El ectric Charges and Fiel ds



1. What is the force between two small charged spheres of charges 2×10^{-7} C and 3×10^{-7} C placed 30cm apart in the air? Ans: We are given the following information:

Ans. we are given the following information

Repulsive force of magnitude, $F = 6 \times 10^{-3} N$

Charge on the first sphere, $q_1 = 2 \times 10^{-7} C$

Charge on the second sphere, $q_2 = 3 \times 10^{-7}$ C Distance between the two spheres, r = 30 cm = 0.3m

Electrostatic force between the two spheres is given by Coulomb's law as,

$$F = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2}$$

Where, ε_0 is the permittivity of free space and,

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \, \text{Nm}^2 \text{C}^{-2}$$

Now on substituting the given values, Coulomb's law becomes,

F =
$$\frac{9 \times 10^9 \times 2 \times 10^{-7} \times 3 \times 10^{-7}}{(0.3)^2}$$

∴ F = 6×10⁻³ N

Therefore, we found the electrostatic force between the given charged spheres to be $F = 6 \times 10^{-3}$ N. Since the charges are of the same nature, we could say that the force is repulsive.

2. The electrostatic force on a small sphere of charge 0.4μ C due to another small sphere of charge -0.8μ C in the air is 0.2N.

a) What is the distance between the two spheres?

Ans: Electrostatic force on the first sphere is given to be, F = 0.2N

Charge of the first sphere is, $q_1 = 0.4 \mu C = 0.4 \times 10^{-6} C$ Charge of the second sphere is, $q_2 = -0.8 \mu C = -0.8 \times 10^{-6} C$

We have the electrostatic force given by Coulomb's law as,

$$F = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2}$$
$$\Rightarrow r = \sqrt{\frac{q_1 q_2}{4\pi\varepsilon_0 F}}$$

Substituting the given values in the above equation, we get,

$$\Rightarrow r = \sqrt{\frac{0.4 \times 10^{-6} \times 8 \times 10^{-6} \times 9 \times 10^{9}}{0.2}}$$
$$\Rightarrow r = \sqrt{144 \times 10^{-4}}$$
$$\therefore r = 0.12m$$

Therefore, we found the distance between charged spheres to be r = 0.12m.

b) What is the force on the second sphere due to the first?

Ans: From Newton's third law of motion, we know that every action has an equal and opposite reaction.

Thus, we could say that the given two spheres would attract each other with the same force.

So, the force on the second sphere due to the first sphere will be 0.2N.

3. Check whether the ratio
$$\frac{ke^2}{Gm_em_p}$$
 is dimensionless. Look up a table of

physical constants and hence determine the value of the given ratio. What does the ratio signify?

Ans: We are given the ratio, $\frac{ke^2}{Gm_em_p}$.

Here, G is the gravitational constant which has its unit Nm^2kg^{-2} ;

 $m_{_{e}} \text{and} \ m_{_{p}}$ are the masses of electron and proton in kg respectively;

e is the electric charge in C;

k is a constant given by
$$k = \frac{1}{4\pi\varepsilon_0}$$
.

In the expression for k, ϵ_0 is the permittivity of free space which has its unit Nm^2C^{-2} .

Now, we could find the dimension of the given ratio by considering their units as follows:

$$\frac{\mathrm{k}\mathrm{e}^{2}}{\mathrm{Gm}_{\mathrm{e}}\mathrm{m}_{\mathrm{p}}} = \frac{\left[\mathrm{Nm}^{2}\mathrm{C}^{-2}\right]\left[\mathrm{C}\right]^{2}}{\left[\mathrm{Nm}^{2}\mathrm{kg}^{-2}\right]\left[\mathrm{kg}\right]\left[\mathrm{kg}\right]} = \mathrm{M}^{0}\mathrm{L}^{0}\mathrm{T}^{0}$$

Clearly, it is understood that the given ratio is dimensionless.

Now, we know the values for the given physical quantities as,

$$e = 1.6 \times 10^{-19} C$$

$$G = 6.67 \times 10^{-11} Nm^{2} kg^{-2}$$

$$m_{e} = 9.1 \times 10^{-31} kg$$

$$m_{p} = 1.66 \times 10^{-27} kg$$

Substituting these values into the required ratio, we get,

$$\frac{\mathrm{ke}^{2}}{\mathrm{Gm}_{\mathrm{e}}\mathrm{m}_{\mathrm{p}}} = \frac{9 \times 10^{9} \times (1.6 \times 10^{-19})^{2}}{6.67 \times 10^{-11} \times 9.1 \times 10^{-3} \times 1.67 \times 10^{-22}}$$
$$\Rightarrow \frac{\mathrm{ke}^{2}}{\mathrm{Gm}_{\mathrm{e}}\mathrm{m}_{\mathrm{p}}} \approx 2.3 \times 10^{39}$$

We could infer that the given ratio is the ratio of electrical force to the gravitational force between a proton and an electron when the distance between them is kept constant.

4.

a) Explain the meaning of the statements 'electric charge of a body is quantized'.

Ans: The given statement 'Electric charge of a body is quantized' means that only integral number (1, 2, 3, ..., n) of electrons can be transferred from one body to another.

That is, charges cannot be transferred from one body to another in fraction.

b) Why can one ignore quantization of electric charge when dealing with macroscopic i.e., large scale charges?

Ans: On a macroscopic scale or large-scale, the number of charges is as large as the magnitude of an electric charge.

So, quantization is considered insignificant at a macroscopic scale for an electric charge and electric charges are considered continuous.

5. When a glass rod is rubbed with a silk cloth, charges appear on both. A similar phenomenon is observed with many other pairs of bodies. Explain how this observation is consistent with the law of conservation of charge. Ans: Rubbing two objects would produce charges that are equal in magnitude and opposite in nature on the two bodies.

This happens due to the reason that charges are created in pairs. This phenomenon is called charging by friction.

The net charge of the system however remains zero as the opposite charges equal in magnitude annihilate each other.

So, rubbing a glass rod with a silk cloth creates opposite charges of equal magnitude on both of them and this observation is found to be consistent with the law of conservation of charge.

6. Four point charges $q_A = 2\mu C$, $q_B = -5\mu C$, $q_C = 2\mu C$ and $q_D = -5\mu C$ are located at the corners of a square ABCD with side 10cm. What is the force on 1 μ C charge placed at the centre of this square?

Ans: Consider the square of side length 10cm given below with four charges at its corners and let O be its centre.



From the figure we find the diagonals to be,

$$AC = BD = 10\sqrt{2}cm$$
$$\Rightarrow AO = OC = DO = OB = 5\sqrt{2}cm$$

Now the repulsive force at O due to charge at A,

$$F_{AO} = k \frac{q_A q_O}{OA^2} = k \frac{(+2\mu C)(1\mu C)}{(5\sqrt{2})^2} \dots \dots (1)$$

And the repulsive force at O due to charge at D,

$$F_{DO} = k \frac{q_D q_O}{OD^2} = k \frac{(+2\mu C)(1\mu C)}{(5\sqrt{2})^2} \dots (2)$$

And the attractive force at O due to charge at B,

$$F_{BO} = k \frac{q_B q_O}{OB^2} = k \frac{(-5\mu C)(1\mu C)}{(5\sqrt{2})^2} \dots (3)$$

And the attractive force at O due to charge at C,

$$F_{\rm CO} = k \frac{q_{\rm C} q_{\rm O}}{OC^2} = k \frac{(-5\mu C)(1\mu C)}{(5\sqrt{2})^2} \dots (4)$$

We find that (1) and (2) are of same magnitude but they act in the opposite direction and hence they cancel out each other.

Similarly, (3) and (4) are of the same magnitude but in the opposite direction and hence they cancel out each other too.

Hence, the net force on charge at centre O is found to be zero.

7.

a) An electrostatic field line is a continuous curve. That is, a field line cannot have sudden breaks. Why not?

Ans: An electrostatic field line is a continuous curve as the charge experiences a continuous force on being placed in an electric field.

As the charge doesn't jump from one point to the other, field lines will not have sudden breaks.

b) Explain why two field lines never cross each other at any point?

Ans: If two field lines are seen to cross each other at a point, it would imply that the electric field intensity has two different directions at that point, as two different tangents (representing the direction of electric field intensity at that point) can be drawn at the point of intersection.

This is however impossible and thus, two field lines never cross each other.

8. Two point charges $q_A = 3\mu C$ and $q_B = -3\mu C$ are located 20cm apart in vacuum.

a) What is the electric field at the midpoint O of the line AB joining the two charges?

Ans: The situation could be represented in the following figure. Let O be the midpoint of line AB.



We are given:

AB = 20cm

AO = OB = 10cm

Take E to be the electric field at point O, then,

The electric field at point O due to charge $+3\mu$ C would be,

$$E_{1} = \frac{3 \times 10^{-6}}{4\pi\varepsilon_{0} (AO)^{2}} = \frac{3 \times 10^{-6}}{4\pi\varepsilon_{0} (10 \times 10^{-2})^{2}} NC^{-1} along OB$$

The electric field at point O due to charge $-3\mu C$ would be,

$$E_{2} = \left| \frac{3 \times 10^{-6}}{4\pi\epsilon_{0} (OB)^{2}} \right| = \frac{3 \times 10^{-6}}{4\pi\epsilon_{0} (10 \times 10^{-2})^{2}} NC^{-1} along OB$$

The net electric field,

$$\Rightarrow E = E_1 + E_2$$
$$\Rightarrow E = 2 \times \frac{9 \times 10^9 \times 3 \times 10^{-6}}{\left(10 \times 10^{-2}\right)^2}$$

 \Rightarrow E = 5.4×10⁶ NC⁻¹

Therefore, the electric field at mid-point O is $E = 5.4 \times 10^6 \text{ NC}^{-1}$ along OB.

b) If a negative test charge of magnitude 1.5×10^{-19} C is placed at this point, what is the force experienced by the test charge?

Ans: We have a test charge of magnitude 1.5×10^{-9} C placed at mid-point O and we found the electric field at this point to be $E = 5.4 \times 10^{6}$ NC⁻¹.

So, the force experienced by the test charge would be F,

$$\Rightarrow$$
 F = qE

$$\Rightarrow F = 1.5 \times 10^{-9} \times 5.4 \times 10^{6}$$

$$\Rightarrow$$
 F = 8.1×10⁻³ N

This force will be directed along OA since like charges repel and unlike charges attract.

9. A system has two charges $q_A = 2.5 \times 10^{-7}$ C and $q_B = -2.5 \times 10^{-7}$ C located at points A : (0,0,-15cm) and B : (0,0,+15cm) respectively. What are the total charge and electric dipole moment of the system?

Ans: The figure given below represents the system mentioned in the question:



The charge at point A, $q_A = 2.5 \times 10^{-7} C$

The charge at point B, $q_B = -2.5 \times 10^{-7} C$

Then, the net charge would be, $q = q_A + q_B = 2.5 \times 10^{-7} \text{ C} - 2.5 \times 10^{-7} \text{ C} = 0$

The distance between two charges at A and B would be,

$$d = 15 + 15 = 30 cm$$

d = 0.3m

The electric dipole moment of the system could be given by,

$$P = q_A \times d = q_B \times d$$
$$\implies P = 2.5 \times 10^{-7} \times 0.3$$

 $\therefore P = 7.5 \times 10^{-8} Cm$ along +z axis.

Therefore, the electric dipole moment of the system is found to be 7.5×10^{-8} Cm and it is directed along positive z-axis.

10. An electric dipole with dipole moment 4×10^{-9} Cm is aligned at 30° with direction of a uniform electric field of magnitude 5×10^4 NC⁻¹. Calculate the magnitude of the torque acting on the dipole.

Ans: We are given the following:

Electric dipole moment, $\vec{p} = 4 \times 10^{-9} \text{Cm}$

Angle made by \vec{p} with uniform electric field, $\theta = 30^{\circ}$

Electric field, $\vec{E} = 5 \times 10^4 \text{ NC}^{-1}$

Torque acting on the dipole is given by

$$\tau = pE\sin\theta$$

Substituting the given values we get,

$$\Rightarrow \tau = 4 \times 10^{-9} \times 5 \times 10^4 \times \sin 30^\circ$$

$$\Rightarrow \tau = 20 \times 10^{-5} \times \frac{1}{2}$$

 $\therefore \tau = 10^{-4} \, \text{Nm}$

Thus, the magnitude of the torque acting on the dipole is found to be 10^{-4} Nm.

11. A polythene piece rubbed with wool is found to have a negative charge of

3×10⁻⁷C

a) Estimate the number of electrons transferred (from which to which?)

Ans: When polythene is rubbed against wool, a certain number of electrons get transferred from wool to polythene.

As a result of which wool becomes positively charged on losing electrons and polythene becomes negatively charged on gaining them.

We are given:

Charge on the polythene piece, $q = -3 \times 10^{-7} C$

Charge of an electron, $e = -1.6 \times 10^{-19} C$

Let n be the number of electrons transferred from wool to polythene, then, from the property of quantization we have, q = ne

$$\Rightarrow$$
 n = $\frac{q}{e}$

Now, on substituting the given values, we get,

$$\Rightarrow n = \frac{-3 \times 10^{-7}}{-1.6 \times 10^{-19}}$$
$$\therefore n = 1.87 \times 10^{12}$$

 1.87×10^{12} .

Therefore, the number of electrons transferred from wool to polythene would be

b) Is there a transfer of mass from wool to polythene?

Ans: Yes, during the transfer of electrons from wool to polythene, along with charge, mass is transferred too.

Let m be the mass being transferred in the given case and m_e be the mass of the electron, then,

$$m = m_e \times n$$

$$\Rightarrow m = 9.1 \times 10^{-31} \times 1.85 \times 10^{12}$$

∴ m = 1.706 × 10⁻¹⁸ kg

Thus, we found that a negligible amount of mass does get transferred from wool to polythene.

12.

a) Two insulated charged copper spheres A and B have their centres separated by a distance of 50cm. What is the mutual force of electrostatic repulsion if the charge on each is 6.5×10^{-7} C? The radii of A and B are negligible compared to the distance of separation.

Ans: We are given:

Charges on spheres A and B are equal,

 $q_{\rm A} = q_{\rm B} = 6.5 \times 10^{-7} \rm C$

Distance between the centres of the spheres is given as,

r = 50 cm = 0.5 m

It is known that the force of repulsion between the two spheres would be given by Coulomb's law as,

$$F = \frac{q_A q_B}{4\pi\epsilon_0 r^2}$$

Where, ε_0 is the permittivity of the free space

Substituting the known values into the above expression, we get,

$$F = \frac{9 \times 10^9 \times (6.5 \times 10^{-7})^2}{(0.5)^2} = 1.52 \times 10^{-2} N$$

Thus, the mutual force of electrostatic repulsion between the two spheres is found to be $F = 1.52 \times 10^{-2} N$.

b) What is the force of repulsion if each sphere is charged double the above amount, and the distance between them is halved?

Ans: It is told that the charges on both the spheres are doubled and the distance between the centres of the spheres is halved. That is,

$$q_{\rm A}' = q_{\rm B}' = 2 \times 6.5 \times 10^{-7} = 13 \times 10^{-7} \,\mathrm{C}$$

 $r' = \frac{1}{2}(0.5) = 0.25 \,\mathrm{m}$

Now, we could substitute these values in Coulomb's law to get,

$$F' = \frac{q_A' q_B'}{4\pi\varepsilon_0 r'^2}$$
$$\Rightarrow F = \frac{9 \times 10^9 \times (13 \times 10^{-7})^2}{(0.25)^2}$$

 \Rightarrow F = 0.243N

The new mutual force of electrostatic repulsion between the two spheres is found to be 0.243N.

13. Figure below shows tracks taken by three charged particles in a uniform electrostatic field. Give the signs of the three charges and also mention which particle has the highest charge to mass ratio?



Ans: From the known properties of charges, we know that the unlike charges attract and like charges repel each other.

So, the particles 1 and 2 that move towards the positively charged plate while repelling away from the negatively charged plate would be negatively charged and the particle 3 that moves towards the negatively charged plate while repelling away from the positively charged plate would be positively charged.

Now, we know that the charge to mass ratio (which is generally known as emf) is directly proportional to the displacement or the amount of deflection for a given velocity.

Since the deflection of particle 3 is found to be maximum among the three, it would have the highest charge to mass ratio.

14. Consider a uniform electric field $E = 3 \times 10^3 \text{ iN} / \text{C}$.

a) Find the flux of this field through a square of side **10cm** whose plane is parallel to the y-z plane.

Ans: We are given:

Electric field intensity, $\vec{E} = 3 \times 10^3 \hat{i} N / C$

Magnitude of electric field intensity, $|\vec{E}| = 3 \times 10^3 \text{ N/C}$

Side of the square, a = 10cm = 0.1m

Area of the square, $A = a^2 = 0.01m^2$

Since the plane of the square is parallel to the y-z plane, the normal to its plane would be directed in the x direction. So, angle between normal to the plane and the electric field would be, $\theta = 0^{\circ}$

We know that the flux through a surface is given by the relation,

$$\phi = |\mathbf{E}||\mathbf{A}|\cos\theta$$

Substituting the given values, we get,

$$\Rightarrow \phi = 3 \times 10^3 \times 0.01 \times \cos 0^\circ$$

 $\therefore \phi = 30 \text{Nm}^2 / \text{C}$

Thus, we found the net flux through the given surface to be $\phi = 30 \text{Nm}^2 / \text{C}$.

b) What would be the flux through the same square if the normal to its plane makes 60° angle with the x-axis?

Ans: When the plane makes an angle of 60° with the x-axis, the flux through the given surface would be,

$$\phi = |\mathbf{E}| |\mathbf{A}| \cos \theta$$
$$\Rightarrow \phi = 3 \times 10^3 \times 0.01 \times \cos 60^\circ$$
$$\Rightarrow \phi = 30 \times \frac{1}{2}$$
$$\Rightarrow \phi = 15 \text{Nm}^2 / \text{C}$$

So, we found the flux in this case to be, $\phi = 15 \text{Nm}^2 / \text{C}$.

15. What is the net flux of the uniform electric field of exercise 1.15 through a cube of side 20cm oriented so that its faces are parallel to the coordinate planes?

Ans: We are given that all the faces of the cube are parallel to the coordinate planes.

Clearly, the number of field lines entering the cube is equal to the number of field lines entering out of the cube. As a result, the net flux through the cube would be zero.

16. Careful measurement of the electric field at the surface of a black box indicates that the net outward flux through the surface of the box is $8.0 \times 10^3 \text{Nm}^2/\text{C}$.

a) What is the net charge inside the box? Ans: We are given that:

Net outward flux through surface of the box, $\phi = 8.0 \times 10^3 \text{ Nm}^2 / \text{ C}$

For a body containing of net charge q, flux could be given by

$$\phi = \frac{q}{\epsilon_0}$$

Where, $\varepsilon_0 = 8.854 \times 10^{-12} \text{ N}^{-1} \text{C}^2 \text{m}^{-2}$ = Permittivity of free space

Therefore, the charge q is given by

$$q = \phi \varepsilon_0$$

$$\Rightarrow q = 8.854 \times 10^{-12} \times 8.0 \times 10^3$$

$$\Rightarrow q = 7.08 \times 10^{-8}$$

$$\Rightarrow q = 0.07 \mu C$$

Therefore, the net charge inside the box is found to be $0.07\mu C$.

b) If the net outward flux through the surface of the box were zero, could you conclude that there were no charges inside the box? Why or why not? Ans: No, the net flux entering out through a body depends on the net charge contained within the body according to Gauss's law.

So, if the net flux is given to be zero, then it can be inferred that the net charge inside the body is zero.

However, the net charge of the body being zero only implies that the body has equal amount of positive and negative charges and thus, we cannot conclude that there were no charges inside the box.

17. A point charge $+10\mu$ C is a distance 5cm directly above the centre of a square of side 10cm, as shown in Figure below. What is the magnitude of the electric flux through the square? (Hint: Think of the square as one face of a cube with edge 10cm)

Ans: Consider the square as one face of a cube of edge length 10cm with a charge q at its centre, according to Gauss's theorem for a cube, total electric flux is through all its six faces.



The electric flux through one face of the cube could be now given by, $\phi = \frac{\phi_{\text{total}}}{6}$.

$$\phi = \frac{1}{6} \frac{q}{\varepsilon_0}$$

 $\varepsilon_0 = 8.854 \times 10^{-12} \text{ N}^{-1} \text{C}^2 \text{m}^{-2} = \text{Permittivity of free space}$

The net charge enclosed would be, $q = 10\mu C = 10 \times 10^{-6} C$

Substituting the values given in the question, we get,

$$\phi = \frac{1}{6} \times \frac{10 \times 10^{-6}}{8.854 \times 10^{-12}}$$

$$\therefore \phi = 1.88 \times 10^5 \,\mathrm{Nm}^2 \mathrm{C}^{-1}$$

Therefore, electric flux through the square is found to be $1.88 \times 10^5 \text{ Nm}^2 \text{C}^{-1}$.

18. A point charge of 2.0μ C is kept at the centre of a cubic Gaussian surface of edge length 9cm. What is the net electric flux through this surface? Ans: Let us consider one of the faces of the cubical Gaussian surface considered (square).

Since a cube has six such square faces in total, we could say that the flux through one surface would be one-sixth the total flux through the gaussian surface considered.



The net flux through the cubical Gaussian surface by Gauss's law could be given by,

$$\phi_{\text{total}} = \frac{q}{\epsilon_0}$$

So, the electric flux through one face of the cube would be, $\phi = \frac{\phi_{\text{total}}}{6}$

$$\Rightarrow \phi = \frac{1}{6} \frac{q}{\varepsilon_0} \dots (1)$$

But we have,

$$\varepsilon_0 = 8.854 \times 10^{-12} \text{ N}^{-1} \text{C}^2 \text{m}^{-2} = \text{Permittivity of free space}$$

Charge enclosed, $q = 10\mu C = 10 \times 10^{-6} C$

Substituting the given values in (1) we get,

$$\phi = \frac{1}{6} \times \frac{10 \times 10^{-6}}{8.854 \times 10^{-12}}$$
$$\Rightarrow \phi = 1.88 \times 10^5 \,\mathrm{Nm}^2 \mathrm{C}^{-1}$$

Therefore, electric flux through the square surface is $1.88 \times 10^5 \text{ Nm}^2 \text{C}^{-1}$.

19. A point charge causes an electric flux of $-1.0 \times 10^3 \text{Nm}^2/\text{C}$ to pass through a spherical Gaussian surface of 10cm radius centred on the charge. a) If the radius of the Gaussian surface were doubled, how much flux could pass through the surface?

Ans: We are given:

Electric flux due to the given point charge, $\phi = -1.0 \times 10^3 \text{ Nm}^2 / \text{ C}$

Radius of the Gaussian surface enclosing the point charge, r = 10.0cm

Electric flux piercing out through a surface depends on the net charge enclosed by the surface according to Gauss's law and is independent of the dimensions of the arbitrary surface assumed to enclose this charge.

Hence, if the radius of the Gaussian surface is doubled, then the flux passing through the surface remains the same i.e., $-10^3 \text{Nm}^2 / \text{C}$.

b) What is the magnitude of the point charge?

Ans: Electric flux could be given by the relation,

$$\phi_{\text{total}} = \frac{q}{\epsilon_0}$$

Where, q = net charge enclosed by the spherical surface

$$\varepsilon_0 = 8.854 \times 10^{-12} \text{ N}^{-1} \text{C}^2 \text{m}^{-2} = \text{Permittivity of free space}$$

$$\Rightarrow q = \phi \varepsilon_0$$

Substituting the given values,

$$\Rightarrow$$
 q = -1.0×10³×8.854×10⁻¹² = -8.854×10⁻⁹C

$$\Rightarrow$$
q = -8.854nC

Thus, the value of the point charge is found to be -8.854nC.

20. A conducting sphere of radius 10cm has an unknown charge. If the electric field at a point 20cm from the centre of the sphere of magnitude $1.5 \times 10^3 \text{ N/C}$ is directed radially inward, what is the net charge on the sphere?

Ans: We have the relation for electric field intensity E at a distance (d) from the centre of a sphere containing net charge q is given by,

$$E = \frac{q}{4\pi\varepsilon_0 d^2} \dots \dots (1)$$

Where,

Net charge, $q = 1.5 \times 10^3 \text{ N} / \text{ C}$

Distance from the centre, d = 20cm = 0.2m

 $\varepsilon_0 = 8.854 \times 10^{-12} \text{ N}^{-1} \text{C}^2 \text{m}^{-2} = \text{Permittivity of free space}$

$$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \,\mathrm{Nm}^2\mathrm{C}^{-2}$$

From (1), the unknown charge would be,

$$q = E(4\pi\varepsilon_0)d^2$$

Substituting the given values we get,

$$\Rightarrow q = \frac{1.5 \times 10^3 \times (0.2)^2}{9 \times 10^9} = 6.67 \times 10^{-9} \text{C}$$
$$\Rightarrow q = 6.67 \text{nC}$$

Therefore, the net charge on the sphere is found to be 6.67nC.

21. A uniformly charged conducting sphere of 2.4m diameter has a surface charge density of $80.0 \mu C\,/\,m^2$.



a) Find the charge on the sphere.

Ans: Given that,

Diameter of the sphere, d = 2.4m.

Radius of the sphere, r = 1.2m.

Surface charge density,

 $\sigma \,{=}\, 80.0 \mu C \,{/}\,m^2 \,{=}\, 80 \,{\times}\, 10^{-6} C \,{/}\,m^2$

Total charge on the surface of the sphere,

 $Q = Charge density \times Surface area$

$$\Rightarrow Q = \sigma \times 4\pi r^2 = 80 \times 10^{-6} \times 4 \times 3.14 \times (1.2)^2$$

$$\Rightarrow$$
 Q = 1.447 × 10⁻³ C

Therefore, the charge on the sphere is found to be 1.447×10^{-3} C.

b) What is the total electric flux leaving the surface of the sphere?

Ans: Total electric flux (ϕ_{total}) leaving out the surface containing net charge Q is given by Gauss's law as,

$$\phi_{\text{total}} = \frac{Q}{\varepsilon_0} \dots \dots (1)$$

Where, permittivity of free space,

 $\epsilon_0 = 8.854 \times 10^{-12} \, N^{-1} C^2 m^{-2}$

We found the charge on the sphere to be,

$$Q = 1.447 \times 10^{-3} C$$

Substituting these in (1), we get,

$$\phi_{\text{total}} = \frac{1.447 \times 10^{-3}}{8.854 \times 10^{-12}}$$
$$\Rightarrow \phi_{\text{total}} = 1.63 \times 10^{-8} \,\text{NC}^{-1}\text{m}^{2}$$

Therefore, the total electric flux leaving the surface of the sphere is found to be $1.63 \times 10^{-8} \text{NC}^{-1} \text{m}^2$.

22. An infinite line charge produces a field of magnitude $9 \times 10^4 \text{ N/C}$ at a distance of 2cm. Calculate the linear charge density.

Ans: Electric field produced by the given infinite line charge at a distance d having linear charge density λ could be given by the relation,

$$E = \frac{\lambda}{2\pi\epsilon_0 d}$$

$$\Rightarrow \lambda = 2\pi\epsilon_0 Ed \dots (1)$$

We are given:

$$d = 2cm = 0.02m$$

$$E = 9 \times 10^4 N / C$$

Permittivity of free space,

$$\epsilon_0 = 8.854 \times 10^{-12} N^{-1} C^2 m^{-2}$$

Substituting these values in

Substituting these values in (1) we get, $\Rightarrow \lambda = 2\pi (8.854 \times 10^{-12}) (9 \times 10^4) (0.02)$

 $\Rightarrow \lambda = 10 \times 10^{-8} \mathrm{C} / \mathrm{m}$

Therefore, we found the linear charge density to be 10×10^{-8} C / m.

23. Two large, thin metal plates are parallel and close to each other. On their inner faces, the plates have surface charge densities of opposite signs and of magnitude 17.0×10^{-22} Cm⁻². What is E in the outer region of the first plate? What is E in the outer region of the second plate? What is E between the plates?

Ans: The given nature of metal plates is represented in the figure below:



Here, A and B are two parallel plates kept close to each other. The outer region of plate A is denoted as I, outer region of plate B is denoted as III, and the region between the plates, A and B, is denoted as II.

It is given that:

Charge density of plate A, $\sigma = 17.0 \times 10^{-22} \text{ C} / \text{m}^2$

Charge density of plate B, $\sigma = -17.0 \times 10^{-22} \text{ C} / \text{m}^2$

In the regions I and III, electric field E is zero. This is because the charge is not enclosed within the respective plates.

Now, the electric field E in the region II is given by

$$E = \frac{|\sigma|}{\epsilon_0}$$

Where,

Permittivity of free space $\epsilon_0 = 8.854 \times 10^{-12} \text{ N}^{-1} \text{C}^2 \text{m}^{-2}$

Clearly,

$$E = \frac{17.0 \times 10^{-22}}{8.854 \times 10^{-12}}$$

$$\Rightarrow$$
 E = 1.92 × 10⁻¹⁰ N / C

Thus, the electric field between the plates is $1.92 \times 10^{-10} \, N \, / \, C$.