Atoms

12 Chapter

1. Fill in the blanks using the given options:

a) The size of the atoms in Thomson's model are _______ the atomic size in Rutherford's model (much greater than/no different from/much lesser than).

Ans: The sizes of the atoms in Thomson's model are no different from the atomic size in Rutherford's model.

b) In the ground state of _______ electrons are in stable equilibrium, while in _______ electrons always experience a net force. (Thomson's model/Rutherford's model)

Ans: In the ground state of Thomson's model, the electrons are in stable equilibrium, while in Rutherford's model, electrons always experience a net force.

c) A classical atom based on _______ is doomed to collapse. (Thomson's model/ Rutherford's model.)

Ans: A classical atom based on Rutherford's model is doomed to collapse.

d) An atom features a nearly continuous mass distribution in _______ but features a highly non- uniform mass distribution in _______ (Thomson's model/ Rutherford's model.)

Ans: An atom features a nearly continuous mass distribution in Thomson's model, but features a highly non-uniform mass distribution in Rutherford's model.

e) The positively charged part of the atom possesses most of the mass in _______ (Rutherford's model/ Thomson's model /both the models.)

Ans: The positively charged part of the atom possesses most of the mass in both the models.

2. If you' re given a chance to repeat the $\alpha-$particle scattering experiment employing a thin sheet of solid hydrogen instead of the gold foil (Hydrogen is a solid at temperatures below 14 K). What results would you expect?

Ans: In the $\alpha-$particle scattering experiment, when a thin sheet of solid hydrogen is replaced with the gold foil, the scattering angle would not turn out to be large enough.

This is because the mass of hydrogen is smaller than the mass of incident $\alpha-$particles. Also, the mass of the scattering particle is more than the target nucleus (hydrogen).

As a consequence, the $\alpha-$particles would not bounce back when solid hydrogen is utilized in the $\alpha-$particle scattering experiment and hence, we cannot evaluate the size of the hydrogen nucleus.

3. The two energy levels in an atom are separated by a difference of 2.3eV. What is the frequency of radiation emitted when the atom makes a transition from the higher level to the lower level?

Ans: Given that the distance between the two energy levels in an atom is $E = 2.3\text{eV}$.

$\Rightarrow E = 2.3 \times 1.6 \times 10^{-19}$

$\Rightarrow E = 3.68 \times 10^{-19}\,\text{J}$

Let ν be the frequency of radiation emitted when the atom jumps from the upper level to the lower level.

The relation for energy is given as;

$E = h\nu$

Here,

$h =$ Planck's constant $= 6.6 \times 10^{-34}\,\text{Js}$

$$\Rightarrow \nu = \frac{E}{h}$$

$$\Rightarrow \nu = \frac{3.38 \times 10^{-19}}{6.62 \times 10^{-32}}$$

$\Rightarrow \nu = 5.55 \times 10^{14}\,\text{Hz}$

Clearly, the frequency of the radiation is $5.6 \times 10^{14}\,\text{Hz}$.

4. The ground state energy of hydrogen atom is –13.6 eV. What are the kinetic and potential energies of the electron in this state?

Ans: Provided that the ground state energy of hydrogen atom, $E = -13.6\text{eV}$ which is the total energy of a hydrogen atom.

Here, kinetic energy is equal to the negative of the total energy.

Kinetic energy $= -E = -(-13.6) = 13.6\text{eV}$

The potential energy is the same as the negative of two times kinetic energy.

Potential energy $= -2 \times (13.6) = -27.2\text{eV}$

$\therefore$ The kinetic energy of the electron is 13.6eV and the potential energy is -27.2eV .

5. A hydrogen atom absorbs a photon when it is in the ground level, this excites it to the $n = 4$ level. Find out the wavelength and frequency of the photon.

Ans: It is known that for ground level absorption, $n_1 = 1$

Let E_1 be the energy of this level. It is known that E_1 is related with n_1 as;

$$E_1 = \frac{-13.6}{n_1^2}\text{eV}$$

$$\Rightarrow E_1 = \frac{-13.6}{1^2} = -13.6\text{eV}$$

When the atom jumps to a higher level, $n_2 = 4$.

Let E_2 be the energy of this level.

$$\Rightarrow E_2 = \frac{-13.6}{n_2^2}\text{eV}$$

$$\Rightarrow E_2 = \frac{-13.6}{4^2} = \frac{-13.6}{16}\text{eV}$$

The amount of energy absorbed by the photon is given as;

$$E = E_1 - E_2$$

$$\Rightarrow E = \left(\frac{-13.6}{16}\right) - \left(\frac{-13.6}{1}\right)$$

$$\Rightarrow E = \frac{13.6\times 15}{16}\text{eV}$$

$$\Rightarrow E = \frac{13.6\times 16}{16}\times 1.6\times 10^{-19}$$

$$\Rightarrow E = 2.04\times 10^{-18}\,\text{J}$$

For a photon of wavelength λ , the expression of energy is written as;

$$E = \frac{hc}{\lambda}$$

Here,

$h =$ Planck’s constant $= 6.6\times 10^{-34}\,\text{Js}$

$c =$ speed of light $= 3\times 10^{8}\,\text{m/s}$

$$\Rightarrow \lambda = \frac{hc}{E}$$

$$\Rightarrow \lambda = \frac{6.6\times 10^{-34}\times 3\times 10^{8}}{2.04\times 10^{-18}}$$

$$\Rightarrow \lambda = 9.7\times 10^{-8}\,\text{m}$$

$$\Rightarrow \lambda = 97\,\text{nm}$$

Also, frequency of a photon is given by the relation,

$$\nu = \frac{c}{\lambda}$$

$$\Rightarrow \nu = \frac{3\times 10^{8}}{9.7\times 10^{-8}} \approx 3.1\times 10^{15}\,\text{Hz}$$

Clearly, the wavelength of the photon is 97nm whereas the frequency is $3.1\times 10^{15}\,\text{Hz}$.

6. Answer the following questions.

a) Using the Bohr’s model, calculate the speed of the electron in a hydrogen atom in the $n = 1, 2$ and 3 levels.

Ans: Consider ν_1 to be the orbital speed of the electron in a hydrogen atom in the ground state level $n_1 = 1$. For charge (e) of an electron, ν_1 is given by the relation,

$$\nu_1 = \frac{e^2}{n_1 4\pi \in_0 \left(\frac{h}{2\pi}\right)}$$

$$\Rightarrow \nu_1 = \frac{e^2}{2 \in_0 h}$$

Here,

$e = 1.6 \times 10^{-19} C$

$\in_0 =$ Permittivity of free space $= 8.85 \times 10^{-12} N^{-1} C^2 m^{-2}$

$h =$ Planck's constant $= 6.6 \times 10^{-34} Js$

$$\Rightarrow \nu_1 = \frac{\left(1.6 \times 10^{-19}\right)^2}{2 \times 8.85 \times 10^{-12} \times 6.62 \times 10^{-34}}$$

$$\Rightarrow \nu_1 = 0.0218 \times 10^8$$

$$\Rightarrow \nu_1 = 2.18 \times 10^6 m/s$$

For level $n_2 = 2$, we can write the relation for the corresponding orbital speed as;

$$\nu_2 = \frac{e^2}{n_2 2 \in_0 h}$$

$$\Rightarrow \nu_2 = \frac{\left(1.16 \times 10^{-19}\right)^2}{2 \times 2 \times 8.85 \times 10^{-12} \times 6.62 \times 10^{-34}}$$

$$\Rightarrow \nu_2 = 1.09 \times 10^6 m/s$$

And, for $n_3 = 3$, we can write the relation for the corresponding orbital speed as;

$$\nu_3 = \frac{e^2}{n_3 2 \in_0 h}$$

$$\Rightarrow \nu_3 = \frac{\left(1.16 \times 10^{-19}\right)^2}{3 \times 2 \times 8.85 \times 10^{-12} \times 6.62 \times 10^{-34}}$$

$$\Rightarrow \nu_3 = 7.27 \times 10^5 m/s$$

Clearly, the speeds of the electron in a hydrogen atom in the levels $n = 1, 2$ and 3 are $2.18 \times 10^6 m/s$, $1.09 \times 10^6 m/s$ and $7.27 \times 10^5 m/s$ respectively.

b) Calculate the orbital period in each of these levels.

Ans: Consider T_1 to be the orbital period of the electron when it is in level $n_1 = 1$.

It is known that the orbital period is related to the orbital speed as

$$T_1 = \frac{2\pi r_1}{\nu_1}$$

Here,

$r_1 =$ Radius of the orbit in $n_1 = \frac{n_1^2 h^2 \in_0}{\pi m e^2}$

$h =$ Planck's constant $= 6.6 \times 10^{-34}$ Js

$e =$ Charge of an electron $= 1.6 \times 10^{-19}$ C

$\in_0 =$ Permittivity of free space $= 8.85 \times 10^{-12}\ N^{-1}C^2m^{-2}$

$m =$ Mass of an electron $= 9.1 \times 10^{-31}$ kg

$$\Rightarrow T_1 = \frac{2\pi \times (1)^2 \times \left(6.62 \times 10^{-34}\right)^2 \times 8.85 \times 10^{-12}}{2.18 \times 10^6 \times \pi \times 9.1 \times 10^{-31} \times \left(1.6 \times 10^{-19}\right)^2}$$

$$\Rightarrow T_1 = 15.27 \times 10^{-17}$$

$$\Rightarrow T_1 = 1.527 \times 10^{-16} s$$

For level $n_2 = 2$, we can write the orbital period as;

$$T_2 = \frac{2\pi r_2}{\nu_2}$$

Here,

$r_2 =$ Radius of the orbit in $n_2 = \frac{n_2^2 h^2 \in_0}{\pi m e^2}$

$$\Rightarrow T_1 = \frac{2\pi \times (2)^2 \times \left(6.62 \times 10^{-34}\right)^2 \times 8.85 \times 10^{-12}}{1.09 \times 10^6 \times \pi \times 9.1 \times 10^{-31} \times \left(1.6 \times 10^{-19}\right)^2} = 1.22 \times 10^{-15} s$$

And for the level $n_3 = 3$, we can write the orbital period as;

$$T_3 = \frac{2\pi r_3}{\nu_3}$$

Here,

$r_3 =$ Radius of the orbit in $n_3 = \frac{n_3^2 h^2 \in_0}{\pi m e^2}$

$$\Rightarrow T_3 = \frac{2\pi \times (3)^2 \times \left(6.62 \times 10^{-34}\right)^2 \times 8.85 \times 10^{-12}}{7.27 \times 10^5 \times \pi \times 9.1 \times 10^{-31} \times \left(1.6 \times 10^{-19}\right)^2} = 4.12 \times 10^{-15} s$$

Hence, the orbital periods in the levels $n = 1, 2$ and 3 are $1.527 \times 10^{-16} s$, $1.22 \times 10^{-15} s$ and $4.12 \times 10^{-15} s$ respectively.

7. The innermost electron orbit of a hydrogen atom has a radius of $5.3\times10^{-11}\,m$. What are the radii of the $n=2$ and $n=3$ orbits?

Ans: Provided that the innermost radius, $r_1 = 5.3\times10^{-11}\,m$.

Let r_2 be the radius of the orbit at $n=2$. It is related to the radius of the innermost orbit as;

$r_2 = (n)^2 r_1$

$\Rightarrow r_2 = (2)^2 \times 5.3\times10^{-11} = 2.1\times10^{-10}\,m$

Similarly, for $n=3$;

$r_3 = (n)^2 r_1$

$\Rightarrow r_3 = (3)^2 \times 5.3\times10^{-11} = 4.77\times10^{-10}\,m$

Clearly, the radii of the $n=2$ and $n=3$ orbits are $2.1\times10^{-10}\,m$ and $4.77\times10^{-10}\,m$ respectively.

8. A $12.5\,eV$ electron beam is used to bombard gaseous hydrogen at room temperature. What series of wavelengths will be emitted?

Ans: It is provided that the energy of the electron beam used to bombard gaseous hydrogen at room temperature is $12.5\,eV$.

It is also known that the energy of the gaseous hydrogen in its ground state at room temperature is $-13.6\,eV$.

When gaseous hydrogen is bombarded with an electron beam at room temperature, the energy of the gaseous hydrogen becomes $-13.6+12.5\,eV = -1.1\,eV$.

Now, the orbital energy is related to orbit level (n) as;

$$E = \frac{-13.6}{(n)^2}\,eV$$

For $n=3; E = \frac{-13.6}{9} = -1.5\,eV$

This energy is approximately equal to the energy of gaseous hydrogen.

So, it can be concluded that the electron has excited from $n=1$ to $n=3$ level.

During its de-excitation, the electrons can jump from $n=3$ to $n=1$ directly, which forms a line of the Lyman series of the hydrogen spectrum.

The formula for wave number for Lyman series is given as;

$$\frac{1}{\lambda} = R_y\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$$

Here,

$R_y =$ Rydberg constant $= 1.097\times10^7\,m^{-1}$

$\lambda =$ Wavelength of radiation emitted by the transition of the electron
Using this relation for $n=3$ we get,

$$\frac{1}{\lambda}=1.097\times 10^{7}\left(\frac{1}{1^{2}}-\frac{1}{3^{2}}\right)$$

$$\Rightarrow \frac{1}{\lambda}=1.097\times 10^{7}\left(1-\frac{1}{9}\right)$$

$$\Rightarrow \frac{1}{\lambda}=1.097\times 10^{7}\left(\frac{8}{9}\right)$$

$$\Rightarrow \lambda =\frac{9}{8\times 1.097\times 10^{7}}=102.55\text{nm}$$

If the transition takes place from $n=3$ to $n=2$, and then from $n=2$ to $n=1$, then the wavelength of the radiation emitted in transition from $n=3$ to $n=2$ is given as;

$$\frac{1}{\lambda}=1.097\times 10^{7}\left(\frac{1}{2^{2}}-\frac{1}{3^{2}}\right)$$

$$\Rightarrow \frac{1}{\lambda}=1.097\times 10^{7}\left(\frac{1}{4}-\frac{1}{9}\right)$$

$$\Rightarrow \frac{1}{\lambda}=1.097\times 10^{7}\left(\frac{5}{36}\right)$$

$$\Rightarrow \lambda =\frac{36}{5\times 1.097\times 10^{7}}=656.33\text{nm}$$

This radiation corresponds to the Balmer series of the hydrogen spectrum.
Now, the wavelength of the radiation when the transition takes place from $n=2$ to $n=1$ is given as;

$$\frac{1}{\lambda}=1.097\times 10^{7}\left(\frac{1}{1^{2}}-\frac{1}{2^{2}}\right)$$

$$\Rightarrow \frac{1}{\lambda}=1.097\times 10^{7}\left(1-\frac{1}{4}\right)$$

$$\Rightarrow \frac{1}{\lambda}=1.097\times 10^{7}\left(\frac{3}{4}\right)$$

$$\Rightarrow \lambda =\frac{4}{3\times 1.097\times 10^{7}}=121.54\text{nm}$$

Clearly, in the Lyman series, two wavelengths are emitted i.e., 102.5nm and 121.5nm whereas in the Balmer series, only one wavelength is emitted i.e., 656.33nm.

9. In accordance with the Bohr's model, what is the quantum number that characterizes the earth's revolution around the sun in an orbit of radius $1.5\times10^{11}m$ with an orbital speed of $3\times10^{4}m/s$. The mass of the earth is given as $6\times10^{24}kg$.

Ans: Here, it is provided that,

Radius of the earth's orbit around the sun, $r = 1.5\times10^{11}m$

Orbital speed of the earth, $v = 3\times10^{4}m/s$

Mass of the earth, $m = 6\times10^{24}kg$

With respect to the Bohr's model, angular momentum is quantized and is given as;

$$mvr = \frac{nh}{2\pi}$$

Here,

$h =$ Planck's constant $= 6.6\times10^{-34}Js$

$n =$ Quantum number

$$n = \frac{mvr^2\pi}{h}$$

$$\Rightarrow n = \frac{2\pi\times6\times10^{24}\times3\times10^{4}\times1.5\times10^{11}}{6.62\times10^{-34}}$$

$$\Rightarrow n = 25.61\times10^{73} = 2.6\times10^{74}$$

Clearly, the quantum number that characterizes the earth's revolution around the sun is 2.6×10^{74} .