Mechanical Properties of Solids

8
Chapter

1. A steel wire of length 4.7 m and cross-sectional area $3.0\times10^{-5}m^2$ stretches by the same amount as a copper wire of length 3.5 m and cross-sectional area $4.0\times10^{-5}m^2$
of steel to that of copper?

Ans: In the above question it is given that:

Length of the steel wire is $L_1 = 4.7m$.

Area of cross-section of the steel wire is $A_1 = 3.0\times10^{-5}m^2$.

Length of the copper wire is $L_2 = 3.5m$.

Area of cross-section of the copper wire is $A_2 = 4.0\times10^{-5}m^2$.

Now,

The change in length is given by:

$\Delta L = L_1 - L_2 = 4.7 - 3.5 = 1.2m$

Let the force applied in both the cases be F .

$$Y_1 = \frac{F}{A_1}\times\frac{L_1}{\Delta L} = \frac{F\times4.7}{3.0\times10^{-5}\times1.2}$$

copper wire is given by:

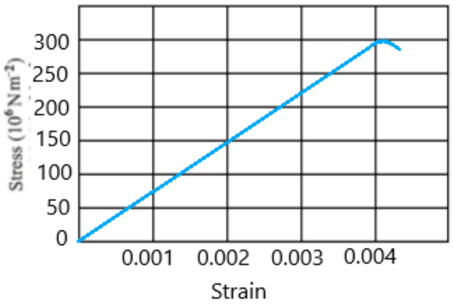
$$Y_2 = \frac{F}{A_2}\times\frac{L_2}{\Delta L} = \frac{F\times3.5}{4.0\times10^{-5}\times1.2}$$

Therefore,

$$\frac{Y_1}{Y_2} = \frac{F\times4.7\times4.0\times10^{-5}\times1.2}{3.0\times10^{-5}\times1.2\times F\times3.5} = \frac{1.79}{1}$$

$1.79:1$.

2. **Figure shows the strain-stress curve for a given material. What are a)**



Ans: From the graph given in the above question it is clear that:

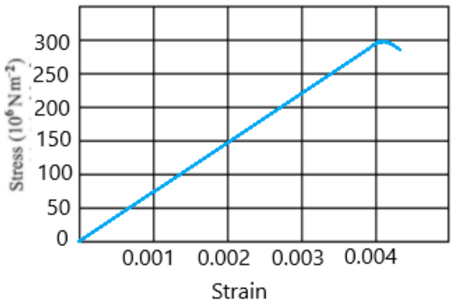
Stress for a given material is $150 \times 10^6 \text{ N/m}^2$ and strain is 0.002.

$$Y = \frac{\text{Stress}}{\text{Strain}}$$

$$\Rightarrow Y = \frac{150 \times 10^6 \text{ N/m}^2}{0.002} = 7.5 \times 10^{10} \text{ N/m}^2$$

$7.5 \times 10^{10} \text{ N/m}^2$.

b) approximate yield strength for this material?



Ans: The yield strength of the material is the maximum stress it sustains without crossing the elastic limit.

From the graph given in the above question, it is clear that approximate yield strength for this material is $300 \times 10^6 \text{ N/m}^2$ or $3 \times 10^8 \text{ N/m}^2$.

3. **The stress -strain graphs for materials A and B are shown in figure.**

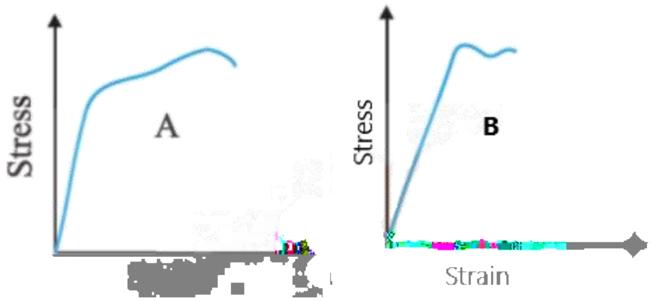


Fig 9.12

The graphs are drawn to the same scale.

a)

Ans: In the two graphs it is that given that stress for A is more than that of B.

As,

$$\text{Young's modulus} = \frac{\text{Stress}}{\text{Strain}},$$

Therefore, material A has greater Young's modulus.

b) Which of the two is the stronger material?

Ans: The strength of a material is determined by the amount of stress required for fracturing a material, corresponding to its fracture point.

Fracture point is defined as the extreme point in a stress-strain curve.

From the graph it is clear that material A can withstand more strain than material B.

Therefore, material A is stronger than material B.

4. **Read the following two statements below carefully and state, with reasons, if it is true or false.**

a) **steel.**

Ans: The given statement is false.

As there is more strain in rubber than steel and modulus of elasticity is inversely

of rubber.

b) The stretching of a coil is determined by its shear modulus.

Ans: The given statement is true.

As the shear modulus of a coil relates with the change in shape of the coil and the stretching of coil changes its shape without any change in the length. Therefore, the shear modulus of elasticity is involved. Hence the stretching of a coil is determined by its shear modulus.

5. **Two wires of diameter 0.25 cm, one made of steel and the other made of brass are loaded as shown in figure. The unloaded length of steel wire is 1.5 m and that of brass wire is 1.0 m. Compute the elongations of the steel and the brass wires.**

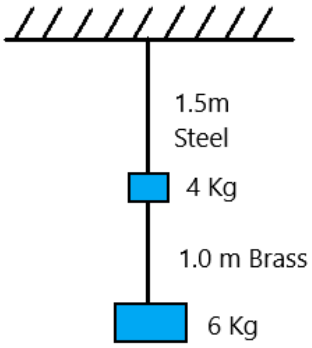


Fig 9.13

Ans: In the above question it is given that:

Diameter of the wires is $d = 0.25\text{m}$.

Hence $r = 0.125\text{m}$.

Length of the steel wire is $L_1 = 1.5\text{m}$.

Length of the brass wire is $L_2 = 1.0\text{m}$.

Total force exerted on the steel wire is $F_1 = (4+6)g = 10g$

$\therefore F_1 = 10 \times 9.8 = 98\text{N}$.

$$Y_1 = \frac{F_1}{A_1} \times \frac{L_1}{\Delta L_1}$$

Where,

ΔL_1 is the change in the length of the steel wire.

And A_1 is the area of cross-section of the steel wire.

$\therefore A_1 = \pi r_1^2$

We have,

$$Y_1 = 2.0 \times 10^{11} \text{Pa}.$$

$$\Rightarrow \Delta L_1 = \frac{F_1 \times L_1}{A_1 \times Y_1}$$

$$\Rightarrow \Delta L_1 = \frac{98 \times 1.5}{\pi(0.125)^2 \times 2.0 \times 10^{11}} = 1.49 \times 10^{-4} \text{m}.$$

Total force on the brass wire is $F_2 = 6 \times 9.8 = 58.8\text{N}$.

$$Y_2 = \frac{F_2}{A_2} \times \frac{L_2}{\Delta L_2}$$

Where,

ΔL_2 is the change in the length of the brass wire.

And A_2 is the area of cross-section of the brass wire.

$\therefore A_2 = \pi r_2^2$

We have,

$$Y_2 = 0.91 \times 10^{11} \text{Pa}.$$

$$\Rightarrow \Delta L_2 = \frac{F_2 \times L_2}{A_2 \times Y_2}$$

$$\Rightarrow \Delta L_2 = \frac{58.8 \times 1}{\pi(0.125)^2 \times 0.91 \times 10^{11}} = 1.3 \times 10^{-4} \text{m}.$$

Clearly, the elongation of the steel wire is $1.49 \times 10^{-4}\text{m}$ and that of the brass wire is $1.3 \times 10^{-4}\text{m}$.

6. **The edge of an aluminium cube is 10 cm long. One face of the cube is firmly fixed to a vertical wall. A mass of 100 kg is then attached to the opposite face of the cube. The shear modulus of aluminium is 25 GPa. What is the vertical deflection of this face?**

Ans: In the above question it is given that:

Edge of the aluminium cube is $L = 10cm = 0.1m$.

The mass attached to the cube is $m = 100kg$.

Shear modulus η of aluminium is $25GPa = 25\times10^{10}Pa$.

We know that:

$$\text{Shear modulus}(\eta) = \frac{\text{Shear stress}}{\text{Shear strain}}$$

$$\Rightarrow \eta = \frac{\left(\frac{F}{A}\right)}{\left(\frac{L}{\Delta L}\right)}$$

Where,

F is the applied force.

$$\therefore F = mg = 100\times9.8 = 980N$$

Area of one of the faces of the cube is $A = 0.1\times0.1 = 0.01m^2$.

Vertical deflection of the cube is ΔL.

$$\Delta L = \frac{FL}{A\eta}$$

$$\Rightarrow \Delta L = \frac{980\times0.1}{0.01\times25\times10^9} = 3.92\times10^{-7}m$$

Clearly, the vertical deflection of this face of the cube is $3.92\times10^{-7}m$.

7. **Four identical hollow cylindrical columns of mild steel support a big structure of mass 50,000 kg. The inner and outer radii of each column are 30 cm and 60 cm respectively. Assuming the load distribution to be uniform, calculate the compressional strain of each column.**

Ans: In the above question it is given that:

Mass of the big structure is $M = 50000\text{kg}$.

Inner radius of the column is $r = 30\text{cm} = 0.3\text{m}$.

Outer radius of the column is $R = 60\text{cm} = 0.6\text{m}$

$$Y = 2 \times 10^{11}\text{Pa}$$

The total force exerted is $F = Mg = 50000 \times 9.8\text{N}$.

Stress = Force exerted on a single column

$$\Rightarrow \text{Stress} = \frac{50000 \times 9.8}{4} = 122500\text{N}$$

$$Y = \frac{\text{Stress}}{\text{Strain}}$$

$$\Rightarrow \text{Strain} = \frac{\left(\frac{F}{A}\right)}{Y}$$

Where,

Area is given by

$$A = \pi\left(R^2 - r^2\right) = \pi\left((0.6)^2 - (0.3)^2\right).$$

$$\Rightarrow \text{Strain} = \frac{\left(\frac{50000 \times 9.8}{\pi\left((0.6)^2 - (0.3)^2\right)}\right)}{2 \times 10^{11}} = 7.22 \times 10^{-7}$$

Therefore, the compressional strain of each column is 7.22×10^{-7}.

8. A piece of copper having a rectangular cross-section of 15.2 mm × 19.1 mm is pulled in tension with 44,500 N force, producing only elastic deformation. Calculate the resulting strain?

Ans: In the above question it is given that:

Length of the piece of copper is $l = 19.1\text{mm} = 19.1 \times 10^{-3}\text{m}$.

Breadth of the piece of copper is $b = 15.2\text{mm} = 15.2 \times 10^{-3}\text{m}$
Area of the copper piece will be:

$$A = l \times b$$

$$\Rightarrow A = 19.1 \times 10^{-3} \times 15.2 \times 10^{-3} = 2.9 \times 10^{-4}\text{m}^2$$

Tension force applied on the piece of copper is $F = 44500\text{N}$.

Modulus of elasticity of copper is $\eta = 42 \times 10^9 \text{N/m}^2$.

We know that :

$$\text{Modulus of elasticity}(\eta) = \frac{\text{Stress}}{\text{Strain}}$$

$$\Rightarrow \eta = \frac{\left(\frac{F}{A}\right)}{\text{Strain}}$$

$$\Rightarrow \text{Strain} = \frac{F}{A\eta}$$

$$\Rightarrow \text{Strain} = \frac{44500}{2.9 \times 10^{-4} \times 42 \times 10^9} = 3.65 \times 10^{-3}$$

Hence, the resulting strain is 3.65×10^{-3}.

9. A steel cable with a radius of 1.5 cm supports a chairlift at a ski area. If the maximum stress is not to exceed 10^8N/m^2, what is the maximum load the cable can support?

Ans: In the above question it is given that:

Radius of the steel cable is $r = 1.5\text{cm} = 0.015\text{m}$.

Maximum allowable stress is 10^8N/m^2.

We know that:

$$\text{Maximum force} = \text{Maximum stress} \times \text{Area of cross-section}$$

$$\Rightarrow \text{Maximum force} = 10^8 \times \pi(0.015)^2 = 7.065 \times 10^4 \text{N}$$

Therefore, the cable can support the maximum load of $7.065 \times 10^4 \text{N}$.

10. A rigid bar of mass 15 kg is supported symmetrically by three wires each 2.0 m long. Those at each end are of copper and the middle one is of iron. Determine the ratio of their diameters if each is to have the same tension.

Ans: In the above question it is given that:

Tension force acting on each wire is the same.

Therefore the extension produced in each wire is the same.

As the length of both wires is the same, the strain in both wires is also the same.

$$Y = \frac{\text{Stress}}{\text{Strain}}$$

$$\Rightarrow Y = \frac{\left(\frac{F}{A}\right)}{\text{Strain}} = \frac{\frac{4F}{\pi d^2}}{\text{Strain}}$$

Where,

F is the Tension force,

A is the area of cross-section and

d is the diameter of the wire

From equation (1), it is clear that $Y \propto \frac{1}{d^2}$.

$$Y_1 = 190 \times 10^9 \text{ Pa}.$$

Let diameter of the iron wire be d_1.

$$Y_2 = 100 \times 10^9 \text{ Pa}.$$

Let diameter of the copper wire be d_2.

Therefore, the ratio of their diameters is given as:

$$\Delta l = \frac{220.1 \times 1}{0.065 \times 10^{-4} \times 2 \times 10^{11}} = 1.53 \times 10^{-4} \text{ m}$$

Thus, the elongation of the wire is 1.53×10^{-4} m.

11. A 14.5 kg mass, fastened to the end of a steel wire of unstretched length 1.0 m, is whirled in a vertical circle with an angular velocity of 2 rev/s at the bottom of the circle.
The cross-sectional area of the wire is 0.065 cm2.
Calculate the elongation of the wire when the mass is at the lowest point of its path.

Ans: Mass, m = 14.5kg
Length of the steel wire, l = 1.0m
Angular velocity, = 2 rev/s = 2×2 π rad/s = 12.56 rad/s
Cross-sectional area of the wire, A = 0.065cm^2 = 0.065×10^{-4} m^2
Let Δl be the elongation of the wire when the mass is at the lowest point of its path.
When the mass is placed at the position of the vertical circle, the total force on the mass is:
$F = mg + m\omega^2 l$
$= 14.5 \times 9.8 + 14.5 \times 1 \times (12.56)^2$
= 2429.53 N
Young's modulus = Stress / Strain
Y = (F/A)/(Δl/l)
Δl = Fl/AY
Youngs modulus for steel = 2×10^{11}Pa
$\Delta l = 2429.53 \times 1/(0.065 \times 10^{-4} \times 2 \times 10^{11}) = 1.87 \times 10^{-3}$ m
Hence, the elongation of the wire is 1.87×10^{-3} m.

12. Compute the bulk modulus of water from the following data: Initial volume = 100.0 litre, Pressure increase = 100.0 atm$\left(1\text{atm} = 1.013 \times 10^5\,\text{Pa}\right)$. Final volume = 100.5 litre. Compare the bulk modulus of water with that of air (at constant temperature). Explain in simple terms why the ratio is so large.

Ans: In the above question it is given that:

Initial volume is $V_1 = 100.0l = 100 \times 10^{-3} m^3$.

Final volume is $V_2 = 100.5l = 100.5 \times 10^{-3} m^3$.

Thus, the increase in volume is $V_2 - V_1 = 0.5 \times 10^{-3} m^3$.

Increase in pressure is $\Delta p = 100\text{atm} = 100 \times 1.013 \times 10^5\,\text{Pa}$.

The formula for bulk modulus is

$$\text{Bulk Modulus} = \frac{\Delta p}{\left(\dfrac{\Delta V}{V_1}\right)} = \frac{\Delta p V_1}{\Delta V}$$

$$\Rightarrow \text{Bulk Modulus} = \frac{100 \times 1.013 \times 10^5 \times 100 \times 10^{-3}}{0.5 \times 10^{-3} m^3} = 2.026 \times 10^9\,\text{Pa}$$

We know that Bulk modulus of air is $1 \times 10^5\,\text{Pa}$.

$$\Rightarrow \frac{\text{Bulk modulus of water}}{\text{Bulk modulus of air}} = \frac{2.026 \times 10^9}{1 \times 10^5} = 2.026 \times 10^4$$

This ratio is very high because air is more compressible than water.

13. What is the density of water at a depth where pressure is 80.0 atm, given that its density at the surface is $1.03\times10^3 kg/m^3$?

Ans: In the above question it is given that:

Pressure at the given depth is $p=80.0atm=80\times1.01\times10^5 Pa$.

Consider the given depth to be h.

Density of water at the surface is $\rho_1=1.03\times10^3 kg/m^3$

Consider ρ_2 to be the density of water at the depth h.

Consider V_1 to be the volume of water of mass m at the surface.

Consider V_2 to be the volume of water of mass m at the depth h.

Consider ΔV to be the change in volume.

$$\Delta V=V_1-V_2$$

$$\Rightarrow \Delta V=m\left[\left(\frac{1}{\rho_1}\right)-\left(\frac{1}{\rho_2}\right)\right]$$

Now,

$$\text{Volumetric strain}=m\left[\left(\frac{1}{\rho_1}\right)-\left(\frac{1}{\rho_2}\right)\right]\times\left(\frac{\rho_1}{m}\right)$$

$$\Rightarrow \frac{\Delta V}{V_1}=1-\left(\frac{\rho_1}{\rho_2}\right)$$

Bulk modulus is given by:

$$\text{Bulk modulus}=\frac{pV_1}{\Delta V}$$

$$\Rightarrow \frac{\Delta V}{V_1}=\frac{p}{B}$$

Compressibility of water is given by:

$$\frac{1}{B}=45.8\times10^{-11}Pa^{-1}$$

$$\Rightarrow \frac{\Delta V}{V_1}=80\times1.013\times10^5\times45.8\times10^{-11}=3.71\times10^{-3}$$

From equations (1) and (2) we get:

$$1-\left(\frac{\rho_1}{\rho_2}\right)=3.71\times10^{-3}$$

$$\Rightarrow \rho_2=\frac{1.03\times10^3}{\left[1-\left(3.71\times10^{-3}\right)\right]}=1.034\times10^3\,kgm^{-3}$$

Clearly, the density of water at the given depth (h) is $1.034\times10^3\,kgm^{-3}$.

14. Compute the fractional change in volume of a glass slab, when subjected to a hydraulic pressure of 10 atm.

Ans: In the above question it is given that:

The hydraulic pressure exerted on the glass slab is $p=10atm=10\times1.013\times10^5\,Pa$.

Also, we know that bulk modulus of glass is $B=37\times10^9\,N/m^2$.

Bulk modulus is given by the relation:

$$B=\frac{p}{\left(\frac{\Delta V}{V}\right)}$$

Where,

$\frac{\Delta V}{V}$ is the fractional change in volume.

$$\Rightarrow\left(\frac{\Delta V}{V}\right)=\frac{p}{B}$$

$$\Rightarrow\left(\frac{\Delta V}{V}\right)=\frac{10\times1.013\times10^5}{37\times10^9}=2.73\times10^{-5}$$

Clearly, the fractional change in the volume of the glass slab is 2.73×10^{-5}.

15. Determine the volume contraction of a solid copper cube, 10 cm on an edge, when subjected to a hydraulic pressure of $7\times10^6\,Pa$.

Ans: In the above question it is given that:

The length of an edge of the solid copper cube is $l=10cm=0.1m$.

Hydraulic pressure is $p = 7\times10^{6}\,\text{Pa}$.

Bulk modulus of copper is $B = 140\times10^{9}\,\text{Pa}$.

Bulk modulus is given by the relation:

$$B = \frac{p}{\left(\frac{\Delta V}{V}\right)}$$

Where,

$\frac{\Delta V}{V}$ is the volumetric strain

ΔV is the change in volume.

V is the original volume.

$$\Rightarrow \Delta V = \frac{pV}{B}$$

The original volume of the cube is $V = l^{3}$.

$$\Rightarrow \Delta V = \frac{pl^{3}}{B}$$

$$\Rightarrow \Delta V = \frac{7\times10^{6}\times(0.1)^{3}}{140\times10^{9}} = 5\times10^{-8}\,\text{m}^{3} = 5\times10^{-2}\,\text{cm}^{3}$$

Clearly, the volume contraction of the solid copper cube is $5\times10^{-2}\,\text{cm}^{3}$.

16. How much should the pressure on a litre of water be changed to compress it by 0.10%? carry one quarter of the load.

Ans: In the above question it is given that:

Volume of water is $V = 1\text{L}$.

The water is to be compressed by 0.10%.

$$\therefore \text{Fractional change} = \frac{\Delta V}{V} = \frac{0.1}{100\times1} = 10^{-3}$$

Bulk modulus is given by the relation:

$$B = \frac{p}{\left(\frac{\Delta V}{V}\right)}$$

$$\Rightarrow p = B \times \left(\frac{\Delta V}{V}\right)$$

We know that, bulk modulus of water is $B = 2.2 \times 10^9 \, N/m^2$

$$\Rightarrow p = 2.2 \times 10^9 \times 10^{-3} = 2.2 \times 10^6 \, N/m^2.$$

Clearly, the pressure on water should be $2.2 \times 10^6 \, N/m^2$.