linear programming

12
Chapter

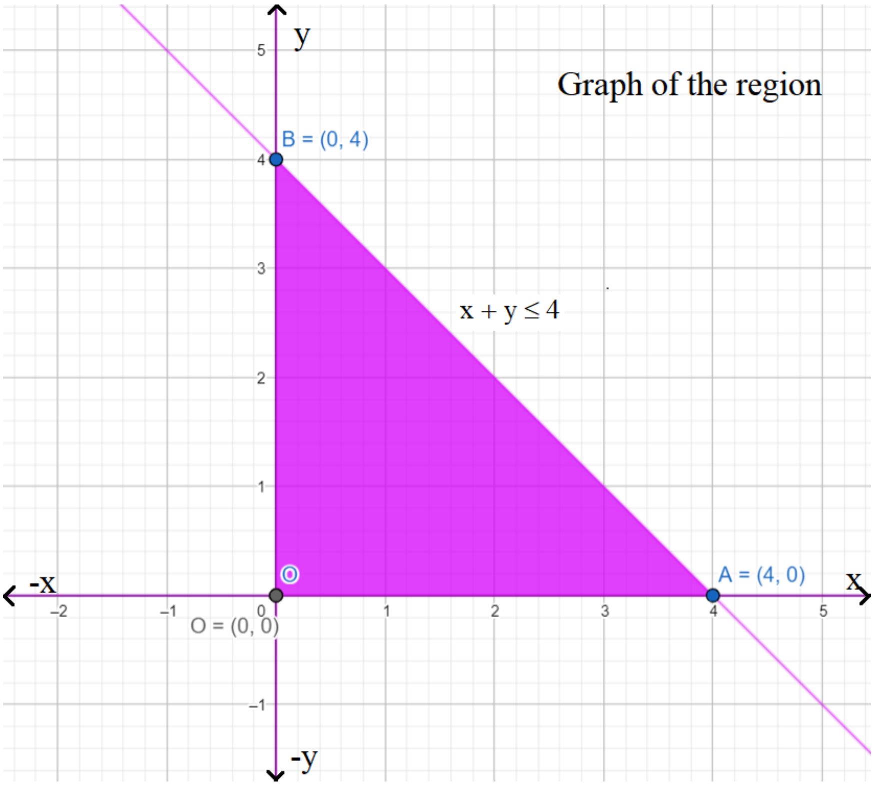
Exercise 12.1

1. **Maximize Z= 3x+ 4y**

Subject to the constraints: $x+y\le 4$, $x\ge 0$, $y\ge 0$

Ans:

The given constraints are, $x+y\le 4$, $x\ge 0$, $y\ge 0$, and the feasible region which is in accordance with the given constraints is



The points at the corners in the feasible region are O (0, 0), A (4, 0), and B (0, 4). Z assumes the following values on these points.

Corner point	$Z = 3x + 4y$	
O(0,0)	0	
A(4,0)	12	
B(0,4)	16	$\rightarrow$Maximum

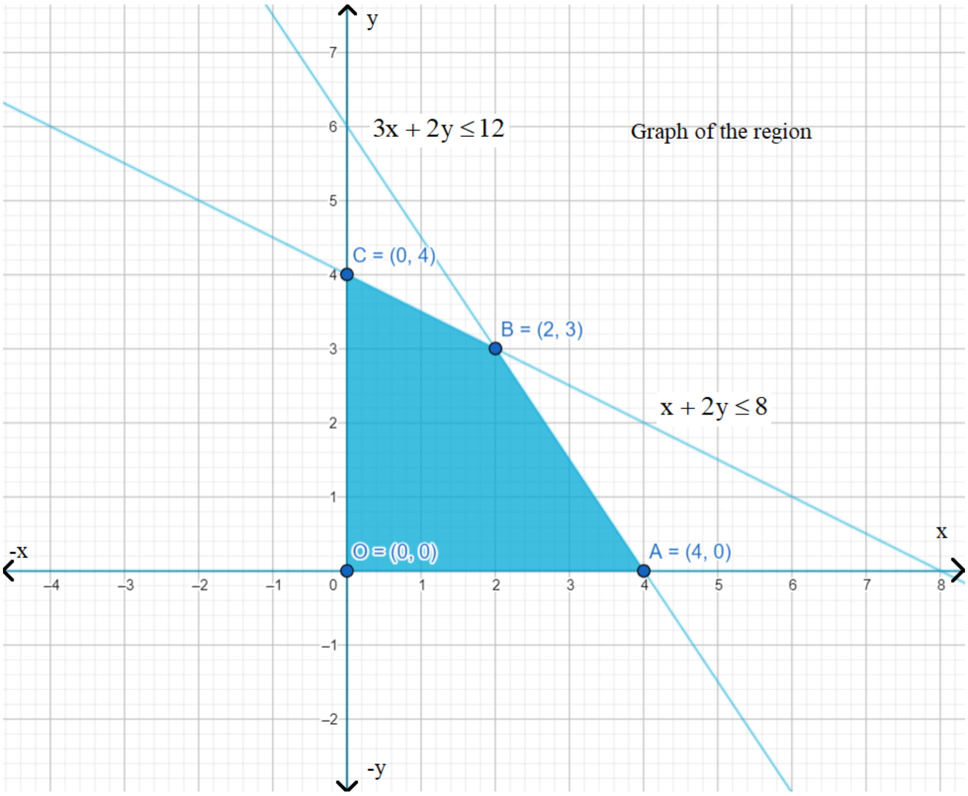
Table of values

Thus, the maximum value of Z is 16 at the point B (0,4).

2. **Minimize $Z = -3x + 4y$ subject to $x + 2y \le 8$, $3x + 2y \le 12$, $x \ge 0$, $y \ge 0$**

Ans:

The given constraints are, $x + 2y \le 8$, $3x + 2y \le 12$, $x \ge 0$ and $y \ge 0$, and the feasible region which is in accordance with the given constraints is



The points at the corners in the feasible region are O (0, 0), A (4, 0), B (2, 3), and C (0, 4). Z assumes the following values on these points.

Corner point	$Z = -3x + 4y$	
O (0, 0)	0	
A (4, 0)	-12	$\rightarrow$Minimum
B (2, 3)	6	
C (0,4)	16	

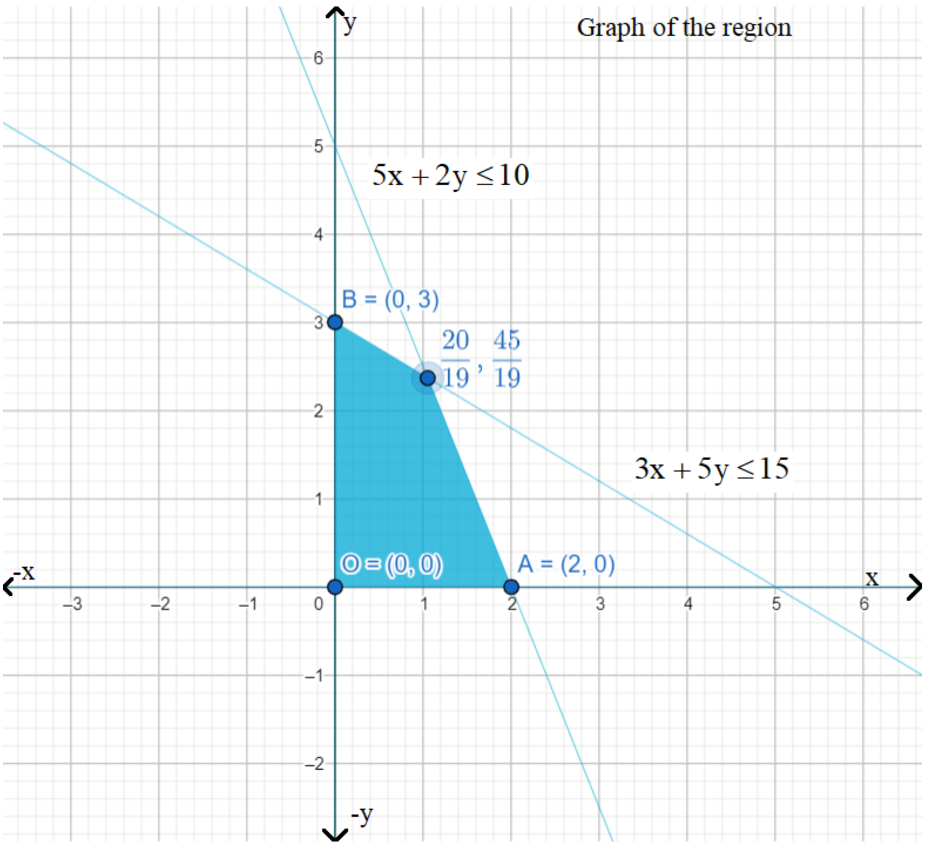
Table of values

Thus, the minimum value of Z is -12 at the point (4, 0).

3. **Maximize $Z = 5x + 3y$ subject to $3x + 5y \le 15$, $5x + 2y \le 10$, $x \ge 0$, $y \ge 0$.**

Ans:

The given constraints are, $3x + 5y \le 15$, $5x + 2y \le 10$, $x \ge 0$, and $y \ge 0$, and the feasible region which is in accordance with the given constraints is



The points at the corners in the feasible region are O (0, 0), A (2, 0), B (0, 3), and $C\left(\frac{20}{19},\frac{45}{19}\right)$. Z assumes the following values on these points.

Corner point	$\mathbf{Z = 5x + 3y}$	
O (0, 0)	0	
A (2, 0)	10	
B (0, 3)	9	
$C\left(\frac{20}{19},\frac{45}{19}\right)$	$\frac{235}{19}$	$\rightarrow$ Maximum

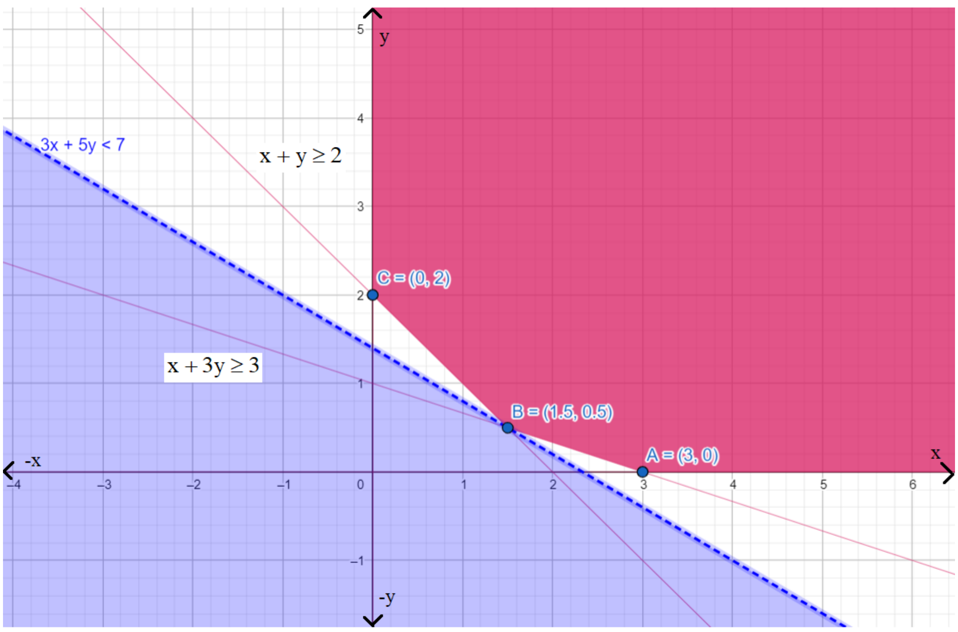
Table of values

Thus, the maximum value of Z is $\frac{235}{19}$ at the point $\left(\frac{20}{19},\frac{45}{19}\right)$.

4. **Minimize $Z = 3x + 5y$ such that $x + 3y \geq 3$, $x + y \geq 2$, $x, y \geq 0$**

Ans:

The given constraints are, $x + 3y \geq 3$, $x + y \geq 2$, and $x, y \geq 0$, and the feasible region which is in accordance with the given constraints is



Graph of the region

We can see that the feasible region is not bounded.

The points at the corners in the feasible region are A (3, 0), $B\left(\frac{3}{2},\frac{1}{2}\right)$, and C (0, 2). Z assumes the following values on these points.

Corner point	$\mathbf{Z = 3x + 5y}$	
A (3,0)	9	
$B\left(\frac{3}{2},\frac{1}{2}\right)$	7	$\rightarrow$ Smallest
C (0,2)	10	

Table of values

Since the feasible region is unbounded, we cannot be sure that 7 is the minimum value of Z. To confirm this, we need to sketch the graph of the inequality, $3x+5y<7$, and see if the resulting plane has any point in common with the feasible region.

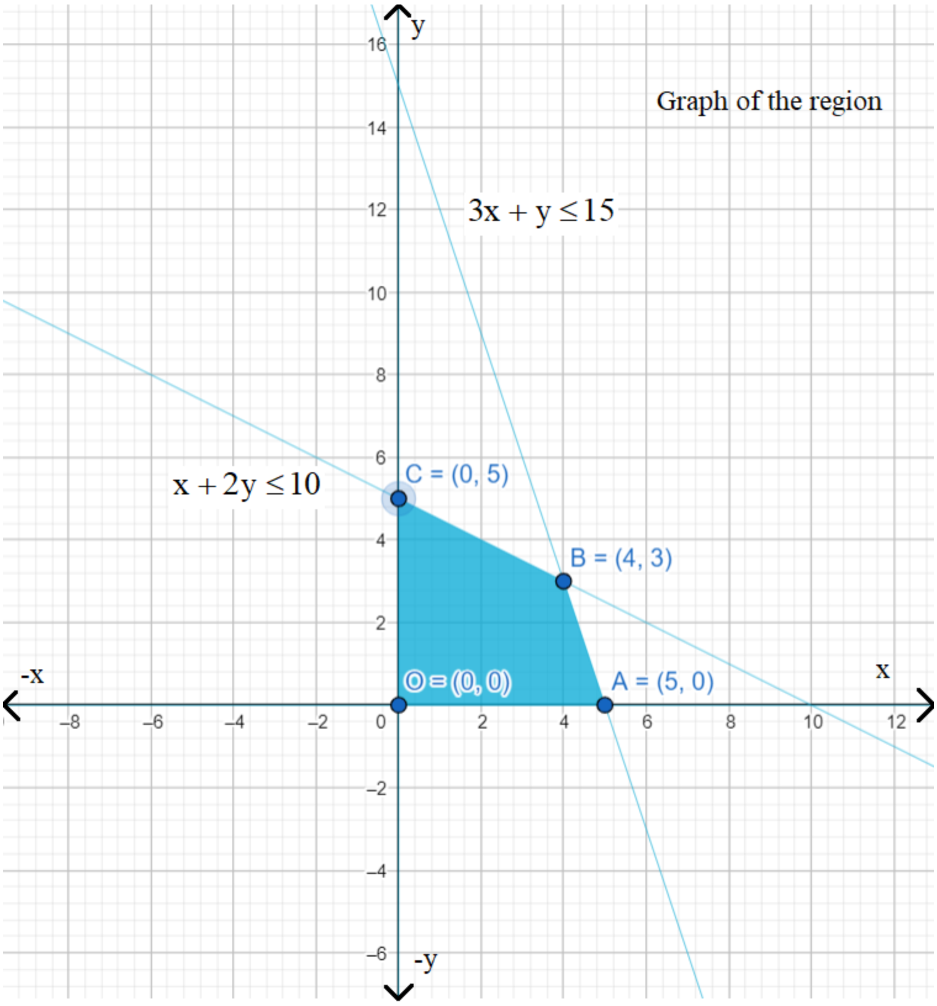
From the graph that we sketched, we can see that there is no common point between feasible regions and the sketched inequality $3x+5y<7$.

Z achieves minimum value 7 at $\left(\frac{3}{2},\frac{1}{2}\right)$.

5. **Maximize $\mathbf{Z = 3x + 2y}$ subject to $\mathbf{x + 2y \le 10}$, $\mathbf{3x + y \le 15}$, $\mathbf{x, y \ge 0}$.**

Ans:

The given constraints are, $x+2y \le 10$, $3x+y \le 15$, $x \ge 0$, and $y \ge 0$, and the feasible region which is in accordance with the given constraints is



The points at the corners in the feasible region are A (5, 0), B (4, 3), and C (0, 5).
Z assumes the following values on these points.

Corner point	$\mathbf{Z = 3x + 2y}$	
A (5,0)	15	
B (4,3)	18	$\rightarrow$Maximum
C (0,5)	10	

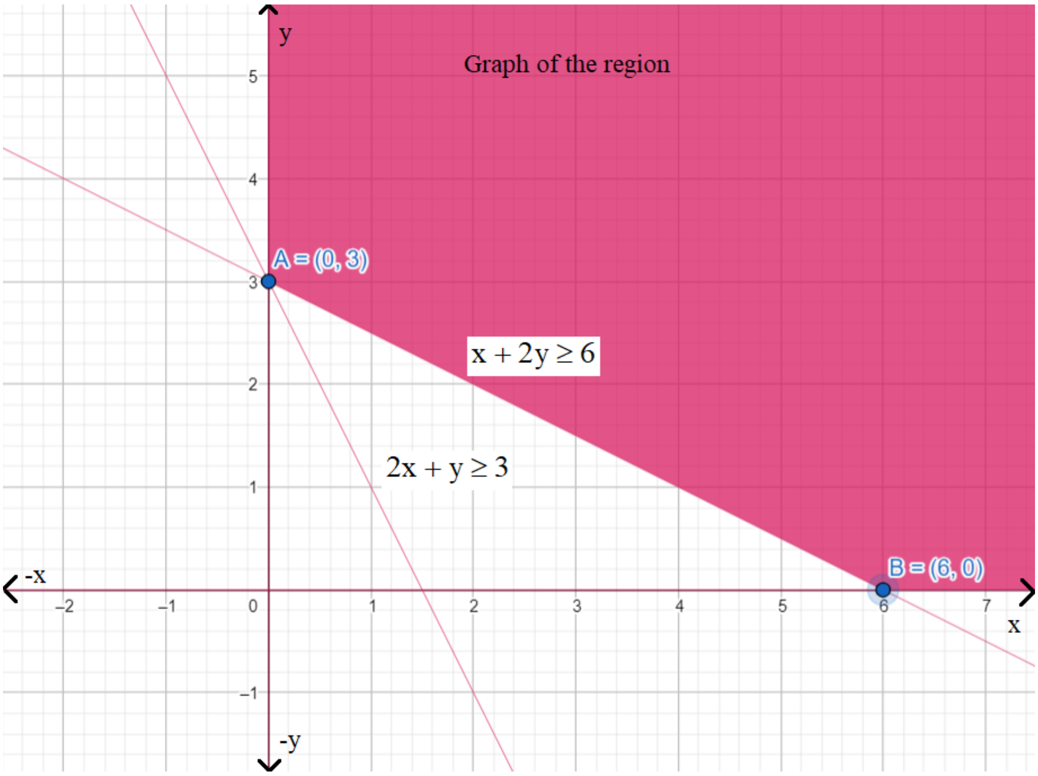
Table of values

Z achieves maximum value 18 at (4,3).

6. **Minimize $Z = x + 2y$ subject to $2x + y \ge 3$, $x + 2y \ge 6$, $x, y \ge 0$.**

Ans:

The given constraints are, $2x + y \ge 3$, $x + 2y \ge 6$, $x \ge 0$, and $y \ge 0$, and the feasible region which is in accordance with the given constraints is



The points at the corners in the feasible region are A (6, 0) and B (0, 3). Z assumes the following values on these points.

Corner point	$Z = x + 2y$	
A (6,0)	6	
B (0,3)	6	

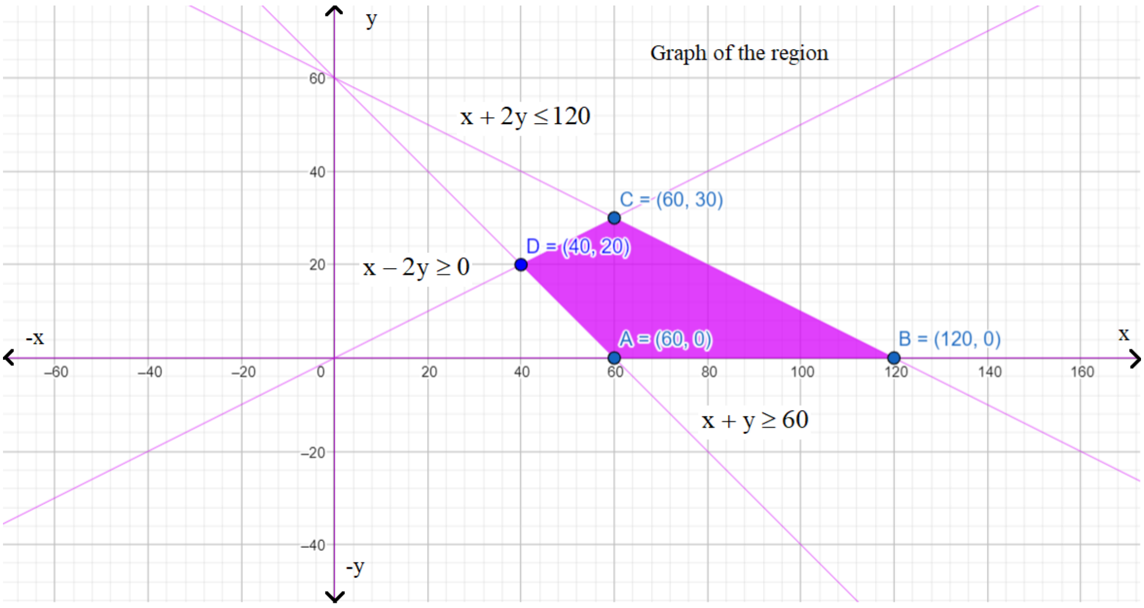
Table of values

We can see that the value of Z is the same on both A and B hence, we will need to check on the other point on the line $x + 2y = 6$ as well. The value of Z is 6 at point (2,2) also. Hence, the minimum value of Z Occurs at more than two points Thus, the value of Z is minimum at every point on the line, $x + 2y = 6$

7. **Minimize and Maximize $Z = 5x + 10y$ subject to $x + 2y \le 120$, $x + y \ge 60$, $x - 2y \ge 0$, $x, y \ge 0$.**

Ans:

The given constraints are, $x + 2y \le 120$, $x + y \ge 60$, $x - 2y \ge 0$, $x \ge 0$, and $y \ge 0$, and the feasible region which is in accordance with the given constraints is



The points at the corners in the feasible region are A (60, 0), B (120, 0), C (60, 30), and D (40, 20). Z assumes the following values on these points.

Corner point	$\mathbf{Z = 5x + 10y}$	
A (60,0)	300	$\rightarrow$Minimum
B (120,0)	600	$\rightarrow$Maximum
C (60,30)	600	$\rightarrow$Maximum
D (40,20)	400	

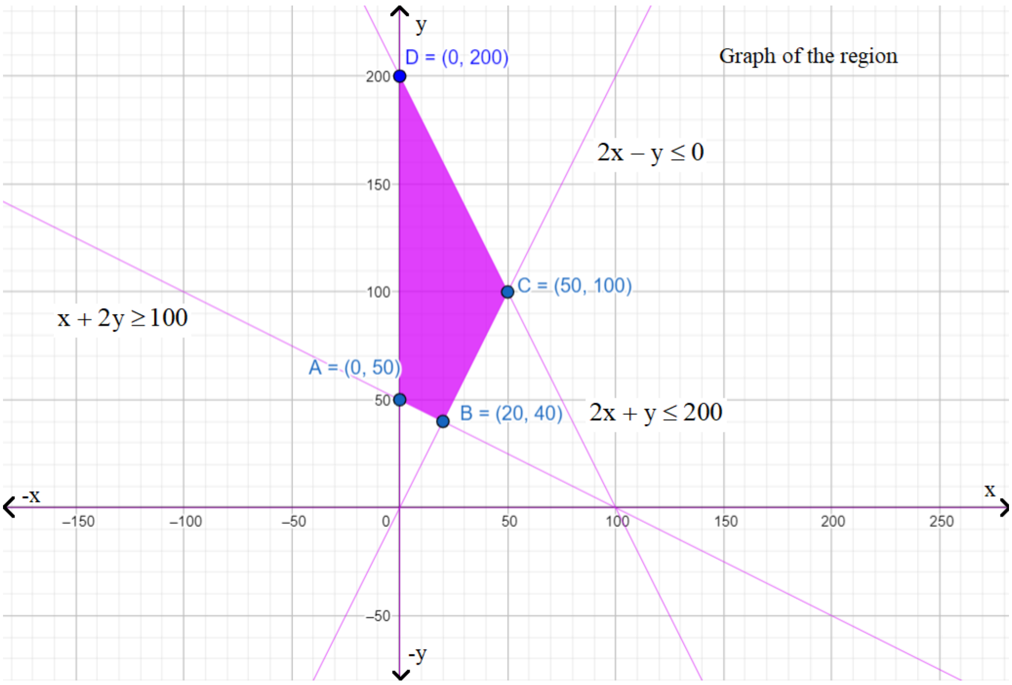
Table of values

Z achieves maximum and minimum values as 600 and 300 respectively. The point of maximum value is all the points on the line segment joining (120, 0) and (60, 30) and minimum value is (60,0).

8. **Minimize and Maximize $Z = x + 2y$ subject to $x + 2y \geq 100$, $2x - y \leq 0$, $2x + y \leq 200$, $x, y \geq 0$.**

Ans:

The given constraints are, $x + 2y \geq 100$, $2x - y \leq 0$, $2x + y \leq 200$, $x \geq 0$, and $y \geq 0$, and the feasible region which is in accordance with the given constraints is



The points at the corners in the feasible region are A(0, 50), B(20, 40), C(50, 100), and D(0, 200). Z assumes the following values on these points.

Corner point	$Z = x + 2y$	
A (0,50)	100	$\rightarrow$Minimum
B (20,40)	100	$\rightarrow$Minimum
C (50,100)	250	
D (0,200)	400	$\rightarrow$Maximum

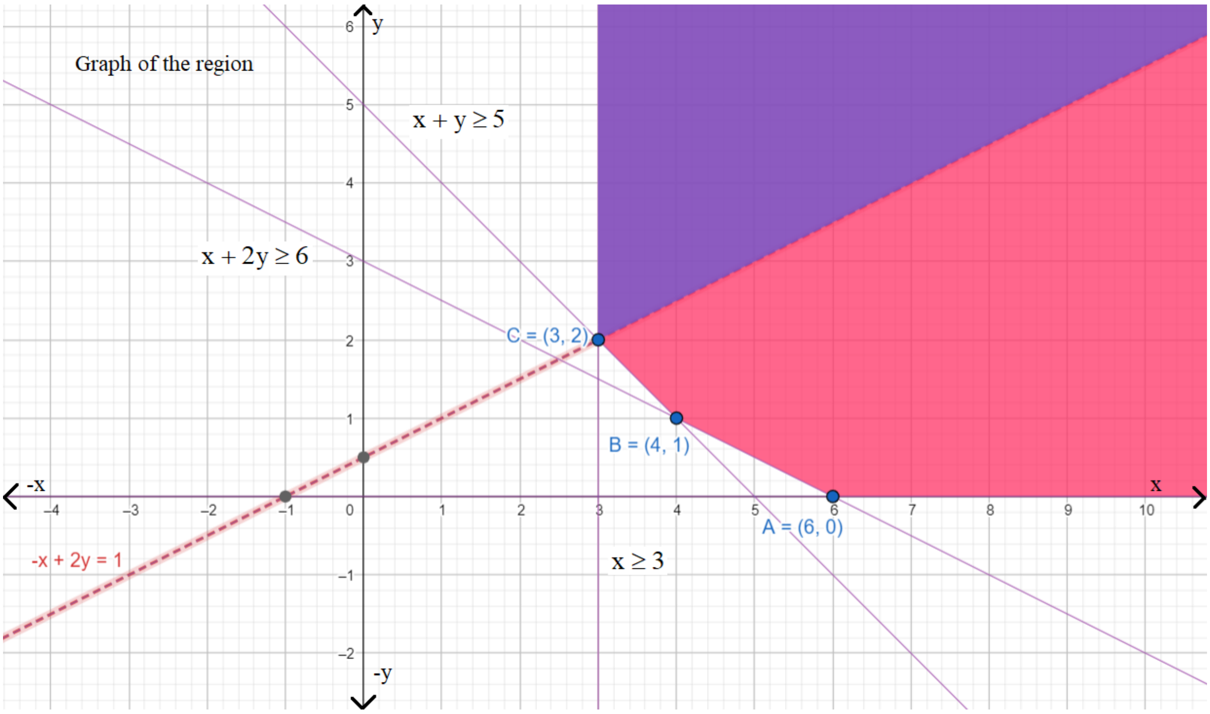
Table of values

Z achieves maximum and minimum values as 400 and 100 respectively. The point of maximum value is (0,200) and minimum value is all points on the line joining the points (0, 50) and (20, 40).

9. **Maximize $Z = -x + 2y$, subject to the constraints: $x \geq 3$, $x + y \geq 5$, $x + 2y \geq 6$, $y \geq 0$.**

Ans:

The given constraints are, $x \geq 3$, $x + y \geq 5$, $x + 2y \geq 6$, and $y \geq 0$ and the feasible region which is in accordance with the given constraints is



We can see that the feasible region is not bounded.
The points at the corners in the feasible region are A (6, 0), B (4, 1), and C (3, 2) . Z assumes the following values on these points.

Corner point	$Z = -x + 2y$
A (6,0)	Z = -6
B (4,1)	Z = -2
C (3,2)	Z = 1

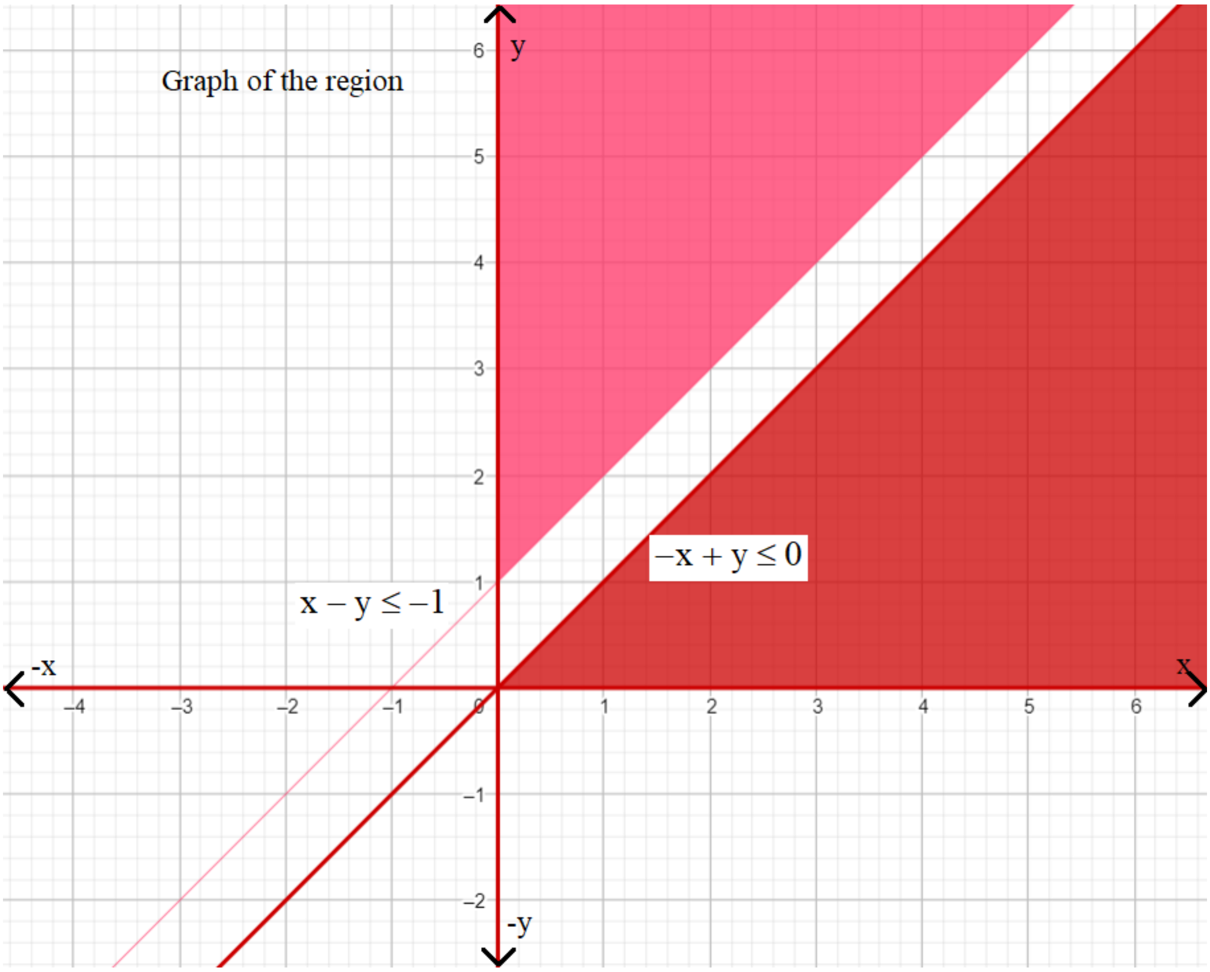
Table of values

Since the feasible region is unbounded, we cannot be sure that 1 is the maximum value of Z. To confirm this, we need to sketch the graph of the inequality, $-x + 2y > 1$, and see if the resulting plane has any point in common with the feasible region.
From the graph that we sketched, we can see that there are common points between feasible regions and the sketched inequality. Thus, Z = 1 is not the maximum value. Z has no maximum value.

10. **Maximize $Z = x + y$, subject to $x - y \leq -1$, $-x + y \leq 0$, $x, y \geq 0$.**

Ans:

The given constraints are, $x - y \leq -1$, $-x + y \leq 0$, $x, y \geq 0$ and the feasible region which is in accordance with the given constraints is



We can see from the graph that there is no feasible region, hence Z has no maximum value.