Rightarrow a_5 = \text{Rs}\,5000$$

Hence, the amount decreases as the number of winner increases.

3. Rehman is making a wheel using spokes. He wants to fix equal spokes in such a way that the angles between any pair of consecutive spokes are equal. Help him by completing the following table.

Numbers of spokes	4	6	8	10	12
Angle between a pair of consecutive spokes	90^o	60^o	….	….	….

(i) Are the number of spokes and the angles formed between the pairs of consecutive spokes in inverse proportion?

(ii) Calculate the angle between a pair of consecutive spokes on a wheel with 15 spokes.

(iii) How many spokes would be needed, if the angle between a pair of consecutive spokes is $40°$?

Ans: We know, the more the number of spokes will be, the lesser the angle between two spokes will be. This implies that these are in inverse proportion.

Now, let us assume the angle between two consecutive spokes as $a_1, a_2,$ and a_3 .

Therefore,

$4 \times 90° = 8 \times a_1$

$\Rightarrow a_1 = \frac{4 \times 90}{8}$

$\Rightarrow a_1 = 45°$

Similarly,

$4 \times 90° = 10 \times a_2$

$\Rightarrow a_2 = \frac{4 \times 90}{10}$

$\Rightarrow a_2 = 36°$

Now, we will calculate a_3

$4 \times 90° = 12 \times a_3$

$\Rightarrow a_3 = \frac{4 \times 90}{12}$

$\Rightarrow a_3 = 30°$

Hence, table will be as –

Number of spokes	4	6	8	10	12
Angle between a pair of consecutive spokes	90º	60º	45º	36º	30º

(i) Are the number of spokes and the angles formed between the pairs of consecutive spokes in inverse proportion?

Ans: As the number of spokes increases, we can observe that the angle between the pair of consecutive spokes decreases. Therefore, this implies they are in inverse proportion.

(ii) Calculate the angle between a pair of consecutive spokes on a wheel with 15 spokes.

Ans: Let us assume the angle as $x°$ of consecutive spokes on a wheel with 15 spokes.

Therefore,

$4 \times 90° = 15 \times x$

$\Rightarrow x = \frac{4 \times 90}{15}$

$\Rightarrow x = 24°.$

Hence, the angle will be $24°$ between a pair of consecutive spokes on a wheel with 15 spokes.

(iii) How many spokes would be needed, if the angle between a pair of consecutive spokes is $40°$?

Ans: Let us assume the number of spokes be y.

Therefore,

$4 \times 90° = y \times 40°$

$\Rightarrow y = \frac{4 \times 90}{40}$

$\Rightarrow y = 9.$

Hence, 9 spokes would be needed if the angle between a pair of consecutive spokes is $40°$.

4. If a box of sweets is divided among 24 children, they will get 5 sweets each. How many would each get, if the number of the children is reduced by 4?

Ans: As, the number of children is reduced by 4. Hence, the remaining children will be $24 - 4 = 20$ children.

Let us assume that the number of sweets each would get is a. We know that the number of sweets each child gets increases whenever the number of children is decreased.

Therefore,

$24 \times 5 = 20 \times a$

$\Rightarrow a = \frac{24 \times 5}{20}$

$\Rightarrow a = 6.$

Hence, each children will get 6 sweets.

5. A farmer has enough food to feed 20 animals in his cattle for 6 days. How long would the food last if there were 10 more animals in his cattle?

Ans: Let us assume that the number of days the food will last be x days.

He has 10 more animals in his cattle. Hence, total animals will be $20 + 10 = 30$

We know, the more is the number of animals the food will last for less days. This implies these are in inverse proportion.

Therefore,

$20 \times 6 = 30 \times x$

$\Rightarrow x = \frac{20 \times 6}{30}$

$\Rightarrow x = 4.$

Hence, the food will last for 4 days if there were 10 more animals in his cattle.

6. A contractor estimates that 3 persons could rewire Jasminder's house in 4 days. If, he uses 4 persons instead of three, how long should they take to complete the job?

Ans: Let us assume the number of days for the persons to complete the job be x days.

We know that the number of days and number of persons are in inverse proportion.

Therefore,

$3 \times 4 = 4 \times x$

$$\Rightarrow x = \frac{3 \times 4}{4}$$

$$\Rightarrow x = 3.$$

Hence, the number of days will be 3 days if he uses 4 persons instead of three to complete the job.

7. A batch of bottles was packed in 25 boxes with 12 bottles in each box. If the same batch is packed using 20 bottles in each box, how many boxes would be filled?

Ans: Let us assume the number of boxes needed to be filled be x.

We know that the number of boxes and number of bottles in each box are in inverse proportion.

Therefore,

$$25 \times 12 = x \times 20$$

$$\Rightarrow x = \frac{25 \times 12}{20}$$

$$\Rightarrow x = 15.$$

Hence, the number of boxes needed to be filled if the same batch is packed using 20 bottles in each box will be 15 boxes.

8. A factory required 42 machines to produce a given number of articles in 63 days. How many machines would be required to produce the same number of articles in 54 days?

Ans: Let us assume the number of machines required to produce the articles be x.

We know that the number of machines and number of days are in inverse proportion.

Therefore,

$$42 \times 63 = x \times 54$$

$$\Rightarrow x = \frac{42 \times 63}{54}$$

$\Rightarrow x = 49$.

Hence, the number of machines required to produce the same number of articles in 54 days will be 49 machines.

9. A car takes 2 hours to reach a destination by travelling at the speed of 60 km/h. how long will it take when the car travels at the speed of 80 km/h ?

Ans: Let us assume the number of hours the car will take to reach the destination be x hrs.

We know that the number of hours and speed of the car are in inverse proportion.

Therefore,

$2 \times 60 = x \times 80$

$\Rightarrow x = \frac{2 \times 60}{80}$

$\Rightarrow x = \frac{3}{2}$.

Hence, the time taken when the car travels at the speed of 80 km/h will be 1.5 hrs.

10. Two persons could fit new windows in house in 3 days.

(i) One of the persons fell ill before the work started. How long would the job take now?

Ans: As, we know that the number of persons and the number of days is in inverse proportion. Let us assume that the number of days for one person to get the job done be x days.

Therefore,

$2 \times 3 = 1 \times x$

$\Rightarrow x = \frac{2 \times 3}{1}$

$\Rightarrow x = 6.$

Hence, if one of the persons fell ill before the work started, it will take 6 days to fit new windows.

(ii) How many people would be needed to fit the windows in one day?

Ans: As, we know that the number of persons and the number of days is in inverse proportion. Let us assume that the number of persons for one person needed to fit the windows be y.

Therefore,

$2 \times 3 = y \times 1$

$\Rightarrow y = \frac{2 \times 3}{1}$

$\Rightarrow y = 6.$

Hence, 6 persons will be needed to fit the windows in one day.

11. A school has 8 periods a day each of 45 minutes duration. How long would each period be, if the school has 9 periods a day, assuming the number of school hours to be the same?

Ans: We know that the time for each period will decrease if we increase the number of periods a day. This implies these are in inverse proportion.

So, let us assume that each period will be of x minutes.

Therefore,

$8 \times 45 = 9 \times x$

$\Rightarrow x = \frac{8 \times 45}{9}$

$\Rightarrow x = 40.$

Hence, each period will be of 40 minutes if the school has 9 periods a day, assuming the number of school hours to be the same.