Gravitation

7
Chapter

1. **Answer the following:**
   a) **You can shield a charge from electrical forces by putting it inside a hollow conductor. Can you shield a body from the gravitational influence of nearby matter by putting it inside a hollow sphere or by some other means?**

Ans: No.
The gravitational force of matter on nearby objects cannot be eliminated by any means. This is because gravitational force is independent of the nature of the medium's material.

   b) **An astronaut inside a small spaceship orbiting around the earth cannot detect gravity. If the space station orbiting around the earth has a large size, can he hope to detect gravity?**

Ans: Yes.
If the space station's size is large enough, then the astronaut will identify the change in Earth's gravity.

   c) **If you compare the gravitational force on the earth due to the sun to that due to the moon, you would find that the Sun's pull is greater than the moon's pull. (You can check this yourself using the data available in the succeeding exercises). However, the tidal effect of the moon's pull is greater than the tidal effect of the sun. Why?**

Ans: The tidal effect depends on the inverse of distance's cube while gravitational force depends on the inverse of distance's square.
Since the distance between the Moon and the Earth is shorter than the distance between the Sun and the Earth, the tidal influence of the Moon's pull is higher than the tidal effect of the Sun's influence.

2. **Choose the correct alternative:**
   a) **Acceleration due to gravity increases/decreases with increasing altitude.**

Ans: Acceleration due to gravity decreases with increasing altitude.
Acceleration due to gravity at height h is given by:

$$g_h = \left(1 - \frac{2h}{R_e}\right) g$$

Where, $R_e =$ Radius of the Earth

$g =$ Acceleration due to gravity on the surface of the Earth

It is clear from the relation that acceleration due to gravity lowers with a height increment.

b) Acceleration due to gravity increases/decreases with increasing depth. (Assume the earth to be a sphere of uniform density).

Ans: Acceleration due to gravity decreases with increasing depth.

Acceleration due to gravity at depth d is given by:

$$g_d = \left(1 - \frac{d}{R_e}\right) g$$

It is clear from the relation that acceleration due to gravity lowers with a depth increment.

c) Acceleration due to gravity is independent of mass of the earth/mass of the body.

Ans: Acceleration due to gravity is independent of mass of the body.

Acceleration due to gravity of body having mass m is given by: $g = \frac{GM}{R^2}$

Where, G = Universal gravitational constant

M = Mass of the Earth

R = Radius of the Earth

Hence, it is clear that acceleration due to gravity is not dependent on the body's mass.

d) The formula $-GMm\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$ is more/less accurate than the formula $mg(r_2 - r_1)$ for the difference of potential energy between two points and distance away from the centre of the earth.

Ans: Gravitational potential energy of two points at r_2 and r_1 distance away from the Earth centre is given by:

$$V(r_1) = -\frac{GmM}{r_1} \text{ and}$$

$$V(r_2) = -\frac{GmM}{r_2}$$

$\therefore$ Difference in potential energy,

$$V = V(r_2) - V(r_1)$$

$$\Rightarrow V = -GMm\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

Hence, this formula is more reliable than the formula $mg(r_2 - r_1)$.

3. Suppose there existed a planet that went around the sun twice as fast as the earth. What would be its orbital size as compared to that of the earth?

Ans: Time taken by the Earth to complete one revolution, $T_e = 1\text{ year}$.

Orbital radius of the Earth in its orbit, $R_e = 1\text{AU}$.

Time taken by the planet to complete one revolution, $T_p = \frac{1}{2}T_e = \frac{1}{2}\text{year}$.

Orbital radius of the planet $= R_p$.

From Kepler's third law of planetary motion, we can write:

$$\left(\frac{R_p}{R_e}\right)^3 = \left(\frac{T_p}{T_e}\right)^2$$

$$\Rightarrow \frac{R_p}{R_e} = \left(\frac{T_p}{T_e}\right)^{\frac{2}{3}}$$

$$\Rightarrow \frac{R_p}{R_e} = \left(\frac{\frac{1}{2}}{1}\right)^{\frac{2}{3}}$$

$$\Rightarrow \frac{R_p}{R_e} = (0.5)^{\frac{2}{3}}$$

$$\Rightarrow \frac{R_p}{R_e} = 0.63$$

Hence, the orbital radius of the planet will be 0.63 times smaller than that of the Earth.

4. $\mathbf{I_O}$**, one of the satellites of Jupiter, has an orbital period of 1.769 days and the radius of the orbit is $\mathbf{4.22\times10^8 m}$. Show that the mass of Jupiter is about one-thousandth that of the sun.**

Ans: Given that,

Orbital period of $I_O = T_{I_O} = 1.769\text{days} = 1.769\times24\times60\times60\text{s}$

Orbital radius of $I_O = R_{I_O} = 4.22\times10^8\text{m}$

Satellite I_O is revolving around Jupiter.

Mass of the Jupiter is given by:

$$M_J = \frac{4\pi^2 R_{I_o}^3}{GT_{I_o}^2} \text{......(i)}$$

Where, M_J = Mass of Jupiter

G is the Universal gravitational constant

Orbital period of the Earth,

$T_e = 365.25\text{days} = 365.25\times25\times60\times60\text{s}$

Orbital radius of the Earth,

$R_e = 1\text{AU} = 1.496\times10^{11}\text{m}$

Mass of sun is given as:

$$M_s = \frac{4\pi^2 R_e^3}{GT_e^2} \text{......(ii)}$$

$$\Rightarrow \frac{M_S}{M_J} = \frac{4\pi^2 R_e^3}{GT_e^2}\times\frac{GT_{Io}^2}{4\pi^2 R_{Io}^3} = \frac{R_e^3}{R_{Io}^3}\times\frac{T_{Io}^2}{T_e^2}$$

$$\Rightarrow \frac{M_S}{M_J} = \left(\frac{1.769\times24\times60\times60}{365.25\times24\times60\times60}\right)^2\times\left(\frac{1.496\times10^{11}}{4.22\times10^8}\right)^3$$

$$\Rightarrow \frac{M_S}{M_J} = 1045.04$$

$$\therefore \frac{M_s}{M_J} \sim 1000$$

We get,

$M_s \sim 1000\times M_J$

Hence, it can be concluded that the mass of Jupiter is about one-thousandth that of the Sun.

5. Let us assume that our galaxy consists of $\mathbf{2.5\times10^{11}}$ stars each of one solar mass. How long will a star at a distance of $\mathbf{50,000ly}$ from the

galactic centre to complete one revolution? Take the diameter of the Milky Way to be 10^5 ly.

Ans: Mass of galaxy Milky way, $M = 2.5\times10^{11}$ solar mass

Solar mass = Mass of sun $= 2.0\times10^{36}$ kg

Mass of our galaxy, $M = 2.5\times10^{11}\times2\times10^{35}$

$M = 5\times10^{41}$ kg

Diameter of Milky Way, $d = 10^5$ ly

Radius of Milky Way, $r = 5\times10^4$ ly

$1\text{ ly} = 9.46\times10^{15}$ m

$r = 5\times10^4\times9.46\times10^{15}$

$\Rightarrow r = 4.73\times10^{20}$ m

Since a star rotates around the galactic centre of the Milky Way, its time period is given by:

$$T = \left(\frac{4\pi^2r^3}{GM}\right)^{\frac{1}{2}} = \left(\frac{4\times(3.14)^2\times(4.73)^3\times10^{60}}{6.67\times10^{-11}\times5\times10^{41}}\right)$$

$$\Rightarrow T = \left(125.27\times10^{30}\right)^{\frac{1}{2}}$$

$$\Rightarrow T = 1.12\times10^{16}\text{s}$$

As we know,

$1\text{ year} = 365\times24\times60\times60\text{s}$

We get,

$$1\text{s} = \frac{1}{365\times24\times60\times60}\text{years}$$

$$\Rightarrow 1.12\times10^{16}\text{s} = \frac{1.12\times10^{16}}{365\times24\times60\times60}$$

$$\therefore 1.12\times10^{16}\text{s} = 3.55\times10^8\text{years}$$

Star will take 3.55×10^8 years to complete one revolution.

6. Choose the correct alternative:

a) If the zero of potential energy is at infinity, the total energy of an orbiting satellite is negative of its kinetic/potential energy.

Ans: Kinetic energy.

The sum of kinetic energy and potential energy is the total mechanical energy. The satellite's gravitational potential energy is zero at infinity. As we know, the Earth-satellite system is a bound system; then, the satellite's total energy is negative.

Thus, at infinity, the orbiting satellite's total energy is equal to the negative of its kinetic energy.

b) The energy required to launch an orbiting satellite out of earth's gravitational influence is more/less than the energy required to project a stationary object at the same height (as the satellite) out of earth's influence.

Ans: Less.

An orbiting satellite gets a certain amount of energy that permits it to rotate around the Earth. Its orbit gives this energy. It requires comparatively lesser energy to go out of the influence of the Earth's gravitational field than a fixed object on the Earth's surface that initially holds no energy.

7. Does the escape speed of a body from the earth depend on

a) the mass of the body?

Ans: No.

Escape velocity of a body from the Earth is given by the relation:

$$v = \sqrt{2gR}$$

The escape speed of a body does not depend on the mass of the body.

b) the location from where it is projected?

Ans: No.

Escape velocity of a body from the Earth is given by the relation:

$$v = \sqrt{2gR}$$

The escape speed of a body does not depend on the location from where it is projected.

c) the direction of projection?

Ans: No.

Escape velocity of a body from the Earth is given by the relation:

$$v = \sqrt{2gR}$$

The escape speed of a body does not depend on the direction of projection.

d) the height of the location from where the body is launched?

Ans: Yes.

Escape velocity of a body from the Earth is given by the relation:

$$v = \sqrt{2gR}$$

The escape speed of a body does slightly depend on the height of the location from where the body is launched. This is because escape velocity is slightly dependent on the gravitational potential at a certain height.

8. A comet orbits the Sun in a highly elliptical orbit. Does the comet have a constant

a) linear speed?

Ans: No.

The expression for angular momentum is:

$$L = \frac{mv^2}{r}$$

As angular momentum is constant, linear speed varies as **r** varies.

b) angular speed?

Ans: No.

The expression for angular momentum is:

$$L = mw^2 r$$

As angular momentum is constant, angular speed varies as **r** varies.

c) angular momentum?

Ans. Yes.

There is no external torque and thus, the angular momentum is constant.

d) kinetic energy?

Ans: No.

The expression for kinetic energy is:

$$K = \frac{1}{2}mv^2$$

Kinetic energy is not constant because linear speed varies.

e) potential energy?

Ans: No.

By law of conservation of energy, total energy is constant and it is the sum of potential energy and kinetic energy.

As kinetic energy is not constant, potential energy is also not constant.

f) **total energy throughout its orbit? Neglect any mass loss of the comet when it comes very close to the Sun.**

Ans: Yes.
By law of conservation of energy, the total energy is constant.

9. Which of the following symptoms is likely to afflict an astronaut in space

a) **swollen feet**

Ans: Legs hold the whole mass in a standing place due to gravitational force.
In space, an astronaut appears weightless because of the absence of gravity. Therefore, swollen feet of a spaceman do not affect him/her in space.

b) **swollen face**

Ans: A swollen face is usually caused because of seeming weightlessness in space.
Sense organs such as the nose, eyes, ears, and mouth establish a person's face. This symptom can affect a spacewalker in space.

c) **headache**

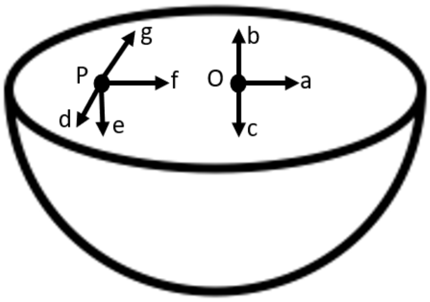
Ans: Headaches are because of mental stress. It can influence the working of an astronaut in space.

d) **orientational problem**

Ans: Space has diverse orientations.
Therefore, orientational difficulty can affect an astronaut in space.

10. Choose the correct answer from among the given ones:
The gravitational intensity at the centre of a hemispherical shell of uniform mass density has the direction indicated by the arrow (see Fig 8.12).

i. **a,**
ii. **b,**
iii. **c,**
iv. **O**

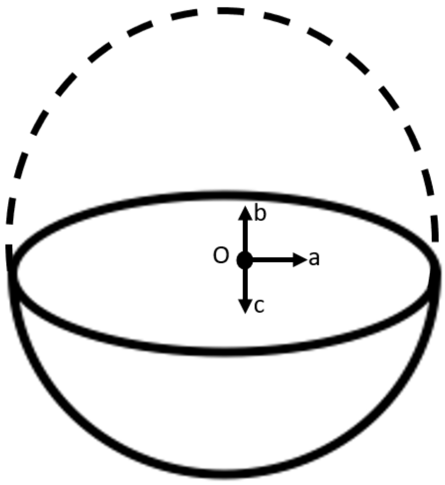


Ans: Option (iii) is correct.

Gravitational potential (V) is fixed at all points in a spherical shell. Hence, the potential gravitational gradient $\left(\frac{dV}{dr}\right)$ is zero everywhere inside the spherical surface.

The potential gravitational gradient is equivalent to the negative of gravitational intensity. Hence, intensity is also zero at all locations inside the spherical shell. This shows that gravitational forces operating at a point in a spherical shell are symmetric.

If the top half of a spherical shell is cut out, then the net gravitational force working on a particle located at centre O will be downward.



Since gravitational intensity is described as the gravitational force per unit mass at that location, it will also act downward. Thus, the gravitational intensity at the centre of the given hemispherical shell has the direction indicated by the arrow c.

11. Choose the correct answer from among the given ones:

For the problem 8.10, the direction of the gravitational intensity at an arbitrary point P is indicated by the arrow

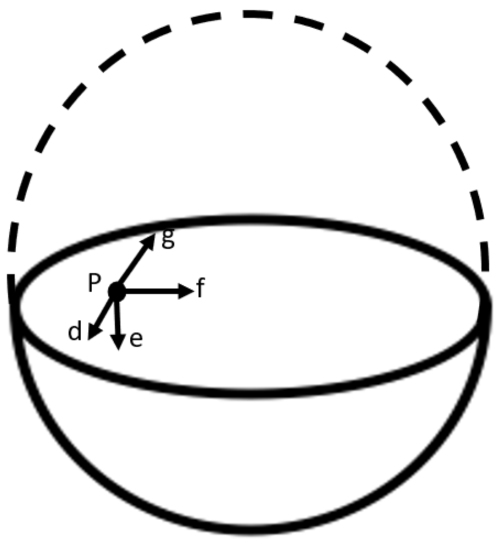
(i) **d,**
(ii) **e,**
(iii) **f,**
(iv) **g.**

Ans: Option (ii) is correct.

Gravitational potential (V) is fixed at all points in a spherical shell. Hence, the potential gravitational gradient $\left(\frac{dV}{dr}\right)$ is zero everywhere inside the spherical surface.

The potential gravitational gradient is equivalent to the negative of gravitational intensity. Hence, intensity is also zero at all locations inside the spherical shell. This shows that gravitational forces operating at a point in a spherical shell are symmetric.

If the top half of a spherical shell is cut out, then the net gravitational force working on a particle located at centre P will be downward.



Since gravitational intensity is described as the gravitational force per unit mass at that location, it will also act downward.

Thus, the gravitational intensity at the arbitrary point P of the given hemispherical shell has the direction indicated by the arrow e.

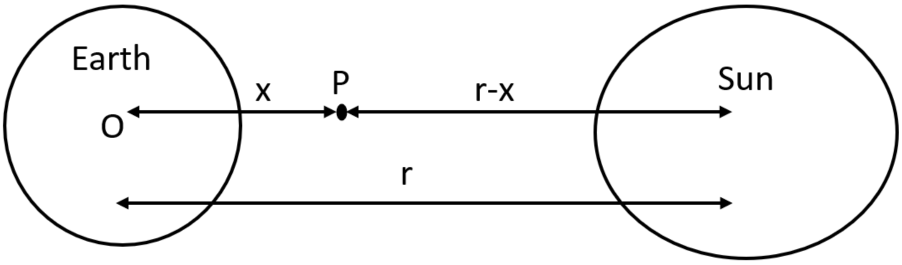
12. **A rocket is fired from the earth towards the sun. At what distance from the earth's centre is the gravitational force on the rocket zero? Mass of the sun $= 2\times10^{30}$ kg, mass of the earth $= 6\times10^{24}$ kg. Neglect the effect of other planets etc. (orbital radius $= 1.5\times10^{11}$ m).**

Ans: Mass of the Sun, $M_s = 2\times10^{30}\,kg$

Mass of the Earth, $M_e = 6 \times 10^{24}$ kg

Orbital radius, $r = 1.5 \times 10^{11}$ m

Mass of the rocket= m



Let x be the distance from the Earth centre where the gravitational force working on satellite P becomes zero.

From Newton's law of gravitation, we can equalize gravitational forces acting on satellite P under the effect of the Sun and the Earth as:

$$\frac{GmM_s}{(r-x)^2} = Gm\frac{M_e}{x^2}$$

We get,

$$\left(\frac{r-x}{x}\right)^2 = \frac{M_s}{M_e}$$

$$\Rightarrow \frac{r-x}{x} = \left(\frac{2 \times 10^{30}}{60 \times 10^{24}}\right)^{\frac{1}{2}} = 577.35$$

$$\Rightarrow 1.5 \times 10^{11} - x = 577.35x$$

$$\Rightarrow 578.35x = 1.5 \times 10^{11}$$

$$\Rightarrow x = \frac{1.5 \times 10^{11}}{578.35} = 2.59 \times 10^8 \text{m}$$

Therefore, 2.59×10^8m is the distance from the earth's centre at which the gravitational force on the rocket is zero.

13. How will you 'weigh the sun', that is, estimate its mass? The mean orbital radius of the earth around the sun is 1.5×10^8km.

Ans: Orbital radius of the Earth around the Sun, $r = 1.5 \times 10^{11}$m.

Time taken by the Earth to cover one revolution around the Sun,

$T = 1$ year

$\Rightarrow$ T=365.25 days

We get,

T=365.25 $\times$ 24 $\times$ 60 $\times$ 60s

Universal gravitational constant,

$G = 6.67 \times 10^{-11} \text{Nm}^2\text{kg}^{-2}$.

Thus, we can use the relation given below for calculating the mass of the Sun,

$$M = \frac{4\pi^2 r^3}{GT^2}$$

$$\Rightarrow M = \frac{4 \times (3.14)^2 \times (1.5 \times 10^{11})^3}{6.67 \times 10^{-11} \times (365.25 \times 24 \times 60 \times 60)^2}$$

$$\Rightarrow M = \frac{133.24 \times 10^{33}}{6.64 \times 10^4}$$

$$\Rightarrow M = 2.0 \times 10^{30} \text{kg}$$

Hence, the mass of the Sun is $2 \times 10^{30} \text{kg}$.

14. A Saturn year is 29.5 times the earth year. How far is the Saturn from the sun if the earth is $1.50 \times 10^8 \text{km}$ away from the sun?

Ans: Distance of the Earth from the Sun, $r_e = 1.5 \times 10^8 \text{km} = 1.5 \times 10^{11} \text{m}$

Time period of the Earth $= T_e$

Time period of Saturn, $T_s = 29.5 T_e$

Distance of Saturn from the Sun $= r_s$

From Kepler's third law of planetary motion, we have,

$$T = \left(\frac{4\pi^2 r^3}{GM}\right)^{\frac{1}{2}}$$

For Saturn and Sun, we can write,

$$\frac{r_s{}^3}{r_e{}^3} = \frac{T_s{}^2}{T_e{}^2}$$

$$\Rightarrow r_s = r_e \left(\frac{T_s}{T_e}\right)^{\frac{2}{3}}$$

$$\Rightarrow r_s = 1.5 \times 10^{11} \left(\frac{29.5 T_e}{T_e}\right)^{\frac{2}{3}}$$

$$\Rightarrow r_s = 1.5 \times 10^{11} (29.5)^{\frac{2}{3}}$$

$$\Rightarrow r_s = 1.5 \times 10^{11} \times 9.55$$

$$\Rightarrow r_s = 14.32 \times 10^{11} \text{m}$$

Hence, the distance between Sun and Saturn is $1.43 \times 10^{12} \text{m}$.

15. A body weighs $63N$ on the surface of the earth. What is the gravitational force on it due to the earth at a height equal to half the radius of the earth?

Ans: Weight of the body, $W = 63N$

Acceleration due to gravity at height h

$R_e =$ Radius of the Earth

For $h = \frac{R_e}{2}$, gravity at h is given by:

$$g_h = \frac{g}{\left(1+\frac{h}{R_e}\right)^2}$$

$$\Rightarrow g_h = \frac{g}{\left(1+\frac{1}{2}\right)^2}$$

$$\Rightarrow g_h = \frac{4}{9}g$$

Weight of a body of mass m at height h is given as:

$$W' = mg_h$$

$$\Rightarrow W' = m \times \frac{4}{9}g$$

$$\Rightarrow W' = \frac{4}{9} \times mg$$

$$\Rightarrow W' = \frac{4}{9}W$$

$$\Rightarrow W' = \frac{4}{9} \times 63$$

$$\Rightarrow W' = 28N$$

Thus, the gravitational force on the body due to the earth at a height equal to half the radius of the earth is $28N$.

16. Assuming the earth to be a sphere of uniform mass density, how much would a body weigh half way down to the centre of the earth if it weighed $250N$ on the surface?

Ans: Weight of a body of mass m at the Earth's surface is given by,

$W = mg = 250N$

Body of mass m is located at depth, $d = \frac{1}{2}R_e$

Where, R_e is the radius of the Earth.

Acceleration due to gravity at depth g_d is given by the relation:

$$g_d = \left(1 - \frac{d}{R_e}\right)g$$

$$\Rightarrow \left(1 - \frac{R_e}{2 \times R_e}\right)g = \frac{1}{2}g$$

Weight of the body at depth d,

$W' = mg$

$$\Rightarrow W' = m \times \frac{1}{2}g = \frac{1}{2}mg = \frac{1}{2}W$$

$$\Rightarrow W' = \frac{1}{2} \times 250 = 125N$$

The weight of body half way down to the centre of the earth is $125N$.

17. A rocket is fired vertically with a speed of $5kms^{-1}$ from the earth's surface. How far from the earth does the rocket go before returning to the earth? Mass of the earth $= 6.0 \times 10^{24}kg$; mean radius of the earth $= 6.4 \times 10^6 m; G = 6.67 \times 10^{-11} Nm^2kg^{-2}$.

Ans: Distance from the centre of the Earth $= 8 \times 10^6 m$

Velocity of the rocket, $v = 5kms^{-1} = 5 \times 10^3 ms^{-1}$

Mass of the Earth, $M_e = 6.0 \times 10^{24} kg$

Radius of the Earth, $R_e = 6.4 \times 10^6 m$

Height reached by rocket mass m, is h.

At the surface of the Earth,

Total energy of the rocket = Kinetic energy + Potential energy

$$= \frac{1}{2}mv^2 + \left(\frac{-GM_em}{R_e}\right)$$

At highest point h, $v = 0$

And, Potential energy $= -\dfrac{GM_em}{R_e + h}$

Total energy of the rocket $= 0 + \left(\dfrac{GM_em}{R_e + h}\right) = -\dfrac{GM_em}{R_e + h}$

From the law of energy conservation, we have

Total energy of the rocket at the Earth's surface = Total energy of rocket at height h.

We have,

$$\frac{1}{2}mv^2 + \left(-\frac{GM_e m}{R_e}\right) = -\frac{GM_e m}{R_e + h}$$

$$\Rightarrow \frac{1}{2}v^2 = GM_e\left(\frac{1}{R_e} - \frac{1}{R_e + h}\right)$$

$$\Rightarrow \frac{1}{2}v^2 = GM_e\left(\frac{R_e + h - R_e}{R_e(R_e + h)}\right)$$

$$\Rightarrow \frac{1}{2}v^2 = \frac{GM_e h}{R_e(R_e + h)} \times \frac{R_e}{R_e}$$

$$\Rightarrow \frac{1}{2}v^2 = \frac{gR_e h}{R_e + h}$$

Where, $g = \frac{GM}{R_e^2} = 9.8ms^{-2}$ **,** is the

surface.

Clearly,

$$v^2(R_e + h) = 2gR_e h$$

$$\Rightarrow v^2 R_e = h(2gR_e - v^2)$$

$$\Rightarrow h = \frac{R_e v^2}{2gR_e - v^2}$$

$$\Rightarrow h = \frac{6.4 \times 25 \times 10^{12}}{100.44 \times 10^6}$$

$$\Rightarrow h = 1.6 \times 10^6 m$$

Height achieved by the rocket with respect to the centre of the Earth, H is given by: $H = R_e + h$

$$\Rightarrow H = 6.4 \times 10^6 + 1.6 \times 10^6$$

We get,

$$H = 8.0 \times 10^6 m$$

The distance from the earth is $8.0 \times 10^6 m$ where the rocket goes before returning to the earth.

18. The escape speed of a projectile on the earth's surface is $11.2kms^{-1}$. A body is projected out with thrice this speed. What is the speed of the body far away from the earth? Ignore the presence of the sun and other planets.

Ans: $v_{esc} = 11.2kms^{-1}$

Projection velocity of the projectile, $v_p = 3_{v_{esc}}$

Mass of the projectile $= m$

$$= v_f$$

$$= \frac{1}{2}mv_p^{\ 2} - \frac{1}{2}mv_{esc}^{\ 2}$$

The projectile's gravitational potential energy far away from the Earth is zero.

Total energy of the projectile far away from the Earth $= \frac{1}{2}mv_f^{\ 2}$

From the law of energy conservation, we have

$$\frac{1}{2}mv_p^{\ 2} - \frac{1}{2}mv_{esc}^{\ 2} = \frac{1}{2}mv_f^{\ 2}$$

We get,

$$v_f = \sqrt{v_p^{\ 2} - v_{esc}^{\ 2}}$$

$$\Rightarrow v_f = \sqrt{(3v_{esc})^2 - (v_{esc})^2}$$

$$\Rightarrow v_f = \sqrt{8}v_{esc}$$

$$\Rightarrow v_f = \sqrt{8} \times 11.2$$

$$\Rightarrow v_f = 31.68\text{kms}^{-1}$$

Thus, the speed of the body far away from the earth is 31.68kms^{-1}.

19. A satellite orbits the earth at a height of 400km above the surface. How much energy must be expended to rocket the satellite out of the earth's gravitational influence?
Mass of the satellite $= 200\text{kg}$;
mass of the earth $= 6.0 \times 10^{24}\text{kg}$;
radius of the earth $= 6.4 \times 10^6$; $G = 6.67 \times 10^{-11}\text{Nm}^2\text{kg}^{-2}$.

Ans: Given that,

Mass of the Earth, $M_e = 6.0 \times 10^{24}\text{kg}$

Mass of the satellite, $m = 200\text{kg}$

Radius of the Earth, $R_e = 6.4 \times 10^6\text{m}$

Universal gravitational constant, $G = 6.67 \times 10^{-11}\text{Nm}^2\text{kg}^{-2}$

Height of the satellite, $h = 400\text{km} = 4 \times 10^5\text{m} = 0.4 \times 10^6\text{m}$

Total energy of the satellite at height h,

$$TE = \frac{1}{2}mv^2 + \left(\frac{-GM_e m}{R_e + h}\right)$$

Orbital velocity of the satellite, $v=\sqrt{\frac{GM_e}{R_e+h}}$

Total energy at height h, $TE=\frac{1}{2}m\left(\frac{GM_e}{R_e+h}\right)-\frac{GM_em}{R_e+h}=-\frac{1}{2}\left(\frac{GM_em}{R_e+h}\right)$

The negative sign explains that the satellite is attached to the Earth. This is called the bound energy of the satellite.

The energy needed to send the satellite out of its orbit is equal to the negative of Bound energy.

Bound energy is, $BE=\frac{1}{2}\left(\frac{GM_em}{R_e+h}\right)$

Where, M_e is the mass of the Earth.

R_e is the radius of the Earth.

h is the height.

m is the mass of the satellite.

$$\Rightarrow BE=\frac{1}{2}\times\frac{6.67\times10^{-11}\times6.0\times10^{24}\times200}{\left(6.4\times10^{6}+0.4\times10^{6}\right)}$$

$$\Rightarrow BE=\frac{1}{2}\times\frac{6.67\times6\times2\times10}{6.4\times10^{6}}$$

$$\Rightarrow BE=5.9\times10^{9}\,J$$

Clearly, $5.9\times10^{9}\,J$ energy must be expended to rocket the satellite out of the earth's gravitational influence.

20. Two stars each of one solar mass ($=2\times10^{30}$kg) are approaching each other for a head on collision. When they are a distance 10^{9}km, their speeds are negligible. What is the speed with which they collide? The radius of each star is 10^{4}km. Assume the stars to remain undistorted until they collide. (Use the known value of G).

Ans: Mass of each star, $M=2\times10^{30}\,kg$

Radius of each star, $R=10^{4}\,km=10^{7}\,m$

Distance between the stars, $r=10^{9}\,km=10^{12}\,m$

For negligible speeds, $v=0$

Total energy of two stars separated at distance r is given by

$$TE=\frac{-GMM}{r}+\frac{1}{2}mv^{2}$$

$$\Rightarrow TE = \frac{GMM}{r} + 0.$$

Now, consider the case when the stars are about to collide:

Velocity of the stars $= v$

Distance between the centres of the stars $= 2R$

Total kinetic energy of both stars $= \frac{1}{2}Mv^2 + \frac{1}{2}Mv^2 = Mv^2$

Total potential energy of both stars $= \frac{-GMM}{2R}$

Total energy of the two stars $= Mv^2 - \frac{GMM}{2R}$ ....(ii)

Using the law of conservation of energy, we can write:

$$Mv^2 - \frac{GMM}{2R} = \frac{-GMM}{R}$$

$$\Rightarrow v^2 = \frac{-GM}{r} + \frac{GM}{2r} = GM\left(-\frac{1}{r} + \frac{1}{2R}\right)$$

$$\Rightarrow v^2 = 6.67 \times 10^{-10} \times 2 \times 10^{30}\left[-\frac{1}{10^{12}} + \frac{1}{2 \times 10^{7}}\right]$$

$$\Rightarrow v^2 = 13.34 \times 10^{19}\left[-10^{-12} + 5 \times 10^{-8}\right]$$

$$\Rightarrow v^2 \sim 13.34 \times 10^{19} \times 5 \times 10^{-8}$$

$$\Rightarrow v^2 \sim 6.67 \times 10^{12}$$

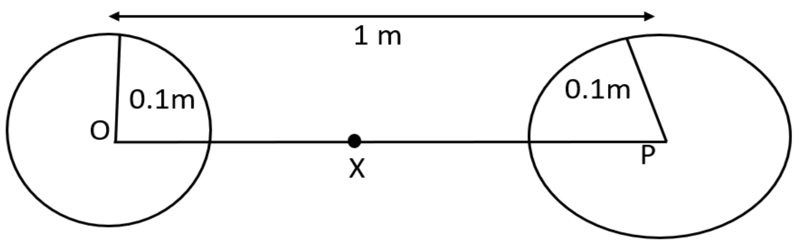
$$\Rightarrow v = \sqrt{6.67 \times 10^{12}}$$

$$\Rightarrow v = 2.58 \times 10^{6}\,m/s$$

The speed with which the two stars collide is $2.58 \times 10^{6}\,m/s$.

21. Two heavy spheres each of mass $100kg$ and radius $0.10m$ are placed $1.0m$ apart on a horizontal table. What is the gravitational force and potential at the midpoint of the line joining the centres of the spheres? Is an object placed at that point in equilibrium? If so, is the equilibrium stable or unstable?

Ans: The situation is represented in the given figure:



Mass of each sphere, $M = 100\text{kg}$

Separation between the spheres, $r = 1\text{m}$

X is the midpoint between the spheres. Gravitational force at point X will be zero. This is because the gravitational force applied by each sphere will act in opposite directions.

Gravitational potential at point $X = \frac{-GM}{\left(\frac{r}{2}\right)} - \frac{GM}{\left(\frac{r}{2}\right)}$

$\Rightarrow PE = -4\frac{GM}{r}$

$\Rightarrow PE = \frac{4 \times 6.67 \times 10^{-11} \times 100}{r}$

$\Rightarrow PE = -2.67 \times 10^{-8} \text{J / kg}$

Any object placed at point X will be in equilibrium state, but the equilibrium is unstable. This is because any change in the position of the object will vary the effective force in that direction.