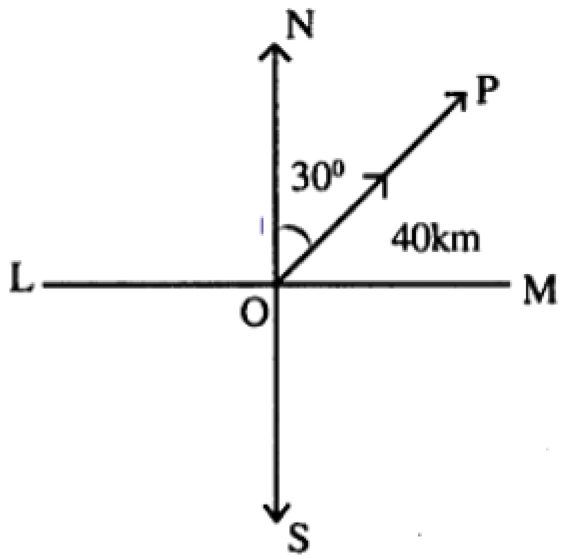
vector alegbra

10
Chapter

Exercise 10.1

1. Represent graphically a displacement of $40\,\text{km}, 30^\circ$ east of north.

Ans:



Here, vector OP is representing the displacement of $40\,\text{km}, 30^\circ$ $10\,\text{m}$ in East of North direction. $20\,\text{m/s}^2$

2. Classify the following measures as scalars and vectors.

(i) $10\,\text{kg}$ (ii) 2 meters north-west (iii) 40 (iv) 40 watt (v) 10^{-9} coulomb (vi)

Ans: (i) $10\,\text{kg}$ is a scalar quantity because it has only magnitude not direction.

(ii) 2 meters north-west is a vector quantity because it has both magnitude as well as direction.

(iii) 10^{-19} is a scalar quantity because it has only magnitude not direction.

(iv) 40 watts is a scalar quantity because it has only magnitude not direction.

(v) 40° Coulomb is a scalar quantity because it has only magnitude not direction.

(vi) $20\,m/s^2$ is a vector quantity because it has both magnitude as well as direction.

3. Classify the following as scalar and vector quantities:-

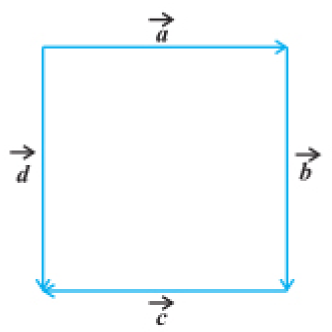
(i) Time period (ii) distance (iii) force (iv) velocity (v) work done

Ans: 3

(i) Time period is a scalar quantity because it has only magnitude.
(ii) Distance is a scalar quantity because it has only magnitude.
(iii) Force is a vector quantity because it has both magnitude as well as direction.
(iv) Velocity is a vector quantity because it has both magnitude as well as direction.
(v) Work done is a scalar quantity because it has only magnitude.

4. In Figure, identify the following vectors.

(i) Co-initial (ii) Equal (iii) Collinear but not equal



Ans:

(i) Vectors $\vec{a}$ and $\vec{d}$ are co-initial.

(ii) Vectors $\vec{b}$ and $\vec{d}$ are equal.

(iii) Vectors $\vec{a}$ and $\vec{c}$ are collinear but not equal.

5. Answer the following as true or false.

(i) $\vec{a}$ **and** $-\vec{a}$ **and are collinear.**

(ii) Two collinear vectors are always equal in magnitude.

(iii) Two vectors having same magnitude are collinear.

(iv) Two collinear vectors having the same magnitude are equal.

Ans: (i) True

(ii) False

(iii) False

(iv) False

Exercise 10.2

1. Compute the magnitude of the following vectors:-

$$\vec{a} = \hat{i} + \hat{j} + \hat{k} \,;\, \vec{b} = 2\hat{i} - 7\hat{j} - 3\hat{k} \,;\, \vec{c} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} - \frac{1}{\sqrt{3}}\hat{k}$$

Ans:

$$|\vec{a}| = \sqrt{(1)^2 + (1)^2 + (1)^2} = \sqrt{3}$$

$$|\vec{b}| = \sqrt{(2)^2 + (-7)^2 + (-3)^2}$$

$$= \sqrt{4 + 49 + 9}$$

$$= \sqrt{62}$$

$$|\vec{c}| = \sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2 - \left(\frac{1}{\sqrt{3}}\right)^2}$$

$$= \sqrt{\frac{1}{3} + \frac{1}{3} + \frac{1}{3}} = 1$$

2. Write two different vectors having same magnitude.

Ans:

$\vec{a} = (\hat{i} - 2\hat{j} + 3\hat{k})$ and $\vec{b} = (2\hat{i} + \hat{j} - 3\hat{k})$.

$$|\vec{a}| = \sqrt{1^2 + (-2)^2 + 3^2} = \sqrt{1+4+9} = \sqrt{14}$$

$$|\vec{b}| = \sqrt{2^2 + 1^2 + (-3)^2} = \sqrt{4+1+9} = \sqrt{14}$$

But $\vec{a} \neq \vec{b}$

3. Write two different vectors having same direction.

Ans: $p = (\hat{i} + \hat{j} + \hat{k})$ and $q = (2\hat{i} + 2\hat{j} + 2\hat{k})$.

The Direction Cosines of $\vec{p}$ are

$$a = \frac{1}{\sqrt{1^2 + 1^2 + 1^2}} = \frac{1}{\sqrt{3}}, b = \frac{1}{\sqrt{1^2 + 1^2 + 1^2}} = \frac{1}{\sqrt{3}}, c = \frac{1}{\sqrt{1^2 + 1^2 + 1^2}} = \frac{1}{\sqrt{3}}$$

The Direction Cosines of $\vec{q}$ are

$$a = \frac{2}{\sqrt{2^2 + 2^2 + 2^2}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}, b = \frac{2}{\sqrt{2^2 + 2^2 + 2^2}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}, c = \frac{2}{\sqrt{2^2 + 2^2 + 2^2}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}$$

The Direction Cosines of $\vec{p}$ and $\vec{q}$ are equal but $\vec{p} \neq \vec{q}$.

4. Find the values of x and y so that the vectors $2\hat{i} + 3\hat{j}$ and $x\hat{i} + y\hat{j}$ are equal.

Ans: $2\hat{i} + 3\hat{j} = x\hat{i} + y\hat{j} \Rightarrow x = 2, y = 3$

5. **Find the scalar and vector components of the vector with initial point** $(2,1)$ **and terminal point** $(-5,7)$**.**

Ans: Let the points be $P(2,1)$ and $Q(-5,7)$

$$\overrightarrow{PQ} = (-5-2)\hat{i} + (7-1)\hat{j}$$

$$\Rightarrow \overrightarrow{PQ} = -7\hat{i} + 6\hat{j}$$

So, scalar components of required vector are -7 and 6 and the vector components are $-7\hat{i}$ and $6\hat{j}$.

6. **Find the sum of the vectors** $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}$ **and** $\vec{c} = \hat{i} - 6\hat{j} - 7\hat{k}$**.**

Ans: $\vec{a} + \vec{b} + \vec{c} = (1-2+1)\hat{i} + (-2+4-6)\hat{j} + (1+5-7)\hat{k} = 0\hat{i} - 4\hat{j} - 1\hat{k} = -4\hat{j} - \hat{k}$

7. **Find the unit vector in the direction of the vector** $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$**.**

Ans: $|a| = \sqrt{1^2 + 1^2 + 2^2} = \sqrt{1+1+4} = \sqrt{6}$

$$\therefore \hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{6}} = \frac{1}{\sqrt{6}}\hat{i} + \frac{1}{\sqrt{6}}\hat{j} + \frac{2}{\sqrt{6}}\hat{k}$$

8. **Find the unit vector in the direction of vector** $\overrightarrow{PQ}$**, where** P **and** Q **are the points** $(1,2,3)$ **and** $(4,5,6)$**, respectively.**

Ans: $\overrightarrow{PQ} = (4-1)\hat{i} + (5-2)\hat{j} + (6-3)k = 3\hat{i} + 3\hat{j} + 3k$

$$|\overrightarrow{PQ}| = \sqrt{3^2 + 3^2 + 3^2} = \sqrt{9+9+9} = \sqrt{27} = 3\sqrt{3}$$

So, unit vector $= \frac{\overrightarrow{PQ}}{|\overrightarrow{PQ}|} = \frac{3\hat{i} + 3\hat{j} + 3\hat{k}}{3\sqrt{3}} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$

9. **For given vectors, $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} - \hat{k}$, find the unit vector in the direction of the vector $\vec{a} + \vec{b}$.**

Ans: $\vec{a} + \vec{b} = (2-1)\hat{i} + (-1+1)\hat{j} + (2-1)\hat{k} = 1\hat{i} + 0\hat{j} + 1\hat{k} = \hat{i} + \hat{k}$

$|\vec{a} + \vec{b}| = \sqrt{1^2 + 1^2} = \sqrt{2}$

So, unit vector $= \dfrac{(\vec{a} + \vec{b})}{|\vec{a} + \vec{b}|} = \dfrac{\hat{i} + \hat{k}}{\sqrt{2}} = \dfrac{1}{\sqrt{2}}\hat{i} + \dfrac{1}{\sqrt{2}}\hat{k}$

10. **Find a vector in the direction of vector $5\hat{i} - \hat{j} + 2\hat{k}$ which has magnitude 8 units.**

Ans: $\vec{a} = 5\hat{i} - \hat{j} + 2\hat{k}$

$|\vec{a}| = \sqrt{5^2 + (-1)^2 + 2^2} = \sqrt{25 + 1 + 4} = \sqrt{30}$

$\therefore \hat{a} = \dfrac{\vec{a}}{|\vec{a}|} = \dfrac{5\hat{i} - \hat{j} + 2\hat{k}}{\sqrt{30}}$

So, a vector in direction of $5\hat{i} - \hat{j} + 2\hat{k}$ with magnitude 8 units is: $8\hat{a} = 8\left(\dfrac{5\hat{i} - \hat{j} + 2\hat{k}}{\sqrt{30}}\right) = \dfrac{40}{\sqrt{30}}\hat{i} - \dfrac{8}{\sqrt{30}}\hat{j} + \dfrac{16}{\sqrt{30}}\hat{k}$

11. **Show that the vectors $2\hat{i} - 3\hat{j} + 4\hat{k}$ and $-4\hat{i} + 6\hat{j} - 8\hat{k}$ are collinear.**

Ans: $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ and $\vec{b} = -4\hat{i} + 6\hat{j} - 8\hat{k}$

$\vec{b} = -4\hat{i} + 6\hat{j} - 8\hat{k} = -2(2\hat{i} - 3\hat{j} + 4\hat{k}) = -2\vec{a}$

$\therefore \vec{b} = \lambda\vec{a},\ \lambda = -2$

So, the given vectors are collinear.

12. **Find the direction cosines of the vector $\hat{i}+2\hat{j}+3\hat{k}$.**

Ans: $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$.

$|\vec{a}|=\sqrt{1^2+2^2+3^2}=\sqrt{1+4+9}=\sqrt{14}$

So, the Direction cosines of $\vec{a}$ are $\left(\frac{1}{\sqrt{14}},\frac{2}{\sqrt{14}},\frac{3}{\sqrt{14}}\right)$

13. **Find the direction cosines of the vector joining the points $A(1,2,-3)$ and $B(-1,-2,1)$ directed from A to B.**

Ans: $\overrightarrow{AB}=(-1-1)\hat{i}+(-2-2)\hat{j}+\{1-(-3)\}\hat{k}$

$\Rightarrow \overrightarrow{AB}=-2\hat{i}-4\hat{j}+4\hat{k}$

$|\overrightarrow{AB}|=\sqrt{(-2)^2+(-4)^2+4^2}=\sqrt{4+16+16}=\sqrt{36}=6$

So, the Direction cosines of $\overrightarrow{AB}$ are $\left(-\frac{2}{6},-\frac{4}{6},\frac{4}{6}\right)=\left(-\frac{1}{3},-\frac{2}{3},\frac{2}{3}\right)$

14. **Show that the vector $\hat{i}+\hat{j}+\hat{k}$ is equally inclined to the axes OX, OY and OZ .**

Ans: $a=\hat{i}+\vec{\hat{j}}+\hat{k}$

$|\vec{a}|=\sqrt{1^2+1^2+1^2}=\sqrt{3}$

So, the Direction Cosines of $\vec{a}$ are $\left(\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right)$

$\cos\alpha=\frac{1}{\sqrt{3}},\cos\beta=\frac{1}{\sqrt{3}},\cos\gamma=\frac{1}{\sqrt{3}}$

So, the vector is equally inclined to OX, OY, and OZ.

15. Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $\hat{i}+2\hat{j}-\hat{k}$ and $-\hat{i}+\hat{j}+\hat{k}$ respectively, in the ratio 2 : 1 ,

(i) Internally

(ii) Externally.

Ans: $\overrightarrow{OP}=\hat{i}+2\hat{j}-\hat{k}$ and $\overrightarrow{OQ}=-\hat{i}+\hat{j}+\hat{k}$

(i) The position vector of R is

$$\overrightarrow{OR}=\frac{2(-\hat{i}+\hat{j}+\hat{k})+1(\hat{i}+2\hat{j}-\hat{k})}{2+1}=\frac{(-2\hat{i}+2\hat{j}+2\hat{k})+(\hat{i}+2\hat{j}-\hat{k})}{3}$$

$$=\frac{-\hat{i}+4\hat{j}+\hat{k}}{3}=-\frac{1}{3}\hat{i}+\frac{4}{3}\hat{j}+\frac{1}{3}\hat{k}$$

(ii) The position vector of R is

$$\overrightarrow{OR}=\frac{2(-\hat{i}+\hat{j}+\hat{k})-1(\hat{i}+2\hat{j}-\hat{k})}{2-1}=(-2\hat{i}+2\hat{j}+2\hat{k})-(\hat{i}+2\hat{j}-\hat{k})$$

$$=-3\hat{i}+3\hat{k}$$

16. Find the position vector of the mid point of the vector joining the points P(2,3,4) and Q(4,1,-2).

Ans: The position vector of R is

$$\overrightarrow{OR}=\frac{(2\hat{i}+3\hat{j}+4\hat{k})+(4\hat{i}+\hat{j}-2\hat{k})}{2}=\frac{(2+4)\hat{i}+(3+1)\hat{j}+(4-2)\hat{k}}{2}$$

$$=\frac{6\hat{i}+4\hat{j}+2\hat{k}}{2}=3\hat{i}+2\hat{j}+\hat{k}$$

17. Show that the points A, B and C with position vectors, $\vec{a}=3\hat{i}-4\hat{j}-4\hat{k}$, $\vec{b}=2\hat{i}-\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}-3\hat{j}-5\hat{k}$, respectively form the vertices of a right angled triangle.

Ans:

$$\overrightarrow{AB} = \vec{b} - \vec{a} = (2-3)\hat{i} + (-1+4)\hat{j} + (1+4)\hat{k} = -\hat{i} + 3\hat{j} + 5\hat{k}$$

$$\overrightarrow{BC} = \vec{c} - \vec{b} = (1-2)\hat{i} + (-3+1)\hat{j} + (-5-1)\hat{k} = -\hat{i} + 2\hat{j} - 6\hat{k}$$

$$\overrightarrow{CA} = \vec{a} - \vec{c} = (3-1)\hat{i} + (-4+3)\hat{j} + (-4+5)\hat{k} = 2\hat{i} - \hat{j} + \hat{k}$$

$$|\overrightarrow{AB}|^2 = (-1)^2 + 3^2 + 5^2 = 1 + 9 + 25 = 35$$

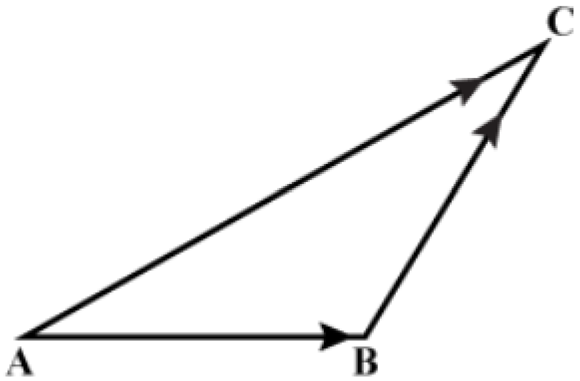
$$|\overrightarrow{BC}|^2 = (-1)^2 + (-2)^2 + (-6)^2 = 1 + 4 + 36 = 41$$

$$|\overrightarrow{CA}|^2 = 2^2 + (-1)^2 + 1^2 = 4 + 1 + 1 = 6$$

$$|\overrightarrow{AB}|^2 + |\overrightarrow{CA}|^2 = 35 + 6 = 41 = |\overrightarrow{BC}|^2$$

So, ABC is a right angled triangle.

18. In triangle ABC, which of the following is not true?



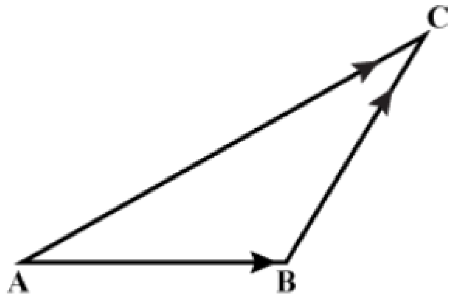
A. $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$

B. $\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{AC} = \vec{0}$

C. $\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{CA} = \vec{0}$

D. $\overrightarrow{AB} - \overrightarrow{CB} + \overrightarrow{CA} = \vec{0}$

Ans:



$\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}$

$\Rightarrow \overrightarrow{AB}+\overrightarrow{BC}=-\overrightarrow{CA}$

$\Rightarrow \overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}=\vec{0}$

$\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}$

$\Rightarrow \overrightarrow{AB}+\overrightarrow{BC}-\overrightarrow{AC}=\vec{0}$

$\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}=\vec{0}$

$\overrightarrow{AB}-\overrightarrow{CB}+\overrightarrow{CA}=\vec{0}$

If $\overrightarrow{AB}+\overrightarrow{BC}-\overrightarrow{CA}=\vec{0}$

$\overrightarrow{AC}=\overrightarrow{CA}$

$\Rightarrow \overrightarrow{AC}=-\overrightarrow{AC}$

$\Rightarrow \overrightarrow{AC}+\overrightarrow{AC}=\vec{0}$

$\Rightarrow 2\overrightarrow{AC}=\vec{0}$

$\Rightarrow \overrightarrow{AC}=\vec{0}$**,** which is not true.

So, the equation given in option C is False.

Hence, the correct answer is C.

19. If $\vec{a}$ and $\vec{b}$ are two collinear vectors, then which of the following are incorrect?

A. $\vec{b}=\lambda\vec{a}$**, for some scalar** λ

B. $a = \pm b$

C. the respective components of $\vec{a}$ and $\vec{b}$ are proportional

D. both the vectors $\vec{a}$ and $\vec{b}$ have same direction, but different magnitudes

Ans: If $\vec{a}$ and $\vec{b}$ are collinear vectors, they are parallel. $\vec{b} = \lambda \vec{a}$(scalar λ)

If $\lambda = \pm 1$, then $\vec{a} = \pm \vec{b}$

If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$,

$$\vec{b} = \lambda\vec{a} \Rightarrow b_1\hat{i} + b_2\hat{j} + b_3\hat{k} = \lambda\left(a\hat{i} + a_2\hat{j} + a_3\hat{k}\right)$$

$$\Rightarrow b_1\hat{i} + b_2\hat{j} + b_3\hat{k} = (\lambda a_1)\hat{i} + (\lambda a_2)\hat{j} + (\lambda a_3)\hat{k}$$

$$\Rightarrow b_1 = \lambda a_1, b_2 = \lambda a_2, b_3 = \lambda a_3$$

$$\Rightarrow \frac{b_1}{a_1} = \frac{b_2}{a_2} = \frac{b_3}{a_3} = \lambda$$

Thus, respective components of $\vec{a}$ and $\vec{b}$ are proportional.

But, $\vec{a}$ and $\vec{b}$ may have different directions. So, option D is incorrect. The correct answer is D.

Exercise 10.3

1. Find the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes $\sqrt{3}$ and 2, respectively having $\vec{a} \cdot \vec{b} = \sqrt{6}$

Ans: $|\vec{a}| = \sqrt{3}, |\vec{b}| = 2, \vec{a}.\vec{b} = \sqrt{6}$

$$\therefore \sqrt{6} = \sqrt{3} \times 2 \times \cos\theta$$

$$\Rightarrow \cos\theta = \frac{\sqrt{6}}{\sqrt{3} \times 2}$$

$$\Rightarrow \cos\theta = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \theta = \frac{\pi}{4}$$

2. Find the angle between the vectors $i - 2\hat{j} + 3\hat{k}$ **and** $3\hat{i} - 2\hat{j} + \hat{k}$**.**

Ans: $|\vec{a}| = \sqrt{1^2 + (-2)^2 + 3^2} = \sqrt{1+4+9} = \sqrt{14}$

$|\vec{b}| = \sqrt{3^2 + (-2)^2 + 1^2} = \sqrt{9+4+1} = \sqrt{14}$

$\vec{a}\,.\,\vec{b} = (i - 2\hat{j} + 3\hat{k})(3\hat{i} - 2\hat{j} + \hat{k})$

$= 1.3 + (-2)(-2) + 3.1$

$= 3 + 4 + 3$

$= 10$

$\therefore 10 = \sqrt{14}\sqrt{14}\cos\theta$

$$\Rightarrow \cos\theta = \frac{10}{14}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{5}{7}\right)$$

3. Find the projection of the vecto r $\hat{i} - \hat{j}$ **on the vector** $i + \hat{j}$**.**

Ans: $a = \hat{i} - \vec{\hat{j}}$ and $b = \hat{i} + \vec{\hat{j}}$

Projection of $\vec{a}$ on $\vec{b}$ is $\frac{1}{|\vec{b}|}(\vec{a}\cdot\vec{b}) = \frac{1}{\sqrt{1+1}}\{1.1 + (-1)(1)\} = \frac{1}{\sqrt{2}}(1-1) = 0$

4. Find the projection of the vector $\hat{i} + 3\hat{j} + 7\hat{k}$ **on the vector** $7\hat{i} - \hat{j} + 8\hat{k}$**.**

Ans: $\vec{a} = \hat{i} + 3\hat{j} + 7\hat{k}$ and $b = 7\hat{i} \vec{-} \hat{j} + 8\hat{k}$

Projection of $\vec{a}$ on $\vec{b}$ is $\frac{1}{|\vec{b}|}(\vec{a}\cdot\vec{b}) = \frac{1}{\sqrt{7^2 + (-1)^2 + 8^2}}\{1(7) + 3(-1) + 7(8)\} = \frac{7 - 3 + 56}{\sqrt{49 + 1 + 64}} = \frac{60}{\sqrt{114}}$

5. Show that each of the given three vectors is a unit vector, which are mutually perpendicular to each other.

$\frac{1}{7}(2\hat{i} + 3\hat{j} + 6\hat{k})$, $\frac{1}{7}(3\hat{i} - 6\hat{j} + 2\hat{k})$, $\frac{1}{7}(6\hat{i} + 2\hat{j} - 3\hat{k})$

Ans:

$$\vec{a} = \frac{1}{7}(2\hat{i} + 3\hat{j} + 6\hat{k}) = \frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k},$$

$$\vec{b} = \frac{1}{7}(3\bar{i} - 6\hat{j} + 2\hat{k}) = \frac{3}{7}\hat{i} - \frac{6}{7}\bar{j} + \frac{2}{7}\hat{k}$$

$$\vec{c} = \frac{1}{7}(6\hat{i} + 2\hat{j} - 3\hat{k}) = \frac{6}{7}\hat{i} + \frac{2}{7}\hat{j} - \frac{3}{7}\hat{k}$$

$$|\vec{a}| = \sqrt{\left(\frac{2}{7}\right)^2 + \left(\frac{3}{7}\right)^2 + \left(\frac{6}{7}\right)^2} = \sqrt{\frac{4}{49} + \frac{9}{49} + \frac{36}{49}} = 1$$

$$|\vec{b}| = \sqrt{\left(\frac{3}{7}\right)^2 + \left(-\frac{6}{7}\right)^2 + \left(\frac{2}{7}\right)^2} = \sqrt{\frac{9}{49} + \frac{36}{49} + \frac{4}{49}} = 1$$

$$|\vec{c}| = \sqrt{\left(\frac{6}{7}\right)^2 + \left(\frac{2}{7}\right)^2 + \left(-\frac{3}{7}\right)^2} = \sqrt{\frac{36}{49} + \frac{4}{49} + \frac{9}{49}} = 1$$

So, each of the vectors is a unit vector.

$$\vec{a}\,.\,\vec{b} = \frac{2}{7}\times\frac{3}{7} + \frac{3}{7}\times\left(\frac{-6}{7}\right) + \frac{6}{7}\times\frac{2}{7} = \frac{6}{49} - \frac{18}{49} + \frac{12}{49} = 0$$

$$\vec{b}\cdot\vec{c} = \frac{3}{7}\times\frac{6}{7} + \left(\frac{-6}{7}\right)\times\frac{2}{7} + \frac{2}{7}\times\left(\frac{-3}{7}\right) = \frac{18}{49} - \frac{12}{49} - \frac{6}{49} = 0$$

$$\vec{c}\cdot\vec{a}=\frac{6}{7}\times\frac{2}{7}+\frac{2}{7}\times\frac{3}{7}+\left(\frac{-3}{7}\right)\times\frac{6}{7}=\frac{12}{49}+\frac{6}{49}-\frac{18}{49}=0$$

So, given vectors are mutually perpendicular to each other.

6. **Find** $|\vec{a}|$ **and** $|\vec{b}|$**, if** $(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b})=8$ **and** $|\vec{a}|=8|\vec{b}|$ **.**

Ans: $(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b})=8$

$$\Rightarrow \vec{a}\vec{a}-\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{a}-\vec{b}\vec{b}=8$$

$$\Rightarrow |\vec{a}|^2-|\vec{b}|^2=8$$

$$\Rightarrow (8|\vec{b}|)^2-|\vec{b}|^2=8$$

$$\Rightarrow 64|\vec{b}|^2-|\vec{b}|^2=8$$

$$\Rightarrow 63|\vec{b}|^2=8$$

$$\Rightarrow |\vec{b}|^2=\frac{8}{63}$$

$$\Rightarrow |\vec{b}|=\sqrt{\frac{8}{63}}$$

$$\Rightarrow |\vec{b}|=\frac{2\sqrt{2}}{3\sqrt{7}}$$

$$|\vec{a}|=8|\vec{b}|=\frac{8\times 2\sqrt{2}}{3\sqrt{7}}=\frac{16\sqrt{2}}{3\sqrt{7}}$$

7. **Evaluate the product** $(3\vec{a}-5\vec{b})\cdot(2\vec{a}+7\vec{b})$

Ans: $(3\vec{a}-5\vec{b})\cdot(2\vec{a}+7\vec{b})$

$$=3\vec{a}\cdot 2\vec{a}+3\vec{a}.7\vec{b}-5\vec{b}\cdot 2\vec{a}-5\vec{b}\cdot 7\vec{b}$$

$$=6\vec{a}\vec{a}+21\vec{a}\vec{b}-10\vec{a}\vec{b}-35\vec{b}\vec{b}$$

$= 6|\vec{a}|^2 + 11\vec{a}\vec{b} - 35|\vec{b}|^2$

8. Find the magnitude of two vectors $\vec{a}$ and $\vec{b}$, having the same magnitude and such that the angle between them is 60° and their scalar product is $\frac{1}{2}$

Ans: Let θ be angle between $\vec{a}$ and $\vec{b}$.

$|\vec{a}| = |\vec{b}|, \vec{a} \cdot \vec{b} = \frac{1}{2}$, and $\theta = 60^\circ$

$\therefore \frac{1}{2} = |\vec{a}||\vec{b}| \cos 60^\circ$

$\Rightarrow \frac{1}{2} = |\vec{a}|^2 \times \frac{1}{2}$

$\Rightarrow |\vec{a}|^2 = 1$

$\Rightarrow |\vec{a}| = |\vec{b}| = 1$

9. Find $|\vec{x}|$, if for a unit vector $\vec{a}$, $(\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 12$

Ans: $(\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 12$

$\Rightarrow \vec{x} \cdot \vec{x} + \vec{x}\vec{a} - \vec{a}\vec{x} - \vec{a} \cdot \vec{a} = 12$

$\Rightarrow |\vec{x}|^2 - |\vec{a}|^2 = 12$

$\Rightarrow |\vec{x}|^2 - 1 = 12$

$\Rightarrow |\vec{x}|^2 = 13$

$\therefore |\vec{x}| = \sqrt{13}$

10. If $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}, \vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$ are such that $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$, then find the value of λ.

Ans: $\vec{a}+\lambda\vec{b}=(2\hat{i}+2\hat{j}+3\hat{k})+\lambda(-\hat{i}+2\hat{j}+\dot{k})=(2-\lambda)\hat{i}+(2+2\lambda)\hat{j}+(3+\lambda)\hat{k}$

$(\vec{a}+\lambda\vec{b})\cdot\vec{c}=0$

$\Rightarrow[(2+-)\hat{i}+(2+2\lambda)\hat{j}+(3+\lambda)\hat{k}]\cdot(3\hat{i}+j)=0$

$\Rightarrow 3(2-\lambda)+(2+2\lambda)+0(3+\lambda)=0$

$\Rightarrow 6-3\lambda+2+2\lambda=0$

$\Rightarrow -\lambda+8=0$

$\Rightarrow \lambda=8$

11. **Show that** $\vec{a}|\vec{b}+|\vec{b}|\vec{a}$ **is perpendicular to** $|\vec{a}|\vec{b}-|\vec{b}|\vec{a}$ **, For any two nonzero vectors** $\vec{a}$ **and** $\vec{b}$ **.**

Ans: $(|a|b+|\vec{b}|\vec{a})\cdot(|\vec{a}|b-|\vec{b}|\vec{a})$ $\vec{}$ $\vec{}$

$=|\vec{a}|^2\vec{b}\vec{b}-|\vec{a}||\vec{b}|\vec{b}\vec{a}+|\vec{b}||a|\vec{a}\vec{b}-|\vec{b}|^2\ \vec{a}\cdot\vec{a}$

$=|\vec{a}|^2|\vec{b}|^2-|\vec{b}|^2|\vec{a}|^2\ =\ 0$

12. **If** $\vec{a}\cdot\vec{a}=0$ **and** $\vec{a}\vec{b}=0$**, then what can be concluded above the vector** $\vec{b}$ **?**

Ans: $\vec{a}\cdot\vec{a}=0\Rightarrow|\vec{a}|^2=0\Rightarrow|\vec{a}|=0$

$\therefore\vec{a}$ is the zero vector. Thus, any vector $\vec{b}$ can satisfy $\vec{a}\,.\,\vec{b}=0$.

13. **If** $\vec{a},\vec{b},\vec{c}$ **are unit vectors such that** $\vec{a}+\vec{b}+\vec{c}=0$**, find the value of** $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\,.\,\vec{a}$**.**

Ans: $\vec{a}+\vec{b}+\vec{c}|^2=(\vec{a}+\vec{b}+\vec{c})\cdot(\vec{a}+\vec{b}+\vec{c})=|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2(\vec{a}\vec{b}+\vec{b}\vec{c}+\vec{c}\cdot\vec{a})$

$\Rightarrow 0=1+1+1+2(\vec{a}\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})$

$\Rightarrow(\vec{a}\,.\,\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\,.\,\vec{a})=\dfrac{-3}{2}$

14. If either vector $\vec{a}=\vec{0}$ or $\vec{b}=\vec{0}$, then $\vec{a}\cdot\vec{b}=0$. But the converse need not be true. Justify your answer with an example.

Ans: $\vec{a}=2\hat{i}+4\hat{j}+3k$ and $\vec{b}=3\hat{i}+3\hat{j}-6k$

$\vec{a}\cdot\vec{b}=2.3+4.3+3(-6)=6+12-18=0$

$|a|=\sqrt{2^2+4^2+3^2}=\sqrt{29}$

$\therefore \vec{a}\neq\vec{0}$

$|\vec{b}|=\sqrt{3^2+3^2+(-6)^2}=54$

$\therefore \vec{b}\neq\overline{0}$

So, it is clear from above example that the converse of the given statement need not be true.

15. If the vertices A, B, C of a triangle ABC are $(1,2,3),(-1,0,0),(0,1,2)$, respectively, then find $\angle ABC$. [$\angle ABC$ is the angle between the vectors $\overrightarrow{BA}$ and $\overrightarrow{BC}$]

Ans: $\overrightarrow{BA}=\{1-(-1)\hat{i}+(2-0)\hat{j}+(3-0)\hat{k}=2\hat{i}+2\hat{j}+3\hat{k}$

$\overrightarrow{BC}=\{0-(-1)\}\hat{i}+(1-0)\hat{j}+(2-0)\hat{k}=\hat{i}+\hat{j}+2\hat{k}$

$\overrightarrow{BA}\cdot\overrightarrow{BC}=(2\hat{i}+2\hat{j}+3\hat{k})\cdot(\hat{i}+\hat{j}+2\hat{k})=2\times1+2\times1+3\times2=2+2+6=10$

$|\overrightarrow{BA}|=\sqrt{2^2+2^2+3^2}=\sqrt{4+4+9}=\sqrt{17}$

$|\overrightarrow{BC}|=\sqrt{1+1+2^2}=\sqrt{6}$

$\overrightarrow{BA}\cdot\overrightarrow{BC}=|\overrightarrow{BA}||\overrightarrow{BC}|\cos(\angle ABC)$

$$\therefore 10 = \sqrt{17} \times \sqrt{6} \cos(\angle ABC)$$

$$\Rightarrow \cos(\angle ABC) = \frac{10}{\sqrt{17} \times \sqrt{6}}$$

$$\Rightarrow (\angle ABC) = \cos^{-1}\left(\frac{10}{\sqrt{102}}\right)$$

16. Show that the points $A(1,2,7), B(2,6,3)$ and $C(3,10,-1)$ are collinear.

Ans:

$$\overrightarrow{AB} = (2-1)\hat{i} + (6-2)\hat{j} + (3-7)\hat{k} = \hat{i} + 4\hat{j} - 4\hat{k}$$

$$\overrightarrow{BC} = (3-2)\hat{i} + (10-6)\hat{j} + (-1-3)\hat{k} = \hat{i} + 4\hat{j} - 4\hat{k}$$

$$\overrightarrow{AC} = (3-1)\hat{i} + (10-2)\hat{j} + (-1-7)\hat{k} = 2\hat{i} + 8\hat{j} - 8\hat{k}$$

$$|\overrightarrow{AB}| = \sqrt{1^2 + 4^2 + (-4)^2} = \sqrt{1+16+16} = \sqrt{33}$$

$$|\overrightarrow{BC}| = \sqrt{1^2 + 4^2 + (-4)^2} = \sqrt{1+16+16} = \sqrt{33}$$

$$|\overrightarrow{AC}| = \sqrt{2^2 + 8^2 + 8^2} = \sqrt{4+64+64} = 2\sqrt{33}$$

$$|\overrightarrow{AC}| = |\overrightarrow{AB}| + |\overrightarrow{BC}|$$

Hence, the given points are collinear.

17. Show that the vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$ form the vertices of a right angled triangle.

Ans: $\overrightarrow{OA} = 2\hat{i} - \hat{j} + \hat{k}$, $\overrightarrow{OB} = \hat{i} - 3\hat{j} - 5\hat{k}$, $\overrightarrow{OC} = 3\hat{i} - 4\hat{j} - 4\hat{k}$

$$\overrightarrow{AB} = (1-2)\hat{i} + (-3+1)\hat{j} + (-5-1)\hat{k} = -\hat{i} - 2\hat{j} - 6\hat{k}$$

$$\overrightarrow{BC} = (3-1)\hat{i} + (-4+3)\hat{j} + (-4+5)\hat{k} = 2\hat{i} - \hat{j} + \hat{k}$$

$\overrightarrow{CA} = (2-3)\hat{i} + (-1+4)\hat{j} + (1+4)\hat{k} = -\hat{i} + 3\hat{j} + 5\hat{k}$

$|\overrightarrow{AB}| = \sqrt{(-1)^2 + (-2)^2 + (-6)^2} = \sqrt{1+4+36} = \sqrt{41}$

$|\overrightarrow{BC}| = \sqrt{2^2 + (-1)^2 + 1^2} = \sqrt{4+1+1} = \sqrt{6}$

$|\overrightarrow{AC}| = \sqrt{(-1)^2 + 3^2 + 5^2} = \sqrt{1+9+25} = \sqrt{35}$

$\therefore |\overrightarrow{BC}|^2 + |\overrightarrow{AC}|^2 = 6 + 35 = 41 = |\overrightarrow{AB}|^2$

Hence, ΔABC is a right triangle.

18. **If $\vec{a}$ is a nonzero vector of magnitude 'a' and λ a nonzero scalar. then $\lambda\vec{a}$ is unit vector if**

(A) $\lambda = 1$

(B) $\lambda = -1$

(C) $a = |\lambda|$

(D) $a = \frac{1}{|\lambda|}$

Ans: $|\lambda \vec{a}| = 1$

$\Rightarrow |\lambda| |\vec{a}| = 1$

$\Rightarrow |\vec{a}| = \frac{1}{|\lambda|}$

$\Rightarrow a = \frac{1}{|\lambda|}$

Exercise 10.4

1. **Find $|\vec{a} \times \vec{b}|$, if $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$**

Ans: We have, $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -7 & 7 \\ 3 & -2 & 2 \end{vmatrix}$$

$$= \hat{i}(-14+14) - \hat{j}(2-21) + \hat{k}(-2+21) = 19\hat{j} + 19\hat{k}$$

$$\therefore |\vec{a} \times \vec{b}| = \sqrt{(19)^2 + (19)^2} = \sqrt{2 \times (19)^2} = 19\sqrt{2}$$

2. **Find a unit vector perpendicular to each of the vector** $\vec{a} + \vec{b}$ **and** $\vec{a} - \vec{b}$**, where** $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$ **and** $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$**.**

Ans: $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$

$$\vec{a} + \vec{b} = 4\hat{i} + 4\hat{j}, \quad \vec{a} - \vec{b} = 2\hat{i} + 4\hat{k}$$

$$(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 4 & 0 \\ 2 & 0 & 4 \end{vmatrix} = \hat{i}(16) - \hat{j}(16) + \hat{k}(-8) = 16\hat{i} - 16\hat{j} - 8\hat{k}$$

$$|(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})| = \sqrt{16^2 + (-16)^2 + (-8)^2}$$

$$= \sqrt{2^2 \times 8^2 + 2^2 \times 8^2 + 8^2}$$

$$= 8\sqrt{2^2 + 2^2 + 1} = 8\sqrt{9} = 8 \times 3 = 24$$

So, the unit vector is $= \pm \dfrac{(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})}{|(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})|} = \pm \dfrac{16\hat{i} - 16\hat{j} - 8\hat{k}}{24}$

$$= \pm \frac{2\hat{i} - 2\hat{j} - \hat{k}}{3} = \pm \frac{2}{3}\hat{i} \mp \frac{2}{3}\hat{j} \mp \frac{1}{3}\hat{k}$$

3. **If a unit vector $\vec{a}$ makes an angle $\frac{\pi}{3}$ with $\hat{i}, \frac{\pi}{4}$ with $\hat{j}$ and an acute angle θ with $\hat{k}$, then find θ and hence, the components of $\vec{a}$**

Ans: $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$|\vec{a}| = 1$. $\cos\frac{\pi}{3} = \frac{a_1}{|\vec{a}|}$

$\Rightarrow \frac{1}{2} = a_1$

$\cos\frac{\pi}{4} = \frac{a_2}{|\vec{a}|}$

$\Rightarrow \frac{1}{\sqrt{2}} = a_2$

$\cos\theta = \frac{a_3}{|\vec{a}|}$

$\Rightarrow a_3 = \cos\theta$

$\Rightarrow \sqrt{a_1^2 + a_2^2 + a_3^2} = 1$

$\Rightarrow \left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \cos^2\theta = 1$

$\Rightarrow \frac{1}{4} + \frac{1}{2} + \cos^2\theta = 1$

$\Rightarrow \frac{3}{4} + \cos^2\theta = 1$

$\Rightarrow \cos^2\theta = 1 - \frac{3}{4} = \frac{1}{4}$

$\Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

$\therefore a_3 = \cos\frac{\pi}{3} = \frac{1}{2}$

So, $\theta = \frac{\pi}{3}$ and components of $\vec{a}$ are $\left(\frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{1}{2}\right)$

4. **Show that** $(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b}) = 2(\vec{a} \times \vec{b})$

Ans: $(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b})$

$= (a - b) \times \vec{a} + (a - b) \times b$

$= a \times \vec{a} - \vec{b} \times \vec{d} + a \times \vec{b} - \vec{b} \times \vec{b}$

$= 0 + \vec{a} \times \vec{b} + \vec{a} \times \vec{b} - 0$

$= 2\ (\vec{a} \times \vec{b})$

5. **Find** λ **and** μ **if** $(2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + \lambda\hat{j} + \mu\hat{k}) = \vec{0}$

Ans: $(2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + \lambda\hat{j} + \mu\hat{k}) = 0$

$$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 6 & 27 \\ 1 & \lambda & \mu \end{vmatrix} = 0\hat{i} + 0\hat{j} + 0\hat{k}$$

$\Rightarrow \hat{i}(6\mu - 27\lambda) - \hat{j}(2\mu - 27) + \hat{k}(2\lambda - 6) = 0\hat{i} + 0\hat{j} + 0\hat{k}$

$6\mu - 27\lambda = 0$

$2\mu - 27 = 0$

$2\lambda - 6 = 0$

$\lambda = 3$

$\mu = \frac{27}{2}$

6. Given that $\vec{a}\,.\,\vec{b}=0$ and $\vec{a}\times\vec{b}=0$. What can you conclude about $\vec{a}$ and $\vec{b}$?

Ans: $\vec{a}\,.\,\vec{b}=0$

(i) $|\vec{a}|=0$ or $|\vec{b}|=0$ or $\vec{a}\perp\vec{b}$ (if $|\vec{a}|\neq 0$ and $|\vec{b}|\neq 0$)

$\vec{a}\times\vec{b}=0$

(ii) $|\vec{a}|=0$ or $|\vec{b}|=0$ or $\vec{a}\parallel\vec{b}$ (if $|\vec{a}|\neq 0$ and $|\vec{b}|\neq 0$)

But $\vec{a}$ and $\vec{b}$ cannot be parallel and perpendicular at same time.

So, $|\vec{a}|=0$ or $|\vec{b}|=0$

7. Let the vectors $\vec{a},\vec{b},\vec{c}$ given as $a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$, $b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$, $c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$.

Then show that $=\vec{a}\times(\vec{b}+\vec{c})=\vec{a}\times\vec{b}+\vec{a}\times\vec{c}$

Ans: $(\vec{b}+\vec{c})=(b_1+c_1)\hat{i}+(b_2+c_2)\hat{j}+(b_3+c_3)\hat{k}$

Now, $\vec{a}\times(\vec{b}+\vec{c})\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & a_2 & a_3\\ b_1+c_1 & b_2+c_2 & b_3+c_3\end{vmatrix}$

$=i\left[a_2(b_3+c_3)-a_3(b_2+c_2)\right]-\hat{j}\left[a_1(b_3+c_3)-a_3(b_1+c_i)\right]+\hat{k}\left[a_1(b_2+c_2)-a_2(b_1+c_1)\right]$

$=i\left[a_2b_3+a_2c_3-a_3b_2-a_3c_2\right]+\hat{j}\left[-a_1b_3-a_1c_3+a_3b_1+a_3c_1\right]+\hat{k}\left[a_1b_2+a_1c_2-a_2b_1-a_2c_i\right]$

$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix}$

$=i\left[a_2b_3-a_3b_2\right]+\hat{j}\left[b_1a_3-ab_3\right]+\hat{k}\left[a_1b_2-a_2h\right]$

$\vec{a}\times\vec{c}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & a_2 & a_3\\ c_1 & c_2 & c_3\end{vmatrix}$

$= \hat{i}\left[a_2c_3 - a_3c_2\right] + \hat{j}\left[a_3c_1 - a_1c_3\right] + \hat{k}\left[ab_2 - a_2b\right]$

$(\vec{a}\times\vec{b}) + (\vec{a}\times\vec{c}) = \hat{i}\left[a_2b_3 + a_2c_3 - a_3b_2 - a_3c_2\right] + \hat{j}[b_1a_3 + a_3c_1 - a_1b_3 - a_i\}_3\Big]$

$+\hat{k}\left[a,b,+a_1c,-a,b_1 - a,c_1\right]$

$\vec{a}\times(\vec{b}+\vec{c}) = \vec{a}\times\vec{b} + \vec{a}\times\vec{c}$

Hence, proved.

8. If either $\vec{a}=0$ or $b=0$, then $\vec{a}\times b=0$. Is the converse true? Justify your answer with an example.

Ans: Let $\hat{}\vec{a} = \hat{2}i + \hat{3}j + 4k$, $\hat{}\vec{b} = \hat{4}i + \hat{6}j + 8k$, $\vec{a}\times\vec{b} = \vec{0}$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} \,\hat{} & \hat{j} & k \\ 2 & 3 & 4 \\ 4 & 6 & 8 \end{vmatrix} = \hat{i}(24-24) - \hat{j}(16-16) + \hat{k}(12-12) = 0\hat{i} + 0\hat{j} + 0\hat{k}$$

$|\vec{a}| = \sqrt{2^2 + 3^2 + 4^2} = \sqrt{29}$

$\therefore \vec{a} \neq \vec{0}$

$|\vec{b}| = \sqrt{4^2 + 6^2 + 8^2} = \sqrt{116}$

$\therefore \vec{b} \neq \vec{0}$

Hence, converse of the statement need not be true.

9. Find the area of the triangle with vertices A $(1,1,2)$, B $(2,3,5)$ and $C(1,5,5)$.

Ans: $\overrightarrow{AB} = (2-1)\hat{i} + (3-1)\hat{j} + (5-2)\hat{k} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\overrightarrow{BC} = (1-2)\hat{i} + (5-3)\hat{j} + (5-5)\hat{k} = -\hat{i} + 2\hat{j}$

Area $= \frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{BC}|$

$$\overrightarrow{AB}\times\overrightarrow{BC}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & 3\\ -1 & 2 & 0\end{vmatrix}=\hat{i}(-6)-\hat{j}(3)+\hat{k}(2+2)=-6\hat{i}-3\hat{j}+4\hat{k}$$

$$|\overrightarrow{AB}\times\overrightarrow{BC}|=\sqrt{(-6)^2+(-3)^2+4^2}=\sqrt{36+9+16}=\sqrt{61}$$

So, area of ΔABC is $\frac{\sqrt{61}}{2}$ sq units.

10. Find the area of the parallelogram whose adjacent sides are determined by the vector $\vec{a}=\hat{i}-\hat{j}+3\hat{k}$ and $\vec{b}=2\hat{i}-7\hat{j}+\hat{k}$

Ans:

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & 3\\ 2 & -7 & 1\end{vmatrix}=\hat{i}(-1+21)-\hat{j}(1-6)+\hat{k}(-7+2)=20\hat{i}+5\hat{j}-5\hat{k}$$

$$\therefore|\vec{a}\times\vec{b}|=\sqrt{20^2+5^2+5^2}=\sqrt{400+25+25}=15\sqrt{2}$$

So, area of parallelogram is $15\sqrt{2}$ sq units

11. Let the vectors $\vec{a}$ and $\vec{b}$ be such that $|\vec{a}|=3$ and $|\vec{b}|=\frac{\sqrt{2}}{3}$, then $\vec{a}\times\vec{b}$ is a unit vector, if the angle between $\vec{a}$ and $\vec{b}$ is:

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{2}$

Ans: $|\vec{a}\times\vec{b}|=1$

$\Rightarrow || a || \vec{b} | \sin\theta |=1$

$\Rightarrow |\vec{a}||\vec{b}|| \sin\theta |=1$

$\Rightarrow 3\times\frac{\sqrt{2}}{3}\times\sin\theta=1$

$\Rightarrow \sin\theta=\frac{1}{\sqrt{2}}$

$\Rightarrow \theta=\frac{\pi}{4}$

12. **Area of a rectangle having vertices** $\mathrm{A,B,C}$**, and** D **with position vectors** $-\hat{i}+\frac{1}{2}\hat{j}+4\hat{k}, \hat{i}+\frac{1}{2}\hat{j}+4\hat{k}, \hat{i}-\frac{1}{2}\hat{j}+4\hat{k}$ **and** $-\hat{i}-\frac{1}{2}\hat{j}+4\hat{k}$ **respectively is**

(A) $\mathbf{\frac{1}{2}}$

(B) 1

(C) 2

(D) 4

Ans: $\overrightarrow{\mathrm{AB}}=(1+1)\hat{i}+\left(\frac{1}{2}-\frac{1}{2}\right)\hat{j}+(4-4)\hat{k}=2\hat{i}$

$\overrightarrow{\mathrm{BC}}=(1-1)\hat{i}+\left(-\frac{1}{2}-\frac{1}{2}\right)\hat{j}+(4-4)\hat{k}=-\hat{j}$

$\overrightarrow{\mathrm{AB}}\times\overrightarrow{\mathrm{BC}}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 0 & 0\\ 0 & -1 & 0\end{vmatrix}=\hat{k}(-2)=-2\hat{k}$

$|\overrightarrow{\mathrm{AB}}\times\overrightarrow{\mathrm{BC}}|=\sqrt{(-2)^2}=2$

So, area of the required rectangle is 2 square units.

Miscellaneous Exercise

1. Write down a unit vector in XY-plane, making an angle of 30° with the positive direction of x axis.

Ans: Unit vector is $\vec{r} = \cos\theta\hat{i} + \sin\theta\hat{j}$, where θ is angle with positive X axis.

$$\vec{r} = \cos 30^\circ \hat{i} + \sin 30^\circ \hat{j} = \frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}$$

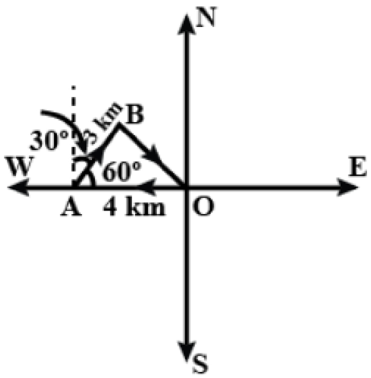
2. Find the scalar components and magnitude of the vector joining the points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$

Ans: $\overrightarrow{PQ} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$

$$|\overrightarrow{PQ}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

3. A girl walks 4 km towards west, then she walks 3 km in a direction 30° east of north and stops. Determine the girl's displacement from her initial point of departure.

Ans:



$\overrightarrow{OA} = -4\hat{i}$

$$\overrightarrow{AB} = \hat{i}\,|\overrightarrow{AB}|\cos 60^\circ + \hat{j}\,|\overrightarrow{AB}|\sin 60^\circ$$

$$= \hat{i}3 \times \frac{1}{2} + \hat{j}3 \times \frac{\sqrt{3}}{2}$$

$$= \frac{3}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$$

$$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$$

$$= (-4\hat{i}) + \left(\frac{3}{2}\hat{i} + \frac{3\sqrt{3}}{2}j\right)$$

$$= \left(-4 + \frac{3}{2}\right)\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$$

$$= \left(\frac{-8+3}{9}\right)\hat{i} + \frac{3\sqrt{3}}{9}\hat{j}$$

$$= \frac{-5}{2}\vec{i} + \frac{3\sqrt{3}}{2}\hat{j}$$

4. If $\vec{a} = \vec{b} + \vec{c}$, then is it true that $|\vec{a}| = |\vec{b}| + |\vec{c}|$? Justify your answer.

Ans: In ΔABC, $\overrightarrow{CB} = \vec{a}$, $\overrightarrow{CA} = b$, $\overrightarrow{AB} = \vec{c}$

$\vec{a} = \vec{b} + \vec{c}$, by triangle law of addition for vectors.

$|\vec{a}| < |\vec{b}| + |\vec{c}|$, by triangle inequality law of lengths.

Hence, it's not true that $|\vec{a}| = |\vec{b}| + |\vec{c}|$

5. Find the value of x for which $x(\hat{i} + \hat{j} + \hat{k})$ unit vector.

Ans: $|x(\hat{i} + \hat{j} + \hat{k})| = 1$

$$\Rightarrow \sqrt{x^2 + x^2 + x^2} = 1$$

$$\Rightarrow \sqrt{3x^2} = 1$$

$$\Rightarrow \sqrt{3}x = 1$$

$$\Rightarrow x = \pm\frac{1}{\sqrt{3}}$$

6. Find a vector of magnitude 5 units, and parallel to the resultant of the vectors $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$

Ans: $\vec{c} = \vec{a} + \vec{b} = (2+1)\hat{i} + (3-2)\hat{j} + (-1+1)\hat{k} = 3\hat{i} + \hat{j}$

$$|\vec{c}| = \sqrt{3^2 + 1^2} = \sqrt{9+1} = \sqrt{10}$$

$$\therefore \hat{c} = \frac{\vec{c}}{|\vec{c}|} = \frac{(3\hat{i} + \hat{j})}{\sqrt{10}}$$

So, a vector of magnitude 5 and parallel to the resultant of $\vec{a}$ and $\vec{b}$ is $\pm 5(\hat{c}) = \pm 5\left(\frac{1}{\sqrt{10}}(3\hat{i} + \hat{j})\right) = \pm\frac{3\sqrt{10}}{2}\hat{i} \pm \frac{\sqrt{10}}{2}\hat{j}$

7. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$, find a unit vector parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c}$.

Ans: $2\vec{a} - \vec{b} + 3\vec{c} = 2(\hat{i} + \hat{j} + \hat{k}) - (2\hat{i} - \hat{j} + 3\hat{k}) + 3(\hat{i} - 2\hat{j} + \hat{k})$

$$= 2\hat{i} + 2\hat{j} + 2\hat{k} - 2\hat{i} + j - 3\hat{k} + 3\hat{i} - 6\hat{j} + 3\hat{k}$$

$$= 3\hat{i} - 3\hat{j} + 2\hat{k}$$

$$|2\vec{a} - \vec{b} + 3c| = \sqrt{3^2 + (-3)^2 + 2^2} = \sqrt{9+9+4} = \sqrt{22}$$

Thus ,required unit vector is $\frac{2\vec{a} - \vec{b} + 3\vec{c}}{|2\vec{a} - \vec{b} + 3\vec{c}|} = \frac{3\hat{i} - 3\hat{j} + 2\hat{k}}{\sqrt{22}} = \frac{3}{\sqrt{22}}\hat{i} - \frac{3}{\sqrt{22}}\hat{j} + \frac{2}{\sqrt{22}}\hat{k}$

8. Show that the points $A(1,-2,-8), B(5,0,-2)$ and $C(11,3,7)$ are collinear, and find the ratio in which B divides AC.

Ans: $\overrightarrow{AB} = (5-1)\hat{i} + (0+2)\hat{j} + (-2+8)\hat{k} = 4\hat{i} + 2\hat{j} + 6\hat{k}$

$\overrightarrow{BC} = (11-5)\hat{i} + (3-0)\hat{j} + (7+2)\hat{k} = 6\hat{i} + 3\hat{j} + 9\hat{k}$

$\overrightarrow{AC} = (11-1)\hat{i} + (3+2)\hat{j} + (7+8)\hat{k} = 10\hat{i} + 5\hat{j} + 15\hat{k}$

$|\overrightarrow{AB}| = \sqrt{4^2 + 2^2 + 6^2} = \sqrt{16+4+36} = \sqrt{56} = 2\sqrt{14}$

$|\overrightarrow{BC}| = \sqrt{6^2 + 3^2 + 9^2} = \sqrt{36+9+81} = \sqrt{126} = 3\sqrt{14}$

$|\overrightarrow{AC}| = \sqrt{10^2 + 5^2 + 15^2} = \sqrt{100+25+225} = \sqrt{350} = 5\sqrt{14}$

$\therefore\ |\overrightarrow{AC}| = |\overrightarrow{AB}| + |\overrightarrow{BC}|$

So, the points are collinear.

Let B divide AC in ratio $\lambda:1$. $\overrightarrow{OB} = \dfrac{\lambda\overrightarrow{OC} + \overrightarrow{OA}}{(\lambda+1)}$

$\Rightarrow 5\hat{i} - 2\hat{k} = \dfrac{\lambda(11\hat{i} + 3\hat{j} + 7\hat{k}) + (\hat{i} - 2\hat{j} - 8\hat{k})}{\lambda+1}$

$\Rightarrow (\lambda+1)(5\hat{i} - 2\hat{k}) = 11\lambda\hat{i} + 3\lambda\hat{j} + 7\lambda\hat{k} + \hat{i} - 2\hat{j} - 8\hat{k}$

$\Rightarrow 5(\lambda+1)\hat{i} - 2(\lambda+1)\hat{k} = (11\lambda+1)\hat{i} + (3\lambda-2)\hat{j} + (7\lambda-8)\hat{k}$

$\Rightarrow \lambda = \dfrac{2}{3}$

So, the required ratio is 2:3

9. Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $(2\vec{a} + \vec{b})$ and $(\vec{a} - 3\vec{b})$ externally in the ratio 1: 2. Also, show that P is the mid point of the line segment RQ.

Ans: $\overrightarrow{OP} = 2a + b, \overrightarrow{OQ} = \vec{a} - 3\vec{b}$

$$\overrightarrow{OR} = \frac{2(2\vec{a}+\vec{b})-(\vec{a}-3\vec{b})}{2-1} = \frac{4\vec{a}+2\vec{b}-\vec{a}-3\vec{b}}{1} = 3\vec{a}+5\vec{b}$$

So, the position vector of R is $3\vec{a}+5\vec{b}$

Position vector of midpoint of $RQ = \frac{\overrightarrow{OQ}+\overrightarrow{OR}}{2}$

$$= \frac{(a\sqrt{6})+(3\vec{a}+5\bar{b})}{2}$$

$$= 2\vec{a}+\vec{b}$$

$$= \overrightarrow{OP}$$

Thus, P is midpoint of line segment RQ

10. The two adjacent sides of a parallelogram are $2\hat{i}-4\hat{j}+5\hat{k}$ and $\hat{i}-2\hat{j}-3\hat{k}$. Find the unit vector parallel to its diagonal. Also, find its area.

Ans: Diagonal of a parallelogram is $\vec{a}+\vec{b}$

$$\vec{a}+\vec{b} = (2+1)\hat{i}+(-4-2)\hat{j}+(5-3)\hat{k} = 3\hat{i}-6\hat{j}+2\hat{k}$$

So, the unit vector parallel to diagonal is

$$\frac{\vec{a}+\vec{b}}{|\vec{a}+\vec{b}|} = \frac{3\hat{i}-6\hat{j}+2\hat{k}}{\sqrt{3^2+(-6)^2+2^2}} = \frac{3\hat{i}-6\hat{j}+2\hat{k}}{\sqrt{9+36+4}} = \frac{3\hat{i}-6\hat{j}+2\hat{k}}{7} = \frac{3}{7}\hat{i}-\frac{6}{7}\hat{j}+\frac{2}{7}\hat{k}$$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 2 & -4 & 3 \\ 1 & -2 & -3 \end{vmatrix}$$

$$= \hat{i}(12+10)-\hat{j}(-6-5)+\bar{k}(-4+4)$$

$$= 22\hat{i}+11\hat{j}$$

$$= 11(2\hat{i}+\hat{j})$$

$\therefore |\vec{a}\times\vec{b}| = 11\sqrt{2^2+1^2} = 11\sqrt{5}$

So, area of parallelogram is $11\sqrt{5}$ sq units

11. Show that the direction cosines of a vector equally inclined to the axes OX, OY and OZ are $\frac{1}{\sqrt{3}}$ $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$.

Ans: Let a vector be equally inclined to OX, OY, and OZ at an angle α.

So, the Direction Cosines of the vector are $\cos\alpha, \cos\alpha$ and $\cos\alpha$.

$\cos^2\alpha + \cos^2\alpha + \cos^2\alpha = 1$

$\Rightarrow 3\cos^2\alpha = 1$

$\Rightarrow \cos\alpha = \frac{1}{\sqrt{3}}$

So, the DCs of the vector are $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$.

12. Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c}\cdot\vec{d} = 15$

Ans: $d = d_1\vec{\hat{i}} + d_2\hat{j} + d_3\hat{k}$

$\vec{d}\,.\,\vec{a} = 0 \Rightarrow d_1 + 4d_2 + 2d_3 = 0$

$\vec{d}\,.\,\vec{b} = 0 \Rightarrow 3d_1 - 2d_2 + 7d_3 = 0$

$\vec{c}\cdot\vec{d} = 15 \Rightarrow 2d_1 - d_2 + 4d_3 = 15$

Solving these equations, we get $d_1 = \frac{160}{3}, d_2 = -\frac{5}{3}, d_3 = -\frac{70}{3}$

$\therefore \vec{d} = \frac{160}{3}\hat{i} + \frac{5}{3}j + \frac{70}{3}\hat{k} = \frac{1}{3}(160\hat{i} + 5\hat{j} + 70\hat{k})$

13. **The scalar product of the vector $\hat{i}+\hat{j}+\hat{k}$ with a unit vector along the sum of veclors $2\hat{i}+4\hat{j}-5\hat{k}$ änd $\lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to one. Find the value of λ.**

Ans: $(2\hat{i}+4\hat{j}-5\hat{k})+(\lambda\hat{i}+2\hat{j}+3\hat{k})=(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}$

So, unit vector along $(2\hat{i}+4\hat{j}-5\hat{k})+(\lambda\hat{i}+2\hat{j}+3\hat{k})$ is $\left(\frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{\lambda^2+4\lambda+44}}\right)$

$$(\hat{i}+\hat{j}+\hat{k})\cdot\left(\frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{\lambda^2+4\lambda+44}}\right)=1$$

$$\Rightarrow \frac{(2+\lambda)+6-2}{\sqrt{\lambda^2+4\lambda+44}}=1$$

$$\Rightarrow \sqrt{\lambda^2+4\lambda+44}=\lambda+6$$

$$\Rightarrow \lambda^2+4\lambda+44=(\lambda+6)^2$$

$$\Rightarrow \lambda^2+4\lambda+44=\lambda^2+12\lambda+36$$

$$\Rightarrow 8\lambda=8$$

$$\Rightarrow \lambda=1$$

14. **If $\vec{a},b,\vec{c}$ are mutually perpendicular vectors of equal magnitudes, show that the vector $\vec{a}+\vec{b}+\vec{c}$ is equally inclined to $\vec{a}$, $\vec{b}$ and $\vec{c}$.**

Ans: $\vec{a}\,.\,b=b\,.\,\vec{c}=\vec{c}\,.\,\vec{a}=0$

$|\vec{a}|=|\vec{b}|=|\vec{c}|$

Let $\vec{a}+\vec{b}+\vec{c}$ be inclined to $\vec{a},\vec{b},\vec{c}$ at angles $\theta_1,\theta_2,\theta_3$ respectively.

$$\cos\theta_1=\frac{(\vec{a}+\vec{b}+\vec{c})\cdot\vec{a}}{|\vec{a}+\vec{b}+\vec{c}||\vec{a}|}=\frac{\vec{a}\cdot\vec{a}+\vec{b}\cdot\vec{a}+\vec{c}\cdot\vec{a}}{|\vec{a}+\vec{b}+\vec{c}||\vec{a}|}=\frac{|\vec{a}|^2}{|\vec{a}+\vec{b}+\vec{c}||\vec{a}|}=\frac{|\vec{a}|}{|\vec{a}+\vec{b}+\vec{c}|}$$

$$\cos\theta_2=\frac{(\vec{a}+\vec{b}+\vec{c})\vec{b}}{|\vec{a}+\vec{b}+\vec{c}||\vec{b}|}=\frac{\vec{a}\vec{b}+\vec{b}\vec{b}+\vec{c}\vec{b}}{|\vec{a}+\vec{b}+\vec{c}||\vec{b}|}=\frac{|\vec{b}|^2}{|\vec{a}+\vec{b}+\vec{c}||\vec{b}|}=\frac{|\vec{b}|}{|\vec{a}+\vec{b}+\vec{c}|}$$

$$\text{os}\,\theta_3=\frac{(\vec a+\vec b+\vec c)\cdot\vec c}{|\vec a+\vec b+\vec c||\vec c|}=\frac{\vec a\vec c+\vec b\vec c+\vec c\vec c}{|\vec a+\vec b+\vec c||\vec c|}=\frac{|\vec c|^2}{|\vec a+\vec b+\vec c||\vec c|}=\frac{|\vec c|}{|\vec a+\vec b+\vec c|}$$

Since, $|\vec a|=|\vec b|=|\vec c|\Rightarrow\cos\theta_1=\cos\theta_2=\cos\theta_3$, So, $\theta_1=\theta_2=\theta_3$

15. **Prove that,** $(\vec a+\vec b)\cdot(\vec a+\vec b)=|\vec a|^2+|\vec b|^2$ **if and only if** $\vec a,\vec b$ **are perpendicular, given** $\vec a\neq\vec 0,\vec b\neq 0$

Ans: $(a+b)\cdot(a+b)=|a|^2+|b|^2$

$\Rightarrow\vec a\cdot\vec a+\vec a\,.\,\vec b+\vec b\cdot\vec a+\vec b\,.\,\vec b=|\vec a|^2+|\vec b|^2$

$\Rightarrow|\vec a|^2+2\vec a\,.\,\vec b+|\vec b|^2=|a|^2+|\vec b|^2$

$\Rightarrow 2\vec a\,.\,\vec b=0$

$\Rightarrow\vec a\,.\,\vec b=0$

So $\vec a$ and $\vec b$ are perpendicular.

16. **If** θ **is the angle between two vectors** a **and** b**, then** $ab\geq\vec 0$ **only when**

(A) $0<\theta<\frac{\pi}{2}$

(B) $0\leq\theta\leq\frac{\pi}{2}$

(C) $0<\theta<\pi$

(D) $0\leq\theta\leq\pi$

Ans: $\therefore ab\geq 0$

$\Rightarrow|\vec a||\vec b|\cos\theta\geq 0$

$\Rightarrow\cos\theta\geq 0\quad\because[|\vec a|\geq 0$ and $|\vec b|\geq 0]$

$\Rightarrow 0 \le \theta \le \frac{\pi}{2}$

$\vec{a} \cdot \vec{b} \ge 0$ if $0 \le \theta \le \frac{\pi}{2}$

So the right answer is B

17. **Let** a **and** b **be two unit vectors and** θ **is the angle between them. Then** $\vec{a} + \vec{b}$ **is a unit vector if**

(A) $\theta = \frac{\pi}{4}$

(B) $\theta = \frac{\pi}{3}$

(C) $\theta = \frac{\pi}{2}$

(D) $\theta = \frac{2\pi}{3}$

Ans: $|\vec{a}| = |\vec{b}| = 1$

$|\vec{a} + \vec{b}| = 1$

$\Rightarrow (\vec{a} + \vec{b})(\vec{a} + \vec{b}) = 1$

$\Rightarrow \vec{a} \cdot \vec{a} + \vec{a} \,.\, \vec{b} + \vec{b} \,.\, \vec{a} + \vec{b} \,.\, \vec{b} = 1$

$\Rightarrow |\vec{a}|^2 + 2\vec{a} \,.\, \vec{b} + |\vec{b}|^2 = 1$

$\Rightarrow 1^2 + 2|\vec{a}||\vec{b}|\cos\theta + 1^2 = 1$

$\Rightarrow 1 + 2.1.1\cos\theta + 1 = 1$

$\Rightarrow \cos\theta = -\frac{1}{2}$

$\Rightarrow \theta = \frac{2\pi}{3}$

So, $\vec{a} + \vec{b}$ is unit vector if $\theta = \frac{2\pi}{3}$

The correct answer is D

18. **The value of** $\hat{i}.(\hat{j}\times\hat{k})+\hat{j}\cdot(\hat{i}\times\hat{k})+\hat{k}\cdot(\hat{i}\times\hat{j})$ **is**

(A) 0 (B) -1 (C) 1 (D) 3

Ans:

$\hat{i}\cdot(\hat{j}\times\hat{k})+\hat{j}\cdot(\hat{i}\times\hat{k})+\hat{k}\cdot(\hat{i}\times\hat{j})$

$=\hat{i}\,.\,\hat{i}+\hat{j}\cdot(-\hat{j})+\hat{k}\,.\,\hat{k}$

$=1-1+1$

$=1$

The correct answer is C

19. **If** θ **is the angle between any two vectors** $\vec{a}$ **and** $\vec{b}$**, then** $|\vec{a}\vec{b}|=|\vec{a}\times\vec{b}|$ **when** θ **is equal to**

(A) 0

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{2}$

(D) n

Ans: $|\vec{a}b|=|\vec{a}\times\vec{b}|$

$\Rightarrow|\vec{a}|b|\cos\theta=|\vec{a}||b|\sin\theta$

$\Rightarrow\cos\theta=\sin\theta$

$\Rightarrow\tan\theta=1$

$\Rightarrow\theta=\frac{\pi}{4}$

The correct answer is B