Current Electricity

3 Chapter

1. The storage battery of a car has an emf of 12 V. If the internal resistance of the battery is 0.4Ω , what is the maximum current that can be drawn from the battery?

Ans: In the above question it is given that:
Emf of the battery, E= 12V
Internal resistance of the battery, $r = 0.4\Omega$
Consider the maximum current drawn from the battery to be I.
Therefore, using Ohm's law,

$$E = Ir$$

$$\Rightarrow I = \frac{E}{r}$$

$$\Rightarrow I = \frac{12}{0.4}$$

$$\Rightarrow I = 30A$$

Clearly, the maximum current drawn from the given battery is $30A$.

2. A battery of emf 10V and internal resistance 3Ω is connected to a resistor. If the current in the circuit is 0.5 A, what is the resistance of the resistor? What is the terminal voltage of the battery when the circuit is closed?

Ans: In the above question it is given that:
Emf of the battery, E = 10 V
Internal resistance of the battery, $r = 3\Omega$
Current in the circuit, $I = 0.5A$
Consider the resistance of the resistor to be R.
Therefore, using Ohm's law,

$$I = \frac{E}{R + r}$$

$$R + r = \frac{E}{I}$$

$$\Rightarrow R + r = \frac{10}{0.5}$$

$$\Rightarrow R + r = 20$$

$\Rightarrow R = 20 - 3 = 17\Omega$
Let the terminal voltage of the resistor be V.
Using the Ohm's law,
$V = IR$
$\Rightarrow V = 0.5 \times 17 = 8.5V$
Thus, the resistance of the resistor is 17Ω and the terminal voltage is $8.5V$.

3. At room temperature $27.0^\circ C$, the resistance of a heating element is 100Ω What is the temperature of the element if the resistance is found to be 117Ω given that the temperature coefficient of the material of the resistor is $1.70 \times 10^{-4 \circ} C^{-1}$?

Ans: In the above question it is given that at room temperature $(T = 27.0^\circ C)$, the resistance of the heating element is 100Ω (say R).
Also, the heating element's temperature coefficient is given to be $\alpha = 1.70 \times 10^{-4 \circ} C^{-1}$.
Now, it is said that the resistance of the heating element at an increased temperature (say T_1) is 117Ω (say R_1). To compute this unknown increased temperature T_1, the formula for temperature coefficient of a material can be used.
It is known that temperature co-efficient of a material provides information on the nature of that material with respect to its change in resistance with temperature. Mathematically,

$$\alpha = \frac{R_1 - R}{R(T_1 - T)}$$

$$\Rightarrow T_1 - T = \frac{R_1 - R}{R\alpha}$$

Substituting the given values,

$$\Rightarrow T_1 - 27 = \frac{117 - 100}{100 \times 1.70 \times 10^{-4}}$$

$$\Rightarrow T_1 - 27 = 1000$$

$$\Rightarrow T_1 = 1027^\circ C$$

Clearly, it is at $1027^\circ C$ when the resistance of the element is 117Ω.

4. A negligibly small current is passed through a wire of length 15 m and uniform cross-section $6.0\times10^{-7}m^2$, and its resistance is measured to be 5.0Ω. What is the resistivity of the material at the temperature of the experiment?

Ans: In the above question it is given that:

Length of the wire, $l = 15m$

Area of cross-section of the wire, $a = 6.0\times10^{-7}m^2$

Resistance of the material of the wire, $R = 5.0\Omega$

Let resistivity of the material of the wire be ρ

It is known that, resistance is related with the resistivity as:

$$R = \rho\frac{l}{A}$$

$$\Rightarrow \rho = \frac{RA}{l}$$

$$\Rightarrow \rho = \frac{5\times6.0\times10^{-7}}{15}$$

$$\Rightarrow \rho = 2\times10^{-7}m^2$$

Therefore, the resistivity of the material is $2\times10^{-7}m^2$.

5. A silver wire has a resistance of 2.1Ω at $27.5^\circ C$, and a resistance of 2.7Ω at $100^\circ C$. Determine the temperature coefficient of resistivity of silver.

Ans: In the above question it is given that:

Temperature, $T_1 = 27.5^\circ C$.

Resistance of the silver wire at T_1 is $R_1 = 2.1\Omega$.

Temperature, $T_2 = 100^\circ C$.

Resistance of the silver wire at T_2 is $R_2 = 2.7\Omega$.

Let the temperature coefficient of silver be α . It is known that temperature co-efficient of a material provides information on the nature of that material with respect to its change in resistance with temperature. Mathematically, it is related with temperature and resistance by the formula:

$$\alpha = \frac{R_2 - R_1}{R_1(T_2 - T_1)}$$

$$\Rightarrow \alpha = \frac{2.7 - 2.1}{2.1(100 - 27.5)} = 0.0039^\circ C^{-1}$$

Clearly, the temperature coefficient of silver is $0.0039^\circ C^{-1}$.

6. A heating element using nichrome connected to a 230 V supply draws an initial current of 3.2A which settles after a few seconds to a steady value of 2.8 A. What is the steady temperature of the heating element if the room

temperature is $27^\circ C$? Temperature coefficient of resistance of nichrome averaged over the temperature range involved is $1.70\times10^{-4\circ}C^{-1}$.

Ans: In the above question it is given that:

Supply voltage is $V = 230V$

Initial current drawn is $I_1 = 3.2A$.

Let the initial resistance be R_1 .

Therefore, using Ohm's law,

$$R_1 = \frac{V}{I_1}$$

$$\Rightarrow R_1 = \frac{230}{3.2} = 71.87\Omega$$

Steady state value of the current is $I_2 = 2.8A$.

Let the resistance of the steady state be R_2 .

Therefore, using Ohm's law.

$$R_2 = \frac{V}{I_2}$$

$$\Rightarrow R_2 = \frac{230}{2.8} = 82.14\Omega$$

Temperature co-efficient of nichrome is $\alpha = 1.70\times10^{-4\circ}C^{-1}$.

Initial temperature of nichrome is $T_1 = 27^\circ C$.

Let steady state temperature reached by nichrome be T_2 .

Now, it is known that temperature co-efficient of a material provides information on the nature of that material with respect to its change in resistance with temperature. Mathematically, it is given by

$$\alpha = \frac{R_2 - R_1}{R_1(T_2 - T_1)}$$

$$\Rightarrow (T_2 - T_1) = \frac{R_2 - R_1}{R_1\alpha}$$

Substituting the given values,

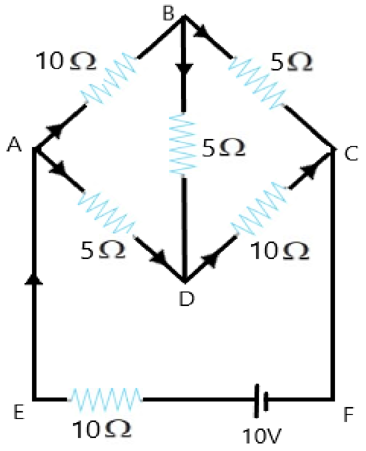
$$\Rightarrow (T_2 - 27) = \frac{82.14 - 71.87}{71.87\times1.70\times10^{-4}}$$

$$\Rightarrow T_2 - 27 = 840.5$$

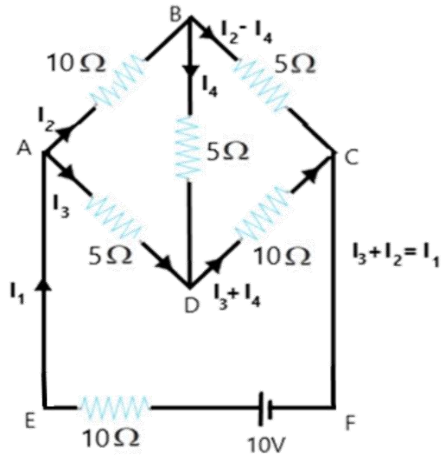
$$\Rightarrow T_2 = 867.5^\circ C$$

Clearly, the steady temperature of the heating element is $867.5^\circ C$.

7. Determine the current in each branch of the network shown in figure:



Ans: Current flowing through various branches of the circuit is represented in the given figure.



Consider

$I_1 =$ Current flowing through the outer circuit

$I_2 =$ Current flowing through branch AB

$I_3 =$ Current flowing through branch AD

$I_2 - I_4 =$ Current flowing through branch BC

$I_3 + I_4 =$ Current flowing through branch CD

$I_4 =$ Current flowing through branch BD

For the closed circuit ABDA, potential is zero i.e.,

$10I_2 + 5I_4 - 5I_3 = 0$

$2I_2 + I_4 - I_3 = 0$

$I_3 = 2I_2 + I_4$ ...... (1)

For the closed circuit BCDB, potential is zero i.e.,

$5(I_2 - I_4) - 10(I_3 + I_4) - 5I_4 = 0$

$$5I_2 + 5I_4 - 10I_3 - 10I_4 - 5I_4 = 0$$
$$5I_2 - 10I_3 - 20I_4 = 0$$
$$I_2 = 2I_3 + 4I_4 \qquad \ldots\ldots (2)$$
For the closed circuit ABCFEA, potential is zero i.e.,
$$-10 + 10(I_1) + 10(I_2) + 5(I_2 - I_4) = 0$$
$$10 = 15I_2 + 10I_1 - 5I_4$$
$$3I_3 + 2I_1 - I_4 = 2 \qquad \ldots\ldots (3)$$
From equations (1) and (2), we obtain
$$I_3 = 2(2I_3 + 4I_4) + I_4$$
$$I_3 = 4I_3 + 8I_4 + I_4$$
$$-3I_3 = 9I_4$$
$$-3I_4 = +I_3 \qquad \ldots\ldots (4)$$
Putting equation (4) in equation (1), we obtain
$$I_3 = 2I_2 + I_4$$
$$-4I_4 = 2I_2 \qquad \ldots\ldots (5)$$
It is evident from the given figure that,
$$I_1 = I_3 + I_2 \qquad \ldots\ldots. (6)$$
Putting equation (6) in equation (1), we obtain
$$3I_2 + 2(I_3 + I_2) - I_4 = 2$$
$$5I_2 + 2I_3 - I_4 = 2 \qquad \ldots\ldots (7)$$
Putting equations (4) and (5) in equation (7), we obtain
$$5(-2I_4) + 2(-3I_4) - I_4 = 2$$
$$-10I_4 - 6I_4 - I_4 = 2$$
$$17I_4 = -2$$
$$I_4 = -\frac{2}{17}\,\text{A}$$
Equation (4) reduces to
$$I_3 = -3(I_4)$$
$$I_3 = -3\left(-\frac{2}{17}\right) = \frac{6}{17}\,\text{A}$$
$$I_2 = -2(I_4)$$
$$I_2 = -2\left(-\frac{2}{17}\right) = \frac{4}{17}\,\text{A}$$

$$I_2 - I_4 = \frac{4}{17} - \left(-\frac{2}{17}\right) = \frac{6}{17}$$

$$I_3 + I_4 = \frac{6}{17} + \left(\frac{-2}{17}\right) = \frac{4}{17}A$$

$$I_1 = I_3 + I_2$$

$$\therefore I_1 = \frac{6}{17} + \frac{4}{17} = \frac{10}{17}A$$

Therefore, current in branch AB $= \frac{4}{17}A$

Current in branch BC $= \frac{6}{17}A$

Current in branch CD $= \frac{-4}{17}A$

Current in branch AD $= \frac{6}{17}A$

Current in branch BD $= \left(\frac{-2}{17}\right)A$

Total current $= \frac{4}{17} + \frac{6}{17} + \frac{-4}{17} + \frac{6}{17} + \frac{-2}{17} = \frac{10}{17}A$.

8. A storage battery of emf 8.0 V and internal resistance 0.5Ω is being charged by a 120 V DC supply using a series resistor of 15.5Ω. What is the terminal voltage of the battery during charging? What is the purpose of having a series resistor in the charging circuit?

Ans: In the above question it is given that:

Emf of the storage battery is $E = 0.8V$.

Internal resistance of the battery is $r = 0.5\Omega$.

DC supply voltage is $V = 120V$

Resistance of the resistor is $R = 15.5\Omega$.

Consider effective voltage in the circuit to be V', which would be the difference in the supply voltage and the emf of the battery.

$$V' = V - E$$

$$\Rightarrow V' = 120 - 8 = 112V$$

Now, current flowing in the circuit is I and the resistance R is connected in series to the storage battery.

Therefore, using Ohm’s law,

$$I = \frac{V'}{R + r}$$

$$\Rightarrow I = \frac{112}{15.5 + 0.5} = 7A$$

Thus, voltage across resistor R would be:

$IR = 7 \times 15.5 = 108.5V$

DC supply voltage = Terminal voltage of battery + Voltage drop across R

Terminal voltage of battery $= 120 - 108.5 = 11.5V$

A series resistor in a charging circuit takes the responsibility for controlling the current drawn from the external source. Excluding this series resistor is dangerous as the current flow would be extremely high if so.

9. The number density of free electrons in a copper conductor estimated in Example 3.1 is $8.5 \times 10^{28} m^{-3}$. How long does an electron take to drift from one end of a wire 3.0 m long to its other end? The area of cross-section of the wire is $2.0 \times 10^{-6} m^2$ and it is carrying a current of 3.0 A.

Ans: In the above question it is given that:

Number density of free electrons in a copper conductor is $n = 8.5 \times 10^{28} m^{-3}$.

Length of the copper wire is $l = 3.0m$.

Area of cross-section of the wire is $A = 2.0 \times 10^{-6} m^2$.

Current carried by the wire is $I = 3.0A$.

Now we know that:

$I = nAeV_d$

Where,

e is the electric charge of magnitude $1.6 \times 10^{-19} C$.

V_d is the drift velocity and

$$\text{Drift velocity} = \frac{\text{Length of the wire } (l)}{\text{Time taken to cover } (t)}$$

$$I = nAe\frac{l}{t}$$

$$\Rightarrow t = \frac{nAel}{I}$$

$$\Rightarrow t = \frac{3 \times 8.5 \times 10^{28} \times 2 \times 10^{-6} \times 1.6 \times 10^{-19}}{3.0}$$

$\therefore t = 2.7 \times 10^4 s$.

Hence the time taken by an electron to drift from one end of the wire to the other is $2.7 \times 10^4 s$.