

Electrostatic Potential -Capacitance

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Chapter

1. Two charges $5 \times 10^{-8} \text{ C}$ and $-3 \times 10^{-8} \text{ C}$ are located 16 cm apart. At what point(s) on the line joining the two charges is the electric potential zero? Take the potential at infinity to be zero.

Ans: It is provided that,

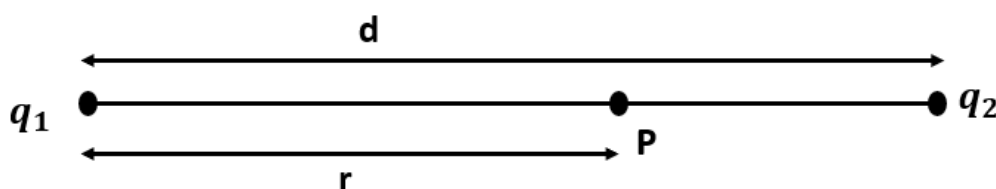
First charge, $q_1 = 5 \times 10^{-8} \text{ C}$

Second charge, $q_2 = -3 \times 10^{-8} \text{ C}$

Distance between the two given charges, $d = 16 \text{ cm} = 0.16 \text{ m}$

Case 1. When point P is inside the system of two charges.

Consider a point named P on the line connecting the two charges.



r is the distance of point P from q_1 .

Potential at point P will be,

$$V = \frac{q_1}{4\pi\epsilon_0 r} + \frac{q_2}{4\pi\epsilon_0 (d - r)}$$

Where, ϵ_0 is the Permittivity of free space

But $V = 0$ so,

$$0 = \frac{q_1}{4\pi\epsilon_0 r} + \frac{q_2}{4\pi\epsilon_0 (d - r)}$$

$$\Rightarrow \frac{q_1}{4\pi\epsilon_0 r} = -\frac{q_2}{4\pi\epsilon_0 (d - r)}$$

$$\Rightarrow \frac{q_1}{r} = -\frac{q_2}{(d - r)}$$

$$\Rightarrow \frac{5 \times 10^{-8}}{r} = -\frac{-3 \times 10^{-8}}{(0.16 - r)}$$

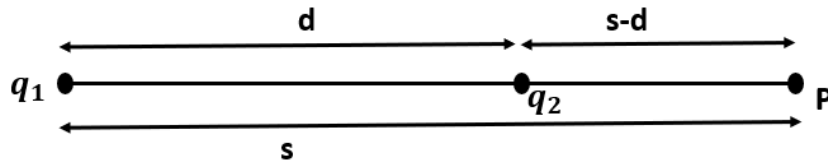
$$\Rightarrow \frac{0.16}{r} = \frac{8}{5}$$

We get,

$$r = 0.1\text{m} = 10\text{cm}$$

Therefore, the potential is zero at 10 cm distance from the positive charge.

Case 2. When point P is outside the system of two charges.



Potential at point P will be,

$$V = \frac{q_1}{4\pi\epsilon_0 s} + \frac{q_2}{4\pi\epsilon_0 (s-d)}$$

Where, ϵ_0 is the Permittivity of free space

But $V = 0$ so,

$$0 = \frac{q_1}{4\pi\epsilon_0 s} + \frac{q_2}{4\pi\epsilon_0 (s-d)}$$

$$\Rightarrow \frac{q_1}{4\pi\epsilon_0 s} = -\frac{q_2}{4\pi\epsilon_0 (s-d)}$$

$$\Rightarrow \frac{q_1}{s} = -\frac{q_2}{(s-d)}$$

$$\Rightarrow \frac{5 \times 10^{-8}}{s} = -\frac{-3 \times 10^{-8}}{(s-0.16)}$$

$$\Rightarrow \frac{0.16}{s} = \frac{2}{5}$$

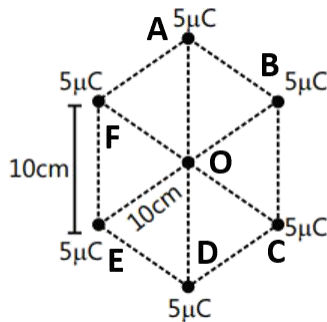
We get,

$$s = 0.4\text{m} = 40\text{cm}$$

Therefore, the potential is zero at 40 cm distance from the positive charge.

2. A regular hexagon of side 10 cm has a charge $5\mu\text{C}$ at each of its vertices. Calculate the potential at the centre of the hexagon.

Ans: The given figure represents six equal charges, $q = 5 \times 10^{-6}\text{C}$, at hexagon's vertices.



Sides of the hexagon, $AB = BC = CD = DE = EF = FA = 10\text{cm}$

The distance of O from each vertex, $d = 10\text{cm}$

Electric potential at point O,

$$V = \frac{6q}{4\pi\epsilon_0 d}$$

Where, ϵ_0 is the Permittivity of free space

$$\text{Value of } \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ NC}^{-2}\text{m}^{-2}$$

$$\Rightarrow V = \frac{6 \times 9 \times 10^9 \times 5 \times 10^{-6}}{0.1}$$

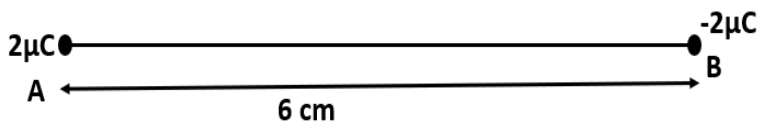
$$\Rightarrow V = 2.7 \times 10^6 \text{ V}$$

Clearly, the potential at the hexagon's centre is $2.7 \times 10^6 \text{ V}$.

3. Two charges $2\mu\text{C}$ and $-2\mu\text{C}$ are placed at points A and B, 6 cm apart.

a) Identify an equipotential surface of the system.

Ans: The given figure represents two charges.



An equipotential surface is defined as that plane on which electric potential is equal at every point. One such plane is normal to line AB. The plane is placed at the mid-point of line AB because the magnitude of charges is equal.

b) What is the direction of the electric field at every point on this surface?

Ans: The electric field's direction is perpendicular to the plane in the line AB direction at every location on this surface.

4. A spherical conductor of radius 12 cm has a charge of $1.6 \times 10^{-7} \text{ C}$ distributed uniformly on its surface. What is the electric field,

a) inside the sphere?

Ans: It is provided that,

Spherical conductor's radius, $r = 12\text{cm} = 0.12\text{m}$

The charge is evenly distributed across the conductor. The electric field within a spherical conductor is zero because the total net charge within a conductor is zero.

b) just outside the sphere?

Ans: Just outside the conductor, Electric field E is given by

$$E = \frac{q}{4\pi\epsilon_0 r^2}$$

Where, ϵ_0 is the Permittivity of free space

$$\text{Value of } \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ NC}^{-2}\text{m}^{-2}$$

$$\Rightarrow E = \frac{1.6 \times 10^{-7} \times 9 \times 10^9}{(0.12)^2}$$

$$\Rightarrow E = 10^5 \text{ NC}^{-1}$$

Clearly, the electric field just outside the sphere is 10^5 NC^{-1} .

c) at a point 18 cm from the centre of the sphere?

Ans: Let electric field at a given point which is 18 cm from the sphere centre = E_1

Distance of the given point from the centre, $d = 18 \text{ cm} = 0.18\text{m}$

The formula for electric field is given by,

$$E_1 = \frac{q}{4\pi\epsilon_0 d^2}$$

Where, ϵ_0 is the Permittivity of free space

$$\text{Value of } \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ NC}^{-2}\text{m}^{-2}$$

$$\Rightarrow E = \frac{1.6 \times 10^{-7} \times 9 \times 10^9}{(0.18)^2}$$

$$\Rightarrow E = 4.4 \times 10^4 \text{ NC}^{-1}$$

Therefore, the electric field at a given point 18 cm from the sphere centre is $4.4 \times 10^4 \text{ NC}^{-1}$.

5. A parallel plate capacitor with air between the plates has a capacitance of 8pF. What will be the capacitance if the distance between the plates is reduced by half, and the space between them is filled with a substance of dielectric constant 6?

Ans: It is provided that,

Capacitance between the capacitor's parallel plates, $C = 8\text{pF}$

Originally, the distance separating the parallel plates was d , and the air was filled in it.

Dielectric constant of air, $k = 1$

The formula for Capacitance is given by:

$$C = \frac{k\epsilon_0 A}{d}$$

Here, $k = 1$, so,

$$C = \frac{\epsilon_0 A}{d}$$

Where, ϵ_0 is the Permittivity of free space

A is the area of each plate.

If the distance separating the plates is decreased to half and the substance has a dielectric constant of 6 filled in between the plates.

Then,

$$k' = 6, d' = \frac{d}{2}$$

Hence, capacitor's capacitance becomes,

$$C' = \frac{k'\epsilon_0 A}{d'}$$

$$\Rightarrow C' = \frac{6\epsilon_0 A}{\frac{d}{2}}$$

$$\Rightarrow C' = 12C$$

$$\Rightarrow C' = 12 \times 8 = 96\text{pF}$$

Therefore, the capacitance when substance of dielectric constant 6 is filled between the plates is 96 pF.

6. Three capacitors each of capacitance 9 pF are connected in series.

a) What is the total capacitance of the combination?

Ans: It is provided that,

Capacitance of each three capacitors, $C = 9\text{pF}$

The formula for equivalent capacitance (C') of the capacitors' series combination is given by

$$\frac{1}{C'} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C}$$

$$\Rightarrow \frac{1}{C'} = \frac{1}{9} + \frac{1}{9} + \frac{1}{9}$$

$$\Rightarrow \frac{1}{C'} = \frac{3}{9}$$

$$\Rightarrow C' = 3\text{pF}$$

Clearly, total capacitance of the combination of the capacitors is 3pF.

b) What is the potential difference across each capacitor if the combination is connected to a 120 V supply?

Ans: Provided that,

Supply voltage, $V = 120 \text{ V}$

Potential difference (V') across each capacitor will be one-third of the supply voltage.

$$V' = 120 / 3 = 40\text{V}$$

Clearly, the potential difference across each capacitor is 40 V.

7. Three capacitors of capacitance 2pF, 3 pF and 4pF are connected in parallel.

a) What is the total capacitance of the combination?

Ans: Provided that,

Capacitances of the given capacitors are,

$$C_1 = 2\text{pF} ; C_2 = 3\text{pF} ; C_3 = 4\text{pF}$$

The formula for equivalent capacitance (C') of the capacitors' parallel combination is given by

$$C' = C_1 + C_2 + C_3$$

$$\Rightarrow C' = 2 + 3 + 4 = 9\text{pF}$$

Therefore, total capacitance of the combination is 9pF.

b) Determine the charge on each capacitor if the combination is connected to a 100 V supply.

Ans: We have,

Supply voltage, $V = 100 \text{ V}$

Charge on a capacitor with capacitance C and potential difference V is given by,

$$q = CV \dots\dots(i)$$

For $C = 2\text{pF}$,

$$\text{Charge} = VC = 100 \times 2 = 200\text{pF}$$

For $C = 3\text{pF}$,

$$\text{Charge} = VC = 100 \times 3 = 300\text{pF}$$

For $C = 4\text{pF}$,

$$\text{Charge} = VC = 100 \times 4 = 400 \text{ pF}$$

8. In a parallel plate capacitor with air between the plates, each plate has an area of $6 \times 10^{-3} \text{ m}^2$ and the distance between the plates is 3 mm. Calculate the capacitance of the capacitor. If this capacitor is connected to a 100 V supply, what is the charge on each plate of the capacitor?

Ans: It is provided that,

Area of parallel plate capacitor's each plate, $A = 6 \times 10^{-3} \text{ m}^2$

Distance separating the plates, $d = 3 \text{ mm} = 3 \times 10^{-3} \text{ m}$

Supply voltage, $V = 100 \text{ V}$

The formula for parallel plate capacitor's Capacitance is given by,

$$C = \frac{\epsilon_0 A}{d}$$

Where, ϵ_0 is the Permittivity of free space

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\Rightarrow C = \frac{8.854 \times 10^{-12} \times 6 \times 10^{-3}}{3 \times 10^{-3}}$$

$$\Rightarrow C = 17.71 \times 10^{-12} \text{ F}$$

$$\Rightarrow C = 17.71 \text{ pF}$$

The formula for Potential V is related with charge q and capacitance C is given by,

$$V = \frac{q}{C}$$

$$\Rightarrow q = CV = 100 \times 17.71 \times 10^{-12}$$

$$\Rightarrow q = 1.771 \times 10^{-9} \text{ C}$$

Clearly, capacitor's capacitance is 17.71 pF and charge on each plate is $1.771 \times 10^{-9} \text{ C}$.

9. Explain what would happen if in the capacitor given in Exercise 8, a 3 mm thick mica sheet (of dielectric constant = 6) were inserted between the plates, a) while the voltage supply remained connected.

Ans: It is provided that,

Mica sheet's Dielectric constant, $k = 6$

Initial capacitance, $C = 17.71 \times 10^{-12} \text{ F}$

New capacitance, $C' = kC = 6 \times 17.71 \times 10^{-12} = 106 \text{ pF}$

Supply voltage, $V = 100 \text{ V}$

New charge, $q' = C'V = 106 \times 100 \text{ pC} = 1.06 \times 10^{-8} \text{ C}$

Potential across the plates will remain 100 V.

b) after the supply was disconnected.

Ans: It is provided that,

Mica sheet's Dielectric constant, $k = 6$

Initial capacitance, $C = 17.71 \times 10^{-12} \text{ F}$

New capacitance, $C' = kC = 6 \times 17.71 \times 10^{-12} = 106 \text{ pF}$

If supply voltage is disconnected, then there will be no influence on the charge amount on the plates.

The formula for potential across the plates is given by,

$$V' = \frac{q}{C'}$$

$$V' = \frac{1.771 \times 10^{-9}}{106 \times 10^{-12}} = 16.7 \text{ V}$$

The potential across the plates when the supply was removed is 16.7V .

10. A 12pF capacitor is connected to a 50 V battery. How much electrostatic energy is stored in the capacitor?

Ans: It is provided that,

Capacitance of the capacitor, $C = 12 \times 10^{-12} \text{ F}$

Potential difference, $V = 50 \text{ V}$

The formula for stored electrostatic energy in the capacitor is given by,

$$E = \frac{1}{2} CV^2$$

$$\Rightarrow E = \frac{1}{2} \times 12 \times 10^{-12} \times 50^2$$

$$\Rightarrow E = 1.5 \times 10^{-8} \text{ J}$$

Therefore, the stored electrostatic energy in the capacitor is $1.5 \times 10^{-8} \text{ J}$.

11. A 600 pF capacitor is charged by a 200 V supply. It is then disconnected from the supply and is connected to another uncharged 600 pF capacitor. How much electrostatic energy is lost in the process?

Ans: It is provided that,

Capacitance of the capacitor, $C = 600 \text{ pF}$

Potential difference, $V = 200 \text{ V}$

The formula for stored electrostatic energy in the capacitor is given by,

$$E = \frac{1}{2} CV^2$$

$$\Rightarrow E = \frac{1}{2} \times 600 \times 10^{-12} \times 200^2$$

$$\Rightarrow E = 1.2 \times 10^{-5} \text{ J}$$

If supply is removed from the capacitor and another capacitor of capacitance $C = 600 \text{ pF}$ is joined to it, then equivalent capacitance (C') of the series combination is given by

$$\frac{1}{C'} = \frac{1}{C} + \frac{1}{C}$$

$$\Rightarrow \frac{1}{C'} = \frac{1}{600} + \frac{1}{600}$$

$$\Rightarrow \frac{1}{C'} = \frac{2}{600}$$

$$\Rightarrow C' = 300 \text{ pF}$$

New electrostatic energy will be,

$$E' = \frac{1}{2} C' V^2$$

$$\Rightarrow E' = \frac{1}{2} \times 300 \times 10^{-12} \times 200^2$$

$$\Rightarrow E' = 0.6 \times 10^{-5} \text{ J}$$

Loss in electrostatic energy = $E - E'$

$$\Rightarrow E - E' = 1.2 \times 10^{-5} - 0.6 \times 10^{-5} = 0.6 \times 10^{-5} \text{ J}$$

$$\Rightarrow E - E' = 6 \times 10^{-6} \text{ J}$$

Clearly, the lost electrostatic energy in the process is $6 \times 10^{-6} \text{ J}$.