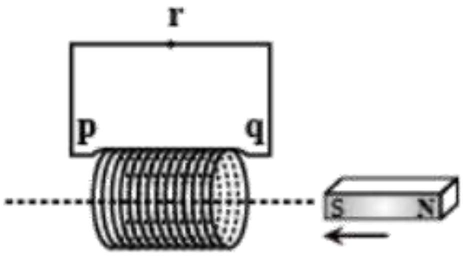
Electromagnetic Induction

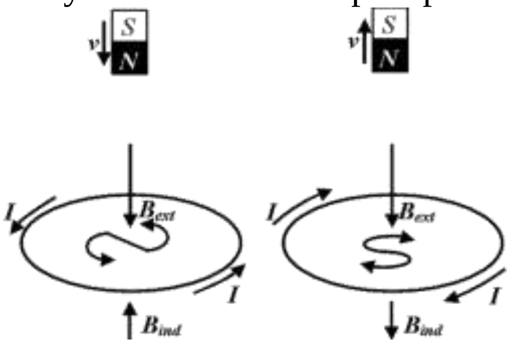
Chapter 6

1. Predict the direction of induced current in the situations described by the following figures (a) to (f).

a)

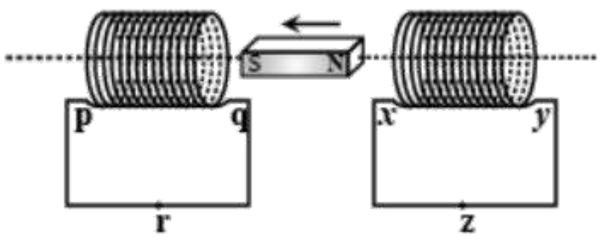


Ans: The direction of the induced current in a closed loop could be given by Lenz's law. The following pairs of figures show the direction of the induced current when the North pole of a bar magnet is moved towards and away from a closed loop respectively.



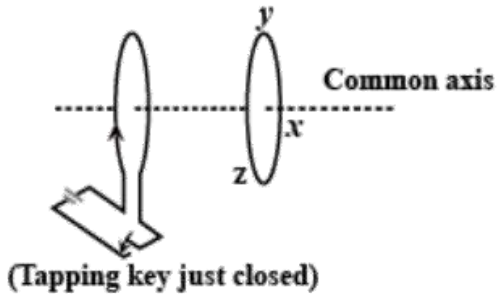
Now, by using Lenz's rule, the direction of the induced current in the given situation is found to be along qrpq.

b)



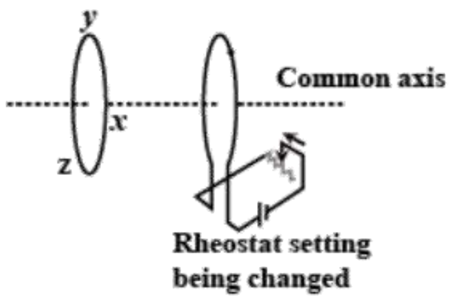
Ans: On using Lenz's law, we find the direction of the induced current here to be along prqp.

c)



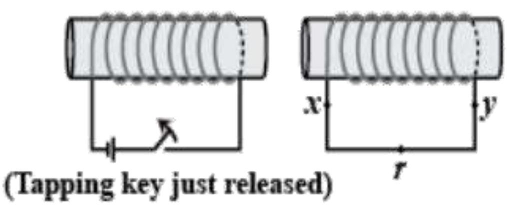
Ans: Using Lenz's law, we find the direction of the induced current to be along yzxy.

d)



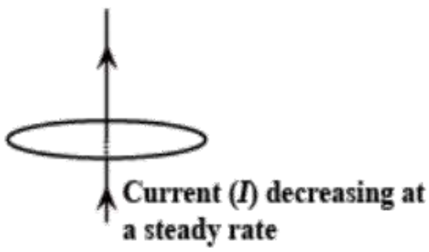
Ans: Using Lenz's law, we find the direction of the induced current to be along zyxz.

e)



Ans: Using Lenz's law, we found the direction of the induced current to be along xryx.

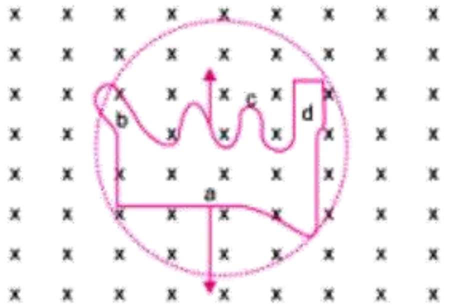
f)



Ans: Here we find that, no current is induced since the field lines are lying in the same plane as that of the closed loop.

2. Use Lenz's law to determine the direction of induced current in the situations described by Figure:

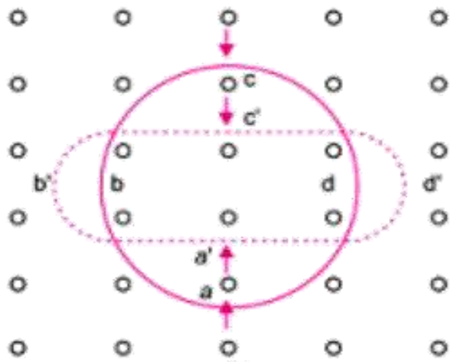
a) A wire of irregular shape turning into a circular shape;



Ans: According to the Lenz's law, the direction of the induced emf is such that it tends to produce a current that would oppose the change in the magnetic flux that produced it.

The wire is here is expanding to form a circle, which means that force would be acting outwards on each part of wire because of the magnetic field (acting in the downwards direction). Now, the direction of induced current should be such that it will produce magnetic field in the upward direction (towards the reader). Therefore, the force on wire will be towards inward direction, i.e., induced current would be flowing in anticlockwise direction in the loop from cbad.

b) A circular loop being deformed into a narrow straight wire.



Ans: On deforming the shape of a circular loop into a narrow straight wire, the flux piercing the surface decreases. Therefore, the induced current flows along abcd according to Lenz's law.

3. A long solenoid with 15 turns per cm has a small loop of area 2.0cm^2 placed inside the solenoid normal to its axis. If the current carried by the solenoid changes steadily from 2.0A to 4.0A in 0.1s, what is the induced emf in the loop while the current is changing?

Ans: We are given the following information:
Number of turns on the solenoid $= 15\text{turns/cm} = 1500\text{turns/m}$
Number of turns per unit length, $n = 1500$ turns
The solenoid has a small loop of area, $A = 2.0\text{cm}^2 = 2\times10^{-4}\text{m}^2$
Current carried by the solenoid changes from 2A to 4A.
Now, the change in current in the solenoid, $di = 4 - 2 = 2\text{A}$
Change in time, $dt = 0.1$ s
Induced emf in the solenoid could be given by Faraday's law as:

$$\varepsilon = \frac{d\phi}{dt} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Where, induced flux through the small loop, $\phi = BA \ldots\ldots\ldots\ldots\ldots (2)$
Equation (1) would now reduce to:

$$\varepsilon = \frac{d}{dt}(BA) = A\mu_0 n \times \left(\frac{di}{dt}\right)$$

Substituting the given values into this equation, we get,

$$\varepsilon = 2\times10^{-4}\times4\pi\times10^{-7}\times1500\times\frac{2}{0.1}$$

$$\therefore \varepsilon = 7.54\times10^{-6}\text{V}$$

Therefore, the induced voltage in the loop is found to be, $\varepsilon = 7.54\times10^{-6}\text{V}$.

4. A rectangular wire loop of sides 8cm and 2cm with a small cut is moving out of a region of uniform magnetic field of magnitude 0.3T directed normal to the loop. What is the emf developed across the cut if the velocity of the loop is in a direction normal to the:

a) longer side? For how long does the induced voltage last in this case?

Ans: We are given the following,
Length of the rectangular wire, $l = 8\text{cm} = 0.08$ m
Width of the rectangular wire, $b = 2\text{cm} = 0.02\text{m}$
Now, the area of the rectangular loop,

$$A = lb = 0.08\times0.02 = 16\times10^{-4}\text{m}^2$$

Magnetic field strength, $B = 0.3\text{T}$
Velocity of the loop, $v = 1$ cm/s $= 0.01$ m/s
Emf developed in the loop could be given as:

$$\varepsilon = Blv$$

Substituting the given values,

$$\varepsilon = 0.3\times0.08\times0.01 = 2.4\times10^{-4}\text{V}$$

Time taken to travel along the width, $t = \frac{\text{Distance travelled}}{\text{velocity}} = \frac{b}{v}$

$\Rightarrow t = \frac{0.02}{0.01} = 2s$

Therefore, the induced voltage is found to be $2.4 \times 10^{-4}V$ which lasts for 2s.

b) shorter side of the loop? For how long does the induced voltage last in this case?

Ans: We know that, Emf developed in the loop could be given as:

$\varepsilon = Blv$

Substituting the given values,

$\varepsilon = 0.3 \times 0.02 \times 0.01 = 0.6 \times 10^{-4}V$

Time taken to travel along the width, $t = \frac{\text{Distance travelled}}{\text{velocity}} = \frac{l}{v}$

$\Rightarrow t = \frac{0.08}{0.01} = 8s$

Therefore, the induced voltage is found to be $0.6 \times 10^{-4}V$ which lasts for 8s.

5. A $1.0m$ long metallic rod is rotated with an angular frequency of $400 rad s^{-1}$ about an axis normal to the rod passing through its one end. The other end of the rod is in contact with a circular metallic ring. A constant and uniform magnetic field of $0.5T$ parallel to the axis exists everywhere. Calculate the emf developed between the centre and the ring.

Ans: We are given the following:

Length of the rod, $l = 1m$

Angular frequency, $\omega = 400 rad/s$

Magnetic field strength, $B = 0.5T$

One end of the rod has zero linear velocity, while the other end has a linear velocity of $l\omega$.

Average linear velocity of the rod, $v = \frac{l\omega + 0}{2} = \frac{l\omega}{2}$

Emf developed between the centre and the ring,

$\varepsilon = Blv = Bl\left(\frac{l\omega}{2}\right) = \left(\frac{Bl^2\omega}{2}\right)$

On substituting the given values,

$\therefore \varepsilon = \frac{0.5 \times (1)^2 \times 400}{2} = 100V$

Therefore, the emf developed between the centre and the ring is $100V$.

6. A horizontal straight wire $10m$ long extending from east to west is falling with a speed of $5.0ms^{-1}$, at right angles to the horizontal component of the earth's magnetic field, $0.30\times10^{-4}Wbm^{-2}$.

a) What is the instantaneous value of the emf induced in the wire?

Ans: We are given the following:

Length of the wire, $l = 10m$

Falling speed of the wire, $v = 5.0m/s$

Magnetic field strength, $B = 0.3\times10^{-4}Wbm^{-2}$

Emf induced in the wire is thus found to be,

$\varepsilon = Blv$

$\Rightarrow \varepsilon = 0.3\times10^{-4}\times5\times10$

$\therefore \varepsilon = 1.5\times10^{-3}V$

Hence, the emf induced in the wire is $\varepsilon = 1.5\times10^{-3}V$.

b) What is the direction of the emf?

Ans: Using Fleming's rule, we find that the direction of the induced emf is from West to East.

c) Which end of the wire is at the higher electrical potential?

Ans: The eastern end of the wire is the end that is at higher potential.

7. Current in a circuit falls from $5.0A$ to $0.0A$ in $0.1s$. If an average emf of $200V$ induced, give an estimate of the self-inductance of the circuit.

Ans: We are given the following:

Initial current, $I_1 = 5.0A$

Final current, $I_2 = 0.0A$

Change in current, $dI = I_1 - I_2 = 5A$

Time taken for the change, $t = 0.1s$

Average emf, $\varepsilon = 200V$

For self-inductance (L) of the coil, we have the relation for average emf that could be given as:

$$\varepsilon = L\frac{di}{dt}$$

$$\Rightarrow L=\frac{\varepsilon}{\left(\frac{di}{dt}\right)}$$

Substituting the given values we get,

$$\therefore L=\frac{200}{\left(\frac{5}{0.1}\right)}=4H$$

Therefore, we found the self induction in the coil to be 4H.

8. A pair of adjacent coils has a mutual inductance of $1.5H$. If the current in one coil changes from 0 to $20A$ in $0.5s$, what is the change of flux linkage with the other coil?

Ans: We are given the following,

Mutual inductance of a pair of coils, $\mu=1.5H$

Initial current, $I_1=0A$

Final current, $I_2=20A$

Change in current, $dI=I_2-I_1=20-0=20A$

Time taken for the change, $t=0.5s$

Induced emf, $\varepsilon=\frac{d\phi}{dt}$ …………………. (1)

Where, $d\phi$ is the change in the flux linkage with the coil.

Emf is related with mutual inductance could be given as:

$\varepsilon=\mu\frac{dI}{dt}$ …………………….. (2)

Equating equations (1) and (2), we get,

$$\frac{d\phi}{dt}=\mu\frac{dI}{dt}$$

$$\Rightarrow d\phi=1.5\times(20)$$

$$\therefore d\phi=30Wb$$

Hence, we found the change in the flux linkage to be 30Wb.