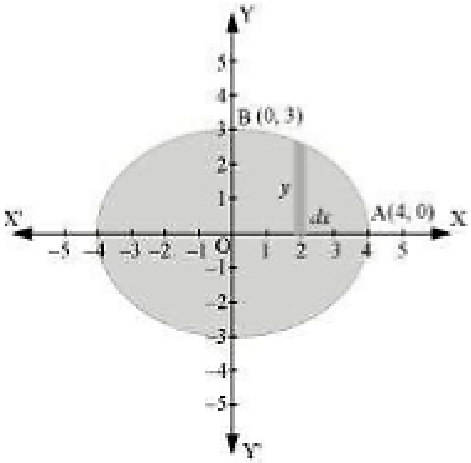
application of integrals

8 chapter

EXERCISE 8.1

1: Find the area of the region bounded by the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$

Ans:



Area of ellipse = 4 × Area of OAB

Area of OAB = $\int_0^4 y dx$

$=\int_0^4 3\sqrt{1-\frac{x^2}{16}}dx$

$=\frac{3}{4}\int_0^4 \sqrt{16-x^2}dx$

$=\frac{3}{4}\left[\frac{x}{2}\sqrt{16-x^2}+\frac{16}{2}\sin^{-1}\frac{x}{4}\right]_0^4$

$=\frac{3}{4}\left[2\sqrt{16-16}+8\sin^{-1}(1)-0-8\sin^{-1}(0)\right]$

$=\frac{3}{4}\left[\frac{8\pi}{2}\right]$

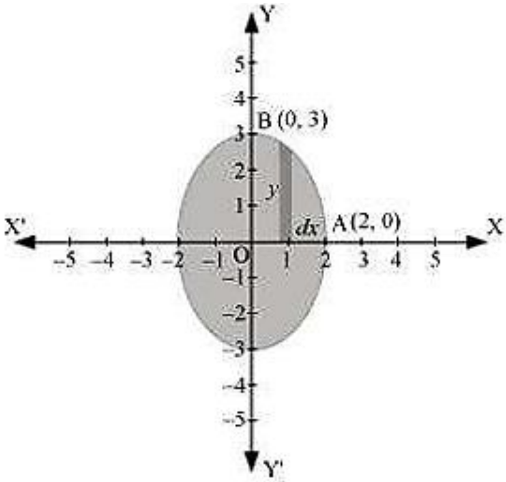
$=\frac{3}{4}[4\pi]$

$=3\pi$

area of ellipse = 4 × 3π = 12π units

2: Find the area of the region bounded by the ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$

Ans:



$$\frac{x^2}{4}+\frac{y^2}{9}=1$$

$$\Rightarrow y=3\sqrt{1-\frac{x^2}{4}}$$

Area of ellipse = 4 × Area OAB

Area of OAB= $\int_0^2$ ydx

$$=\int_0^2 3\sqrt{1-\frac{x^2}{4}} \qquad [\text{Using (1)}]$$

$$=\frac{3}{2}\int_0^2 \sqrt{4-x^2}$$

$$=\frac{3}{2}\left[\frac{x}{2}\sqrt{4-x^2}+\frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2$$

$$=\frac{3}{2}\left[\frac{2\pi}{2}\right]$$

$$=\frac{3\pi}{2}$$

area of ellipse = $4\times\frac{3\pi}{2}=6\pi$ units

3: Area lying in the first quadrant and bounded by the circle $x^2 + y^2 = 4$ and the lines x = 0 and x = 2 is

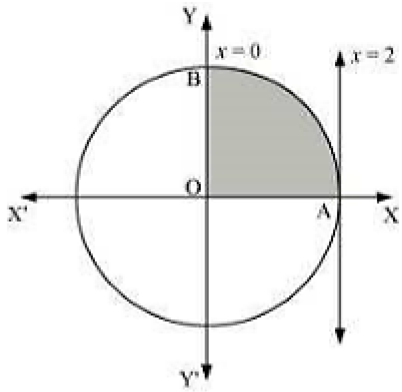
A. π

B. $\frac{\pi}{2}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{4}$

Ans:



$\text{Area OAB} = \int_0^2 y dx$

$= \int_0^2 \sqrt{4 - x^2} dx$

$= \left[\frac{x}{2}\sqrt{4 - x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2} \right]_0^2$

$= 2\left(\frac{\pi}{2}\right)$

$= \pi$ units

correct answer is A.

4: Area of the region bounded by the curve $y^2 = 4x$, y-axis and the line y = 3 is

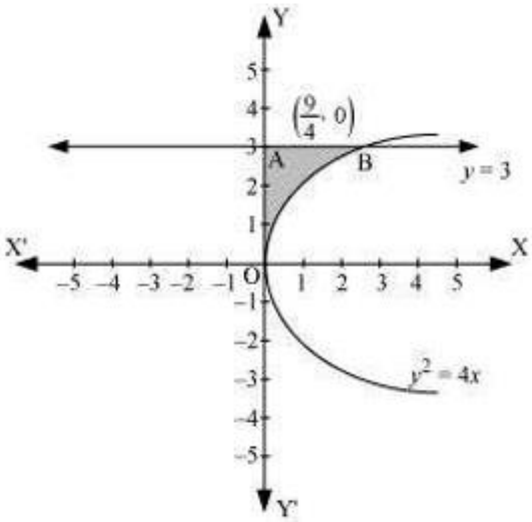
A. 2

B. $\frac{9}{4}$

C. $\frac{9}{3}$

D. $\frac{9}{2}$

Ans:



Area OAB = $\int_{o}^{3} x dy$

$= \int_0^3 \frac{y^2}{4} dy$

$= \frac{1}{4}\left[\frac{y^3}{4}\right]_0^3$

$= \frac{1}{12}(27)$

$= \frac{9}{4}$ units

correct answer is B.

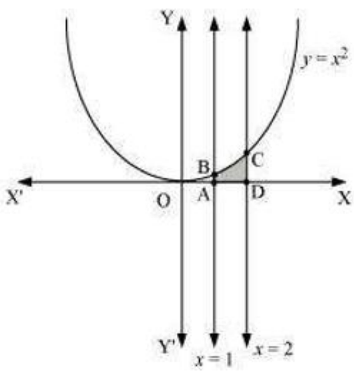
Miscellaneous EXERCISE

1: Find the area under the given curves and given lines:

(i) $y = x^2$, x = 1, x = 2 and x-axis

(ii) $y = x^4$, x = 1, x = 5 and x –axis

Ans: i.



Area ADCBA = $\int_1^2 y dx$

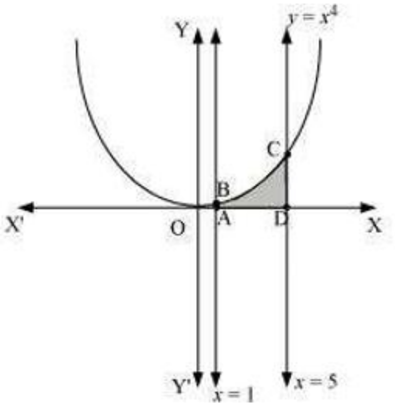
$= \int_1^2 x^2 dx$

$= \left[\frac{x^3}{3}\right]_1^2$

$= \frac{8}{3} - \frac{1}{3}$

$= \frac{7}{3}$ units

ii.



Area of ADCBA = $\int_1^5 x^4 dx$

$= \left[\frac{x^5}{5}\right]_1^5$

$= \frac{(5)^5}{5} - \frac{1}{5}$

$= (5)^4 - \frac{1}{5}$

$= 625 - \frac{1}{5}$

$= 624.8$ units

$\therefore$ Area ABCD=$\int_1^4 x dx$

$= \int_1^4 \frac{\sqrt{4}}{2} dx$

$= \frac{1}{2}\left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^4$

$$= \frac{1}{3}\left[(4)^{\frac{3}{2}} - 1\right]$$

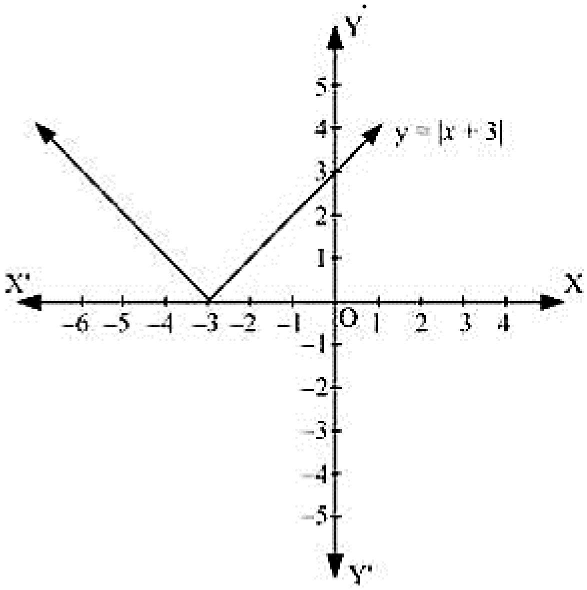
$$= \frac{1}{3}[8-1]$$

$$= \frac{7}{3} \text{ units}$$

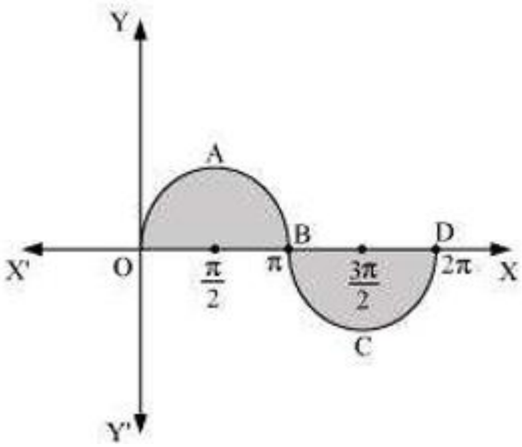
2: Sketch the graph of $y = |x+3|$ and evaluate $\int_{-6}^{0} |x+3|\, dx$

Ans:

x	− 6	− 5	− 4	− 3	− 2	− 1	0
y	3	2	1	0	1	2	3



$(x+3) \le 0$ for $-6 \le x \le -3$ and $(x+3) \ge 0$ for $-3 \le x \le 0$



$\therefore$ area = Area OABO + Area BCDB

$= \int_0^{\pi} \sin x dx + \left| \int_{\pi}^{2\pi} \sin x dx \right|$

$= [-\cos x]_0^{\pi} + \left| [-\cos x]_{\pi}^{2\pi} \right|$

$= [-\cos \pi + \cos 0] + |-\cos 2\pi + \cos \pi|$

$= 1 + 1 + |(-1-1)|$

$= 2 + |-2|$

$= 2 + 2 = 4$ units

4: Area bounded by the curve y = x^3, the x-axis and the ordinates x = –2 and x = 1 is

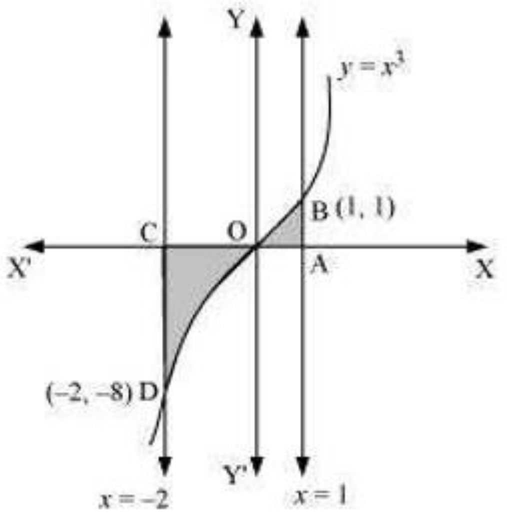
A. – 9

B. $-\frac{15}{4}$

C. $\frac{15}{4}$

D. $\frac{17}{4}$

Ans:



Required area = $\int_{-2}^{1} y dx$

$= \int_{-2}^{1} x^3 dx$

$$= \left[\frac{x^4}{4}\right]_{-2}^{1}$$

$$\left[\frac{1}{4} - \frac{(-2)^4}{4}\right]$$

$$\left(\frac{1}{4} - 4\right) = -\frac{15}{4} units$$

correct answer is B.

5: The area bounded by the curve $y = x|x|$, x-axis and the ordinates x = –1 and x = 1 is given by

[Hint: y = x^2 if x > 0 and y = $-x^2$ if x < 0]

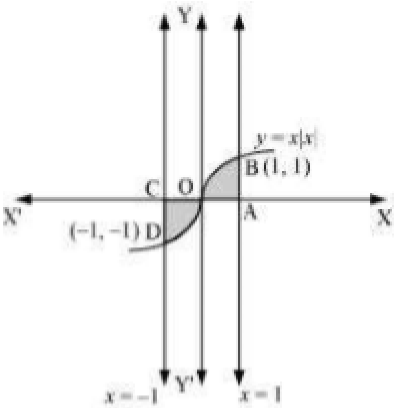
A. 0

B. $\frac{1}{3}$

C. $\frac{2}{3}$

D. $\frac{4}{3}$

Ans:



Required area = $\int_{-1}^{1} y dx$

$$= \int_{-1}^{1} x|x| dx$$

$$= \int_{-1}^{0} x^2 dx + \int_{0}^{1} x^2 dx$$

$$= \left[\frac{x^3}{3}\right]_{-1}^{0} + \left[\frac{x^3}{3}\right]_{0}^{1}$$

$$= -\left(-\frac{1}{3}\right) + \frac{1}{3}$$

$$= \frac{2}{3} \text{ units}$$

correct answer is C.