matrices

3 Chapter

Exercise 3.1

1. In the matrix $A=\begin{bmatrix} 2 & 5 & 19 & -7 \\ 35 & -2 & \frac{5}{2} & 12 \\ \sqrt{3} & 1 & -5 & 17 \end{bmatrix}$**, write**

i. The order of the matrix.

Ans: The order of a matrix is $m\times n$ where m is the number of rows and n is the number of columns. Therefore, here the order is 3×4.

ii. The number of elements.

Ans: Since the order of the given matrix is 3×4 therefore, the number of elements in it is $3\times 4=12$.

iii. Write the elements $a_{13},a_{21},a_{33},a_{24},a_{23}$

Ans: The elements are given as a_{mn} . Therefore, here $a_{13}=19$, $a_{21}=35$,

$a_{33}=-5$, $a_{24}=12$, $a_{23}=\frac{5}{2}$.

2. If a matrix has 24 **elements, what are the possible order it can have? What if it has** 13 **elements?**

Ans: The order of a matrix is $m\times n$ where m is the number of rows and n is the number of columns. To find the possible orders of a matrix, we have to find all the ordered pairs of natural numbers whose product is 24 .

$\therefore (1\times 24),(24\times 1),(2\times 12),(12\times 2),(3\times 8),(8\times 3),(4\times 6),(6\times 4)$ are all the possible ordered pairs here.

If the matrix had 13 elements, then the ordered pairs would be (1×13) and (13×1).

3. If a matrix has 18 elements, what are the possible orders it can have? What if it has 5 elements?

Ans: The order of a matrix is $m\times n$ where m is the number of rows and n is the number of columns. To find the possible orders of a matrix, we have to find all the ordered pairs of natural numbers whose product is 18.

$\therefore (1\times18),(18\times1),(2\times9),(9\times2),(3\times6),(6\times3)$ are all the possible ordered pairs here.

If the matrix had 5 elements, then the ordered pairs would be (1×5) and (5×1)

4. Construct a 2 × 2 matrix, A = [a_{ij}], whose elements are given by:

(i) $a_{ij} = \frac{(i+j)^2}{2}$

(ii) $a_{ij} = \frac{i}{j}$

(iii) $a_{ij} = \frac{(i+2j)^2}{2}$

Ans: (i) Since it is a 2×2 matrix

Let matrix be A

Where A = $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

Now it is given that

$a_{ij} = \frac{(i+j)^2}{2}$

a_{ij} i, j $a_{ij} = \frac{(i+j)^2}{2}$

a_{11} 1,1 $a_{11} = \frac{(1+1)^2}{2} = \frac{(2)^2}{2} = 2$

a_{12} 1,2 $a_{12} = \frac{(1+2)^2}{2} = \frac{(3)^2}{2} = \frac{9}{2}$

a_{21} 2,1 $a_{21} = \frac{(2+1)^2}{2} = \frac{(3)^2}{2} = \frac{9}{2}$

a_{22} 2,2 $a_{22} = \frac{(2+2)^2}{2} = \frac{(4)^2}{2} = 8$

Hence, the required matrix A is

A = $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 2 & \frac{9}{2} \\ \frac{9}{2} & 8 \end{bmatrix}$

(ii): Since it is a 2×2 matrix.
Let matrix be A

Where A = $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

Now it is given that $a_{ij} = \frac{i}{j}$

a_{ij} i,j $a_{ij} = \frac{i}{j}$

a_{11} $^{i = 1,}_{j = 1}$ $a_{11} = \frac{1}{1} = 1$

a_{12} $^{i = 1,}_{j = 2}$ $a_{12} = \frac{1}{2}$

a_{21} $^{i = 2,}_{j = 1}$ $a_{21} = \frac{2}{1} = 2$

a_{22} $^{i = 2,}_{j = 2}$ $a_{22} = \frac{2}{2} = 1$

Hence, the required matrix A is

A = $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{2} \\ 2 & 1 \end{bmatrix}$

(iii): Since it is a 2×2 matrix.
Let matrix be A

Where A = $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

Now it is given that

$a_{ij} = \frac{(i+2j)^2}{2}$

i,j $a_{ij} = \frac{(i+2j)^2}{2}$

1,1 $a_{11} = \frac{(1+2(1))^2}{2} = \frac{3^2}{2} = \frac{9}{2}$

1,2 $a_{12} = \frac{(1+2(2))^2}{2} = \frac{(5)^2}{2} = \frac{25}{2}$

2,1 $a_{21} = \frac{(2+2(1))^2}{2} = \frac{(4)^2}{2} = \frac{16}{2} = 8$

2,2 $a_{22} = \frac{(2+2(2))^2}{2} = \frac{(6)^2}{2} = \frac{36}{2} = 18$

Hence, the required matrix A is

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} \frac{9}{2} & \frac{25}{2} \\ 8 & 18 \end{bmatrix}$$

5. Construct a 3×4 matrix, whose elements are given by

i. $a_{ij} = \frac{1}{2}|-3i+j|$

Ans: A 3×4 matrix is given by $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$

Given that $a_{ij} = \frac{1}{2}|-3i+j|$,

$\therefore a_{11} = \frac{1}{2}|-3\times 1+1| = 1$

$a_{21} = \frac{1}{2}|-3\times 2+1| = \frac{5}{2}$

$a_{31} = \frac{1}{2}|-3\times 3+1| = 4$

$$a_{12} = \frac{1}{2}\left|-3\times1+2\right| = \frac{1}{2}$$

$$a_{22} = \frac{1}{2}\left|-3\times2+2\right| = 2$$

$$a_{32} = \frac{1}{2}\left|-3\times3+2\right| = \frac{7}{2}$$

$$a_{13} = \frac{1}{2}\left|-3\times1+3\right| = 0$$

$$a_{23} = \frac{1}{2}\left|-3\times2+3\right| = \frac{3}{2}$$

$$a_{33} = \frac{1}{2}\left|-3\times3+3\right| = 3$$

$$a_{14} = \frac{1}{2}\left|-3\times1+4\right| = \frac{1}{2}$$

$$a_{24} = \frac{1}{2}\left|-3\times2+4\right| = 1$$

$$a_{34} = \frac{1}{2}\left|-3\times3+4\right| = \frac{5}{2}$$

Thus, the required matrix is $A = \begin{bmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{5}{2} & 2 & \frac{3}{2} & 1 \\ 4 & \frac{7}{2} & 3 & \frac{5}{2} \end{bmatrix}$.

ii. $a_{ij} = 2i - j$

Ans: A 3×4 matrix is given by $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$

Given that $a_{ij} = 2i - j$,

$\therefore a_{11} = 2\times1-1=1$

$a_{21} = 2\times2-1=3$

$a_{31} = 2\times3-1=5$

$a_{12} = 2\times1-2=0$

$a_{22} = 2\times2-2=4$

$a_{32} = 2\times3-2=4$

$a_{13} = 2\times1-3=-1$

$a_{23} = 2\times2-3=1$

$a_{33} = 2\times3-3=3$

$a_{14} = 2\times1-4=-2$

$a_{24} = 2\times2-4=0$

$a_{34} = 2\times3-4=2$

Thus, the required matrix is $A=\begin{bmatrix}1 & 0 & -1 & -2\\ 3 & 2 & 1 & 0\\ 5 & 4 & 3 & 2\end{bmatrix}$.

6. Find the value of x, y, z from the following equation:

i. $\begin{bmatrix}\mathbf{4} & \mathbf{3}\\ \mathbf{x} & \mathbf{5}\end{bmatrix}=\begin{bmatrix}\mathbf{y} & \mathbf{z}\\ \mathbf{1} & \mathbf{5}\end{bmatrix}$

Ans: Given $\begin{bmatrix}4 & 3\\ x & 5\end{bmatrix}=\begin{bmatrix}y & z\\ 1 & 5\end{bmatrix}$

Comparing the corresponding elements we get,

$x=1, y=4, z=3$

ii. $\begin{bmatrix} \mathbf{x+y} & \mathbf{2} \\ \mathbf{5+z} & \mathbf{xy} \end{bmatrix} = \begin{bmatrix} \mathbf{6} & \mathbf{2} \\ \mathbf{5} & \mathbf{8} \end{bmatrix}$

Ans: Given $\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$

Comparing the corresponding elements we get,

$x+y=6, xy=8, 5+z=5$

Now, $\because 5+z=5$

$\Rightarrow z=0$

We know that, $(x-y)^2 = (x+y)^2 - 4xy$

$\Rightarrow (x-y)^2 = 36-32$

$\Rightarrow (x-y) = \pm 2$

When $(x-y)=2$ and $(x+y)=6$,

We get $x=4, y=2$

When $(x-y)=-2$ and $(x+y)=6$,

We get $x=2, y=4$

$\therefore x=4, y=2, z=0$ or $\therefore x=2, y=4, z=0$

iii. $\begin{bmatrix} \mathbf{x+y+z} \\ \mathbf{x+z} \\ \mathbf{y+z} \end{bmatrix} = \begin{bmatrix} \mathbf{9} \\ \mathbf{5} \\ \mathbf{7} \end{bmatrix}$

Ans: Given $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$

Comparing the corresponding elements we get,

$x+y+z=9 \qquad \ldots(1)$

$x+z=5 \quad \ldots(2)$

$y+z=7 \quad \ldots(3)$

From equation (1) and (2),

$y+5=9$

$\Rightarrow y=4$

From equation (3) we have,

$4+z=7$

$\Rightarrow z=3$

$x+z=5$

$\Rightarrow x=2$

$\therefore x=2, y=4, z=3$

7. Find the value of a,b,c,d from the equation:

$$\begin{bmatrix} \mathbf{a-b} & \mathbf{2a+c} \\ \mathbf{2a-b} & \mathbf{3c+d} \end{bmatrix} = \begin{bmatrix} \mathbf{-1} & \mathbf{5} \\ \mathbf{0} & \mathbf{13} \end{bmatrix}$$

Ans: Given $\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$

Comparing the corresponding elements we get,

$a-b=-1 \quad \ldots(1)$

$2a-b=0 \quad \ldots(2)$

$2a+c=5 \quad \ldots(3)$

$3c+d=13 \quad \ldots(4)$

From equation (2),

$b=2a$

From equation (1),

$a-2a=-1$

$\Rightarrow a = 1$

$\Rightarrow b = 2$

From equation (3),

$2 \times 1 + c = 5$

$\Rightarrow c = 3$

From equation (4),

$3 \times 3 + d = 13$

$\Rightarrow d = 4$

$\therefore a = 1, b = 2, c = 3, d = 4$

8. **$A = [a_y]_{m \times n}$ is a square matrix, if**

A. $m < n$

B. $m > n$

C. $m = n$

D. None of these

Ans: A given matrix is said to be a square matrix if the number of rows is equal to the number of columns.

$\therefore A = [a_y]_{m \times n}$ is a square matrix if, $m = n$.

Thus, option (C) is correct.

9. **Which of the given values of x and y make the following pair of matrices equal**

$$\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix} = \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$$

A. $x = \frac{-1}{3}, y = 7$

B. Not possible to find

C. $\mathbf{y = 7, x = \frac{-2}{3}}$

D. $\mathbf{x = \frac{-1}{3}, y = \frac{-2}{3}}$

Ans: Given $\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix} = \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$

Comparing the corresponding elements we get,

$3x + 7 = 0$

$\Rightarrow x = -\frac{7}{3}$

$y - 2 = 5$

$\Rightarrow y = 7$

$y + 1 = 8$

$\Rightarrow y = 7$

$2 - 3x = 4$

$\Rightarrow x = -\frac{2}{3}$

Since we get two different values of x ,which is not possible. It is not possible to find the values of x and y for which the given matrices are equal.

Thus, the correct option is (B).

10. The number of all possible matrices of order 3×3 with each entry 0 or 1 is:

A. 27

B. 18

C. 81

D. 512

Ans: Given matrix is of the order 3×3 has nine elements and each of these elements can be either 0 or 1.

Now, each of the nine elements can be filled in two possible ways.

Therefore, the required number of possible matrices is $2^9 = 512$.

Exercise 3.2

1. **Let** $\mathbf{A}=\begin{bmatrix}\mathbf{2} & \mathbf{4}\\ \mathbf{3} & \mathbf{2}\end{bmatrix},\mathbf{B}=\begin{bmatrix}\mathbf{1} & \mathbf{3}\\ \mathbf{-2} & \mathbf{5}\end{bmatrix},\mathbf{C}=\begin{bmatrix}\mathbf{-2} & \mathbf{5}\\ \mathbf{3} & \mathbf{4}\end{bmatrix}$

Find each of the following

i. $\mathbf{A+B}$

Ans: Given $A=\begin{bmatrix}2 & 4\\ 3 & 2\end{bmatrix},B=\begin{bmatrix}1 & 3\\ -2 & 5\end{bmatrix}$

$$\therefore A+B=\begin{bmatrix}2 & 4\\ 3 & 2\end{bmatrix}+\begin{bmatrix}1 & 3\\ -2 & 5\end{bmatrix}$$

$$\Rightarrow A+B=\begin{bmatrix}2+1 & 4+3\\ 3-2 & 2+5\end{bmatrix}$$

$$\Rightarrow A+B=\begin{bmatrix}3 & 7\\ 1 & 7\end{bmatrix}$$

ii. $\mathbf{A-B}$

Ans: Given $A=\begin{bmatrix}2 & 4\\ 3 & 2\end{bmatrix},B=\begin{bmatrix}1 & 3\\ -2 & 5\end{bmatrix}$

$$\therefore A-B=\begin{bmatrix}2 & 4\\ 3 & 2\end{bmatrix}-\begin{bmatrix}1 & 3\\ -2 & 5\end{bmatrix}$$

$$\Rightarrow A-B=\begin{bmatrix}2-1 & 4-3\\ 3+2 & 2-5\end{bmatrix}$$

$$\Rightarrow A-B=\begin{bmatrix}1 & 1\\ 5 & -3\end{bmatrix}$$

iii. 3A – C

Ans: Given $A=\begin{bmatrix}2 & 4\\ 3 & 2\end{bmatrix}, C=\begin{bmatrix}-2 & 5\\ 3 & 4\end{bmatrix}$

$$\therefore 3A-C=3\begin{bmatrix}2 & 4\\ 3 & 2\end{bmatrix}-\begin{bmatrix}-2 & 5\\ 3 & 4\end{bmatrix}$$

$$\Rightarrow 3A-C=\begin{bmatrix}6 & 12\\ 9 & 6\end{bmatrix}-\begin{bmatrix}-2 & 5\\ 3 & 4\end{bmatrix}$$

$$\Rightarrow 3A-C=\begin{bmatrix}6+2 & 12-5\\ 9-3 & 6-4\end{bmatrix}$$

$$\Rightarrow 3A-C=\begin{bmatrix}8 & 7\\ 6 & 2\end{bmatrix}$$

iv. AB

Ans: Given $A=\begin{bmatrix}2 & 4\\ 3 & 2\end{bmatrix}, B=\begin{bmatrix}1 & 3\\ -2 & 5\end{bmatrix}$

$$\therefore AB=\begin{bmatrix}2 & 4\\ 3 & 2\end{bmatrix}\begin{bmatrix}1 & 3\\ -2 & 5\end{bmatrix}$$

$$\Rightarrow AB=\begin{bmatrix}2(1)+4(-2) & 2(3)+4(5)\\ 3(1)+2(-2) & 3(3)+2(5)\end{bmatrix}$$

$$\Rightarrow AB=\begin{bmatrix}-6 & 26\\ -1 & 19\end{bmatrix}$$

v. BA

Ans: Given $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$

$$\therefore BA = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$$

$$\Rightarrow BA = \begin{bmatrix} 1(2)+3(3) & 1(4)+3(2) \\ -2(2)+5(3) & -2(4)+5(2) \end{bmatrix}$$

$$\Rightarrow BA = \begin{bmatrix} 11 & 10 \\ 11 & 2 \end{bmatrix}$$

2. Compute the following:

i. $\begin{bmatrix} \mathbf{a} & \mathbf{b} \\ \mathbf{-b} & \mathbf{a} \end{bmatrix} + \begin{bmatrix} \mathbf{a} & \mathbf{b} \\ \mathbf{b} & \mathbf{a} \end{bmatrix}$

Ans: We have to find $\begin{bmatrix} a & b \\ -b & a \end{bmatrix} + \begin{bmatrix} a & b \\ b & a \end{bmatrix}$

$$\therefore \begin{bmatrix} a & b \\ -b & a \end{bmatrix} + \begin{bmatrix} a & b \\ b & a \end{bmatrix} = \begin{bmatrix} a+a & b+b \\ -b+b & a+a \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a & b \\ -b & a \end{bmatrix} + \begin{bmatrix} a & b \\ b & a \end{bmatrix} = \begin{bmatrix} 2a & 2b \\ 0 & 2a \end{bmatrix}$$

ii. $\begin{bmatrix} \mathbf{a^2+b^2} & \mathbf{b^2+c^2} \\ \mathbf{a^2+c^2} & \mathbf{a^2+b^2} \end{bmatrix} + \begin{bmatrix} \mathbf{2ab} & \mathbf{2bc} \\ \mathbf{-2ac} & \mathbf{-2ab} \end{bmatrix}$

Ans: we have to find $\begin{bmatrix} a^2+b^2 & b^2+c^2 \\ a^2+c^2 & a^2+b^2 \end{bmatrix} + \begin{bmatrix} 2ab & 2bc \\ -2ac & -2ab \end{bmatrix}$

$$\therefore \begin{bmatrix} a^2+b^2 & b^2+c^2 \\ a^2+c^2 & a^2+b^2 \end{bmatrix} + \begin{bmatrix} 2ab & 2bc \\ -2ac & -2ab \end{bmatrix} = \begin{bmatrix} a^2+b^2+2ab & b^2+c^2+2bc \\ a^2+c^2-2ac & a^2+b^2-2ab \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a^2+b^2 & b^2+c^2 \\ a^2+c^2 & a^2+b^2 \end{bmatrix} + \begin{bmatrix} 2ab & 2bc \\ -2ac & -2ab \end{bmatrix} = \begin{bmatrix} (a+b)^2 & (b+c)^2 \\ (a-c)^2 & (a-b)^2 \end{bmatrix}$$

iii. $\begin{bmatrix} \mathbf{-1} & \mathbf{4} & \mathbf{-6} \\ \mathbf{8} & \mathbf{5} & \mathbf{16} \\ \mathbf{2} & \mathbf{8} & \mathbf{5} \end{bmatrix} + \begin{bmatrix} \mathbf{12} & \mathbf{7} & \mathbf{6} \\ \mathbf{8} & \mathbf{0} & \mathbf{5} \\ \mathbf{3} & \mathbf{2} & \mathbf{4} \end{bmatrix}$

Ans: we have to find $\begin{bmatrix} -1 & 4 & -6 \\ 8 & 5 & 16 \\ 2 & 8 & 5 \end{bmatrix} + \begin{bmatrix} 12 & 7 & 6 \\ 8 & 0 & 5 \\ 3 & 2 & 4 \end{bmatrix}$

$$\therefore \begin{bmatrix} -1 & 4 & -6 \\ 8 & 5 & 16 \\ 2 & 8 & 5 \end{bmatrix} + \begin{bmatrix} 12 & 7 & 6 \\ 8 & 0 & 5 \\ 3 & 2 & 4 \end{bmatrix} = \begin{bmatrix} -1+12 & 4+7 & -6+6 \\ 8+8 & 5+0 & 16+5 \\ 2+3 & 8+2 & 5+4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -1 & 4 & -6 \\ 8 & 5 & 16 \\ 2 & 8 & 5 \end{bmatrix} + \begin{bmatrix} 12 & 7 & 6 \\ 8 & 0 & 5 \\ 3 & 2 & 4 \end{bmatrix} = \begin{bmatrix} 11 & 11 & 0 \\ 16 & 5 & 21 \\ 5 & 10 & 9 \end{bmatrix}$$

iv. $\begin{bmatrix} \mathbf{\cos^2 x} & \mathbf{\sin^2 x} \\ \mathbf{\sin^2 x} & \mathbf{\cos^2 x} \end{bmatrix} + \begin{bmatrix} \mathbf{\sin^2 x} & \mathbf{\cos^2 x} \\ \mathbf{\cos^2 x} & \mathbf{\sin^2 x} \end{bmatrix}$

Ans: we have to find $\begin{bmatrix} \cos^2 x & \sin^2 x \\ \sin^2 x & \cos^2 x \end{bmatrix} + \begin{bmatrix} \sin^2 x & \cos^2 x \\ \cos^2 x & \sin^2 x \end{bmatrix}$

$$\therefore \begin{bmatrix} \cos^2 x & \sin^2 x \\ \sin^2 x & \cos^2 x \end{bmatrix} + \begin{bmatrix} \sin^2 x & \cos^2 x \\ \cos^2 x & \sin^2 x \end{bmatrix} = \begin{bmatrix} \cos^2 x + \sin^2 x & \sin^2 x + \cos^2 x \\ \sin^2 x + \cos^2 x & \cos^2 x + \sin^2 x \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \cos^2 x & \sin^2 x \\ \sin^2 x & \cos^2 x \end{bmatrix} + \begin{bmatrix} \sin^2 x & \cos^2 x \\ \cos^2 x & \sin^2 x \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

3. **Compute the indicated products**

i. $\begin{bmatrix} \mathbf{a} & \mathbf{b} \\ \mathbf{-b} & \mathbf{a} \end{bmatrix}\begin{bmatrix} \mathbf{a} & \mathbf{-b} \\ \mathbf{b} & \mathbf{a} \end{bmatrix}$

Ans: we have to find $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$

$$\therefore \begin{bmatrix} a & b \\ -b & a \end{bmatrix}\begin{bmatrix} a & -b \\ b & a \end{bmatrix} = \begin{bmatrix} a(a)+b(b) & a(-b)+b(a) \\ -b(a)+a(b) & -b(-b)+a(a) \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a & b \\ -b & a \end{bmatrix}\begin{bmatrix} a & -b \\ b & a \end{bmatrix} = \begin{bmatrix} a^2+b^2 & 0 \\ 0 & a^2+b^2 \end{bmatrix}$$

ii. $\begin{bmatrix} \mathbf{1} \\ \mathbf{2} \\ \mathbf{3} \end{bmatrix}\begin{bmatrix} \mathbf{2} & \mathbf{3} & \mathbf{4} \end{bmatrix}$

Ans: We have to find $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\begin{bmatrix} 2 & 3 & 4 \end{bmatrix}$

$$\therefore \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\begin{bmatrix} 2 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 1(2) & 1(3) & 1(4) \\ 2(2) & 2(3) & 2(4) \\ 3(2) & 3(3) & 3(4) \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\begin{bmatrix} 2 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 6 & 8 \\ 6 & 9 & 12 \end{bmatrix}$$

iii. $\begin{bmatrix} \mathbf{1} & \mathbf{-2} \\ \mathbf{2} & \mathbf{3} \end{bmatrix}\begin{bmatrix} \mathbf{1} & \mathbf{2} & \mathbf{3} \\ \mathbf{2} & \mathbf{3} & \mathbf{1} \end{bmatrix}$

Ans: we have to find $\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$

$$\therefore \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 1(1)-2(2) & 1(2)-2(3) & 1(3)-2(1) \\ 2(1)+3(2) & 2(2)+3(3) & 2(3)+3(1) \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix} = \begin{bmatrix} -3 & -4 & 1 \\ 8 & 13 & 9 \end{bmatrix}$$

iv. $\begin{bmatrix} \mathbf{2} & \mathbf{3} & \mathbf{4} \\ \mathbf{3} & \mathbf{4} & \mathbf{5} \\ \mathbf{4} & \mathbf{5} & \mathbf{6} \end{bmatrix}\begin{bmatrix} \mathbf{1} & \mathbf{-3} & \mathbf{5} \\ \mathbf{0} & \mathbf{2} & \mathbf{4} \\ \mathbf{3} & \mathbf{0} & \mathbf{5} \end{bmatrix}$

Ans: we have to find $\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}\begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix}$

$$\therefore \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}\begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix} =$$

$$\begin{bmatrix} 2(1)+3(0)+4(3) & 2(-3)+3(2)+4(0) & 2(5)+3(4)+4(5) \\ 3(1)+4(0)+5(3) & 3(-3)+4(2)+5(0) & 3(5)+4(4)+5(5) \\ 4(1)+5(0)+5(3) & 4(-3)+5(2)+6(0) & 4(5)+5(4)+6(5) \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}\begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix} = \begin{bmatrix} 14 & 0 & 42 \\ 18 & -1 & 56 \\ 22 & -2 & 70 \end{bmatrix}$$

v. $\begin{bmatrix} \mathbf{2} & \mathbf{1} \\ \mathbf{3} & \mathbf{2} \\ \mathbf{-1} & \mathbf{1} \end{bmatrix}\begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{1} \\ \mathbf{-1} & \mathbf{2} & \mathbf{1} \end{bmatrix}$

Ans: we have to find $\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix}$

$$\therefore \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 2(1)+1(-1) & 2(0)+1(2) & 2(1)+1(1) \\ 3(1)+2(-1) & 3(0)+2(2) & 3(1)+2(1) \\ -1(1)+1(-1) & -1(0)+1(2) & -1(1)+1(1) \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 5 \\ -2 & 2 & 0 \end{bmatrix}$$

vi. $\begin{bmatrix} \mathbf{3} & \mathbf{-1} & \mathbf{3} \\ \mathbf{-1} & \mathbf{0} & \mathbf{2} \end{bmatrix}\begin{bmatrix} \mathbf{2} & \mathbf{-3} \\ \mathbf{1} & \mathbf{0} \\ \mathbf{3} & \mathbf{1} \end{bmatrix}$

Ans: we have to find $\begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ 1 & 0 \\ 3 & 1 \end{bmatrix}$

$$\therefore \begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ 1 & 0 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 3(2)-1(1)+3(3) & 3(-3)-1(0)+3(1) \\ -1(2)+0(1)+2(3) & -1(-3)+0(0)+2(1) \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ 1 & 0 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 14 & -6 \\ 4 & 5 \end{bmatrix}$$

4. **If** $\mathbf{A} = \begin{bmatrix} \mathbf{1} & \mathbf{2} & \mathbf{-3} \\ \mathbf{5} & \mathbf{0} & \mathbf{2} \\ \mathbf{1} & \mathbf{-1} & \mathbf{1} \end{bmatrix}, \mathbf{B} = \begin{bmatrix} \mathbf{3} & \mathbf{-1} & \mathbf{2} \\ \mathbf{4} & \mathbf{2} & \mathbf{5} \\ \mathbf{2} & \mathbf{0} & \mathbf{3} \end{bmatrix}, \mathbf{C} = \begin{bmatrix} \mathbf{4} & \mathbf{1} & \mathbf{2} \\ \mathbf{0} & \mathbf{3} & \mathbf{2} \\ \mathbf{1} & \mathbf{-2} & \mathbf{3} \end{bmatrix}$ **,then**

Compute $\mathbf{(A+B)}$ **and** $\mathbf{(B-C)}$ **.Also, verify that** $\mathbf{A+(B-C)=(A+B)-C}$ **.**

Ans: Given $A = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}, B = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix}, C = \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$

$$\therefore A+B=\begin{bmatrix}1 & 2 & -3\\5 & 0 & 2\\1 & -1 & 1\end{bmatrix}+\begin{bmatrix}3 & -1 & 2\\4 & 2 & 5\\2 & 0 & 3\end{bmatrix}$$

$$\Rightarrow A+B=\begin{bmatrix}1+3 & 2-1 & -3+2\\5+4 & 0+2 & 2+5\\1+2 & -1+0 & 1+3\end{bmatrix}$$

$$\Rightarrow A+B=\begin{bmatrix}4 & 1 & -1\\9 & 2 & 7\\3 & -1 & 4\end{bmatrix}$$

$$\text{And } B-C=\begin{bmatrix}3 & -1 & 2\\4 & 2 & 5\\2 & 0 & 3\end{bmatrix}-\begin{bmatrix}4 & 1 & 2\\0 & 3 & 2\\1 & -2 & 3\end{bmatrix}$$

$$\Rightarrow B-C=\begin{bmatrix}3-4 & -1-1 & 2-2\\4-0 & 2-3 & 5-2\\2-1 & 0+2 & 3-3\end{bmatrix}$$

$$\therefore B-C=\begin{bmatrix}-1 & -2 & 0\\4 & -1 & 3\\1 & 2 & 0\end{bmatrix}$$

Now,

$$A+\left(B-C\right)=\begin{bmatrix}1 & 2 & -3\\5 & 0 & 2\\1 & -1 & 1\end{bmatrix}+\begin{bmatrix}-1 & -2 & 0\\4 & -1 & 3\\1 & 2 & 0\end{bmatrix}$$

$$\Rightarrow A+\left(B-C\right)=\begin{bmatrix}1-1 & 2-2 & -3+0\\5+4 & 0-1 & 2+3\\1+1 & -1+2 & 1+0\end{bmatrix}$$

$$\therefore A+(B-C)=\begin{bmatrix}0 & 0 & -3\\ 9 & -1 & 5\\ 2 & 1 & 1\end{bmatrix} \qquad \ldots(1)$$

$$\text{And } (A+B)-C=\begin{bmatrix}4 & 1 & -1\\ 9 & 2 & 7\\ 3 & -1 & 4\end{bmatrix}-\begin{bmatrix}4 & 1 & 2\\ 0 & 3 & 2\\ 1 & -2 & 3\end{bmatrix}$$

$$\Rightarrow (A+B)-C=\begin{bmatrix}4-4 & 1-1 & -1-2\\ 9-0 & 2-3 & 7-2\\ 3-1 & -1+2 & 4-3\end{bmatrix}$$

$$\therefore (A+B)-C=\begin{bmatrix}0 & 0 & -3\\ 9 & -1 & 5\\ 2 & 1 & 1\end{bmatrix} \qquad \ldots(2)$$

Thus, from equation (1) and (2),

$$A+(B-C)=(A+B)-C$$

Hence proved.

5. If $A=\begin{bmatrix}\frac{2}{3} & 1 & \frac{5}{3}\\ \frac{1}{3} & \frac{2}{3} & \frac{4}{3}\\ \frac{7}{3} & 2 & \frac{2}{3}\end{bmatrix}$ and $B=\begin{bmatrix}\frac{2}{5} & \frac{3}{5} & 1\\ \frac{1}{5} & \frac{2}{5} & \frac{4}{5}\\ \frac{7}{5} & \frac{6}{5} & \frac{2}{5}\end{bmatrix}$ then compute $3A-5B$.

Ans: Given that $A=\begin{bmatrix}\frac{2}{3} & 1 & \frac{5}{3}\\ \frac{1}{3} & \frac{2}{3} & \frac{4}{3}\\ \frac{7}{3} & 2 & \frac{2}{3}\end{bmatrix}$ and $B=\begin{bmatrix}\frac{2}{5} & \frac{3}{5} & 1\\ \frac{1}{5} & \frac{2}{5} & \frac{4}{5}\\ \frac{7}{5} & \frac{6}{5} & \frac{2}{5}\end{bmatrix}$

$$\therefore 3A-5B=3\begin{bmatrix}\frac{2}{3} & 1 & \frac{5}{3}\\ \frac{1}{3} & \frac{2}{3} & \frac{4}{3}\\ \frac{7}{3} & 2 & \frac{2}{3}\end{bmatrix}-5\begin{bmatrix}\frac{2}{5} & \frac{3}{5} & 1\\ \frac{1}{5} & \frac{2}{5} & \frac{4}{5}\\ \frac{7}{5} & \frac{6}{5} & \frac{2}{5}\end{bmatrix}$$

$$\Rightarrow 3A-5B=\begin{bmatrix}2 & 3 & 5\\ 1 & 2 & 4\\ 7 & 6 & 2\end{bmatrix}-\begin{bmatrix}2 & 3 & 5\\ 1 & 2 & 4\\ 7 & 6 & 2\end{bmatrix}$$

$$\Rightarrow 3A-5B=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

6. Simplify $\cos\theta\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}+\sin\theta\begin{bmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{bmatrix}$.

Ans: we have to simplify $\cos\theta\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}+\sin\theta\begin{bmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{bmatrix}$

$$\therefore \cos\theta\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}+\sin\theta\begin{bmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{bmatrix}=$$

$$\begin{bmatrix}\cos^2\theta & \cos\theta\sin\theta\\ -\cos\theta\sin\theta & \cos^2\theta\end{bmatrix}+\begin{bmatrix}\sin^2\theta & -\sin\theta\cos\theta\\ \sin\theta\cos\theta & \sin^2\theta\end{bmatrix}$$

$$\Rightarrow \cos\theta\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}+\sin\theta\begin{bmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{bmatrix}=$$

$$\begin{bmatrix}\cos^2\theta+\sin^2\theta & \cos\theta\sin\theta-\cos\theta\sin\theta\\ -\cos\theta\sin\theta+\cos\theta\sin\theta & \cos^2\theta+\sin^2\theta\end{bmatrix}$$

$$\Rightarrow \cos\theta\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}+\sin\theta\begin{bmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

7. Find X and Y, if

i. $\mathbf{X+Y=\begin{bmatrix}7 & 0\\2 & 5\end{bmatrix}}$ **and** $\mathbf{X-Y=\begin{bmatrix}3 & 0\\0 & 3\end{bmatrix}}$

Ans: Given:

$$X+Y=\begin{bmatrix}7 & 0\\2 & 5\end{bmatrix} \qquad \ldots(1)$$

$$X-Y=\begin{bmatrix}3 & 0\\0 & 3\end{bmatrix} \qquad \ldots(2)$$

Adding equations (1) and (2), we get,

$$2X=\begin{bmatrix}7 & 0\\2 & 5\end{bmatrix}+\begin{bmatrix}3 & 0\\0 & 3\end{bmatrix}$$

$$\Rightarrow 2X=\begin{bmatrix}10 & 0\\2 & 8\end{bmatrix}$$

$$\Rightarrow X=\frac{1}{2}\begin{bmatrix}10 & 0\\2 & 8\end{bmatrix}$$

$$\therefore X=\begin{bmatrix}5 & 0\\1 & 4\end{bmatrix}$$

Now, since $X+Y=\begin{bmatrix}7 & 0\\2 & 5\end{bmatrix}$

$$\Rightarrow \begin{bmatrix}5 & 0\\1 & 4\end{bmatrix}+Y=\begin{bmatrix}7 & 0\\2 & 5\end{bmatrix}$$

$$\Rightarrow Y=\begin{bmatrix}7 & 0\\2 & 5\end{bmatrix}-\begin{bmatrix}5 & 0\\1 & 4\end{bmatrix}$$

$$\therefore Y=\begin{bmatrix}2 & 0\\1 & 1\end{bmatrix}$$

$$\therefore X=\begin{bmatrix}5 & 0\\ 1 & 4\end{bmatrix}, Y=\begin{bmatrix}2 & 0\\ 1 & 1\end{bmatrix}$$

ii. $\mathbf{2X+3Y=\begin{bmatrix}2 & 3\\ 4 & 0\end{bmatrix}}$ **and** $\mathbf{3X+2Y=\begin{bmatrix}2 & -2\\ -1 & 5\end{bmatrix}}$

Ans: Given:

$$2X+3Y=\begin{bmatrix}2 & 3\\ 4 & 0\end{bmatrix} \quad \ldots(1)$$

$$3X+2Y=\begin{bmatrix}2 & -2\\ -1 & 5\end{bmatrix} \quad \ldots(2)$$

Multiplying equation (1) with two we get,

$$2(2X+3Y)=2\begin{bmatrix}2 & 3\\ 4 & 0\end{bmatrix}$$

$$\Rightarrow (4X+6Y)=\begin{bmatrix}4 & 6\\ 8 & 0\end{bmatrix} \quad \ldots(3)$$

Multiplying equation (2) with three we get,

$$3(3X+2Y)=3\begin{bmatrix}2 & -2\\ -1 & 5\end{bmatrix}$$

$$\Rightarrow (9X+6Y)=\begin{bmatrix}6 & -6\\ -3 & 15\end{bmatrix} \quad \ldots(4)$$

From equation (3) and (4),

$$(4X+6Y)-(9X+6Y)=\begin{bmatrix}4 & 6\\ 8 & 0\end{bmatrix}-\begin{bmatrix}6 & -6\\ -3 & 15\end{bmatrix}$$

$$\Rightarrow -5X=\begin{bmatrix}-2 & 12\\ 11 & -15\end{bmatrix}$$

$$\Rightarrow X = \frac{-1}{5}\begin{bmatrix} -2 & 12 \\ 11 & -15 \end{bmatrix}$$

$$\therefore X = \begin{bmatrix} \frac{2}{5} & \frac{-12}{5} \\ \frac{-11}{5} & 3 \end{bmatrix}$$

Now, since $2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$

$$\therefore 2\begin{bmatrix} \frac{2}{5} & \frac{-12}{5} \\ \frac{-11}{5} & 3 \end{bmatrix} + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \frac{4}{5} & \frac{-24}{5} \\ \frac{-22}{5} & 6 \end{bmatrix} + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$$

$$\Rightarrow 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} - \begin{bmatrix} \frac{4}{5} & \frac{-24}{5} \\ \frac{-22}{5} & 6 \end{bmatrix}$$

$$\Rightarrow 3Y = \begin{bmatrix} \frac{6}{5} & \frac{39}{5} \\ \frac{42}{5} & -6 \end{bmatrix}$$

$$\therefore Y = \begin{bmatrix} \frac{2}{5} & \frac{13}{5} \\ \frac{14}{5} & -2 \end{bmatrix}$$

$$\therefore X = \begin{bmatrix} \frac{2}{5} & \frac{-12}{5} \\ \frac{-11}{5} & 3 \end{bmatrix}, Y = \begin{bmatrix} \frac{2}{5} & \frac{13}{5} \\ \frac{14}{5} & -2 \end{bmatrix}$$

8. Find X ,if $Y = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ and $2X + Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$.

Ans: Given $Y = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$

And $2X + Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$

$$\Rightarrow 2X + \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} -2 & -2 \\ -4 & -2 \end{bmatrix}$$

$$\Rightarrow X = \frac{1}{2}\begin{bmatrix} -2 & -2 \\ -4 & -2 \end{bmatrix}$$

$$\therefore X = \begin{bmatrix} -1 & -1 \\ -2 & -1 \end{bmatrix}$$

9. Find X and Y ,if $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

Ans: Given: $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 2 & 6 \\ 0 & 2x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2+y & 6+0 \\ 0+1 & 2x+2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2+y & 6 \\ 1 & 2x+2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

Comparing the corresponding elements of these two matrices, we get:

$2+y=5$

$\Rightarrow y=3$

$2x+2=8$

$\Rightarrow x=3$

$\therefore x=3, y=3$

10. Solve the equation for $\mathbf{X, Y, Z}$ and $\mathbf{t}$ if

$$2\begin{bmatrix} \mathbf{x} & \mathbf{z} \\ \mathbf{y} & \mathbf{t} \end{bmatrix} + 3\begin{bmatrix} \mathbf{1} & \mathbf{-1} \\ \mathbf{0} & \mathbf{2} \end{bmatrix} = 3\begin{bmatrix} \mathbf{3} & \mathbf{5} \\ \mathbf{4} & \mathbf{6} \end{bmatrix}$$

Ans: Given: $2\begin{bmatrix} x & z \\ y & t \end{bmatrix} + 3\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 2x & 2z \\ 2y & 2t \end{bmatrix} + \begin{bmatrix} 3 & -3 \\ 0 & 6 \end{bmatrix} = \begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x+3 & 2z-3 \\ 2y & 2t+6 \end{bmatrix} = \begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix}$$

Equating the corresponding elements of these two matrices, we get:

$2x+3=9$

$\Rightarrow x=3$

$2y=12$

$\Rightarrow y = 6$

$2z - 3 = 15$

$\Rightarrow z = 9$

$2t + 6 = 18$

$\Rightarrow t = 6$

$\therefore x = 3, y = 6, z = 9, t = 6$

11. **If $x\begin{bmatrix} 2 \\ 3 \end{bmatrix} + y\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$,find values of x and y .**

Ans: Given: $x\begin{bmatrix} 2 \\ 3 \end{bmatrix} + y\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2x \\ 3x \end{bmatrix} + \begin{bmatrix} -y \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2x - y \\ 3x + y \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$

Equating the corresponding elements of these two matrices, we get:

$2x - y = 10$ and

$3x + y = 5$

Adding these two equations, we get:

$5x = 15$

$\Rightarrow x = 3$

Now, since $3x + y = 5$

$\Rightarrow y = 5 - 3x$

$\Rightarrow y = 5 - 9$

$\Rightarrow y = -4$

$\therefore x = 3, y = -4$

12. **Given** $3\begin{bmatrix} \mathbf{x} & \mathbf{y} \\ \mathbf{z} & \mathbf{w} \end{bmatrix} = \begin{bmatrix} \mathbf{x} & \mathbf{6} \\ \mathbf{-1} & \mathbf{2w} \end{bmatrix} + \begin{bmatrix} \mathbf{4} & \mathbf{x+y} \\ \mathbf{z+w} & \mathbf{2w+3} \end{bmatrix}$ **,find the values of** $\mathbf{x, y, z}$ **and** $\mathbf{w}$ **.**

Ans: Given: $3\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 2w+3 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 3x & 3y \\ 3z & 3w \end{bmatrix} = \begin{bmatrix} x+4 & 6+x+y \\ -1+z+w & 2w+3 \end{bmatrix}$$

Equating the corresponding elements of these two matrices, we get:

$3x = x + 4$

$\Rightarrow x = 2$

$3y = 6 + x + y$

$\Rightarrow y = 4$

$3w = 2w + 3$

$\Rightarrow w = 3$

$3z = -1 + z + w$

$\Rightarrow z = 1$

$\therefore x = 2, y = 4, z = 1, w = 3$

13. **If** $\mathbf{F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}}$ **, show that** $\mathbf{F(x)F(y) = F(x+y)}$ **.**

Ans: Here, $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$F(y) = \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore F(x)F(y)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos y & -\sin y & 0\\ \sin y & \cos y & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$\Rightarrow F(x)F(y)=\begin{bmatrix}\cos x\cos y-\sin x\sin y+0 & -\cos x\sin y-\sin x\cos y+0 & 0\\ \sin x\cos y-\cos x\sin y+0 & -\sin x\sin y+\cos x\cos y+0 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$\Rightarrow F(x)F(y)=\begin{bmatrix}\cos(x+y) & -\sin(x+y) & 0\\ \sin(x+y) & \cos(x+y) & 0\\ 0 & 0 & 1\end{bmatrix} \qquad \ldots(1)$$

$$\text{And } F(x+y)=\begin{bmatrix}\cos(x+y) & -\sin(x+y) & 0\\ \sin(x+y) & \cos(x+y) & 0\\ 0 & 0 & 1\end{bmatrix} \qquad \ldots(2)$$

From equation (1) and (2),

$$F(x)F(y)=F(x+y)$$

Hence proved.

14. Show that

i. $\begin{bmatrix}5 & -1\\ 6 & 7\end{bmatrix}\begin{bmatrix}2 & 1\\ 3 & 4\end{bmatrix}\neq\begin{bmatrix}2 & 1\\ 3 & 4\end{bmatrix}\begin{bmatrix}5 & -1\\ 6 & 7\end{bmatrix}$

Ans: LHS: $\begin{bmatrix}5 & -1\\ 6 & 7\end{bmatrix}\begin{bmatrix}2 & 1\\ 3 & 4\end{bmatrix}=\begin{bmatrix}5(2)-1(3) & 5(1)-1(4)\\ 6(2)+7(3) & 6(1)+7(4)\end{bmatrix}$

$$\Rightarrow\begin{bmatrix}5 & -1\\ 6 & 7\end{bmatrix}\begin{bmatrix}2 & 1\\ 3 & 4\end{bmatrix}=\begin{bmatrix}7 & 1\\ 33 & 34\end{bmatrix} \qquad \ldots(1)$$

RHS: $\begin{bmatrix}2 & 1\\ 3 & 4\end{bmatrix}\begin{bmatrix}5 & -1\\ 6 & 7\end{bmatrix}=\begin{bmatrix}2(5)+1(6) & 2(-1)+1(7)\\ 3(5)+4(6) & 3(-1)+4(7)\end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} = \begin{bmatrix} 16 & 5 \\ 39 & 25 \end{bmatrix} \qquad \ldots(2)$$

From equation (1) and (2),

$\text{LHS} \neq \text{RHS}$

Hence proved.

ii. $$\begin{bmatrix} \mathbf{1} & \mathbf{2} & \mathbf{3} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{1} & \mathbf{1} & \mathbf{0} \end{bmatrix}\begin{bmatrix} \mathbf{-1} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{-1} & \mathbf{1} \\ \mathbf{2} & \mathbf{3} & \mathbf{4} \end{bmatrix} \neq \begin{bmatrix} \mathbf{-1} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{-1} & \mathbf{1} \\ \mathbf{2} & \mathbf{3} & \mathbf{4} \end{bmatrix}\begin{bmatrix} \mathbf{1} & \mathbf{2} & \mathbf{3} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{1} & \mathbf{1} & \mathbf{0} \end{bmatrix}$$

Ans: LHS:

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} =$$

$$\begin{bmatrix} 1(-1)+2(0)+3(2) & 1(1)+2(-1)+3(3) & 1(0)+2(1)+3(4) \\ 0(-1)+1(0)+0(2) & 0(1)+1(-1)+0(3) & 0(0)+1(1)+0(4) \\ 1(-1)+1(0)+0(2) & 1(1)+1(-1)+0(3) & 1(0)+1(1)+0(4) \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 5 & 8 & 14 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \qquad \ldots(1)$$

RHS:

$$\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} =$$

$$\begin{bmatrix} -1(1)+1(0)+0(1) & -1(2)+1(1)+0(1) & -1(3)+1(0)+0(0) \\ 0(1)-1(0)+1(1) & 0(2)-1(1)+1(1) & 0(3)-1(0)+1(0) \\ 2(1)+3(0)+4(1) & 2(2)+3(1)+4(1) & 2(3)+3(0)+4(0) \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & -1 & 3 \\ 1 & 0 & 0 \\ 6 & 11 & 6 \end{bmatrix} \qquad \ldots(2)$$

From equation (1) and (2),

$\text{LHS} \neq \text{RHS}$

Hence proved.

15. Find $\mathbf{A^2 - 5A + 6I}$ if $\mathbf{A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}}$.

Ans: Given: $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$

$A^2 = AA$

$$\therefore A^2 = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 2(2)+0(2)+1(1) & 2(0)+0(1)+1(-1) & 2(1)+0(3)+1(0) \\ 2(2)+1(2)+3(1) & 2(0)+1(1)+3(-1) & 2(1)+1(3)+3(0) \\ 1(2)-1(2)+0(1) & 1(0)-1(1)+0(-1) & 1(1)-1(3)+0(0) \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix}$$

$$\therefore A^2 - 5A + 6I = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - 5\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} + 6\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow A^2-5A+6I=\begin{bmatrix}5 & -1 & 2\\ 9 & -2 & 5\\ 0 & -1 & -2\end{bmatrix}-\begin{bmatrix}10 & 0 & 5\\ 10 & 5 & 15\\ 5 & -5 & 0\end{bmatrix}+\begin{bmatrix}6 & 0 & 0\\ 0 & 6 & 0\\ 0 & 0 & 6\end{bmatrix}$$

$$\Rightarrow A^2-5A+6I=\begin{bmatrix}5-10 & -1-0 & 2-5\\ 9-10 & -2-5 & 5-15\\ 0-5 & -1+5 & -2-0\end{bmatrix}+\begin{bmatrix}6 & 0 & 0\\ 0 & 6 & 0\\ 0 & 0 & 6\end{bmatrix}$$

$$\Rightarrow A^2-5A+6I=\begin{bmatrix}-5 & -1 & -3\\ -1 & -7 & -10\\ -5 & 4 & -2\end{bmatrix}+\begin{bmatrix}6 & 0 & 0\\ 0 & 6 & 0\\ 0 & 0 & 6\end{bmatrix}$$

$$\Rightarrow A^2-5A+6I=\begin{bmatrix}1 & -1 & -3\\ -1 & -1 & -10\\ -5 & 4 & 4\end{bmatrix}$$

16. **If $A=\begin{bmatrix}1 & 0 & 2\\ 0 & 2 & 1\\ 2 & 0 & 3\end{bmatrix}$, prove that $A^3-6A^2+7A+21=0$.**

Ans: Given: $A=\begin{bmatrix}1 & 0 & 2\\ 0 & 2 & 1\\ 2 & 0 & 3\end{bmatrix}$

$A^2=AA$

$$\therefore A^2=\begin{bmatrix}1 & 0 & 2\\ 0 & 2 & 1\\ 2 & 0 & 3\end{bmatrix}\begin{bmatrix}1 & 0 & 2\\ 0 & 2 & 1\\ 2 & 0 & 3\end{bmatrix}$$

$$\Rightarrow A^2=\begin{bmatrix}1+0+4 & 0+0+0 & 2+0+6\\ 0+0+2 & 0+4+0 & 0+2+3\\ 2+0+6 & 0+0+0 & 4+0+9\end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}$$

Now, $A^3 = A^2 \cdot A$

$$\therefore A^3 = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

$$\Rightarrow A^3 = \begin{bmatrix} 5+0+16 & 0+0+0 & 10+0+24 \\ 2+0+10 & 0+8+0 & 4+4+15 \\ 8+0+26 & 0+0+0 & 16+0+39 \end{bmatrix}$$

$$\Rightarrow A^3 = \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix}$$

$\therefore A^3 - 6A^2 + 7A + 21 =$

$$\begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - 6\begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} + 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow A^3 - 6A^2 + 7A + 21 =$

$$\begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - \begin{bmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{bmatrix} + \begin{bmatrix} 7 & 0 & 14 \\ 0 & 14 & 7 \\ 14 & 0 & 21 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\Rightarrow A^3 - 6A^2 + 7A + 21 = \begin{bmatrix} 21-30+7+2 & 0-0+0+0 & 34-48+14+0 \\ 12-12+0+0 & 8-24+14+2 & 23-30+7+0 \\ 34-48+14+0 & 0-0+0+0 & 55-78+21+2 \end{bmatrix}$$

$$\Rightarrow A^3 - 6A^2 + 7A + 21 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\therefore A^3 - 6A^2 + 7A + 21 = 0$$

Hence proved.

17. If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find k so that $A^2 = kA - 21$.

Ans: Given: $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$

$$A^2 = AA$$

$$\Rightarrow A^2 = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 9-8 & -6+4 \\ 12-8 & -8+4 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix}$$

Now, $A^2 = kA - 21$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = k\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k-2 & -2k \\ 4k & -2k-2 \end{bmatrix}$$

Equating the corresponding elements, we have:

$$3k - 2 = 1$$

$\Rightarrow k=1$

Thus, the value of k is 1 .

18. **If** $\mathbf{A}=\begin{bmatrix} \mathbf{0} & -\mathbf{tan}\frac{\boldsymbol{\alpha}}{\mathbf{2}} \\ \mathbf{tan}\frac{\boldsymbol{\alpha}}{\mathbf{2}} & \mathbf{0} \end{bmatrix}$ **and** $\mathbf{I}$ **is the identity matrix of order** $\mathbf{2}$ **, show that**

$$\mathbf{I}+\mathbf{A}=(\mathbf{I}-\mathbf{A})\begin{bmatrix} \mathbf{cos}\boldsymbol{\alpha} & -\mathbf{sin}\boldsymbol{\alpha} \\ \mathbf{sin}\boldsymbol{\alpha} & \mathbf{cos}\boldsymbol{\alpha} \end{bmatrix}$$

Ans: Given: $A=\begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$

LHS: $I+A=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}+\begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$

$$\Rightarrow I+A=\begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix} \qquad \ldots(1)$$

RHS: $(I-A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}=\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}-\begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}\right)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$$\Rightarrow (I-A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}=\begin{bmatrix} 1 & \tan\frac{\alpha}{2} \\ -\tan\frac{\alpha}{2} & 1 \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$\Rightarrow (I-A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} \cos\alpha + \sin\alpha\tan\frac{\alpha}{2} & -\sin\alpha + \cos\alpha\tan\frac{\alpha}{2} \\ -\cos\alpha\tan\frac{\alpha}{2} + \sin\alpha & \sin\alpha\tan\frac{\alpha}{2} + \cos\alpha \end{bmatrix}$$

$$\Rightarrow (I-A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} =$$

$$\begin{bmatrix} 1-2\sin^2\frac{\alpha}{2} + 2\sin\frac{\alpha}{2} - \cos\frac{\alpha}{2}\tan\frac{\alpha}{2} & -2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} + \left(2\cos^2\frac{\alpha}{2} - 1\right)\tan\frac{\alpha}{2} \\ -\left(2\cos^2\frac{\alpha}{2} - 1\right)\tan\frac{\alpha}{2} + 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} & 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\tan\frac{\alpha}{2} + 1 - 2\sin^2\frac{\alpha}{2} \end{bmatrix}$$

$$\Rightarrow (I-A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} =$$

$$\begin{bmatrix} 1-2\sin^2\frac{\alpha}{2} + 2\sin^2\frac{\alpha}{2} & -2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} + 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} - \tan\frac{\alpha}{2} \\ -\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} + \tan\frac{\alpha}{2} + 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} & 2\sin^2\frac{\alpha}{2} + 1 - 2\sin^2\frac{\alpha}{2} \end{bmatrix}$$

$$\Rightarrow (I-A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix} \qquad \ldots(2)$$

Thus, from equation (1) and (2).

LHS $=$ RHS

Hence proved.

19. A trust fund has Rs 30,000 that must be invested in two different types of bonds. The first bond pays 5% interest per year, and the second bond pays 7% interest per year. Using matrix multiplication, determine how to divide Rs 30,000 among the two types of bonds. If the trust fund must obtain an annual total interest of:

i. Rs 1,800

Ans: Let Rs x be invested in the first round.

Then, the sum of money invested in the second bond pays Rs $(30000-x)$

It is given that the first bond pays 5% interest per year, and the second bond pays 7% interest per year.

We know that,

Simple interest for one year is $\frac{\text{principle} \times \text{rate}}{100}$.

Therefore, in order to obtain an annual total interest of Rs $1{,}800$,

$$\begin{bmatrix} x & (30000-x) \end{bmatrix} \begin{bmatrix} \frac{5}{100} \\ \frac{7}{100} \end{bmatrix} = 1800$$

$$\Rightarrow \frac{5x}{100} + \frac{7(30000-x)}{100} = 1800$$

$$\Rightarrow 210000 - 2x = 180000$$

$$\Rightarrow x = 15000$$

Thus, in order to obtain an annual total interest of Rs $1{,}800$, the trust fund should invest Rs 15000 in the first bond and the remaining Rs 15000 in the second bond.

ii. Rs $2{,}000$

Ans: Let Rs x be invested in the first round.

Then, the sum of money invested in the second bond pays Rs $(30000-x)$

It is given that the first bond pays 5% interest per year, and the second bond pays 7% interest per year.

We know that,

Simple interest for one year is $\frac{\text{principle} \times \text{rate}}{100}$.

Therefore, in order to obtain an annual total interest of Rs $2{,}000$,

$$\begin{bmatrix} x & (30000-x) \end{bmatrix}\begin{bmatrix} \frac{5}{100} \\ \frac{7}{100} \end{bmatrix} = 2000$$

$$\Rightarrow \frac{5x}{100} + \frac{7(30000-x)}{100} = 2000$$

$$\Rightarrow 210000 - 2x = 200000$$

$$\Rightarrow x = 5000$$

Thus, in order to obtain an annual total interest of Rs $2,000$, the trust fund should invest Rs 5000 in the first bond and the remaining Rs 25000 in the second bond.

20. The bookshop of a particular school has 10 dozen chemistry books, 8 dozen physics books, 10 dozen economics books. Their selling prices are Rs 80, Rs 60 and Rs 40 each respectively. Find the total amount the bookshop will receive from selling all the books using matrix algebra.

Ans: The total amount of money that will be received from the sale of all these books can be represented in the form of a matrix as:

$$12\begin{bmatrix} 10 & 8 & 10 \end{bmatrix}\begin{bmatrix} 80 \\ 60 \\ 40 \end{bmatrix}$$

$$\Rightarrow 12[10\times 80 + 8\times 60 + 10\times 40]$$

$$\Rightarrow 12[1680]$$

$$\Rightarrow 20160$$

Therefore, the bookshop will receive Rs 20160 from the sale.

21. Assume $\mathbf{X, Y, Z, W}$ and $\mathbf{P}$ are the matrices of order $\mathbf{2\times n, 3\times k, 2\times p, n\times 3}$ and $\mathbf{p\times k}$ respectively. The restriction on $\mathbf{n, k, p}$ so that $\mathbf{PY + WY}$ will be defined are:

A. $\mathbf{k = 3, p = n}$

B. $\mathbf{k}$ **is arbitrary,** $\mathbf{p=2}$

C. $\mathbf{p}$ **is arbitrary,** $\mathbf{k=3}$

D. $\mathbf{k=2, p=3}$

Ans: Matrices P and Y are of the orders $p\times k$ and $3\times k$ respectively.

Therefore, matrix PY will be defined if $k=3$.

Also, PY will be of order $p\times k$.

Since the number of columns in matrix W is equal to the number of rows in matrix Y .

Therefore, matrix WY is well-defined and is of the order $n\times k$.

Moreover, Matrices PY and WY can be added only when their orders are the same.

But PY is of the order $p\times k$ and WY is of the order $n\times k$.

Therefore, we must have $p=n$.

$\therefore p=n, k=3$ are the restrictions on n,k,p so that $PY+WY$ will be defined.

Thus, option (A) is correct.

22. Assume $\mathbf{X,Y,Z,W}$ **and** $\mathbf{P}$ **are matrices of order** $\mathbf{2\times n, 3\times k, 2\times p, n\times 3}$ **and** $\mathbf{p\times k}$ **respectively. If** $\mathbf{p=n}$ **, then the order of the matrix** $\mathbf{7X-5Z}$ **is:**

A. $\mathbf{p\times 2}$

B. $\mathbf{2\times n}$

C. $\mathbf{n\times 3}$

D. $\mathbf{p\times n}$

Ans: Matrix X is of the order $2\times n$.

Thus, matrix $7X$ is also of the same order.

Since, $p=n$

Matrix Z is of the order $2\times p$ or $2\times n$.

Thus, matrix $5Z$ is also of the same order.

Since, both the matrices $7X$ and $5Z$ are of the same order $2\times n$.

Therefore, $7X-5Z$ is well defined and is of the order $2\times n$.

Thus, option (B) is correct.

Exercise 3.3

1. Find the transpose of each of the following matrices:

i. $\begin{bmatrix} \mathbf{5} \\ \mathbf{\frac{1}{2}} \\ \mathbf{-1} \end{bmatrix}$

Ans: The transpose of a matrix is obtained by changing its rows into columns and its columns into rows.

Thus, if $A=\begin{bmatrix} 5 \\ \frac{1}{2} \\ -1 \end{bmatrix}$, then $A^T=\begin{bmatrix} 5 & \frac{1}{2} & -1 \end{bmatrix}$

ii. $\begin{bmatrix} \mathbf{1} & \mathbf{-1} \\ \mathbf{2} & \mathbf{3} \end{bmatrix}$

Ans: The transpose of a matrix is obtained by changing its rows into columns and its columns into rows.

Thus, if $A=\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$, then $A^T=\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$

iii. $\begin{bmatrix} \mathbf{-1} & \mathbf{5} & \mathbf{6} \\ \mathbf{\sqrt{3}} & \mathbf{5} & \mathbf{6} \\ \mathbf{2} & \mathbf{3} & \mathbf{-1} \end{bmatrix}$

Ans: The transpose of a matrix is obtained by changing its rows into columns and its columns into rows.

Thus, if $A = \begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix}$, then $A^T = \begin{bmatrix} -1 & \sqrt{3} & 2 \\ 5 & 5 & 3 \\ 6 & 6 & -1 \end{bmatrix}$

2. If $\mathbf{A} = \begin{bmatrix} \mathbf{-1} & \mathbf{2} & \mathbf{3} \\ \mathbf{5} & \mathbf{7} & \mathbf{9} \\ \mathbf{-2} & \mathbf{1} & \mathbf{1} \end{bmatrix}$ **and** $\mathbf{B} = \begin{bmatrix} \mathbf{-4} & \mathbf{1} & \mathbf{-5} \\ \mathbf{1} & \mathbf{2} & \mathbf{0} \\ \mathbf{1} & \mathbf{3} & \mathbf{1} \end{bmatrix}$**, then verify that**

i. $\mathbf{(A+B)' = A' + B'}$

Ans: Given that: $A = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$

Thus, we have $A' = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix}$ and $B' = \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix}$

$$(A+B) = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix} + \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$$

$$\therefore (A+B) = \begin{bmatrix} -5 & 3 & -2 \\ 6 & 9 & 9 \\ -1 & 4 & 2 \end{bmatrix}$$

$$\Rightarrow (A+B)' = \begin{bmatrix} -5 & 6 & -1 \\ 3 & 9 & 4 \\ -2 & 9 & 2 \end{bmatrix} \quad \ldots(1)$$

$$\text{And } A'+B' = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix} + \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix}$$

$$\therefore A'+B' = \begin{bmatrix} -5 & 6 & -1 \\ 3 & 9 & 4 \\ -2 & 9 & 2 \end{bmatrix} \quad \ldots(2)$$

Thus, from equation (1) and (2),

$(A+B)' = A'+B'$

Hence proved.

ii. $(A-B)' = A'-B'$

Ans: Given that: $A = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$

Thus, we have $A' = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix}$ and $B' = \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix}$

$$(A-B) = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix} - \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$$

$$\therefore (A-B) = \begin{bmatrix} 3 & 1 & 8 \\ 4 & 5 & 9 \\ -3 & -2 & 0 \end{bmatrix}$$

$$\Rightarrow (A-B)' = \begin{bmatrix} 3 & 4 & -3 \\ 1 & 5 & -2 \\ 8 & 9 & 0 \end{bmatrix} \qquad \ldots(1)$$

$$\text{And } A' - B' = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix} - \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix}$$

$$\therefore A' - B' = \begin{bmatrix} 3 & 4 & -3 \\ 1 & 5 & -2 \\ 8 & 9 & 0 \end{bmatrix} \qquad \ldots(2)$$

Thus, from equation (1) and (2),

$(A-B)' = A' - B'$

Hence proved.

3. If $A' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, then verify that

i. $(A+B)' = A' + B'$

Ans: We know that, $A = (A')'$

Thus, we have: $A = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$

$$\text{And } B' = \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$$

$$\therefore A + B = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$$

$$\Rightarrow A+B=\begin{bmatrix}2 & 1 & 1\\ 5 & 4 & 4\end{bmatrix}$$

$$\therefore (A+B)'=\begin{bmatrix}2 & 5\\ 1 & 4\\ 1 & 4\end{bmatrix} \qquad \ldots(1)$$

$$A'+B'=\begin{bmatrix}3 & 4\\ -1 & 2\\ 0 & 1\end{bmatrix}+\begin{bmatrix}-1 & 1\\ 2 & 2\\ 1 & 3\end{bmatrix}$$

$$\therefore A'+B'=\begin{bmatrix}2 & 5\\ 1 & 4\\ 1 & 4\end{bmatrix} \qquad \ldots(2)$$

Thus, from equation (1) and (2),

$(A+B)'=A'+B'$

Hence proved.

ii. $(A-B)'=A'-B'$

Ans: We know that, $A=(A')'$

Thus, we have: $A=\begin{bmatrix}3 & -1 & 0\\ 4 & 2 & 1\end{bmatrix}$

And $B'=\begin{bmatrix}-1 & 1\\ 2 & 2\\ 1 & 3\end{bmatrix}$

$$\therefore A-B=\begin{bmatrix}3 & -1 & 0\\ 4 & 2 & 1\end{bmatrix}-\begin{bmatrix}-1 & 2 & 1\\ 1 & 2 & 3\end{bmatrix}$$

$$\Rightarrow A-B=\begin{bmatrix}4 & -3 & -1\\ 3 & 0 & -2\end{bmatrix}$$

$$\therefore (A-B)' = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix} \qquad \ldots(1)$$

$$A' - B' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$$

$$\therefore A' - B' = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix} \qquad \ldots(2)$$

Thus, from equation (1) and (2),

$$(A-B)' = A' - B'$$

Hence proved.

4. If $A' = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$, then $(A+2B)'$.

Ans: We know that, $A = (A')'$

$$\therefore A = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix}$$

$$\therefore A + 2B = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix} + 2\begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$$

$$\Rightarrow A + 2B = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} -2 & 0 \\ 2 & 4 \end{bmatrix}$$

$$\Rightarrow A + 2B = \begin{bmatrix} -4 & 1 \\ 5 & 6 \end{bmatrix}$$

$$\therefore (A+2B)' = \begin{bmatrix} -4 & 5 \\ 1 & 6 \end{bmatrix}$$

5. **For the matrices $\mathbf{A}$ and $\mathbf{B}$, verify that $(\mathbf{AB})' = \mathbf{B'A'}$ where**

i. $\mathbf{A} = \begin{bmatrix} \mathbf{1} \\ \mathbf{-4} \\ \mathbf{3} \end{bmatrix}, \mathbf{B} = \begin{bmatrix} \mathbf{-1} & \mathbf{2} & \mathbf{1} \end{bmatrix}$

Ans: LHS: $AB = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix} \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$

$$\Rightarrow AB = \begin{bmatrix} -1 & 2 & 1 \\ 4 & -8 & -4 \\ -3 & 6 & 3 \end{bmatrix}$$

$$\therefore (AB)' = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix} \qquad \ldots(1)$$

Now, $A' = \begin{bmatrix} 1 & -4 & 3 \end{bmatrix}$

And $B' = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$

RHS: $\therefore B'A' = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & -4 & 3 \end{bmatrix}$

$$\therefore B'A' = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix} \qquad \ldots(2)$$

Thus, from equation (1) and (2),

$(AB)' = B'A'$

Hence proved.

ii. $\mathbf{A}=\begin{bmatrix}\mathbf{0}\\ \mathbf{1}\\ \mathbf{2}\end{bmatrix},\mathbf{B}=\begin{bmatrix}\mathbf{1} & \mathbf{5} & \mathbf{7}\end{bmatrix}$

Ans: LHS: $AB=\begin{bmatrix}0\\ 1\\ 2\end{bmatrix}\begin{bmatrix}1 & 5 & 7\end{bmatrix}$

$$\Rightarrow AB=\begin{bmatrix}0 & 1 & 2\\ 0 & 5 & 10\\ 0 & 7 & 14\end{bmatrix}$$

$$\therefore (AB)'=\begin{bmatrix}0 & 1 & 2\\ 0 & 5 & 10\\ 0 & 7 & 14\end{bmatrix} \qquad \ldots(1)$$

Now, $A'=\begin{bmatrix}0 & 1 & 2\end{bmatrix}$

And $B'=\begin{bmatrix}1\\ 5\\ 7\end{bmatrix}$

RHS: $\therefore B'A'=\begin{bmatrix}1\\ 5\\ 7\end{bmatrix}\begin{bmatrix}0 & 1 & 2\end{bmatrix}$

$$\therefore B'A'=\begin{bmatrix}0 & 1 & 2\\ 0 & 5 & 10\\ 0 & 7 & 14\end{bmatrix} \qquad \ldots(2)$$

Thus, from equation (1) and (2),

$(AB)'=B'A'$

Hence proved.

6. If

i. $\mathbf{A}=\begin{bmatrix}\cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha\end{bmatrix}$**, then verify that** $\mathbf{A'A=I}$

Ans: Here, $A'=\begin{bmatrix}\cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha\end{bmatrix}$

$$\therefore A'A=\begin{bmatrix}\cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha\end{bmatrix}\begin{bmatrix}\cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha\end{bmatrix}$$

$$\Rightarrow A'A=\begin{bmatrix}\cos^2\alpha+\sin^2\alpha & \sin\alpha\cos\alpha-\sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha-\sin\alpha\cos\alpha & \cos^2\alpha+\sin^2\alpha\end{bmatrix}$$

$$\therefore A'A=\begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix}$$

$$\therefore A'A=I$$

Hence proved.

ii. $\mathbf{A}=\begin{bmatrix}\sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha\end{bmatrix}$**, then verify that** $\mathbf{A'A=I}$

Ans: Here, $A'=\begin{bmatrix}\sin\alpha & -\cos\alpha \\ \cos\alpha & \sin\alpha\end{bmatrix}$

$$\therefore A'A=\begin{bmatrix}\sin\alpha & -\cos\alpha \\ \cos\alpha & \sin\alpha\end{bmatrix}\begin{bmatrix}\sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha\end{bmatrix}$$

$$\Rightarrow A'A=\begin{bmatrix}\cos^2\alpha+\sin^2\alpha & \sin\alpha\cos\alpha-\sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha-\sin\alpha\cos\alpha & \cos^2\alpha+\sin^2\alpha\end{bmatrix}$$

$$\therefore A'A=\begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix}$$

$$\therefore A'A=I$$

Hence proved.

7. Show that

i. The matrix $A=\begin{bmatrix}1 & -1 & 5\\ -1 & 2 & 1\\ 5 & 1 & 3\end{bmatrix}$ **is a symmetric matrix.**

Ans: Given $A=\begin{bmatrix}1 & -1 & 5\\ -1 & 2 & 1\\ 5 & 1 & 3\end{bmatrix}$

$$\therefore A'=\begin{bmatrix}1 & -1 & 5\\ -1 & 2 & 1\\ 5 & 1 & 3\end{bmatrix}$$

Here, we have $A'=A$

Thus, A is a symmetric matrix.

ii. The matrix $A=\begin{bmatrix}0 & 1 & -1\\ -1 & 0 & 1\\ 1 & -1 & 0\end{bmatrix}$ **is a skew symmetric matrix.**

Ans: Given $A=\begin{bmatrix}0 & 1 & -1\\ -1 & 0 & 1\\ 1 & -1 & 0\end{bmatrix}$

$$\therefore A'=\begin{bmatrix}0 & -1 & 1\\ 1 & 0 & -1\\ -1 & 1 & 0\end{bmatrix}$$

$$\Rightarrow A'=-\begin{bmatrix}0 & 1 & -1\\ -1 & 0 & 1\\ 1 & -1 & 0\end{bmatrix}$$

Here, we have $A'=-A$

Thus, A is a skew symmetric matrix.

8. For the matrix $A=\begin{bmatrix}1 & 5\\ 6 & 7\end{bmatrix}$, verify that

i. $(A+A')$ is a symmetric matrix.

Ans: Given $A=\begin{bmatrix}1 & 5\\ 6 & 7\end{bmatrix}$

$$\therefore A'=\begin{bmatrix}1 & 6\\ 5 & 7\end{bmatrix}$$

$$A+A'=\begin{bmatrix}1 & 5\\ 6 & 7\end{bmatrix}+\begin{bmatrix}1 & 6\\ 5 & 7\end{bmatrix}$$

$$\Rightarrow A+A'=\begin{bmatrix}2 & 11\\ 11 & 14\end{bmatrix}$$

$$(A+A')'=\begin{bmatrix}2 & 11\\ 11 & 14\end{bmatrix}$$

Here, we have $A+A'=(A+A')'$

Thus, $A+A'$ is a symmetric matrix.

ii. $(A-A')$ is a skew symmetric matrix.

Ans: Given $A=\begin{bmatrix}1 & 5\\ 6 & 7\end{bmatrix}$

$$\therefore A'=\begin{bmatrix}1 & 6\\ 5 & 7\end{bmatrix}$$

$$A-A'=\begin{bmatrix}1 & 5\\ 6 & 7\end{bmatrix}-\begin{bmatrix}1 & 6\\ 5 & 7\end{bmatrix}$$

$$\Rightarrow A-A'=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$$

$$(A - A')' = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$\Rightarrow (A - A')' = -\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

Here, we have $A - A' = -(A - A')'$

Thus, $A - A'$ is a skew symmetric matrix.

9. **Find $\frac{1}{2}(A + A')$ and $\frac{1}{2}(A - A')$, when $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$.**

Ans: Given $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$

$$\therefore A' = \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix}$$

$$A + A' = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix} + \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix}$$

$$\Rightarrow A + A' = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\therefore \frac{1}{2}(A + A') = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\text{Now, } A - A' = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix} - \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix}$$

$$(A - A') = \begin{bmatrix} 0 & 2a & 2b \\ -2a & 0 & 2c \\ -2b & -2c & 0 \end{bmatrix}$$

$$\therefore \frac{1}{2}(A - A') = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$$

10. **Express the following matrices as the sum of a symmetric and a skew symmetric matrix:**

i. $\begin{bmatrix} \mathbf{3} & \mathbf{5} \\ \mathbf{1} & \mathbf{-1} \end{bmatrix}$

Ans: Let $A = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$ and $A' = \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}$

$$\text{Now, } A + A' = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}$$

$$\Rightarrow A + A' = \begin{bmatrix} 6 & 6 \\ 6 & -2 \end{bmatrix}$$

$$\text{Let } P = \frac{1}{2}(A + A')$$

$$\Rightarrow P = \frac{1}{2}\begin{bmatrix} 6 & 6 \\ 6 & -2 \end{bmatrix}$$

$$\Rightarrow P = \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix}$$

Now, $P' = \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix}$

Here, $P = P'$

$\therefore P = \frac{1}{2}(A + A')$ is a symmetric matrix.

Now, $A - A' = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}$

$\Rightarrow A - A' = \begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix}$

Let $Q = \frac{1}{2}(A - A')$

$\Rightarrow Q = \frac{1}{2}\begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix}$

$\Rightarrow Q = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$

Now, $Q' = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$

Here, $Q = -Q'$

$\therefore Q = \frac{1}{2}(A - A')$ is a skew symmetric matrix.

Thus, A is the sum of matrices $P = \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix}$ and $Q = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$.

ii. $\begin{bmatrix} \mathbf{6} & \mathbf{-2} & \mathbf{2} \\ \mathbf{-2} & \mathbf{3} & \mathbf{-1} \\ \mathbf{2} & \mathbf{-1} & \mathbf{3} \end{bmatrix}$

Ans: Let $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ and $A' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

Now, $A + A' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} + \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

$\Rightarrow A + A' = \begin{bmatrix} 12 & -4 & 4 \\ -4 & 6 & -2 \\ 4 & -2 & 6 \end{bmatrix}$

Let $P = \frac{1}{2}(A + A')$

$\Rightarrow P = \frac{1}{2}\begin{bmatrix} 12 & -4 & 4 \\ -4 & 6 & -2 \\ 4 & -2 & 6 \end{bmatrix}$

$\Rightarrow P = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

Now, $P' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

Here, $P = P'$

$\therefore P = \frac{1}{2}(A + A')$ is a symmetric matrix.

Now, $A - A' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} - \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

$$\Rightarrow A - A' = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Let $Q = \frac{1}{2}(A - A')$

$$\Rightarrow Q = \frac{1}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow Q = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Now, $Q' = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Here, $Q = -Q'$

$\therefore Q = \frac{1}{2}(A - A')$ is a skew symmetric matrix.

Thus, A is the sum of matrices $P = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ and $Q = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

iii. $\begin{bmatrix} \mathbf{3} & \mathbf{3} & \mathbf{-1} \\ \mathbf{-2} & \mathbf{-2} & \mathbf{1} \\ \mathbf{-4} & \mathbf{-5} & \mathbf{2} \end{bmatrix}$

Ans: Let $A = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$ and $A' = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$

$$\text{Now, } A+A'=\begin{bmatrix}3 & 3 & -1\\ -2 & -2 & 1\\ -4 & -5 & 2\end{bmatrix}+\begin{bmatrix}3 & -2 & -4\\ 3 & -2 & -5\\ -1 & 1 & 2\end{bmatrix}$$

$$\Rightarrow A+A'=\begin{bmatrix}6 & 1 & -5\\ 1 & -4 & -4\\ -5 & -4 & 4\end{bmatrix}$$

$$\text{Let } P=\frac{1}{2}(A+A')$$

$$\Rightarrow P=\frac{1}{2}\begin{bmatrix}6 & 1 & -5\\ 1 & -4 & -4\\ -5 & -4 & 4\end{bmatrix}$$

$$\Rightarrow P=\begin{bmatrix}3 & \frac{1}{2} & \frac{-5}{2}\\ \frac{1}{2} & -2 & -2\\ \frac{-5}{2} & -2 & 2\end{bmatrix}$$

$$\text{Now, } P'=\begin{bmatrix}3 & \frac{1}{2} & \frac{-5}{2}\\ \frac{1}{2} & -2 & -2\\ \frac{-5}{2} & -2 & 2\end{bmatrix}$$

Here, $P=P'$

$\therefore P=\frac{1}{2}(A+A')$ is a symmetric matrix.

$$\text{Now, } A-A'=\begin{bmatrix}3 & 3 & -1\\ -2 & -2 & 1\\ -4 & -5 & 2\end{bmatrix}-\begin{bmatrix}3 & -2 & -4\\ 3 & -2 & -5\\ -1 & 1 & 2\end{bmatrix}$$

$$\Rightarrow A-A'=\begin{bmatrix}0 & 5 & 3\\ -5 & 0 & 6\\ -3 & -6 & 0\end{bmatrix}$$

Let $Q=\frac{1}{2}\left(A-A'\right)$

$$\Rightarrow Q=\frac{1}{2}\begin{bmatrix}0 & 5 & 3\\ -5 & 0 & 6\\ -3 & -6 & 0\end{bmatrix}$$

$$\Rightarrow Q=\begin{bmatrix}0 & \frac{5}{2} & \frac{3}{2}\\ \frac{-5}{2} & 0 & 3\\ \frac{-3}{2} & -3 & 0\end{bmatrix}$$

Now, $Q'=\begin{bmatrix}0 & -\frac{5}{2} & -\frac{3}{2}\\ \frac{5}{2} & 0 & -3\\ \frac{3}{2} & 3 & 0\end{bmatrix}$

Here, $Q=-Q'$

$\therefore Q=\frac{1}{2}\left(A-A'\right)$ is a skew symmetric matrix.

Thus, A is the sum of matrices $P=\begin{bmatrix}3 & \frac{1}{2} & \frac{-5}{2}\\ \frac{1}{2} & -2 & -2\\ \frac{-5}{2} & -2 & 2\end{bmatrix}$ and $Q=\begin{bmatrix}0 & \frac{5}{2} & \frac{3}{2}\\ \frac{-5}{2} & 0 & 3\\ \frac{-3}{2} & -3 & 0\end{bmatrix}$.

iv. $\begin{bmatrix} \mathbf{1} & \mathbf{5} \\ \mathbf{-1} & \mathbf{2} \end{bmatrix}$

Ans: Let $A = \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}$ and $A' = \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}$

Now, $A + A' = \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}$

$\Rightarrow A + A' = \begin{bmatrix} 2 & 4 \\ 4 & 4 \end{bmatrix}$

Let $P = \frac{1}{2}(A + A')$

$\Rightarrow P = \frac{1}{2}\begin{bmatrix} 2 & 4 \\ 4 & 4 \end{bmatrix}$

$\Rightarrow P = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$

Now, $P' = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$

Here, $P = P'$

$\therefore P = \frac{1}{2}(A + A')$ is a symmetric matrix.

Now, $A - A' = \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}$

$\Rightarrow A - A' = \begin{bmatrix} 0 & 6 \\ -6 & 0 \end{bmatrix}$

Let $Q = \frac{1}{2}(A - A')$

$$\Rightarrow Q=\frac{1}{2}\begin{bmatrix}0 & 6\\ -6 & 0\end{bmatrix}$$

$$\Rightarrow Q=\begin{bmatrix}0 & 3\\ -3 & 0\end{bmatrix}$$

Now, $Q'=\begin{bmatrix}0 & -3\\ 3 & 0\end{bmatrix}$

Here, $Q=-Q'$

$\therefore Q=\frac{1}{2}\left(A-A'\right)$ is a skew symmetric matrix.

Thus, A is the sum of matrices $P=\begin{bmatrix}1 & 2\\ 2 & 2\end{bmatrix}$ and $Q=\begin{bmatrix}0 & 3\\ -3 & 0\end{bmatrix}$.

11. If A,B are symmetric matrices of same order, then $AB-BA$ is a:

A.Skew symmetric matrix

B.Symmetric matrix

C.Zero matrix

D.Identity matrix

Ans: A and B are symmetric matrix, therefore, we have:

$A'=A$ and $B'=B$ …(1)

Here, $\left(AB-BA\right)'=\left(AB\right)'-\left(BA\right)'$

$\Rightarrow\left(AB-BA\right)'=B'A'-A'B'$

From (1),

$\Rightarrow\left(AB-BA\right)'=BA-AB$

$\Rightarrow\left(AB-BA\right)'=-\left(AB-BA\right)$ …(2)

From equation (2),

$AB-BA$ is a skew symmetric matrix.

12. **If $A=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$, then $A+A'=I$, if the value of α is**

A. $\frac{\pi}{6}$

B. $\frac{\pi}{3}$

C. **n**

D. $\frac{3\pi}{2}$

Ans: Given $A=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$

$$\Rightarrow A'=\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$$

Now, $\because A+A'=I$

$$\therefore \begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}+\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}2\cos\alpha & 0\\ 0 & 2\cos\alpha\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

Equating the corresponding elements of the two matrices, we have:

$$\cos\alpha=\frac{1}{2}$$

$$\Rightarrow \alpha=\cos^{-1}\left(\frac{1}{2}\right)$$

$$\Rightarrow \alpha=\frac{\pi}{3}$$

Thus, option (B) is correct.

Exercise 3.4

1. Matrices A and B will be inverse of each other only if

A. $AB = BA$

B. $AB = 0, BA = I$

C. $AB = BA = 0$

D. $AB = BA = I$

Ans: Since, if A is a square matrix of order m, and if there exists another square matrix B of the same order m, such that $AB = BA = I$, then B is said to be the inverse of A. In such case, it is clear that A is the inverse of B.

Thus, matrices A and B will be inverse of each other only if $AB = BA = I$.

Thus, option (D) is correct.

Miscellaneous Solutions

1. If A and B are symmetric matrices, prove that $AB - BA$ is a skew symmetric matrix.

Ans: A and B are symmetric matrix, therefore, we have:

$A' = A$ and $B' = B$ …(1)

Here, $(AB - BA)' = (AB)' - (BA)'$

$\Rightarrow (AB - BA)' = B'A' - A'B'$

From (1),

$\Rightarrow (AB - BA)' = BA - AB$

$\Rightarrow (AB - BA)' = -(AB - BA)$ …(2)

From equation (2),

$AB - BA$ is a skew symmetric matrix.

2. **Show that the matrix $B'AB$ is symmetric or skew symmetric accordingly as A is symmetric or skew symmetric.**

Ans: Let A be a symmetric matrix, then $A' = A$ …(1)

$(B'AB)' = \{B'(AB)\}'$

$\Rightarrow (B'AB)' = (AB)'B'$

$\Rightarrow (B'AB)' = B'(A'B)$

From (1),

$\Rightarrow (B'AB)' = B'(AB)$

Thus, if A is a symmetric matrix, then $B'AB$ is a symmetric matrix.

Let A be a skew symmetric matrix, then $A' = -A$ …(2)

$(B'AB)' = \{B'(AB)\}'$

$\Rightarrow (B'AB)' = (AB)'B'$

$\Rightarrow (B'AB)' = B'(A'B)$

From (2),

$\Rightarrow (B'AB)' = B'(-AB)$

$\Rightarrow (B'AB)' = -B'AB$

Thus, if A is a skew-symmetric matrix, then $B'AB$ is a skew-symmetric matrix.

Therefore, the matrix $B'AB$ is symmetric or skew symmetric accordingly as A is symmetric or skew symmetric.

3. **Find the value of x, y, z if the matrix A = $\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ satisfy the equation A'A = I.**

Ans: We have, A = $\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ and A'= $\begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix}$

Also, A’ = A^{-1}

⇒ $AA^1 = AA^{-1}$

⇒ $AA^1 = I$ …… [∵ $AA^{-1} = I$]

⇒ $\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

⇒ $\begin{bmatrix} 4y^2 + z^2 & 2y^2 - z^2 & -2y^2 + z^2 \\ 2y^2 - z^2 & x^2 + y^2 + z^2 & x^2 + y^2 - z^2 \\ -2y^2 + z^2 & x^2 - y^2 - z^2 & x^2 + y^2 + z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

⇒ $2y^2 - z^2 = 0$

⇒ $2y^2 - z^2$

⇒ $4y^2 + z^2 = 1$

⇒ $2z^2 + z^2 = 1$

Z = $\pm\frac{1}{\sqrt{3}}$

∴ $y^2 = \frac{z^2}{2}$

⇒ y = $\pm\frac{1}{\sqrt{3}}$

Also, $x^2 + y^2 + z^2 = 1$

⇒ $x^2 = 1 - y^2 - z^2 = 1 - \frac{1}{6} - \frac{1}{3}$

$= 1 - \frac{3}{6} = \frac{1}{2}$

⇒ x = $\pm \frac{1}{\sqrt{2}}$

∴ x = $\pm\frac{1}{\sqrt{2}}$, $y = \pm\frac{1}{\sqrt{6}}$ *and* $z = \pm\frac{1}{\sqrt{3}}$

4. **For what values of x, $\begin{bmatrix}1 & 2 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 0\\ 2 & 0 & 1\\ 1 & 0 & 2\end{bmatrix}\begin{bmatrix}0\\ 2\\ x\end{bmatrix}=O$?**

Ans: Given $\begin{bmatrix}1 & 2 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 0\\ 2 & 0 & 1\\ 1 & 0 & 2\end{bmatrix}\begin{bmatrix}0\\ 2\\ x\end{bmatrix}=O$

$$\Rightarrow \begin{bmatrix}1+4+1 & 2+0+0 & 0+2+2\end{bmatrix}\begin{bmatrix}0\\ 2\\ x\end{bmatrix}=O$$

$$\Rightarrow \begin{bmatrix}6 & 2 & 4\end{bmatrix}\begin{bmatrix}0\\ 2\\ x\end{bmatrix}=O$$

$$\Rightarrow \left[6(0)+2(2)+4(x)\right]=O$$

$$\Rightarrow [4+4x]=[0]$$

$$\Rightarrow 4+4x=0$$

$$\therefore x=-1$$

Thus, the required value of x is -1.

5. **If $A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$, show that $A^2-5A+7I=O$.**

Ans: Given $A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$

$$\therefore A^2=A\cdot A$$

$$\Rightarrow A^2=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 3(3)+1(-1) & 3(1)+1(2) \\ -1(3)+2(-1) & -1(1)+2(2) \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

LHS: $A^2 - 5A + 7I$

$$\therefore \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\Rightarrow O$

RHS: $\Rightarrow O$

$LHS = RHS$

$\therefore A^2 - 5A + 7I = O$ hence proved.

6. Find X , if $\begin{bmatrix} \mathbf{x} & \mathbf{-5} & \mathbf{-1} \end{bmatrix} \begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{2} \\ \mathbf{0} & \mathbf{2} & \mathbf{1} \\ \mathbf{2} & \mathbf{0} & \mathbf{3} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{4} \\ \mathbf{1} \end{bmatrix} = \mathbf{O}$ **.**

Ans: Given $\begin{bmatrix} x & -5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$

$$\Rightarrow \begin{bmatrix} x-2 & -10 & 2x-8 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$$

$$\Rightarrow \left[x(x-2) - 40 + 2x - 8 \right] = O$$

$$\Rightarrow \left[x^2 - 48 \right] = [0]$$

$$\Rightarrow x^2 - 48 = 0$$

$$\therefore x = \pm 4\sqrt{3}$$

Thus, the required value of x is $\pm 4\sqrt{3}$.

7. **A manufacture produces three products X, Y, Z which he sells in two markets.**

Annual sales are indicated below:

Market	**Products**		
I	**10000**	**2000**	**18000**
II	**6000**	**20000**	**8000**

a) If unit sale prices of X, Y, Z are Rs 2.50 , Rs 1.50 and Rs 1.00 , respectively, find the total revenue in each market with the help of matrix algebra.

Ans: Here the total revenue in market I can be represented in the form of a matrix as:

$$\begin{bmatrix} 10000 & 2000 & 18000 \end{bmatrix} \begin{bmatrix} 2.50 \\ 1.50 \\ 1.00 \end{bmatrix}$$

$$\Rightarrow 1000 \times 2.50 + 2000 \times 1.50 + 18000 \times 1.00$$

$$\Rightarrow 46000$$

And, the total revenue in market II can be represented in the form of a matrix as:

$$[6000 \quad 20000 \quad 8000]\begin{bmatrix} 2.50 \\ 1.50 \\ 1.00 \end{bmatrix}$$

$\Rightarrow 6000 \times 2.50 + 20000 \times 1.50 + 8000 \times 1.00$

$\Rightarrow 53000$

Thus, the total revenue in market I is Rs 46000 and the same in market II is Rs 53000 .

b) If the unit costs of the above three commodities are Rs 2.00 , Rs 1.00 and 50 paise respectively. Find the gross profit.

Ans: Here, the total cost prices of all the products in the market I can be

represented in the

form of a matrix as:

$$[10000 \quad 2000 \quad 18000]\begin{bmatrix} 2.00 \\ 1.00 \\ 0.50 \end{bmatrix}$$

$\Rightarrow 1000 \times 2.00 + 2000 \times 1.00 + 18000 \times 0.50$

$\Rightarrow 31000$

As, the total revenue in market I is Rs 46000 , the gross profit in this market is

$\text{Rs}46000 - \text{Rs}31000 = \text{Rs}15000$.

And, the total revenue in market II can be represented in the form of a matrix as:

$$[6000 \quad 20000 \quad 8000]\begin{bmatrix} 2.00 \\ 1.00 \\ 0.50 \end{bmatrix}$$

$\Rightarrow 6000 \times 2.00 + 20000 \times 1.00 + 8000 \times 0.50$

$\Rightarrow 36000$

As, the total revenue in market II is Rs 53000 , the gross profit in this market is

$\text{Rs}53000 - \text{Rs}36000 = \text{Rs}17000$.

8. Find the matrix X so that $X\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\ 2 & 4 & 6\end{bmatrix}$.

Ans: Given $X\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\ 2 & 4 & 6\end{bmatrix}$

Here, X has to be a 2×2 matrix.

Now, let $X=\begin{bmatrix}a & c\\ b & d\end{bmatrix}$

Thus, we have:

$$\begin{bmatrix}a & c\\ b & d\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\ 2 & 4 & 6\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}a+4c & 2a+5c & 3a+6c\\ b+4d & 2b+5d & 3b+6d\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\ 2 & 4 & 6\end{bmatrix}$$

Comparing the corresponding elements of two matrices, we have:

$a+4c=-7$

$2a+5c=-8$

$3a+6c=-9$

$b+4d=2$

$2b+5d=4$

$3b+6d=6$

Now, solving the above equations we get,

$\therefore a=1, b=2, c=-2, d=0$

Thus, the required matrix X is $\begin{bmatrix}1 & -2\\ 2 & 0\end{bmatrix}$.

9. Choose the correct answer in the following questions:

If $A=\begin{bmatrix}\alpha & \beta\\ \gamma & -\alpha\end{bmatrix}$ is such that $A^2=I$ then

A. $1+\alpha^2+\beta\gamma=0$

B. $1-\alpha^2+\beta\gamma=0$

C. $1-\alpha^2-\beta\gamma=0$

D. $1+\alpha^2-\beta\gamma=0$

Ans: Given $A=\begin{bmatrix}\alpha & \beta\\ \gamma & -\alpha\end{bmatrix}$

$\therefore A^2=A\cdot A$

$\Rightarrow A^2=\begin{bmatrix}\alpha & \beta\\ \gamma & -\alpha\end{bmatrix}\begin{bmatrix}\alpha & \beta\\ \gamma & -\alpha\end{bmatrix}$

$\Rightarrow A^2=\begin{bmatrix}\alpha^2+\beta\gamma & \alpha\beta-\alpha\beta\\ \alpha\gamma-\alpha\gamma & \beta\gamma+\alpha^2\end{bmatrix}$

Now, $A^2=I$

$\Rightarrow \begin{bmatrix}\alpha^2+\gamma & 0\\ 0 & \beta\gamma+\alpha^2\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

Equating the corresponding elements, we get:

$\beta\gamma+\alpha^2=1$

$\Rightarrow \alpha^2+\beta\gamma-1=0$

$\Rightarrow 1-\alpha^2-\beta\gamma=0$

10. If the matrix A is both symmetric and skew symmetric, then

A. A is a diagonal matrix

B. A is a zero matrix

C. A is a square matrix

D. None of these

Ans: If a matrix A is both symmetric and skew symmetric, then

$A'=A$ and $A'=-A$

$\Rightarrow A=-A$

$\Rightarrow A+A=O$

$\Rightarrow A=O$

Thus, option (B) is correct.

11. **If A is a square matrix such that $A^2 = A$, then $(I+A)^3 - 7A$ is equal to**

A. A

B. $I - A$

C. I

D. $3A$

Ans: $(I+A)^3 - 7A = I^3 + A^3 + 3A + 3A^2 - 7A$

$\Rightarrow (I+A)^3 - 7A = I + A^3 + 3A + 3A^2 - 7A$

Given that $A^2 = A$,

$\Rightarrow (I+A)^3 - 7A = I + A^2 \cdot A + 3A + 3A^2 - 7A$

$\Rightarrow (I+A)^3 - 7A = I + A \cdot A - A$

$\Rightarrow (I+A)^3 - 7A = I$

Thus, option (C) is correct.