Dual Nature of Radiation & Matter



EXERCISE

1:

Find the

- a) maximum frequency, and
- b) Minimum wavelength of X-rays produced by 30 kV electrons.

Solution:

a) We know,

$$K_{\text{max}} = hv - \Phi_0$$

$$Or eV_0 = hv - \Phi_0$$

$$v_{\text{max}} = eV_0 / h$$

Substituting the required values

$$v_{\text{max}} = (1.6 \times 10^{-19}).(30000)/(6.63 \times 10^{-34})$$

$$=7.24\times10^{18}\,Hz$$

(b) By the relation
$$\lambda = c / v_{\text{max}}$$
$$= 0.041 nm$$

2:

The work function of caesium metal is 2.14 eV, When light of frequency 6×10^{14} Hz is incident on the metal surface, photoemission of electrons occurs.

- a) What is the maximum kinetic energy of the emitted electrons,
- b) Stopping potential, and
- c) Maximum speed of the emitted photoelectrons?

Solution:

Given:

$$\Phi_0 = 2.14V$$

$$v = 6 \times 10^{-14} \, Hz$$

a) Maximum kinetic energy is the difference between the photon energy and the work function of the metal. For caesium it is given to be 2.14eV

Using the expression

$$K_{\text{max}} = hv - \Phi_0$$

Substitution yields

$$K_{\text{max}} = \left[\left(6.63 \times 10^{-34} \right) \cdot \left(6 \times 10^{14} \right) \right] - \left[\left(2.114 \right) \cdot \left(1.6 \times 10^{19} \right) \right]$$

 $=5.54\times10^{-20}J$

=0.346eV

b) Stopping potential can be deduced by maximum kinetic energy, by the expression

 $eVo = K \max$

Or $Vo = K \max / e$

Putting the required values

$$Vo = 5.54 \times 10^{-20} / 1.6 \times 10^{-19}$$

= 0.34 V

c) Maximum speed of photoelectron is given by the maximum kinetic energy it possesses,

$$Kmax = 5.54 \times 10^{-20}$$

Or
$$mv^2/2 = 5.54 \times 10^{-20}$$

Or
$$v_{max} = 344 \text{ km/s}$$

3:

The photoelectric cut-off voltage in a certain experiment is 1.5 V. What is the maximum kinetic energy of photoelectrons emitted?

Solution:

Maximum kinetic energy,

Kmax = eVo

Putting values and solving

$$=2.4\times10^{-19}J$$

4:

Monochromatic light of wavelength 632.8 nm is produced by a helium-neon laser. The power emitted is 9.42 m W.

- a) Find the energy and momentum of each photon in the light beam,
- b) How many photons per second, on the average, arrive at a target irradiated by this beam? (Assume the beam to have uniform cross-section which is less than the target area), and
- c) How fast does a hydrogen atom have to travel in order to have the same momentum as that of the photon?

Solution:

Given:

Wavelength of monochromatic light

 $\lambda = 632.8nm$

Power of He-be laser

P = 9.42mW

(a) Energy of a photon is given by

E = hv

Or $E = hc / \lambda$

Which gives

$$E = 3.14 \times 10^{-19} J$$

Now momentum of a photon

$$p = h / \lambda$$

$$p = 1.05 \times 10^{-27} kg \ m \ / \ s$$

b) For a beam of uniform cross-section having cross-sectional area less than target area

 $P = E \times N$

where

P = power emitted

E = energy of photon

N = number of photons

Therefore

N = P / E

Substitution gives

$$N = 3 \times 10^{16}$$
 photons / second

(c) Momentum of He-Ne laser = 1.05×10^{-27} kg m/s

For this much momentum of a hydrogen atom

$$mv = 1.05 \times 10^{-27}$$

Or
$$v = 1.05 \times 10^{-27} / 1.6 \times 10^{-27}$$

Or
$$v = 0.63 \text{ m/s}$$

The required speed for hydrogen atom is 0.63 m/s.

5:

In an experiment on photoelectric effect, the slope of the cut-off voltage versus frequency of incident light is found to be $4.12 \times 10^{-15}~V$ s. Calculate the value of Planck's constant.

Solution:

Slope =
$$4.12 \times 10^{-15}$$

Or
$$h/e = 4.12 \times 10^{-15}$$

Putting value of e and solving

$$h = 6.592 \times 10^{-34} JS$$

The threshold frequency for a certain metal is 3.3×10^{14} Hz. If light of frequency 8.2×10^{14} Hz is incident on the metal, predict the cutoff-voltage for the photoelectric emission.

Solution:

Given:

$$v_0 = 3.3 \times 10^{14} Hz$$

$$v = 8.2 \times 10^{14} Hz$$

Cut=off voltage

$$eV_0 = h(v - v_0)$$

Substituting the values

$$V_0 = 2.03 \, eV$$

7:

The work function for a certain metal is 4.2 eV. Will this metal give photoelectric emission for incident radiation of wavelength 330nm?

Solution:

$$\phi_0 = 4.2 eV$$

Or
$$v_0 h = 4.2$$

Or
$$v_0 = 1.01 \times 10^{15} Hz$$

Incident wavelength = 330nm

Or
$$v = 6 \times 10^{-19} Hz$$

Since v is less than v_0 , there will be no photoelectric emission.

8:

Light of frequency 7.21×10^{14} Hz is incident on a metal surface. Electrons with a maximum speed of 6.0×10^5 m/s are ejected from the surface. What is the threshold frequency for photoemission of electrons?

Solution:

Given:

$$v = 7.21 \times 10^{14} Hz$$

$$v = 6 \times 10^5 \, m/s$$

Kinetic energy = $h(v-v_0)$

$$Or mv^2 / 2 = h(v - v_0)$$

Solving the above equation

$$v_0 = 4.73 \times 10^{14} Hz$$

9:

Light of wavelength 488 nm is produced by an argon laser which is used in the photoelectric effect. When light from this spectral line is incident on the emitter, the stopping (cut-off) potential of photoelectrons is 0.38 V. Find the work function of the material from which the emitter is made.

Solution:

Given:

$$\lambda = 488nm$$

$$V_0 = 0.38V$$

We Know

$$eVo = hv - \Phi o$$

Or

$$\Phi o = hc / \lambda - eVo$$

$$=3.46\times10^{-19}J$$

$$= 2.16 eV$$

10:

What is the de Broglie wavelength of

- a) a bullet of mass 0.040 kg travelling at the speed of 1.0 km/s,
- b) a ball of mass 0.060 kg moving at a speed of 1.0 m/s, and
- c) a dust particle of mass 1.0×10^{-9} kg drifting with a speed of 2.2 m/s?

Solution:

a)
$$m = 0.04 \text{ kg}$$

$$v = 1 \text{ km/s}$$

$$\lambda = h / mv$$

$$=1.66\times10^{-35}m$$

b)
$$m = 0.06 \text{ kg}$$

$$v = 1 \text{ m/s}$$

$$\lambda = 1.1 \times 10^{-32} m$$

c)
$$m = 1 \times 10^{-9} kg$$

$$v = 2.2m/s$$

$$\lambda = 3 \times 10^{-25} m$$

11:

Show that the wavelength of electromagnetic radiation is equal to the de Broglie wavelength of its quantum (photon).

Solution:

For a photon,

$$p = hv/c$$

Therefore,

$$h/p = c/v = \lambda$$

That is, the de Broglie wavelength of a photon equals the wavelength of electromagnetic radiation of which the photon is a quantum of energy and momentum.