Ray Optics & Optical Instruments

Chapter 9

1. A small candle, 2.5 cm in size is placed at 27 cm in front of a concave mirror of radius of curvature 36cm. At what distance from the mirror should a screen be placed in order to obtain a sharp image? Describe the nature and size of the image. If the candle is moved closer to the mirror, how would the screen have to be moved?

Ans: We have,

Size of the candle, $h = 2.5\text{cm}$

Let image size $= h'$

Object distance, $u = -27\text{cm}$

concave mirror's curvature radius, $R = -36\text{cm}$

Focal length of the concave mirror, $f = \frac{R}{2} = -18\text{cm}$

Using mirror formula,

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

Where, u is the object distance,

v is the image distance and

f is the focal length.

Now, we put given values,

$$\Rightarrow \frac{1}{v} = \frac{1}{-18} - \frac{1}{-27}$$

$$\Rightarrow \frac{1}{v} = \frac{-3+2}{54}$$

$$\Rightarrow \frac{1}{v} = \frac{-1}{54}$$

$$\Rightarrow v = -54\text{cm}$$

Therefore, the screen should be 54 cm away from the mirror to get a sharp image.

The formula for magnification of image is given by:

$$m = \frac{h'}{h} = -\frac{v}{u}$$

$\therefore h' = -\frac{v}{u} \times h$

$\Rightarrow h' = -\frac{-54}{-27} \times 2.5$

$\Rightarrow h' = -5cm$

The height of the image of the candle is $5\ cm$. The negative sign shows that the image is inverted and real.

If the candle is moved nearer to the mirror, then the screen will have to be moved far from the mirror in order to get the image.

2. **A $4.5\ cm$ needle is placed $12cm$ away from a convex mirror of focal length $15cm$. Give the location of the image and the magnification. Describe what happens as the needle is moved farther from the mirror.**

Ans: Given that,

Height of the needle, $h_1 = 4.5cm$

Object distance, $u = -12cm$

Focal length of the convex mirror, $f = 15cm$

Image distance $= v$

Using mirror formula,

$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

$\Rightarrow \frac{1}{v} = \frac{1}{f} - \frac{1}{u}$

Now, we put given values,

$\Rightarrow \frac{1}{v} = \frac{1}{15} - \frac{1}{-12}$

$\Rightarrow \frac{1}{v} = \frac{4+5}{60}$

$\Rightarrow \frac{1}{v} = \frac{9}{60}$

$\Rightarrow v = 6.7cm$

Hence, the needle's image is $6.7\ cm$ away from the mirror and it is on the mirror's other side.

The formula for magnification of image is given by:

$m = \frac{h_2}{h_1} = -\frac{v}{u}$

$$\therefore h_2 = -\frac{v}{u} \times h_1$$

$$\Rightarrow h_2 = -\frac{6.7}{-12} \times 4.5$$

$$\Rightarrow h_2 = 2.5\text{cm}$$

The image's height is 2.5cm. The positive sign shows that the image is virtual, erect, and diminished.If the needle is moved away from the mirror, the image will also move farther from the mirror, and the size of the image will decrease gradually.

3. A t ank is filled with water to a height of 12.5 cm. The apparent depth of a needle lying at the bottom of the tank is measured by a microscope to be 9.4cm. What is the refractive index of water? If water is replaced by a liquid of refractive index 1.63 up to the same height, by what distance would the microscope have to be moved to focus on the needle again?

Ans: Given that,

Actual depth of the needle in water, $h_1 = 12.5\text{cm}$

Apparent depth of the needle in water, $h_2 = 9.4\text{cm}$

Refractive index of water $= \mu$

The formula for refractive index is given by:

$$\mu = \frac{h_1}{h_2}$$

Put the given values,

$$\mu = \frac{12.5}{9.4}$$

$$\mu \approx 1.33$$

Hence, the water's refractive index is about 1.33.

When water is replaced by a liquid of refractive index, $\mu' = 1.63$.

The actual depth of the needle will be the same, but it's apparent depth will vary.

Let y be the new apparent depth of the needle.

We will use the relation given below:

$$\mu' = \frac{h_1}{y}$$

$$\therefore y = \frac{h_1}{\mu'}$$

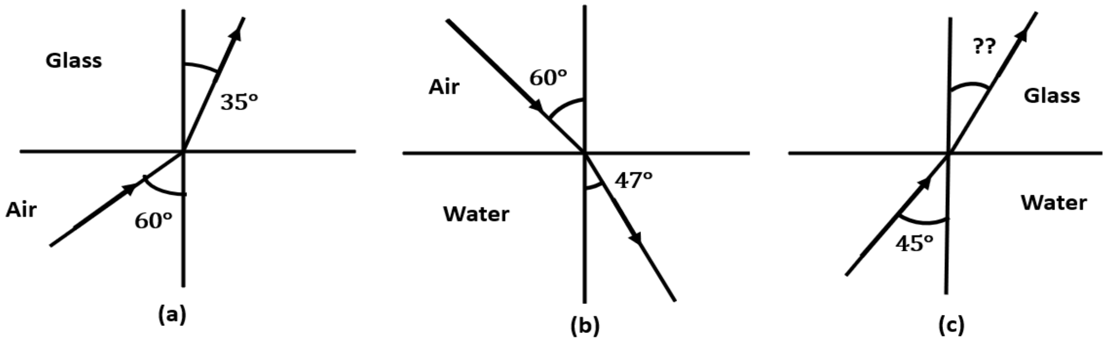
$\Rightarrow y = \dfrac{12.5}{1.63}$

$\Rightarrow y = 7.67\text{cm}$

We get the new apparent depth of the needle to be 7.67cm. It is less than h_2. Therefore, the microscope should be moved up to focus the needle again.

The distance by which the microscope would be moved up $= 9.4 - 7.67 = 1.73\text{cm}$.

4. Figures (a) and (b) show refraction of a ray in air incident at $60°$ with the normal to a glass-air and water air interface, respectively. Predict the angle of refraction in glass when the angle of incidence in water is $45°$ with the normal to a water-glass interface [figure (c)].



Ans: Given that,

For the glass -air interface,

Angle of incidence, $i = 60°$

Angle of refraction, $r = 35°$

We can use Snell's law,

$\mu_g^a = \dfrac{\sin i}{\sin r}$

$\Rightarrow \mu_g^a = \dfrac{\sin 60°}{\sin 35°}$

$\Rightarrow \mu_g^a = 1.51 \ldots\ldots (i)$

For the air-water interface,

Angle of incidence, $i = 60°$

Angle of refraction, $r = 47°$

We can use Snell's law,

$\mu_w^a = \dfrac{\sin i}{\sin r}$

$$\Rightarrow \mu_w^a = \frac{\sin 60°}{\sin 47°}$$

$$\Rightarrow \mu_w^a = 1.184 \ldots\ldots\text{(ii)}$$

Using (i) and (ii), the relative refractive index of glass with respect to water can be derived as:

$$\mu_g^w = \frac{\mu_g^a}{\mu_w^a}$$

$$\Rightarrow \mu_g^w = \frac{1.51}{1.184}$$

$$\Rightarrow \mu_g^w = 1.275$$

For the glass - water interface,
Angle of incidence, $i = 45°$
Angle of refraction, $= r$
We can use Snell's law,

$$\mu_g^w = \frac{\sin i}{\sin r}$$

$$\Rightarrow 1.275 = \frac{\sin 45°}{\sin r}$$

$$\Rightarrow \sin r = 0.5546$$

$$\Rightarrow r = \sin^{-1}(0.5546)$$

$$\Rightarrow r = 38.68°$$

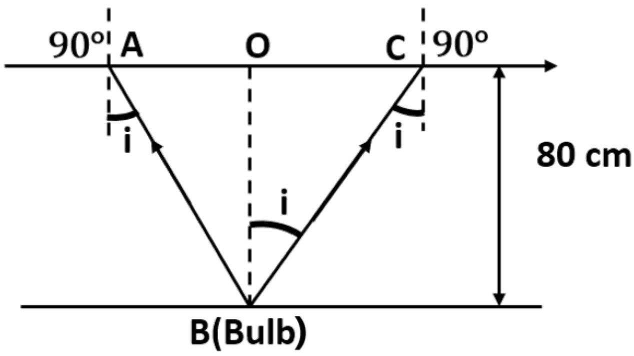
Hence, the angle of refraction at the water – glass interface is $38.68°$.

5. A small bulb is placed at the bottom of a tank containing water to a depth of 80 cm. What is the area of the surface of water through which light from the bulb can emerge out? Refractive index of water is 1.33. (Consider the bulb to be a point source.)

Ans: Provided that,
Bulb's actual depth in water, $d_1 = 80\text{cm} = 0.8\text{m}$

Water's refractive index, $\mu = 1.33$

The following diagram represents the given setup:



Where, i is the Angle of incidence

r is the Angle of refraction, $r = 90°$

As the bulb acts as a point source, the emergent light would be considered as a circle of radius, $R = \frac{AC}{2} = AO = OB$

Snell's law may be used as follows:

$$\mu = \frac{\sin r}{\sin i}$$

$$\Rightarrow 1.33 = \frac{\sin 90°}{\sin i}$$

$$\Rightarrow \sin i = \frac{1}{1.33}$$

$$\Rightarrow i = 48.75°$$

Considering the given diagram, we have the relation:

$$\tan i = \frac{OC}{OB} = \frac{R}{d_1}$$

$$\Rightarrow R = \tan 48.75° \times 0.8$$

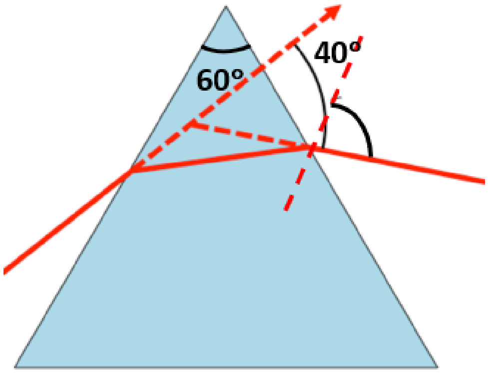
$$\Rightarrow R = 0.91\text{m}$$

Area of the surface of water $= \pi R^2 = \pi(0.91)^2 = 2.61\text{m}^2$

Clearly, the area of the water surface through which the light from the bulb could project is about 2.61m^2.

6. **A prism is made of glass of unknown refractive index. A parallel beam of light is incident on a face of the prism. The angle of minimum deviation is measured to be $40°$. What is the refractive index of the material of the prism? The refracting angle of the prism is $60°$. If the prism is placed in water (refractive index 1.33), predict the new angle of minimum deviation of a parallel beam of light.**

Ans: The minimum deviation angle and the angle of prism is as shown in figure given below:



Angle of minimum deviation, $\delta_m = 40°$

Angle of the prism, $A = 60°$

Refractive index of water, $\mu = 1.33$

Refractive index of the material of the prism $= \mu'$

The relation between angle of deviation with refractive index is given by:

$$\mu' = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

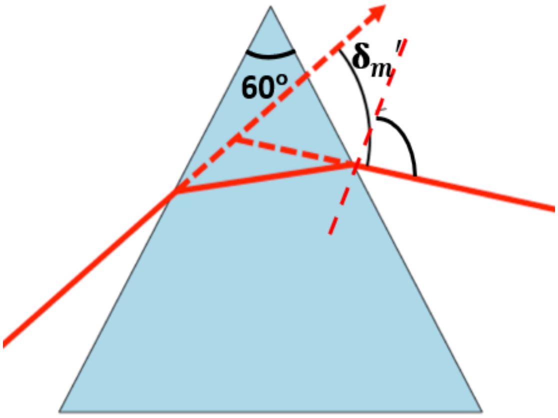
$$\Rightarrow \mu' = \frac{\sin\left(\frac{60° + 40°}{2}\right)}{\sin\left(\frac{60°}{2}\right)} = \frac{\sin 50°}{\sin 30°}$$

$$\Rightarrow \mu' = 1.532$$

Hence, the refractive index of the prism is 1.532.

Since the prism is placed in water, let δ'_m be the new angle of minimum deviation for the same prism.

The below figure shows the angle of prism and the unknown minimum deviation angle.



The refractive index of glass with respect to water is given by the relation:

$$\mu_g^w = \frac{\mu'}{\mu} = \frac{\sin\left(\frac{A+\delta'_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$\Rightarrow \frac{\mu'}{\mu}\sin\left(\frac{A}{2}\right) = \sin\left(\frac{A+\delta'_m}{2}\right)$$

$$\Rightarrow \sin\left(\frac{A+\delta'_m}{2}\right) = \frac{1.532}{1.33}\sin\left(\frac{60°}{2}\right) = 0.5759$$

$$\Rightarrow \frac{A+\delta'_m}{2} = \sin^{-1}(0.5759) = 35.16°$$

$$\Rightarrow 60° + \delta'_m = 70.32°$$

$$\Rightarrow \delta'_m = 10.32°$$

Hence, the new minimum angle of deviation is $10.32°$.

7. Double-convex lenses are to be manufactured from a glass of refractive index 1.55, with both faces of the same radius of curvature. What is the radius of curvature required if the focal length is to be 20 cm?

Ans: Given that,

Refractive index of glass, $\mu = 1.55$

Focal length of the double-convex lens, $f = 20$ cm

Radius of curvature of one face of the lens $= R_1$

Radius of curvature of the other face of the lens $= R_2$

Radius of curvature of the double-convex lens $= R$

$\therefore R_1 = R;\ R_2 = -R$

We can use this formula:

$$\frac{1}{f}=(\mu-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$$

$$\Rightarrow \frac{1}{20}=(1.55-1)\left[\frac{1}{R}+\frac{1}{R}\right]$$

$$\Rightarrow \frac{1}{20}=(0.55)\left[\frac{2}{R}\right]$$

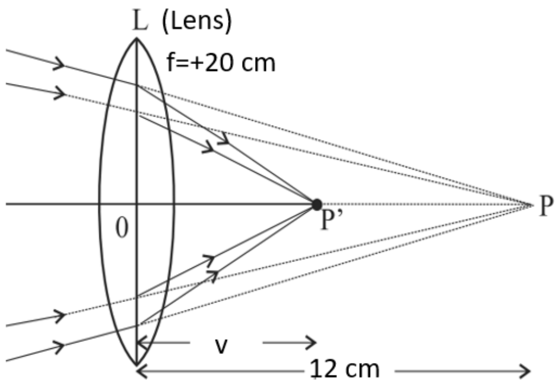
$$\therefore R=22\text{cm}$$

Hence, the radius of curvature of the double-convex lens is 22cm.

8. A beam of light converges at a point P. Now a lens is placed in the path of the convergent beam 12 cm from P. At what point does the beam converge if the lens is

a) a convex lens of focal length 20cm?

Ans: Consider the given setup of a convex lens of focal length 20cm.



Here,

Object distance, $u=+12\text{ cm}$

Focal length of the convex lens, $f=20\text{ cm}$

Image distance $=v$

Using lens formula,

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\Rightarrow \frac{1}{v}-\frac{1}{12}=\frac{1}{20}$$

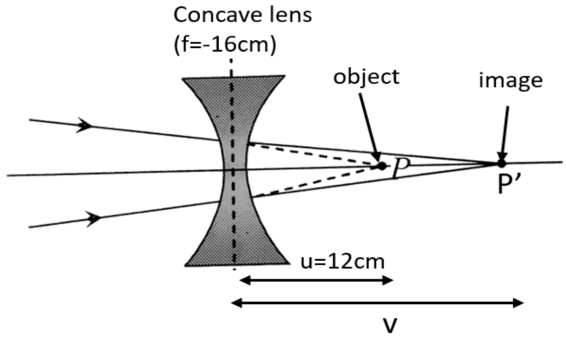
$$\Rightarrow \frac{1}{v}=\frac{1}{20}+\frac{1}{12}=\frac{8}{60}$$

$$\Rightarrow v=\frac{60}{8}=7.5\text{cm}$$

Clearly, the image is formed 7.5cm away from the lens, toward its right.

b) a concave lens of focal length 16cm?

Ans: Consider the given setup of a concave lens if focal length 16cm.



Here,

Focal length of the concave lens, $f = -16\text{cm}$

Image distance $= v$

Using lens formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{v} = -\frac{1}{16} + \frac{1}{12}$$

$$\Rightarrow \frac{1}{v} = \frac{-3+4}{48}$$

$$\Rightarrow \frac{1}{v} = \frac{1}{48}$$

$\therefore v = 48\text{cm}$

Clearly, the image is formed 48cm away from the lens, toward its right.

9. An object of size 3.0 cm is placed 14 cm in front of a concave lens of focal length 21cm. Describe the image produced by the lens. What happens if the object is moved further away from the lens?

Ans: Given that,

Size of the object, $h_1 = 3$ cm

Object distance, $u = -14$ cm

Focal length of the concave lens, $f = -21$ cm

Image distance $= v$

Using lens formula,

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\Rightarrow \frac{1}{v}=-\frac{1}{21}-\frac{1}{14}$$

$$\Rightarrow \frac{1}{v}=\frac{-2-3}{42}$$

$$\Rightarrow \frac{1}{v}=\frac{-5}{42}$$

$$\therefore v=\frac{-42}{5}=-8.4\text{cm}$$

Hence, the image is formed on the same side of the lens as the object, 8.4 cm away from it. The negative sign indicates that the image is erect and virtual.

The formula for magnification of the image is given as:

$$m=\frac{h_2}{h_1}=\frac{v}{u}$$

$$\therefore h_2=\frac{-8.4}{-14}\times 3=1.8\text{cm}$$

$$\Rightarrow h_2=1.8\text{cm}$$

Hence, the height of the image is 1.8 cm.

If the object is moved further away from the lens, then the virtual image will move towards the lens focus, but not beyond it. The image size will decrease with the increase in the distance of the object.

10. What is the focal length of a convex lens of focal length 30 cm in contact with a concave lens of focal length 20 cm? Is the system a converging or a diverging lens? Ignore thickness of the lenses.

Ans: Given that,

Focal length of the convex lens, $f_1=30$ cm

Focal length of the concave lens, $f_2=-20$ cm

Focal length of the system of lenses $=$ f

The equivalent focal length of two lenses system in contact is given by:

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}$$

$$\Rightarrow \frac{1}{f}=\frac{1}{30}-\frac{1}{20}=\frac{-1}{60}$$

$$\Rightarrow f=-60\text{cm}$$

Hence, the focal length of the combination of lenses is 60 cm. The negative sign shows that the system of lenses acts as a diverging lens.

11. A compound microscope consists of an objective lens of focal length 2.0 cm and an eyepiece of focal length 6.25 cm separated by a distance of 15 cm. How far from the objective should an object be placed in order to obtain the final image at:

a) the least distance of distinct vision (25 cm)? What is the magnifying power of the microscope?

Ans: Given that,

Focal length of the objective lens, $f_1 = 2.0\text{ cm}$

Focal length of the eyepiece, $f_2 = 6.25\text{ cm}$

Distance between the objective lens and the eyepiece, $d = 15\text{ cm}$

Least distance of distinct vision, $d' = 25\text{ cm}$

lmage distance for the eyepiece, $v_2 = -25\text{ cm}$

Object distance for the eyepiece $= u_2$

Using lens formula,

$$\frac{1}{v_2} - \frac{1}{u_2} = \frac{1}{f_2}$$

$$\Rightarrow \frac{1}{u_2} = \frac{1}{v_2} - \frac{1}{f_2}$$

$$\Rightarrow \frac{1}{u_2} = \frac{1}{-25} - \frac{1}{6.25}$$

$$\Rightarrow \frac{1}{u_2} = \frac{-5}{25}$$

$$\Rightarrow u_2 = -5\text{cm}$$

The distance of image for the objective lens, $v_1 = d + u_2 = 15 - 5 = 10\text{cm}$.

The distance of object for the objective lens $= u_1$

Using lens formula,

$$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f_1}$$

$$\Rightarrow \frac{1}{u_1} = \frac{1}{v_1} - \frac{1}{f_1}$$

$$\Rightarrow \frac{1}{u_1} = \frac{1}{10} - \frac{1}{2}$$

$$\Rightarrow \frac{1}{u_1} = \frac{-4}{10}$$

$$\Rightarrow u_1 = -2.5\text{cm}$$

Magnitude of the object distance, $|u_1| = 2.5\text{cm}$.

The compound microscope's magnifying power is given by the relation:

$$m = \frac{v_1}{|u_1|}\left(1 + \frac{d'}{f_2}\right)$$

$$\Rightarrow m = \frac{10}{2.5}\left(1 + \frac{25}{6.25}\right)$$

$$\Rightarrow m = 4(1+4) = 20$$

Hence, the magnifying power of the microscope is 20.

b) at infinity? What is the magnifying power of the microscope?

Ans: Given that, the final image is formed at infinity.

The distance of image of the eyepiece, $v_2 = \infty$

The distance of object of the eyepiece $= u_2$

Using lens formula,

$$\frac{1}{v_2} - \frac{1}{u_2} = \frac{1}{f_2}$$

$$\Rightarrow \frac{1}{u_2} = \frac{1}{v_2} - \frac{1}{f_2}$$

$$\Rightarrow \frac{1}{u_2} = \frac{1}{\infty} - \frac{1}{6.25}$$

$$\Rightarrow u_2 = -6.25\text{cm}$$

The distance of image for the objective lens, $v_1 = d + u_2 = 15 - 6.25 = 8.75\text{cm}$.

The distance of object for the objective lens $= u_1$

Using lens formula,

$$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f_1}$$

$$\Rightarrow \frac{1}{u_1} = \frac{1}{v_1} - \frac{1}{f_1}$$

$$\Rightarrow \frac{1}{u_1} = \frac{1}{8.75} - \frac{1}{2}$$

$$\Rightarrow u_1 = -2.59\text{cm}$$

Magnitude of the object distance, $|u_1| = 2.59\text{cm}$.

The compound microscope's magnifying power is given by the relation:

$$m = \frac{v_1}{|u_1|}\left(\frac{d'}{|u_2|}\right)$$

$$\Rightarrow m = \frac{8.75}{2.59}\left(\frac{25}{6.25}\right)$$

$$\Rightarrow m = 13.51$$

Hence, the magnifying power of the microscope is 13.51.

12. A person with a normal near point (25 cm) using a compound microscope with objective of focal length 8.0 mm and an eyepiece of focal length 2.5 cm can bring an object placed at 9.0 mm from the objective in sharp focus. What is the separation between the two lenses? Calculate the magnifying power of the microscope.

Ans: Given that,

Focal length of the objective lens, $f_o = 8\text{ mm} = 0.8\text{ cm}$

Focal length of the eyepiece, $f_e = 2.5\text{ cm}$

The distance of the object for the Objective lens, $u_o = -9.0\text{ mm} = -0.9\text{ cm}$

Least distance of distant vision, $d = 25\text{ cm}$

Image distance for the eyepiece, $v_e = -d = -25\text{ cm}$

Object distance for the eyepiece, u_e

Using lens formula,

$$\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e}$$

$$\Rightarrow \frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e}$$

$$\Rightarrow \frac{1}{u_e} = \frac{1}{-25} - \frac{1}{2.5}$$

$$\Rightarrow \frac{1}{u_e} = \frac{-11}{25}$$

$\Rightarrow u_e = -2.27\text{cm}$

Using lens formula, we can obtain image distance for the objective lens, v_o, is given by:

$$\frac{1}{v_o} - \frac{1}{u_o} = \frac{1}{f_o}$$

$$\Rightarrow \frac{1}{v_o} = \frac{1}{u_o} + \frac{1}{f_o}$$

$$\Rightarrow \frac{1}{v_o} = \frac{1}{0.8} - \frac{1}{0.9}$$

$$\Rightarrow \frac{1}{v_o} = \frac{0.1}{0.72}$$

$\Rightarrow v_o = 7.2\text{cm}$

The distance between the objective lens and the eyepiece, $|u_e| + v_o = 2.27 + 7.2 = 9.47\text{cm}$

The microscope's magnifying power is given by the relation:

$$m = \frac{v_o}{|u_o|}\left(1 + \frac{d}{f_e}\right)$$

$$\Rightarrow m = \frac{7.2}{0.9}\left(1 + \frac{25}{2.5}\right)$$

$\Rightarrow m = 8(1+10) = 88$

Hence, the magnifying power of the microscope is 88.

13. A small telescope has an objective lens of focal length 144 cm and an eyepiece of focal length 6.0 cm. What is the magnifying power of the telescope? What is the separation between the objective and the eyepiece?

Ans: Given that,

Focal length of the objective lens, $f_o = 144\text{ cm}$

Focal length of the eyepiece, $f_e = 6.0\text{ cm}$

The telescope's magnifying power is given by:

$$m = \frac{f_o}{f_e}$$

$$\Rightarrow m = \frac{144}{6}$$

$\Rightarrow m = 24$

The separation between the objective lens and the eyepiece is given by:

$x = f_o + f_e$

$\Rightarrow x = 144 + 6 = 150cm$

Hence, the magnifying power of the telescope is 24 and the distance between the objective lens and the eyepieces is $150cm$.

14. a) A giant refracting telescope at an observatory has an objective lens of focal length $15\ m$. If an eyepiece of focal length $1.0\ cm$ is used, what is the angular magnification of the telescope?

Ans: Given that,

Focal length of the objective lens, $f_o = 15m = 15 \times 100 = 1500cm$

Focal length of the eyepiece, $f_e = 1.0\ cm$

The telescope's angular magnification is given as:

$$\alpha = \frac{f_o}{f_e}$$

$$\Rightarrow \alpha = \frac{1500}{1} = 1500$$

Hence, the refracting telescope's angular magnification is 1500.

b) If this telescope is used to view the moon, what is the diameter of the image of the moon formed by the objective lens? The diameter of the moon is $3.48 \times 10^6\ m$, and the radius of lunar orbit is $3.8 \times 10^8\ m$.

Ans: Given that,

Diameter of the moon, $d_o = 3.48 \times 10^6 m$

Radius of the lunar orbit, $r_o = 3.8 \times 10^8 m$

Let d' be the diameter of the moon image formed by the objective lens.

The angle subtended by the moon's diameter is equal to the angle subtended by the image.

$$\frac{d_o}{r_o} = \frac{d'}{f_o}$$

$$\Rightarrow \frac{3.48 \times 10^6}{3.8 \times 10^8} = \frac{d'}{15}$$

$$\Rightarrow d' = 13.74 \times 10^{-2} m = 13.74cm$$

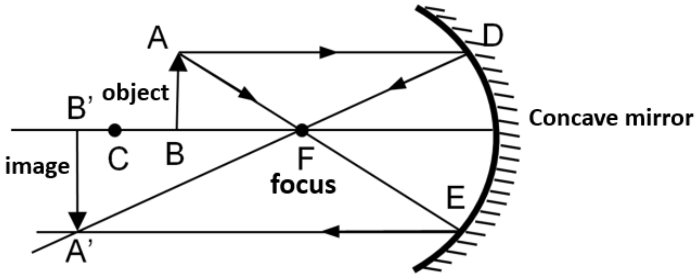
Hence, the diameter of the moon's image formed by the objective lens is $13.74\ cm$.

15. Use the mirror equation to deduce that:

a) an object placed between f and $2f$ of a concave mirror produces a real image beyond $2f$.

Ans: For a concave mirror, $f<0$.

When the object is placed on the left side of the mirror, then $u<0$.



Using mirror formula,

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$$

$$\Rightarrow \frac{1}{v}=\frac{1}{f}-\frac{1}{u} \ldots\ldots(1)$$

If the object placed between f and $2f$ i.e., $2f<u<f$:

$$\Rightarrow \frac{1}{2f}>\frac{1}{u}>\frac{1}{f}$$

$$\Rightarrow -\frac{1}{2f}<-\frac{1}{u}<-\frac{1}{f}$$

$$\Rightarrow \frac{1}{f}-\frac{1}{2f}<\frac{1}{f}-\frac{1}{u}<0 \ldots\ldots(2)$$

Using equation (1), we get

$$\Rightarrow \frac{1}{2f}<\frac{1}{v}<0$$

Since $\frac{1}{v}$ is negative, v is negative.

$$\Rightarrow \frac{1}{2f}<\frac{1}{v}$$

$$\Rightarrow 2f>v$$

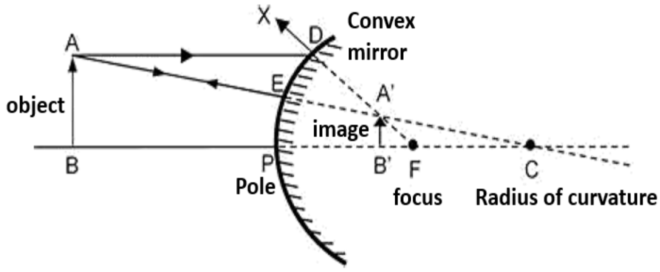
$$\Rightarrow -v>-2f$$

Therefore, image will lie beyond $2f$.

b) a convex mirror always produces a virtual image independent of the location of the object.

Ans: For a convex mirror, $f > 0$.

When the object is placed on the left side of the mirror, then $u < 0$.



Using mirror formula,

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

Using equation (2), we get:

$$\frac{1}{v} < 0$$

$$\Rightarrow v > 0$$

Thus, the image is formed on the mirror's back side.

Hence, a convex mirror always gives a virtual image, regardless of the object distance.

c) the virtual image produced by a convex mirror is always diminished in size and is located between the focus and the pole.

Ans: For a convex mirror, $f > 0$.

Using mirror formula,

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

But we have, $u < 0$

$$\therefore \frac{1}{v} > \frac{1}{f}$$

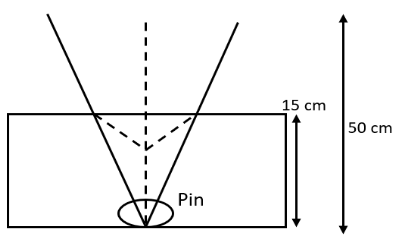
$\Rightarrow v < f$
Hence, the image formed is diminished and is located between the focus and the pole.

d) An object placed between the pole and focus of a concave mirror produces a virtual and enlarged image.

Ans: For a concave mirror, $f < 0$.
When the object is placed on the left side of the mirror, then $u < 0$.
The object is placed between the focus and the pole.
$\therefore f > u > 0$
$$\Rightarrow \frac{1}{f} < \frac{1}{u} < 0$$
$$\Rightarrow \frac{1}{f} - \frac{1}{u} < 0$$
$$\therefore \frac{1}{v} < 0$$
$\Rightarrow v > 0$
The image is formed on the mirror's right side. Hence, it is a virtual image.
For $u < 0, v > 0$
$$\Rightarrow \frac{1}{u} > \frac{1}{v}$$
$\Rightarrow v > u$
Magnification, $m = \frac{v}{u} > 1$
Hence, the formed image is enlarged.

16. A small pin fixed on a table top is viewed from above from a distance of 50 cm. By what distance would the pin appear to be raised if it is viewed from the same point through a 15 cm thick glass slab held parallel to the table? Refractive index of glass 1.5. Does the answer depend on the location of the slab?

Ans: According to question,



Actual depth of the pin, $d = 15$ cm
Apparent depth of the pin $= d'$
Refractive index of glass, $\mu = 1.5$
The refractive index of glass is equal to the ratio of actual depth to the apparent depth, that is,

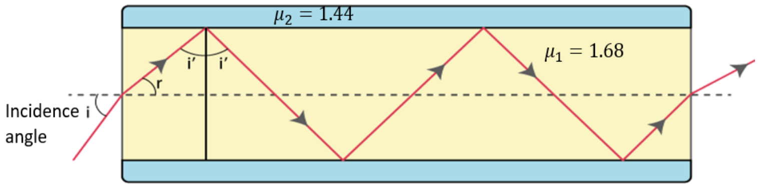
$$\mu = \frac{d}{d'}$$

$$\therefore d' = \frac{d}{\mu}$$

$$\Rightarrow d' = \frac{15}{1.5} = 10\text{cm}$$

The distance at which the pin appears to be raised $= d' - d = 15 - 10 = 5\text{cm}$.
For a small incidence angle, this distance does not depend upon the slab location.

17. **a) Figure shows a cross-section of a 'light pipe' made of a glass fibre of refractive index 1.68. The outer covering of the pipe is made of a material of refractive index 1.44. What is the range of the angles of the incident rays with the axis of the pipe for which total reflections inside the pipe take place, as shown in the figure?**



Ans: Given that,
Refractive index of the glass fibre, $\mu_1 = 1.68$
Refractive index of the outer covering of the pipe, $\mu_2 = 1.44$
Angle of incidence $=$ i
Angle Of refraction $=$ r
Angle of incidence at the interface $=$ i'
The refractive index (μ) of the inner core - outer core interface is given as:

$$\mu = \frac{\mu_2}{\mu_1} = \frac{1}{\sin i'}$$

$$\Rightarrow \sin i' = \frac{\mu_1}{\mu_2}$$

$$\Rightarrow \sin i' = \frac{1.44}{1.68} = 0.8571$$

For the critical angle, total internal reflection (TIR) takes place only when $i > i'$
That is, when $i > 59°$.

Maximum angle of reflection, $r_{max} = 90° - i' = 90° - 59° = 31°$.

Let i_{max} be the maximum incidence angle.

The refractive index at the air – glass interface, $\mu_1 = 1.68$.

We can use the relation for the maximum angles of incidence and reflection as:

$$\mu_1 = \frac{\sin i_{max}}{\sin r_{max}}$$

$$\Rightarrow \sin i_{max} = \mu_1 \times \sin r_{max}$$

$$\Rightarrow \sin i_{max} = 1.68 \times \sin 31°$$

$$\Rightarrow \sin i_{max} = 0.8652$$

$$\Rightarrow i_{max} \approx 60°$$

Thus, all the ray's incident at angles lying in the range $0 < i < 60°$ will suffer total internal reflection.

b) What is the answer if there is no outer covering of the pipe?

Ans: If the outer covering of the pipe is not present, then:

Refractive index of the outer pipe $= \mu_1$

Refractive index of air $= 1$

For the angle of incidence $i = 90°$, we can use Snell's law at the air – pipe interface as

$$\mu_2 = \frac{\sin i}{\sin r}$$

$$\Rightarrow 1.68 = \frac{\sin 90°}{\sin r}$$

$$\Rightarrow \sin r = \frac{1}{1.68}$$

$$\Rightarrow r = 36.5°$$

$$\therefore i' = 90° - 36.5° = 53.5°$$

Since, $i' > r$, all incident rays will suffer total internal reflection.

18. The image of a small electric bulb fixed on the wall of a room is to be obtained on the opposite wall 3 m away by means of a large convex lens. What is the maximum possible focal length of the lens required for the purpose?

Ans: Given that,

Distance between the object and the image, $d = 3\text{ m}$

Maximum focal length of the convex lens $= f_{max}$

The maximum focal length is given by for real image:

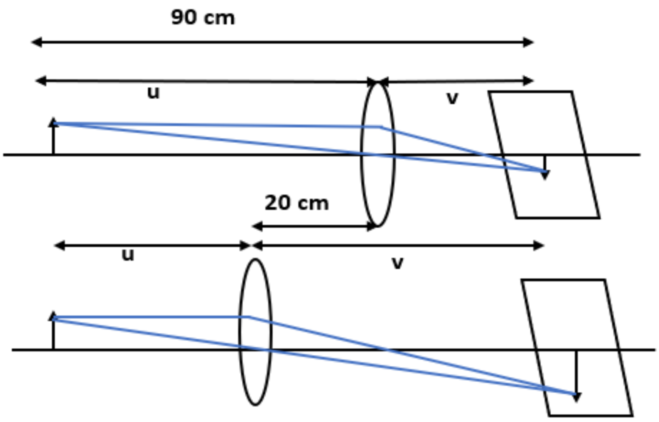
$$f_{max} = \frac{d}{4}$$

$$\Rightarrow f_{max} = \frac{3}{4} = 0.75\text{m}$$

Hence, for the required purpose, the maximum possible focal length of the convex lens is 0.75 m.

19. A screen is placed 90 cm from an object. The image of the object on the screen is formed by a convex lens at two different locations separated by 20 cm. Determine the focal length of the lens.

Ans: The figure shows the given arrangement of convex lens.



Here,

Distance between the image (screen) and the object, $D = 90\text{ cm}$.

Distance between two locations of the convex lens, $d = 20\text{ cm}$.

Focal length of the lens $= f$

Focal length is related to d and D by:

$$f = \frac{D^2 - d^2}{4D}$$

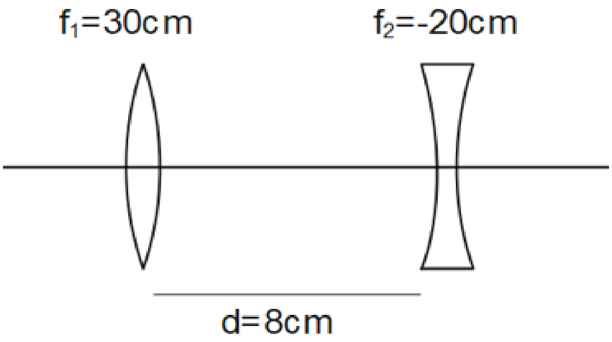
$$\Rightarrow f = \frac{90^2 - 20^2}{4 \times 90} = \frac{770}{36}$$

$$\Rightarrow f = 21.39 cm$$

Clearly, the focal length of the convex lens is $21.39 cm$.

20. **a) Determine the 'effective focal length' of the combination of the two lenses in Exercise 9.10, if they are placed 8.0 cm apart with their principal axes coincident. Does the answer depend on which side of the combination a beam of parallel light is incident? Is the notion of effective focal length of this system useful at all?**

Ans: Consider the diagram below which represents the combination of two lenses.



Here,

Focal length of the convex lens, $f_1 = 30$ cm

Focal length of the concave lens, $f_2 = -20$ cm

Distance between the two lenses, $d = 8.0$ cm

First, consider the case when the parallel beam of light falls on the convex lens.

Using lens formula,

$$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f_1}$$

Where, object distance, $u_1 = \infty$

Image distance $= v_1$

$$\Rightarrow \frac{1}{v_1} = \frac{1}{30} - \frac{1}{\infty} = \frac{1}{30}$$

$$\Rightarrow v_1 = 30 cm$$

The image will serve as a virtual object for the concave lens.

Using lens formula,

$$\frac{1}{v_2}-\frac{1}{u_2}=\frac{1}{f_2}$$

Where, object distance $=u_2$.

$u_2=30-8=22\text{cm}$

Image distance $=v_2$.

$$\Rightarrow \frac{1}{v_2}=\frac{1}{22}-\frac{1}{20}=\frac{-1}{220}$$

$\Rightarrow v_2=-220\text{cm}$

The parallel incident beam seems to diverge from a point, that is, $220-\frac{d}{2}=220-4=216\text{cm}$ from the centre of the combination of the two lenses.

Secondly, when the parallel beam of light falls, from the left, on the concave lens;

Using lens formula,

$$\frac{1}{v_2}-\frac{1}{u_2}=\frac{1}{f_2}$$

Where, object distance, $u_2=-\infty$.

Image distance $=v_2$.

$$\Rightarrow \frac{1}{v_2}=\frac{1}{-20}-\frac{1}{\infty}=\frac{1}{-20}$$

$\Rightarrow v_2=-20\text{cm}$

The image will serve as a real object for the convex lens.

Using lens formula,

$$\frac{1}{v_1}-\frac{1}{u_1}=\frac{1}{f_1}$$

Where, object distance, $=u_1$.

$u_1=-(20+8)=-28\text{cm}$.

Image distance $=v_1$.

$$\Rightarrow \frac{1}{v_1}=\frac{1}{30}+\frac{1}{-28}=\frac{-1}{420}$$

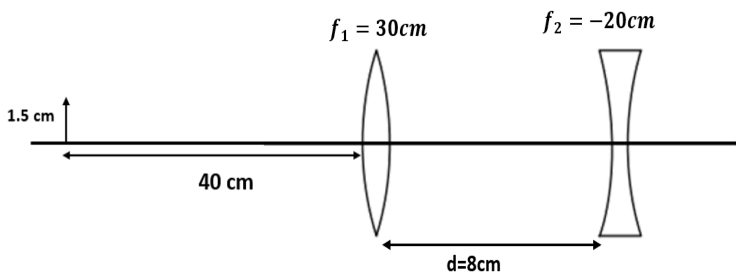
$\Rightarrow v_1=-420\text{cm}$

Hence, the parallel incident beam seems to diverge from a point, that is $=420-4=416\text{cm}$ from the left of the centre of the combination of the two lenses.

Thus, the answer does depend on the combination side at which the parallel beam of light is incident. The notion of effective focal length does not appear to be useful for this combination.

b)An object 1.5 cm in size is placed on the side of the convex lens in the arrangement (a) above. The distance between the object and the convex lens is 40cm. Determine the magnification produced by the two-lens system, and the size of the image.

Ans: Consider the given diagram of the previous arrangement as follows:



Here, it is said that,

Height of the image, $h_1 = 1.5\text{ cm}$

Object distance from the side of the convex lens, $u_1 = -40\text{cm}$

$|u_1| = 40\text{cm}$

Using lens formula,

$$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f_1}$$

$$\Rightarrow \frac{1}{v_1} = \frac{1}{30} + \frac{1}{-40} = \frac{1}{120}$$

$$\Rightarrow v_1 = 120\text{cm}$$

Magnification, $m = \frac{v_1}{|u_1|}$

$$\Rightarrow m = \frac{120}{40} = 3$$

Hence, the magnification due to the convex lens is 3.

The image made by the convex lens acts as an object for the concave lens.

Using lens formula,

$$\frac{1}{v_2} - \frac{1}{u_2} = \frac{1}{f_2}$$

Where, object distance $= u_2$.

$u_2 = 120 - 8 = 112\text{cm}$.

Image distance $= v_2$.

$$\Rightarrow \frac{1}{v_2} = \frac{1}{-20} + \frac{1}{112} = \frac{-92}{2240}$$

$$\Rightarrow v_2 = -\frac{2240}{92}\text{cm}$$

Magnification, $m' = \frac{|v_1|}{|u_1|}$

$$\Rightarrow m' = \frac{2240}{92} \times \frac{1}{112} = \frac{20}{92}$$

Hence, the magnification due to the concave lens is $\frac{20}{92}$.

The magnification due to the combination of the two lenses is calculated as:

$$m \times m' = 3 \times \frac{20}{92} = 0.652$$

Thus,

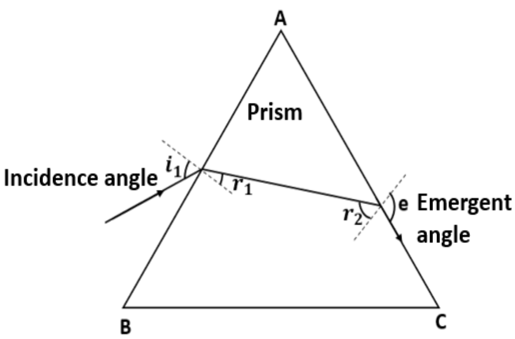
$$\frac{h_2}{h_1} = 0.652$$

$$\Rightarrow h_2 = 0.652 \times 1.5 = 0.98\text{cm}$$

Clearly, the height of the image is 0.98cm.

21. At what angle should a ray of light be incident on the face of a prism of refracting angle $60°$ so that it just suffers total internal reflection at the other face? The refractive index of the material of the prism is 1.524.

Ans: Consider the given figure below:



Angle of prism, $A = 60°$

i_1 is the incidence angle.
r_1 is the refracted angle.
r_2 is the angle of incidence at the face AC.
e is the emergent angle, $e = 90°$.
Using Snell's law,

$$\mu = \frac{\sin e}{\sin r_2}$$

$$\Rightarrow 1.524 = \frac{\sin 90°}{\sin r_2}$$

$$\Rightarrow \sin r_2 = 0.6562$$

$$\Rightarrow r_2 \approx 41°$$

For refraction through prism, angle $A = r_1 + r_2$.
We get,

$$r_1 = A - r_2 = 60° - 41°$$

$$\therefore r_1 = 19°$$

Using Snell's law,

$$\mu = \frac{\sin i_1}{\sin r_1}$$

$$\Rightarrow 1.524 = \frac{\sin i_1}{\sin 19°}$$

$$\Rightarrow \sin i_1 = 0.496$$

$$\Rightarrow i_1 = 29.75°$$

Hence, the incidence angle is $29.75°$.

22. **A card sheet divided into squares each of size $1\ \mathrm{mm}^2$ is being viewed at a distance of $9\ \mathrm{cm}$ through a magnifying glass (a converging lens of focal length $9\ \mathrm{cm}$) held close to the eye.**

a)What is the magnification produced by the lens? How much is the area of each square in the virtual image?

Ans: Given that,

Area of each square, $A = \mathbf{1\ mm}^2$

Object distance, $u = -9\mathrm{cm}$

Focal length, $f = 10\mathrm{cm}$

Using lens formula,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\Rightarrow \frac{1}{10} = \frac{1}{v} - \frac{1}{-9}$$

$$\Rightarrow v = -90\mathrm{cm}$$

Magnification, $m = \frac{v}{u}$

$$\Rightarrow m = \frac{-90}{-10} = 9$$

Area of each square in the virtual image $= 10^2 A$

$10^2 \times 1 = 100\mathrm{mm}^2 = 1\mathrm{cm}^2$

b). What is the angular magnification (magnifying power) of the lens?

Ans: Magnifying power of the lens, $m = \frac{d}{|u|}$

$$\Rightarrow m = \frac{25}{9} = 2.8$$

c) Is the magnification in (a) equal to the magnifying power in (b)? Explain.

Ans: The magnification in **(a)** is not the same as the magnifying power in **(b)**.

The magnification magnitude is $\left|\frac{v}{u}\right|$ and the magnifying power is $\frac{d}{|u|}$.

The two quantities will be same when the image is formed at the close point $(25\mathrm{cm})$.

23. **a) At what distance should the lens be held from the figure in Exercise 9.29 in order to view the squares distinctly with the maximum possible magnifying power?**

Ans: The maximum possible magnification got when the image is made at the near point $(d = 25\text{ cm})$.

Image distance, $v = -d = -25\text{ cm}$

Focal length, $f = 10\text{ cm}$

Object distance $= u$

Using lens formula,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\Rightarrow \frac{1}{10} = \frac{1}{-25} - \frac{1}{u}$$

$$\Rightarrow u = \frac{-50}{7}\text{cm} = -7.14\text{cm}$$

Hence, to view the squares distinctly, the lens should be kept 7.14cm away from them.

b) What is the magnification in this case?

Ans: Magnification, $m = \left|\frac{v}{u}\right|$

$$\Rightarrow m = \frac{25}{\frac{50}{7}} = 3.5$$

c) Is the magnification equal to the magnifying power in this case? Explain.

Ans: Magnifying power, $m' = \frac{d}{u}$

$$\Rightarrow m' = \frac{25}{\frac{50}{7}} = 3.5 = m$$

Since the image is made at the near point $(\mathbf{25\ cm})$, the magnifying power is equal to the magnification magnitude.

24. What should be the distance between the Object in Exercise 9.30 and the magnifying glass if the virtual image of each square in the figure is to have an area of $6.25\ mm^2$. Would you be able to see the squares distinctly with your eyes very close to the magnifier?

Ans: Given that,

Area of the virtual image of each square, $A = 6.25\ mm^2$

Area of each square, $A_o = 1\ mm^2$

Hence, the linear magnification is given by:

$$m = \sqrt{\frac{A}{A_o}}$$

$$m = \sqrt{\frac{6.25}{1}} = 2.5$$

But,

$$m = \frac{v}{u}$$

$$\Rightarrow v = 2.5u$$

Focal length of the magnifying glass, $f = 10cm$

Using lens formula,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\Rightarrow \frac{1}{10} = \frac{1}{2.5u} - \frac{1}{u}$$

$$\Rightarrow u = \frac{-1.5 \times 10}{2.5} cm = -6cm$$

And,

$$v = 2.5u$$

$$\Rightarrow v = 2.5 \times -6 = -15cm$$

The virtual image is made at a distance of $15\ cm$, which is less than the near point of a normal eye. Hence it cannot be visible by the eye distinctly.

25. a) The angle subtended at the eye by an object is equal to the angle subtended at the eye by the virtual image produced by a magnifying glass. In what sense then does a magnifying glass provide angular magnification?

Ans: Though the image size is larger than the object, the angular size of the image is equivalent to the angular size of the object. A magnifying glass supports one seeing the objects closer than the least distance of distinct vision. A closer object produces a larger angular size. A magnifying glass gives angular magnification. Without magnification, the object cannot be located closer to the eye. With magnification, the object can be set much closer to the eye.

b) In viewing through a magnifying glass, one usually positions one's eyes very close to the lens. Does angular magnification change if the eye is moved back?

Ans: Yes, the angular magnification varies. When the length between the eye and a magnifying glass rises, the angular magnification reduces slightly because the subtended angle at the eye is imperceptibly less than the lenses. Image distance does not have any impact on angular magnification.

c) Magnifying power of a simple microscope is inversely proportional to the focal length of the lens. What then stops us from using a convex lens of smaller and smaller focal length and achieving greater and greater magnifying power?

Ans: The convex lens's focal length cannot be reduced by a more significant amount. This is because making lenses having tiny focal lengths is not easy. Spherical and chromatic aberrations are created by a convex lens having a petite focal length.

d) Why must both the objective and the eyepiece of a compound microscope have short focal lengths?

Ans: The angular magnification produced by the compound microscope's eyepiece is:

$$\left[\left(\frac{25}{f_e}\right)+1\right]$$

Where, f_e is the eyepiece's focal length.

It can be seen that if f_e is small, then angular magnification of the eyepiece will be great.

The angular magnification of the compound microscope's objective lens is given as $\frac{f_o}{u_o}$.

u_o: Object distance for the objective lens

f_o: Objective's focal length

The magnification is large when $u_o > f_o$.

In the case of a microscope, the object is placed close to the objective lens. Hence, the object distance is tiny. Since u_o is small, f_o will be smaller. Therefore, f_e and f_o are both small for the given condition.

e) When viewing through a compound microscope, our eyes should be positioned not on the eyepiece but a short distance away from it for best viewing. Why? How much should be that short distance between the eye and eyepiece?

Ans: We cannot collect much-refracted light when we put our eyes too close to the compound microscope's eyepiece. As a result, the field view reduces substantially. Hence, the image's clarity gets blurred. The best view of the eye for seeing through a compound microscope is at the eye-ring connected to the eyepiece. The exact location of the eye depends on the separation between the objective lens and the eyepiece.

26. An angular magnification (magnifying power) of 30X is desired using an objective of focal length 1.25 cm and an eyepiece of focal length 5 cm. How will you set up the compound microscope?

Ans: Given that,

Focal length of the objective lens, $f_o = 1.25\text{cm}$

Focal length of the eyepiece, $f_e = 5\text{ cm}$

Least distance of distinct vision, $d = 25\text{ cm}$

Angular magnification of the compound microscope $= 30X$

Total magnifying power of the compound microscope, $m = 30$

The eyepiece's angular magnification is given by:

$$m_e = 1 + \frac{d}{f_e}$$

$$\Rightarrow m_e = 1 + \frac{25}{5}$$

$$\Rightarrow m_e = 6$$

The objective lens angular magnification is given by:

$$m = m_o m_e$$

$$\Rightarrow m_o = \frac{m}{m_e}$$

$$\Rightarrow m_o = \frac{30}{6} = 5$$

We have,

$$m_o = \frac{v_o}{-u_o}$$

$$\Rightarrow 5 = \frac{v_o}{-u_o}$$

$$\Rightarrow v_o = -5u_o \ldots\ldots(1)$$

Using lens formula,

$$\frac{1}{f_o} = \frac{1}{v_o} - \frac{1}{u_o}$$

$$\Rightarrow \frac{1}{1.25} = \frac{1}{-5u_o} - \frac{1}{u_o}$$

$$\Rightarrow u_o = -1.5\text{cm}$$

And, $v_o = -5u_o = -5 \times -1.5 = 7.5\text{cm}$

The object should be placed 1.5cm away from the objective lens to get the desired magnification.

Using lens formula,

$$\frac{1}{f_e} = \frac{1}{v_e} - \frac{1}{u_e}$$

Where, $v_e = -d = -25\text{cm}$

$$\Rightarrow \frac{1}{5} = \frac{1}{-25} - \frac{1}{u_e}$$

$$\Rightarrow u_e = -4.17\text{cm}$$

Separation between the eyepiece and the objective lens $= |u_e| + |v_o|$

$$|u_e| + |v_o| = 4.17 + 7.5 = 11.67\text{cm}$$

Therefore, the separation between the eyepiece and the objective lens should be 11.67cm.

27. A small telescope has an objective lens of focal length 140 cm and an eyepiece of focal length 5.0 cm. What is the magnifying power of the telescope for viewing distant objects when

a) the telescope is in normal adjustment (i.e., when the final image is at infinity)?

Ans: Given that,

Focal length of the objective lens, $f_o = 140\text{cm}$

Focal length of the eyepiece, $f_e = 5$ cm

Least distance of distinct vision, $d = 25$ cm

When the telescope is in normal adjustment, formula for magnifying power is given by:

$$m = \frac{f_o}{f_e}$$

$$\Rightarrow m = \frac{140}{5} = 28$$

b) the final image is formed at the least distance of distinct vision (25 cm)?

Ans: When the final image is formed at d, the formula for magnifying power of telescope is given by:

$$m = \frac{f_o}{f_e}\left(1 + \frac{f_e}{d}\right)$$

$$\Rightarrow m = \frac{140}{5}\left(1 + \frac{5}{25}\right) = 28[1 + 0.2]$$

$$\Rightarrow m = 33.6$$

28. a) For the telescope described in Exercise 9.34 (a), what is the separation between the objective lens and the eyepiece?

Ans: Given that,

Focal length of the objective lens, $f_o = 140$ cm

Focal length of the eyepiece, $f_e = 5$ cm

The separation between the objective lens and the eyepiece in normal adjustment $= f_o + f_e = (140 + 5)$ cm $= 145$cm

b) If this telescope is used to view a 100 m tall tower 3 km away, what is the height of the image of the tower formed by the objective lens?

Ans: Given that,

Height of the tower, $h_1 = 100$ m

Distance of the tower from the telescope, $u = 3$ km $= 3000$ m

The angle subtended by the tower at the telescope is given as:

$$\theta = \frac{h_1}{u}$$

$$\Rightarrow \theta = \frac{100}{3000} = \frac{1}{30} \text{rad}$$

The angle subtended by the image made by the objective lens is given by:

$$\theta = \frac{h_2}{f_o}$$

$$\Rightarrow \frac{1}{30} = \frac{h_2}{140}$$

$$\therefore h_2 = 4.7\text{cm}$$

Therefore, the objective lens forms a 4.7cm tall image of the tower.

c) What is the height of the final image of tower if it is formed at 25 cm?

Ans: Given that,

Image is formed at a distance, $d = 25\text{cm}$

The eyepiece's magnification is given by:

$$m = 1 + \frac{d}{f_e}$$

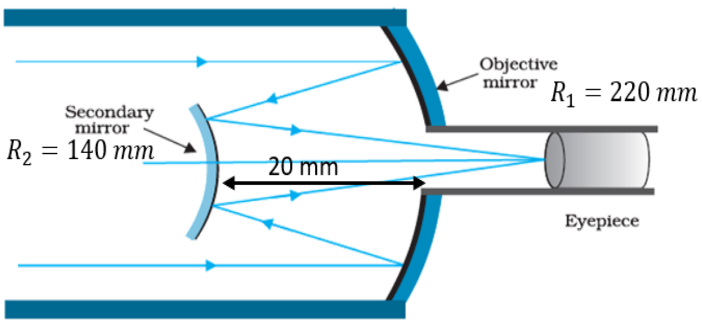
$$\Rightarrow m = 1 + \frac{25}{5}$$

$$\Rightarrow m = 6$$

Height of the final image, $= m \times h_2 = 6 \times 4.7 = 28.2\text{cm}$

Hence, the final image' height of the tower is 28.2cm.

29. A Cassegrain telescope uses two mirrors as shown in figure. Such a telescope is built with the mirrors 20 mm apart. If the radius of curvature of the large mirror is 220 mm and these all mirror is 140mm, where will the final image of an object at infinity be?



Ans: A Cassegrain telescope contains two mirrors, one is objective mirror and second one is secondary mirror. We need to find the distance of final image from secondary mirror.

Given: distance between the objective mirror and the secondary mirror, $d = 20mm$

Radius of curvature of the objective mirror, $R_1 = 220$ mm

Hence, objective mirror's focal length, $f_1 = \frac{R_1}{2} = 110mm$

Secondary mirror's radius of curvature, $R_2 = 140$ mm

Secondary mirror's focal length, $f_2 = \frac{R_2}{2} = 70mm$

The image of an object set at infinity, made by the objective mirror, will serve as a virtual object for the secondary mirror.

Hence, the secondary mirror's virtual object distance, $u = f_1 - d$

$\Rightarrow u = 110 - 20 = 90mm$

Using mirror formula,

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f_2}$$

$$\Rightarrow \frac{1}{v} = \frac{1}{f_2} - \frac{1}{u}$$

Now, we put given values,

$$\Rightarrow \frac{1}{v} = \frac{1}{70} - \frac{1}{90}$$

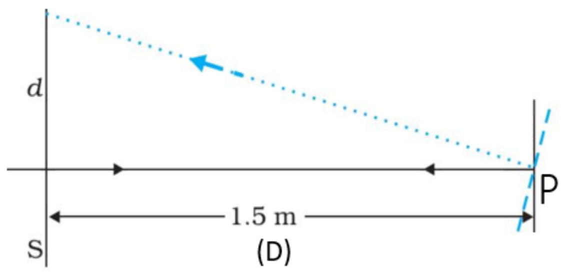
$$\Rightarrow \frac{1}{v} = \frac{9-7}{630}$$

$$\Rightarrow \frac{1}{v} = \frac{2}{630}$$

$\Rightarrow v = 315mm$

Clearly, the final image will be made 315 mm away from the secondary mirror.

30. Light incident normally on a plane mirror attached to a galvanometer coil retraces backwards as shown in figure. A current in the coil produces a deflection of $3.5°$ from the mirror. What is the displacement of the reflected spot of light on a screen placed 1.5 m away?



Ans: Angle of deflection, $\theta = \mathbf{3.5°}$

Screen distance from the mirror, $D = 1.5$ m

The reflected rays will deflect by an amount twice the deflection angle, $2\theta = 7°$

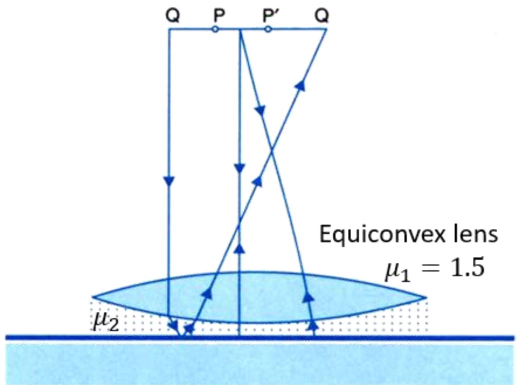
The displacement of the light's reflected spot on the screen is given as:

$$\tan 2\theta = \frac{d}{1.5}$$

$$\therefore d = 1.5 \times \tan 7° = 0.184\text{m} = 18.4\text{cm}$$

Hence, the displacement of the reflected spot of light is 18.4 cm.

31. Figure shows an equiconvex lens (of refractive index 1.50) in contact with a liquid layer on top of a plane mirror. A small needle with its tip on the principal axis is moved along the axis until its inverted image is found at the position of the needle. The distance of the needle from the lens is measured to be 45.0 cm. The liquid is removed and the experiment is repeated. The new distance is measured to be 30.0 cm. What is the refractive index of the liquid?



Ans: An equiconvex lens is in touch with a liquid layer on a plane mirror top. A small needle placed on the principal axis is moved along the axis until its inverted image will be at the position of the needle.

Convex lens focal length, $f_1 = 30$ cm

The liquid will behave as a mirror.
Liquid's focal length $= f_2$
System's focal length, $f = 45$ cm
The equivalent focal length is given as:

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\Rightarrow \frac{1}{f_2} = \frac{1}{f} - \frac{1}{f_1}$$

$$\Rightarrow \frac{1}{f_2} = \frac{1}{45} - \frac{1}{30} = -\frac{1}{90}$$

$$\therefore f_2 = -90\text{cm}$$

Let the lens's refractive index be μ_1 and curvature radius of one surface be R.
Hence, the curvature radius of the other surface is $-R$.
We use,

$$\frac{1}{f_1} = (\mu_1 - 1)\left[\frac{1}{R} - \frac{1}{-R}\right]$$

$$\Rightarrow \frac{1}{30} = (1.5 - 1)\left[\frac{2}{R}\right]$$

$$\therefore R = 30\text{cm}$$

Let μ_2 be the liquid's refractive index.
Liquid's curvature radius on the side of the plane mirror $= \infty$.
Curvature radius of the liquid on the side of the lens, $R = -30$ cm.
We use,

$$\frac{1}{f_2} = (\mu_2 - 1)\left[\frac{1}{R} - \frac{1}{\infty}\right]$$

$$\Rightarrow \frac{-1}{90} = (\mu_2 - 1)\left[\frac{1}{-30} - 0\right]$$

$$\Rightarrow (\mu_2 - 1) = \frac{1}{3}$$

$$\Rightarrow \mu_2 = 1.33$$

Clearly, the liquid's refractive index is 1.33.