Alternating Current

Chapter 7

1. A 100Ω resistor is connected to a $220V$, $50Hz$ ac supply.

a) What is the rms value of current in the circuit?

Ans: It is given that,

Resistance, $R = 100\Omega$

Voltage, $V = 220V$

Frequency, $f = 50Hz$

It is known that,

$$I_{rms} = \frac{V_{rms}}{R}$$

$$\Rightarrow I_{rms} = \frac{220}{100} = 2.2A$$

Therefore, the rms value of current in the circuit is $I_{rms} = 2.2A$.

b) What is the net power consumed over a full cycle?

Ans: It is known that,

$$Power = V \times I$$

$$\Rightarrow Power = 220 \times 2.2$$

$$\Rightarrow Power = 484W$$

Therefore, the net power consumed over a full cycle is $484W$.

2.

a) The peak voltage of an ac supply is $300V$. What is the rms voltage?

Ans: It is given that,

Peak voltage of the ac supply, $V_0 = 300V$

It is known that,

$$V_{rms} = \frac{V_0}{\sqrt{2}}$$

$$\Rightarrow V_{rms} = \frac{300}{\sqrt{2}}$$

$\Rightarrow V_{rms} = 212.1V$

Therefore, the rms voltage is $212.1V$.

b) The rms value of current in an ac circuit is $10A$. What is the peak current?

Ans: It is given that,

Rms value of current in an ac circuit, $I_{rms} = 10A$

It is known that,

$I_0 = \sqrt{2} \times I_{rms}$

$\Rightarrow I_0 = 1.414 \times 10$

$\Rightarrow I_0 = 14.14A$

Therefore, the peak current is $14.14A$.

3. A $44mH$ inductor is connected to $220V, 50Hz$ ac supply. Determine the rms value of the current in the circuit.

Ans: It is known that,

Inductance, $L = 44mH = 44 \times 10^{-3}H$

Voltage, $V = 220V$

Frequency, $f_L = 50Hz$

Angular frequency, $\omega_L = 2\pi f_L$

It is known that,

Inductive reactance, $X_L = \omega_L L = 2\pi f_L L$

$\Rightarrow X_L = 2 \times 3.14 \times 50 \times 44 \times 10^{-3}\Omega$

$\Rightarrow X_L = 13.8\Omega$

$I_{rms} = \dfrac{V}{X_L}$

$\Rightarrow I_{rms} = \dfrac{220}{13.82}$

$\Rightarrow I_{rms} = 15.92A$

Therefore, the rms value of the current in the circuit is $15.92A$.

4. A $60\mu F$ capacitor is connected to a $110V, 60Hz$ ac supply. Determine the rms value of the current in the circuit.

Ans: It is given that,

Capacitance, $C = 60\mu F = 60 \times 10^{-6} F$

Voltage, $V = 110V$

Frequency, $f_C = 60Hz$

It is known that,

$$I_{rms} = \frac{V}{X_C}$$

$$X_C = \frac{1}{\omega_C C} = \frac{1}{2\pi f_C C}$$

$$\Rightarrow X_C = \frac{1}{2 \times 3.14 \times 60 \times 60 \times 10^{-6}}$$

$$\Rightarrow X_C = 44.248\Omega$$

$$\Rightarrow I_{rms} = \frac{110}{44.28}$$

$$\Rightarrow I_{rms} = 2.488A$$

Therefore, the rms value of the current in the circuit is $2.488A$.

5. In exercises 4 and 5 What is the net power absorbed by each circuit over a complete cycle? Explain your answer.

Ans: From the inductive circuit,

Rms value of current, $I_{rms} = 15.92A$

Rms value of voltage, $V_{rms} = 220V$

It is known that,

Net power absorbed, $P = V_{rms} \times I_{rms} \cos\phi$

Where,

ϕ is the phase difference between voltage and current

For a pure inductive circuit, the phase difference between alternating voltage and current is 90^0 i.e., $\phi = 90^0$

$$\Rightarrow P = 220 \times 15.92 \cos 90^0 = 0$$

Therefore, net power absorbed is zero in a pure inductive circuit.

In a capacitive circuit,

Rms value of current, $I_{rms} = 2.49A$

Rms value of voltage, $V_{rms} = 110V$

It is known that,

Net power absorbed, $P = V_{rms} \times I_{rms} \cos\phi$

Where,

ϕ is the phase difference between voltage and current

For a pure capacitive circuit, the phase difference between alternating voltage and current is 90^0 i.e., $\phi = 90^0$

$\Rightarrow P = 110 \times 2.49 \cos 90^0 = 0$

Therefore, net power absorbed is zero in a pure capacitive circuit.

6. A charged $30\mu F$ capacitor is connected to a $27mH$ inductor. What is the angular frequency of free oscillations of the circuit?

Ans: It is given that,

Capacitance, $C = 30\mu F = 30 \times 10^{-6} F$

Inductance, $L = 27mH = 27 \times 10^{-3} H$

It is known that,

Angular frequency of free oscillations, $\omega_r = \dfrac{1}{\sqrt{LC}}$

$$\Rightarrow \omega_r = \frac{1}{\sqrt{27 \times 10^{-3} \times 30 \times 10^{-6}}}$$

$$\Rightarrow \omega_r = \frac{1}{9 \times 10^{-4}}$$

$$\Rightarrow \omega_r = 1.11 \times 10^3 rad/s$$

Therefore, the angular frequency of free oscillations of the circuit is $1.11 \times 10^3 rad/s$.

7. A series LCR circuit with $R = 20\Omega$, $L = 1.5H$ and $C = 35\mu F$ is connected to a variable frequency $200V$ ac supply. When the frequency of the supply equals the natural frequency of the circuit, what is the average power transferred to the circuit in one complete cycle?

Ans: It is known that,

Resistance, $R = 20\Omega$

Inductance, $L = 1.5H$

Capacitance, $C = 35\mu F = 35 \times 10^{-6} F$

Voltage, $V = 200V$

It is known that,

Impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

At resonance, $X_L = X_C$

$\Rightarrow Z = R = 20\Omega$

$I = \frac{V}{Z} = \frac{200}{20}$

$\Rightarrow I = 10A$

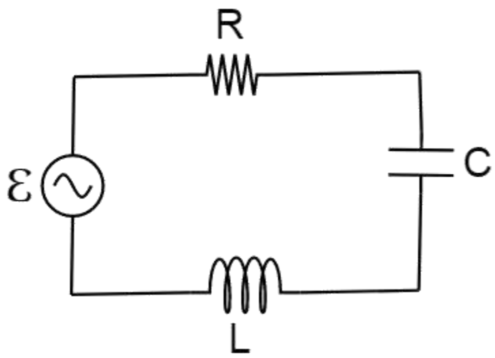
Average power, $P = I^2 R$

$\Rightarrow P = 10^2 \times 20$

$\Rightarrow P = 2000W$

Therefore, the average power transferred is $2000W$.

8. Figure shows a series LCR circuit connected to a variable frequency $230V$ source. $L = 5.0H$, $C = 80\mu F$, $R = 40\Omega$.



a) Determine the source frequency which drives the circuit in resonance.

Ans: It is given that,

Voltage, $V = 230V$

Inductance, $L = 5.0H$

Capacitance, $C = 80\mu F = 80 \times 10^{-6} F$

Resistance, $R = 40\Omega$

It is known that,

Source frequency at resonance $= \frac{1}{\sqrt{LC}}$

$\Rightarrow \frac{1}{\sqrt{5\times 80\times 10^{-6}}} = 50\text{rad/s}$

Therefore, the source frequency of the circuit in resonance is 50rad/s.

b) Obtain the impedance of the circuit and the amplitude of current at the resonating frequency.

Ans: It is known that,

At resonance, Impedance, $Z =$ Resistance, R

$\Rightarrow Z = R = 40\Omega$

$I = \frac{V}{Z}$

$\Rightarrow I = \frac{230}{40} = 5.75\text{A}$

Amplitude, $I_0 = 1.414\times I$

$\Rightarrow I_0 = 1.414\times 5.75$

$\Rightarrow I_0 = 8.13\text{A}$

Therefore, the impedance of the circuit is 40Ω and the amplitude of current at resonating frequency is 8.13A.

c) Determine the rms potential drops across the three elements of the circuit. Show that the potential drop across the LC combination is zero at the resonating frequency.

Ans: It is known that,

Potential drop, $V = IR$

Across resistor, $V_R = IR$

$\Rightarrow V_R = 5.75\times 40 = 230\text{V}$

Across capacitor, $V_C = IX_C = \frac{I}{\omega C}$

$\Rightarrow V_C = 5.75\times \frac{1}{50\times 80\times 10^{-6}}$

$\Rightarrow V_C = 1437.5\text{V}$

Across Inductor, $V_L = IX_L = I\omega L$

$\Rightarrow V_L = 5.75 \times 50 \times 5$

$\Rightarrow V_L = 1437.5V$

Across LC combination, $V_{LC} = I(X_L - X_C)$

At resonance, $X_L = X_C$

$\Rightarrow V_{LC} = 0$

Therefore, the rms potential drop across Resistor is $230V$, Capacitor is $1437.5V$, Inductor is $1437.5V$ and the potential drop across LC combination is zero at resonating frequency.