probability

13 Chapter

Exercise 13.1

1. Given that E and F are events such that $P(E)=0.6$, and $P(F)=0.3$ $P(E\cap F)=0.3$, find P(E|F) and P(F|E).

Ans: It is given in the question that $P(E)=0.6$ = 0.6, $P(F)=0.3$, and $P(E\cap F)=0.2$

Now P(E|F) is given by

$$P(E|F)=\frac{P(E\cap F)}{P(F)}$$

$$\Rightarrow P(E|F)=\frac{0.2}{0.3}$$

Hence we found that

$$P(E|F)=\frac{2}{3}$$

With similar idea and the same formula we can proceed to find P(F|E) as shown

$$P(F|E)=\frac{P(F\cap E)}{P(E)}$$

Now we know that $P(F\cap E)$ and $P(E\cap F)$ is same

$$\therefore P(F|E)=\frac{0.2}{0.6}$$

Hence we found that

$$P(F|E)=\frac{1}{3}$$

Thus $P(E|F)=\frac{2}{3}$ and $P(F|E)=\frac{1}{3}$

2. **Compute P(A|B), if $P(B)=0.5$ and $P(A\cap B)=0.32$**

Ans: Given in the question $P(B)=0.5$ and $P(A\cap B)=0.32$

So to find P(A|B we use the formula

$$P(A|B)=\frac{P(A\cap B)}{P(B)}$$

$$P(A|B)=\frac{0.32}{0.5}$$

$$=\frac{32}{50}$$

$$=\frac{16}{25}$$

Hence we found that $P(A|B)=\frac{16}{25}$

3. **If $P(A)=0.8$, $P(B)=0.5$ and $P(B|A)=0.4$ find**

(i) $P(A\cap B)$

Ans: It is given that $P(A)=0.8$, $P(B)=0.5$, $P(B)=0.5$ and $P(B|A)=0.4$

Put all the data in the following formula

$$P(B|A)=\frac{P(A\cap B)}{P(A)}$$

$$P(A\cap B)=P(A|B).P(B)$$
$$P(A\cap B)=0.4\times 0.8$$

Thus we found that $P(A\cap B)=0.32$

(ii) $P(A|B)$

Ans: Given in the question $P(A)=0.8$, $P(B)=0.5$, $P(B)=0.5$ and $P(B|A)=0.4$

We know that $P(A|B)$ is the probability of occurrence of A when B has already happened

$$\therefore P(A|B)=\frac{P(A\cap B)}{P(B)}$$

Now put $P(A\cap B)=0.32$, $P(B)=0.5$ in the above equation as shown

$$P(A|B)=\frac{0.32}{0.5}$$

$$=0.64$$

Thus we found that $P(A|B)=0.64$

(iii) $P(A\cup B)$

Ans: Given in the question $P(A)=0.8$, $P(B)=0.5$, $P(B)=0.5$ and $P(B|A)=0.4$

Now we have the formula as shown

$$P(A\cup B)=P(A)+P(B)-P(A\cap B)$$

Put $P(A)=0.8$,

$P(B)=0.5$,

$P(A\cap B)=0.32$ in the above as shown

$$\therefore P(A\cup B)=0.8+0.5-0.32$$

$$\Rightarrow P(A\cup B)=0.98$$

Thus we found that $P(A\cup B)=0.98$

4. Evaluate $P(A\cup B)$ if $2P(A)=P(B)=\frac{5}{13}$ and $P(A|B)=\frac{2}{5}$

Ans: It is Given that

$$2P(A)=P(B)=\frac{5}{13} \text{ and}$$

$$P(A|B)=\frac{2}{5}$$

$$\Rightarrow P(A)=\frac{5}{26}$$

Now we know the formula

$$P(A|B)=\frac{P(A\cap B)}{P(B)}$$

$$\Rightarrow \frac{2}{5}=\frac{13P(A\cap B)}{5} \text{ (since } P(B)=\frac{5}{13},\ P(A|B)=\frac{2}{5}\text{)}$$

$$\Rightarrow P(A\cap B)=\frac{2}{13}$$

Also $P(A\cup B)$ is given by the formula

$$P(A\cup B)=P(A)+P(B)-P(A\cap B)$$

$$\Rightarrow P(A\cup B)=\frac{5}{26}+\frac{5}{13}-\frac{2}{13}$$

$$\Rightarrow P(A\cup B)=\frac{11}{26}$$

Thus we found that $P(A\cup B)=\frac{11}{26}$

5. If $P(A)=\frac{6}{11}$, $P(B)=\frac{5}{11}$ and $P(A\cup B)=\frac{7}{11}$ find

(i) $P(A\cap B)$

Ans: Given $P(A)=\frac{6}{11}$, $P(B)=\frac{5}{11}$

Also it is given that

$$P(A\cup B)=\frac{7}{11}$$

And we know that it is given by the formula

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow \frac{7}{11} = \frac{6}{11} + \frac{5}{11} - P(A \cap B)$$

$$\Rightarrow P(A \cap B) = \frac{11}{11} - \frac{7}{11}$$

$$\Rightarrow P(A \cap B) = \frac{4}{11}$$

Thus we found that $P(A \cap B) = \frac{4}{11}$

(ii) $P(A|B)$

Ans: Given $P(A) = \frac{6}{11}$, $P(B) = \frac{5}{11}$

Also we know that $P(A|B)$ is given by

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow P(A|B) = \frac{4}{11} \times \frac{11}{5} \text{ (since } P(A \cap B) = \frac{4}{11}\text{)}$$

Thus we found that $P(A|B) = \frac{4}{5}$

(iii) $P(B|A)$

Ans: Given $P(A) = \frac{6}{11}$, $P(B) = \frac{5}{11}$

Also we know that $P(B|A)$ is given by

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$\Rightarrow P(B|A) = \frac{4}{11} \times \frac{11}{6}$

Thus we found that $P(B|A) = \frac{2}{3}$

6. A coin is tossed three times, where

(i) E: head on third toss, F: heads on first two tosses

Ans: Sample space is given by

$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$ and the events E and F and their probabilities are given by

$E = \{HHH, HTH, THH, TTH\}$

Therefore $P(E) = \frac{4}{8}$

$F = \{HHH, HHT\}$

Therefore $P(F) = \frac{2}{8}$

$\therefore E \cap F = \{HHH\}$

Therefore $P(E \cap F) = \frac{1}{8}$

And hence $P(E|F)$ is given by

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$$\Rightarrow P(E|F) = \frac{\frac{1}{8}}{\frac{2}{8}}$$

Thus $P(E|F) = \frac{1}{2}$

(ii) E: at least two heads, F: at most two heads

Ans: Sample space is given by

$S=\{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$ and the events E and F and their probabilities are given by

$E=\{HHH, HHT, THH, HTH\}$

Therefore $P(E)=\frac{4}{8}$

$F=\{HHT, HTH, HTT, THH, THT, TTH, TTT\}$

Therefore $P(F)=\frac{7}{8}$

$E\cap F=\{HHT, HTH, THH\}$

Therefore $P(E\cap F)=\frac{3}{8}$

And hence $P(E|F)$ is given by

$$P(E|F)=\frac{P(E\cap F)}{P(F)}$$

$$\Rightarrow P(E|F)=\frac{\frac{3}{8}}{\frac{7}{8}}$$

Thus $P(E|F)=\frac{3}{7}$

(iii) E: at most two tails, F: at least one tail.

Ans: Sample space is given by

$S=\{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$ and the events E and F and their probabilities are given by

$$E=\{HHH, HHT, THH, HTH, THH, THT, TTH\}$$

Therefore $P(E)=\frac{7}{8}$

$$F=\{HHT, HTT, HTH, THH, THT, TTH, TTT\}$$

Therefore $P(F)=\frac{7}{8}$

$$E\cap F=\{HHT, HTT, HTH, THH, THT, TTH\}$$

Therefore $P(E\cap F)=\frac{6}{8}$

And hence $P(E|F)$ is given by

$$P(E|F)=\frac{P(E\cap F)}{P(F)}$$

$$\Rightarrow P(E|F)=\frac{\frac{6}{8}}{\frac{7}{8}}$$

Thus $P(E|F)=\frac{6}{7}$

7. Two coins are tossed once, where

(i) E: tail appears on one coin, F: one coin shows head

Ans: Sample Space is given by

$$S=\{HH, HT, TH, TT\}$$

The events E and F and their probabilities are given by

$$E=\{HT, TH\}$$

Therefore $P(E)=\frac{2}{4}$

$F=\{HT,TH\}$

Therefore $P(F)=\frac{2}{4}$

Also $E\cap F$ is given by

$E\cap F=\{HT,TH\}$

We know that $P(E|F)$ is given by

$$P(E|F)=\frac{P(E\cap F)}{P(F)}$$

$$\Rightarrow P(E|F)=\frac{\frac{2}{4}}{\frac{2}{4}}$$

Thus we found that $P(E|F)=1$

That is, $(E|F)$ is a sure event

(ii) E: not tail appears, F: no head appears

Ans: Sample Space is given by

$S=\{HH,HT,TH,TT\}$

The events E and F and their probabilities are given by

$E=\{HH\}$

Therefore $P(E)=\frac{1}{4}$

$F=\{TT\}$

Therefore $P(F)=\frac{1}{4}$

Therefore $E\cap F$ is given by

$E\cap F=\phi$

We know that $P(E|F)$ is given by

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$$\Rightarrow P(E|F) = \frac{0}{\frac{1}{4}}$$

Thus we found that $P(E|F) = 0$

8. A die is thrown three times,

E: 4 appears on the third toss, F: 6 and 5 appears respectively on first two tosses

Ans: Number of elements in Sample space is given by 216

The events E and F and their probabilities are given by

$$E = \begin{Bmatrix} (1,1,4),(1,2,4)......(1,6,4) \\ (2,1,4),(2,2,4)........(2,6,4) \\ (3,1,4),(3,2,4)..........(3,6,4) \\ (4,1,4),(4,2,4)..........(4,6,4) \\ (5,1,4),(5,2,4)...........(5,6,4) \\ (6,1,4),(6,2,4)...........(6,6,4) \end{Bmatrix}$$

$$F = \{(6,5,1),(6,5,2),(6,5,3),(6,5,4),(6,5,5),(6,5,6)\}$$

Therefore $P(F) = \frac{6}{216}$

And hence $E \cap F = \{(6,5,4)\}$

Therefore $P(E \cap F) = \frac{1}{216}$

We know that $P(E|F)$ is given by

$$P(E|F)=\frac{P(E\cap F)}{P(F)}$$

$$\Rightarrow P(E|F)=\frac{\frac{1}{216}}{\frac{6}{216}}$$

Thus $P(E|F)=\frac{1}{6}$

9. Mother, father and son line up at random for a family picture

E: son on one end, F: father in middle

Ans: Let mother (M), father (F), and son (S) line up for the family picture, then the sample space will be as shown

$$A=\left(MFS, MSF, FMS, FSM, SMF, SFM\right\}$$

The events E and F and their probabilities are

$$E=\{MFS, FMS, SMF, SFM\}$$

$$F=\{MFS, SFM\}$$

Therefore $P(F)=\frac{2}{6}$

Hence $E\cap F=\{MFS, SFM\}$

Therefore $P(E\cap F)=\frac{2}{6}$

We know that $P(E|F)$ is given by

$$\Rightarrow P(E|F)=\frac{P(E\cap F)}{P(F)}$$

$$\Rightarrow P(E|F)=\frac{\frac{4}{6}}{\frac{4}{6}}$$

Thus $P(E|F)=1$

10. A black and a red dice are rolled.

(a) Find the conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5.

Ans: Let the first observation come from black die and second from red die respectively

In the case when two dices are rolled the elements in sample space is 36

The events A and B and their probabilities are given by

$$A=\{(4,6),(5,5),(5,6),(6,4),(6,5),(4,6)(6,6)\}$$

Where A is the event when sum is greater than 9

Similarly,

$$B=\{(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)\}$$

Where B is the event when black die resulted in a 5

Therefore $P(B)=\frac{6}{36}$

And hence $A\cap B=\{(5,5),(5,6)\}$

Therefore the conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5 $P(A|B)$ is given by

$$\Rightarrow P(A|B)=\frac{P(A\cap B)}{P(B)}$$

$$\Rightarrow P(A|B)=\frac{\frac{2}{36}}{\frac{6}{36}}$$

Thus $P(A|B)=\frac{1}{3}$

(b) Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

Ans: Let E and F be the events and their probabilities defined as

E: Sum of the observations is 8

$$E=\{(2,6),(3,5),(4,4),(5,3),(6,2)\}$$

F: red die resulted in a number less than 4

$$F=\left\{\begin{matrix}(1,1),(1,2),(1,3),(2,1),(2,2),(2,2)\\(3,1),(3,2),(3,3),(4,1),(4,2),(4,3)\\(5,1),(5,2),(5,3),(6,1),(6,2),(6,3)\end{matrix}\right\}$$

Therefore $P(F)=\frac{18}{36}$

And hence $E\cap F=\{(5,3),(6,2)\}$

Therefore $P(E\cap F)=\frac{2}{36}$

The conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4 is given $P(E|F)$ as shown

$$P(E|F)=\frac{P(E\cap F)}{p(F)}$$

$$\Rightarrow P(E|F)=\frac{\frac{2}{36}}{\frac{18}{36}}$$

Thus $P(E|F)=\frac{1}{9}$

11. **A fair die is rolled. Consider events E = {1, 3, 5}, F = {2, 3} and G = {2, 3, 4, 5}**

Find

i. $P(E|F)$ and $P(F|E)$

Ans: The sample space is given by

$S = \{1, 2, 3, 4, 5, 6\}$

It is given in the question

$E = \{1, 3, 5\}$

$F = \{2, 3\}$

Therefore $E \cap F = \{3\}$

Hence $P(E \cap F) = \frac{1}{6}$

$\therefore P(E) = \frac{3}{6}$

$\therefore P(F) = \frac{2}{6}$

Hence $P(E|F)$ is given by

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$$\Rightarrow P(E|F) = \frac{\frac{1}{6}}{\frac{2}{6}}$$

Similarly $P(F|E)$ is given by

$$P(F|E) = \frac{\frac{1}{6}}{\frac{3}{6}}$$

Thus $P(E|F)=\frac{1}{2}$ and $P(F|E)=\frac{1}{3}$

ii. $\mathbf{P(E|G)}$ and $\mathbf{P(G|E)}$

Ans: It is given in the question

$E=\{1,3,5\}$

$\therefore P(E)=\frac{3}{6}$

$G=\{2,3,4,5\}$

$\therefore P(G)=\frac{4}{6}$

$\therefore E\cap G=\{3,5\}$

$\therefore P(E\cap G)=\frac{2}{6}$

Therefore $P(E|G)=\frac{(E\cap G)}{P(G)}$

$\Rightarrow P(E|G)=\frac{\frac{2}{6}}{\frac{4}{6}}$

Thus $P(E|G)=\frac{1}{2}$

Similarly

$P(G|E)=\frac{\frac{2}{6}}{\frac{3}{6}}$

Thus $P(G|E)=\frac{2}{3}$

Therefore, $P(E|G) = \frac{1}{2}$ and $P(G|E) = \frac{2}{3}$

iii. P((E ∪ F)|G) and P((E ∩ F)|G)

Ans: The sample space is given by

$S = \{1, 2, 3, 4, 5, 6\}$

$G = \{2, 3, 4, 5\}$

We have $E \cup F = \{1, 2, 3, 5\}$

Therefore

$$(E \cup F) \cap G = \{1, 2, 3, 5\} \cap \{2, 3, 4, 5\}$$
$$= \{2, 3, 5\}$$

Also

$$(E \cap F) \cap G = \{1, 2, 3, 5\} \cap \{3\}$$
$$= \{3\}$$

$$\therefore P(E \cup F) \cap G = \frac{3}{6}$$

$$= \frac{1}{2}$$

$$\therefore P(E \cap F) \cap G = \frac{1}{6}$$

$$\therefore P(E \cup F|G) = \frac{P(E \cup F) \cap G}{P(G)}$$

$$\Rightarrow P(E \cup F|G) = \frac{\frac{3}{6}}{\frac{4}{6}}$$

Thus $P(E \cup F|G) = \frac{3}{4}$

Similarly

$$P(E \cap F|G) = \frac{\frac{1}{6}}{\frac{4}{6}}$$

Thus $P(E \cap F|G) = \frac{1}{4}$

Thus, $P(E \cup F|G) = \frac{3}{4}$ and $P(E \cap F|G) = \frac{1}{4}$

12. Assume that each born child is equally like to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that

i. the youngest is a girl

Ans: The sample space for a family having two children is given by

$S = \{(B,B),(B,G),(G,G),(G,B)\}$

Where B refers to boy child and G refers to girl child

Let an event be defined as

E: Both children are girls

$E = \{(GG)\}$

$\therefore P(E) = \frac{2}{4}$

Let F be an event defined as

F: youngest child is girl

$F = \{(BG),(GG)\}$

$\therefore E \cap F = \{(GG)\}$

$\therefore P(E \cap F) = \frac{1}{4}$

We know that $P(E|F)$ is given by

$$P(E|F)=\frac{P(E\cap F)}{P(F)}$$

$$\Rightarrow P(E|F)=\frac{\frac{1}{4}}{\frac{2}{4}}$$

Thus $P(E|F)=\frac{1}{2}$

ii. at least one is a girl

Ans: The sample space for a family having two children is given by

$$S=\{(B,B),(B,G),(G,G),(G,B)\}$$

Where B refers to boy child and G refers to girl child

Let an event be defined as

E: Both children are girls

$$E=\{(GG)\}$$

$$\therefore P(E)=\frac{2}{4}$$

Let A be an event defined as

$$A=\{(B,G),(G,B),(G,G)\}$$

$$\therefore P(A)=\frac{3}{4}$$

$$\therefore E\cap A=\{(G,G)\}$$

$$\therefore P(E\cap A)=\frac{1}{4}$$

We know that

$P(E|A)$ is given by

$$P(E|A)=\frac{P(\cap A)}{P(A)}$$

$$\Rightarrow P(E|A)=\frac{\frac{1}{4}}{\frac{3}{4}}$$

Thus $P(E|A)=\frac{1}{3}$

13. An instructor has a bank consisting of 300 easy True/False s, 200 difficult True/False s, 500 easy multiple choice s and 400 difficult multiple choice s. If a is selected at random from the bank, what is the probability that it will be an easy given that it is a multiple choice ?

Ans: The given data can be represented as

	True/False	Multiple choice	Total
Easy	300	500	800
Difficult	200	400	600
Total	500	900	1400

Let us have following notations

E for easy questions, M for multiple questions, D for difficult questions and T for true/false questions

It is given that total number of questions is 1400, that of multiple questions is 900

Hence probability for selecting easy multiple choice questions is given by

$$P(E \cap M)=\frac{5}{14} \text{ (since } \frac{500}{1400}=\frac{5}{14})$$

Probability for selecting multiple choice questions is given by

$$P(M)=\frac{9}{14} \text{ (since } \frac{90}{1400}=\frac{9}{14})$$

$$\therefore P(E|M)=\frac{P(\cap M)}{P(M)}$$

$$\Rightarrow P(E|M) = \frac{\frac{5}{14}}{\frac{9}{14}}$$

Thus $P(E|M) = \frac{5}{9}$

14. Given that the two numbers appearing on throwing the two dice are different. Find the probability of the event 'the sum of numbers on the dice is 4'.

Ans: Let A and be events defined as

A: the sum of the numbers on the dice is 4

B: the two numbers appearing on throwing the two dice are different**.**

$\therefore A = \{(1,3),(2,3),(3,1)\}$

$$\therefore B = \begin{Bmatrix} (1,2),(1,3)....(1,6) \\ (2,1),(2,2)....(2,6) \\ (3,1),(3,2)......(3,6) \\ (4,1),(4,2).......(4,6) \\ (5,1),(5,2).........(5,6) \\ (6,1),(6,2).........(6,6) \end{Bmatrix}$$

Therefore $P(B) = \frac{30}{36}$

$\therefore A \cap B = \{(1,3),(3,1)\}$

$P(A \cap B) = \frac{2}{36}$

We know that $P(A|B)$ is given by

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow P(A|B)=\frac{\frac{2}{36}}{\frac{30}{36}}$$

Thus $P(A|B)=\frac{1}{15}$

15. Consider the experiment of throwing a die, if a multiple of 3 comes up, throw the die again and if any other number comes, toss a coin. Find the conditional probability of the event 'the coin shows a tail', given that 'at least one die shows as 3'.

Ans: For this case the sample space is given by

$$S=\left\{\begin{matrix}(1,H),(2,H),(1,T),(1,T),(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)\\(4,H),(4,T),(5,H),(5,T),(6,1),(6,2)......(6,6)\end{matrix}\right\}$$

Let A and B be events defined as

A: The coin shows tail

B: at least one die shows 3

$\therefore A=\{(1,T),(2,T),(4,T),(5,T)\}$

$\therefore B=\{(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),(6,3)\}$

$\therefore P(B)=\frac{7}{36}$

Also it is observable that $A\cap B=\phi$

We know that $P(A|B)$ given by

$$P(A|B)=\frac{P(A\cap B)}{P(B)}$$

$$P(A|B)=\frac{0}{\frac{7}{36}}$$

Thus $P(A|B)=0$

16. **If $P(A)=\frac{1}{2}$ and $P(B)=0$ then $P(A|B)$ is**

A. 0

B. $\frac{1}{2}$

C. Not defined

D. 1

Ans: It is given that $P(A)=\frac{1}{2}$ and $P(B)=0$

We know that $P(A|B)$ is given by

$$P(A|B)=\frac{P(\cap B)}{P(B)}$$

$$\Rightarrow P(A|B)=\frac{P(\cap B)}{0}$$

Thus $P(A|B)$ is not defined

17. **if A and B are events such that $P(A|B)=P(B|A)$ then**

(A) $A\subset$ but $A\neq B$ (B) $A=B$ (C) $A\cap B=\phi$) (D)= $P(A)=P(B)$

Ans: It is given in the question that

$$P(A|B)=P(B|A)$$

$$\Rightarrow \frac{P(A\cap B)}{P(B)}=\frac{P(A\cap B)}{P(A)}$$

$$\Rightarrow \frac{P(A)}{1}=\frac{P(B)}{1}$$

Thus we found that $P(A)=P(B)$

Exercise 13.2

1. If $P(A)=\frac{3}{5}$ and $P(B)=\frac{1}{5}$, find $P(A\cap B)$ if A and B are independent events

Ans: It is given in the question that $P(A)=\frac{3}{5}$ and $P(B)=\frac{1}{5}$

Also it is given that A and B are independent events

$\therefore P(A\cap B)=P(A)P(B)$

$\Rightarrow P(A\cap B)=\frac{3}{5}\times\frac{1}{5}$

Thus $P(A\cap B)=\frac{3}{25}$

2. Two cards are drawn at random and without replacement from a pack of 52 playing cards. Find the probability that both the cards are black.

Ans: Given a deck of 52 cards

We know that the number of black cards are 26

Let us have a notation $P(A)$ denote the probability of picking a black card at first chance

$\therefore P(A)=\frac{26}{52}$

Similarly, let us have a notation $P(B)$ denote the probability of picking a black card at second chance.

Since there is no replacement

$\therefore P(B)=\frac{25}{51}$

$P(A)=\frac{1}{2}$ and $P(B)=\frac{25}{51}$

The probability that the both the cards are black is

$P(A \cap B) = \frac{1}{2} \times \frac{25}{51}$

$= \frac{25}{102}$

The probability that the both the cards are black is $\frac{25}{102}$

3. A box of oranges is inspected by examining three randomly selected oranges drawn without replacement. If all the three oranges are good, the box is approved for sale, otherwise, it is rejected. Find the probability that a box containing 15 oranges out of which 12 are good and 3 are bad ones will be approved for sale.

Ans: Let A,B and C be the events and their probabilities defined as

A: the first orange is good

$\therefore P(A) = \frac{12}{15}$

B: the second orange is good

$\therefore P(B) = \frac{1}{15}$

C: the third orange is good

$\therefore P(C) = \frac{10}{15}$

It is given that the box is approved for sale only when all the oranges are good.

i.e

$\text{probability that the box is approved for sale} = \text{probability of all oranges to be good}$

$\therefore \text{probability of all oranges to be good} = \frac{12}{15} \times \frac{11}{15} \times \frac{10}{15}$

$\text{Thus probability that the box is approved for sale} = \frac{44}{91}$

4. A fair coin and an unbiased die are tossed. Let A be the event 'head appears on the coin' and B be the event 3 on the die'. Check whether A and B are independent events or not.

Ans: The sample space is given by

$$S=\begin{Bmatrix}(H,1),(H,2)......(H,6)\\(T,1),(T,2)........(T,6)\end{Bmatrix}$$

According to the question

$$A=\{(H,1),(H,2)......(H,6)\}$$

$$\therefore P(A)=\frac{6}{12}$$

$$B=\{(H,3),(T,3)\}$$

$$\therefore P(B)=\frac{2}{12}$$

Also $(A\cap B)=\{(H,3)\}$

$$\therefore P(A\cap B)=\frac{1}{12}$$

We know that for A and B to be independent events

$$P(A\cap B)=P(A)P(B)$$

Now $\quad P(A)P(B)=\frac{6}{12}\times\frac{2}{12}$

$$\Rightarrow P(A)P(B)=\frac{1}{12}$$

Which is as same as $P(A\cap B)$

Thus A and B are independent events

5. A die marked $1,2,3$ in red and $4,5,6$ in green is tossed. Let A be the events, 'the number is even,' and B be the event, 'the number is red'. Are A and B independent?

Ans: The sample space is given by

$S=\{1,2,3,4,5,6\}$

According to the question

$A=\{2,4,6\}$

$\therefore P(A)=\frac{3}{6}$

$B=\{1,2,3\}$

$\therefore P(B)=\frac{3}{6}$

Also $(A\cap B)=\{2\}$

$\therefore P(A\cap B)=\frac{1}{6}$

We know that for A and B to be independent events

$P(A\cap B)=P(A)P(B)$

Now $P(A)P(B)=\frac{3}{6}\times\frac{3}{6}$

$\Rightarrow P(A)P(B)=\frac{1}{4}$

Which is not as same as $P(A\cap B)$

Thus A and B are not independent events

6. Let E and F be the vents with $P(E)=\frac{3}{5}$, $P(F)=\frac{3}{10}$ and $P(E\cap F)=\frac{1}{5}$

Are E and F independent?

Ans: Given $P(E)=\frac{3}{5}$, $P(F)=\frac{3}{10}$ and $P(E\cap F)=\frac{1}{5}$

We know that for A and B to be independent events

$P(A\cap B)=P(A)P(B)$

Now $P(A)P(B)=\frac{3}{5}\times\frac{3}{10}$

$\Rightarrow P(A)P(B)=\frac{9}{50}$

Also $P(E\cap F)=\frac{1}{5}$

Thus $P(A)P(B)\neq P(E\cap F)$

And hence A and B are not independent events

7. given that the events A and B are such that $P(A)=\frac{1}{2}$, $P(A\cup B)=\frac{3}{5}$ and $P(B)=p$ find p if they are

(i) mutually exclusive

Ans: It is given in the question that $P(A)=\frac{1}{2}$, $P(B)=p$ and $P(A\cup B)=\frac{3}{5}$

We know that if two events A and B are mutually exclusive then $P(A\cap B)=0$

Also we know that

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\Rightarrow \frac{3}{5}=\frac{1}{2}+p-0$

$\Rightarrow p=\frac{3}{5}-\frac{1}{2}$

$\Rightarrow p=\frac{1}{10}$

(ii) independent

Ans: It is given in the question that $P(A)=\frac{1}{2}$, $P(B)=p$ and $P(A\cup B)=\frac{3}{5}$

We know that if two events A and B are independent then
$P(A\cap B)=P(A)P(B)$

Also we know that

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \frac{3}{5} = \frac{1}{2} + p - \frac{p}{2}$ (since $P(A)P(B) = \frac{p}{2}$)

$\Rightarrow \frac{p}{2} = \frac{3}{5} - \frac{1}{2}$

$\Rightarrow p = \frac{2}{10}$

$= \frac{1}{5}$

The value of p when A and B are independent event is $\frac{1}{5}$

8. Let A and B be independent events with $P(A) = 0.3$ and $P(B) = 0.4$ find

(i) $P(A \cap B)$

Ans: It is given that A and B are independent events with given probabilities as shown

$P(A) = 0.3$

$P(B) = 0.4$

We know that if A and B are independent events then

$P(A \cap B) = P(A)P(B)$

$\Rightarrow P(A \cap B) = 0.3 \times 0.4$

$\Rightarrow P(A \cap B) = 0.12$

(ii) $P(A \cup B)$

Ans: It is given that A and B are independent events with given probabilities as shown

$P(A) = 0.3$

$P(B) = 0.4$

We know that if A and B are independent events then

$P(A \cap B) = P(A)P(B)$

$\Rightarrow P(A \cap B) = 0.3 \times 0.4$

$\Rightarrow P(A \cap B) = 0.12$

Also we know that

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A \cup B) = 0.3 + 0.4 - 0.12$

$\Rightarrow P(A \cup B) = 0.58$

(iii) $\mathbf{P(A|B)}$

Ans: It is given that A and B are independent events with given probabilities as shown

$P(A) = 0.3$

$P(B) = 0.4$

We know that if A and B are independent events then

$P(A \cap B) = P(A)P(B)$

$\Rightarrow P(A \cap B) = 0.3 \times 0.4$

$\Rightarrow P(A \cap B) = 0.12$

Also we know that $P(A|B)$ is given by

$$P(A|B) = \frac{(P(A \cap B))}{P(B)}$$

$$\Rightarrow P(A|B) = \frac{0.12}{0.4}$$

$= 0.3$

Thus we found that $P(A|B) = 0.3$

(iv) $P(B|A)$

Ans: It is given that A and B are independent events with given probabilities as shown

$P(A) = 0.3$

$P(B) = 0.4$

We know that if A and B are independent events then

$P(A \cap B) = P(A)P(B)$

$\Rightarrow P(A \cap B) = 0.3 \times 0.4$

$\Rightarrow P(A \cap B) = 0.12$

Also we know that $P(B|A)$ is given by

$$P(B|A) = \frac{(P(A \cap B))}{P(A)}$$

$$\Rightarrow P(B|A) = \frac{0.12}{0.3}$$

$= 0.4$

Thus we found that $P(B|A) = 0.4$

9. If A and B are two events such that $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{8}$ find $P(\text{not A and not B})$

Ans: It is given that two events A and b have probabilities as shown

$P(A)=\frac{1}{4},\ P(B)=\frac{1}{2}$ and $P(A\cap B)=\frac{1}{8}$

$P(\text{not A and not B})=P(A'\cap B')$

And we know that

$P(A'\cap B')=P(A\cup B)'$

$P(A'\cap B')=1-P(A\cup B)$

Also $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\Rightarrow P(A\cup B)=\frac{1}{4}+\frac{1}{2}-\frac{1}{8}$

$\Rightarrow P(A\cup B)=\frac{5}{8}$

$\therefore P(A'\cap B')=1-\frac{5}{8}$

Thus $P(\text{not A and not B})=\frac{3}{8}$

10. Events A and B are such that $P(A)=\frac{1}{2},\ P(B)=\frac{7}{12}$ and $P(\text{not A or not B})=\frac{1}{4}$. State whether A and B are independent

Ans: It is given that $P(A)=\frac{1}{2},\ P(B)=\frac{7}{12}$ and $P(A'\cup B')=\frac{1}{4}$

According to the question

$1-P(A\cap B)=\frac{1}{4}$

$\Rightarrow P(A\cap B)=\frac{3}{4}$

We know that A and B are independent events if

$P(A)P(B)=P(A\cap B)$

$\therefore P(A)P(B) = \frac{1}{2} \times \frac{7}{12}$

Clearly $P(A)P(B) \neq P(A \cap B)$

Thus A and B are not independent events

11. **G iven two independent events A and B such that $P(A) = 0.3$, $P(B) = 0.6$. Find**

(i) $P(\text{A and B})$

Ans: It is given that $P(A) = 0.3$, $P(B) = 0.6$

Where A and B are independent events

$\therefore \quad P(A \cap B) = P(A)P(B)$

$\Rightarrow P(\text{A and B}) = 0.3 \times 0.6$

Thus we found that $P(\text{A and B}) = 0.18$

(ii) $P(\text{A and not B})$

Ans: It is given that $P(A) = 0.3$, $P(B) = 0.6$

Where A and B are independent events

$\therefore P(A \cap B) = P(A)P(B)$

Now we know that

$P(\text{A and not B}) = P(A \cap B')$

Also we know that

$P(A \cap B') = P(A) - P(A \cap B)$

$P(A \cap B') = 0.3 - 0.18$

Thus we found that $P(\text{A and not B}) = 0.12$

(iii) $P(\text{A or B})$

Ans: It is given that $P(A)=0.3$, $P(B)=0.6$

Where A and B are independent events

$\therefore \quad P(A\cap B)=P(A)P(B)$

$\Rightarrow P(\text{A and B})=0.3\times 0.6$

Thus we found that $P(\text{A and B})=0.18$

Also we know that

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$P(A\cup B)=0.3+0.6-0.18$

Thus we know that $P(\text{A or B})=0.72$

(iv) $\mathbf{P(\text{neither A and nor B})}$

Ans: It is given that $P(A)=0.3$, $P(B)=0.6$

Where A and B are independent events

$\therefore \quad P(A\cap B)=P(A)P(B)$

$\Rightarrow P(\text{A and B})=0.3\times 0.6$

Thus we found that $P(\text{A and B})=0.18$

Also we know that

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$P(A\cup B)=0.3+0.6-0.18$

Thus we know that $P(\text{A or B})=0.72$

$P(\text{neither A nor B})=P(A\cup B)'$

And $P(A\cup B)'=1-P(A\cup B)$

$P(A\cup B)'=1-0.72$

Thus $P(\text{neither A nor B})=0.28$

12. A die tossed thrice. Find the probability of getting an odd number at least once.

Ans: It is given that a die is tossed thrice, the number of elements in sample space for each throw is 6

Now

$\text{probability of getting atleast odd once}=1-\text{probability of getting even numbers in all three events}$

Now we know that

$$\text{probability of getting even numbers in all three events}=\left(\frac{1}{2}\right)^3$$

$$\text{probability of getting atleast odd once}=1-\frac{1}{8}$$

$$\text{Thus probability of getting atleast odd once}=\frac{7}{8}$$

13. Two balls are drawn at random with replacement from a box containing 10 black and 8 red balls.

Find the probability that

(i) Both balls are red.

Ans: It is given in the question that

number of red balls $=8$ and

number of black balls $=10$

$$\text{Now probability of getting red ball at first draw, say } P(A)=\frac{8}{18}$$

$$=\frac{4}{9}$$

$$\text{Also probability of getting red ball at second draw, say } P(B)=\frac{8}{18}$$

$= \frac{4}{9}$

$\therefore P(A \cap B) = \frac{4}{9} \times \frac{4}{9}$ (since the events are independent)

$= \frac{16}{81}$

The probability of getting both balls red is $\frac{16}{81}$

(ii) First ball is black and second is red.

Ans: it is given in the question that

number of red balls $= 8$ and

number of black balls $= 10$

Now probability of getting black ball at first draw $= \frac{10}{18}$

$= \frac{5}{9}$

Also probability of getting red ball at second draw $= \frac{8}{18}$

$= \frac{4}{9}$

Also above two events are independent therefore their intersection is given by

$= \frac{5}{9} \times \frac{4}{9}$

$= \frac{20}{81}$

probability of getting black ball in first draw

and red ball in second draw $= \frac{20}{81}$

(iii) One of them is black and other is red.

Ans: Let us have following notations

A: first ball is black and the other is red

Now probability of getting a black ball in first draw, $\mathrm{P}(\mathrm{A})=\frac{10}{18}$

$=\frac{5}{9}$

As the ball is replaced after first throw, probability of getting a red ball in second draw $=\frac{8}{18}$

$=\frac{4}{9}$

Probability of getting first ball as black and second ball as red $=\frac{5}{9}\times\frac{4}{9}$

$=\frac{20}{81}$

Probability of drawing red ball in first draw $=\frac{8}{18}$

$=\frac{4}{9}$

Probability of getting black ball in second draw $=\frac{10}{81}$

$=\frac{5}{9}$

Probability of drawing first ball as red and second as black $=\frac{5}{9}\times\frac{4}{9}$

$=\frac{20}{81}$

Probability of getting one of them is black and the other is red $=\frac{20}{81}+\frac{20}{81}$

$= \frac{40}{81}$

Probability of getting one of the ball is black and the other is red is $\frac{40}{81}$

14. Probability of solving specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently, find the probability that

(i) the problem is solved

Ans: Given that the two events A and B are independent with probabilities given as shown

$P(A) = \frac{1}{2}$ and $P(B) = \frac{1}{3}$

$\therefore\ P(A \cap B) = P(A)P(B)$

$\Rightarrow P(A \cap B) = \frac{1}{6}$

Probability of problem being solved is given by

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A \cup B) = \frac{1}{2} + \frac{1}{3} - \frac{1}{6}$

Thus probability of problem being solved is $\frac{2}{3}$

(ii) exactly one of them solves the problem

Ans: Given that the two events A and B are independent with probabilities given as shown

$P(A) = \frac{1}{2}$ and $P(B) = \frac{1}{3}$

15. One card is drawn at random from a well shuffled deck of 52 cards. In which of the following cases are the events E and F independent?

(i) E: 'the card drawn is a spade'

F: 'the card drawn is an ace'

Ans: Given a deck of cards 52

We know that 13 cards are spades and 4 are aces

$P(E)=\frac{13}{52}$ and $P(F)=\frac{4}{52}$

Also in the deck only 1 card is ace of spade

I.e $P(E\cap F)=\frac{1}{52}$

It is clearly visible that

$P(E)P(F)=\frac{13}{52}\times\frac{4}{52}$ equals $P(E\cap F)$

Hence the events are independent

(ii) E: 'the card drawn is black'

F: 'the card drawn is a king'

Ans: We know that in the deck of cards 26 cards are black and 4 cards are kings

$P(E)=\frac{26}{52}$ and $P(F)=\frac{4}{52}$

Also there are 2 cards which are black as well as king

i.e $P(E\cap F)=\frac{2}{52}$

It is clearly visible that $\frac{26}{52}\times\frac{4}{52}=\frac{2}{52}$

Hence these are independent events

(iii) E: 'the card drawn is a king and queen'

F: ‘the card drawn is a queen or jack’

Ans: In a deck of 52 cards each king, queen and jack is 4

$$\therefore P(E) = P(F) = \frac{8}{52}$$

There are 4 cards which which are king and queen or jack

Hence its probability is $\frac{4}{52}$

Clearly E and F are not independent

16. In a hostel, 60 percent of the students read Hindi newspaper, 40 percent read English newspaper and 20 percent read both Hindi and English newspapers. A student is selected at random.

a) Find the probability that she reads neither Hindi and English newspapers.

Ans: Let us have following notations

H: Students who read Hindi newspapers

E: Students who read English newspapers

$\therefore P(H) = 0.6$ and

$\therefore P(E) = 0.4$

Also it is given that

$P(H \cap E) = 0.2$

Therefore the required probability is given by

$P(H \cup E') = 1 - P(H \cup E)$

$P(H \cup E') = 1 - (P(H) + P(E) - P(H \cap E))$

$P(H \cup E') = 1 - 0.8$

Hence the probability that she reads neither Hindi nor English is 0.2

b) If she reads Hindi newspaper, find the probability that she reads English newspaper.

Ans: Let us have following notations

H: Students who read Hindi newspapers

E: Students who read English newspapers

$\therefore P(H) = 0.6$ and

$\therefore P(E) = 0.4$

Also it is given that

$P(H \cap E) = 0.2$

Now required probability is given by

$$P(E|H) = \frac{P(H \cap E)}{P(H)}$$

$$\Rightarrow P(E|H) = \frac{0.2}{0.6}$$

Thus the probability that she reads English newspapers if she reads Hindi newspapers is $\frac{1}{3}$

c) If she reads English newspaper, find the probability that she reads Hindi newspaper.

Ans: Let us have following notations

H: Students who read Hindi newspapers

E: Students who read English newspapers

$\therefore P(H) = 0.6$ and

$\therefore P(E) = 0.4$

Also it is given that

$P(H \cap E) = 0.2$

The required probability is given by

$$P(E|H) = \frac{P(H \cap E)}{P(H)}$$

$$\Rightarrow P(E|H) = \frac{0.2}{0.4}$$

Therefore the probability that she reads Hindi newspapers provided she reads English newspapers is $\frac{1}{2}$

17. The probability of obtaining an even prime number on each die, when a pair of dice is rolled is

A. 0

B. $\mathbf{\frac{1}{3}}$

C. $\mathbf{\frac{1}{12}}$

D. $\mathbf{\frac{1}{36}}$

Ans: Out of six numbers in a die the only even prime number is 2

Therefore probability of getting an even prime number is given by

Say $P(E) = \frac{1}{36}$

Hence probability of getting an even prime number is $\frac{1}{36}$

18. Two events A and B will be independently, if

(A) A and B are mutually exclusive

Ans: Let $P(A) = p$ and $P(B) = q \quad 0 < p, q < 1$

It is given that they are mutually exclusive

$\therefore P(A \cap B) = 0$

But $P(A)P(B) = pq$ which is not necessarily zero

Hence they are not independent

(B) $\mathbf{P(A'B') = [1 - P(A)][1 - P(B)]}$

Ans: Two events are said to be independent if $P(AB) = P(A)P(B)$

Let us solve for option B first

$P(A'B') = [1 - P(A)][1 - P(B)]$

$\Rightarrow P(A' \cap B') = 1 - P(A) - P(B) + P(A)P(B)$

$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A)P(B)$

$\Rightarrow P(A) + P(B) - P(A \cap B) = P(A) + P(B) - P(A)P(B)$

Hence we found that $P(AB) = P(A)P(B)$

These are independent events

(C) $\mathbf{P(A) = P(B)}$

Ans: If $P(A) = P(B)$ then

Each of them has probability of $\frac{1}{2}$

Surely it can be sometime true but not a sufficient condition for two events to be independent

(D) $\mathbf{P(A) + P(B) = 1}$

Ans: Similarly If $P(A) + P(B) = 1$ it is not necessary condition for two events to be independent

Exercise 13.3

1. **An urn contains 5 red and 5 black balls. A ball is drawn at random, its colour is noted and is returned to the urn. Moreover, 2 additional balls of the colour drawn are put in the urn and then a ball is drawn at random. What is the probability that the second ball is red?**

Ans: It is given in the question that at random one ball is picked and its colour is noted then two balls of same colour is added, hence we have two cases here

Consider the first case

Let the first ball drawn be Red, its probability is $\frac{5}{10}$

Now two extra red balls are added,

$\therefore \mathrm{P}(\text{drawing red ball at second attempt}) = \frac{7}{12}$

Since these two events are independent therefore required probability for this case is $\frac{5}{10} \times \frac{7}{12}$

Consider the second case

Let black ball be drawn at first attempt, its probability is $\frac{5}{10}$

Now two extra black balls will be added

$\therefore \mathrm{P}(\text{drawing red ball at second attempt}) = \frac{5}{12}$

Since these two events are independent therefore required probability for this case is $\frac{5}{10} \times \frac{5}{12}$

Total probability is given by $\frac{5}{10} \times \frac{7}{12} + \frac{5}{10} \times \frac{5}{12} = \frac{5}{10}$

Thus the probability of drawing second ball red is $\frac{1}{2}$

2. **A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn**

from the bag which is found to be red. Find the probability that the ball is drawn from the first bag.

Ans: Let E and F be the events of selecting first and second bag respectively

$\therefore P(E) = P(F) = \frac{1}{2}$

Let K be the event of getting a red ball

$\therefore P(K|E) = \frac{4}{8}$, where

$(K|E)$: drawing red ball from the first bag

Similarly, $P(K|F) = \frac{2}{8}$, where

$(K|F)$: drawing red ball from the first bag

Now from the Bay's theorem

$$P(E|K) = \frac{P(E)P(K|E)}{P(E)P(K|E) + P(F)P(K|F)}$$

$$\Rightarrow P(E|K) = \frac{\frac{1}{2} \times \frac{1}{2}}{\frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{4}}$$

Thus the probability that the red ball is drawn from the first bag is $\frac{2}{3}$

3. Of the students in a college, it is known that 60 percent reside in hostel and 40 percent are day scholars (not residing in hostel). Previous year results report that 30 percent of all students who reside in hostel attain A grade and 20 percent of day scholars attain A grade in their annual examination. At the end of the year, one student is chosen at random from the college and he has an A grade, what is the probability that the student is hosteler?

Ans: Let us have following notations

E: The event when a student is a hosteler

F: The event when a student is a day scholar

K: The event chosen student gets grade A

$\therefore P(E) = 0.6$

$\therefore P(F) = 0.4$

$\therefore P(K|E) = 0.3$

$\therefore P(K|F) = 0.2$

By Bay's theorem

$$P(E|K) = \frac{P(E)P(K|E)}{P(E)P(K|E) + P(F)P(K|F)}$$

$$\Rightarrow P(E|K) = \frac{0.6 \times 0.3}{0.6 \times 0.3 + 0.4 \times 0.2}$$

Thus the probability that the gets grade A provided he Is a hosteler is $\frac{9}{13}$

4. In answering a on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{4}$ be the probability that he knows the answer and $\frac{1}{4}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{4}$. What is the probability that the student knows the answer given that he answered it correctly?

Ans: Let us have following notations

E: The event when a student knows the answer

F: The event when a student guesses the answer

K: The event that the answer is correct

$\therefore P(E) = \frac{3}{4}$

$\therefore P(F) = \frac{1}{4}$

$\therefore P(K|E)=1$ (since if he knows the answer then he will answer it correctly, it's a trivial case)

where $(K|E)$: the answer is correct provided he knows the answer

$$\therefore P(K|F)=\frac{1}{4}$$

$(K|F)$: the answer is correct provided he guesses the answer

By Bay's theorem

$$P(E|K)=\frac{P(E)P(K|E)}{P(E)P(K|E)+P(F)P(K|F)}$$

$$\Rightarrow P(E|K)=\frac{\frac{3}{4}\times\frac{1}{1}}{\frac{3}{4}\times\frac{1}{1}+\frac{1}{4}\times\frac{1}{4}}$$

Thus the probability that he knows the answer provided he answered correctly is $\frac{12}{13}$

5. A laboratory blood test is 99% effective in detecting a certain disease when it is in fact, present.

However, the test also yield a false positive result for 0.5 percent **of the healthy person tested (that is, if a healthy person is tested, then, with probability 0.005, the test will imply he has the disease). If 0.1** percent **of the population actually has the disease, what is the probability that a person has the disease given that his test result is positive?**

Ans: Let us have following notations

E: The event when the person has disease

F: The event when the person does not have disease

K: The event that the test result is positive

$$\therefore P(E)=0.001$$

It is clear that E and F are complimentary events

$\therefore P(E)+P(F)=1$

Hence

$P(F)=0.999$

$\therefore P(K|E)=0.99$

$(K|E)$: the result is correct provided he has the disease

$\therefore P(K|F)=0.005$

$(K|F)$: the result is correct provided he doesn't have the disease

By Bay's theorem

$$P(E|K)=\frac{P(E)P(K|E)}{P(E)P(K|E)+P(F)P(K|F)}$$

$$\Rightarrow P(E|K)=\frac{0.001\times 0.99}{0.001\times 0.99+0.999\times 0.005}$$

Thus the probability that the person had the disease provided his test result is positive is $\frac{22}{133}$

6. There are three coins. One is two headed coin (having head on both faces), another is a biased coin that comes up heads 75 percent of the time and third is an unbiased coin. One of the three coins is chosen at random and tossed, it shows heads, what is the probability that it was the two headed coin?

Ans: Let us have following notations

E: The event when chosen coin is two-headed

F: The event when choosen coin is biased

G: The event when choosen coin is unbiased

K: The event that the coin shows heads

$\therefore P(E)=\frac{1}{3}$

$P(F) = \frac{1}{3}$

$P(G) = \frac{1}{3}$

$\therefore P(K|E) = 1$ (since it is a trivial case)

$(K|E)$: the coins shows heads provided it is two headed

$\therefore P(K|F) = \frac{3}{4}$

$(K|F)$: the coins shows heads provided it is biased

$\therefore P(K|G) = \frac{1}{2}$

$(K|G)$: the coins shows heads provided it is unbiased

By Bay's theorem

$$P(E|K) = \frac{P(E)P(K|E)}{P(E)P(K|E) + P(F)P(K|F) + P(G)P(K|G)}$$

$$P(E|K) = \frac{\frac{1}{3}\times 1}{\frac{1}{3}\times 1 + \frac{1}{3}\times\frac{3}{4} + \frac{1}{3}\times\frac{1}{2}}$$

Thus the probability that the coin shows heads provided it is two headed coin is $\frac{4}{9}$

7. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of accidents are 0.01 , 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver?

Ans: Let us have following notations

E: The event when the driver drives scooter

F: The event when the driver drives car

G: The event when the driver drives truck

K: The event that the driver meets accident

$\therefore P(E)=\frac{2000}{12000}$

$P(F)=\frac{4000}{12000}$

$P(G)=\frac{6000}{12000}$

$\therefore P(K|E)=\frac{1}{100}$

$(K|E)$: the driver meets accident provided he drives scooter

$\therefore P(K|F)=\frac{3}{100}$

$(K|F)$: the driver meets accident provided he drives car

$\therefore P(K|G)=\frac{15}{100}$

$(K|G)$: the driver meets accident provided he drives truck

By Bay's theorem

$$P(E|K)=\frac{P(E)P(K|E)}{P(E)P(K|E)+P(F)P(K|F)+P(G)P(K|G)}$$

$$P(E|K)=\frac{\frac{1}{6}\times\frac{1}{100}}{\frac{1}{6}\times\frac{1}{100}+\frac{1}{3}\times\frac{3}{100}+\frac{1}{2}\times\frac{15}{100}}$$

Thus the probability that the accidented person is a scooter driver is $\frac{1}{52}$

8. **A factory has two machines A and B. Past record shows that machine A produced 60 percent of the items of output and machine B produced 40 percent of the items. Future, 2 percent of the items produced by machine A and 1 percent produced by machine B were defective. All the items are put into one stockpile and then one item is chosen random from this and is found to be defective. What is the probability that was produced by machine B?**

Ans: Let us have following notations

E: The event when the production is from machine A

F: The event when the production is from machine B

K: The event that the item is defective

$\therefore P(E) = 0.6$

$\therefore P(F) = 0.4$

$\therefore P(K|E) = 0.02$

$(K|E)$: The event that the item is defective provided it from A

$\therefore P(K|F) = 0.01$

$(K|F)$: The event that the item is defective provided it from B

By Bay's theorem

$$P(F|K) = \frac{P(F)P(K|F)}{P(E)P(K|E) + P(F)P(K|F)}$$

$$\Rightarrow P(F|K) = \frac{0.4 \times 0.01}{0.6 \times 0.02 + 0.4 \times 0.01}$$

Thus the probability that that the item is defective provided it from B is $\frac{1}{4}$

9. **Two groups are competing for the position on board of directors of a corporation. The probabilities that the first and the second groups will win are 0.6 and 0.4 respectively. Further, if the first group wins, the probability of introducing a new product is 0.7 and the corresponding probability is 0.3 if the second group wins. Find the probability that the new product introduced was by the second group.**

Ans: Let us have following notations

E: The event when the first group wins

F: The event when the second group wins

K: The event that the new item is produced

$\therefore P(E)=0.6$

$\therefore P(F)=0.4$

$\therefore P(K|E)=0.7$

$(K|E)$: The event that the item is introduced by first group

$\therefore P(K|F)=0.3$

$(K|F)$: The event that item is introduced by second group

By Bay's theorem

$$P(F|K)=\frac{P(F)P(K|F)}{P(E)P(K|E)+P(F)P(K|F)}$$

$$\Rightarrow P(F|K)=\frac{0.4\times 0.3}{0.6\times 0.7+0.4\times 0.3}$$

Thus the probability that that the item is produced from second group is $\frac{2}{9}$

10. Suppose a girl throws a die. If she gets a 5 or 6, she tosses a coin three times and notes the number of heads. If she gets $1,2,3,$ or 4, she tosses a coin once and notes whether a head or tail is obtained. If she obtained exactly one head, what is the probability that she threw $1,2,3,$ or 4 with the die?

Ans: Let us have following notations

E: The event when 5 or 6 come

F: The event when $1,2,3$ or 4 come

K: The event that the coin shows exactly one head

$\therefore P(E)=\frac{2}{6}$

$\therefore P(F)=\frac{4}{6}$

$\therefore P(K|E)=\frac{3}{8}$

$(K|E)$: The event that the coin shows exactly one head provided 5 or 6 come

$\therefore P(K|F)=\frac{1}{2}$

$(K|F)$: The event that the single throw of coin shows exactly one head provided 1,2,3 or 4 come

By Bay's theorem

$$P(F|K)=\frac{P(F)P(K|F)}{P(E)P(K|E)+P(F)P(K|F)}$$

$$\Rightarrow P(F|K)=\frac{\frac{4}{6}\times\frac{1}{2}}{\frac{2}{6}\times\frac{3}{8}+\frac{4}{6}\times\frac{1}{2}}$$

Thus the probability that that the item is produced from second group is $\frac{8}{11}$

11. A manufacture has three machine operators A, B and C. The first operator A produces 1 percent defective items, whereas the other two operators B and C produce 5 percent and 7 percent defective items respectively. A is on the job for 50 Percent of the time, B is on the job for 30 Percent of the time and C is on the job for 20 percent of the time. A defective item is produced, what is the probability that was produced by A?

Ans: Let us have following notations

E: The event when A works

F: The event when B works

G: The event when C works

K: The event that the defective item is produced

$\therefore P(E) = 0.5$

$\therefore P(F) = 0.3$

$\therefore P(G) = 0.2$

$\therefore P(K|E) = 0.01$

$(K|E)$: The event that the defective item is produced from A

$\therefore P(K|F) = 0.05$

$(K|F)$: The event that the defective item is produced from B

$\therefore P(K|G) = 0.07$

The event that the defective item is produced from C

By Bay’s theorem

$$P(E|K) = \frac{P(E)P(K|F)}{P(E)P(K|E) + P(F)P(K|F) + P(G) \times P(K|G)}$$

$$\Rightarrow P(E|K) = \frac{0.5 \times 0.01}{0.5 \times 0.01 + 0.3 \times 0.05 + 0.2 \times 0.07}$$

Thus the probability that that the item is produced from A and found defective $\frac{5}{34}$

12. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be both diamonds. Find the probability of the lost card being a diamond.

Ans: Let us have following notations

E: The event when a card is diamond

F: The event when the card is not diamond

K: The event to denote lost card

Clearly E and F are contemporary Events

$\therefore P(E) = \frac{13}{52}$ (since 13 cards are diamonds)

$\therefore P(F)=\frac{39}{52}$

$(K|E)$: The event of drawing two cards when one diamond card is lost

Since one diamond card is lost therefore left total cards are
51 and left diamond cards are 13

$\therefore P(K|E)=\frac{{}^{12}C_2}{{}^{51}C_2}$

$\Rightarrow P(K|E)=\frac{22}{425}$

$(K|F)$: The event of drawing two cards when one non diamond card is lost

Since one card is lost therefore left total cards are 51

$\therefore P(K|F)=\frac{{}^{13}C_2}{{}^{51}C_2}$

$\Rightarrow P(K|F)=\frac{26}{425}$

By Bay's theorem

$$P(E|K)=\frac{P(E)P(K|F)}{P(E)P(K|E)+P(F)P(K|F)}$$

$$\Rightarrow P(E|K)=\frac{\frac{1}{4}\times\frac{22}{425}}{\frac{1}{4}\times\frac{22}{425}+\frac{3}{4}\times\frac{26}{425}}$$

Thus the probability of the lost card being a diamond is $\frac{11}{50}$

13. **Probability that A speaks truth is $\frac{4}{5}$. A coin is tossed. A reports that a head appears. The probability that actually there was head is**

(A) $\mathbf{\frac{4}{5}}$

(B) $\mathbf{\frac{1}{2}}$

(C) $\mathbf{\frac{1}{5}}$

(D) $\mathbf{\frac{2}{5}}$

Ans: Let us have following notations

E: The event when A speaks truth

F: The event when A speak false

K: The event it is head

Clearly E and F are contemporary Events

$\therefore P(E)=\frac{4}{5}$ (since 13 cards are diamonds)

$\therefore P(F)=\frac{1}{5}$

$(K|E)$: The event it is head and he speaks truth

$\therefore P(K|E)=\frac{1}{2}$

$(K|F)$: The event it is head and he speaks false

$\therefore P(K|F)=\frac{1}{2}$

By Bay's theorem

$$P(E|K)=\frac{P(E)P(K|F)}{P(E)P(K|E)+P(F)P(K|F)}$$

$$\Rightarrow P(E|K) = \frac{\frac{4}{5}\times\frac{1}{2}}{\frac{4}{5}\times\frac{1}{2}+\frac{1}{5}\times\frac{1}{2}}$$

Thus the probability of the lost card being a diamond is $\frac{4}{5}$

14. If A and B are two events such that $A \subset B$ and $P(B) \neq 0$ then which of the following is correct

(A) $\mathbf{P(A|B) = \frac{P(B)}{P(A)}}$

(B) $\mathbf{P(A|B) < P(A)}$

(C) $\mathbf{P(A|B) \geq P(A)}$

(D) None of these

Ans: Given $A \subset B$

$\therefore A \cap B = A$ and $P(A) < P(B)$

Now $P(A|B) = \frac{P(A \cap B)}{P(B)}$

$\Rightarrow P(A|B) = \frac{P(A)}{P(B)}$

Now $P(B) \leq 1$

$\therefore \frac{1}{P(B)} \geq 1$

Hence option C is correct, i.e $P(A|B) \geq P(A)$

Miscellaneous

1. If A and B are two events such that $P(A) \neq 0$. Find $P(B|A)$ if

(i) A is a subset of B

Ans: It is given in the question that $P(A) \neq 0$ and A is a subset of B

$\therefore P(A \cap B) = P(A)$

$$\therefore P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$\Rightarrow P(B|A) = \frac{P(A)}{P(A)} = 1$$

(ii) $A \cap B = \phi$

Ans: since it is given that $A \cap B = \phi$

Therefore $P(A \cap B) = 0$

$$\therefore P(B|A) = \frac{P(A \cap B)}{P(A)} = 0$$

2. A couple has two children,

(i) Find the probability that both children are males, if it is known that at least one of the children is male.

Ans: The sample space for a family to have two children is given by

$S = \{BB, GG, BG, GB\}$

Let us have following notations and their probabilities given as shown

N: Both children are males

$$P(N) = \frac{1}{4}$$

K: at least one of the children is male

$$P(K) = \frac{3}{4}$$

$\because N \cap K = BB$

$$\therefore P(N \cap K) = \frac{1}{4}$$

Now probability that both of the children are males provided atleast one of the child is male is given by

$$P(N|K)=\frac{P(N\cap K)}{P(K)}$$

$$P(N|K)=\frac{\frac{1}{4}}{\frac{3}{4}}$$

Hence probability that both of the children are males provided atleast one of the child is male is $\frac{1}{3}$

(ii) Find the probability that both children are females, if it is known that the elder child is a female.

Ans: The sample space for a family to have two children is given by

$$S=\{BB, GG, BG, GB\}$$

Let us have following notations and their probabilities given as shown

N: Both children are females

$$P(N)=\frac{1}{4}$$

K: elder child is a female

$$P(K)=\frac{2}{4}$$

$$\because N\cap K=GG$$

$$\therefore P(N\cap K)=\frac{1}{4}$$

Now probability that both of the children are males provided atleast one of the child is male is given by

$$P(N|K)=\frac{P(N\cap K)}{P(K)}$$

$$P(N|K)=\frac{\frac{1}{4}}{\frac{2}{4}}$$

Hence probability that both of the children are males provided atleast one of the child is male is $\frac{1}{2}$

3. Suppose that 5% of men and 0.25% of women have grey hair. A haired person is selected at random. What is the probability of this person being male? Assume that there are equal numbers of males and females.

Ans: It is given that 5 percent of males and 0.25 percent of females have grey hair

Therefore total people having grey hair is $(5+0.25)=5.25$ percent

Hence probability of male being haired is $\frac{5}{5.25}$

4. Suppose that 90% of people are right-handed. What is the probability that at most 6 of a random sample of 10 people are right-handed?

Ans: Given probability of a person being right-handed is $p=0.9$

Since a person can only be right-handed or left handed

Therefore it follows binomial distribution with

$$n=10, p=\frac{9}{10}, q=\frac{1}{10}$$

Probability that at least 6 people are right handed is given by

$$\sum_{k=0}^{k=7} {}^{10}C_k \left(\frac{9}{10}\right)^k \left(\frac{1}{10}\right)^{10-k}$$

Therefore probability that at most 6 people are right handed is given by

$$1-\sum_{k=0}^{k=7} {}^{10}C_k \left(\frac{9}{10}\right)^k \left(\frac{1}{10}\right)^{10-k}$$

5. If a leap year is selected at random, what is the change that it will contain 53 Tuesdays?

Ans: In a leap year, we have 366 days i.e., 52 weeks and 2 days.

In 52 weeks, we have 52 Tuesdays.

Therefore, the probability that the leap year will contain 53 Tuesday is equal to the probability that the remaining 2 days will be Tuesdays.

The remaining 2 days can be any of the following

Monday and Tuesday, Tuesday and Wednesday, Wednesday and Thursday, Thursday and Friday, Friday and Saturday, Saturday and Sunday and Sunday and Monday

Total number of cases = 7

$\therefore$Total number of cases = 7ay, Saturday and 53 Tuesdays $= \frac{2}{7}$

6. Suppose we have four boxes. A, B, C and D containing coloured marbles as given below.

Box	**Red**	**White**	**Black**
A	**1**	**6**	**3**
B	**6**	**2**	**2**
C	**8**	**1**	**1**
D	**0**	**6**	**4**

One of the boxes has been selected at random and a single marble is drawn from it. If the marble is red, what is the probability that it was drawn from box A?, box B?, box C?

Ans: Let us have following notations

R be the event of drawing red marble

E be the event of selecting the box A

F be the event of selecting the box B

G be the event of selecting the box C

Total numbers of marbles is 40

Number of red marbles is 15

$\therefore P(R) = \frac{15}{40}$

Number of red marbles in box A i.e $n(R \cap E) = 1$

Number of red marbles in box B i.e $n(R \cap F) = 6$

Number of red marbles in box C i.e $n(R \cap G) = 8$

Now the probability that red marble is picked from box A is given by

$$P(R|E)=\frac{P(R\cap E)}{P(E)}$$

$$\Rightarrow P(R|E)=\frac{\frac{1}{40}}{\frac{15}{40}}$$

Hence E be the event of selecting the box A is $\frac{1}{15}$

Similarly, probability that red marble is picked from box B is given by

$$P(R|F)=\frac{\frac{6}{40}}{\frac{15}{40}}$$

Hence probability that red marble is picked from box B is $\frac{2}{5}$

Similarly, probability that red marble is picked from box C is given by

$$P(R|G)=\frac{\frac{8}{40}}{\frac{15}{40}}$$

Hence probability that red marble is picked from box C is $\frac{8}{15}$

7. Assume that the changes of the patient having a heart attack are 40%. It is also assumed that a meditation and yoga course reduce the risk of heart attack by 30% and prescription of certain drug reduces its changes by 25%. At a time a patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga?

Ans: Let us have following notations

E: The events when the person took yoga and meditation courses

$$\therefore P(E)=\frac{1}{2}$$

F:The events when the person took drugs

$\therefore P(F) = \frac{1}{2}$

G: the person suffered heart attack

$\therefore P(G) = 0.4$

From the question also we have

$P(G|E) = 0.4 \times 0.7 = 0.28$

$P(G|F) = 0.4 \times 0.75 = 0.30$

Now probability that found person has heart attack despite of having yoga and meditation courses is given by

$$P(E|G) = \frac{P(E) \times P(G|E)}{P(E) \times P(G|E) + P(F) \times P(G|F)}$$

$$P(E|G) = \frac{0.5 \times 0.28}{P0.5 \times 0.28 + 0.5 \times 0.3}$$

Hence probability that found person has heart attack despite of having yoga and meditation courses is $\frac{14}{29}$

8. If each element of a second order determinant is either zero or one, what is the probability that the value of the determinant is positive? (Assume that the individual entries of the determinant are chosen independently, each value being assumed with probability $\frac{1}{2}$)

Ans: It is clear that total number of determinants of second order entries being 0's or 1's is 2^4

The value of determinant is positive for cases as shown

$$\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}$$

i.e favourable case is 3

Hence probability that value of determinant is positive is $\frac{3}{16}$

9. **An electronic assembly consists of two subsystems, say, A and B. From previous testing procedures, the following probabilities are assumed to be known:**

P (A fails) = 0.2

P (B fails alone) = 0.15

P (A and B fails) = 0.15

Evaluate the following probabilities

(i) P (A fails | B has failed)

Ans: Let us have following notations

E: A fails

F: B fails

Given in the question

$$P(E) = 0.2$$

$$P(E \cap F) = 0.15$$

$$P(E' \cap F) = 0.15$$

We know that

$$P(E' \cap F) = P(E) - P(E \cap F)$$

$$\Rightarrow P(E) = 0.3$$

Now the probability that A fails given B has failed is given by

$$P(E|F) = \frac{0.15}{0.3}$$

Hence the probability that A fails given B has failed is 0.5

(ii) P (A fails alone)

Ans: Probability that A fails alone is given by

$$P(E \cap F') = P(F) - P(E \cap F)$$

$$P(E \cap F') = 0.05$$

Hence Probability that A fails alone is 0.05

10. Bag I contains 3 red and 4 black balls and Bag II contains 4 red and 5 black balls. One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II. The ball so drawn is found to be red in colour. Find the probability that the transferred ball is black.

Ans: Let us have following notations that

E: red ball is transferred

$$P(E)=\frac{3}{7}$$

F: black ball is transferred

$$P(F)=\frac{4}{7}$$

G: red ball is drawn

When a red ball is transferred

$$P(G|E)=\frac{5}{10}$$

Similarly, When a black ball is transferred

$$P(G|F)=\frac{4}{10}$$

$$P(F|G)=\frac{\frac{4}{7}\times\frac{4}{10}}{\frac{4}{7}\times\frac{4}{10}+\frac{3}{7}\times\frac{5}{10}}$$

Hence probability that the transferred ball is black is $\frac{16}{31}$

11. If A and B are two events such that P(A) ≠ 0 and P(B | A) = 1, then

Ans: The correct option is **A** $A\subset B$

$P(A)\neq O$ and $P(B|A)=1$

$$P(B|A)=\frac{P(B\cap A)}{P(A)}$$

$$1=\frac{P(B\cap A)}{P(A)}$$

$$P(A)=P(B\cap A)$$

Hence $A \subset B$
Thus, the correct answer is A.

12. If $P(A|B) > P(A)$), then what is correct?

Ans: Given $P(A|B) > P(A)$

And we know that

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow \frac{P(A \cap B)}{P(B)} > P(A)$$

Hence we found that $P(B|A) > P(B)$

13. If A and B are any two events such that $P(A) + P(B) - P(A \text{ and } B) = P(A)$ then what result can follow?

Ans: Given in the question we have

$$P(A) + P(B) - P(A \cap B) = P(A)$$

$$\Rightarrow P(B) = P(A \cap B)$$

Hence

$$P(A|B) = 1$$