Nuclei

13 Chapter

1. Obtain the binding energy (in MeV) of a nitrogen nucleus $\left({}^{14}_{7}N\right)$, given $m\left({}^{14}_{7}N\right) = 14.00307u$

Ans: We are given:

Atomic mass of nitrogen $\left({}_{7}N^{14}\right)$, $m = 14.00307u$

A nucleus of ${}_{7}N^{14}$ nitrogen contains 7 protons and 7 neutrons.

Hence, the mass defect of this nucleus would be, $\Delta m = 7m_H + 7m_n - m$

Where,

Mass of a proton, $m_H = 1.007825u$

Mass of a neutron, $m_n = 1.008665u$

Substituting these values into the above equation, we get,

$\Delta m = 7 \times 1.007825 + 7 \times 1.008665 - 14.00307$

$\Rightarrow \Delta m = 7.054775 + 7.06055 - 14.00307$

$\therefore \Delta m = 0.11236u$

But we know that, $1u = 931.5MeV/c^2$

$\Rightarrow \Delta m = 0.11236 \times 931.5MeV/c^2$

Now, we could give the binding energy as,

$E_b = \Delta mc^2$

Where, $c =$ speed of light $= 3 \times 10^8 ms^{-2}$

Now, $E_b = 0.11236 \times 931.5\left(\frac{MeV}{c^2}\right) \times c^2$

$\therefore E_b = 104.66334MeV$

Therefore, we found the binding energy of a Nitrogen nucleus to be $104.66334MeV$.

2. **Obtain the binding energy of the nuclei $^{56}_{26}Fe$ and $^{209}_{83}Bi$ in units of MeV from the following data: $m\left(^{56}_{26}Fe\right) = 55.934939u$, $m\left(^{209}_{83}Bi\right) = 208.980388u$**

Ans: We are given the following:

Atomic mass of $^{56}_{26}Fe$, $m_1 = 55.934939u$

$^{56}_{26}Fe$ nucleus has 26 protons and $56 - 26 = 30$ neutrons

Hence, the mass defect of the nucleus would be, $\Delta m = 26 \times m_H + 30 \times m_n - m_1$

Where, Mass of a proton, $m_H = 1.007825u$

Mass of a neutron, $m_n = 1.008665u$

Substituting these values into the above equation, we get,

$\Delta m = 26 \times 1.007825 + 30 \times 1.008665 - 55.934939$

$\Rightarrow \Delta m = 26.20345 + 30.25995 - 55.934939$

$\therefore \Delta m = 0.528461u$

But we have, $1u = 931.5 MeV / c^2$

$\Delta m = 0.528461 \times 931.5 MeV / c^2$

The binding energy of this nucleus could be given as,

$E_{b1} = \Delta mc^2$

Where, c = Speed of light

$$\Rightarrow E_{b1} = 0.528461 \times 931.5\left(\frac{MeV}{c^2}\right) \times c^2$$

$\therefore E_{b1} = 492.26 MeV$

Now, we have the average binding energy per nucleon to be,

$$B.E = \frac{492.26}{56} = 8.79 MeV$$

Also, atomic mass of $^{209}_{83}Bi$, $m_2 = 208.980388u$

We know that, $^{209}_{83}Bi$ nucleus has 83 protons and $209 - 83 = 126$ neutrons

Where,

Mass of a proton, $m_H = 1.007825u$

Mass of a neutron, $m_n = 1.008665u$

$\Delta m' = 83 \times 1.007825 + 126 \times 1.008665 - 208.980388$

$\Rightarrow \Delta m' = 83.649475 + 127.091790 - 208.980388$

$\therefore \Delta m' = 1.760877u$

But we know, $1u = 931.5 MeV / c^2$

Hence, the binding energy of this nucleus could be given as,

$$E_{b2} = \Delta m' c^2 = 1.760877 \times 931.5\left(\frac{MeV}{c^2}\right) \times c^2$$

$\therefore E_{b2} = 1640.26 MeV$

Average binding energy per nucleon is found to be $= \frac{1640.26}{209} = 7.848 MeV$

Hence, the average binding energy per nucleon is found to be $7.848 MeV$.

3. A given coin has a mass of 3.0 g. Calculate the nuclear energy that would be required to separate all the neutrons and protons from each other. For simplicity assume that the coin is entirely made of $^{63}_{29}Cu$ atoms (of mass $62.92960u$).

Ans: We are given:

Mass of a copper coin, $m' = 3g$

Atomic mass of $_{29}Cu^{63}$ atom, $m = 62.92960u$

The total number of $^{63}_{29}Cu$ atoms in the coin, $N = \frac{N_A \times m'}{\text{Mass number}}$

Where, $N_A =$ Avogadro's number $= 6.023 \times 10^{23}$ atoms / g

Mass number = 63g

$\Rightarrow N = \frac{6.023 \times 10^{23} \times 3}{63} = 2.868 \times 10^{22}$

$_{29}Cu^{63}$ nucleus has 29 protons and $(63 - 29) = 34$ neutrons

Mass defect of this nucleus would be, $\Delta m' = 29 \times m_H + 34 \times m_n - m$

Where,

Mass of a proton, $m_H = 1.007825u$

Mass of a neutron, $m_n = 1.008665u$

$\Delta m' = 29 \times 1.007825 + 34 \times 1.008665 - 62.9296 = 0.591935u$

Mass defect of all the atoms present in the coin would be,

$\Delta m = 0.591935 \times 2.868 \times 10^{22} = 1.69766958 \times 10^{22} u$

But we have, $1u = 931.5 MeV / c^2$

$\Rightarrow \Delta m = 1.69766958 \times 10^{22} \times 931.5 MeV / c^2$

Hence, the binding energy of the nuclei of the coin could be given as:

$E_b = \Delta mc^2 = 1.69766958 \times 10^{22} \times 931.5 \left(\frac{MeV}{c^2}\right) \times c^2$

$\therefore E_b = 1.581 \times 10^{25} MeV$

But, $1 MeV = 1.6 \times 10^{-13} J$

$\Rightarrow E_b = 1.581 \times 10^{25} \times 1.6 \times 10^{-13}$

$\therefore E_b = 2.5296 \times 10^{12} J$

This much energy is needed to separate all the neutrons and protons from the given coin.

4. Obtain approximately the ratio of the nuclear radii of the gold isotope $^{197}_{79}Au$ and the silver isotope $^{107}_{47}Ag$.

Ans: We know that,

Nuclear radius of the gold isotope ${}_{79}Au^{197} = R_{Au}$

Nuclear radius of the silver isotope ${}_{47}Ag^{107} = R_{Ag}$

Mass number of gold, $A_{Au} = 197$

Mass number of silver, $A_{Ag} = 107$

We also know that the ratio of the radii of the two nuclei is related with their mass numbers as:

$$\frac{R_{Au}}{R_{Ag}} = \left(\frac{A_{Au}}{A_{Ag}}\right)^{\frac{1}{3}} = 1.2256$$

Hence, the ratio of the nuclear radii of the gold and silver isotopes is found to be about 1.23.

5. The Q-value of a nuclear reaction $A + b \rightarrow C + d$ is defined by $Q = \left[m_A + m_b - m_C - m_d\right]c^2$ where the masses refer to the respective nuclei. Determine from the given data the Q-value of the following reactions and state whether the reactions are exothermic or endothermic.

Atomic masses are given to be: $m\left({}_1^2H\right) = 2.014102u$, $m\left({}_1^3H\right) = 3.016049u$, $m\left({}_6^{12}C\right) = 12.000000u$, $m\left({}_{10}^{20}Ne\right) = 19.992439u$

a) ${}_1^1H + {}_1^3H \rightarrow {}_1^2H + {}_1^2H$

The given nuclear reaction is:

${}_1^1H + {}_1^3H \rightarrow {}_1^2H + {}_1^2H$

Atomic mass of ${}_1^1H = 1.007825u$

Atomic mass of ${}_1^3H = 3.0164049u$

Atomic mass of ${}_1^2H = 2.014102u$

According to the question, the Q-value of the reaction could be written as:

$$Q = \left[m\left({}_1^1H\right) + m\left({}_1^3H\right) - 2m\left({}_1^2H\right)\right]c^2$$

$$\Rightarrow Q = \left[1.007825 + 3.016049 - 2 \times 2.014102\right]c^2 = \left(-0.00433c^2\right)u$$

But we know, $1u = 931.5 MeV / c^2$

$\therefore Q = -0.00433 \times 931.5 = -4.0334 MeV$

The negative Q-value of this reaction shows that the given reaction is endothermic.

b) ${}_6^{12}C + {}_6^{12}C \rightarrow {}_{10}^{20}Ne + {}_2^4He$

We are given that,

Atomic mass of ${}_6^{12}C = 12.0u$

Atomic mass of ${}^{12}_{10}\text{Ne} = 19.992439\text{u}$

Atomic mass of ${}^{4}_{2}\text{He} = 4.002603\text{u}$

The Q-value here could be given as,

$$Q = \left[2m\left({}^{12}_{6}\text{C}\right) - m\left({}^{20}_{10}\text{Ne}\right) - m\left({}^{4}_{2}\text{He}\right)\right]c^2$$

$$\Rightarrow Q = [2\times 12.0 - 19.992439 - 4.002603]c^2 = \left(0.004958c^2\right)\text{u} = 0.004958\times 931.5$$

$$\therefore Q = 4.618377\text{MeV}$$

Since the Q-value is found to be positive, the reaction could be considered exothermic.

6. Suppose, we think of fission of a ${}^{56}_{26}\text{Fe}$ nucleus into two equal fragments of ${}_{13}\text{Al}^{28}$. Is the fission energetically possible? Argue by working out Q of the process. Given: $m\left({}^{56}_{26}\text{Fe}\right) = 55.93494\text{u}$ and $m\left({}^{28}_{13}\text{Al}\right) = 27.98191\text{u}$

Ans: We know that the fission of ${}^{56}_{26}\text{Fe}$ could be given as,

$${}^{56}_{26}\text{Fe} \to 2\,{}^{28}_{13}\text{Al}$$

We are also given, atomic masses of ${}^{56}_{26}\text{Fe}$ and ${}^{28}_{13}\text{Al}$ as 55.93494u and 27.98191u respectively.

The Q-value here would be given as,

$$Q = \left[m\left({}^{56}_{26}\text{Fe}\right) - 2m\left({}^{28}_{13}\text{Al}\right)\right]c^2$$

$$\Rightarrow Q = [55.93494 - 2\times 27.98191]c^2 = \left(-0.02888c^2\right)\text{u}$$

But, $1\text{u} = 931.5\text{MeV}/c^2$

$$\therefore Q = -0.02888\times 931.5 = -26.902\text{MeV}$$

The Q value is found to be negative and hence we could say that the fission is not possible energetically. In order for a reaction to be energetically possible, the Q-value must be positive.

7. The fission properties of ${}^{239}_{94}\text{Pu}$ are very similar to those of ${}^{235}_{92}\text{U}$ .The average energy released per fission is 180MeV. How much energy, in MeV, is released if all the atoms in 1kg of pure ${}^{239}_{94}\text{Pu}$ undergo fission?

Ans: We are given that the average energy released per fission of ${}^{239}_{94}\text{Pu}$,

$E_{av} = 180\text{MeV}$

The amount of pure ${}_{94}\text{Pu}^{239}$, $m = 1\text{kg} = 1000\text{g}$

Avogadro number, $N_A = 6.023\times 10^{23}$

Mass number of ${}^{239}_{94}\text{Pu} = 239\text{g}$

1 mole of ${}_{94}Pu^{239}$ contains Avogadro number of atoms.

1g of ${}_{94}Pu^{239}$ contains $\left(\frac{N_A}{\text{mass number}} \times m\right)$ atoms

$$\Rightarrow \left(\frac{6.023 \times 10^{23}}{239} \times 1000\right) = 2.52 \times 10^{24} \text{atoms}$$

Total energy released during the fission of 1kg of ${}^{239}_{94}Pu$ could be calculated as:

$$E = E_{av} \times 2.52 \times 10^{24} = 180 \times 2.52 \times 10^{24} = 4.536 \times 10^{26} \text{MeV}$$

Therefore, 4.536×10^{26} MeV is released if all the atoms in 1kg of pure ${}_{94}Pu^{239}$ undergo fission.

8. How long can an electric lamp of 100W be kept glowing by fusion of 2.0kg of deuterium? Take the fusion reaction as

$$\mathbf{{}^2_1H + {}^2_1H \rightarrow {}^3_2He + n + 3.27MeV}$$

Ans: The fusion reaction is given to be:

$${}^2_1H + {}^2_1H \rightarrow {}^3_2He + n + 3.27\text{MeV}$$

Amount of deuterium, $m = 2$ kg

1 mole, i.e., 2 g of deuterium contains 6.023×10^{23} atoms.

2.0 kg of deuterium contains $\frac{6.023 \times 10^{23}}{2} \times 2000 = 6.023 \times 10^{26}$ atoms atoms

It could be inferred from the given reaction that when two atoms of deuterium fuse, 3.27MeV energy is released.

Therefore, the total energy per nucleus released in the fusion reaction would be:

$$E = \frac{3.27}{2} \times 6.023 \times 10^{26} \text{MeV} = \frac{3.27}{2} \times 6.023 \times 10^{26} \times 1.6 \times 10^{-19} \times 10^{6}$$

$$\therefore E = 1.576 \times 10^{14} \text{J}$$

Power of the electric lamp is given to be, $P = 100W = 100J/s$, that is, the energy consumed by the lamp per second is 100J.

Now, the total time for which the electric lamp glows could be calculated as,

$$t = \frac{1.576 \times 10^{14}}{100} = \frac{1.576 \times 10^{14}}{100 \times 60 \times 60 \times 24 \times 365}$$

$$\therefore t \approx 4.9 \times 10^{4} \text{years}$$

Hence, the total time for which the electric lamp glows is found to be 4.9×10^{4} years.

9. Calculate the height of the potential barrier for a head on collision of two deuterons. (Hint: The height of the potential barrier is given by the Coulomb repulsion between the two deuterons when they just touch each other. Assume that they can be taken as hard spheres of radius 2.0fm.)

Ans: When two deuterons collide head-on, the distance between their centres, d could be given as:

Radius of 1st deuteron $+$ Radius of 2nd deuteron

Radius of a deuteron nucleus $= 2\text{fm} = 2\times10^{-15}\text{m}$

$\Rightarrow d = 2\times10^{-15} + 2\times10^{-15} = 4\times10^{-15}\text{m}$

Also, charge on a deuteron $=$ Charge on an electron $= e = 1.6\times10^{-19}\text{C}$

Potential energy of the two-deuteron system could be given by,

$$V = \frac{e^2}{4\pi\varepsilon_0 d}$$

Where, ε_0 is the permittivity of free space.

Also, $\frac{1}{4\pi\varepsilon_0} = 9\times10^9\,\text{Nm}^2\text{C}^{-2}$

$$\Rightarrow V = \frac{9\times10^9\times\left(1.6\times10^{19}\right)^2}{4\times10^{15}}\text{J} = \frac{9\times10^9\times\left(1.6\times10^{-19}\right)^2}{4\times10^{-15}\times\left(1.6\times10^{-19}\right)}\text{eV}$$

$\therefore V = 360\text{keV}$

Therefore, we found the height of the potential barrier of the two-deuteron system to be 360keV.

10. From the relation $\mathbf{R = R_0A^{\frac{1}{3}}}$, where $\mathbf{R_0}$ is a constant and A is the mass number of a nucleus, show that the nuclear matter density is nearly constant (i.e., independent of A).

Ans: We know the expression for nuclear radius to be:

$$R = R_0A^{\frac{1}{3}}$$

Where, R_0 is a Constant and A is the mass number of the nucleus

Nuclear matter density would be,

$$\rho = \frac{\text{Mass of the nucleus}}{\text{Volume of the nucleus}}$$

Now, let m be the average mass of the nucleus, then, mass of the nucleus $= mA$

Nuclear density,

$$\rho = \frac{mA}{\frac{4}{3}\pi R^3} = \frac{3mA}{4\pi\left(R_0A^{\frac{1}{3}}\right)^3} = \frac{3mA}{4\pi {R_0}^3 A}$$

$$\therefore \rho = \frac{3m}{4\pi {R_0}^3}$$

Therefore, we found the nuclear matter density to be independent of A and it is found to be nearly constant.