inverse trigonometric function

2
Chapter

Exercise 2.1

1. The principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$ is

Ans: Assuming $\sin^{-1}\left(-\frac{1}{2}\right)=x$

Further solving,

$\sin x=\left(-\frac{1}{2}\right)$

$=\sin\left(-\frac{\pi}{6}\right)$

The principal value of $\sin^{-1}x$ lies in the range of $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Therefore, the principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$ is $-\frac{\pi}{6}$

2. The principal value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is

Ans: Assuming $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)=x$

Further solving,

$\cos x=\left(\frac{\sqrt{3}}{2}\right)$

$=\cos\left(\frac{\pi}{6}\right)$

The principal value of $\cos^{-1}x$ lies in the range of $[0,\pi]$

Therefore, the principal value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is $\frac{\pi}{6}$

3. The principal value of $\operatorname{cosec}^{-1}(2)$ is

Ans: Assuming $\operatorname{cosec}^{-1}(2)=x$

Further solving,

$\operatorname{cosec}(x)=2$

$=\operatorname{cosec}\left(\frac{\pi}{6}\right)$

The principal value of $\operatorname{cosec}^{-1}x$ lies in the range of $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]-\{0\}$

Therefore, the principal value of $\operatorname{cosec}^{-1}(2)$ is $\frac{\pi}{6}$

4. The principal value of $\tan^{-1}\left(-\sqrt{3}\right)$ is

Ans: Assuming $\tan^{-1}\left(-\sqrt{3}\right)=x$

Further solving,

$\tan x=-\sqrt{3}$

$=\tan\left(-\frac{\pi}{3}\right)$

The principal value of $\tan^{-1}x$ lies in the range of $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Therefore, the principal value of $\tan^{-1}\left(-\sqrt{3}\right)$ is $-\frac{\pi}{3}$

5. The principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$ is

Ans: Assuming $\cos^{-1}\left(-\frac{1}{2}\right)=x$

Further solving,

$\cos x=-\frac{1}{2}$

$=\cos\left(\frac{2\pi}{3}\right)$

The principal value of $\cos^{-1}x$ lies in the range of $[0,\pi]$

Therefore, the principal value of $\cos^{-1}\left(\frac{1}{2}\right)$ is $\frac{2\pi}{3}$

6. The principal value of $\tan^{-1}(-1)$ is

Ans: Assuming $\tan^{-1}(-1)=x$

Further solving,

$\tan x=-1$

$=\tan\left(-\frac{\pi}{4}\right)$

The principal value of $\tan^{-1}x$ lies in the range of $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Therefore, the principal value of $\tan^{-1}(-1)$ is $-\frac{\pi}{4}$

7. The principal value of $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$ is

Ans: Assuming $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)=x$

Further solving,

$\sec x=\frac{2}{\sqrt{3}}$

$=\sec\left(\frac{\pi}{6}\right)$

The principal value of $\sec^{-1}x$ lies in the range of $[0,\pi]$

Therefore, the principal value of $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$ is $\frac{\pi}{6}$

8. The principal value of $\cot^{-1}\left(\sqrt{3}\right)$ is

Ans: Assuming $\cot^{-1}\left(\sqrt{3}\right)=x$

Further solving,

$\cot x=\sqrt{3}$

$=\cot\left(\frac{\pi}{6}\right)$

The principal value of $\cot^{-1}x$ lies in the range of $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Therefore, the principal value of $\cot^{-1}\left(\sqrt{3}\right)$ is $\frac{\pi}{6}$

9. The principal value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ is

Ans: Assuming $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)=x$

Further solving,

$\cos x=-\frac{1}{\sqrt{2}}$

$=\cos\left(\frac{3\pi}{4}\right)$

The principal value of $\cos^{-1}x$ lies in the range of $[0,\pi]$

Therefore, the principal value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ is $\frac{3\pi}{4}$

10. The principal val ue of $\mathbf{cosec^{-1}\left(-\sqrt{2}\right)}$ is

Ans: Assuming $\text{cosec}^{-1}\left(-\sqrt{2}\right)=x$

Further solving,

$\text{cosec}x=-\sqrt{2}$

$=\text{cosec}\left(-\frac{\pi}{4}\right)$

The principal value of $\cos^{-1}x$ lies in the range of $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]-\{0\}$

Therefore, the principal value of $\text{cosec}^{-1}\left(-\sqrt{2}\right)$ is $-\frac{\pi}{4}$

11. Evaluate$\mathbf{\tan^{-1}(1)+\cos^{-1}\left(-\frac{1}{2}\right)+\sin^{-1}\left(-\frac{1}{2}\right)}$

Ans: Assuming that $\tan^{-1}(1)=a$

$\tan a=1$

$=\tan\frac{\pi}{4}$

$\tan^{-1}(1)=\frac{\pi}{4}$ -----(1)

Assuming that $\cos^{-1}\left(\frac{1}{2}\right)=b$

$\cos b=\frac{1}{2}$

$=\cos\frac{\pi}{3}$

$\cos^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{3}$ -----(2)

Assuming that $\sin^{-1}\left(\frac{1}{2}\right)=c$

$\sin c=\frac{1}{2}$

$=\sin\frac{\pi}{6}$

$\sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{6}$ -----(3)

From Equations (1), (2) and (3),

$\tan^{-1}(1)+\cos^{-1}\left(-\frac{1}{2}\right)+\sin^{-1}\left(-\frac{1}{2}\right)$

$=\frac{\pi}{4}+\frac{2\pi}{3}-\frac{\pi}{6}$

$=\frac{3\pi}{4}$

Therefore, $\tan^{-1}(1)+\cos^{-1}\left(-\frac{1}{2}\right)+\sin^{-1}\left(-\frac{1}{2}\right)=\frac{3\pi}{4}$

12. **Evaluate** $\mathbf{\cos^{-1}\left(\frac{1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right)}$

Ans: Assuming that $\cos^{-1}\left(\frac{1}{2}\right)=a$

$\cos a=\frac{1}{2}$

$=\cos\frac{\pi}{3}$

$\cos^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{3}$ -----(1)

Assuming that $2\sin^{-1}\left(\frac{1}{2}\right)=b$

$\sin b=\frac{1}{2}$

$=\sin\frac{\pi}{6}$

$\sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{6}$

$2\sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{3}$ -----(2)

From Equations (1) and (2),

$\cos^{-1}\left(\frac{1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right)$

$=\frac{\pi}{3}+\frac{\pi}{3}$

$=\frac{2\pi}{3}$

Therefore, $\cos^{-1}\left(\frac{1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right)=\frac{2\pi}{3}$

13. Range of y if $\sin^{-1}x=y$ is

(A) $0\le y\le\pi$ (B) $-\frac{\pi}{2}\le y\le\frac{\pi}{2}$ (C) $0<y<\pi$ (D) $-\frac{\pi}{2}<y<\frac{\pi}{2}$

Ans: Given that $\sin^{-1}x=y$

Since, the range of $\sin^{-1}x$ is $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

Therefore the range of y is $-\frac{\pi}{2}\le y\le\frac{\pi}{2}$

14. Evaluate $\tan^{-1}\left(\sqrt{3}\right)-\sec^{-1}(-2)$

Ans: Assuming that $\tan^{-1}\left(\sqrt{3}\right)=a$

$\tan a=\sqrt{3}$

$=\tan\frac{\pi}{3}$

$\tan^{-1}\left(\sqrt{3}\right)=\frac{\pi}{3}$ -----(1)

Assuming that $\sec^{-1}(-2)=b$

$\sec b=-2$

$=\sec\left(\frac{2\pi}{3}\right)$

$\sec^{-1}(-2)=\frac{2\pi}{3}$ -----(2)

From Equations (1) and (2),

$\tan^{-1}\left(\sqrt{3}\right)-\sec^{-1}(-2)$

$=\frac{\pi}{3}-\frac{2\pi}{3}$

$=-\frac{\pi}{3}$

Therefore, $\tan^{-1}\left(\sqrt{3}\right)-\sec^{-1}(-2)=-\frac{\pi}{3}$

Exercise 2.2

1. Using inverse trigonometric identities, prove that

$$\mathbf{3\sin^{-1}x = \sin^{-1}\left(3x - 4x^3\right), x \in \left[-\frac{1}{2}, \frac{1}{2}\right]}$$

Ans: Assuming $\sin^{-1}x=\alpha$

$\sin\alpha=x$

We know that $\sin3\theta=3\sin\theta-4\sin^3\theta$

Consider

$\sin^{-1}\left(3x-4x^3\right)$

$=\sin^{-1}\left(3\sin\alpha-4\sin^3\alpha\right)$

$=\sin^{-1}\left(\sin3\alpha\right)$

$=3\alpha$

$=3\sin^{-1}x$

Hence it is proved that $3\sin^{-1}x=\sin^{-1}\left(3x-4x^3\right)$

2. Using in verse trigonometric identities, prove that

$$\mathbf{3\cos^{-1}x = \cos^{-1}\left(4x^3 - 3x\right), x \in \left[\frac{1}{2}, 1\right]}$$

Ans: Assuming $\cos^{-1}x=\alpha$

$\cos\alpha=x$

We know that $\cos3\theta=4\cos^3\theta-3\cos\theta$

Consider

$\cos^{-1}\left(4x^3-3x\right)$

$=\cos^{-1}\left(4\cos^3\alpha-3\cos\alpha\right)$

$=\cos^{-1}\left(\cos3\alpha\right)$

$=3\alpha$

$=3\cos^{-1}x$

Hence it is proved that $3\cos^{-1}x=\cos^{-1}\left(4x^3-3x\right)$

3. Simplify $\mathbf{\tan^{-1}\frac{\sqrt{1+x^2}-1}{x}, x \neq 0}$

Ans: Consider $\tan^{-1}\frac{\sqrt{1+x^2}-1}{x}$

Assuming

$x=\tan\theta$

$\beta=\tan^{-1}x$

$\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$

$=\tan^{-1}\left(\frac{\sqrt{1+\tan^2\beta}-1}{\tan\beta}\right)$

$=\tan^{-1}\left(\frac{\sec\beta-1}{\tan\beta}\right)$

$=\tan^{-1}\left(\frac{1-\cos\beta}{\sin\beta}\right)$

$=\tan^{-1}\left(\tan\left(\frac{\beta}{2}\right)\right)$

$=\frac{\beta}{2}$

$=\frac{1}{2}\tan^{-1}x$

Therefore, $\tan^{-1}\frac{\sqrt{1+x^2}-1}{x}=\frac{1}{2}\tan^{-1}x$

4. Simplify $\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right), x<\pi$

Ans: Consider $\tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}$

$\tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}$

$=\tan^{-1}\sqrt{\frac{2\sin^2\left(\frac{x}{2}\right)}{2\cos^2\left(\frac{x}{2}\right)}}$

$=\tan^{-1}\left(\tan\left(\frac{x}{2}\right)\right)$

$=\frac{x}{2}$

Therefore, $\tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}=\frac{x}{2}$

5. **Simplify** $\tan^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right), 0<x<\pi$

Ans: Consider $\tan^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right)$

$$\tan^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right)$$

$$=\tan^{-1}\left(\frac{1-\tan x}{1+\tan x}\right)$$

$$=\tan^{-1}\left(\tan\left(\frac{\pi}{4}-x\right)\right)$$

$$=\frac{\pi}{4}-x$$

Therefore, $\tan^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right)=\frac{\pi}{4}-x$

6. **Simplify** $\tan^{-1}\frac{x}{\sqrt{a^2-x^2}}, |x|<a$

Ans: Consider $\tan^{-1}\frac{x}{\sqrt{a^2-x^2}}$

Assuming

$x=a\sin\alpha$

$$\tan^{-1}\frac{x}{\sqrt{a^2-x^2}}$$

$$=\tan^{-1}\left(\frac{a\sin\alpha}{\sqrt{a^2-a^2\sin^2\alpha}}\right)$$

$$=\tan^{-1}(\tan\alpha)$$

$$=\alpha$$

$$=\sin^{-1}\left(\frac{x}{a}\right)$$

Therefore, $\tan^{-1}\frac{x}{\sqrt{a^2-x^2}}=\sin^{-1}\left(\frac{x}{a}\right)$

7. **Simplify** $\tan^{-1}\left(\frac{3a^2x-x^3}{a^3-3ax^2}\right), a>0; \frac{-a}{\sqrt{3}}\le x\le\frac{a}{\sqrt{3}}$

Ans: Consider $\tan^{-1}\left(\frac{3a^2x-x^3}{a^3-3ax^2}\right)$

Assuming
$x = a\tan\alpha$

$$\tan^{-1}\left(\frac{3a^2x-x^3}{a^3-3ax^2}\right)$$

$$=\tan^{-1}\left(\frac{3a^3\tan\alpha-a^3\tan\alpha}{a^3-3a^3\tan^2\alpha}\right)$$

$$=\tan^{-1}\left(\tan3\alpha\right)$$

$$=3\alpha$$

$$=3\tan^{-1}\left(\frac{x}{a}\right)$$

Therefore, $\tan^{-1}\left(\frac{3a^2x-x^3}{a^3-3ax^2}\right)=3\tan^{-1}\left(\frac{x}{a}\right)$

8. Evaluate $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$

Ans: Assuming $\sin^{-1}\frac{1}{2}=a$

$$\sin a=\frac{1}{2}$$

$$=\sin\left(\frac{\pi}{6}\right)$$

$$\sin^{-1}\frac{1}{2}=\frac{\pi}{6} \text{ -----(1)}$$

Further solving,
From equation (1)

$$\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$$

$$=\tan^{-1}\left[2\cos\left(2\times\frac{\pi}{6}\right)\right]$$

$$=\tan^{-1}\left[2\cos\frac{\pi}{3}\right]$$

$$=\tan^{-1}(1)$$

$$=\frac{\pi}{4}$$

Therefore, $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]=\frac{\pi}{4}$

9. Evaluate $\tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1-x^2}+\cos^{-1}\frac{1-y^2}{1+y^2}\right], |x|<1, y>0$ and $xy<1$

Ans: Assuming $x=\tan\alpha$

Consider

$$\sin^{-1}\left(\frac{2x}{1-x^2}\right)$$

$$=\sin^{-1}\left(\frac{2\tan\alpha}{1-\tan^2\alpha}\right)$$

$$=\sin^{-1}(\sin2\alpha)$$

$$=2\alpha$$

$$=2\tan^{-1}x \text{ ----(1)}$$

Consider $\cos^{-1}\left(\frac{1-y^2}{1+y^2}\right)$

Assuming $y=\tan\beta$

$$\cos^{-1}\left(\frac{1-y^2}{1+y^2}\right)$$

$$=\cos^{-1}\left(\frac{1-\tan^2\beta}{1+\tan^2\beta}\right)$$

$$=\cos^{-1}(\cos2\beta)$$

$$=2\beta$$

$$=2\tan^{-1}y \text{ -----(2)}$$

From equations (1) and (2),

$$\tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1-x^2}+\cos^{-1}\frac{1-y^2}{1+y^2}\right]$$

$$=\tan\frac{1}{2}\left[2\tan^{-1}x+2\tan^{-1}y\right]$$

$$=\tan\left(\tan^{-1}x+\tan^{-1}y\right)$$

$$=\frac{x+y}{1-xy}$$

Therefore, $\tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1-x^2}+\cos^{-1}\frac{1-y^2}{1+y^2}\right]=\frac{x+y}{1-xy}$

10. Evaluate $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

Ans: We know that $\sin(\theta)=\sin(\pi-\theta)$

Consider $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

$\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

$=\sin^{-1}\left(\sin\left(\pi-\frac{2\pi}{3}\right)\right)$

$=\sin^{-1}\left(\sin\left(\frac{\pi}{3}\right)\right)$

$=\frac{\pi}{3}$

Therefore, $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)=\frac{\pi}{3}$

11. Evaluate $\tan^{-1}\left(\tan\left(\frac{3\pi}{4}\right)\right)$

Ans: We know that $\tan(\theta)=-\tan(-\theta)$

Consider $\tan^{-1}\left(\tan\left(\frac{3\pi}{4}\right)\right)$

$\tan^{-1}\left(\tan\left(\frac{3\pi}{4}\right)\right)$

$=\tan^{-1}\left(-\tan\left(-\frac{3\pi}{4}\right)\right)$

$=\tan^{-1}\left(\tan\left(-\frac{\pi}{4}\right)\right)$

$=-\frac{\pi}{4}$

Therefore, $\tan^{-1}\left(\tan\left(\frac{3\pi}{4}\right)\right)=\frac{\pi}{4}$

12. **Evaluate** $\tan\left(\sin^{-1}\frac{3}{5}+\cot^{-1}\frac{3}{2}\right)$

Ans: Assuming $\sin^{-1}\frac{3}{5}=a$

$$\sin^{-1}\frac{3}{5}=a$$

$$\sin a=\frac{3}{5}$$

$$\cos a=\sqrt{1-\left(\frac{3}{5}\right)^2}$$

$$\cos a=\frac{4}{5}$$

$$\tan a=\frac{3}{4}$$

$$a=\tan^{-1}\left(\frac{3}{4}\right)$$

$$\sin^{-1}\left(\frac{3}{5}\right)=\tan^{-1}\left(\frac{3}{4}\right) \text{ ----(1)}$$

$$\cot^{-1}\frac{3}{2}=\tan^{-1}\frac{2}{3} \text{ -----(2)}$$

Further solving,

$$\tan\left(\sin^{-1}\frac{3}{5}+\cot^{-1}\frac{3}{2}\right)$$

$$=\tan\left(\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{2}{3}\right)$$

$$=\tan\left(\tan^{-1}\frac{\frac{3}{4}+\frac{2}{3}}{1-\left(\frac{3}{4}\times\frac{2}{3}\right)}\right)$$

$$=\tan\left(\tan^{-1}\frac{17}{6}\right)$$

$$=\frac{17}{6}$$

Therefore, $\tan\left(\sin^{-1}\frac{3}{5}+\cot^{-1}\frac{3}{2}\right)=\frac{17}{6}$

13. **Value of $\cos\left(\cos^{-1}\frac{7\pi}{6}\right)$ is**

(A) $\frac{7\pi}{6}$ (B) $\frac{5\pi}{6}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{6}$

Ans: Consider $\cos\left(\cos^{-1}\frac{7\pi}{6}\right)$

$=\cos\left(\cos^{-1}\frac{-7\pi}{6}\right)$

$=\cos^{-1}\left[\cos\frac{5\pi}{6}\right]$

$=\frac{5\pi}{6}$

Therefore, $\cos\left(\cos^{-1}\frac{7\pi}{6}\right)=\frac{5\pi}{6}$

The correct option is B

14. **Value of $\sin\left(\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right)$ is**

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) 1

Ans: Assuming $\sin^{-1}\left(-\frac{1}{2}\right)=a$

$\sin a=-\frac{1}{2}$

$=-\sin\left(\frac{\pi}{6}\right)$

$=\sin\left(-\frac{\pi}{6}\right)$

$\sin^{-1}\left(-\frac{1}{2}\right)=-\frac{\pi}{6}$

Further solving,

$\sin\left(\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right)$

$=\sin\left(\frac{\pi}{3}+\frac{\pi}{6}\right)$

$=\sin\left(\frac{\pi}{2}\right)$

$=1$

Therefore, $\sin\left(\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right)=1$

The correct option is D

15. **$\tan^{-1}\left(\sqrt{3}\right)-\cot^{-1}\left(\sqrt{-3}\right)$ is equal to**

(A) 0

(B) $2\sqrt{3}$

(C) $-\frac{\pi}{2}$

(D) π

Ans: The correct option is **C**

$-\frac{\pi}{2}$

Explanation for correct option

Evaluating the given expression:

Given, $\tan^{-1}\left(\sqrt{3}\right)-\cot^{-1}\left(\sqrt{-3}\right)$

We know that, $\pi-tan^{-1}(\theta)=tan^{-1}(-\theta)$

So,

$\pi-tan^{-1}\left(\sqrt{3}\right)=tan^{-1}\left(\sqrt{-3}\right)$

$\Rightarrow\ tan^{-1}\left(\sqrt{3}\right)=\pi-tan^{-1}\left(\sqrt{-3}\right)$

Therefore, we get:

$=\ \pi-tan^{-1}\left(\sqrt{-3}\right)-\cot^{-1}\left(\sqrt{-3}\right)$

$=\ \pi-\left(tan^{-1}\left(\sqrt{-3}\right)+cot^{-1}\left(\sqrt{-3}\right)\right)$

$=\ \pi-\frac{\pi}{2}\left(\because\ tan^{-1}\theta+cot^{-1}\theta=\frac{\pi}{2}\right)$

$=\ -\left(\frac{\pi}{2}\right)$

Hence, the correct answer is Option (C).

Miscellaneous Solutions

1. **Evaluate** $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$

Ans: Consider $\cos\frac{13\pi}{6}$

$\cos\frac{13\pi}{6}$

$=\cos\left(2\pi+\frac{\pi}{6}\right)$

$=\cos\left(\frac{\pi}{6}\right)$

Further solving,

$\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$

$=\cos^{-1}\left(\cos\left(\frac{\pi}{6}\right)\right)$

$=\frac{\pi}{6}$

Therefore, $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)=\frac{\pi}{6}$

2. **Evaluate** $\tan^{-1}\left(\tan\frac{7\pi}{6}\right)$

Ans: Consider $\tan\frac{7\pi}{6}$

$\tan\frac{7\pi}{6}$

$=\tan\left(2\pi-\frac{7\pi}{6}\right)$

$=\tan\left(\frac{\pi}{6}\right)$

Further solving,

$\tan^{-1}\left(\tan\frac{7\pi}{6}\right)$

$$=\tan^{-1}\left(\tan\left(\frac{\pi}{6}\right)\right)$$

$$=\frac{\pi}{6}$$

Therefore, $\tan^{-1}\left(\tan\frac{7\pi}{6}\right)=\frac{\pi}{6}$

3. Using inverse trigonometric identities, prove that $2\sin^{-1}\frac{3}{5}=\tan^{-1}\frac{24}{7}$

Ans: Assuming $2\sin^{-1}\frac{3}{5}=\alpha$ ----(1)

$$\sin\frac{\alpha}{2}=\frac{3}{5}$$

$$\cos\frac{\alpha}{2}=\sqrt{1-\left(\frac{3}{5}\right)^2}$$

$$\cos\frac{\alpha}{2}=\frac{4}{5}$$

Hence,

$$\sin\alpha=2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right)$$

$$\sin\alpha=\frac{24}{25}$$

$$\cos\alpha=\sqrt{1-\left(\frac{24}{25}\right)^2}$$

$$\cos\alpha=\frac{7}{25}$$

Therefore,

$$\tan\alpha=\frac{24}{7}$$

$$\alpha=\tan^{-1}\left(\frac{24}{7}\right)$$

From Equation (1), it is proved that

$$2\sin^{-1}\frac{3}{5}=\tan^{-1}\left(\frac{24}{7}\right)$$

4. **Using inverse trigonometric identities, prove that** $\sin^{-1}\frac{8}{17}+\sin^{-1}\frac{3}{5}=\tan^{-1}\frac{77}{36}$

Ans: Assuming $\sin^{-1}\frac{8}{17}=\alpha$

$$\sin\alpha=\frac{8}{17}$$

$$\cos\alpha=\sqrt{1-\left(\frac{8}{17}\right)^2}$$

$$\cos\alpha=\frac{15}{17}$$

Hence,

Therefore,

$$\tan\alpha=\frac{8}{15}$$

$$\alpha=\tan^{-1}\left(\frac{8}{15}\right)$$

$$\sin^{-1}\left(\frac{8}{17}\right)=\tan^{-1}\left(\frac{8}{15}\right) \text{ ----(1)}$$

Assuming that $\sin^{-1}\frac{3}{5}=\beta$

$$\sin\beta=\frac{3}{5}$$

$$\cos\beta=\sqrt{1-\left(\frac{3}{5}\right)^2}$$

$$\cos\beta=\frac{4}{5}$$

Therefore,

$$\tan\beta=\frac{3}{4}$$

$$\beta=\tan^{-1}\left(\frac{3}{4}\right)$$

$$\sin^{-1}\left(\frac{3}{5}\right)=\tan^{-1}\left(\frac{3}{4}\right) \text{ ----(2)}$$

Consider

$\sin^{-1}\left(\frac{8}{17}\right)+\sin^{-1}\left(\frac{3}{5}\right)$

From Equations (1) and (2),

$\sin^{-1}\left(\frac{8}{17}\right)+\sin^{-1}\left(\frac{3}{5}\right)$

$=\tan^{-1}\left(\frac{8}{15}\right)+\tan^{-1}\left(\frac{3}{4}\right)$

$=\tan^{-1}\left(\frac{\frac{8}{15}+\frac{3}{4}}{1-\left(\frac{8}{15}\times\frac{3}{4}\right)}\right)$

$=\tan^{-1}\left(\frac{77}{36}\right)$

Hence, it is proved that $\sin^{-1}\frac{8}{17}+\sin^{-1}\frac{3}{5}=\tan^{-1}\frac{77}{36}$

5. Using inverse trigonometric identities, prove that $\cos^{-1}\frac{4}{5}+\cos^{-1}\frac{12}{13}=\cos^{-1}\frac{33}{65}$

Ans: Assuming $\cos^{-1}\frac{4}{5}=\alpha$

$\cos\alpha=\frac{4}{5}$

$\sin\alpha=\sqrt{1-\left(\frac{4}{5}\right)^2}$

$\sin\alpha=\frac{3}{5}$

Therefore,

$\tan\alpha=\frac{3}{4}$

$\alpha=\tan^{-1}\left(\frac{3}{4}\right)$

$\cos^{-1}\left(\frac{4}{5}\right)=\tan^{-1}\left(\frac{3}{4}\right)$ ----(1)

Assuming that $\cos^{-1}\frac{12}{13}=\beta$

$$\cos\beta=\frac{12}{13}$$

$$\sin\beta=\sqrt{1-\left(\frac{12}{13}\right)^2}$$

$$\sin\beta=\frac{5}{13}$$

Therefore,

$$\tan\beta=\frac{5}{12}$$

$$\beta=\tan^{-1}\left(\frac{5}{12}\right)$$

$$\cos^{-1}\left(\frac{12}{13}\right)=\tan^{-1}\left(\frac{5}{12}\right) \text{ ----(2)}$$

Consider

$$\cos^{-1}\left(\frac{4}{5}\right)+\cos^{-1}\left(\frac{12}{13}\right)$$

From Equations (1) and (2),

$$\cos^{-1}\left(\frac{4}{5}\right)+\cos^{-1}\left(\frac{12}{13}\right)$$

$$=\tan^{-1}\left(\frac{3}{4}\right)+\tan^{-1}\left(\frac{5}{12}\right)$$

$$=\tan^{-1}\left(\frac{\frac{3}{4}+\frac{5}{12}}{1-\left(\frac{3}{4}\times\frac{5}{12}\right)}\right)$$

$$=\tan^{-1}\left(\frac{56}{33}\right)$$

$$=\cos^{-1}\left(\frac{33}{65}\right)$$

Hence, it is proved that $\cos^{-1}\frac{4}{5}+\cos^{-1}\frac{12}{13}=\cos^{-1}\frac{33}{65}$

6. **Using inverse trigonometric identities, prove that** $\mathbf{\cos^{-1}\frac{12}{13}+\sin^{-1}\frac{3}{5}=\sin^{-1}\frac{56}{65}}$

Ans: Assuming $\cos^{-1}\frac{12}{13}=\alpha$

$\cos\alpha=\frac{12}{13}$

$\sin\alpha=\sqrt{1-\left(\frac{12}{13}\right)^2}$

$\sin\alpha=\frac{5}{13}$

Therefore,

$\tan\alpha=\frac{5}{12}$

$\alpha=\tan^{-1}\left(\frac{5}{12}\right)$

$\cos^{-1}\left(\frac{12}{13}\right)=\tan^{-1}\left(\frac{5}{12}\right)$ ----(1)

Assuming that $\sin^{-1}\frac{3}{5}=\beta$

$\sin\beta=\frac{3}{5}$

$\cos\beta=\sqrt{1-\left(\frac{3}{5}\right)^2}$

$\cos\beta=\frac{4}{5}$

Therefore,

$\tan\beta=\frac{3}{4}$

$\beta=\tan^{-1}\left(\frac{3}{4}\right)$

$\sin^{-1}\left(\frac{3}{5}\right)=\tan^{-1}\left(\frac{3}{4}\right)$ ----(2)

Consider

$\cos^{-1}\left(\frac{12}{13}\right)+\sin^{-1}\left(\frac{3}{5}\right)$

From Equations (1) and (2),

$\cos^{-1}\left(\frac{12}{13}\right)+\sin^{-1}\left(\frac{3}{5}\right)$

$=\tan^{-1}\left(\frac{5}{12}\right)+\tan^{-1}\left(\frac{3}{4}\right)$

$$=\tan^{-1}\left(\frac{\frac{3}{4}+\frac{5}{12}}{1-\left(\frac{3}{4}\times\frac{5}{12}\right)}\right)$$

$$=\tan^{-1}\left(\frac{56}{33}\right)$$

$$=\sin^{-1}\left(\frac{56}{65}\right)$$

Hence, it is proved that $\cos^{-1}\frac{12}{13}+\sin^{-1}\frac{3}{5}=\sin^{-1}\frac{56}{65}$

7. Using inverse trigonometric identities, prove that $\tan^{-1}\frac{63}{16}=\sin^{-1}\frac{5}{13}+\cos^{-1}\frac{3}{5}$

Ans: Assuming $\sin^{-1}\frac{5}{13}=\alpha$

$$\sin\alpha=\frac{5}{13}$$

$$\cos\alpha=\sqrt{1-\left(\frac{5}{13}\right)^2}$$

$$\cos\alpha=\frac{12}{13}$$

Therefore,

$$\tan\alpha=\frac{5}{12}$$

$$\alpha=\tan^{-1}\left(\frac{5}{12}\right)$$

$$\sin^{-1}\left(\frac{5}{13}\right)=\tan^{-1}\left(\frac{5}{12}\right) \quad ----(1)$$

Assuming that $\cos^{-1}\frac{3}{5}=\beta$

$$\cos\beta=\frac{3}{5}$$

$$\sin\beta=\sqrt{1-\left(\frac{3}{5}\right)^2}$$

$$\sin\beta=\frac{4}{5}$$

Therefore,

$\tan\beta=\frac{4}{3}$

$\beta=\tan^{-1}\left(\frac{4}{3}\right)$

$\cos^{-1}\left(\frac{3}{5}\right)=\tan^{-1}\left(\frac{4}{3}\right)$ ----(2)

Consider

$\sin^{-1}\left(\frac{5}{13}\right)+\cos^{-1}\left(\frac{3}{5}\right)$

From Equations (1) and (2),

$\sin^{-1}\left(\frac{5}{13}\right)+\cos^{-1}\left(\frac{3}{5}\right)$

$=\tan^{-1}\left(\frac{5}{12}\right)+\tan^{-1}\left(\frac{4}{3}\right)$

$=\tan^{-1}\left(\frac{\frac{4}{3}+\frac{5}{12}}{1-\left(\frac{4}{3}\times\frac{5}{12}\right)}\right)$

$=\tan^{-1}\left(\frac{63}{16}\right)$

Hence, it is proved that $\tan^{-1}\frac{63}{16}=\sin^{-1}\frac{5}{13}+\cos^{-1}\frac{3}{5}$

8. Using inverse trigonometric identities, prove that

$$\tan^{-1}\sqrt{x}=\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right), x\in[0,1]$$

Ans: Assuming $x=\tan^2\alpha$

Consider

$\tan^{-1}\sqrt{x}$

$=\tan^{-1}\sqrt{\tan^2\alpha}$

$=\tan^{-1}\tan\alpha$

$=\alpha$

$\tan^{-1}\sqrt{x}=\alpha$ ----(1)

Consider

$\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)$

$=\frac{1}{2}\cos^{-1}\left(\frac{1-\tan^2\alpha}{1+\tan^2\alpha}\right)$

$=\frac{1}{2}\cos^{-1}\cos 2\alpha$

$=\alpha$

$\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)=\alpha$ -----(2)

From Equations (1) and (2),

It is proved that $\tan^{-1}\sqrt{x}=\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)$

9. Using inverse trigonometric identities, prove that

$$\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)=\frac{x}{2}, x\in\left(0,\frac{\pi}{4}\right)$$

Ans: We know that

$\sqrt{1+\sin x}=\cos\frac{x}{2}+\sin\frac{x}{2}$

$\sqrt{1-\sin x}=\cos\frac{x}{2}-\sin\frac{x}{2}$

Consider

$$\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)$$

$$=\cot^{-1}\left(\frac{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)+\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)-\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)}\right)$$

$$\cot^{-1}\left(\cot\frac{x}{2}\right)$$

$$=\frac{x}{2}$$

It is proved that $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)=\frac{x}{2}$

10. Using inverse trigonometric ident ities, prove that

$$\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x, -\frac{1}{\sqrt{2}}\le x\le 1$$

Ans: Assuming

$x=\cos 2\beta$

$\sqrt{1+x}$

$=\sqrt{1+\cos 2\beta}$

$=\sqrt{2}\cos\beta$

$\sqrt{1-x}$

$=\sqrt{1-\cos 2\beta}$

$=\sqrt{2}\sin\beta$

Consider

$$\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$$

$$=\tan^{-1}\left(\frac{\sqrt{2}\cos\beta-\sqrt{2}\sin\beta}{\sqrt{2}\cos\beta+\sqrt{2}\sin\beta}\right)$$

$$=\tan^{-1}\left(\frac{1-\tan\beta}{1+\tan\beta}\right)$$

$$=\tan^{-1}\left(\tan\left(\frac{\pi}{4}-\beta\right)\right)$$

$$=\frac{\pi}{4}-\beta$$

$$=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x$$

It is proved that $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x$

11. For what value of x does the equation $2\tan^{-1}(\cos x)=\tan^{-1}(2\operatorname{cosec}x)$ satisfy?

Ans: Consider $2\tan^{-1}(\cos x)=\tan^{-1}(2\operatorname{cosec}x)$

$$2\tan^{-1}(\cos x)=\tan^{-1}(2\operatorname{cosec}x)$$

$$\tan^{-1}\left(\frac{2\cos x}{1-\cos^2 x}\right)=\tan^{-1}(2\operatorname{cosec}x)$$

$$\frac{2\cos x}{1-\cos^2 x}=2\operatorname{cosec}x$$

$$\sin x\cos x=\sin^2 x$$

$$\sin x(\cos x-\sin x)=0$$

$$\sin x=0,\cos x-\sin x=0$$

$$x=0,\frac{\pi}{4}$$

Therefore, $2\tan^{-1}(\cos x)=\tan^{-1}(2\operatorname{cosec}x)$ is satisfied for $x=0,\frac{\pi}{4}$

12. For what value of x does the equation $\tan^{-1}\frac{1-x}{1+x}=\frac{1}{2}\tan^{-1}x,(x>0)$ satisfy?

Ans: Consider $\tan^{-1}\frac{1-x}{1+x}=\frac{1}{2}\tan^{-1}x$

$$\tan^{-1}\frac{1-x}{1+x}=\frac{1}{2}\tan^{-1}x$$

$$\tan^{-1}\left(\tan\left(\frac{\pi}{4}-\tan^{-1}x\right)\right)=\frac{1}{2}\tan^{-1}x$$

$$\frac{\pi}{4}-\tan^{-1}x=\frac{1}{2}\tan^{-1}x$$

$$\frac{\pi}{4}=\frac{3}{2}\tan^{-1}x$$

$$\tan^{-1}x=\frac{\pi}{6}$$

$$x=\frac{1}{\sqrt{3}}$$

Therefore, $\tan^{-1}\frac{1-x}{1+x}=\frac{1}{2}\tan^{-1}x$ is satisfied for $x=\frac{1}{\sqrt{3}}$

13. The expression $\sin(\tan^{-1}x),|x|<1$ is equal to

(A) $\frac{x}{\sqrt{1-x^2}}$ **(B)** $\frac{1}{\sqrt{1-x^2}}$ **(C)** $\frac{1}{\sqrt{1+x^2}}$ **(D)** $\frac{x}{\sqrt{1+x^2}}$

Ans: Assuming

$x=\tan\beta$

$\beta=\tan^{-1}x$

$\sin\left(\tan^{-1}x\right)$

$=\sin\beta$

$=\dfrac{x}{\sqrt{1+x^2}}$

Hence, $\sin\left(\tan^{-1}x\right)=\dfrac{x}{\sqrt{1+x^2}}$

Therefore, the correct option is option D

14. For what value of x does the equation $\sin^{-1}\left(1-x\right)-2\sin^{-1}x=\dfrac{\pi}{2}$ satisfy?

(A) $0,\dfrac{1}{2}$ **(B) $1,\dfrac{1}{2}$** **(C) 0** **(D) $\dfrac{1}{2}$**

Ans: Consider

$\sin^{-1}\left(1-x\right)-2\sin^{-1}x=\dfrac{\pi}{2}$

$-2\sin^{-1}x=\dfrac{\pi}{2}-\sin^{-1}\left(1-x\right)$

$-2\sin^{-1}x=\cos^{-1}\left(1-x\right)$ -----(1)

Assuming

$\cos^{-1}\left(1-x\right)=\beta$

$\cos\beta=1-x$

$\sin\beta=\sqrt{1-\left(1-x\right)^2}$

$\sin\beta=\sqrt{2x-x^2}$

$\beta=\sin^{-1}\sqrt{2x-x^2}$

$\cos^{-1}1-x=\sin^{-1}\sqrt{2x-x^2}$ -----(2)

Substituting Equation (2) in (1)

$-2\sin^{-1}x=\sin^{-1}\sqrt{2x-x^2}$

$\sin^{-1}\left(-2x\sqrt{1-x^2}\right)=\sin^{-1}\sqrt{2x-x^2}$

$-2x\sqrt{1-x^2}=\sqrt{2x-x^2}$

$4x^2-4x^4=2x-x^2$

$4x^4-5x^2+2x=0$

$x=0,\dfrac{1}{2},\dfrac{-1\pm\sqrt{17}}{4}$

But considering the given options and also when $x=\frac{1}{2}$, the equation $\sin^{-1}(1-x)-2\sin^{-1}x=\frac{\pi}{2}$ doesn't satisfy

Thus, $x=0$ is the only solution.

Hence the correct option is C