Wave Optics

10 Chapter

1. Monochromatic light of wavelength 589nm is incident from air on a water surface. What are the wavelength, frequency and speed of

(a) Reflected light?

Ans: Wavelength of incident monochromatic light is given as,

$\lambda = 589\text{nm} = 589 \times 10^{9}\text{m}$

Speed of light in air is $c = 3 \times 10^{8}\text{m}$

Refractive index of water is $\mu = 1.33$

In this case, the ray will reflect in the same medium as that of incident ray. Hence, the wavelength, speed, and frequency of the reflected ray will be the equal to that of the incident ray.

Frequency of light can be given by the relation,

$$\nu = \frac{c}{\lambda}$$

Where,

$\nu =$ Frequency of light

$c =$ Speed of light

$\lambda =$ Wavelength of light

$$\Rightarrow \nu = \frac{3 \times 10^{8}}{589 \times 10^{-9}}$$

$$\Rightarrow \nu = 5.09 \times 10^{14}\text{Hz}$$

Hence, the speed, frequency, and wavelength of the reflected light are $3 \times 10^{8}\text{m/s}$, $5.09 \times 10^{14}\text{Hz}$ and 589nm respectively.

(b) refracted light? Refractive index of water is 1.33.

Ans: The frequency of light is independent of the property of the medium in which it is travelling.

Hence, the frequency of the refracted ray in water will be equal to the frequency of the incident light or reflected light in air.

Frequency of the refracted light ray, $\nu = 5.09 \times 10^{14}\text{Hz}$

Speed of light in water is related to the refractive index of water as given in the formula below:

$$v = \frac{c}{\mu}$$

$$\Rightarrow v = \frac{3\times10^8}{1.33}$$

$$\Rightarrow v = 2.26\times10^8 m/s$$

The formula below gives the relation of wavelength of light in water and the speed and frequency of light,

$$\lambda = \frac{v}{\nu}$$

$$\Rightarrow \lambda = \frac{2.26\times10^8}{5.09\times10^{14}}$$

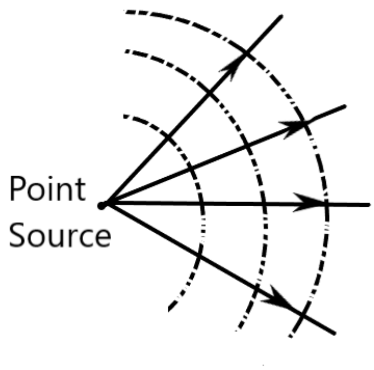
$$\Rightarrow \lambda = 444.007\times10^{-9} m$$

$$\Rightarrow \lambda = 444.01 nm$$

2. What is the shape of the wavefront in each of the following cases:

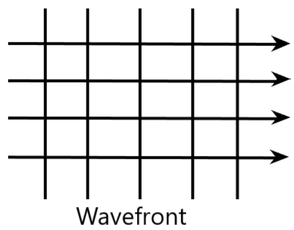
(a) Light diverging from a point source.

Ans: When a light diverges from a point source, the shape of the wavefront in this case is spherical. The wavefront originating from a point source is shown in the given figure.



(b) Light emerging out of a convex lens when a point source is placed at its focus.

Ans: The shape of the wavefront when a light emerging out of a convex lens when a point source is placed at its focus is a parallel grid. This can be represented as shown in the given figure.



(c) The portion of the wavefront of light from a distant star intercepted by the Earth.

Ans: In this case the portion of the wavefront of light from a distant star intercepted by the Earth is a plane.

3. (a) The refractive index of glass is 1.5. What is the speed of light in glass? Speed of light in vacuum is $3.0\times10^{8}\text{m/s}$.

Ans: Refractive index of glass is given as,

$\mu=1.5$

Speed of light, $c=3.0\times10^{8}\text{m/s}$

Speed of light in glass is given by the formula,

$$v=\frac{c}{\mu}$$

$$\Rightarrow v=\frac{3\times10^{8}}{1.5}=2\times10^{8}\text{m/s}$$

Hence, the speed of light in glass is $2\times10^{8}\text{m/s}$.

(b) Is the speed of light in glass independent of the colour of light? If not, which of the two colours red and violet travels slower in a glass prism?

Ans: The speed of light in glass depends on the colour of light.

The refractive index of a violet component of white light is more than the refractive index of a red component. Hence, the speed of violet light is less than the speed of red light in glass as speed and refractive index are inversely related to each other.

Hence, violet light travels slower as compared to red light in a glass prism.

4. In a Young's double-slit experiment, the slits are separated by 0.28mm and the screen is placed 1.4m away. The distance between the central bright fringe and the fourth bright fringe is measured to be 1.2cm. Determine the wavelength of light used in the experiment.

Ans: Distance between the slits is given as, $d=0.28\text{mm}=0.28\times10^{-3}\text{m}$

Distance between the slits and the screen, $D=1.4\text{m}$

Distance between the central fringe and the fourth $(n=4)$ fringe,

$u=1.2\text{cm}=1.2\times10^{-2}\text{m}$

In case of a constructive interference, the relation for the distance between the two fringes can be given as: $u = n\lambda \frac{D}{d}$

where,

$n =$ Order of fringes $= 4$

$\lambda =$ Wavelength of light used

$\lambda = \frac{ud}{nD}$

$\Rightarrow \lambda = \frac{1.2 \times 10^{-2} \times 0.28 \times 10^{-3}}{4 \times 1.4}$

$\Rightarrow \lambda = 6 \times 10^{-7}$

$\Rightarrow \lambda = 600\text{nm}$

Hence, the wavelength of the light is 600nm.

5. In Young's double-slit experiment using monochromatic light of wavelength λ. The intensity of light at a point on the screen where path difference is λ , is K units. What is the intensity of light at a point where path difference is $\frac{\lambda}{3}$?

Ans: The intensity of the two light waves be I_1 and I_2. Their resultant intensities can be evaluated as: $I' = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

Where,

$\phi =$ The phase difference between two waves for monochromatic light waves,

Since $I_1 = I_2$

So $I' = 2I_1 + 2I_1 \cos\phi$

The formula for phase difference can be given as:

$\text{Phase difference} = \frac{2\pi}{\lambda} \times \text{Path difference}$

Since, path difference is λ,

Phase difference is $\phi = 2\pi$

$I' = 2I_1 + 2I_1 = 4I_1$

Given,

$I_1 = \frac{K'}{4}$ …… (1)

When path difference $= \frac{\lambda}{3}$

phase difference, $\phi = \frac{2\pi}{3}$

Hence, resultant intensity,

$$I_R = I_1 + I_1 + 2\sqrt{I_1 I_1}\cos\frac{2\pi}{3}$$

$$\Rightarrow I_R = 2I_1 + 2I_2\left(-\frac{1}{2}\right) = I_1$$

Using equation (1), we can state that

$$I_R = I_1 = \frac{K}{4}$$

Hence, for monochromatic light waves, the intensity of light at a point where the path difference is $\frac{\lambda}{3}$ is $\frac{K}{4}$ units.

6. A beam of light consisting of two wavelengths, 650nm and 520nm, is used to obtain interference fringes in a Young's double-slit experiment.

(a) Find the distance of the third bright fringe on the screen from the central maximum for wavelength 650nm.

Ans: Given that,

Wavelength of the first light beam, $\lambda_1 = 650\text{nm}$

Wavelength of second light beam, $\lambda_2 = 520\text{nm}$

Distance of the slits from the screen $= D$

Distance between the two slits $= d$

Distance of the n^{th} bright fringe on the screen from the central maximum is given by the formula below,

$$x = n\lambda_1\left(\frac{D}{d}\right)$$

For the third bright fringe, $n = 3$

$x = 3 \times 650 \times \frac{D}{d} = 1950\left(\frac{D}{d}\right)\text{nm}$, which is nothing but the distance of the third bright fringe on the screen from the central maximum.

(b) What is the least distance from the central maximum where the bright fringes due to both the wavelengths coincide?

Ans: In this case, let the n bright fringe due to wavelength λ^{th} and $_2$ $(n-1)^{th}$ bright fringe due to wavelength λ_1 coincide on the screen. Equate the conditions for bright fringes as follows:

$$n\lambda_2 = (n-1)\lambda_1$$
$$\Rightarrow 520n = 650n - 650$$
$$\Rightarrow 650 = 130n$$
$$\Rightarrow n = 5$$

Hence, the least distance from the central maximum can be attained by the relation:

$$x = n\lambda_2 \frac{D}{d}$$
$$\Rightarrow x = 5 \times 520 \frac{D}{d} = 2600 \frac{D}{d} \text{nm}$$

Note: The value of d and D are not given in the question, hence the exact answer cannot be found.