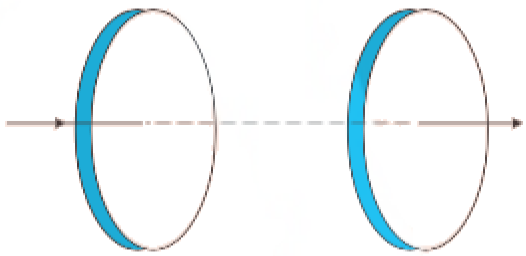
Electromagnetic Waves

Chapter 8

EXERCISE

1. Figure drawn below shows a capacitor made of two circular plates each of radius 12 cm, and separated by 5.0 cm. The capacitor is being charged by an external source (not shown in the figure). The charging current is constant and equal to 0.15 A.



(a) Calculate the capacitance and the rate of change of potential difference between the plates.

Ans: Radius of each circular plates, $r = 12\text{ cm} = 0.12\text{ m}$

Distance between the given plates, $d = 5\text{ cm} = 0.05\text{ m}$

Charging current, $I = 0.15\text{A}$

The permittivity of free space, $\varepsilon_0 = 8.85 \times 10^{-12} C^2N^{-1}m^{-2}$

Capacitance between the two plates can be given as: $C = \frac{\varepsilon_0 A}{d}$

Where, $A = \pi r^2$. Hence,

$$C = \frac{\varepsilon_0 \pi r^2}{d}$$

$$= \frac{8.85 \times 10^{-12} \times (0.12)^2}{0.05}$$

$= 8.0032 \times 10^{-12} F$

$= 80.032 pF$

Charge on each plate is given as, $q = CV$, where,

V = Potential difference across the plates

Differentiating both sides with respect to time (t), we get:

$$\frac{dq}{dt} = C\frac{dV}{dt}$$

As, $\frac{dq}{dt} = I$, so, the rate of change of potential difference between the plates can be given as:

$$\frac{dV}{dt} = \frac{I}{C} = \frac{0.15}{80.032 \times 10^{-12}}$$

$$\therefore \frac{dV}{dt} = 1.87 \times 10^{9} V / s$$

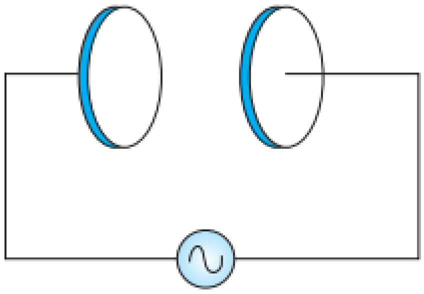
(b) Obtain the displacement current across the plates.

Ans: The displacement current across the plates would be same as the conduction current. Hence, the displacement current, I_d would be $0.15\ A$.

(c) Is Kirchhoff's first rule (junction rule) valid at each plate of the capacitor? Explain.

Ans: Yes, Kirchhoff's first rule would be valid at each plate of the capacitor, provided that sum of conduction and displacement currents, i.e., $I = I_c + I_d$ (junction rule of Kirchhoff's law).

2. A parallel plate capacitor (figure) made of circular plates each of radius $R = 6.0\ cm$ has a capacitance $C = 100\ pF$. The capacitor is connected to a $230\ V$ ac supply with a (angular) frequency of $300 rad s^{-1}$.



(a) What is the rms value of the conduction current?

Ans: Radius of each circular plate, $R = 6.0\text{ cm} = 0.06\text{ m}$

Capacitance of a parallel plate capacitor, $C = 100\text{ pF} = 100 \times 10^{-12}\text{F}$

Supply voltage, $V = 230\text{ V}$

Angular frequency, $\omega = 300\,\text{rad}\,\text{s}^{-1}$

Rms value of conduction current can be given as: $I = \dfrac{V}{X_C}$.

Where, X_C = capacitive currant $= \dfrac{1}{\omega C}$

$\therefore I = V \times \omega C$

$= 230 \times 300 \times 100 \times 10^{-12}$

$= 6.9 \times 10^{-6}$

$= 6.9\mu A$

Therefore, the rms value of conduction current will be $6.9\mu A$.

(b) Is the conduction current equal to the displacement current?

Ans: Yes, conduction current will be equal to displacement current.

(c) Determine the amplitude of B at a point 3.0 cm from the axis between the plates.

Ans: Magnetic field is given as: $B = \dfrac{\mu_0 r}{2\pi R^2} I_0$

Where,

μ_0 = Free space permeability $= 4\pi \times 10^{-7}\,NA^{-2}$

I_0 = Maximum value of current = $\sqrt{2}I$

r = Distance between the plates from the axis = $3.0cm = 0.03cm$.

$$\therefore B = \frac{4\pi \times 10^{-7} \times 0.03 \times \sqrt{2} \times 6.9 \times 10^{-6}}{2\pi \times (0.06)^2}$$

$$= 1.63 \times 10^{-11} T$$

Therefore, the magnetic field at that point will be $1.63 \times 10^{-11} T$.

3. What physical quantity is the same for X-rays of wavelength $10^{-10} m$, red light of wavelength $6800 \overset{\circ}{A}$ and radio waves of wavelength $500\ m$?

Ans: The speed of light ($3 \times 10^8 m/s$) is independent of the wavelength in the vacuum. Hence, it is same for all wavelengths in vacuum.

4. A plane electromagnetic wave travels in vacuum along z-direction. What can you say about the directions of its electric and magnetic field vectors? If the frequency of the wave is $30MHz$, what is its wavelength?

Ans: The electromagnetic wave is travelling along the z-direction, in a vacuum. The electric field (E) and the magnetic field (H) will lie in the x-y plane and they will be mutually perpendicular.

Frequency of the wave, $\nu = 30\ MHz = 30 \times 10^6 s^{-1}$

Speed of light in a vacuum, $c = 3 \times 10^8 m/s$

Wavelength, λ of the wave can be given as: $\lambda = \frac{c}{\nu}$

$$\therefore \lambda = \frac{3\times10^8}{30\times10^6} = 10\text{m}$$

5. A radio can tune in to any station in the 7.5 MHz to 12 MHz band. What is the corresponding wavelength band?

Ans: A radio can tune to minimum frequency, $\nu_1 = 7.5\text{ MHz} = 7.5\times10^6\text{ Hz}$

Maximum frequency, $\nu_2 = 12\text{ MHz} = 12\times10^6\text{ Hz}$

Speed of light, $c = 3\times10^8\text{ m/s}$

Wavelength for frequency, ν_1 can be calculated as:

$$\lambda_1 = \frac{c}{\nu_1}$$

$$= \frac{3\times10^8}{7.5\times10^6}$$

$$= 40\text{m}$$

Wavelength for frequency, ν_2 can be calculated as:

$$\lambda_2 = \frac{c}{\nu_2}$$

$$= \frac{3\times10^8}{12\times10^6}$$

$$= 25\text{m}$$

Therefore, the corresponding wavelength band would be between 40m to 25m.

6. A charged particle oscillates about its mean equilibrium position with a frequency of 10^9Hz. What is the frequency of the electromagnetic waves produced by the oscillator?

Ans: The frequency of an electromagnetic wave produced by the oscillator will be equal to the frequency of charged particle oscillating about its mean position i.e., 10^9Hz.

7. The amplitude of the magnetic field part of a harmonic electromagnetic wave in vacuum is $B_0 = 510\text{nT}$. What is the amplitude of the electric field part of the wave?

Ans: Amplitude of magnetic field of an electromagnetic wave in a vacuum, is:

$$B_0 = 510\text{nT} = 510 \times 10^{-9}\text{T}$$

Speed of light in a vacuum, $c = 3 \times 10^8 \text{m/s}$

Amplitude of electric field of the electromagnetic wave can be given as:

$$E = cB = 3 \times 10^8 \times 510 \times 10^{-9} = 153\text{N/C}.$$

8. Suppose that the electric field amplitude of an electromagnetic wave is $E_0 = 120\text{N/C}$ and that its frequency is $\nu = 50.0 \text{ MHz}$.

(a) Determine, B_0, ω, k, and λ.

Ans: Electric field amplitude, $E_0 = 120\text{N/C}$

Frequency of source, $\nu = 50.0\text{MHz} = 50 \times 10^6 \text{Hz}$

Speed of light, $c = 3 \times 10^8 \text{m/s}$

Magnetic field strength can be given as:

$$B_0 = \frac{E_0}{c}$$

$$= \frac{120}{3 \times 10^8}$$

$$= 4 \times 10^{-7}\text{T}$$

$= 400 nT$

Angular frequency of source can be given as:

$$\omega = 2\pi\nu$$

$$= 2\pi(50 \times 10^6)$$

$$= 3.14 \times 10^8 \text{ rad / s}$$

Propagation constant can be given as:

$$k = \frac{\omega}{c}$$

$$= \frac{3.14 \times 10^8}{3 \times 10^8}$$

$$= 1.05 \text{ rad / m}$$

Wavelength of wave can be given as:

$$\lambda = \frac{c}{\nu}$$

$$= \frac{3 \times 10^8}{50 \times 10^6}$$

$$= 6.0 \text{ m}$$

(b)Find expressions for E and B.

Ans: If the wave propagates in the positive x direction. Then, the electric field vector would be in the positive y direction and the magnetic field vector will lie in the positive z direction. This is because all three vectors are mutually perpendicular.

Equation of electric field vector can be given as:

$$\vec{E} = E_0 \sin(kx - \omega t)\hat{j}$$

$$= 120 \sin\left[1.05x - 3.14 \times 10^8 t\right]\hat{j}$$

And, magnetic field vector can be given as:

$$\vec{B} = B_0 \sin(kx - \omega t)\hat{k}$$

$$= (4 \times 10^{-7}) \sin\left[1.05x - 3.14 \times 10^{8} t\right]\hat{k}$$

9. The terminology of different parts of the electromagnetic spectrum is given in the text. Use the formula $E = h\nu$ (for energy of a quantum of radiation: photon) and obtain the photon energy in units of eV for different parts of the electromagnetic spectrum. In what way are the different scales of photon energies that you obtain related to the sources of electromagnetic radiation?

Ans: The energy of photon can be given as:

$$E = h\nu = \frac{hc}{\lambda}$$

Where,

h = Planck's constant $= 6.6 \times 10^{-34}$ Js

c = speed of light $= 3 \times 10^{8}$ m / s

λ = wavelength of radiation

$$\therefore E = \frac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{\lambda} J$$

$$= \frac{19.8 \times 10^{-26}}{\lambda} J$$

$$= \frac{19.8 \times 10^{-26}}{\lambda \times 1.6 \times 10^{-19}} eV$$

$$= \frac{12.375 \times 10^{-7}}{\lambda} eV$$

For different values of in an electromagnetic spectrum, photon energies are listed in below table:

λ(m)	E(eV)
10^{3}	12.375×10^{-10}
1	12.375×10^{-7}
10^{-3}	12.375×10^{-4}
10^{-6}	12.375×10^{-1}
10^{-8}	12.375×10^{1}
10^{-10}	12.375×10^{3}
10^{-12}	12.375×10^{5}

10. In a plane electromagnetic wave, the electric field oscillates sinusoidally at a frequency of 2×10^{10} Hz and amplitude $48\ \text{Vm}^{-1}$.

(a) What is the wavelength of the wave?

Ans: Frequency of the electromagnetic wave, $\nu = 2.0\times10^{10}\,\text{Hz}$

Electric field amplitude, $E_0 = 48\ \text{Vm}^{-1}$

Speed of light, $c = 3\times10^{8}\,\text{m/s}$

Wavelength of wave can be given as:

$$\lambda = \frac{c}{\nu}$$

$$= \frac{3\times10^{8}}{2\times10^{10}}$$

$$= 0.015\text{m}$$

(b) What is the amplitude of the oscillating magnetic field?

Ans: Magnetic field strength can be given as:

$$B_0 = \frac{E_0}{c}$$

$$= \frac{48}{3 \times 10^8}$$

$$= 1.6 \times 10^{-7} T$$

(c) Show that the average energy density of the E field equals the average energy density of the B field. [$c = 3 \times 10^8 m / s^{-1}$]

Ans: Energy density of the electric field is given as:

$$U_E = \frac{1}{2} \in_0 E^2$$

Energy density of the magnetic field can be given as:

$$U_B = \frac{1}{2\mu_0} B^2$$

Where,

$\in_0$ = Permittivity of free space

μ_0 = Permeability of free space

The relation between E and B can be given as:

$$E = cB \quad \ldots.(1)$$

Where,

$$c = \frac{1}{\sqrt{\in_0 \mu_0}} \quad \ldots.(2)$$

Substituting equation (2) in equation (1), we get:

$$E = \frac{1}{\sqrt{\in_0 \mu_0}} B$$

Squaring both the sides, we get:

$$E^2 = \frac{1}{\epsilon_0\, \mu_0} B^2$$

$$E^2 \epsilon_0 = \frac{B^2}{\mu_0}$$

$$\frac{1}{2} E^2 \epsilon_0 = \frac{1}{2} \frac{B^2}{\mu_0}$$

$$\Rightarrow U_E = U_B \,.$$